Lecture 5

Introduction to Probabilistic models

Machine Learning

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Lecture plan

- Overview and Motivation
- Distributions

Lecture plan

- Overview and Motivation
- Distributions

Problem

An illness, which is spread among 1% of population. This illness test returns true answers in 95% of cases. Someone receives a positive result.

What is the probability, he actually suffers the illness?

PGM: Motivation and Overview

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predisposing
symptoms
test results
diseases
treatment outcomes

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millions of pixels or
thousands of superpixels

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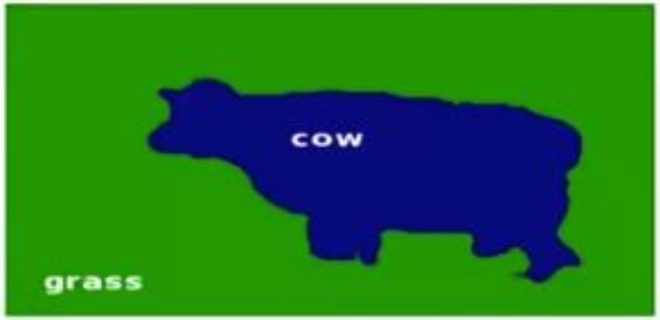
Each, needs to be labeled
{grass, sky, water, cow, horse, …}

PGM: Motivation and Overview



predisposing
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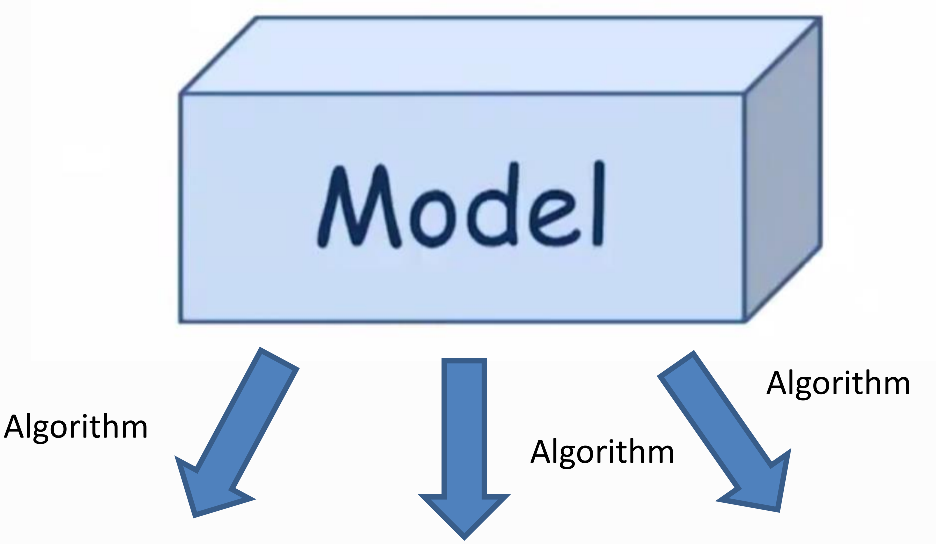
Probabilistic Graphical Models

Models

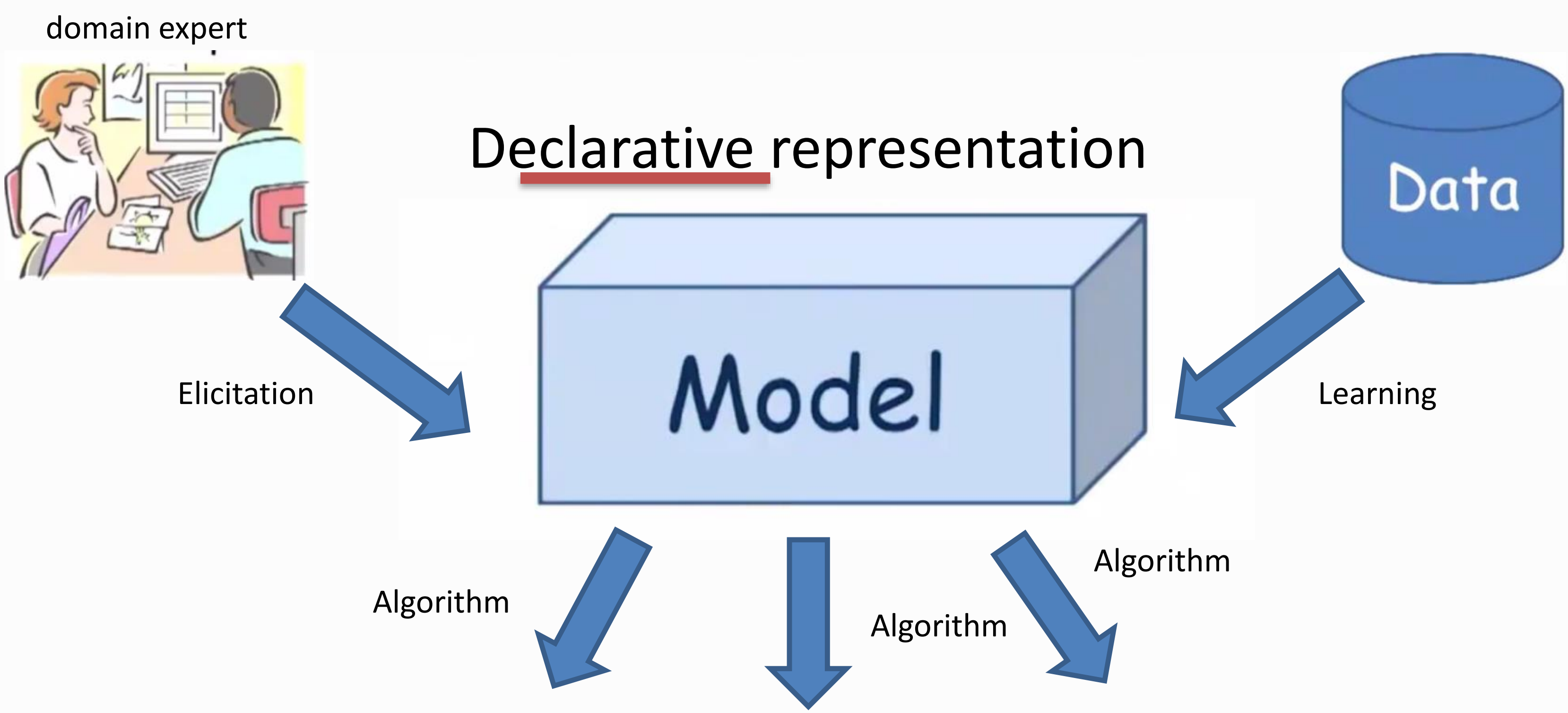
Declarative representation

Models

Declarative representation



Models


domain expert
Declarative representation
Data
Model
Elicitation
Learning
Algorithm
Algorithm
Algorithm

Uncertainty

- Partial knowledge of state of the world

Uncertainty

- Partial knowledge of state of the world
- Noisy observations

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- Noisy observations
- Phenomena not covered by our model

Uncertainty

- Partial knowledge of state of the world
- Noisy observations
- Phenomena not covered by our model
- Inherent stochasticity

Probability Theory

- Declarative representation with clear semantics

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Probability Theory

- Declarative representation with clear semantics
- Powerful reasoning patterns

Conditioning
Decision making

Probability Theory

- Declarative representation with clear semantics
- Powerful reasoning patterns

Conditioning
Decision making

- Established learning methods

Complex Systems

predisposing
symptoms
test results
diseases
treatment outcomes

class labels for
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Random variables $\boxed{X_1, \ldots, X}_n$

Complex Systems

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Random variables $\boxed{X_1, \ldots, X}_n$

Joint distribution $P(X_1, \ldots, X_n)$

Complex Systems

predisposing
symptoms
test results
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class labels for
thousands of superpixels

Random variables $\boxed{X_1, \ldots, X}_n$

$\boxed{\text{Joint}}$ distribution $P(\underline{X_1, \ldots, X_n})$

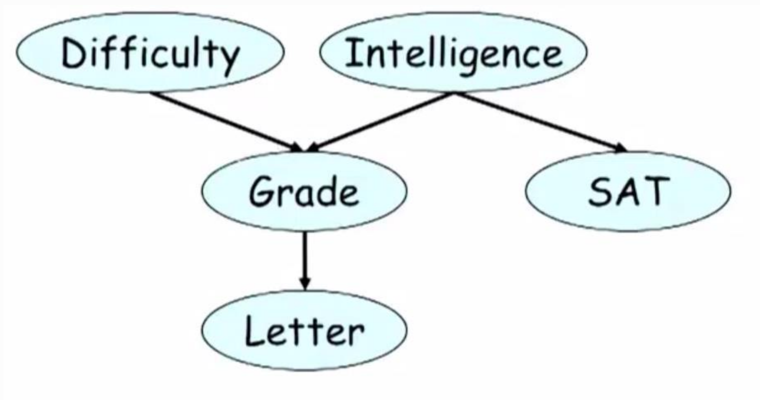
Binary valued distibrution over
2^n possible states

Graphical Models

Bayesian networks

$X_1, \ldots, X_n$ — nodes

Directed graph



Graphical Models

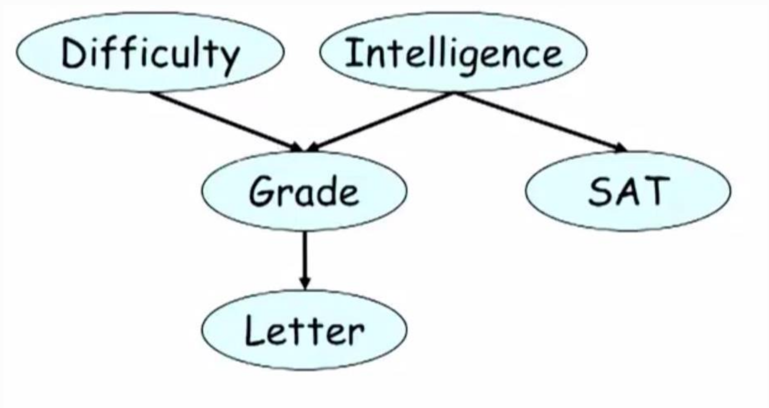
Bayesian networks

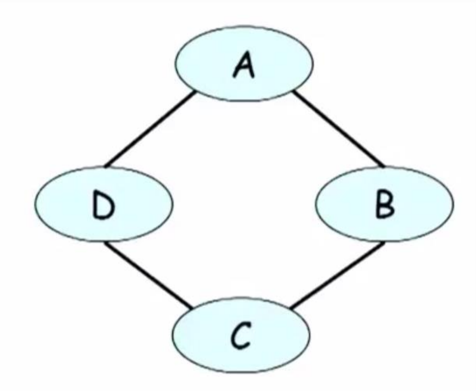
$X_1, \ldots, X_n$ — nodes

Directed graph

Markov networks

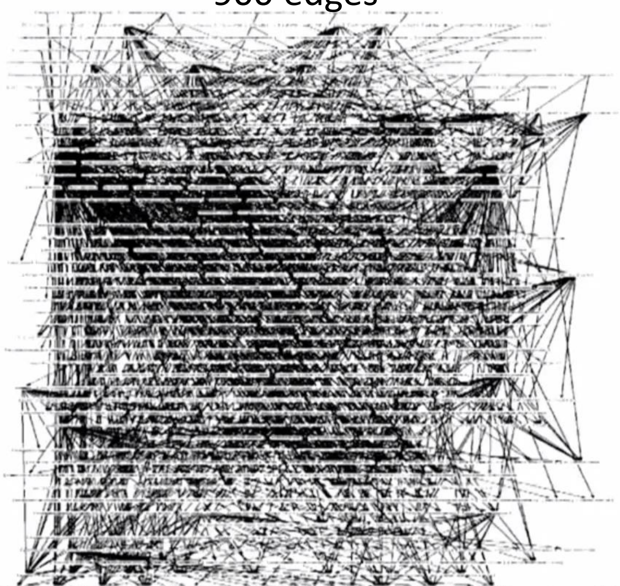
Undirected graph





Graphical Models

CPCS diagnosis
~480 nodes
~900 edges



M. Pradhan, G. Provan, B. Middleton, M. Henrion, UAI 94

Graphical Models

CPCS diagnosis
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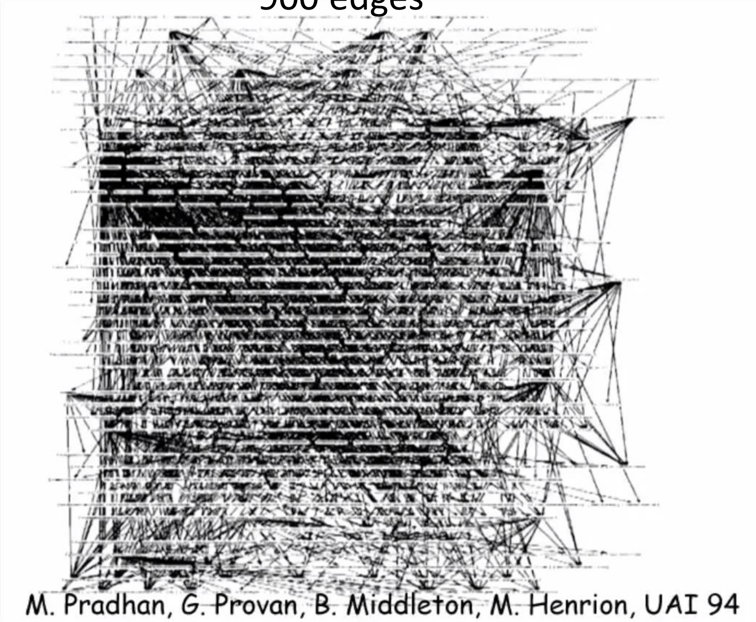
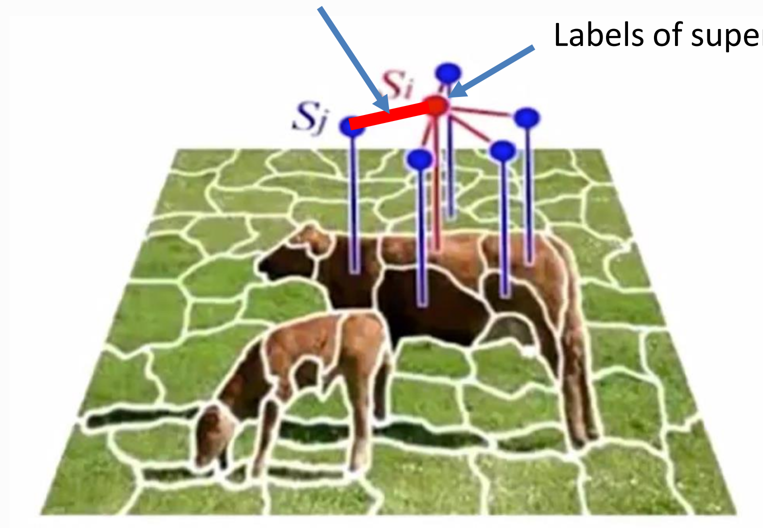


Probabilistic relationships

Labels of superpixels



Graphical Representation

- Intuitive and compact data structure
- Efficient reasoning using general–purpose algorithms
- Sparse parameterization
  - feasible elicitation
  - learning from data

Graphical Representation

- Intuitive and compact data structure
- Efficient reasoning using general–purpose algorithms
- Sparse parameterization
  - feasible elicitation ← by hand
  - learning from data ← automatically

Many Applications

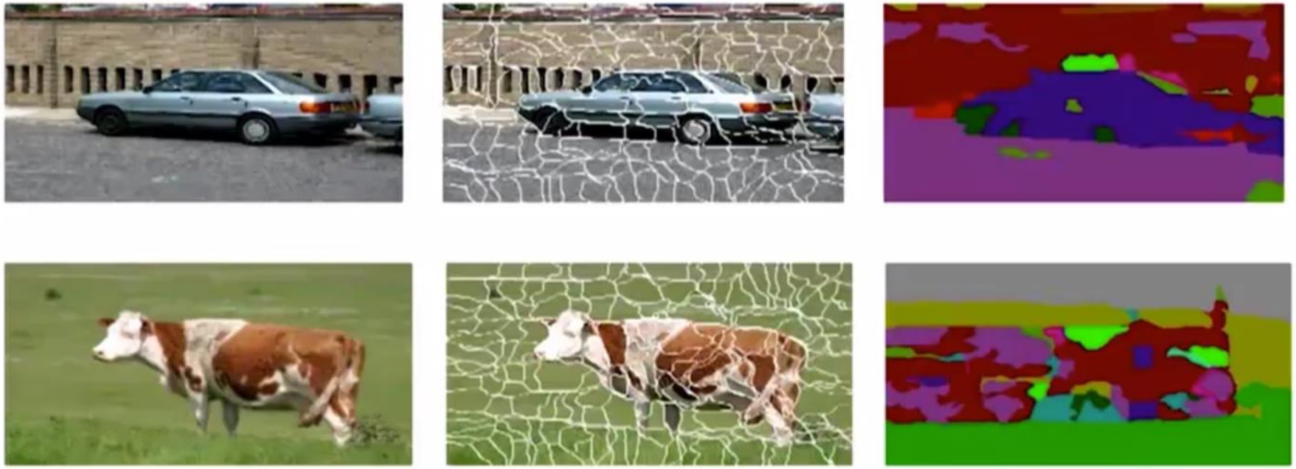
- Medial diagnosis
- Fault diagnosis
- Natural language processing
- Traffic analysis
- Social network models
- Message decoding
- Computer vision
  - Image segmentation
  - 3D reconstruction
  - Holistic scene analysis
- Speech recognition
- Robot localization and mapping

Image Segmentation

Image Segmentation

superpixels

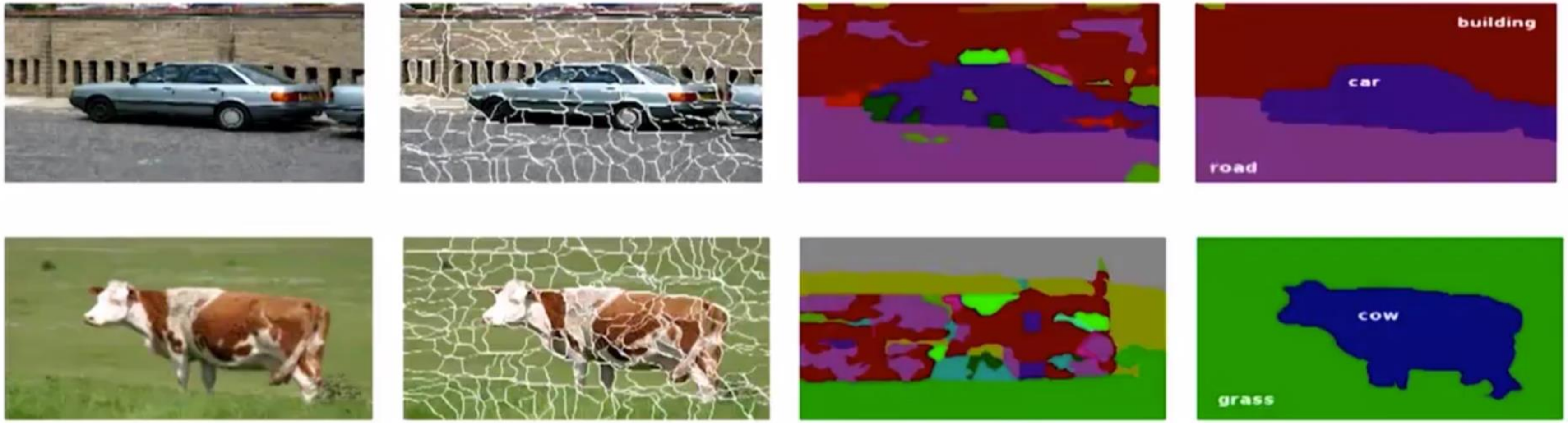
Image Segmentation



superpixels

machine learning
to separate superpixels

Image Segmentation



superpixels

machine learning
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Textual Information Extracion

Mrs. Green spoke today in New York. Green chairs the finance committee.

person

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person location person organization

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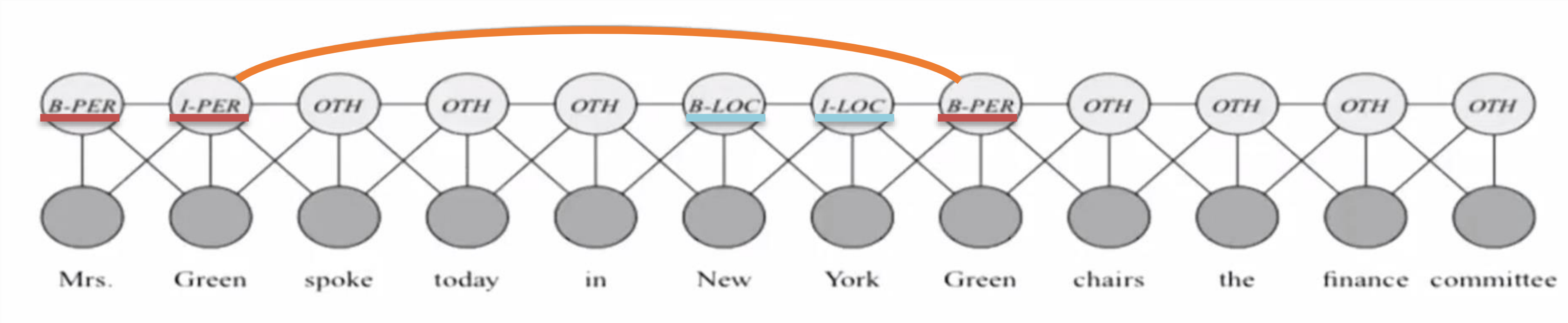
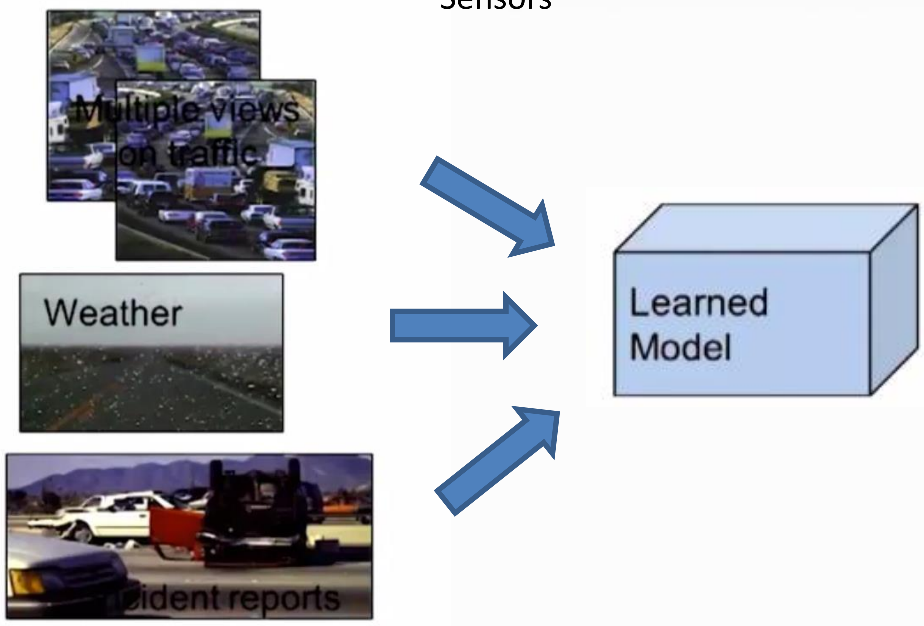


Multi-Sensor Integration

Multi-Sensor Integration

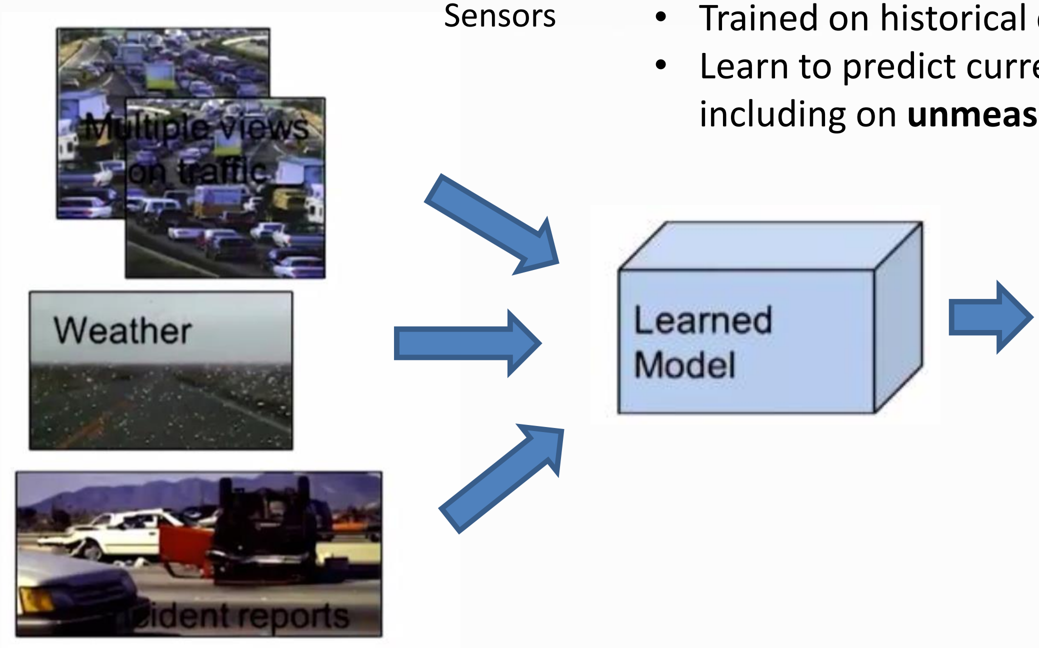
Sensors



Multi-Sensor Integration

Sensors

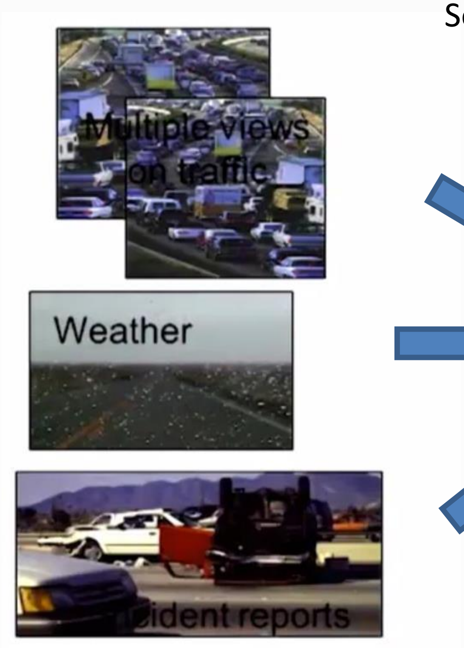
- Trained on historical data
- Learn to predict current and **future** road speed, including on **unmeasured** roads

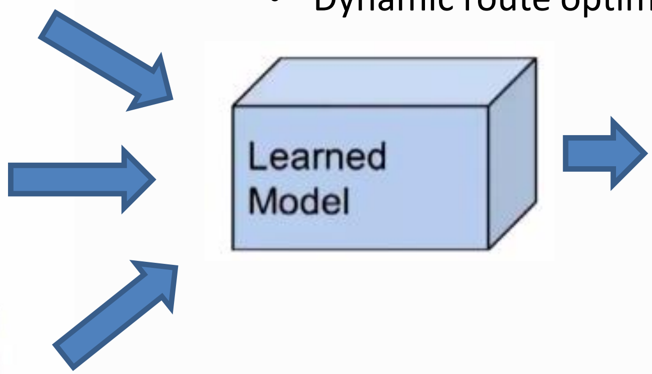


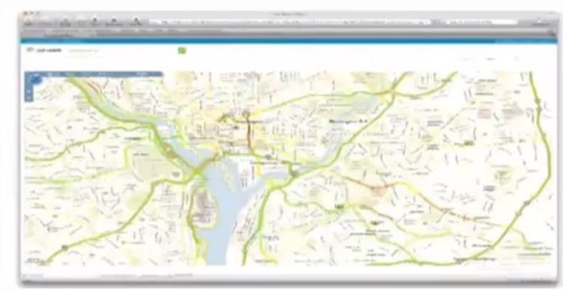
Multi-Sensor Integration

Sensors

- Trained on historical data
- Learn to predict current and **future** road speed, including on **unmeasured** roads
- Dynamic route optimization







Multi-Sensor Integration

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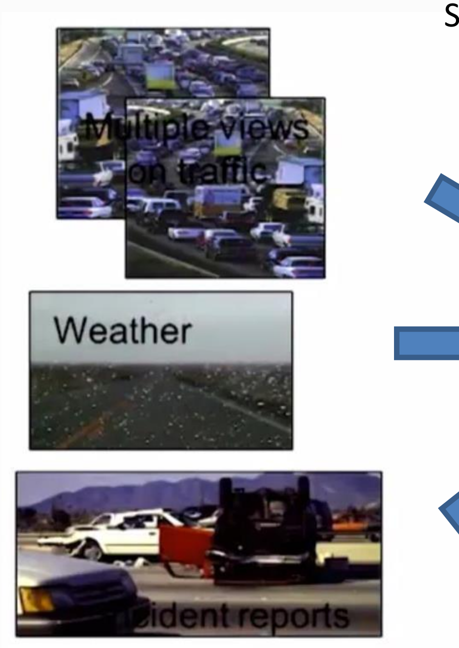

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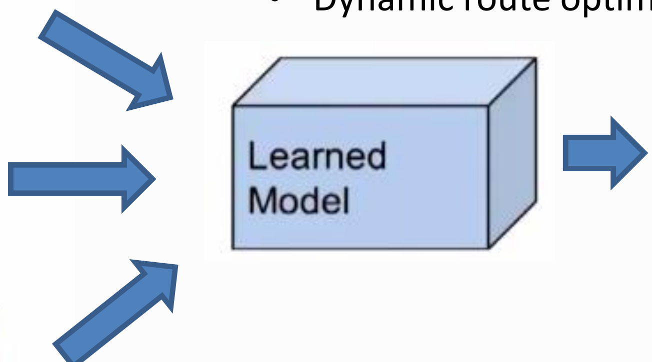
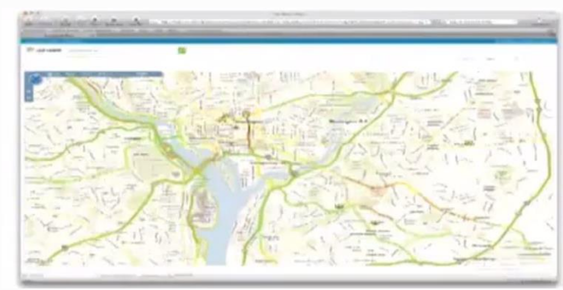
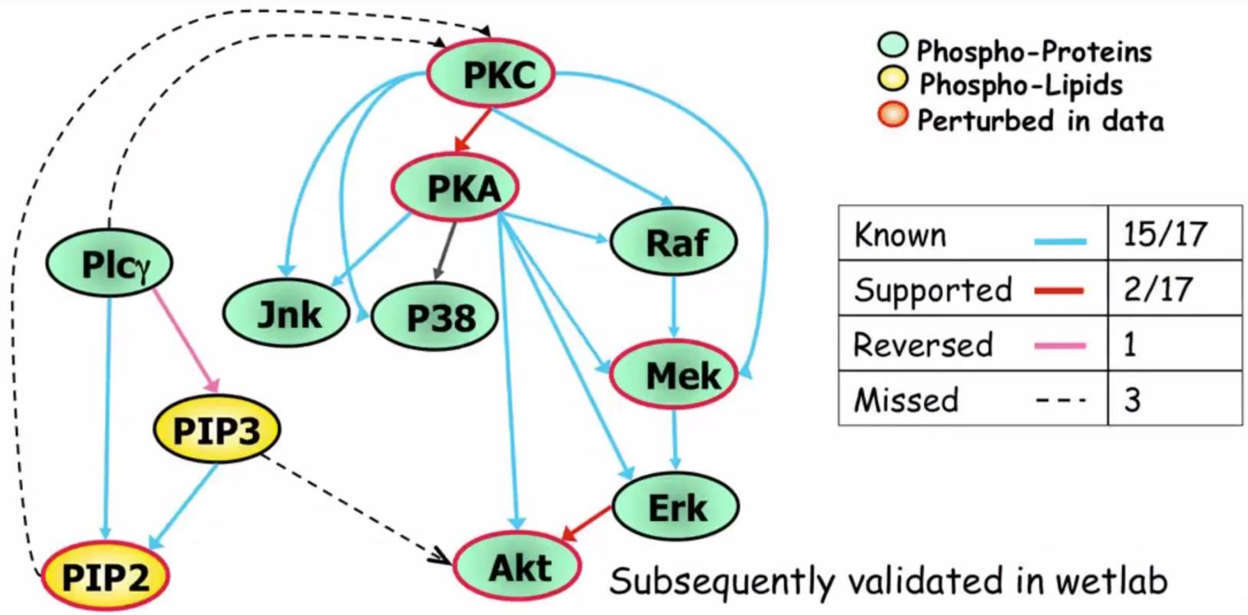




- I95 corridor experiment: accurate to ±5 MPH in 85% of cases
- Fielded in 72 cities

Multi-Sensor Integration



Causal protein-signaling networks derived from multiparameter single-cell data

Multi-Sensor Integration

- Representation
  - Directed and undirected
  - Temporal and plate models
- Inference
  - Exact and approximate
  - Decision making
- Learning
  - Parameters and structure
  - With and without complete data

Lecture Plan

- Overview and Motivation
- Distributions

Joint Distribution

- Intelligence (I)

  i^0 (low), i^1 (high)
- Difficulty (D)

  d^0 (easy), d^1 (hard)
- Grade (G)

  g^1 (A), g^2 (B), g^3 (C)

Joint Distribution

- Intelligence (I) $\leftarrow$ 2

  i^0 (low), i^1 (high)
- Difficulty (D) $\leftarrow$ 2

  d^0 (easy), d^1 (hard)
- Grade (G) $\leftarrow$ 3

  g^1 (A), g^2 (B), g^3 (C)

Joint Distribution

- Intelligence (I) ← **2**
  i^0 (low), i^1 (high)
- Difficulty (D) ← **2**
  d^0 (easy), d^1 (hard)
- Grade (G) ← **3**
  g^1 (A), g^2 (B), g^3 (C)

Parameters:
2x2x3=12

I	D	G	P(I,D,G)
i^0	d^0	g^1	0.126
i^0	d^0	g^2	0.168
i^0	d^0	g^3	0.126
i^0	d^1	g^1	0.009
i^0	d^1	g^2	0.045
i^0	d^1	g^3	0.126
i^1	d^0	g^1	0.252
i^1	d^0	g^2	0.0224
i^1	d^0	g^3	0.0056
i^1	d^1	g^1	0.06
i^1	d^1	g^2	0.036
i^1	d^1	g^3	0.024

Joint Distribution

- Intelligence (I) ← 2
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- Difficulty (D) ← 2
  d^0 (easy), d^1 (hard)
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Parameters: 2x2x3=12

Independent parameters: 11

I	D	G	P(I,D,G)
i^0	d^0	g^1	0.126
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i^1	d^0	g^3	0.0056
i^1	d^1	g^1	0.06
i^1	d^1	g^2	0.036
i^1	d^1	g^3	0.024

sum=1

Conditioning

condition on g^1

I	D	G	P(I,D,G)
i^0	d^0	g^1	0.126
i^0	d^0	g^2	0.168
i^0	d^0	g^3	0.126
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Conditioning

condition on g^1

I	D	G	P(I,D,G)
i^0	d^0	g^1	0.126
~~i^0~~	~~d^0~~	~~g^2~~	~~0.168~~
~~i^0~~	~~d^0~~	~~g^3~~	~~0.126~~
i^0	d^1	g^1	0.009
~~i^0~~	~~d^1~~	~~g^2~~	~~0.045~~
~~i^0~~	~~d^1~~	~~g^3~~	~~0.126~~
i^1	d^0	g^1	0.252
~~i^1~~	~~d^0~~	~~g^2~~	~~0.0224~~
~~i^1~~	~~d^0~~	~~g^3~~	~~0.0056~~
i^1	d^1	g^1	0.06
~~i^1~~	~~d^1~~	~~g^2~~	~~0.036~~
~~i^1~~	~~d^1~~	~~g^3~~	~~0.024~~

Conditioning: Reduction

condition on g^1

I	D	G	P(I,D,G)
i^0	d^0	g^1	0.126
i^0	d^1	g^1	0.009
i^1	d^0	g^1	0.252
i^1	d^1	g^1	0.06

Conditioning: Renormalization

I	D	G	P(I,D,G)
i^0	d^0	g^1	0.126
i^0	d^1	g^1	0.009
i^1	d^0	g^1	0.252
i^1	d^1	g^1	0.06

sum=0.447

P(I,D, g^1)

unnormalized measure

Conditioning: Renormalization

I	D	G	P(I,D, g^1)	
i^0	d^0	g^1	0.126	/0.447
i^0	d^1	g^1	0.009	/0.447
i^1	d^0	g^1	0.252	/0.447
i^1	d^1	g^1	0.06	/0.447

sum=0.447

I	D	P(I,D|g^1)
i^0	d^0	0.282
i^0	d^1	0.02
i^1	d^0	0.564
i^1	d^1	0.134

P(I,D, g^1)

unnormalized measure

Conditioning: Marginalization

Marginalize I

I	D	P(I,D)
i^0	d^0	0.282
i^0	d^1	0.02
i^1	d^0	0.564
i^1	d^1	0.134

Conditioning: Marginalization

Marginalize I

I	D	P(I,D)
i^0	d^0	0.282
i^0	d^1	0.02
i^1	d^0	0.564
i^1	d^1	0.134

D	P(D)
d^0	0.846
d^1	0.154