1

$$\int_1^4 x^2 dx = F(x) \ = \ \frac{x^3}{3}\Big|_1^4 = \ \frac{4^3}{3} - \ \frac{1^3}{3} = \frac{64 \ - \ 1}{3} = 21$$

2

$$\int_{-4}^{-2} \frac{dx}{\sqrt{5-4x-x^2}} = \int_{-4}^{-2} \frac{dx}{\sqrt{9-\left(x^2+4x+4\right)}} = \int_{-4}^{-2} \frac{d(x+2)dx}{\sqrt{3^2-(x+2)^2}} =$$

$$= arcsin\frac{x+2}{3}\Big|_{-4}^{-2} = arcsin\frac{-2+2}{3} - arcsin\frac{-4+2}{3} = arcsin - \frac{2}{3} = arcsin\frac{2}{3}$$

3

$$\int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2}-x\right)dx = \frac{1}{2}\int_0^{\frac{\pi}{2}}\left(1+\cos\left(\frac{\pi}{3}-2x\right)\right)dx = \frac{1}{2}\int_0^{\frac{\pi}{2}}dx \ + \frac{1}{2}\int_0^{\frac{\pi}{2}}\cos\left(\frac{\pi}{3}-2x\right)dx$$

$$\frac{1}{2}x\Big|_0^{\frac{\pi}{2}} - \frac{1}{4}\int_0^{\frac{\pi}{2}}\cos\left(\frac{\pi}{3}-2x\right)d\left(\frac{\pi}{3}-2x\right) = \frac{\pi}{4} - \frac{1}{4}\int_0^{\frac{\pi}{2}}\cos\left(\frac{\pi}{3}-2x\right)d\left(\frac{\pi}{3}-2x\right) =$$

$$= \frac{\pi}{4} - \sin\left(\frac{\pi}{3}-2x\right)_0^{\frac{\pi}{2}} = \frac{\pi}{4} - \left(\sin\left(\frac{\pi}{3}-\pi\right) - sin\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{4} + \sqrt{3}$$

4

$$\int_1^2 \frac{x^4+1}{x^3\left(x^2+1\right)}dx = \frac{x^4+1}{x^3\left(x^2+1\right)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{Dx+E}{x^2+1} =$$

$$= \frac{A\left(x^2+1\right) \ = \ Bx\left(x^2+1\right) + Cx^2\left(x^2+1\right) + x^3(Dx+E)}{x^3\left(x^2+1\right)};$$

$$x^4+1 \ = A\left(x^2+1\right) \ = \ Bx\left(x^2+1\right) + Cx^2\left(x^2+1\right) + x^3(Dx+E)$$

$$x^4+1 \ = (C+D)x^4 + (B+E)x^3 + (A+C)x^2 + Bx + A$$

$$\begin{cases} A = 1 \\ B = 0 \\ A+C = 0 \\ B+E = 0 \\ C+D = 1 \end{cases}$$

$$A = 1, \ B = 0, \ C = -1, \ E = 0, \ D = 2$$

$$\frac{1}{x^3}-\frac{1}{x}+\frac{2x}{x^2+1}=$$

$$=2\int_1^2\frac{x}{x^2+1}dx+\int_1^2\frac{1}{x^3}dx-\int_1^2\frac{1}{x}dx=-\ln|x|+\ln\left(x^2+1\right)-\frac{1}{2x^2}+C=\ln(5)-(1)$$

5

$$\int_0^2 f(x)dx,\ \text{при}\begin{cases} e^x,\ 0\leqslant x<1\\ 2,\ 1\leqslant x\leqslant 2\end{cases}$$

$$x=1$$

$$\int_0^2 f(x)dx=\int_0^1 e^x dx+\int_1^2 2dx=e^x\Big|_0^1+2x\Big|_1^2=(e-1)+(4-2)=e+1$$

6

$$\int_1^9\frac{dx}{5+2\sqrt{x}}=\left[t=\sqrt{x},\ x=t^2,\ dx=2tdt,\ x=1\Longrightarrow t=1\Longrightarrow x=9\right]=$$

$$=\int_1^3\frac{2tdt}{5+2t}=\int_1^3\frac{((5+2t)-5)dt}{5+2t}=\int_1^3 dt-\int_1^3\frac{5dt}{5+2t}=t\Big|_1^3-\frac{5}{2}\int_1^3\frac{d(5+2t)}{5+2t}=$$

$$=2-\frac{5}{2}\ln|5+2t|\Big|_1^3=2-\frac{5}{2}(\ln|11|-\ln|8|)=2-\frac{5}{2}\ln\frac{11}{7}$$

7

$$\int_0^{\frac{\pi}{2}}\frac{dx}{3+2\cos x}$$

$$tg\left(\frac{x}{2}\right)=t$$

$$\cos x=\frac{1-t^2}{1+t^2}$$

$$arctg\left(tg\left(\frac{x}{2}\right)\right)=arctg(t),\ \frac{x}{2}=arctg(t)$$

$$x=2arctg(t),\ dx=\frac{2}{1+x^2}dt$$

$$x=0\Longrightarrow t=0,\ x=\frac{\pi}{2}\Longrightarrow t=1$$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\cos x} = \int_0^1 \frac{2dt}{\left(1+t^2\right)\left(3+2\frac{1-t^2}{1+t^2}\right)} = \int_0^1 \frac{2dt}{t^2+\left(\sqrt{5}\right)^2} = 2\int_0^1 \frac{dt}{t^2+\left(\sqrt{5}\right)^2} =$$

$$= \frac{2}{\sqrt{5}} arctg \frac{t}{\sqrt{5}}\Big|_0^1 = \frac{2}{\sqrt{5}} arctg \frac{1}{\sqrt{5}} - \frac{2}{\sqrt{5}} arctg \frac{0}{\sqrt{5}} = \frac{2}{\sqrt{5}} arctg \frac{1}{\sqrt{5}}$$

8

$$\int_2^3 x(3-x)^7 dx = [z = 3-x, x = 3-z,\ dx = -dz, x = 2 \Longrightarrow z = 1, x = 3 \Longrightarrow z = 0] =$$

$$= \int_1^0 (z-3)z^7 dz = \left[\frac{z^9}{9} - 3\frac{z^8}{8}\right]\Big|_1^0 = -\frac{1}{9} + \frac{3}{8} = \frac{-8+27}{72} = \frac{19}{72}$$