Exercise 1 (for grade) ~ Monday, August 29, 2022 ~ CPSC 535 Fall 2022

Write one submission for your entire group, and write all group members' names on that submission. Turn in your submission before the end of class. The ❌ symbol marks where you should write answers.

Recall that our recommended problem-solving process is:
1. **Understand** the problem definition. What is the input? What is the output?
2. **Baseline** algorithm for comparison
3. **Goal** setting: improve on the baseline how?
4. **Design** a more sophisticated algorithm
5. **Inspiration** (if necessary) from patterns, bottleneck in the baseline algorithm, other algorithms
6. **Analyze** your solution; goal met? Trade-offs?

Follow this process for each of the following computational problems. For each problem, your submission should include:
   a. State are the input variables and what are the output variables
   b. Pseudocode for your baseline algorithm, that needs to include the data type and an explanation for any variable other than input and output variables
   a. The Θ-notation time complexity of your baseline algorithm, with justification.

and if you manage to create an improved algorithm:
   c. Answer the question: how is your improved algorithm different from your baseline; what did you change to make it faster?
   d. Pseudocode for your improved algorithm, that needs to include the data type and an explanation for any variable other than input and output variables
   a. The Θ-notation time complexity of your improved algorithm, with justification.

Today's problems are:

1. *Indicate the class Θ*

For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. Apply all possible rules to simply g(n) as much as possible. Prove your assertions either using definitions or the limit theorem.

a. $\sqrt{4n^2 - 8b + 10}$
b. $2^{n/2-1} + 3^{n/3+1}$

2. *Parallel matrix update of size 3x3*

Given a matrix A of size 3x3, each cell (i.e. an element of the matrix) will be updated based on the values of (existing) immediate neighboring cells. This update must be done in parallel, this means A[0,0] will be updated in parallel with A[0,1], etc.. For a cell A[i,j], the update will be of the form:
A[i][j] = A[i-1][j] + A[i+1][j] + A[i][j-1] + A[i][j+1] - 4*A[i][j]
Some of the terms may be missing if the element is on the border of the matrix: i=0, j=0, i=2, j=2.
For example A[0][0] = A[1][0] + A[0][1] - 4*A[0][0].

In other terms, if $A^{(0)}$ is the initial matrix at time 0, then $A^{(1)}$ will be computed entirely based on the cells of $A^{(0)}$ at time 1, $A^{(2)}$ will be computed entirely based on the cells of $A^{(1)}$ at time 2, and so on.
To compute $A^{(1)}[0][0]$ one needs 3 operations (one addition, one subtraction, and one multiplication).
You need to state the number of simple mathematical operations needed to compute all the cells at time 1.
This value is obtained by adding the number of simple mathematical operations for each cell in the matrix.

3. *Parallel matrix update of size 3x3*

Given a matrix A of size *n* x *n*, each cell (i.e. an element of the matrix) will be updated based on the values of (existing) immediate neighboring cells. This update must be done in parallel, this means A[0,0] will be updated in parallel with A[0,1], etc.. For a cell A[i,j], the update will be of the form:
A[i][j] = A[i-1][j] + A[i+1][j] + A[i][j-1] + A[i][j+1] - 4*A[i][j]
Some of the terms may be missing if the element is on the border of the matrix: i=0, j=0, i=n-1, j=n-1.
For example A[0][0] = A[1][0] + A[0][1] - 4*A[0][0].

In other terms, if $A^{(0)}$ is the initial matrix at time 0, then $A^{(1)}$ will be computed entirely based on the cells of $A^{(0)}$ at time 1, $A^{(2)}$ will be computed entirely based on the cells of $A^{(1)}$ at time 2, and so on.
To compute $A^{(1)}[0][0]$ one needs 3 operations (one addition, one subtraction, and one multiplication).
You need to state the number of simple mathematical operations needed to compute all the cells at time 1.
This value is obtained by adding the number of simple mathematical operations for each cell in the matrix.

Names

Write the names of all group members below.

X Gouri Babasaheb Sabale
Pratishtha Soni
Hetal Patel

Exercise 1: Solve and provide answer

X Pratishtha Soni

f(n) = sqrt(4n^2 - 8b+10)
According to class theta g(n)
c1g(n)<=f(n) < = c2(g(n))

Lets assume c2 = 10 and c1=1 so
And g(n) = n
sqrt(4n^2 - 8b +10) <= 10n
According to properties of asymptotic notations , we can ignore additive constants, so
Sqrt(4n^2) <= 10n
1*n<=2n <=10n
So for every n>=1, this function belongs to theta(g(n))

  Correct assuming that g(n)=n

Gouri's Solution

1. Indicate the class θ

   a. $\sqrt{4n^2 - 8b + 10}$

     Using limit theorem, we have

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{\sqrt{4n^2 - 8b + 10}}{\sqrt{n^2}} \qquad \because\ g(n) = n^2$$

$$= \lim_{n \to \infty} \sqrt{\frac{4n^2 - 8b + 10}{n^2}}$$

$$= \lim_{n \to \infty} \sqrt{\frac{4n^2}{n^2} - \frac{8b}{n^2} + \frac{10}{n^2}}$$

$$= \lim_{n \to \infty} \sqrt{4 - \frac{8b}{n^2} + \frac{10}{n^2}}$$

$$= \lim_{n \to \infty} \sqrt{4} - \lim_{n \to \infty} \sqrt{\frac{8b}{n^2} \frac{10}{n^2}}$$

$$= \sqrt{4} - 0 \qquad \text{₁₁₁ since } \frac{1}{\infty^2} = 0$$

$$= \sqrt{4}$$

$$= \text{constant } C$$

As per rules of limit theorem,

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \text{tit } c > 0, \text{ then } f(n) = \theta(g(n))$$

    Here $g(n) = n^2$

  So

    the function $\sqrt{4n^2 - 8b + 10}$ belongs to $\theta(n^2)$ class.

Incorrect 1a

b. $2^{n/2 - 1} + 3^{n/3 + 1}$

$$= \frac{2^{n/2}}{2} + 3^{n/3} \cdot 3^{1}$$

= Ignore Constant terms

$$= \Theta(2)^{n/2} + \Theta\left(3^{n/3}\right)$$

= Ignore non-dominent terms

$$= \Theta\left(3^{n/3}\right)$$

Correct 1b

Exercise 2: Solve and provide answer
☒ Gouri's Solution:

Arithmetic Operations for
A(0,0)=3
A(2,2)=3

For remaining rows and columns they would be

A(0,1)=5
Then
A(0,2)=5
A(1,0)=5
A(1,1)=5
A(1,2)=5
A(2,0)=5
A(2,1)=5

(-0.5.points): minor computational mistakes

So since we have matrix of order 3,total no operations are 41

Exercise 3: Solve and provide answer

A(0),A(0,0),A[1,0] are getting updated in parallel.
So
For A[0,0] We will have 3 operations
For A[0,1] We will have 5 operations
For A[1,0],We will have 5 operations

SO in total there will 3+5+5=13 operations

When A[1],A[1,1] and A[0,2] gets update in parallel
Each cell takes 5 operations
So we have
5*3 =15 operations

In Total we will have 13+15=28 operations.

(-1.5 points): incorrect exercise 3