Science-1

Het Selarka

UG-2, ECE

$\boxed{1D}$ $\quad \dfrac{dx}{dt} = \alpha x$

$\boxed{2D}$ $\quad \dfrac{d\vec{R}}{dt} = \vec{J}_0 \vec{R}$

$$J(x,y) = \begin{pmatrix} \text{Partial der of } f(x,y) \text{ wrt } x & \text{Par der of } f(x,y) \text{ wrt } y \\ \text{Part der of } g \text{ wrt } x & \text{" " } g \text{ wrt } y \end{pmatrix}$$

$$\therefore J = \begin{pmatrix} \alpha - \beta y & -\beta x \\ \alpha y & \alpha x - \delta \end{pmatrix}$$

for

$$f(x,y) = \alpha x - \beta y x$$
$$g = \gamma x y - \delta y$$

$$\therefore J(0,0) = \begin{pmatrix} \alpha & 0 \\ 0 & -\delta \end{pmatrix} \begin{array}{l} \nearrow \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \vec{v}_1 \\ \searrow -\delta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \vec{v}_2 \end{array}$$

By eigen vector thm,

Any vector can be written as sum of eigen vectors.

$$\therefore \vec{R}(0) = c_1(0)\vec{v}_1 + c_2(0)\vec{v}_2$$

$$\therefore \vec{R}(t) = c_1(t)\vec{v}_1 + c_2(t)\vec{v}_2$$

We know $\dfrac{dR}{dt} = \vec{J}_0 \vec{R}$

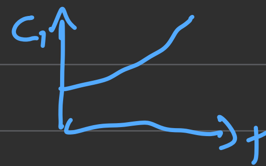
$$\therefore \frac{dc_1}{dt}\vec{v}_1 + \frac{dc_2}{dt}\vec{v}_2 = \vec{J}_0 (c_1\vec{v}_1 + c_2\vec{v}_2)$$

$$-\vec{J}_0 \vec{v}_1 = \lambda_1 \vec{v}_1$$

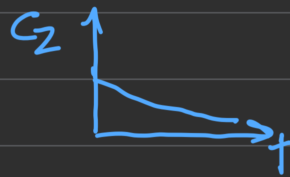
$$\vec{J}_0 \vec{v}_2 = \lambda_2 \vec{v}_2$$

$$\therefore \frac{dc_1}{dt}\vec{v}_1 + \frac{dc_2}{dt}\vec{v}_2 = \lambda_1 c_1 \vec{v}_1 + \lambda_2 c_2 \vec{v}_2$$

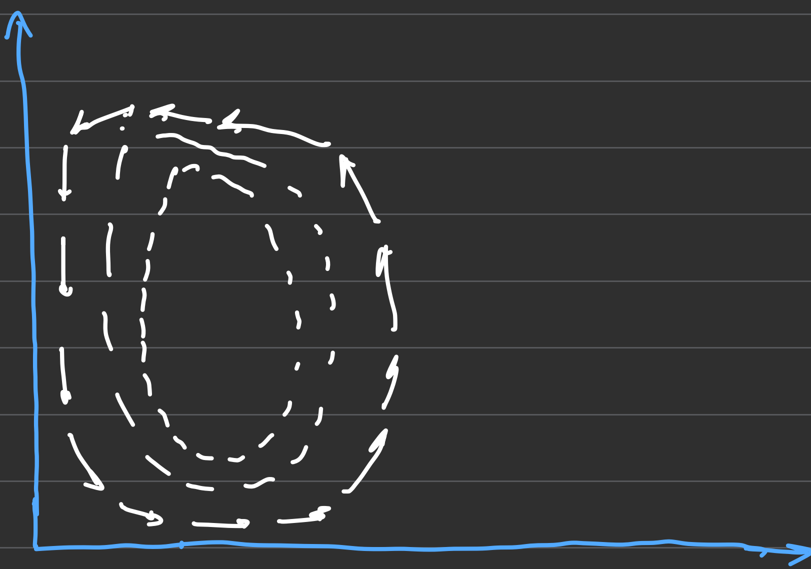
$$\therefore \; C_1(t) = C_1(0)\, e^{\lambda_1 t}$$



$$C_2(t) \simeq C_2(0)\, e^{\lambda_2 t}$$



$\therefore$ Velocity Field for $x(t)$ and $y(t)$ sinusoid functions:



Ex: $\dfrac{d^2x}{dt^2} = f(x,t)$

Soln: Suppose $v(t) = \dfrac{dx}{dt}$

$\therefore \dfrac{dv}{dt} = f(x,t)$

$\therefore$ If $\Phi = \begin{pmatrix} x \\ v \end{pmatrix}$ $\quad \dfrac{d\Phi}{dt} = \begin{pmatrix} v \\ f(x,t) \end{pmatrix}$

And then solve for J

→ HW Find info of 2nd steady state of J → $\left(\dfrac{\delta}{\gamma}, \dfrac{\alpha}{\beta}\right)$