18-10-25

Air University

Mid Semester Examinations: Fall 2025

Student ID: 241539

Subjective Part
(To be solved on Answer Books only)

Subject: Calculus and Analytical Geometry
Class: BSCYS-III
Section(s): A, B (Morning Session)
Course Code: MA-110

Time Allowed: 120 Minutes
Max Marks: 50
FM's Name: Mr. Umair Habib
FM's Signature:

INSTRUCTIONS:
- Attempt responses on the answer book only.
- Nothing is to be written on the question paper.
- Rough work or writing on question paper will be considered as use of unfair means.
- Tables/calculators are allowed.

Q1. (CLO-1) **Marks (5+5)**

a): In 2001, the National Weather Service introduced a new wind chill temperature (WCT)index. For a given outside temperature T and wind speed v, the wind chill temperature index is the equivalent temperature that exposed skin would feel with a wind speed of $v\ mi/h$. Based on a more accurate model of cooling due to wind, the new formula is:

$$WCT = \begin{cases} T & 0 \le v \le 3 \\ 35.74 + 0.6215T - 35.75v^{0.16} + 0.4275Tv^{0.16} & v > 3 \end{cases}$$

Where T is the temperature in °F, v is the wind speed in $\frac{mi}{h}$, and WCT is the equivalent temperature in °F. **Find** the WCT to the nearest degree if $T = 25$°F & $v = 15\ mi/h$. 12.6°F

b): According to the Newton's Law of Universal Gravitation, the weight **"W"** of an object (relative to the Earth) is inversely proportional to the square of the distance **"x"** between the object and the center of the Earth. i.e., $W = \frac{C}{x^2}$.

➢ Assuming that a weather satellite weighs 2000 pounds on the surface of the Earth and that the Earth is a sphere of radius 4000 miles. **Find** the constant **C**. 3.2×10^{10}

➢ **Find** the weight of the satellite when it is 1000 miles above the surface of the Earth. 1280

Q2. (CLO-2) **Marks (4+6+6+4)**

a): Describe (find) the equation for the tangent line to the graph of $y = x^4$ at $(2, 16)$ by using the concept of secant line and limits. $32x - 48$ x_0 y_0

b): Interpret (find) the value of **"k"** (if possible) that makes the given function continuous everywhere. $k=0$

$$f(x) = \begin{cases} 9 - x^2 & x \ge -3 \\ \frac{k}{x^2} & x < -3 \end{cases}$$

$f(x)$ $x \to -3$

c): Consider the functions $f(x) = \begin{cases} 1 & x \neq 4 \\ -1 & x = 4 \end{cases}$ and $g(x) = \begin{cases} 4x - 10 & x \neq 4 \\ -6 & x = 4 \end{cases}$

Discuss the continuity of the function $g(x) - 6f(x)$ at $x = 4$.

d): Let $g(t) = \begin{cases} \frac{2+3t}{5t^2+6} & if\ t < 1{,}000{,}000 \\ \frac{\sqrt{36t^2-100}}{5-t} & if\ t > 1{,}000{,}000 \end{cases}$

Interpret. a): $\lim_{t\to-\infty} g(t)$. 0 b): $\lim_{t\to+\infty} g(t)$. -6

Q3. **(CLO-4)** **Marks (6+7+7)**

a): An automobile is driven down a straight highway such that after $\mathbf{0 \leq t \leq 12}$ seconds it is $s = 4.5\,t^2\ feet$ from its initial position.

- **Calculate** the average velocity of the car over the interval $[0, 12]$. 54
- **Calculate** the instantaneous velocity of the car at $t = 6$. 54

b): Use Implicit differentiation to **calculate** the slope of the Tangent line to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ at the point $(-1, 3\sqrt{3})$.

c): Let $f(x) = Sin(x)$. Then verify that $f'(x) = Cos(x)$ by using the definition of the derivative.