



CHANIA
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Wall-modeled LES of the TUDa Compressor Stage Using a Fourth-order FR Method

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Numerical Method

Governing equations:

unsteady compressible Navier-Stokes equations

$$\frac{\partial U}{\partial t} + \nabla \cdot (\vec{F}_c + \vec{F}_d) = S$$

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \end{bmatrix} \quad \vec{F}_{c,i} = \begin{bmatrix} \rho v_i \\ \rho v_i u + P \delta_{i0} \\ \rho v_i v + P \delta_{i1} \\ \rho v_i w + P \delta_{i2} \\ \rho v_i H \end{bmatrix} \quad \vec{F}_d = \begin{bmatrix} 0 \\ \tau_{i0} \\ \tau_{i1} \\ \tau_{i2} \\ v_j \tau_{ij} + q_i \end{bmatrix}$$

$$\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \frac{\partial v_k}{\partial x_k} \delta_{ij} \right) \quad q_j = -\kappa \frac{\partial T}{\partial x_j}$$

Arbitrary Lagrangian-Eulerian:

$$\vec{F}_{c,i} = \begin{bmatrix} \rho(v_i - v_{g,i}) \\ \rho(v_i - v_{g,i})u + P \delta_{i0} \\ \rho(v_i - v_{g,i})v + P \delta_{i1} \\ \rho(v_i - v_{g,i})w + P \delta_{i2} \\ \rho(v_i - v_{g,i})E + P v_i \end{bmatrix}$$

Turbulence model:

Wall-Adapting Local Eddy-viscosity (WALE) model

$$\bar{\tau}_{ij}^{SM} = \frac{1}{3} \tau_{kk} \delta_{ij} - 2 \nu_{sgs} \bar{S}_{ij}$$

$$\nu_T = (C_w \Delta)^2 \frac{(S_{ij}^d S_{ij}^d)^{3/2}}{(\tilde{S}_{ij} \tilde{S}_{ij})^{5/2} + (S_{ij}^d S_{ij}^d)^{5/4}}$$

$$S_{ij}^d = \frac{1}{2} (\tilde{g}_{ij}^2 + \tilde{g}_{ji}^2) - \frac{1}{3} \delta_{ij} \tilde{g}_{kk}^2 \quad \bar{S}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$$

$$\tilde{g}_{ij} = \partial \tilde{u}_i / \partial x_j \quad \tilde{g}_{ij}^2 = \tilde{g}_{ik} \tilde{g}_{kj} \quad \Delta = (\Delta x \Delta y \Delta z)^{1/3}$$

Wall model:

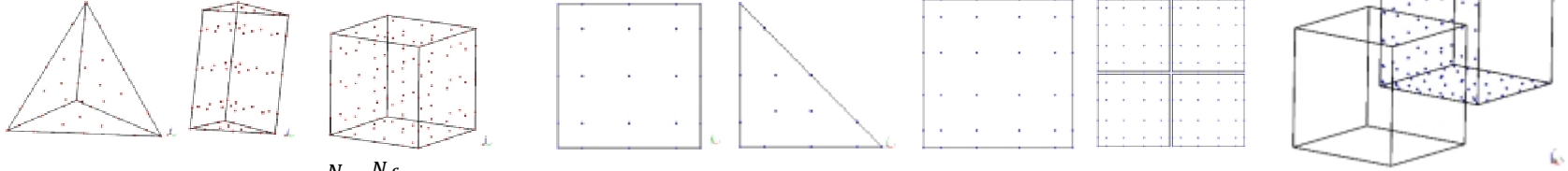
Wall-stress model

$$\frac{U_{wm}(y)}{u_\tau} = \frac{1}{\kappa} \ln \left(\frac{y u_\tau}{\nu} \right) + B.$$

Dimaxer: a STE-KEP-FR Solver

Space-Time Expansion of Kinetic Energy Preserving Flux Reconstruction method

AIAA-2019-3422 & AIAA-2019-3525



$$\frac{\partial \hat{U}_{i,j}}{\partial t} + \left(\nabla^\xi \cdot \vec{\mathbf{F}}^\xi(\mathbf{U}_i) \right)_{i,j} + \sum_{s=1}^{N_s} \sum_{f=1}^{N_f} \alpha_{j,s,f} (\tilde{\mathbf{F}}^\xi|_n - \bar{\mathbf{F}}^\xi|_n)_{j,s,f} = 0 \quad \text{Space-time polynomial: } \frac{dv_{i,j}}{dt} = \mathbb{R}_{i,j}^{Div}(v(\vec{x}_i, t))$$

$$\frac{\partial U_{i,j}}{\partial t} = \mathbb{R}_{i,j}^{Div}(\mathbf{U}_i) + \sum_{s=1}^{N_s} \sum_{f=1}^{N_f} \mathbb{R}_{i,j,s,f}^{Cor}(\mathbf{U}_i, U_{i,s,f}^{adj})$$

$$\mathbb{R}_{i,j}^{Div} = -\frac{1}{V|_{i,j}} \left(\nabla^\xi \cdot \vec{\mathbf{F}}^\xi(\mathbf{U}_i) \right)_{i,j}$$

$$\mathbb{R}_{i,j,s,f}^{Cor} = -\frac{1}{V|_{i,j}} \alpha_{j,s,f} (\tilde{\mathbf{F}}^\xi|_n - \bar{\mathbf{F}}^\xi|_n)_{j,s,f}$$

$$U_{i,j}^{n+1} - U_{i,j}^n = \int_{t^n}^{t^{n+1}} \mathbb{R}_{i,j}^{Div}(\mathbf{U}_i) + \sum_{s=1}^{N_s} \sum_{f=1}^{N_f} \mathbb{R}_{i,j,s,f}^{Cor}(\mathbf{U}_i, U_{i,s,f}^{adj}) dt$$

$$\int_{t^n}^{t^{n+1}} \mathbb{R}_{i,j}^{Div}(\mathbf{v}_i) dt = v_{i,j}(t^{n+1}) - U_{i,j}^n$$

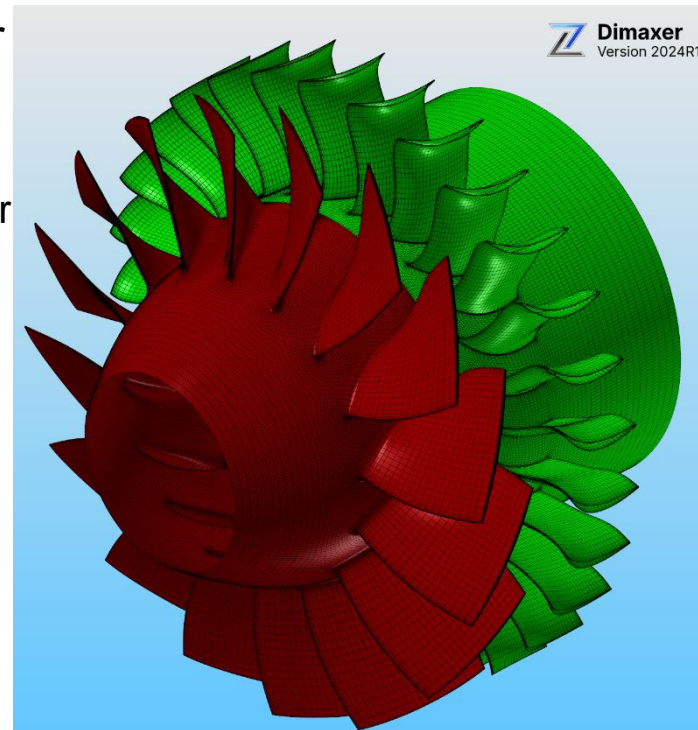
$$U_{i,j}^{n+1} = v_{i,j}(t^{n+1}) + \int_{t^n}^{t^{n+1}} \sum_{s=1}^{N_s} \sum_{f=1}^{N_f} \mathbb{R}_{i,j,s,f}^{Cor}(\mathbf{v}_i(t^{n+1}), v_{i,s,f}^{adj}(t^{n+1})) dt$$

$$U_{i,j}^{n+1} = v_{i,j}(t^{n+1}) + \sum_{s=1}^{N_s} \sum_{f=1}^{N_f} \sum_{qp}^{N_{qp}^t} \Delta t \cdot w_{qp}^t \mathbb{R}_{i,j,s,f}^{Cor}(\mathbf{v}_i(t_{qp}), v_{i,s,f}^{adj}(t_{qp}))$$

Computational setup

Computational domain and mesh generation

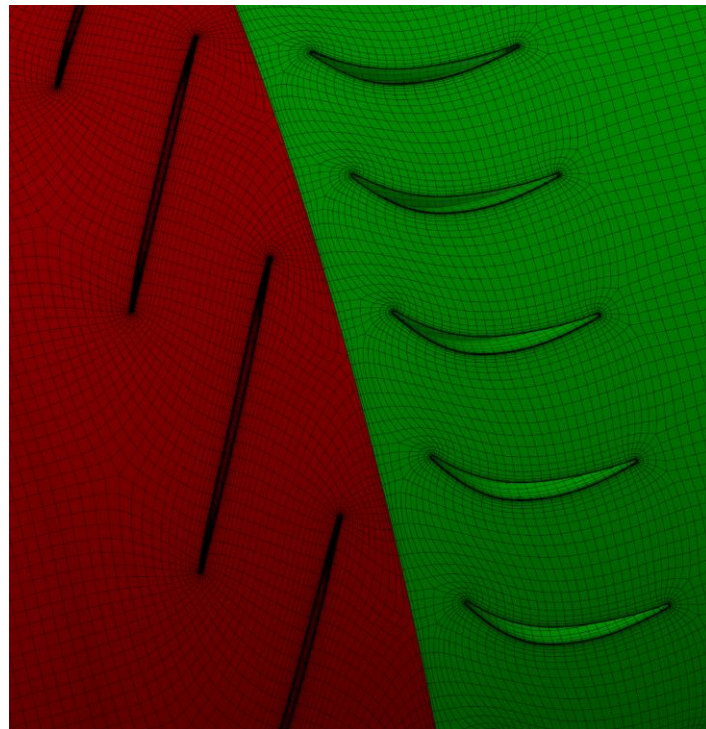
- Full-wheel simulation of the rotor and the stator
 - 1.0 axial chord upstream the rotor leading edge
 - 1.5 axial chord downstream the stator
 - Fillets of the rotor are considered, but not the stator
- Unstructured multi-block O4H mesh
 - Rotor
 - Tip gap radial grid points: 5
 - Total radial grid points: 33
 - Boundary layer grid points: 9
 - Grid growth rate in the boundary layer block: 1.5
 - Grid points around the blade: 76
 - Equivalent y^+ is about 30 per degree of freedom, x^+ and z^+ are less than 50 times y^+



Computational setup

Computational domain and mesh generation

- Unstructured multi-block O4H mesh
 - Rotor
 - Total elements: 827,392
 - Total degrees of freedoms: 52,953,088
 - Stator
 - Total radial grid points: 33
 - Boundary layer grid points: 9
 - Grid growth rate in the boundary layer block: 1.5
 - Grid points around the blade: 76
 - Equivalent y^+ is about 30 per degree of freedom, x^+ and z^+ are less than 50 times y^+
 - Total elements: 1,083,904
 - Total degrees of freedoms: 69,369,856



Computational setup

Boundary conditions

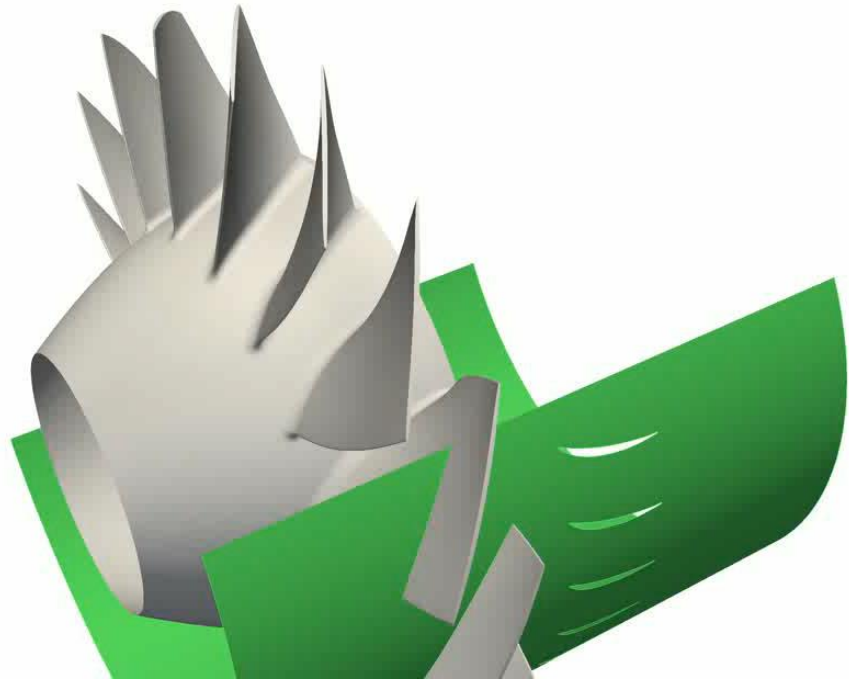
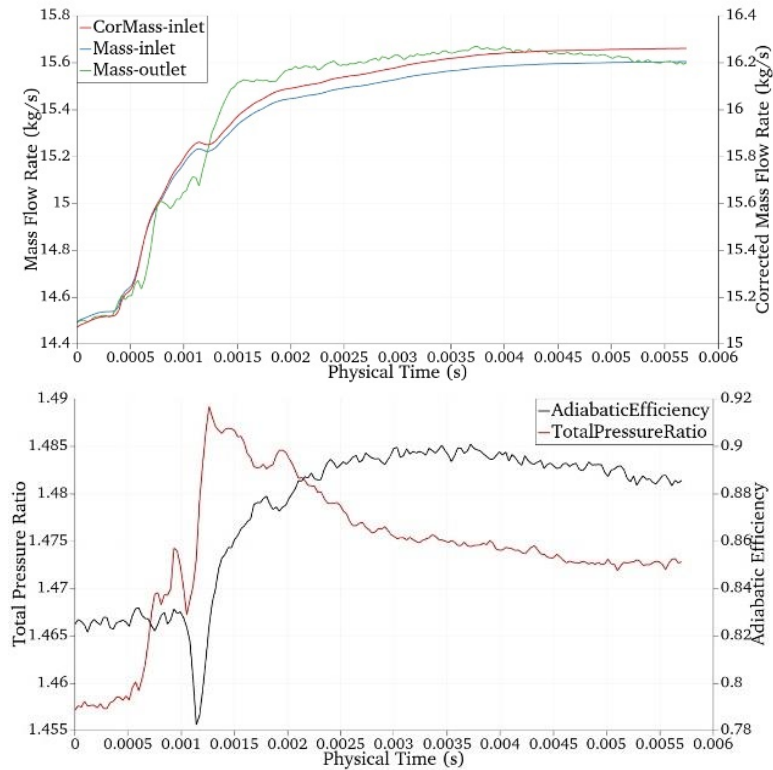
- Peak-efficiency condition (16.00 ± 0.02 kg/s) at 100% design speed (20,000 RPM) is simulated
- Simulation time: 0.006s
- Total pressure inlet:
 - Uniform (laminar inlet), 1-D non-reflect
 - Total pressure: 98,594 Pa
 - Total temperature: 288.15 K
 - Flow direction: Axial
- Pressure outlet:
 - Uniform, 1-D non-reflect
 - Pressure: 123,000 Pa (adjusted to make the mass flow rate meet the experimental value)
- Wall: No-slip adiabatic

Computational setup

Solver methods

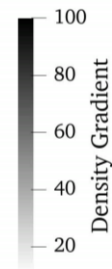
- Governing equations: Unsteady compressible Navier–Stokes equations
- Turbulence model: Wall-modeled LES
- Property of material: Ideal gas
- Spatial discretization method: STE-KEP-FR
- Advection Scheme: HLLC
- Temporal discretization scheme: Runge-Kutta
- Spatial and temporal accuracy: 4th
- Time step: Local time step
- Moving mesh method: Arbitrary Lagrangian-Eulerian
- Rotor-stator interface: Dynamic overset
- Computing resource: 8 Nvidia RTX 4090 GPUs, 1 revolution takes 1 week

Results



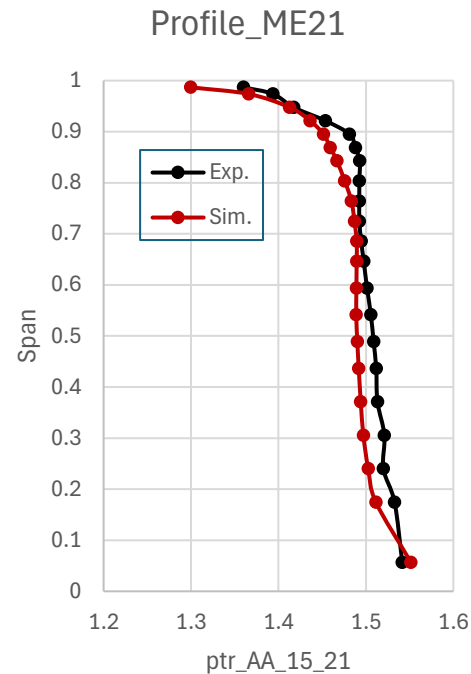
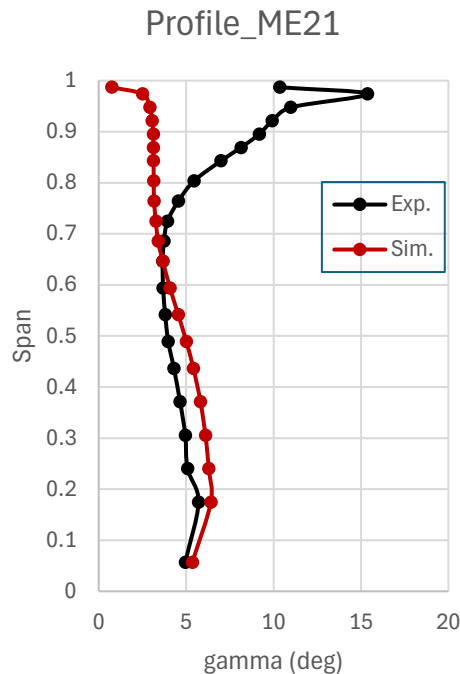
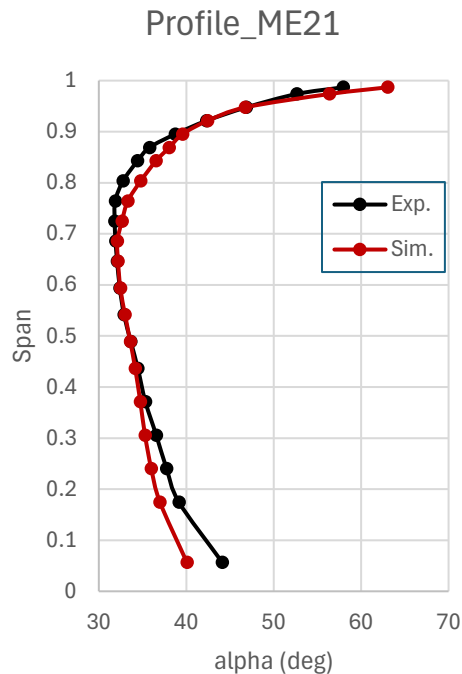
Results

Cylindrical slice where $R = 0.16\text{m}$

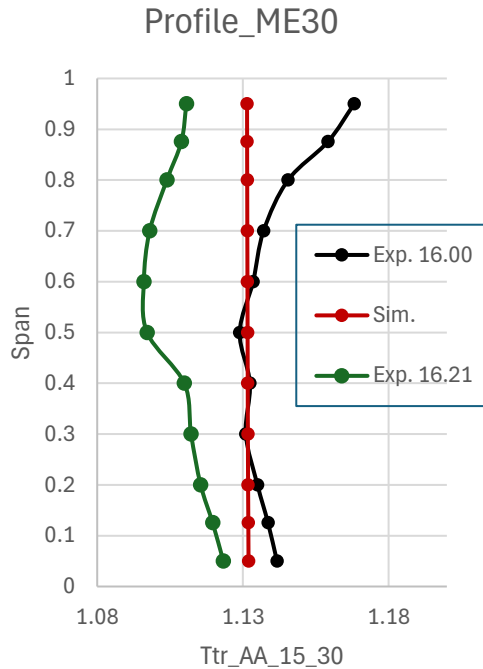
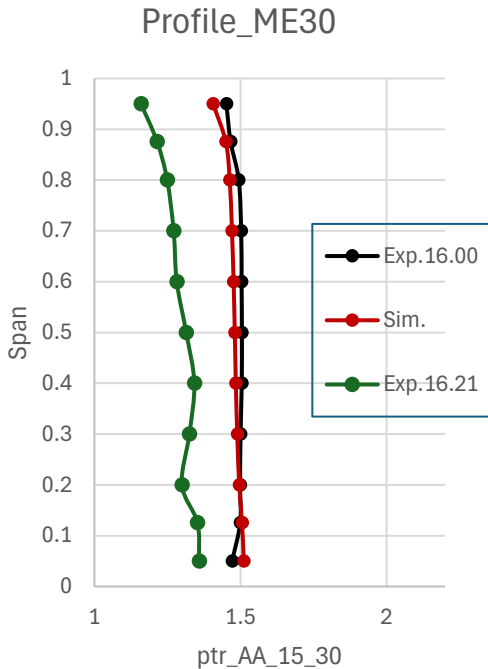


Results

In the Sim., data is calculated using mass-flow averaged method



Results



- The rotor rotates one and a half revolutions from a uniform initial field to a statistical steady state.
- Compared to the experiment, clearance leakage flow has not been fully resolved.
- Under the same mass flow rate, the calculated pressure ratio is lower, which may be due to the shorter inlet and less obvious blockage effect.



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Thank you