Signal Processing

Het The Great
UG-2, ECE

What are signals?

→ Audio
→ Biomedical signals → FMRI
→ ECG
→ EEG
→ Sugar level

→ Light spectrum.

Note: Any circuit with linear circuit components
ex ⇒ L, C, R are LTI system.
Non linear components like diodes don't follow LTI

LTI systems:

Linearity: $x(t) \rightarrow \boxed{} \rightarrow y(t)$, then

$a\,x(t) \rightarrow \boxed{} \rightarrow a\,y(t)$

Time Invariance: Any process in system doesn't change o/p if only time of system execution has changed.

Convolution: Multiplication of entire signals, and not any m individual points of the signals.

Laplace Transform: Time domain → s-domain

$$\mathcal{L}(\underset{\downarrow\,\mathbb{C}}{x}(\underset{\downarrow\,\mathbb{R}}{t})) \longrightarrow \underset{\downarrow\,\mathbb{C}}{X}(\underset{\downarrow\,\mathbb{C}}{s})$$

Fourier Analysis

Periodic, aperiodic, continuous and discrete time signals exist

F Series - periodic and continuous time signal

F Transform - aperiodic, C.T.

Discrete Time FT - aperiodic, DT

DTFS - periodic and DT

DFT - DT and finite length signal

$\xrightarrow{\text{algorithm}}$ FFT

Fourier Series

$x(t)$ is periodic in T

$\omega_0 = \frac{2\pi}{T}$

$$x(t) \overset{\text{trig}}{=} \sum_{k=0}^{\infty} a_k \sin(k\omega_0 t) + \sum_{k=0}^{\infty} b_k \cos(k\omega_0 t)$$

$$\xrightarrow{\text{¢}} \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T} \int_{\langle T \rangle} x(t) \sin(k\omega_0 t)\, dt$$

$$b_k = \frac{1}{T} \int_{\langle T \rangle} x(t) \cos(k\omega_0 t)\, dt$$

$x(t) \xrightarrow{FS} \{a_k, b_k\}$ form or $\{c_k\}$ form

$$c_k = \frac{1}{T} \int_{\langle T \rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

as sq. wave is odd funct.
$a_k = 0$

Q- Compute FS coeff of sq wave.

$$a_k = \frac{1}{2T} \int_0^T (-1) \sin(k\omega t)\, dt + \int_T^{2T} 1 \sin(k\omega t)\, dt$$

$$a_k = \frac{1}{2T} \left[\left[\frac{\cos(k\omega t)}{k\omega} \right]_0^T + \left[\frac{-\cos k\omega t}{k\omega} \right]_T^{2T} \right]$$

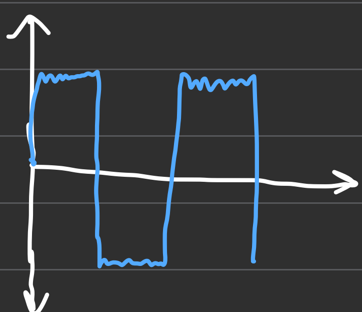
Partial reconstruction / Synthesis

$$\hat{x}(t) = b_0 + \sum_{k=1}^{K} a_k \sin(k\omega_0 t) + \sum_{k=1}^{K} b_k \cos(k\omega_0 t)$$

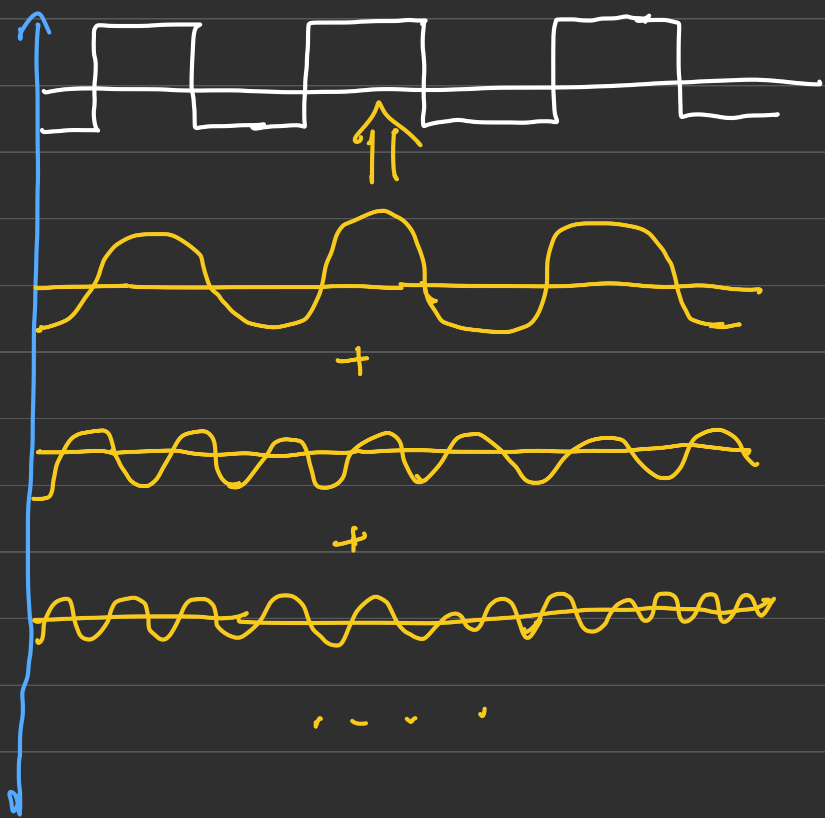
for $x(t)$, $K \to \infty$

$\therefore e(t) = x(t) - \hat{x}(t)$

$\therefore$ If $x(t)$ is a square wave
then $\hat{x}(t)$ is $\rightarrow$



Visual representation of FS:



$$a_k = \frac{2}{2T}\left[\int_0^T (-1)\sin(k\omega_0 t)\,dt + \int_T^{2T} (1)\sin(k\omega_0 t)\,dt\right]$$

$$= \frac{1}{T}\left[\frac{\left[\cos(k\omega t)\right]_0^T}{k\omega} + \left[\frac{-\cos(k\omega_0 t)}{k\omega}\right]_T^{2T}\right]$$

$$= \frac{1}{T}\left[\frac{\cos 2k\pi - \cos 0}{k\omega} + \frac{\left[-\cos 4k\pi + \cos 2k\pi\right]}{k\omega}\right]$$

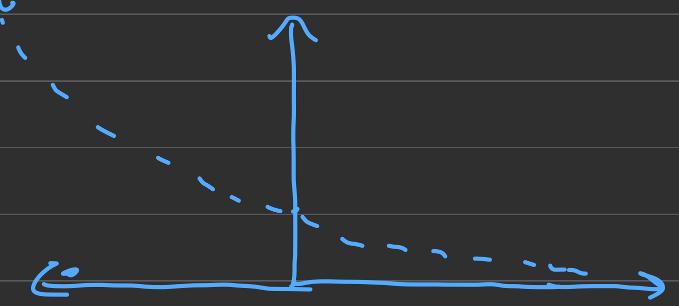
Discrete Time Signal

- → Unit impulse
- → Unit step
- → Exponential signals
- → Sinusoid and complex sinusoid
- → Energy and power of signal
- → Even, Odd signal
- → Periodic

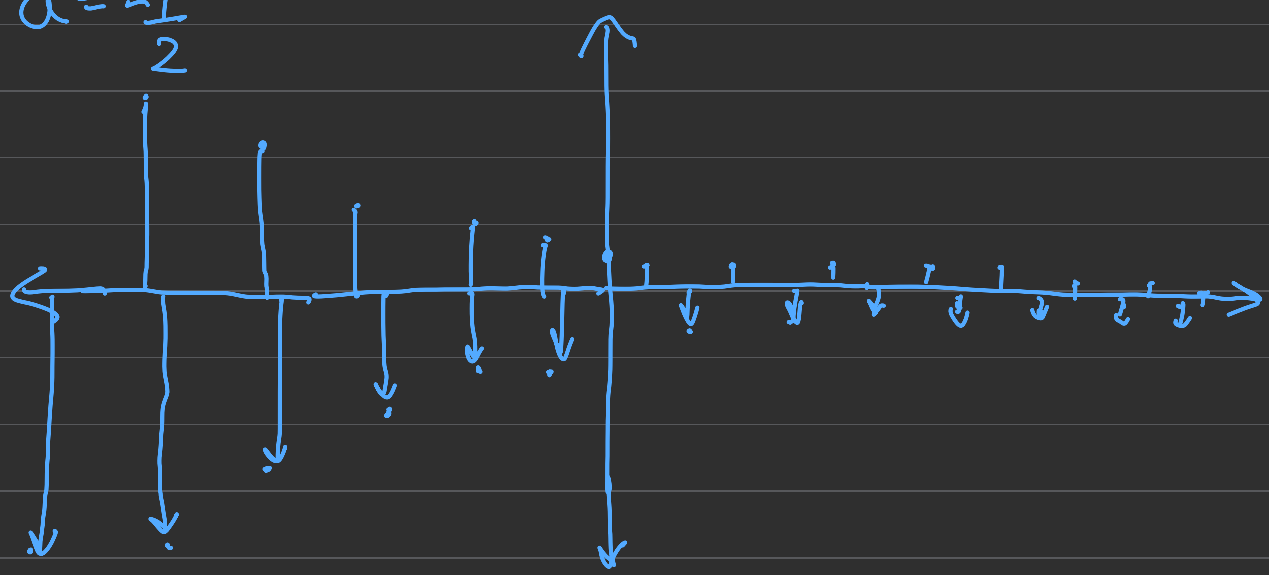
Exp signal:

$x[n] = a^n$

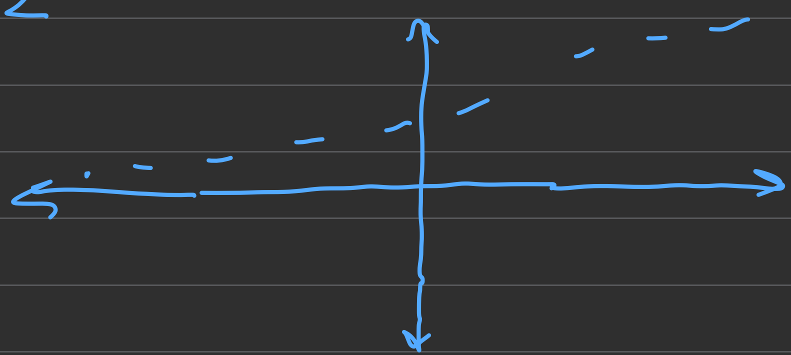
1) $a = 1/2$



2) $a = -\frac{1}{2}$

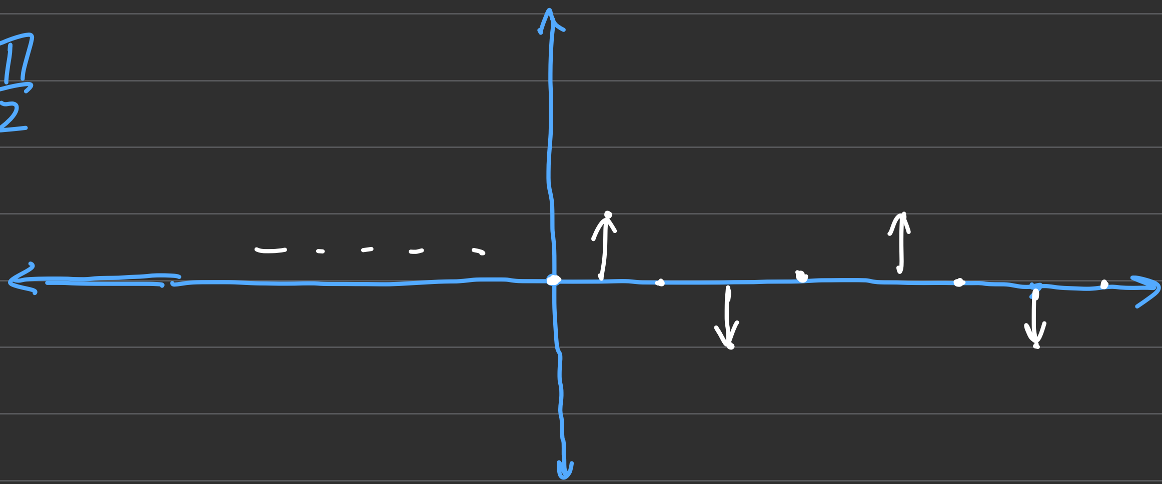


→ $a = 2$

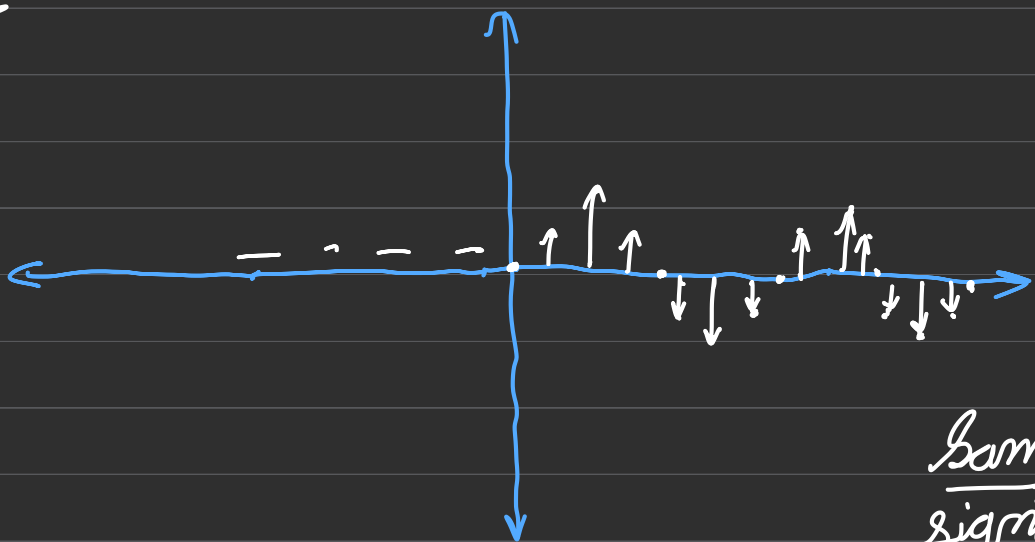


Q- $x[n] = \sin[\omega_0 n]$

$\omega_0 = \frac{\pi}{2}$



$\omega_0 = \frac{\pi}{4}$



Sampling of a time signal

Thus, $\sin[\omega_0 n]$ is periodic only for few ω_0 cases

Periodic: $x[n]$ is periodic if

$$x[n+P] = x[n]$$

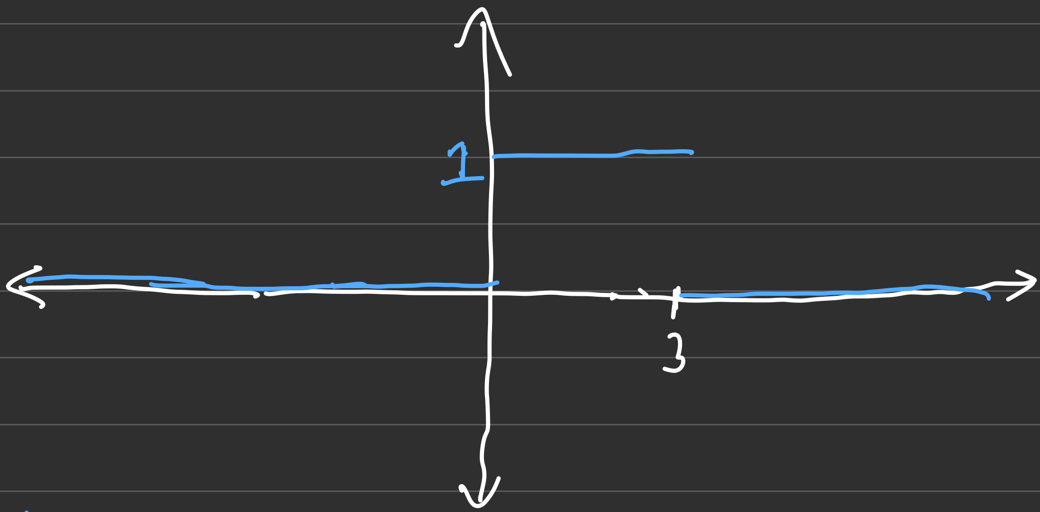
Ex = $e^{j\omega_0 n} = \underbrace{\cos(\omega_0 n)}_{\text{real}} + j\,\underbrace{\sin(\omega_0 n)}_{\text{complex}}$

Energy: $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$ Energy signal

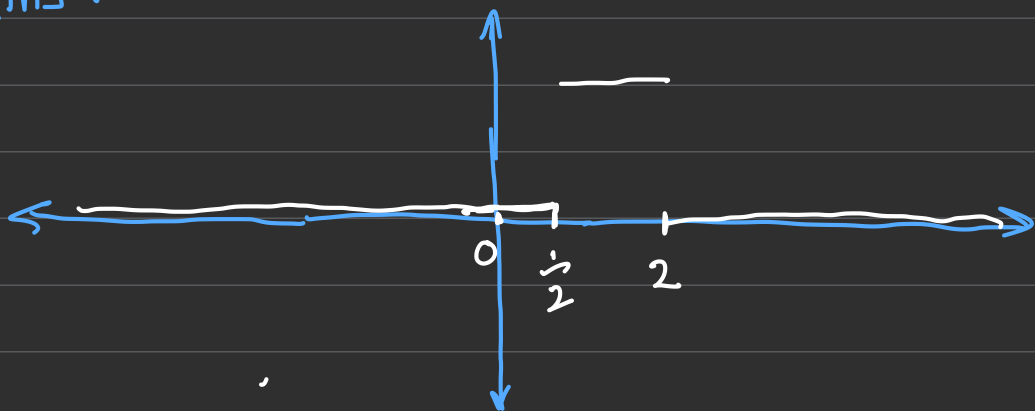
Power: $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$ Power signal

Plot $x[n] = u[n] - u[n-3]$

$y[n] = x[2n-1]$



$y[n]$:



System

$x[n] \longrightarrow \boxed{} \longrightarrow y[n]$

① Delay $y[n] = x[n-\Delta]$ Δ - integer

② Scaling: $y[n] = \alpha x[n]$ $\alpha \in \mathbb{R}, \mathbb{C}$

③ $y[n] = \sum_{n=-\infty}^{\infty} x[n]$

$\cancel{x[}\ y[n] = x[n] - x[n-1]$