ARCHITETTURA DEGLI ELABORATORI

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Università di Napoli Federico II
Corso di Laurea in Informatica

Docenti
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ALGEBRA DI BOOLE E RETI COMBINATORIE

Some Definitions

- **Complement**: variable with a bar over it

  $\bar{A}, \bar{B}, \bar{C}$
- **Literal**: variable or its complement

  $A, \bar{A}, B, \bar{B}, C, \bar{C}$
- **Implicant**: product of literals

  $AB\bar{C}, \bar{A}C, BC$
- **Minterm**: product that includes all input variables

  $AB\bar{C}, A\bar{B}\bar{C}, ABC$
- **Maxterm**: sum that includes all input variables

  $(A+\bar{B}+C), (\bar{A}+B+\bar{C}), (\bar{A}+\bar{B}+C)$

La forma SOP

- I mintermini sono enumerati rigo dopo rigo a partire da 0,1 e così via. Quindi ogni mintermine è denotato dal numero binario della configurazione di input corrispondente.

A	B	Y	minterm	minterm name
0	0	0	$\overline{A}\ \overline{B}$	m_0
0	1	1	$\overline{A}\ B$	m_1
1	0	0	$A\ \overline{B}$	m_2
1	1	0	$A\ B$	m_3

La forma SOP

- Quindi ad ogni tabella di verità corrisponde una espressione booleana ottenuta sommando tutti i mintermini per cui il valore dell'output Y è pari a 1

A	B	Y	minterm	minterm name
0	0	0	$\overline{A}\ \overline{B}$	m_0
0	1	1	$\overline{A}\ B$	m_1
1	0	0	$A\ \overline{B}$	m_2
1	1	1	$A\ B$	m_3

Y= $\bar{A}$B + AB

Y= $\Sigma(1,3)$

La forma POS

- Anche i maxtermini sono enumerati come i mintermini.

A	B	Y	maxterm	maxterm name
0	0	0	$A + B$	M_0
0	1	1	$A + \overline{B}$	M_1
1	0	0	$\overline{A} + B$	M_2
1	1	1	$\overline{A} + \overline{B}$	M_3

- La forma normale POS di una funzione booleana si ottiene come prodotto dei maxtermini per cui la funzione ritorna 0
  - Y=$(A + B) \cdot (\bar{A} + \mathrm{B})$
  - Y=$\prod(0,2)$

Algebra di Boole

- Come abbiamo visto la medesima funzione può essere descritta da espressioni booleane distinte
- Alcune di queste possono essere più semplici di altre

A	B	Y	minterm	minterm name
0	0	0	$\bar{A}\ \bar{B}$	m_0
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1	0	0	$A\ \bar{B}$	m_2
1	1	1	$A\ B$	m_3

Y= $\bar{A}B + AB$
Y= $(A + B) \cdot (\bar{A} + B)$
Y= B

- Come si fa con l'aritmetica, possiamo utilizzare un algebra per semplificare le espressioni

$$\frac{1}{x}\ (x + xy) \Longrightarrow \frac{x}{x}\ (1 + y) \Longrightarrow (1 + y)$$

Boolean Axioms

Number	Axiom	Name
A1	$B = 0$ if $B \neq 1$	Binary Field
A2	$\overline{0} = 1$	NOT
A3	$0 \bullet 0 = 0$	AND/OR
A4	$1 \bullet 1 = 1$	AND/OR
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	AND/OR

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Dual: Replace: $\bullet$ with $+$
0 with 1

Boolean Axioms

Number	Axiom	Dual	Name
A1	$B = 0$ if $B \neq 1$	$B = 1$ if $B \neq 0$	Binary Field
A2	$\overline{0} = 1$	$\overline{1} = 0$	NOT
A3	$0 \bullet 0 = 0$	$1 + 1 = 1$	AND/OR
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- A1 e A1’ ci dicono che il valore di una variabile booleana può essere 0 oppure 1

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- A1 e A1’ ci dicono che il valore di una variabile booleana può essere 0 oppure 1
- A2 e A2’ definiscono l’operatore NOT (di fatto questi assiomi ripropongono la tabella di verità dell’operatore

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A1	B = 0 if B ≠ 1	B = 1 if B ≠ 0	Binary Field
A2	$\overline{0} = 1$	$\overline{1} = 0$	NOT
A3	0 • 0 = 0	1 + 1 = 1	AND/OR
A4	1 • 1 = 1	0 + 0 = 0	AND/OR
A5	0 • 1 = 1 • 0 = 0	1 + 0 = 0 + 1 = 1	AND/OR

- A1 e A1’ ci dicono che il valore di una variabile booleana può essere 0 oppure 1
- A2 e A2’ definiscono l’operatore NOT (di fatto questi assiomi ripropongono la tabella di verità dell’operatore

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A5	$0 \bullet 1 = 1 \bullet 0 = 0$	$1 + 0 = 0 + 1 = 1$	AND/OR

- A3, A4 e A5 definiscono l'operatore AND
- A3', A4' e A5' definiscono l'operatore OR
- Notate che ogni assioma «primato» si ottiene dal corrispondente non primato invertendo, da un lato, OR e AND e dall'altro gli 0 e 1. Questo è un principio generale detto principio di dualità.

Boolean Theorems of One Variable

Number	Theorem	Name
T1	$B \bullet 1 = B$	Identity
T2	$B \bullet 0 = 0$	Null Element
T3	$B \bullet B = B$	Idempotency
T4	$\overline{\overline{B}} = B$	Involution
T5	$B \bullet \overline{B} = 0$	Complements

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T3	$B \bullet B = B$	$B + B = B$	Idempotency
T4	$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	$B + \overline{B} = 1$	Complements

Dual: Replace: • with +
0 with 1

Teoremi ad una variabile

	Theorem		Dual	Name
T1	$B \bullet 1 = B$	T1′	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2′	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3′	$B + B = B$	Idempotency
T4		$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	T5′	$B + \overline{B} = 1$	Complements

Teorema B·1=B
Dimostrazione:
- Supponiamo B=1, allora B·1=1·1=1=B
- Supponiamo B=0, allora B·1=0·1=0=B

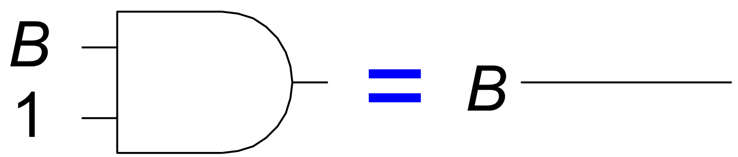
Teorema $B + \bar{B}$=1
Dimostrazione:
- Supponiamo B=1, allora $B + \bar{B}$=1+0=1
- Supponiamo B=0, allora $B + \bar{B}$=0+1=1

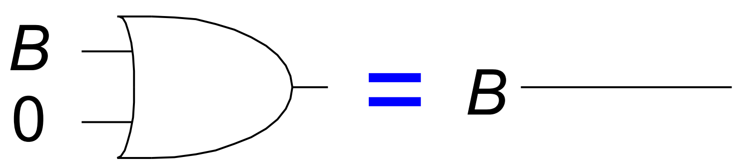
T1: Identity Theorem

- $B \bullet 1 = B$
- $B + 0 = B$

T1: Identity Theorem

- $B \cdot 1 = B$
- $B + 0 = B$



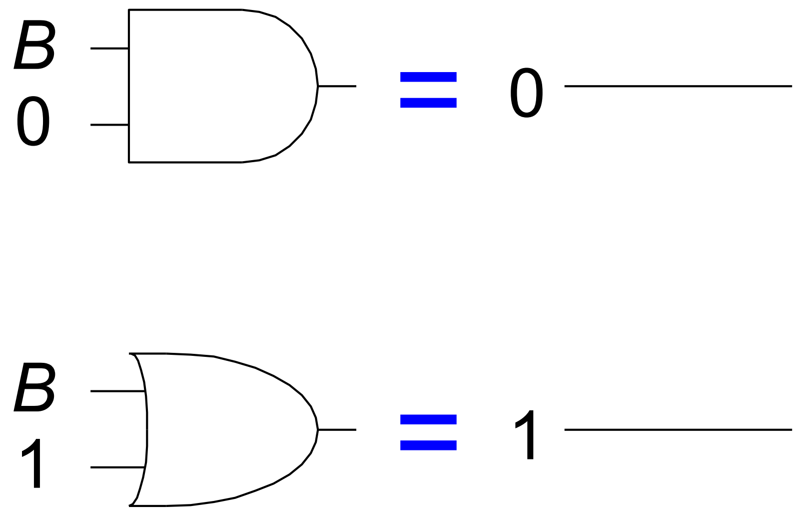


T2: Null Element Theorem

- $B \cdot 0 = 0$
- $B + 1 = 1$

T2: Null Element Theorem

- $B \bullet 0 = 0$
- $B + 1 = 1$

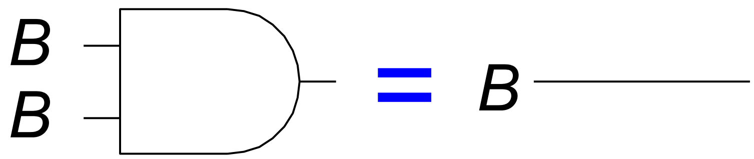


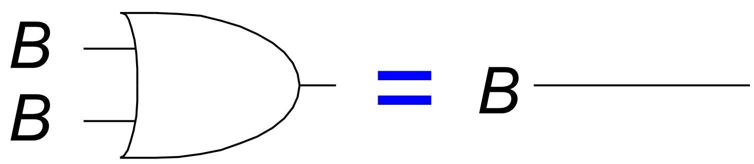
T3: Idempotency Theorem

- $B \bullet B = B$
- $B + B = B$

T3: Idempotency Theorem

- $B \cdot B = B$
- $B + B = B$



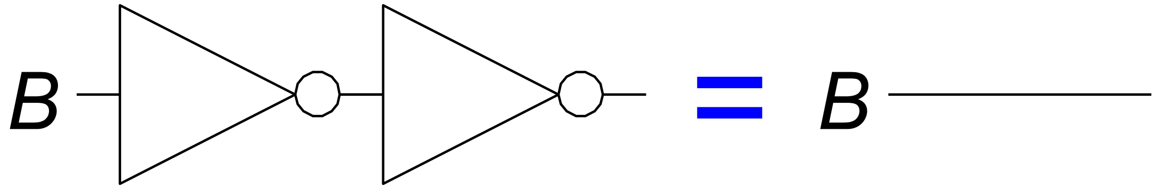


T4: Identity Theorem

- $\overline{\overline{B}} = B$

T4: Identity Theorem

- $\overline{\overline{B}} = B$

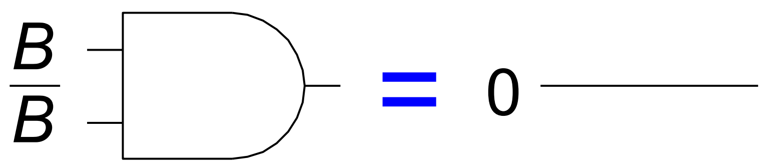


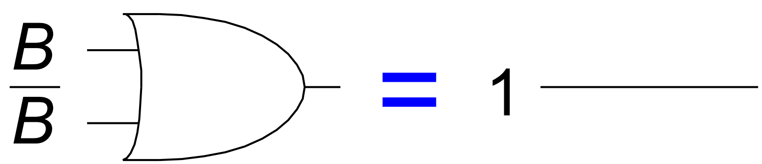
T5: Complement Theorem

- $B \bullet \overline{B} = 0$
- $B + \overline{B} = 1$

T5: Complement Theorem

- $B \bullet \overline{B} = 0$
- $B + \overline{B} = 1$





Recap: Basic Boolean Theorems

Number	Theorem	Dual	Name
T1	$B \bullet 1 = B$	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	$B + B = B$	Idempotency
T4	$\bar{\bar{B}} = B$		Involution
T5	$B \bullet \bar{B} = 0$	$B + \bar{B} = 1$	Complements

Dual: Replace: $\bullet$ with $+$
0 with 1

Boolean Theorems of Several Vars

Number	Theorem	Name
T6	$B \bullet C = C \bullet B$	Commutatività
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	Associatività
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	Distributività
T9	$B \bullet (B+C) = B$	Assorbimento
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	Combinazione
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	Consenso

Teoremi più variabili: dualità

#	Theorem	Dual	Name
T6	$B \bullet C = C \bullet B$	$B+C = C+B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	$(B + C) + D = B + (C + D)$	Associativity
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	$B + (C \bullet D) = (B+C)\ (B+D)$	Distributivity
T9	$B \bullet (B+C) = B$	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	$(B+C) \bullet (B+\overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	$(B+C) \bullet (\overline{B}+D) \bullet (C+D) =$ $(B+C) \bullet (\overline{B}+D)$	Consensus

Principio di dualità: $+ \leftrightarrow \cdot \quad 1 \leftrightarrow 0$

Boolean Theorems of Several Vars

Number	Theorem	Name
T6	$B \bullet C = C \bullet B$	Commutatività
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	Associatività
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	Distributività
T9	$B \bullet (B+C) = B$	Assorbimento
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	Combinazione
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) = (B \bullet C) + (\overline{B} \bullet D)$	Consenso

How do we prove these are true?

Tecniche di dimostrazione

- Vi sono diversi metodi per dimostrare l’equivalenza di due espressioni.
  - Perfect induction: se le tabelle di verità di due espressioni coincidono allora le due espressioni sono equivalenti
  - usare assiomi e teoremi precedentemente provati per manipolare le espressioni fino ad ottenere espressioni uguali

Proof by Perfect Induction

- Also called: **proof by exhaustion**
- Check every possible input value
- If two expressions produce the same value for every possible input combination, the expressions are equal

Example: Proof by Perfect Induction

Number	Theorem	Name
T6	B•C = C•B	Commutativity

B	*C*	*BC*	*CB*
0	0		
0	1		
1	0		
1	1		

Example: Proof by Perfect Induction

Number	Theorem	Name
T6	B•C = C•B	Commutativity

B	*C*	*BC*	*CB*
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

T7: Associativity

Number	Theorem	Name
T7	(B•C) • D = B • (C • D)	Associativity

T8: Distributivity

Number	Theorem	Name
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	Distributivity

T9: Covering

Number	Theorem	Name
T9	$B \bullet (B+C) = B$	Assorbimento

Prove true by:

- **Method 1:** Perfect induction
- **Method 2:** Using other theorems and axioms

T9: Covering

Number	Theorem	Name
T9	$B \bullet (B+C) = B$	Covering

Method 1: Perfect Induction

B	*C*	*(B+C)*	*B(B+C)*
0	0		
0	1		
1	0		
1	1		

T9: Covering

Number	Theorem	Name
T9	B• (B+C) = B	Covering

Method 1: Perfect Induction

B	*C*	*(B+C)*	*B(B+C)*
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

T9: Covering

Number	Theorem	Name
T9	$B \bullet (B+C) = B$	Covering

Method 1: Perfect Induction

B	C	$(B+C)$	$B(B+C)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

T9: Covering

Number	Theorem	Name
T9	B• (B+C) = B	Covering

Method 2: Prove true using other axioms and theorems.

T9: Covering

Number	Theorem	Name
T9	B• (B+C) = B	Assorbimento

Method 2: Prove true using other axioms and theorems.

$B \bullet (B+C)$	$= B \bullet B + B \bullet C$	T8: Distributivity
	$= \mathbf{B} + B \bullet C$	T3: Idempotency
	$= B \bullet (1 + C)$	T8: Distributivity
	$= B \bullet (\mathbf{1})$	T2: Null element
	$= B$	T1: Identity

T10: Combining

Number	Theorem	Name
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	Combinazione

Prove true using other axioms and theorems:

T10: Combining

Number	Theorem	Name
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	Combining

Prove true using other axioms and theorems:

$$B \bullet C + B \bullet \overline{C} = B \bullet (C + \overline{C}) \quad \text{T8: Distributivity}$$

$$= B \bullet (\mathbf{1}) \quad \text{T5': Complements}$$

$$= B \quad \text{T1: Identity}$$

T11: Consensus

Number	Theorem	Name
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) = (B \bullet C) + (\overline{B} \bullet D)$	Consensus

Prove true using (1) perfect induction or (2) other axioms and theorems.

Perfect induction: consensus

Number	Theorem	Name
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	Consensus

B	C	D	$BC + \overline{B}D + CD$	$BC + \overline{B}D$
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

T11: Consensus

Number	Theorem	Name
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	Consensus

Prove using other theorems and axioms:

$B \bullet C + \overline{B} \bullet D + C \bullet D$

$= BC + \overline{B}D + (CDB + CD\overline{B})$ **T10: Combining**

$= BC + \overline{B}D + BCD + \overline{B}CD$ **T6: Commutativity**

$= BC + BCD + \overline{B}D + \overline{B}CD$ **T6: Commutativity**

$= (BC + BCD) + (\overline{B}D + \overline{B}CD)$ **T7: Associativity**

$= BC + \overline{B}D$ **T9': Covering**

Boolean Theorems of Several Vars

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T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	$(B + C) + D = B + (C + D)$	Associativity
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	$B + (C \bullet D) = (B+C)(B+D)$	Distributivity
T9	$B \bullet (B+C) = B$	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	$(B+C) \bullet (B+\overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	$(B+C) \bullet (\overline{B}+D) \bullet (C+D) =$ $(B+C) \bullet (\overline{B}+D)$	Consensus

Warning: T8' differs from traditional algebra:
OR (+) distributes over AND (•)

Limiti del perfect induction

- La tecnica del perfect induction è semplice ma «priva di intelligenza»
- Al crescere della lunghezza delle espressioni diventa sempre più laboriosa
- Al crescere delle variabili che occorrono nelle espressioni diventa estremamente più laboriosa
  - 4→16 checks
  - 5→32 checks
  - 6→64 checks
  - …

Boolean Theorems of Several Vars

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T8	B • (C + D) = (B•C) + (B•D)	B + (C•D) = (B+C) (B+D)	Distributivity
T9	B • (B+C) = B	B + (B•C) = B	Covering
T10	(B•C) + (B•$\overline{C}$) = B	(B+C) • (B+$\overline{C}$) = B	Combining
T11	(B•C) + ($\overline{B}$•D) + (C•D) = (B•C) + ($\overline{B}$•D)	(B+C) • ($\overline{B}$+D) • (C+D) = (B+C) • ($\overline{B}$+D)	Consensus

Axioms and theorems are useful for *simplifying* equations.

Simplifying an Equation

Reducing an equation to the **fewest number of implicants**, where each implicant has the **fewest literals**

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Reducing an equation to the **fewest number of implicants**, where each implicant has the **fewest literals**

Recall:

- Implicant: product of literals

  $A\bar{B}C, \bar{A}C, \bar{B}C$

- Literal: variable or its complement

  $A, \bar{A}, B, \bar{B}, C, \bar{C}$

Simplifying an Equation

Reducing an equation to the **fewest number of implicants**, where each implicant has the **fewest literals**

Recall:

- Implicant: product of literals

  $A\overline{B}C$, $\overline{A}C$, $\overline{B}C$

- Literal: variable or its complement

  A, $\overline{A}$, B, $\overline{B}$, C, $\overline{C}$

*Also called **minimizing** the equation*

Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$

  $B + CD = (B+C)(B+D)$

- **Covering (T9')** $A + AP = A$

- **Combining (T10)** $P\overline{A} + PA = P$

Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$

  $B + CD = (B + C)(B+D)$

- **Covering (T9')** $A + AP = A$

- **Combining (T10)** $P\overline{A} + PA = P$

- **Expansion** $P = P\overline{A} + PA$

  $A = A + AP$

- **Duplication** $A = A + A$

Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$
  $B + CD = (B+C)(B+D)$
- **Covering (T9')** $A + AP = A$
- **Combining (T10)** $P\overline{A} + PA = P$
- **Expansion** $P = P\overline{A} + PA$
  $A = A + AP$
- **Duplication** $A = A + A$
- **"Simplification" theorem** $\overline{P}A + A = P + A$
  $PA + \overline{A} = P + \overline{A}$

Proving the "Simplification" Theorem

"Simplification" theorem

$$PA + \overline{A} = P + \overline{A}$$

Method 1:

$PA + \overline{A}$	$= PA + (\overline{A} + \overline{A}P)$	**T9' Covering**
	$= PA + P\overline{A} + \overline{A}$	**T6 Commutativity**
	$= P(A + \overline{A}) + \overline{A}$	**T8 Distributivity**
	$= P(1) + \overline{A}$	**T5' Complements**
	$= P + \overline{A}$	**T1 Identity**

Proving the "Simplification" Theorem

"Simplification" theorem

$$PA + \overline{A} = P + \overline{A}$$

Method 2: $PA + \overline{A} = (\overline{A} + A)(\overline{A} + P)$ **T8' Distributivity**

$= 1(\overline{A} + P)$ **T5' Complements**

$= \mathbf{\overline{A} + P}$ **T1 Identity**

Boolean Theorems of Several Vars

#	Theorem	Dual	Name
T6	B•C = C•B	B+C = C+B	Commutativity
T7	(B•C) • D = B • (C•D)	(B + C) + D = B + (C + D)	Associativity
T8	B • (C + D) = (B•C) + (B•D)	B + (C•D) = (B+C) (B+D)	Distributivity
T9	B • (B+C) = B	B + (B•C) = B	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	$(B+C) \bullet (B+\overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	$(B+C) \bullet (\overline{B}+D) \bullet (C+D) =$ $(B+C) \bullet (\overline{B}+D)$	Consensus

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Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$
  $B + CD = (B+C)(B+D)$
- **Covering (T9')** $A + AP = A$
- **Combining (T10)** $P\overline{A} + PA = P$
- **Expansion** $P = P\overline{A} + PA$
  $A = A + AP$
- **Duplication** $A = A + A$
- **"Simplification" theorem** $P\overline{A} + A = P + A$
  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 1:

$$Y = AB + A\overline{B}$$

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Simplifying Boolean Equations

Example 1:

$Y = AB + A\overline{B}$

$Y = A$ T10: Combining

or

$= A(B + \overline{B})$ T8: Distributivity

$= A(1)$ T5': Complements

$= A$ T1: Identity

Simplification methods

- **Distributivity (T8, T8')** — **$B(C+D) = BC + BD$**
  $B + CD = (B+C)(B+D)$
- **Covering (T9')** — $A + AP = A$
- **Combining (T10)** — $P\overline{A} + PA = P$
- **Expansion** — $P = P\overline{A} + PA$
  $A = A + AP$
- **Duplication** — $A = A + A$
- **"Simplification" theorem** — $P\overline{A} + A = P + A$
  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 2:

$$Y = A(AB + ABC)$$

Simplifying Boolean Equations

Example 2:

$Y = A(AB + ABC)$

$= A(AB(1 + C))$	T8: Distributivity
$= A(AB(1))$	T2': Null Element
$= A(AB)$	T1: Identity
$= (AA)B$	T7: Associativity
$= AB$	T3: Idempotency

Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$
  $B + CD = (B+C)(B+D)$
- **Covering (T9')** $A + AP = A$
- **Combining (T10)** $P\overline{A} + PA = P$
- **Expansion** $P = P\overline{A} + PA$
  $A = A + AP$
- **Duplication** $A = A + A$
- **"Simplification" theorem** $P\overline{A} + A = P + A$
  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 3:

$Y = A'BC + A'$

Recall: $A' = \overline{A}$

Simplifying Boolean Equations

Example 3:

$Y = A'BC + A'$ **Recall:** $A' = \overline{A}$

$= A'$ T9' Covering: $X + XY = X$

or

$= A'(BC + 1)$ T8: Distributivity

$= A'(1)$ T2': Null Element

$= A'$ T1: Identity

Simplifying Boolean Equations

Example 4:

Y = AB'C + ABC + A'BC

Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$
  $B + CD = (B + C)(B+D)$
- **Covering (T9')** $A + AP = A$
- **Combining (T10)** $P\overline{A} + PA = P$
- **Expansion** $P = P\overline{A} + PA$
  $A = A + AP$
- **Duplication** $A = A + A$
- **"Simplification" theorem** $P\overline{A} + A = P + A$
  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 4:

Y = AB'C + ABC + A'BC

= AB'C + **ABC + ABC** + A'BC   T3': Idempotency

= (AB'C+ABC) + (ABC+A'BC)   T7': Associativity

= AC + BC   T10: Combining

Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$
  $B + CD = (B+C)(B+D)$
- **Covering (T9')** $A + AP = A$
- **Combining (T10)** $P\bar{A} + PA = P$
- **Expansion** $P = P\bar{A} + PA$
  $A = A + AP$
- **Duplication** $A = A + A$
- **"Simplification" theorem** $P\bar{A} + A = P + A$
  $PA + \bar{A} = P + \bar{A}$

Simplifying Boolean Equations

Example 5:

$Y = AB + BC + B'D' + AC'D'$

Method 1:

$Y = AB + BC + B'D' + (ABC'D' + AB'C'D')$	T10: Combining
$= (AB + ABC'D') + BC + (B'D' + AB'C'D')$	T6: Commutativity
	T7: Associativity
$= AB + BC + B'D'$	T9: Covering

Method 2:

$Y = \mathbf{AB} + BC + \mathbf{B'D'} + AC'D' + \mathbf{AD'}$	T11: Consensus
$= AB + BC + B'D' + AD'$	T9: Covering
$= AB + BC + B'D'$	T11: Consensus