Analisi Matematica 1-Informatica-UniNa
Foglio 1

ESERCIZIO 1: Risolvere le seguenti disequazioni algebriche su $\mathbb{R}$:

1. $(x-1)(x^3-x^2-2x)>0$;
2. $\dfrac{x^2-4}{x-1}\leq 0$;
3. $2^{2x+1}<4^{x^2}$;
4. $\left(\dfrac{1}{2}\right)^{2x+1}<\left(\dfrac{1}{2}\right)^{\frac{x^2}{x-1}}$;
5. $\log_{\frac{1}{2}}\left(\dfrac{x^2-1}{x}\right)>0$;
6. $2\log(3x)<\log(3x+2)$;
7. $\sqrt{x^2+4-4x}>x-3$;
8. $\sqrt{x-1}+\sqrt{x+1}>\sqrt{3x}$;
9. $\sqrt[3]{x^2+x}<x$;
10. $\sqrt{x+4}<x+3$;
11. $3^{-2x}-4\cdot 3^{-x}+5\leq 0$;
12. $|x^2+5x+3|>3$;

ESERCIZIO 2: Determinare e disegnare i domini delle seguenti funzioni:

1. $f(x)=\arccos\left(\dfrac{x^2-2}{x}\right)$;
2. $f(x)=\log(\sin(x))$;
3. $f(x)=\sqrt[4]{\dfrac{|x|x-1}{x}}$;
4. $f(x)=\sqrt{\log_2(x)-2}+\dfrac{1}{|x-7|}$;
5. $f(x)=\tan\left(x+\dfrac{\pi}{2}\right)$;
6. $f(x)=\sqrt{\log\left(1+\dfrac{1}{x}\right)}$;
7. $f(x)=\sqrt{\dfrac{x-2}{2x+1}}+\log(\sqrt{x}-1)$;

ESERCIZIO 3: Calcolare sup e inf e, se esistono, max e min dei seguenti insiemi:

$$A=\left\{\frac{1}{n}:n\in\mathbb{N}\right\};$$
$$B=\left\{\frac{2n}{n^2+1}:n\in\mathbb{N}\right\};$$
$$C=\left\{\frac{n^2+1}{n}:n\in\mathbb{N}\right\};$$
$$D=\left\{2^{-n}:n\in\mathbb{N}\right\};$$
$$E=\left\{\frac{n-3}{n^2}:n\in\mathbb{N}\right\}\cup(0,2);$$
$$F=\left\{\frac{n-1}{n+1}:n\in\mathbb{N}\right\};$$
$$G=\left\{\frac{(-1)^n n}{n^2+1}:n\in\mathbb{N}\right\};$$
$$H=\left\{2\log(n)-\frac{1}{4}\log(n^2):n\in\mathbb{N}\right\};$$
$$I=\left\{-\frac{n^2}{n+1}:n\in\mathbb{N}\right\};$$
$$L=\left\{\frac{n^2+(-1)^n}{n}:n\in\mathbb{N}\right\};$$

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\begin{aligned}
M &= \left\{ \frac{(-1)^n + 1}{n} : n \in \mathbb{N} \right\}; \\
N &= \left\{ \frac{n! + 1}{(n+1)!} : n \in \mathbb{N} \right\}; \\
O &= \left\{ \sqrt{n+1} - \sqrt{n} : n \in \mathbb{N} \right\}; \\
P &= \left\{ x \in \mathbb{R} : |2x^2 - 1| < \frac{1}{|x|} \right\}; \\
Q &= \left\{ x \in \mathbb{R} : \log^3_{\frac{1}{2}}(x) - \log_{\frac{1}{2}}(x) > 0 \right\}; \\
R &= \left\{ x \in \mathbb{R} : x^2 - 4x - 3 \geq 0 \wedge x \leq 4 \right\}; \\
S &= \left\{ x \in \mathbb{R} : \sqrt{|x-1|} < x \right\}; \\
T &= \left\{ x \in \mathbb{R} : \frac{1}{|x-2|} \leq \frac{1}{|x-3|} \right\}.
\end{aligned}
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