A Low-Profile Electrically Small Antenna with a Circular Slot for Global Positioning System Applications

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Abstract—A flexible, planar electrically small antenna (ESA) with omnidirectional radiation pattern is designed and fabricated for GPS and WLAN applications resonating at 1.5 GHz and 3.7 GHz. The design consists of a circular loop attached with 3 rectangular bars, and it is fed by a 50 Ω feed line. The circular loop in the antenna provides impedance matching. Generally, these electrically small antennas have narrow bandwidth. Here the antenna is fabricated on a polyimide substrate having a thickness of 0.1 mm, ε_r of 3.4 mm, and it occupies a size of 38 mm× 34 mm. Electrically small antenna is designed at 1.5 GHz, 3.7 GHz, and the parameters that are measured are S_{11}, VSWR, Ka values, quality factor, and radiation patterns.

1. INTRODUCTION

Researchers are becoming interested in electrically small antennas (ESAs) because of their compact size, ease of integration, and multiband capabilities [1]. Nowadays, there is a growing demand for miniature multiband resonating antennas for wireless systems. Numerous singular-band electrically small antennas have been proposed in the literature, and researchers have proposed a number of electrically small antennas for a range of purposes. Researchers have created a 2.4 GHz antenna for biomedical purposes which is electrically small and has a partial ground plane [2]. Electrically small antenna is described in [3] as an active, superconductive array-based antenna for the 200 MHz band. A small slotted electrically small antenna for multiple-input multiple-output (MIMO) applications is designed using a meander line and a partial ground plane antenna, as illustrated in [4]. For 2.40 GHz bluetooth applications, a trident shaped antenna with an electrically small meandered line is recommended [5]. [6] describes a flexible antenna operating at 2.40 GHz which is electrically compact and based on a thin substrate material. [7, 17–20] suggest a small, circularly polarised WiMAX antenna made of Taconic TLY-5 substrate material with a high gain. As devices get more complicated, multiband and dual band antennas become more necessary. Therefore, a large number of dual-band and triple-band antenna types have been described for the usage in several applications. [8, 31] provide a portable, multi-band electrically small antenna that operates at 0.1 GHz and 0.5 GHz and has a planar inverted F-antenna shape. [9] shows a multi-band circularly polarised 4 short stub loaded patch antenna for wireless communication applications, although due to its size it is not categorised as a tiny antenna. For the usage in ultra-wideband (UWB) applications, a novel antenna with an area of 40×40 mm^2 [10] is proposed. As a conductive adhesive, carbon composite is used. A compact, dual band notched UWB slotted antenna is suggested by [12, 22–25]. A reflector array antenna has been designed at millimetric band, and two antennas were there in which an electrically small antenna has been used as the transmitting antenna [21]. [13, 26–28] suggest an electrically small circular antenna with near-field

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resonance and 0.3 GHz broad-band properties using a non-Foster element. When circular polarisation is implemented using cross-shaped fractal slots, 4 cross-shaped slot structures are used to provide a broad band [14, 29, 30]. Metamaterials, single pole antennas with disc structures, alternating current-fed structures, 3D-shaped structures, and ring-type structures are only a few of the architectures that have been proposed to enhance the performance [15, 16]. [32] describes the fundamental elements of an antenna with antenna size limitations.

An electrically small antenna with dual band characteristics is modelled by using three rectangular microstrip transmission lines with a circular slot attached by a feed line having an impedance of 50 Ω. This circular slotted electrically antenna resonates at 1.5 GHz and 3.7 GHz applicable to GPS and WLAN applications. The dimensions are very small for this circular slotted antenna compared to the other electrically small antennas existing in the literature.

2. GEOMETRY AND DESIGN METHODOLOGY

An electrically small antenna is designed by utilizing a circular patch, and a circular slot in the proposed design will provide better return loss. A rectangular bar is attached to the antenna, and it is tapped by an impedance of Z_0 having 50 Ω. The simulated antenna is modelled on a polyimide substrate material having 0.1 mm thickness with (ε_r) of 3.4 and loss tangent (tan δ) of 0.006. The basic structure of an antenna has a feeding path, and it is tapped by an impedance of 50 Ω. This circular slotted electrically small antenna is simulated by using HFSS V13 which is based on finite element method and method of moments, and the ground plane is partial to provide a positive gain value which is
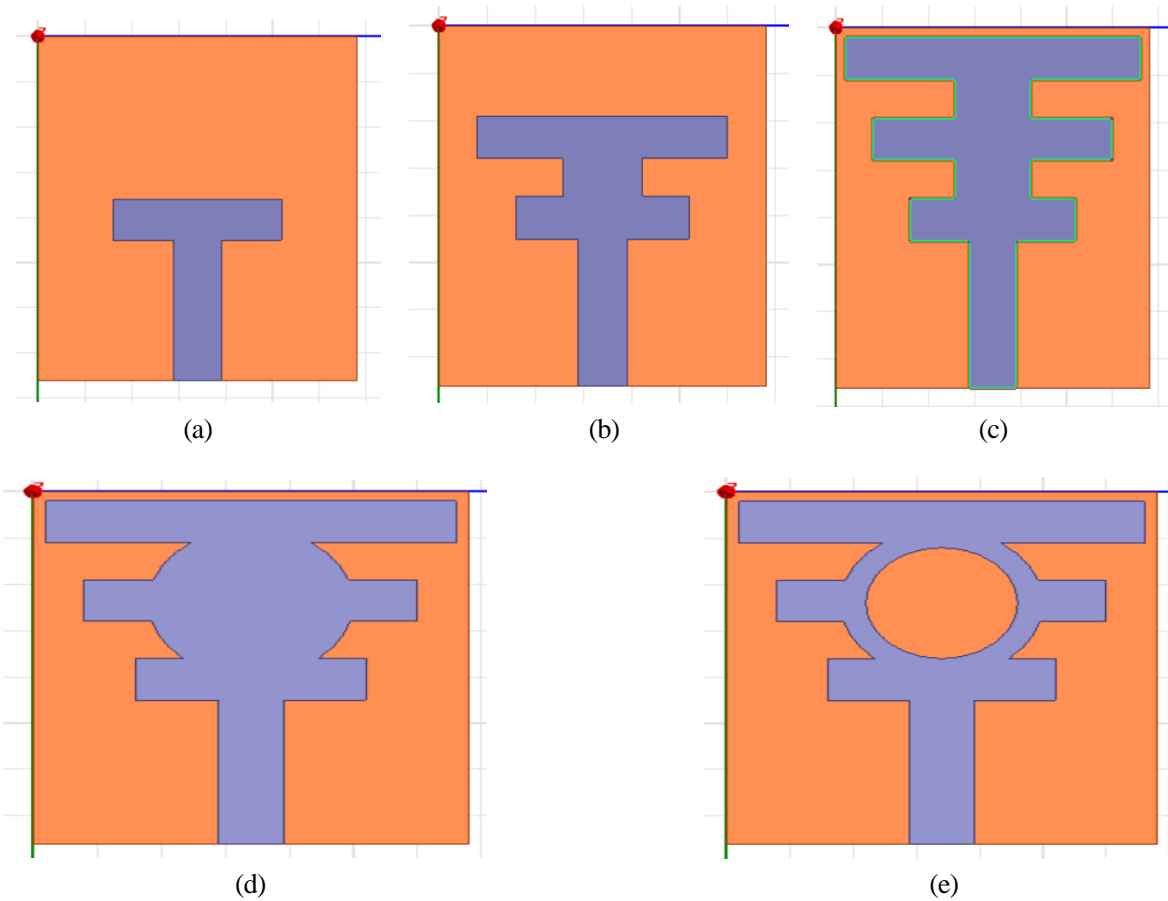

Figure 1. Iterative based design of a circular slotted electrically small antenna. (a) Stage I, (b) Stage II, (c) Stage III, (d) Stage IV, (e) Stage V.
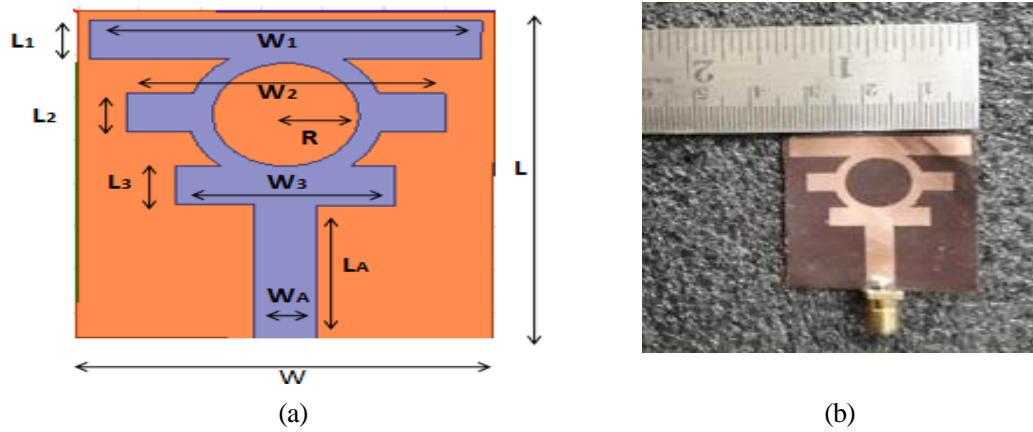
Figure 2. (a) Geometry of circular slotted electrically small antenna. (b) Fabricated prototype.
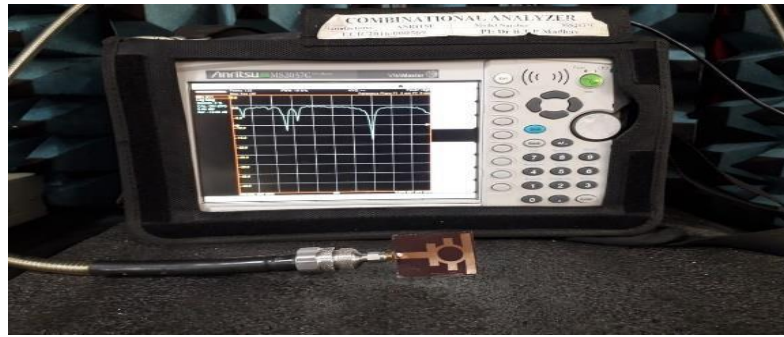

Figure 3. Measurement of circular slotted electrically small antenna with MS2037C combinational analyzer.

under the substrate. This circular slotted antenna resonates at 1.5 GHz, 3.7 GHz and applicable to GPS and Wi-Max applications. The dimensions are very small compared with the other electrically small antennas existing in the literature. Figure 1 represents the stepwise design of an electrically small antenna. At first, a rectangular bar is attached with length and width of 17 mm and 5.05 mm, and it resonates at 1.5 GHz. In iteration 1, two rectangular bars are attached, and this antenna resonates at 1.5 GHz. In iteration 2, three rectangular bars are attached to the feed line, and this antenna resonates at 2.4 GHz and 3.7 GHz with a less amount of return loss values. In iteration 3, a circle of radius 8 mm is attached to the existing antenna structure, and this antenna resonates at 1.5 GHz and 3.7 GHz. The final electrically small antenna is designed by subtracting a circle of radius 6 mm to the existing antenna structure, and the antenna resonates at the above mentioned frequencies with higher return loss values. Figure 2(a) represents the image of a circular slotted electrically small antenna with its dimensions. Figure 2(b) represents the fabricated prototype of the circular slotted electrically small antenna in which the antenna is fabricated on a polyimide material. Figure 3 represents the measurement of a circular slotted electrically small antenna measuring various antenna parameters such as return loss and voltage standing wave ratio (VSWR) by using MS2037C Anrtisu Combinational Analyser. The dimensions utilised in designing the circular slotted antenna are represented in Table 1.

$$W = \frac{V_o}{2F_r} \sqrt{\frac{2}{\varepsilon_r + 1}} \tag{1}$$

$$L = \frac{C}{2F_r} \sqrt{\frac{1}{\varepsilon_{reff}}} - 2\Delta l \tag{2}$$

Table 1. Dimensions of the electrically small antenna with a circular slot.

Parameter	Values (mm)	Parameter	Values (mm)
L	38	L_3	4.5
W	34	W_3	18
L_1	4.5	L_A	17
W_1	32	W_A	5.05
L_2	4.5	T_s	0.1
W_2	26	R	6

$$\Delta l = 0.412h \frac{(\varepsilon_{reff} + 0.03)(w + 0.26h)}{(\varepsilon_{reff} - 0.258)(w + 0.8h)}$$

The dielectric constant of the substrate is shown below

$$\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + \left[\frac{12h}{w}\right]\right]^{-1/2} \tag{3}$$

The length and width of an antenna are calculated as per the formulae shown below

$$L_g = 6h + L \tag{4}$$
$$W_g = 6h + W \tag{5}$$

where L and W are the length and width of the microstrip patch, and h is the thickness of the FR4 substrate.

Radius of the patch is measured by using Equation (6)

$$a = \frac{F}{\sqrt{1 + \frac{2h}{\Pi\varepsilon_r}\left[\ln\left(\frac{\Pi f}{2h}\right) + 1.7726\right]}} \tag{6}$$

$$F = \frac{8.791 * 10^9}{f_r \sqrt{\varepsilon_r}}$$

Three rectangular bars help an antenna function efficiently, while a circular slot inside the antenna creates a capacitance effect. Due to the tiny antenna size, which results in low radiation and high reactance, impedance matching is the ESA's main issue. Capacitance and inductance are provided by this circular slot and three rectangular bars. By altering the size of the circular slot and rectangular bars, the antennas resonant frequencies may be changed. According to that, an electrically small antenna has been designed with a circular slot at the top plane, resonating at 1.5 GHz for WLAN applications and at 3.7 GHz for wireless communication applications like GPS.

3. RESULTS AND DISCUSSIONS

Figure 5 illustrates simulated and manufactured S_{11} for a dual band electrically small antenna, with a circular slit in the middle. Using an Anritsu combinational analyzer MS2037C, the S_{11} of the antenna is measured. It should be recognized that the manufactured and simulated results provide a strong knowledge, and small variance is seen due to manufacturing tolerances. At 1.5 GHz and 3.7 GHz, the circular slotted electrically small antenna offers a return loss of −21 dB and −37 dB, respectively. The circular slotted electrically small antenna offers a bandwidth at 1.25–1.75 GHz and 3.6 GHz–3.75 GHz for return loss less than −10 dB. The ratio of power delivered from the antenna's source to the load, or VSWR, should be typically less than 2. A antenna should generally have a VSWR of less than 2, which can be well defined as the power supplied from the antenna's source to the load. The circular ring's VSWR for an ESA is less than 2 and is equal to 1.2. Figure 11 & Figure 12 show the ESA's
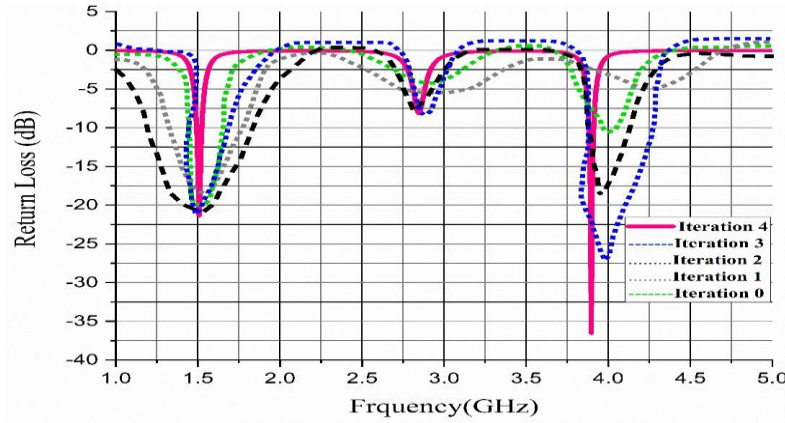
Figure 4. Iterative based return loss of circular slotted Electrically Small Antenna.
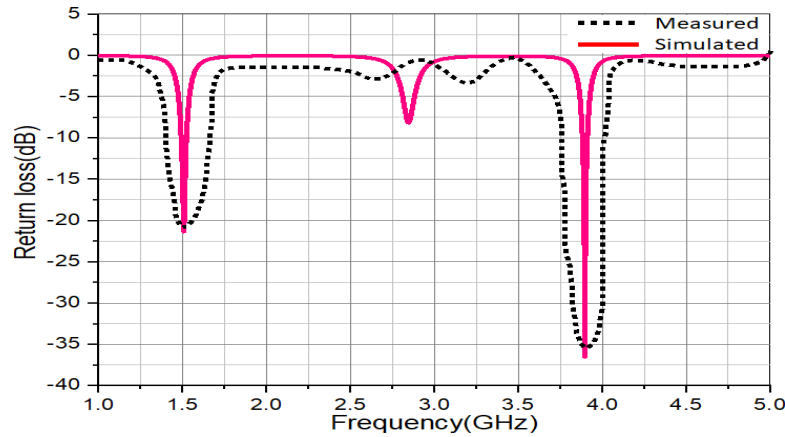

Figure 5. Simulated and measured S_{11} of a circular slotted Electrically Small Antenna.

circular ring radiation pattern. By looking at this pattern, we can readily conclude that the antenna has an omnidirectional radiation pattern for all frequencies between $0°$ and $180°$. This parametric analysis is important because it gives a complete view of the antenna's characteristics and allows for a better understanding of the concept of electrically small antennas' parameters. This analysis has been carried out by using finite element method-based software tool An-soft HFSS. Figure 4 represents the return loss of an electrically small antenna by considering the stepwise analysis. In iteration 0, the electrically small antenna resonates at 1.5 GHz with a return loss of -16 dB. In iteration 1, our antenna resonates at 1.5 GHz with an S_{11} of -18 dB. In iteration 2, our antenna resonates at 1.5 GHz and 3.7 GHz with a return loss of -20 dB and -17 dB, respectively. In iteration 3, ESA resonates at 1.5 GHz and 3.7 GHz with a return loss of -20 dB and -27 dB, respectively. The proposed antenna resonates at 1.5 GHz and 3.7 GHz and offers a return loss of -22 dB and -37 dB, respectively.

The circular slotted electrically small antenna is designed to keep an antenna's bandwidth at $ka = 0.612$ for 1.5 GHz, $ka = 0.75$ for 3.7 GHz. Chu suggested using Eq. (1) to determine the electrically tiny antenna's minimum sphere radius, which is $a = 14.5$ mm [33]. Q_{chu} is 6.0007 at 1.5 GHz and 8.0057 at 3.7 GHz. Chu calculated the smallest radiation quality factor for electrically small antennas under the assumption that the patch antenna should be contained within a closed sphere. Chu developed Eq. (1), which is based on the quality factor [10, 11], to establish the constraint of an electrically tiny antenna.

$$Q = \frac{1}{ka} + \frac{1}{k^3 a^3} \qquad (7)$$

where K is the wave number and varies proportionately to the frequency of proposed antenna. For
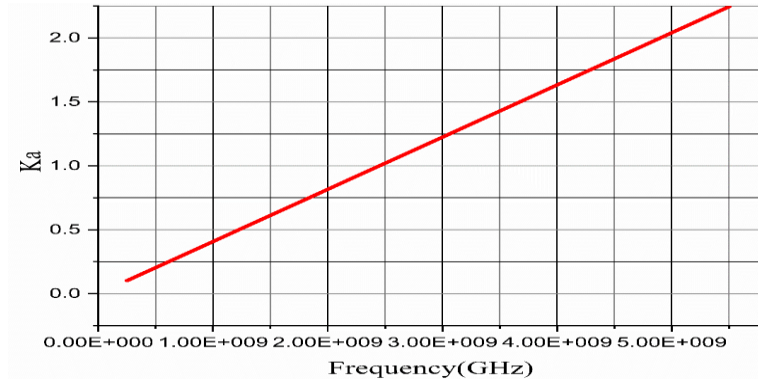
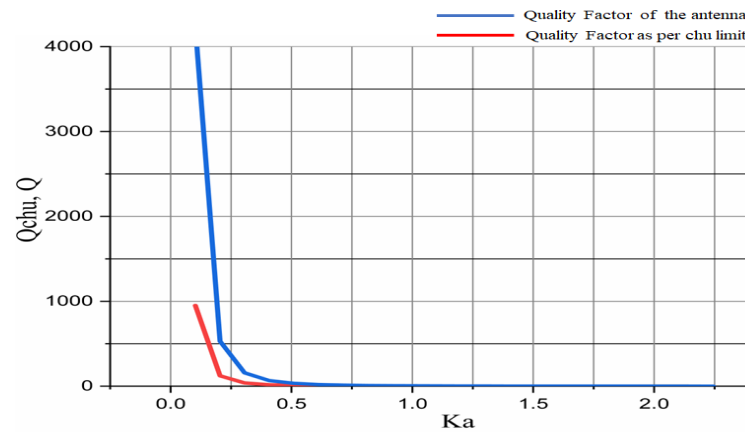**Figure 6.** Variation of Ka with frequency.

Figure 7. Quality factor variation with Ka value.

a small antenna Ka is the most significant parameter and must be less than 1. Figure 6 shows the variation of the Ka with frequency. In small antennas by using miniaturizing techniques, size of the antenna decreases and becomes smaller. Due to reactive input impedance of the small antenna, it exhibits narrow band of operation. Low operating band causes high quality factor (Q), while dealing with small antennas it should require to take care about the quality factor and maintained as low as possible, and at the same time antenna Q must be not less than the Q_{chu}. Figure 7 shows the variation of Q and Q_{chu} with respect to Ka value. At resonant frequency, Ka is 0.46; Q_{chu} is 14.7; and the obtained Q value is 52.4, close to the chu limit. From the figure it is observed that when Ka value increases, Q and Q_{chu} decrease.

Figure 8 shows the variation of Max(G/Q) with the frequency for the proposed ESA. The instance when we move from lower frequency to higher frequency the physical size of the antenna is extremely large compared with the wavelength. Gain of the antenna depends on its physical size, and from the figure it is observed that gain increases with increases in frequency.

Ka is the wave number of the antenna. At high frequencies Ka is large, and for small antenna Ka should be less than 1. For the proposed design, the Ka value is 0.612 for 1.5 GHz and 0.75 for 3.7 GHz. Figure 9 represents the variation of Max(G/Q) to Ka as G/Q value increases with the increase in frequency and is almost zero at small value of Ka.

Figure 10 shows the VSWR variation of the antenna with frequency. The proposed antenna resonates at a frequency of 1.5 GHz and 3.7 GHz. At resonant frequency of 1.5 GHz, simulated VSWR is 0.55, and measured VSWR is 0.7. Similarly at 3.7 GHz, simulated VSWR is 0.77, and measured VSWR is 0.9. The radiation patterns of a circular slotted electrically small antenna are modelled by using HFSS for the resonant frequency ranges of 1.5 GHz and 3.7 GHz, and the results are collected
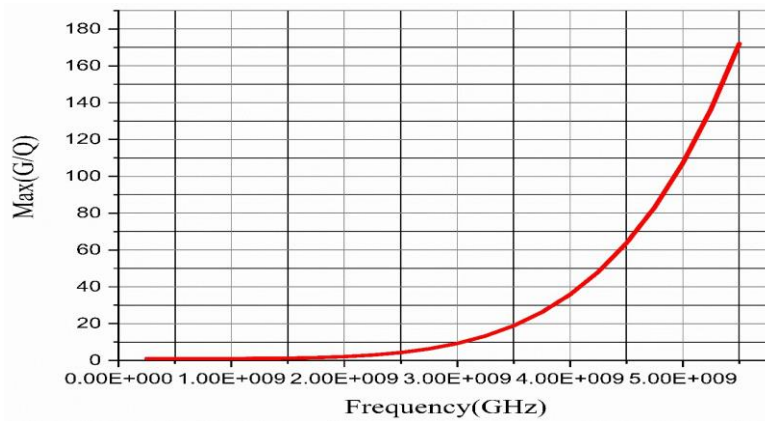
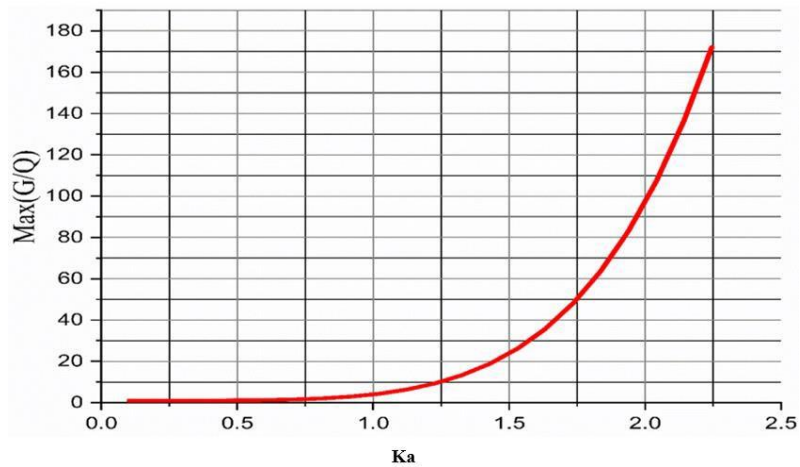**Figure 8.** Max(G/Q) variation with frequency in GHz.

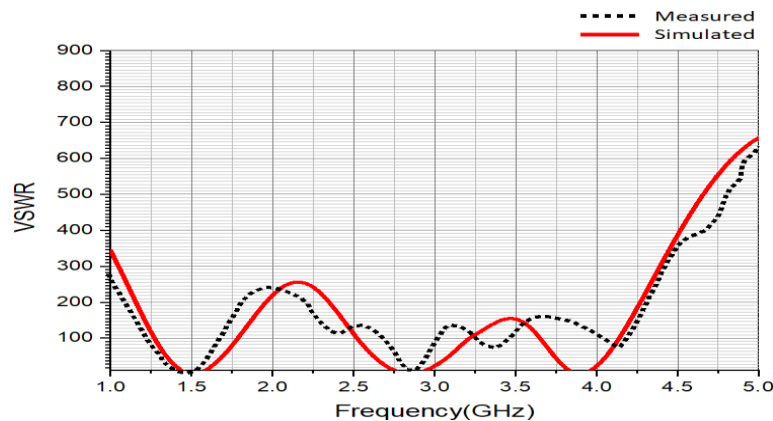**Figure 9.** Variation of Max(G/Q) with Ka value.

Figure 10. Simulated and measured VSWR of circular slotted electrically small antenna.

from an anechoic chamber utilizing antenna measurement equipment. A standard horn antenna is used as the reference antenna.

Antenna efficiency of 0.5 is calculated using the formula by taking the values of incident and radiated power. Radiation pattern is the next important antenna characteristic. Figures 11 and 12
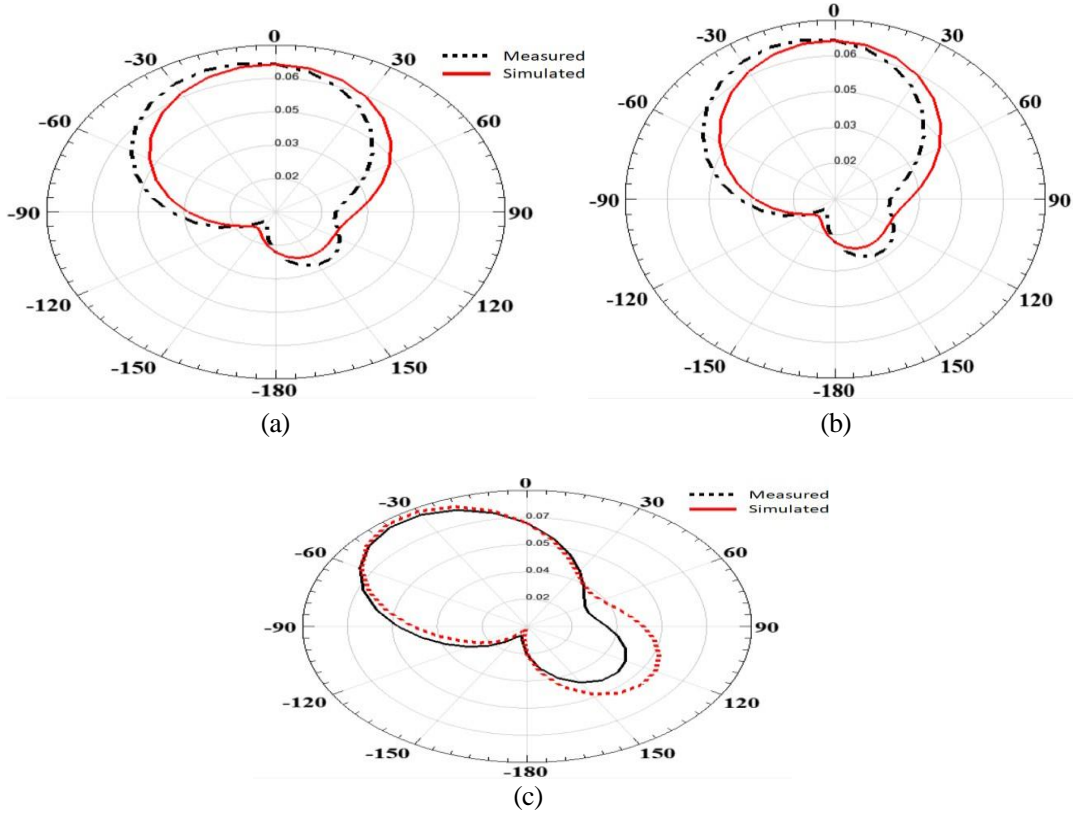
(a)   (b)

(c)

Figure 11. Simulated and measured radiation pattern of electrically small antenna in terms of elevation angle with a circular slot. (a) $\vartheta = 0°$, (b) $\vartheta = 90°$, (c) $\vartheta = 180°$.

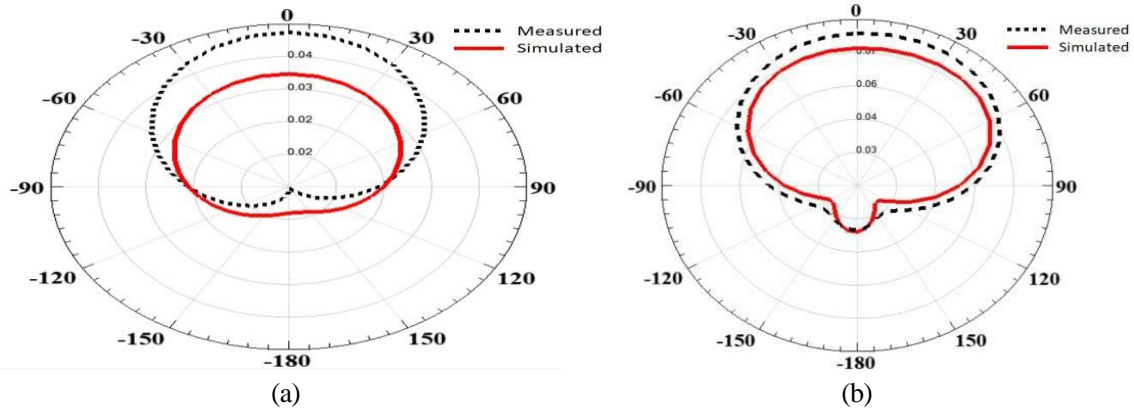(a)   (b)

Figure 12. Simulated and measured azimuthal radiation pattern of electrically small antenna with a circular slot. (a) $\phi = 90°$, (b) $\phi = 180°$.

demonstrate an antenna's gain and the radiation patterns of the antenna, which are plotted at 0, 90, and 180 degrees, respectively. The minor variations between the fabricated and simulated outcomes are caused by measurement and alignment issues. The *E*-field and current distribution of an electrically tiny circular ring antenna are shown in Figure 13. Maximum radiation is indicated by the colour red, minimum radiation indicated by the colour blue, and average distribution of surface currents indicated by the colour green.

Table 2 compares the proposed antenna in this article to various mentioned electrically small
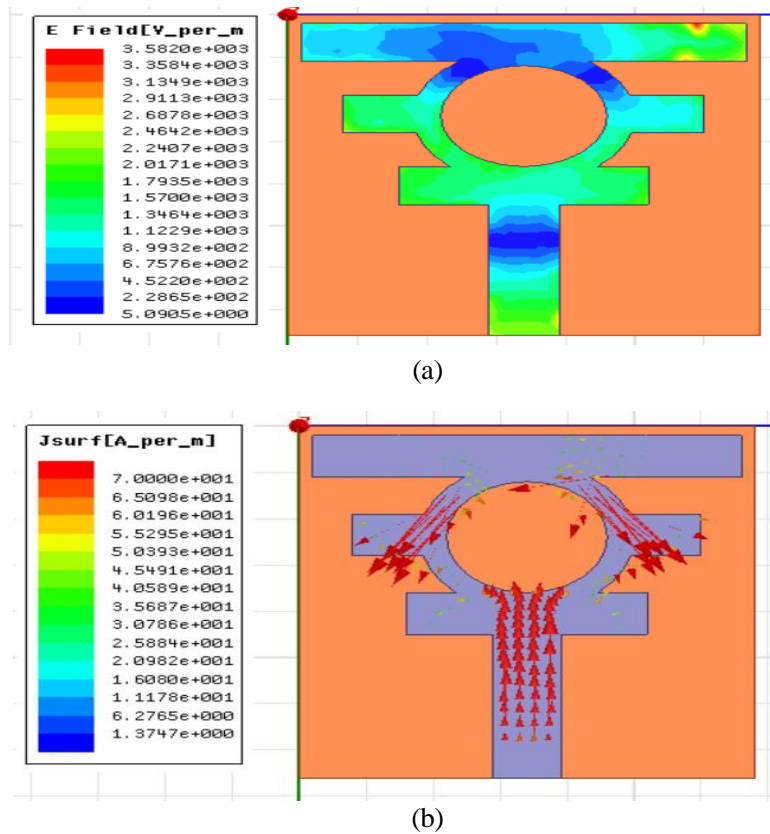
(a)

(b)

Figure 13. *E*-field distributions and current distribution of a circular slotted electrically small antenna.

Table 2. Comparison of the proposed electrically small antenna with the available literary works.

Reference	Antenna Type	Antenna Size (mm²)	VSWR	Return Loss (dB)	Resonating bands
[1]	Dual band	41 × 42	1.21	−18 dB, −22 dB	2.4 GHz, 3.1 GHz
[2]	Dual band	48 × 48	1.425	−22 dB, −19 dB	1.5 GHz, 3.3 GHz
[8]	Dual band	51 × 50	1.356	−17 dB, −19 dB	1.5 GHz, 1.8 GHz
[9]	Dual band	60 × 60	1.78	−15 dB, −19 dB	0.8 GHz, 1.7 GHz
[14]	Dual band	40 × 40	1.69	−12 dB, −15 dB	0.9 GHz, 1.7 GHz
Proposed model	Dual band	38 mm × 34 mm	0.55	−21 dB, −37 dB	1.5 GHz, 3.7 GHz

antennas in terms of results in antenna size, VSWR, and return loss. Furthermore, among the equivalents, the VSWR remains the lowest. When the results are observed, it is evident that the antenna presented in this study occupies less size and has better return loss.

4. CONCLUSION

A dual-band circular slotted electrically small antenna resonating at 1.5 GHz and 3.7 GHz is designed, simulated, and measured by using MS2037C Anritsu Combinational Analyzer. Simulated S_{11} of circular slotted electrically small antenna is– 20 dB at 1.5 GHz and –37 dB at 3.7 GHz. Measured bandwidth of circular slotted electrically small antenna for $S_{11} < $ –10 dB is 1.46–1.67 GHz (bandwidth 0.2 GHz) and 3.72–4.15 GHz (bandwidth 0.43 GHz) of the dual band ESA. There is a good match between the simulated and measured results in terms of VSWR and S_{11}. Radiation pattern is omnidirectional for the two frequencies 1.5 GHz and 3.7 GHz, thus it is suitable for GPS applications and WLAN applications.

REFERENCES

1. Hong, W. B., K.-H. Back, and S. Ko, "Millimeter-wave 5G antennas for smartphones: Overview and experimental demonstration," *IEEE Transactions on Antennas and Propagation*, Vol. 65, No. 12, 6250–6261, December 2017.
2. Maema, H. and T. Fukusako, "Radiation efficiency improvement for electrically small and low-profile antenna by stacked elements," *IEEE Antennas and Wireless Propagation Letters*, Vol. 13, 305–308, January 2014.
3. Tang, M.-C., B. Zhou, and R. W. Ziolkowski, "Low-profile, electrically small, Huygens source antenna with pattern-reconfigurability that covers the entire azimuthal plane," *IEEE Transactions on Antennas and Propagation*, Vol. 65, No. 3, 1063–1072, March 2017.
4. Chlavin, "A new antenna feed having equal *E*- and *H*-plane patterns," *IRE Transactions on Antennas and Propagation*, Vol. 2, No. 3, 113–119, July 1954.
5. Siu, L., H. Wong, and K. M. Luk, "A dual-polarized magneto-electric dipole antenna with dielectric loading," *IEEE Transactions on Antennas and Propagation*, Vol. 57, No. 3, 616–623, March 2009.
6. Wong, H., K. M. Mak, and K. M. Luk, "Wideband shorted bowtie patch antenna with electric dipole," *IEEE Transactions on Antennas and Propagation*, Vol. 56, No. 7, 2098–2101, July 2008.
7. Ge, L. and K. M. Luk, "A low-profile magneto-electric dipole antenna," *IEEE Transactions on Antennas and Propagation*, Vol. 60, No. 4, 1684–1689, January 2012.
8. Ge, L. and K. M. Luk, "linearly polarized and dual-polarized magneto-electric dipole antennas with reconfigurable beamwidth in the *H*-plane," *IEEE Transactions on Antennas and Propagation*, Vol. 64, No. 2, 423–431, February 2016.
9. Ge, L., M. J. Li, Y. J. Li, H. Wong, and K. M. Luk, "Linearly polarized and circularly polarized wideband dipole antennas with reconfigurable beam direction," *IEEE Transactions on Antennas and Propagation*, Vol. 66, No. 4, 1747–1755, April 2018.
10. Wu, F. and K. M. Luk, "Wideband tri-polarization reconfigurable magneto-electric dipole antenna," *IEEE Transactions on Antennas and Propagation*, Vol. 65, No. 4, 1633–1641, April 2017.
11. Tang, M.-C., Z. T. Wu, T. Shi, H. Zeng, W. Lin, and R. W. Ziolkowski, "Dual-linearly-polarized, electrically small, low-profile, broadside radiating, Huygens dipole antenna," *IEEE Transactions on Antennas and Propagation*, Vol. 66, No. 8, 3877–3885, August 2018.
12. Lin, W. and R. W. Ziolkowski, "Electrically small, low-profile, Huygens circularly polarized antenna," *IEEE Transactions on Antennas and Propagation*, Vol. 66, No. 2, 636–643, February 2018.
13. Ouyang, J., Y. M. Pan, and S. Y. Zheng, "Center-fed unilateral and pattern reconfigurable planar antennas with slotted ground plane," *IEEE Transactions on Antennas and Propagation*, Vol. 66, No. 10, 5139–5149, October 2018.
14. Trong, N. N., A. Piotrowski, and C. Fumeaux, "A frequency-reconfigurable dual-band low-profile monopolar antenna," *IEEE Transactions on Antennas and Propagation*, Vol. 65, No. 7, 3336–3343, July 2017.
15. Ge, L. and K. M. Luk, "Frequency-reconfigurable low-profile circular monopolar patch antenna," *IEEE Transactions on Antennas and Propagation*, Vol. 62, No. 7, 3443–3449, July 2014.

16. Mansoul, F. Ghanem, M. R. Hamid, and M. Trabelsi, "A selective frequency-reconfigurable antenna for cognitive radio applications," *IEEE Antennas and Wireless Propagation Letters*, Vol. 13, 515–518, February 2014.

17. Young, M. W., S. Yong, and J. T. Bernhard, "A miniaturized frequency reconfigurable antenna with single bias, dual varactor tuning," *IEEE Transactions on Antennas and Propagation*, Vol. 63, No. 3, 946–951, March 2015.

18. Trong, N. N. and C. Fumeaux, "Tuning range and efficiency optimization of a frequency-reconfigurable patch antenna," *IEEE Antennas and Wireless Propagation Letters*, Vol. 17, No. 1, 150–154, January 2018.

19. Petosa, A., "An overview of tuning techniques for frequency-agile antennas," *IEEE Antennas and Propagation Magazine*, Vol. 54, No. 5, 271–296, October 2012.

20. Imamdi, G., M. Venkata Narayana, A. Navya, A. Venkatesh, S. Charles Spurjeon, S. Sai Venkat, and S. Sanjay, "Design of high directional crossed dipole antenna with metallic sheets for UHF and VHF applications," *International Journal of Engineering & Technology*, Vol. 7, Nos. 1–5, 42–50, February 2018.

21. Imamdi, G., M. Venkata Narayan, A. Navya, and A. Roja, "Reflector array antenna design at millimetric (mm) band for on the move applications," *ARPN Journal of Engineering and Applied Sciences*, Vol. 13, No. 1, 352–359, January 2018.

22. Čvorović, R., L. Lazović, V. Rubežić, and A. Jovanović, "Printed asymmetrical Sierpinski slot antenna for energy harvesting application," *2020 24th International Conference on Information Technology (IT)*, 1–4, IEEE, February 2020.

23. Tang, M. C., C. Li, and Y. Chen, "Electrically small, planar, wideband filtering antenna with dual ring patches," *Electronics Letters*, Vol. 56, No. 2, 61–62, 2020.

24. Guan, D. F., Q. Zhang, P. You, Z. B. Yang, Y. Zhou, and S. W. Yong, "Scanning rate enhancement of leaky-wave antennas using slow-wave substrate integrated waveguide structure," *IEEE Transactions on Antennas and Propagation*, Vol. 66, No. 7, 3747–3751, 2018.

25. Shi, D., T. Aftab, G. Gidion, F. Sayed, and L. M. Reindl, "A novel electrically small ground-penetrating radar patch antenna with a parasitic ring for respiration detection," *Sensors*, Vol. 21, No. 6, 1930, 2021.

26. Ameen, M., A. Mishra, and R. K. Chaudhary, "Asymmetric CPW-fed electrically small metamaterial-inspired wideband antenna for 3.3/3.5/5.5 GHz WiMAX and 5.2/5.8 GHz WLAN applications," *AEU-International Journal of Electronics and Communications*, Vol. 119, 153177, 2020.

27. Li, S., S. Song, M. Cen, and L. Yu, "Design of a high selectivity fractal antenna with triple band-notched characteristics," *2020 International Conference on Microwave and Millimeter Wave Technology (ICMMT)*, 1–3, IEEE, September 2020.

28. Imamdi, G., M. V. Narayana, A. Navya, A. Venkatesh, S. C. Spurjeon, S. S. Venkat, and S. Sanjay, "Design of high directional crossed dipole antenna with metallic sheets for UHF and VHF applications," *International Journal of Engineering and Technology (UAE)*, Vol. 7, Nos. 1–5, 42–50, 2018.

29. Imamdi, G., M. Venkata Narayan, A. Navya, and A. Roja, "Reflector array antenna design at millimetric (mm) band for on the move applications," *ARPN Journal of Engineering and Applied Sciences*, Vol. 13, No. 1, 352–359, 2018.

30. Immadi, G., M. Venkata Narayana, A. Navya, C. Anudeep Varma, A. Abhishek Reddy, A. Manisai Deepika, and K. Kavya, "Analysis of substrate integrated frequency selective surface antenna for IOT applications," *Indonesian Journal of Electrical Engineering and Computer Science*, Vol. 18, No. 2, 875–881, 2020.

31. Venkata Narayana, M., G. Immadi, A. Navya, D. Anirudh, K. Naveen, and M. Sriram, "A non-foster elemental triangular shaped patch antenna for FM applications," *Journal of Critical Reviews*, Vol. 7, No. 13, 468–470, 2020.

32. Davis, W. A., T. Yang, E. D. Caswell, and W. Stutzman, "Fundamental limits on antenna size: A new limit," *IET Microwaves, Antennas* & *Propagation*, Vol. 5, 1297–1302, 2011, 10.1049/iet-map.2010.0604.

33. Xu, X. and Y. E. Wang, "Chu's limit and switched electrically small antennas," *2007 International workshop on Antenna Technology: Small and Smart Antennas Metamaterials and Applications*, 463–466, Cambridge, UK, 2007, doi:10.1109/IWAT.2007.370174.