Exercise 9 (for grade) ~ Monday, December 5, 2022 ~ CPSC 535.01 Fall 2022

Write one submission for your entire group, and write all group members' names on that submission. Turn in your submission before the end of class. The X symbol marks where you should write answers.

Recall that our recommended problem-solving process is:

1. **Understand** the problem definition. What is the input? What is the output?
2. **Baseline** algorithm for comparison
3. **Goal** setting: improve on the baseline how?
4. **Design** a more sophisticated algorithm
5. **Inspiration** (if necessary) from patterns, bottleneck in the baseline algorithm, other algorithms
6. **Analyze** your solution; goal met? Trade-offs?

Follow this process for each of the following computational problems. For each problem, your submission should include:

a. State are the input variables and what are the output variables
b. Pseudocode for your baseline algorithm, that needs to include the data type and an explanation for any variable other than input and output variables
a. The Θ-notation time complexity of your baseline algorithm, with justification.

and if you manage to create an improved algorithm:

c. Answer the question: how is your improved algorithm different from your baseline; what did you change to make it faster?
d. Pseudocode for your improved algorithm, that needs to include the data type and an explanation for any variable other than input and output variables
a. The Θ-notation time complexity of your improved algorithm, with justification.

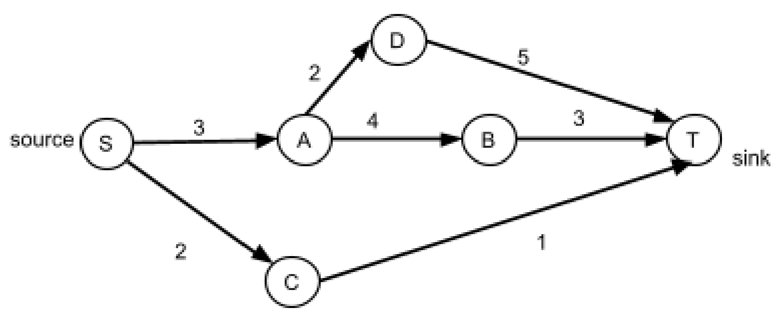
Today's problems are:

1. *(linear programming)*
   A farmer has 110 acres of land that he plans to grow wheat or barley to be sold with the maximum profit. He wants to know how many acres he needs to plant the wheat and how many acres to be used for barley. To plant wheat on one acre, one spends \$100 per acre and uses 10 days of one man skill. To plant barley on one acre, one spends \$200 per acre and uses 30 days of one man skill. The farmer has \$10000 and 1,200 man-days available. The farmer can obtain \$5000 profit per acre of wheat and \$12,000 profit per acre of barley. Express the assignment problem as an integer linear programming problem.
2. *(linear programming)*
   Consider the linear programming problem

minimize	$2x + 3y$
subject to	$x + y \geq 4$
	$x + 3y = 6$
	$x \geq 0$

a. Convert the LP problem from general form to standard form
b. Convert the standard form to slack form and state what are the nonbasic and what are the basic variables
c. Solve the LP problem using simplex method

3. *(linear programming)*
   Given the flow network below:



a. Convert the maximum-flow problem as a linear programming problem.
b. Formulate the linear programming problem into the standard form.
c. Formulate the linear programming problem into the slack form.

Names

Write the names of all group members below.

Dhruti Dilipbhai Patel
Tina Torabinejad
Hetal Patel

Exercise 1: Solve and provide answer

Dhruti Patel's Solution:
Decision Variables:
Let x = units of acre land of wheat
y = units of acre land of barley

Objective Function is to maximize the profit:
Maximize z = 5000x+12000y

Constraints:
i) Total land available,
x+y<=110

ii) Total budget,
100x+200y<=10000

iii) Total man skill,
10x+30y<=1200

iv) Non-negativity,
x,y>=0

The linear programming form for the problem is:
Maximize 5000x+12000y subject to the following constraints given below:
x+y <=110 (total land available)
100x+200y<=10000 (total budget)
10x+30y<=1200 (total man skill)
x,y>=0 (non-negativity)

Exercise 2: Solve and provide answer

Tina Torabinejad's Solutio

2

a)

step 1: convert min to max

object: $-2x - 3y$

$x = x' - x''$, $y = y' - y''$

max $-2x - 3y$

s.t $x' - x'' + y' - y'' \geq 4$

$x' - x'' + 3(y' - y'') = 6$

$x' - x'' \geq 0$

max $-2(x' - x'') - 3(y' - y'')$

s.t $x' - x'' + y' - y'' \geq 4$

$x' - x'' + 3(y' - y'') = 6$

$x' - x'' \geq 0$

$x', x'', y', y'' \geq 0$

change $\geq$ to $\leq$:

max $-2(x' - x'') - 3(y' - y'')$

s.t $-x' + x'' - y' + y'' \leq -4$ a

$x' - x'' + 3(y' - y'') = 6$ b

$-x' + x'' \leq 0$ c

$x', x'', y'', y' \geq 0$

This is a standard form

b)

$Z = -2x' + 2x'' - 3y' + 3y''$

$-x' + x'' - y' + y'' + a = -4$

$x' - x'' + 3y' - 3y'' + b = 6$

$-x' + x'' + c = 0$

basic variables: $a, b, c \geq 0$

non basic variables: $x', x'', y', y'' \geq 0$

c)

$$x\begin{bmatrix}1\\1\\1\end{bmatrix} + y\begin{bmatrix}1\\3\\0\end{bmatrix} + a\begin{bmatrix}1\\0\\0\end{bmatrix} + b\begin{bmatrix}0\\1\\0\end{bmatrix} + c\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-4\\6\\0\end{bmatrix}$$

	x	y	a	b	c	
a	1	1	1	0	0	-4
b	1	3	0	1	0	6
c	3	0	0	0	1	0
	2	3	0	0	0	0

divide row by 3

divide row by 3

	x	y	a	b	c	
a	1	1	1	0	0	-4
b		1	0		0	2
c	1	0	0	0		0
	2	3	0	0	0	0

subtract 2nd row from 1st

x	y	a	b	c	
	0	1			6
	1	0			2
-1	0	0			0
	3	0			0

subtract 2nd row times 3 from 4th rows

x	y	a	b	c	
	0	1			6
	1	0			2
					0
	0	0			6

$x_1 = 6$
$x_2 = 2$
$\max = 6$

2) min $2x + 3y$

Subject to
$$x + y \geq 4$$
$$x + 3y = 6$$
$$x \geq 0$$

(a) Convert LPP from general to standard form :-

<u>Step 1</u> :- Convert min to max

Obj funct :- $-2x - 3y$

<u>Step 2</u> :- Convert negative variable, $y = (y' - y'')$ we get,

max $-2x - 3(y' - y'')$

subj to
$$x + (y' - y'') \geq 4$$
$$x + 3(y' - y'') = 6$$
$$x, y', y'' \geq 0$$

<u>Step 3</u> :- Address $\geq$ & $=$ constraints,

max $-2x - 3y' + 3y''$

subj to
$$-x - (y' - y'') \leq -4$$
$$x + 3(y' - y'') \geq 6$$
$$x + 3(y' - y'') \leq 6$$
$$x, y', y'' \geq 0$$

Address $\geq$ constraints,

max $-2x - 3y' + 3y''$

subj to
$$-x - y' + y'' \leq -4$$
$$\cancel{x + 3y} \quad x + 3y' - 3y'' \leq 6$$
$$-x - 3y' + 3y'' \leq -6$$
$$x, y', y'' \geq 0$$

This is standard form

(b) Slack Form :-

$$Z = \cancel{max} \quad -2x - 3y' + 3y''$$

s.t.

$$-x - y' + y'' + v_1 \leq -4$$

$$x + 3y' - 3y'' + v_2 \leq 6$$

$$-x - 3y' + 3y'' + v_3 \leq -6$$

Basic Variable :- $x, y', y'' \geq 0$

Non-basic Variable :- $v_1, v_2, v_3 \geq 0$

(c) LPP to simplex :-

	x	y'	y''	v_1	v_2	v_3	b
	-1	-1	1	1	0	0	-4
→	1	3	-3	0	1	0	6
	-1	-3	3	0	0	1	-6
	-2	-3	3	0	0	0	0
		↑					

divide 2nd row by 3

x	y'	y''	v_1	v_2	v_3	b
-1	-1	1	1	0	0	-4
1/3	1	-1	0	1/3	0	2
-1	-3	3	0	0	1	-6
-2	-3	3	0	0	0	0

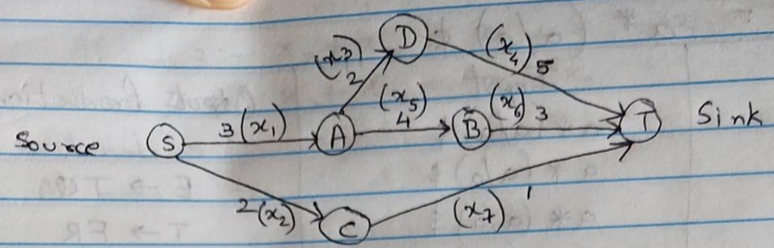
~~and~~ Add 1st & 2nd row

x	y'	y''	v_1	v_2	v_3	b
-2/3	0	0	1	0	0	-3
1/3	1	-1	0	1/3	0	2
-1	-3	3	0	0	1	-6
-2	-3	3	0	0	0	0

Exercise 3: Solve and provide answer

Hetal Patel Solution:

3)



(a) Converting the maximum-flow problem as a linear Programming Problem :-

Step 1 Create a new variable for each edge

$f_{sa} = x_1$; $f_{ad} = x_3$; $f_{ab} = x_5$; $f_{ct} = x_7$

$f_{sc} = x_2$; $f_{dt} = x_4$; $f_{bt} = x_6$

Step 2 :- Formulate the LPP

Objective Function : max $x_1 + x_2$

subject to

$$x_1 \le 3$$
$$x_2 \le 2$$
$$x_3 \le 2$$
$$x_4 \le 5$$
$$x_5 \le 4$$
$$x_6 \le 3$$
$$x_7 \le 1$$

Flow Conversion
$$\left\{\begin{aligned} x_1 &= x_3 + x_5 \\ x_3 &= x_4 \\ x_5 &= x_6 \\ x_2 &= x_7 \end{aligned}\right.$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \ge 0$$

(b) Formulate the LPP into the standard form :-

Step 1 :- Rearrange ~~The given~~ flow conversion equation :-

$$x_1 - x_3 - x_5 = 0$$
$$x_3 - x_4 = 0$$
$$x_5 - x_6 = 0$$
$$x_2 - x_7 = 0$$

Step 2 :- Addressing $\geq$ & $=$ constraints

$$x_1 - x_3 - x_5 \geq 0$$
$$x_1 - x_3 - x_5 \leq 0$$
$$x_3 - x_4 \geq 0$$
$$x_3 - x_4 \leq 0$$
$$x_5 - x_6 \geq 0$$
$$x_5 - x_6 \leq 0$$
$$x_2 - x_7 \geq 0$$
$$x_2 - x_7 \leq 0$$

Addressing $\geq$ constraints,

$$-x_1 + x_3 + x_5 \leq 0$$
$$x_1 - x_3 - x_5 \leq 0$$
$$-x_3 + x_4 \leq 0$$
$$x_3 - x_4 \leq 0$$
$$-x_5 + x_6 \leq 0$$
$$x_5 - x_6 \leq 0$$
$$-x_2 + x_7 \leq 0$$
$$x_2 - x_7 \leq 0$$

$$\max \quad x_1 + x_2$$

subject to

$$x_1 \le 3$$
$$x_2 \le 2$$
$$x_3 \le 2$$
$$x_4 \le 5$$
$$x_5 \le 4$$
$$x_6 \le 3$$
$$x_7 \le 1$$
$$-x_1 + x_3 + x_5 \le 0$$
$$x_1 - x_3 - x_5 \le 0$$
$$-x_3 + x_4 \le 0$$
$$x_3 - x_4 \le 0$$
$$-x_5 + x_6 \le 0$$
$$x_5 - x_6 \le 0$$
$$-x_2 + x_7 \le 0$$
$$x_2 - x_7 \le 0$$
$$x_1, x_2, x_3, x_4, x_5, x_7 \ge 0$$

This is the standard Form

(c) Formulate the LPP into the slack Form:-

$$z = x_1 + x_2$$

$$x_1 + v_1 = 3$$

$$x_2 + v_2 = 2$$

$$x_3 + v_3 = 2$$

$$x_4 + v_4 = 5$$

$$x_5 + v_5 = 4$$

$$x_6 + v_6 = 3$$

$$x_7 + v_7 = 1$$

$$-x_1 + x_3 + x_5 + v_8 = 0$$

$$x_1 - x_3 - x_5 + v_9 = 0$$

$$-x_3 + x_4 + v_{10} = 0$$

$$x_3 - x_4 + v_{11} = 0$$

$$-x_5 + x_6 + v_{12} = 0$$

$$x_5 - x_6 + v_{13} = 0$$

$$-x_2 + x_7 + v_{14} = 0$$

$$x_2 - x_7 + v_{15} = 0$$

Basic Variables :- $x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$

Non-basic Variables :- $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8,$
$v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15} \geq 0$