10

Data Structures – II : Stacks and Queues

10.1 INTRODUCTION

Any good programming language course does cover data structures. A *data structure,* in general, refers to a particular way of storing and organizing data in a computer so that it can be used most efficiently to give optimal performance. Different kinds of *data structures* are designed and used for different kinds of applications. The significance of data structures lies in the fact that they help a computer perform tasks in most efficient and productive manner.

Some data structures are highly specialized to specific tasks. **Stacks** and **Queues** are such data structures. In this chapter, we shall be talking about the basics of these data structures and how these can be implemented in Python.

10.2 STACKS

A stack is a linear structure implemented in LIFO (**Last In First Out**) manner where insertions and deletions are restricted to occur only at one end – ***Stack's top***. LIFO means element last inserted would be the first one to be deleted. Thus, we can say that a stack is a list of data that follows these rules :

1. Data can only be removed from the top (*pop*), *i.e.*, the element at the *top of the stack*. The removal of element from a stack is technically called **POP operation**.
2. A new data element can only be added to the *top of the stack* (*push*). The insertion of element in a stack is technically called **PUSH operation**.

Consider Fig. 10.1 that illustrates the operations (**Push** and **Pop**) on a stack.

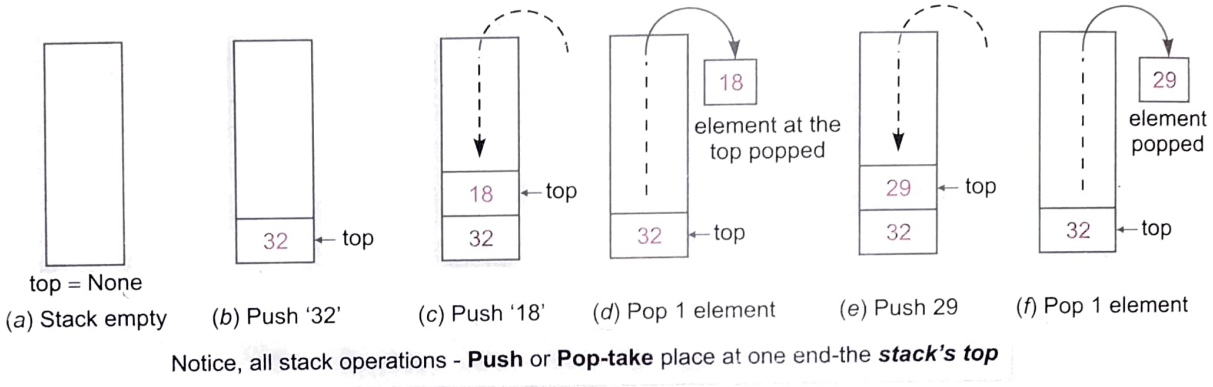


Figure 10.1 Stack operations – *Push* and *Pop*.

STACK

A Stack is a linear list implemented in LIFO – Last In First Out manner where insertions and deletions are restricted to occur only at one end – *Stack's top*.

The stack is a *dynamic data structure* as it can grow (with increase in number of elements) or shrink (with decrease in number of elements). A *static data structure*, on the other hand, is the one that has fixed size.

Other Stack Terms

There are *some* other terms related to stacks, such as *Peek*, *Overflow* and *Underflow*.

Peek Refers to inspecting the value at the ***stack's top*** without removing it. It is also sometimes referred as **inspection**.

Overflow Refers to situation (ERROR) when one tries to push an item in stack that is full. This situation occurs when the size of the stack is fixed and cannot grow further or there is no memory left to accommodate new item.

Underflow Refers to situation (ERROR) when one tries to pop/delete an item from an empty stack. That is, stack is currently having no item and still one tries to pop an item.

NOTE

The technical terms for insertion-in-a-stack and deletion-from-stack are Push and Pop respectively.

Consider some examples illustrating stack-functioning in limited-size stack. (Please note, we have bound fixed the capacity of the stack for understanding purposes.)

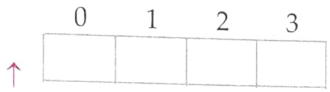
EXAMPLE 10.1 *Given a Bounded Stack of capacity 4 which is initially empty, draw pictures of the stack after each of the following steps. Initially the Stack is empty.*

(*i*) Stack is empty (*ii*) Push 'a' (*iii*) Push 'b' (*iv*) Push 'c'
(*v*) Pop (*vi*) Push 'd' (*vii*) Pop (*viii*) Push 'e'
(*ix*) Push 'f' (*x*) Push (*xi*) Pop (*xii*) Pop
(*xiii*) Pop (*xiv*) Pop (*xv*) Pop

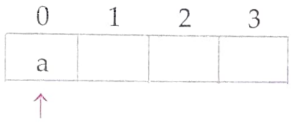
Solution.

top = ↑

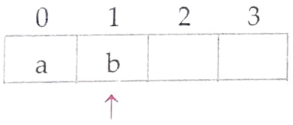
(*i*) Stack is empty (top = None)



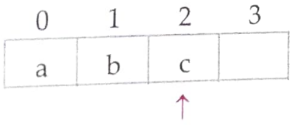
(*ii*) Push 'a' ***top = 0***



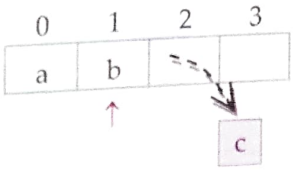
(*iii*) Push 'b' ***top = 1***



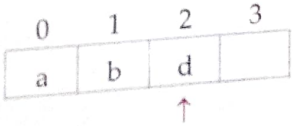
(*iv*) Push 'c' ***top = 2***



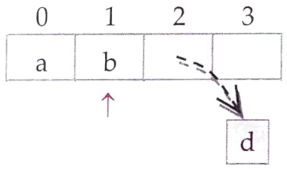
(*v*) Pop ***top = 1***



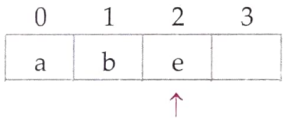
(*vi*) Push 'd' ***top = 2***



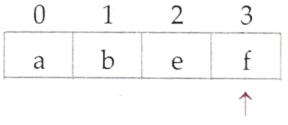
(*vii*) Pop ***top = 1***



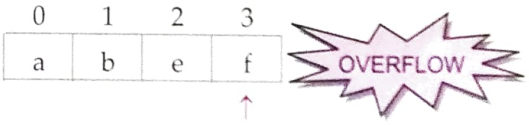
(*viii*) Push 'e' ***top = 2***



(*ix*) Push 'f' ***top = 3***



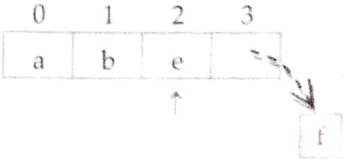
(*x*) Push 'g' ***top = 3***



[OVERFLOW because the stack is bounded, it cannot grow. If it could grow, then there would have been no OVERFLOW until no memory is left.

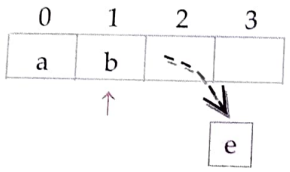
In Python, (for stacks implemented through lists) since Lists can grow, OVERFLOW condition does not arise until all the memory is exhausted.]

(*xi*) Pop ***top = 2***

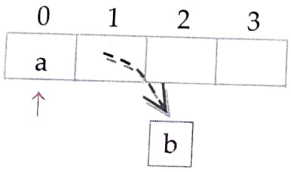


top = ↑

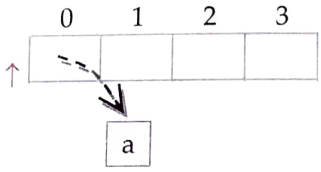
(*xii*) Pop **top = 1**



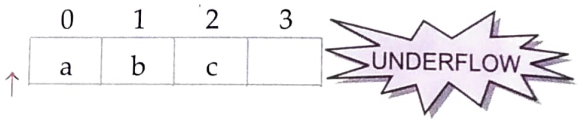
(*xiii*) Pop top = 0



(*xiv*) Pop top = None



(*xv*) Pop top = None



10.2.1 Implementing Stack in Python

In Python, you can use Lists to implement stacks. Python offers us a convenient set of methods to operate lists as stacks.

For various stack operations, we can use a list say ***Stack*** and use Python code as described below :

Peek We can use : `<Stack>[top]`

where **<Stack>** is a list ; ***top*** is an integer having value equal to **len(<Stack>) - 1.**

Push We can use : `<Stack>.append(<item>)`

where **<item>** is the item being pushed in the Stack.

Pop We can use : `<Stack>.pop()`

it removes the last value from the *Stack* and returns it.

Let us now implement a stack of numbers through a program.

```
def Push(stk, item) :
```

Program 10.1 Python program to implement stack operations.

```
############    STACK IMPLEMENTATION    ##############
"""
    Stack: implemented as a list
    top : integer having position of topmost element in Stack
"""
def isEmpty( stk ) :
    if stk == [] :
        return True
    else :
        return False
```

```
def Push(stk, item) :
    stk.append(item)
    top = len(stk) - 1
def Pop(stk) :
    if isEmpty(stk) :
        return "Underflow"
    else :
        item = stk.pop()
        if len (stk) == 0:
            top = None
        else :
            top = len(stk) - 1
        return item
def Peek(stk) :
    if is Empty(stk) :
        return "Underflow"
    else :
        top = len(stk) - 1
        return stk[top]
def Display(stk) :
    if isEmpty(stk) :
        print("Stack empty")
    else :
        top = len(stk) - 1
        print(stk[top], "<- top")
        for a in range(top-1, -1 , -1 ) :
            print(stk[a])
# __main__
Stack = []                          # initially stack is empty
top = None
while True :
    print("STACK OPERATIONS")
    print("1. Push")
    print("2. Pop")
    print("3. Peek")
    print("4. Display stack")
    print("5. Exit")
    ch = int(input("Enter your choice (1-5) :"))
    if ch == 1 :
        item = int(input("Enter item :"))
        Push(Stack, item)
    elif ch == 2 :
        item = Pop(Stack)
        if item == "Underflow" :
            print("Underflow! Stack is empty!")
        else :
            print("Popped item is", item)
```

```
elif ch == 3 :
    item = Peek(Stack)
    if item == "Underflow" :
        print("Underflow! Stack is empty!")
    else :
        print("Topmost item is", item)
elif ch == 4 :
    Display(Stack)
elif ch == 5 :
    break
else :
    print("Invalid choice!")
```

Sample run of the above program is as shown below :

```
STACK OPERATIONS
1. Push
2. Pop
3. Peek
4. Display stack
5. Exit
Enter your choice (1-5) :1
Enter item :6
--------------------
STACK OPERATIONS
1. Push
2. Pop
3. Peek
4. Display stack
5. Exit
Enter your choice (1-5) :1
Enter item :8
--------------------
STACK OPERATIONS
1. Push
2. Pop
3. Peek
4. Display stack
5. Exit
Enter your choice (1-5) :1
Enter item :2
--------------------
STACK OPERATIONS
1. Push
2. Pop
3. Peek
4. Display stack
5. Exit
Enter your choice (1-5) :1
Enter item :4
--------------------
STACK OPERATIONS
1. Push
2. Pop
3. Peek
4. Display stack
5. Exit
Enter your choice (1-5) :4
4 <- top
2
8
6
--------------------
STACK OPERATIONS
1. Push
2. Pop
3. Peek
4. Display stack
5. Exit
Enter your choice ( 1-5) :3
Topmost item is  4
--------------------
STACK OPERATIONS
1. Push
2. Pop
3. Peek
4. Display stack
5. Exit
Enter your choice (1-5) :4
4 <- top
2
8
6
--------------------
STACK OPERATIONS
1. Push
2. Pop
3. Peek
4. Display stack
5. Exit
Enter your choice (1-5) :5
```

Types of Stack–Itemnode

An item stored in a stack is also called item-node sometimes. In the above implemented stack, the stack contained item-nodes containing just integers. If you want to create stack that may contain logically group information such as member details like : *member no, member name, age* etc. For such a stack the item-node will be a list containing the member details and then this list will be entered as an item to the stack. (See figure 10.2 below).

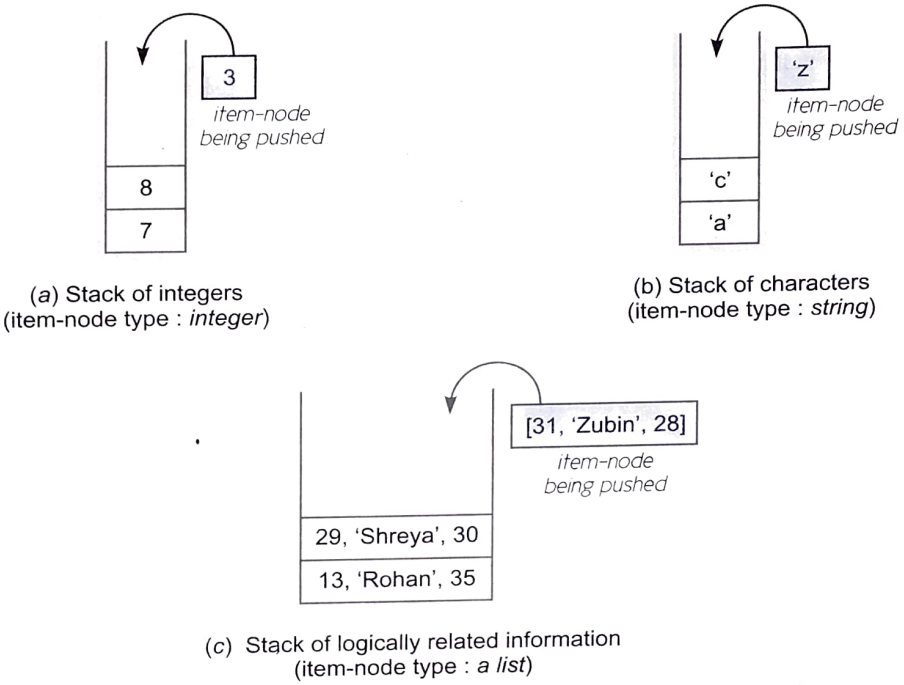


Figure 10.2 Different types of stack item-nodes.

- For stack of Fig. 10.2(*a*), the stack will be implemented as ***Stack of integers*** as *item-node* is of integer type.
- For stack of Fig. 10.2(*b*), the stack will be implemented as ***Stack of strings*** as *item-node* is of string type.
- For stack of Fig. 10.2(*c*), the stack will be implemented as ***Stack of lists*** as *item-node* is of list type. Solved problem 20 implements such a stack.

10.2.2 Stack Applications[1]

There are several applications and uses of stacks. The stacks are basically applied where LIFO (Last In First Out) scheme is required.

10.2.2A Reversing a Line

A simple example of stack application is reversal of a given line. We can accomplish this task by pushing each character on to a stack as it is read. When the line is finished, characters are then

1. Some other applications of stacks include :
(*a*) The compilers use stacks to store the previous state of a program when a function is called, or during recursion.
(*b*) One of the most important applications of Stacks is *backtracking*. Backtracking is used in large number of puzzles like *n*-Queen problem, *Sudoku* etc and optimization problems such as *knapsack problem*.

popped off the stack, and they will come off in the reverse order as shown in Fig. 10.3. The given line is : *Stack*

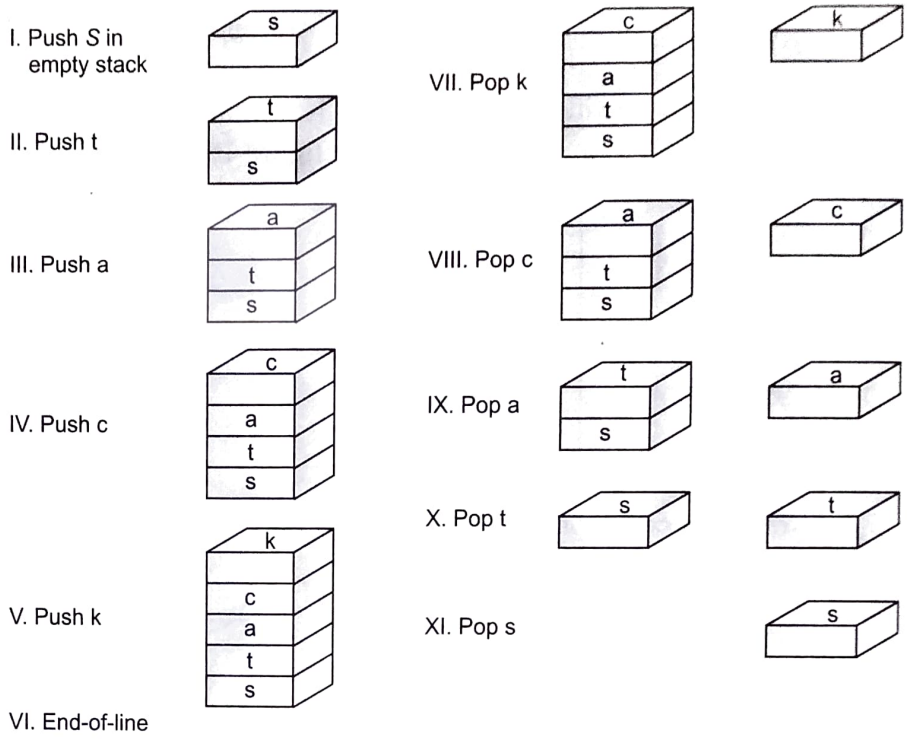


Figure 10.3
Reversal of a line using stack.

10.2.2B Polish Strings

Another application of stacks is in the conversion of arithmetic expressions in high-level programming languages into machine readable form. As our computer system can only understand and work on a binary language, it assumes that an arithmetic operation can take place in two operands only *e.g.*, $A + B$, $C \times D$, D/A etc. But in our usual form an arithmetic expression may consist of more than one operator and two operands *e.g.*, $(A + B) \times C\ (D/(J + D))$. These complex arithmetic operations can be converted into *polish strings* using stacks which then can be executed in *two operands and a operator* form.

Polish string, named after a polish mathematician, *Jan Lukasiewicz*, refers to the notation in which the operator symbol is placed either before its operands (*prefix notation*) or after its operands (*postfix notation*) in contrast to usual form where operator is placed in between the operands (*infix notation*).

Following table shows the three *types* of notations :

Table 10.1 *Expressions in infix, prefix, postfix notations*

Infix notation	Prefix notation	Postfix notation
$A + B$	$+ AB$	$AB +$
$(A - C) \times B$	$\times - ACB$	$AC - B \times$
$A + (B \times C)$	$+ A \times BC$	$ABC \times +$
$(A + B)/(C - D)$	$/ + AB - CD$	$AB + CD - /$
$(A + (B \times C))/(C - (D \times B))$	$/ + A \times BC - C \times DB$	$ABC \times + CDB \times - /$

Conversion of Infix Expression to Postfix (Suffix) Expression

While evaluating an infix expression, there is an evaluation order according to which

I Brackets or Parenthesis,
II Exponentiation,
III Multiplication or Division,
IV Addition or Subtraction

take place in the above specified order. The operators with the same priority (*e.g.*, × and/) are evaluated from left to right.

To convert an infix expression into a postfix expression, this evaluation order is taken into consideration.

An infix expression may be converted into postfix form either manually or using a stack. The manual conversion requires two passes : one for inserting braces and another for conversion. However, the conversion through stack requires single pass only.

The steps to convert an infix expression into a postfix expression manually are given below :

(*i*) Determine the actual evaluation order by inserting braces.
(*ii*) Convert the expression in the innermost braces into postfix notation by putting *the operator after the operands.*
(*iii*) Repeat step (*ii*) until entire expression is converted into postfix notation.

EXAMPLE 10.2 *Convert* $(A+B)\times C/D$ *into postfix notation.*

Solution.

Step I : Determine the actual evaluation order by putting braces

$$=((A+B)\times C)/D$$

Step II : Converting expressions into innermost braces

$$=((AB+)\times C)/D=(AB+C\times)/D=AB+C\times D/$$

EXAMPLE 10.3 *Convert* $((A+B)*C/D+E\wedge F)/G$ *into postfix notation.*

Solution. The evaluation order of given expression will be

$$=((((A+B)*C)/D)+(E\wedge F))/G$$

Converting expressions in the braces, we get

$$=((((AB+)*C)/D)+(EF\wedge))/G$$

$$=(((AB+C*)/D)+EF\wedge)/G$$

$$=((AB+C*D/)+EF\wedge)/G=(AB+C*D/EF\wedge+)/G$$

$$=AB+C*D/EF\wedge\ +G/$$

EXAMPLE 10.4 *Give postfix form of the following expression*

$$A*(B+(C+D)*(E+F)/G)*H$$

Solution. Evaluation order is

$$(A*(B+((C+D)*(E+F))/G))*H$$

Converting expressions in the braces, we get

$$= (A * (B + [(CD +) * (EF +)] / G)) * H = A * (B + (CD + EF + *) / G) * H$$
$$= A * (B + (CD + EF + * G/)) * H = (A * (BCD + EF + * G / +)) * H$$
$$= (ABCD + EF + * G / + *) * H = \mathbf{ABCD + EF + * G / + * H *}$$

EXAMPLE 10.5 *Give postfix form for* $A + [(B + C) + (D + E) * F] / G$.

Solution. Evaluation order is : $A + [\{(B + C) + ((D + E) * F)\} / G]$

Converting expressions in braces, we get

$$= A + [\{(BC +) + (DE +) * F\} / G] = A + [\{(BC +) + (DE + F *)\} / G]$$
$$= A + [\{BC + DE + F * +\} / G] = A + [BC + DE + F * + G/]$$
$$= ABC + DE + F * + G / +$$

EXAMPLE 10.6 *Give postfix form of expression for the following : NOT A OR NOT B NOT C*

Solution. The order of evaluation will be

((NOT A) OR ((NOT B) AND (NOT C)))

(As priority order is NOT, AND, OR)

$$= ((A \text{ NOT}) \text{ OR } ((B \text{ NOT}) \text{ AND } (C \text{ NOT})))$$
$$= ((A \text{ NOT}) \text{ OR } ((B \text{ NOT } C \text{ NOT AND}))$$
$$= A \text{ NOT } B \text{ NOT } C \text{ NOT AND OR}$$

While converting from *infix to prefix form,* operators are put before the operands. Rest of the conversion procedure is similar to that of *infix to postfix* conversion.

Algorithm to Convert Infix Expression to Postfix Form

The following algorithm transforms the infix expression X into its equivalent postfix expression Y. The algorithm uses a stack to temporarily hold operators and left parentheses. The postfix expression Y will be constructed from left to right using the operands from X and the operators which are removed from STACK. We begin by pushing a left parenthesis onto STACK and adding a right parenthesis at the end of X. The algorithm is completed when STACK is empty.

Algorithm *Infix to Postfix Conversion using Stack*

Suppose X is an arithmetic expression written in infix notation. This algorithm finds the equivalent postfix expression Y.

1. Push "(" onto STACK, and add ") " to the end of X.
2. Scan X from left to right and REPEAT Steps 3 to 6 for each element of X UNTIL the STACK is empty :
3. If an operand is encountered, add it to Y.
4. If a left parenthesis is encountered, push it onto STACK.
5. If an operator is encountered, then :

   (a) Repeatedly pop from STACK and add to Y each operator (on the top of STACK) which has the same precedence as or higher precedence than operator.

(b) Add operator to STACK.
'''End of If structure'''

6. If a right parenthesis is encountered, then :
(a) Repeatedly pop from STACK and add to Y each operator (on the top of STACK) until a left parenthesis is encountered.
(b) Remove the left parenthesis. [Do not add the left parenthesis to Y].
'''End of If structure'''
'''End of Step 2 Loop'''

7. END.

EXAMPLE 10.7 *Convert* $X: A+(B*C-(D/E\wedge F)*G)*H$ *into postfix form showing stack status after every step in tabular form.*

Solution.

Symbol Scanned		Stack	Expression Y
1.	A	(	A
2.	+	(+	A
3.	(	(+ (	A
4.	B	(+ (	$A\ B$
5.	*	(+ (*	$A\ B$
6.	C	(+ (*	$A\ B\ C$
7.	–	(+ (–	$A\ B\ C\ *$
8.	(	(+ (– (	$A\ B\ C\ *$
9.	D	(+ (– (	$A\ B\ C\ *\ D$
10.	/	(+ (– (/	$A\ B\ C\ *\ D$
11.	E	(+ (– (/	$A\ B\ C\ *\ D\ E$
12.	^	(+ (– (/ ^	$A\ B\ C\ *\ D\ E$
13.	F	(+ (– (/ ^	$A\ B\ C\ *\ D\ E\ F$
14.	)	(+ (–	$A\ B\ C\ *\ D\ E\ F\ \wedge\ /$
15.	*	(+ (– *	$A\ B\ C\ *\ D\ E\ F\ \wedge\ /$
16.	G	(+ (– *	$A\ B\ C\ *\ D\ E\ F\ \wedge /\ G$
17.	)	(+	$A\ B\ C\ *\ D\ E\ F\ \wedge\ /\ G\ *\ -$
18.	*	(+ *	$A\ B\ C\ *\ D\ E\ F\ \wedge /\ G\ *\ -$
19.	H	(+ *	$A\ B\ C\ *\ D\ E\ F\ \wedge /\ G\ *\ -\ H$
20.	)		$A\ B\ C\ *\ D\ E\ F\ \wedge /\ G\ *\ -\ H\ *\ +$

Advantage of Postfix Expression over Infix Expression

An infix expression is difficult for the machine to know and keep track of precedence of operators. On the other hand, a postfix expression itself determines the precedence of operators (as the placement of operators in a postfix expression depends upon its precedence). Therefore, for the machine it is easier to carry out a postfix expression than an infix expression.

Evaluation of a postfix Expression using stacks

As postfix expression is without parenthesis and can be evaluated as two operands and an operator at a time, this becomes easier for the compiler and the computer to handle. Evaluation rule of a postfix expression states :

- While reading the expression from left to right, push the element in the stack if it is an operand ;
- pop the two operands from the stack, if the element is a binary operator. In case of NOT operator, pop one operand from the stack and then evaluate it (two operands and an operator).
- Push back the result of the evaluation. Repeat it till the end of the expression.

For a binary operator, two operands one popped from stack and for a unary operator, one operand is popped. Then, the result is calculated using operand(s) and the operator, and pushed back into the stack.

Algorithm *Evaluation of Postfix Expression*

```
'''Reading of expression takes place from left to right'''
1.  Read the next element     '''first element for the first time'''
2.  If element is operand then
        Push the element in the stack
3.  If element is operator then
    {
4.      Pop two operands from the stack
            '''POP one operand in case of unary operator'''
5.      Evaluate the expression formed by the two operands and the operator
6.      Push the result of the expression in the stack end
    }
7.  If no-more-elements then
        POP the result
    else
        go to step 1.
8.  END.
```

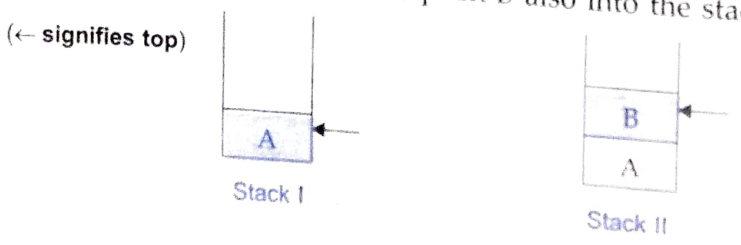
EXAMPLE 10.8 *Evaluate the postfix expression* $AB+C\times D/$ *if* $A=2$, $B=3$, $C=4$ *and* $D=5$.

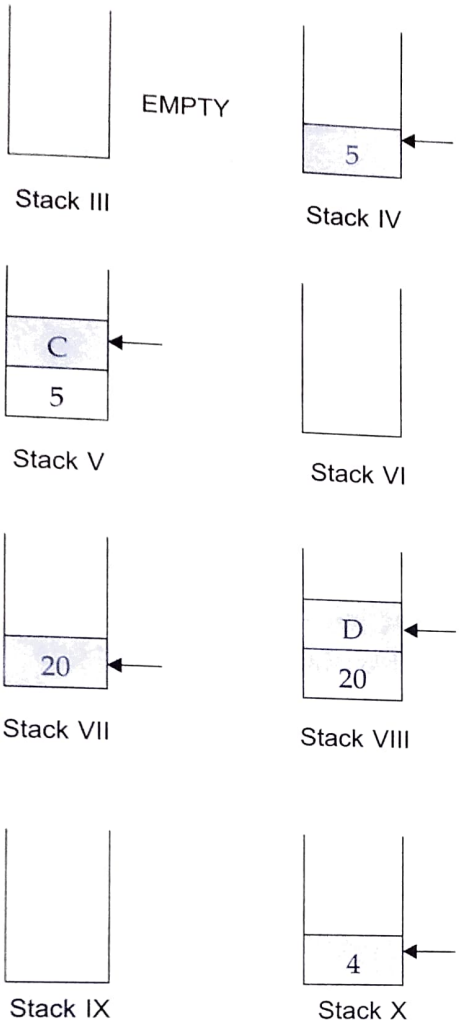
Solution. The expression given is $AB+C\times D/$

Starting from left to right

I First element is operand A, push A into the stack, (see *Stack I*)

II Second element is also operand B, push B also into the stack, (see *Stack II*)





III Third element '+' is an operand, pop 2 operands from the stack *i.e.*, *A* and *B* (see *Stack III*) and evaluate the expression, *i.e.*,

$$A + B = 2 + 3 = 5$$

IV Push the result (5) into the stack, (see *Stack IV*)

V Next element *C* is operand ; pushed into the stack, (see *Stack V*)

VI Next X is an operator, pop 2 operands from the stack *i.e.*, 5 and *C* (see *Stack VI*) and evaluate :

$$5 \times C = 5 \times 4 = 20$$

VII Push the result (20) into the stack, (*Stack VII*)

VIII Next '*D*' is operand ; pushed into the stack, (*Stack VIII*)

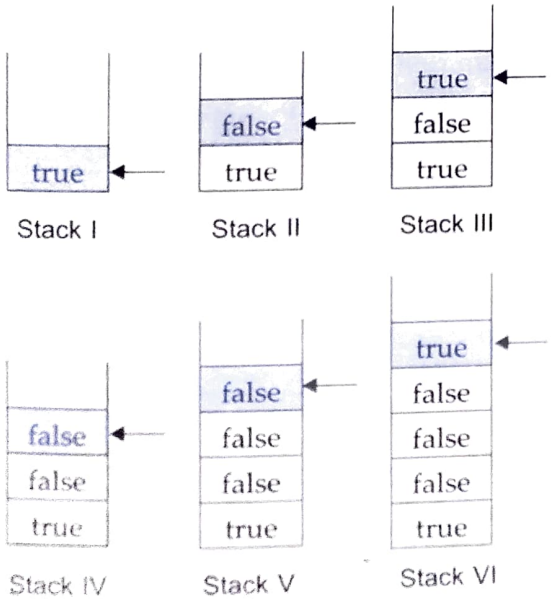
IX Next '/' is operator ; 2 operands popped *i.e.*, *D* and 20 ; (*Stack IX*)

Expression evaluated as follows : $\frac{20}{D} = \frac{20}{5} = 4$, pushed back. (*Stack X*)

X End of the expression, Pop from stack. Thus **the result is 4.**

EXAMPLE 10.9 *Evaluate the following expression in postfix form using a stack and show the contents of the stack after execution of each operation : true false true NOT false true OR NOT AND OR AND.*

Solution. Reading from left to right



I *'true'*, is operand ; pushed into stack (*Stack I*)

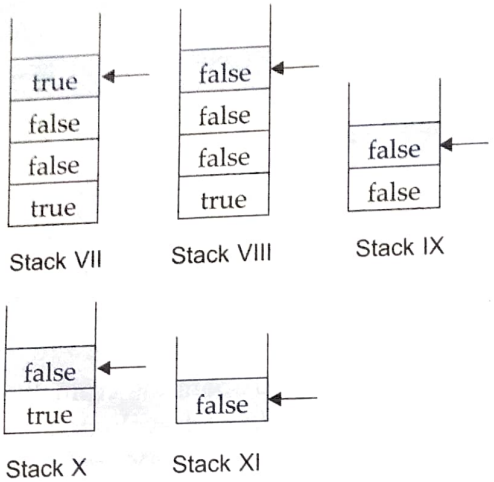
II *'false'* is operand ; pushed into stack (*Stack II*)

III 'true' is operand ; pushed into stack (*Stack III*)

IV 'NOT' is a unary operator. Thus, one operand is popped *i.e.*, *true* ;
Evaluating NOT *true* = *false* ; Pushing the result *'false'* into the stack (*Stack IV*)

V *'false'* is operand ; pushed (*Stack V*)

VI *'true'* is operand, pushed (*Stack VI*)



VII 'OR' is operator, Pop two operands *i.e., true, false.*
Evaluating the expression, we get : *false* OR *true* = *true* ;
Push the result *'true'*, into the stack (*Stack VII*)

VIII 'NOT' is operator ; One operand is popped, *i.e., true.*
Evaluating : NOT *true* = *false*
Push the result *'false'*, into the stack (*Stack VIII*)

IX 'AND' is operator ; Pop two operands, *i.e., false, false.*
Evaluating : *false* AND *false* = *false* ;
Push the result *'false'*, into the stack (*Stack IX*)

X 'OR' is operator ; Pop two operands, *i.e., false, false.*
Evaluating : *false* OR *false* = *false* ; Push the result *'false'* (*Stack X*)

XI 'AND' is operator ; Pop *two* operands *i.e., false, true.*
Evaluate and Push back : *true* AND *false* = *false* (*Stack XI*)

XII No-more-elements Pop from stack, therefore, the **result is 'false'.**

Evaluation of Prefix expression using stacks

The prefix expression is also without parentheses. The prefix expressions are evaluated by the compiler by using *two* stacks :

- one stack (**Symbol Stack**) for holding the *symbols of the expression* (All the *operators* and *operands/values* of the expression are considered ***symbols***) and
- another stack (**Number Stack** or **Operand-Value Stack**) for holding the numbers or values (*i.e.*, the *operands*).

EXAMPLE 10.10 *Evaluate the expression* 5 6 2 + * 12 4 /- *in tabular form showing stack status after every step.*

Solution.

Step	Input Symbol/Element		Stack	Intermediate Calculations Output
1.	5	Push	5	
2.	6	Push	5, 6	
3.	2	Push	5, 6, 2	
4.	+	Pop (2 elements) & evaluate	5	6 + 2 = 8
5.		Push result (8)	5, 8	
6.	*	Pop (2 elements) & evaluate	# empty	5 × 8 = 40
7.		Push result (40)	40	
8.	12	Push	40, 12	
9.	4	Push	40, 12, 4	
10.	/	Pop (2 elements) & evaluate	40	12/4 = 3
11.		Push result (3)	40, 3	
12.	–	Pop (2 elements) & evaluate	#	40 – 3 = 37
13.		Push result (37)	37	
14.		No-more-elements		37 (result)

(top most element is shown in colour)

Check Point 10.1

1. What are the technical names of insertion and deletion in a stack ?
2. What is the situation called, when an insertion/push is attempted in a full stack ?
3. What is the situation called, when a deletion/pop is attempted in an empty stack ?
4. Give some examples of stack applications.
5. What are infix expressions ? What are postfix expressions ?
6. Write equivalent postfix expressions for the following infix expressions :
   (*a*) a + b * d (*b*) p/(q – r)
   (*c*) i**2 + 3
7. Evaluate following postfix expression using a stack : 28, 8, 4, /, +.

WORKING WITH STACKS — Progress In Python 10.1

This PriP session is dedicated to the practice of data structure Stack's concepts and provides practical assignment for the same.

Please check the practical component-book – Progress in Computer Science with Python and fill it there in PriP 10.1 under Chapter 10 after practically doing it on the computer.

>>>❖<<<

10.3 QUEUES

Queues are similar to stacks in that a queue also consists of a sequence of items (a linear list), and there are restrictions about how items can be added to and removed from the list. However, a queue has two ends, called the **front-end** and the **back-end** or **rear-end** of the queue. Items are always added to the queue at the ***rear-end*** and removed from the queue at the ***front-end***. The operations of adding and removing items are called **enqueue** and **dequeue**. An item that is added to the back of the queue will remain on the queue until all the items in front of it have been removed.

QUEUE

A Queue is a linear list implemented in FIFO – First In First Out manner where insertions take place at the rear-end and deletions are restricted to occur only at front end of the queue.

A queue is like a "line" or "queue" of customers waiting for service. Customers are serviced in the order in which they arrive on the queue. Thus a queue is also called a **FIFO list** *i.e.*, **First In First Out** list where the item first inserted is the first to get removed. So, we can say that a queue is a list of data that follows these rules :

1. Data can only be removed from the front end, *i.e.*, the element at the *front-end of the queue*. The removal of element from a queue is technically called **DEQUEUE operation**.
2. A new data element can only be added to the *rear of the queue*. The insertion of element in a stack is technically called **ENQUEUE operation**.

NOTE

The Enqueue operation adds item at the back of the queue (at rear-end) and the Dequeue operation removes the item from the front of the queue (at front-end) and returns it.

Other Queue Terms

There are *some* other terms related to queues, such as *Overflow* and *Underflow*.

Peek Refers to inspecting the value at the ***queue's front*** without removing it. It is also sometimes referred as **inspection**.

Overflow Refers to situation (ERROR) when one tries to enqueue an item in a queue that is full. This situation occurs when the size of the queue is fixed and cannot grow further or there is no memory left to accommodate new item.

Underflow Refers to situation (ERROR) when one tries to Dequeue/delete an item from an empty queue. That is, queue is currently having no item and still one tries to dequeue an item.

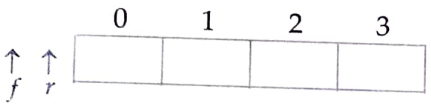
EXAMPLE 10.12 *Given a Bounded Queue of capacity 4 which is initially empty, draw pictures of the queue after each of the following steps :*

(*i*) Queue empty (*ii*) enqueue 'a' (*iii*) enqueue 'b'
(*iv*) enqueue 'c' (*v*) dequeue (*vi*) enqueue 'd'
(*vii*) enqueue 'e' (*viii*) dequeue (*ix*) dequeue
(*x*) dequeue (*xi*) dequeue

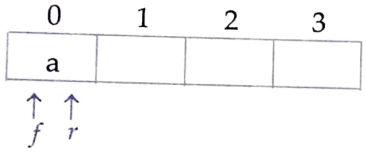
Solution.

Front = $\underset{f}{\uparrow}$; Rear = $\underset{r}{\uparrow}$

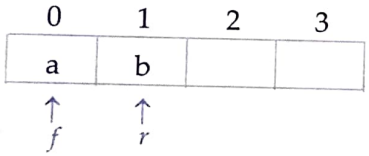
(*i*) Queue is empty **(front = rear = None)**



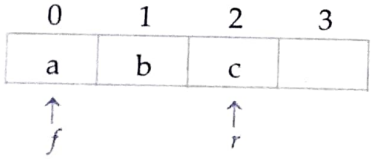
(*ii*) enqueue 'a' **(front = 0, rear = 0)**



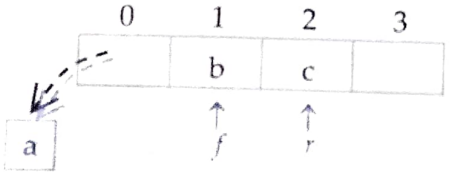
(*iii*) enqueue 'b' **(front = 0, rear = 1)**



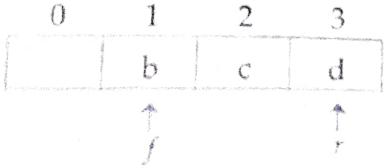
(*iv*) enqueue 'c' **(front = 0, rear = 2)**



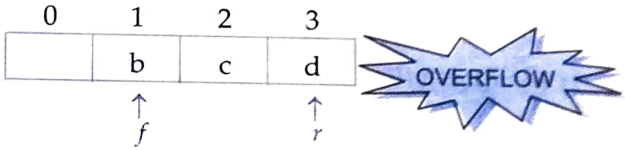
(*v*) dequeue **(front = 1, rear = 2)**



(*vi*) enqueue 'd' **(front = 1, rear = 3)**

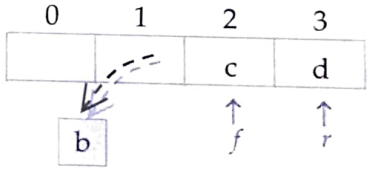


(*vii*) enqueue 'e' **(front = 1, rear = 3)**

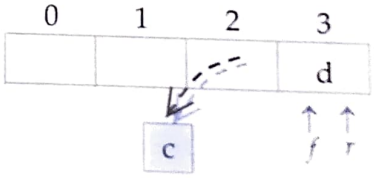


[OVERFLOW because the queue is bounded, it cannot grow. If it could grow, then there would have been no OVERFLOW until no memory is left. In Python, (for queues implemented through lists) since Lists can grow, OVERFLOW condition does not arise until all the memory is exhausted.]

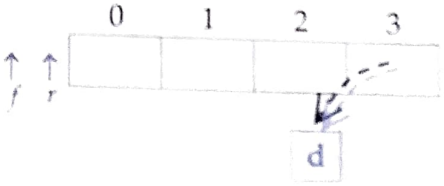
(*viii*) dequeue **(front = 2, rear = 3)**



(*ix*) dequeue **(front = 3, rear = 3)**



(*x*) dequeue **(front = None, rear = None)**



(*xi*) dequeue **(front = None, rear = None)**

10.3.1 Implementing Queues in Python

In Python, you can use ***Lists*** to implement queues. Python offers us a convenient set of methods to operate lists as queues. For various Queue operations, we can use a list say ***Queue*** and use Python code as described below :

Peek We can use : `<Queue>[front]`

where **<Queue>** is a list ; ***front*** is an integer storing the position of first value in the queue.

Enqueue We can use : `<Queue>.append(<item>)`

where **<item>** is the item being pushed in the *Queue*. The item will be added at the rear-end of the queue.

Dequeue We can use : `<Queue>.pop(0)`

it removes the first value from the *Queue* (*i.e.*, the item at the *front-end*) and returns it.

Number of Elements in Queue

We can determine the size of a queue using the formula:

`Number of elements in queue = rear - front + 1`

In Python queues, implemented through lists, the *front* and *rear* are :

`front = 0 and rear = len(<queue>) - 1`

You can also use Python function **len(<queue>)** to get the size of the queue.

Let us now implement a Queue of numbers through a program.

10.2 Program to implement Queue Operations

Program

```
############  queue IMPLEMENTATION  ##############
"""
queue: implemented as a list
front : integer having position of first (frontmost) element in queue
rear : integer having position of last element in queue
"""
def cls():
    print("\n" * 100)

def isEmpty( Qu ) :
    if Qu == [] :
        return True
    else :
        return False

def Enqueue(Qu, item) :
    Qu.append(item)
    if len(Qu) == 1 :
        front = rear = 0
    else :
        rear = len(Qu) - 1
```

```
def Dequeue(Qu) :
    if isEmpty(Qu) :
        return "Underflow"
    else :
        item = Qu.pop(0)
    if len(Qu) == 0 :                #if it was single-element queue
        front = rear = None
    return item

def Peek(Qu) :
    if isEmpty(Qu) :
        return "Underflow"
    else :
        front = 0
    return Qu[front]

def Display(Qu) :
    if isEmpty(Qu) :
        print("Queue Empty!")
    elif len(Qu) == 1:
        print(Qu[0], "<== front, rear")
    else :
        front = 0
        rear = len(Qu) - 1
        print(Qu[front], "<- front")
        for a in range(1, rear ) :
            print(Qu[a])
        print(Qu[rear], "<- rear")

# __main__ program
queue = []                       # initially queue is empty
front = None
while True :
    cls()
    print("QUEUE OPERATIONS")
    print("1. Enqueue")
    print("2. Dequeue")
    print("3. Peek")
    print("4. Display queue")
    print("5. Exit")
    ch = int(input("Enter your choice ( 1-5) : "))
    if ch == 1 :
        item = int(input("Enter item :"))
        Enqueue(queue, item)
        input("Press Enter to continue...")
    elif ch == 2 :
        item = Dequeue(queue)
        if item == "Underflow" :
            print("Underflow! Queue is empty!")
        else :
            print("Dequeue-ed item is", item)
        input("Press Enter to continue...")
```

```
        elif ch == 3 :
            item = Peek(queue)
            if item == "Underflow" :
                print("Queue is empty!")
            else :
                print("Frontmost item is", item)
            input("Press Enter to continue...")
        elif ch == 4 :
            Display(queue)
            input("Press Enter to continue...")
        elif ch == 5 :
            break
        else :
            print("Invalid choice!")
            input("Press Enter to continue...")
```

```
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 1
Enter item :5
Press Enter to continue...
_________________
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 4
5 <== front, rear
Press Enter to continue...
_________________
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 1
Enter item :7
Press Enter to continue...
_________________
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 3
Frontmost item is  5
Press Enter to continue...
_________________
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 1
Enter item :9
Press Enter to continue...
_________________
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 4
5 <- front
7
9 <- rear
Press Enter to continue...
_________
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 2
Dequeue-ed item is  5
Press Enter to continue...
_________________
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 2
Dequeue-ed item is  7
Press Enter to continue...
_________________
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 2
Dequeue-ed item is  9
Press Enter to continue...
_________________
QUEUE OPERATIONS
1. Enqueue
2. Dequeue
3. Peek
4. Display queue
5. Exit
Enter your choice (1-5) : 2
Underflow! Queue is empty!
Press Enter to continue...
```

Queue Item-nodes

Like stacks, queues can also store information of various types in its item-nodes and if information is logically related then item-node is first create as a list (or any other sequence as per your requirements) and then inserted in the queue. Solved problem 21 implements such a queue.

10.3.2 Variations in Queues

Queues can be used in several forms and ways, depending upon the requirements of the program. Two popular variations of queues are **Circular Queues** and **Deque** (Double-Ended queues).

10.3.2A Circular Queues

Circular Queues are the queues implemented in circular form rather than a straight line. These are used in programming languages that allow the use of fixed-size linear structures (such as arrays of C/C++ etc.) as queues. In such queues, after some insertions and deletions, some unutilized space lies in the beginning of the queue. To overcome such problems, circular queues are used that overcome the problem of unutilized spaces in fixed-size linear queues. Figure 10.4 shows a circular queue.

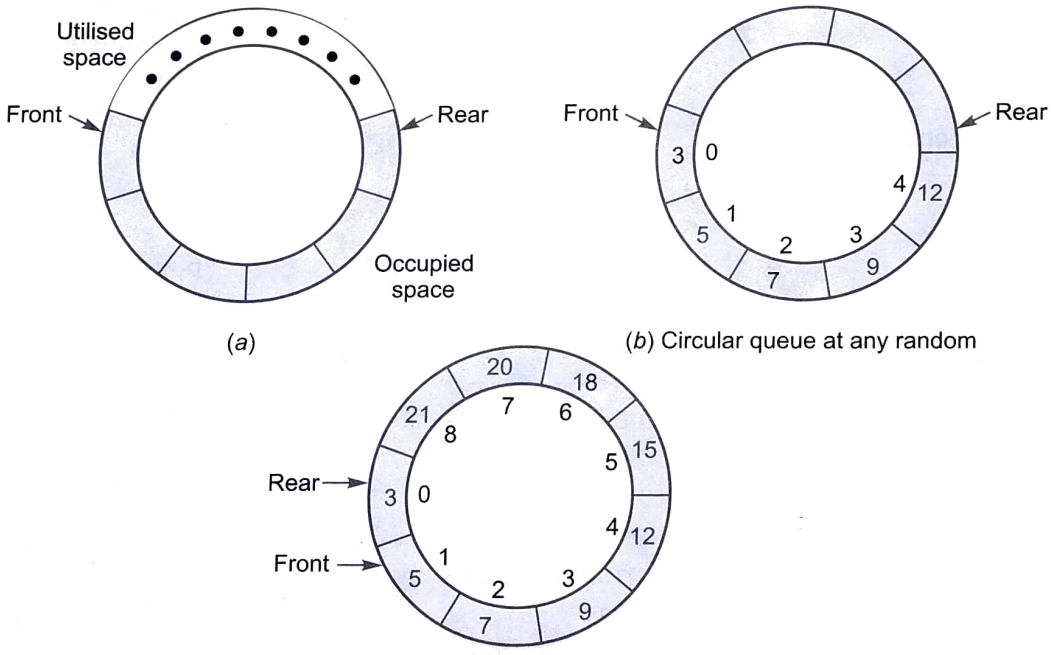


Figure 10.4

Python lists are dynamic structures that can grow and shrink when needed, thus, you don't need circular queues generally when you are implementing queues through Python lists.

10.3.2B Deque (Double-ended Queues)

Deques (double-ended queues) are the refined queues in which elements can be added or removed at either end but not in the middle. There are *two* variations of a deque – an **input restricted deque** and an **output restricted deque**.

- An **Input Restricted Deque** is a deque which allows insertions at only one end but allows deletions at both ends of the list. [see Fig 10.5(*a*)]
- An **Output Restricted Deque** is a deque which allows deletions at only one end of the list but allows insertions at both ends of the list. [see Fig. 10.5(*b*)]

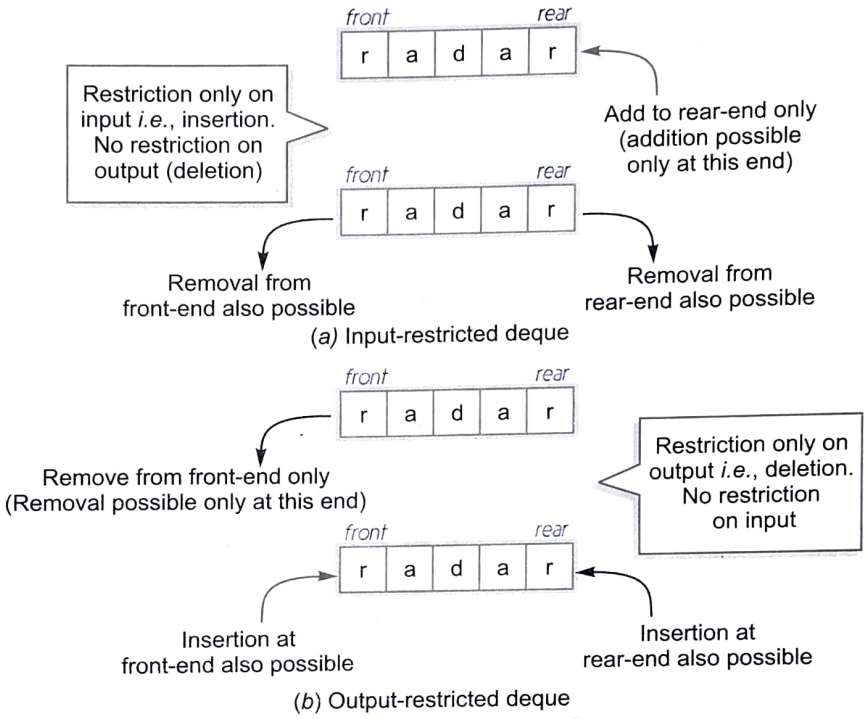


Figure 10.5 Dequeues

10.3.3 Queue Applications

Queue data structure has found application in many diverse areas. We are discussing here only a few of them, for understanding purposes.

Direct Applications

- *Any flow* of ***first-in-first-out*** or ***first-come-first-served*** may be implemented as a queue. For example, the passengers leave a bus as the order (sequence) they hopped on. (*first-in-first-out, last-in-last-out*), or a single-lane vehicles into a tunnel, and come out from the other end – all these are real-life applications of queues.
- When a resource is shared among multiple consumers, queues are the best way to implement this type of sharing. For example, in a computer lab, there can be printer being shared among a number of computers. All the computers can send their printing requests to the shared printer. For all the printing requests, printer will maintain a queue and will print on FIFO basis.
- When you make calls to a call center, the call center phone systems will use a queue to hold people in line until a service representative is free.
- Airport authorities make use of queues in situation of sharing a single runway of airport for both landing and take-off of flights.

Check Point 10.2

1. What are the technical names of insertion and deletion in a queue?
2. What is the situation called, when an insertion/push is attempted in a full queue ?
3. What is the situation called, when a deletion/pop is attempted in an empty queue ?
4. Give some examples of queue applications.
5. What is circular queue ?
6. What is a deque ? What are its types ?

- When you run multiple programs/applications on one CPU, and the CPU has to treat each application equally, then CPU might use queues to process these applications in phased-manner[2]. (Job scheduling)

Indirect Applications

- Queues have found indirect usage in many computer applications as auxiliary data structures and components of other data structures.[3]

LET US REVISE

- *A stack is a linear structure implemented in LIFO (Last In First Out) manner where insertions and deletions take place only at one end i.e., the stack's top.*
- *An insertion in a stack is called pushing and a deletion from a stack is called popping.*
- *When a stack, implemented as an array, is full and no new element can be accommodated, it is called an OVERFLOW.*
- *When a stack is empty and an element's deletion is tried from the stack, it is called an UNDERFLOW.*
- *Polish string refers to the notation in which the operator symbol is placed either before its operands (this form is called prefix notation) or after its operands (this form is called postfix notation). The usual form, in which the operator is placed in between the operands, is called infix notation.*
- *A queue is a linear structure implemented in FIFO (First In First Out) manner where insertions can occur only at the " rear" end and deletions can occur only at the " front" end.*
- *Circular Queues are the queues implemented in circle form rather than a straight line.*
- *Dequeues (double-ended queues) are the refined queues in which elements can be added or removed at either end but not in the middle.*
- *An input restricted deque is a deque which allows insertions at only one end but allows deletions at both ends of the list.*
- *An output restricted deque is a deque which allows deletions at only one end of the list but allows insertions at both ends of the list.*

WORKING WITH QUEUES

PriP — Progress In Python 10.2

This PriP session is dedicated to the practice of data structure Stack's concepts and provides practical assignment for the same.

Please check the practical component-book – Progress in Computer Science with Python and fill it there in PriP 10.2 under Chapter 10 after practically doing it on the computer.

2. This is known as **Round-Robin Scheduling.**
3. Queues also play a very essential role in graph algorithms like *BFS* and *DFS* but these are way ahead of the scope of this book.