3D Vision 1

Week 7

Introduction to 3D Vision

Model Fitting

Announcements

- Where student lab attendance dropped below a threshold for multiple weeks, the School has taken action to merge labs

ORIGINAL					NEW			
Day	Start Time	End Time	Location		Day	Start Time	End Time	Location
Wed	13:00	15:00	Rm N113_CSIT Bldg 108	→	Wed	13:00	15:00	Rm N109_CSIT Bldg 108
Wed	14:00	16:00	Rm N112_CSIT Bldg 108	→	Wed	14:00	16:00	Comp Lab 1.23_Hanna Neumann Bldg 145
Wed	15:00	17:00	Rm N109_CSIT Bldg 108	→	Wed	14:00	16:00	Comp Lab 1.23_Hanna Neumann Bldg 145
Fri	11:00	13:00	Rm N113_CSIT Bldg 108	→	Fri	11:00	13:00	Rm N114_CSIT Bldg 108
Fri	13:00	15:00	Comp Lab 1.23_Hanna Neumann Bldg 145	→	Fri	13:00	15:00	Rm N114_CSIT Bldg 108
Fri	13:00	15:00	Rm N113_CSIT Bldg 108	→	Fri	13:00	15:00	Rm N114_CSIT Bldg 108

Announcements

- **Reminder:** 30-minute tutor drop-in session
  - E.g., for a lab scheduled from 2:00–4:00PM, we have:

Time	Activity
2:00PM–2:30PM	Tutor Drop-in Session
2:30PM–4:00PM	Lab

Announcements

- Tutorial 5, on backpropagation, is available on Wattle and will be covered during the labs this week
- Assignment 2 due 11:59pm Friday 26 April (end of Week 8)
  - This includes a one week extension that has already been applied
  - **Zero** marks if either report or code submitted late (unless extension)
    - Submit early; you can always resubmit an updated version later
    - Depending on your internet connection and load on the TurnItIn servers, uploading can sometimes be slow, so please factor this into your submission schedule
  - Submit your report (PDF) and code (ZIP file) **separately under the correct tab** in the submission box
  - Follow the instructions under Submission Requirements

Weekly Study Plan: Overview

Wk	Starting	Lecture	Lab	Assessment
1	19 Feb	Introduction	✗	
2	26 Feb	Low-level Vision 1	1	
3	4 Mar	Low-level Vision 2	1	
		Mid-level Vision 1		
4	11 Mar	Mid-level Vision 2	1	CLab1 report due Friday
		High-level Vision 1		
5	18 Mar	High-level Vision 2	2	
6	25 Mar	High-level Vision 3[1]	2	
	1 Apr	Teaching break	✗	
	8 Apr	Teaching break	✗	
7	15 Apr	3D Vision 1	2	CLab2 report due Friday
8	22 Apr	3D Vision 2	3	
9	29 Apr	3D Vision 3	3	
10	6 May	3D Vision 4	3	
		Mid-level Vision 3		
11	13 May	High-level Vision 4	✗	CLab3 report due Friday
12	20 May	Course Review	✗	

Weekly Study Plan: Part B

Wk	Starting	Lecture	By
7	15 Apr	3D vision: introduction, camera model, single-view geometry	Dylan
8	22 Apr	3D vision: camera calibration, two-view geometry (homography)	Dylan
9	29 Apr	3D vision: two-view geometry (epipolar geometry, triangulation, stereo)	Dylan
10	6 May	3D vision: multiple-view geometry	Weijian
		Mid-level vision: optical flow, shape-from-X	Dylan
11	13 May	High-level vision: self-supervised learning, detection, segmentation	Dylan
12	20 May	Course review	Dylan

Outline

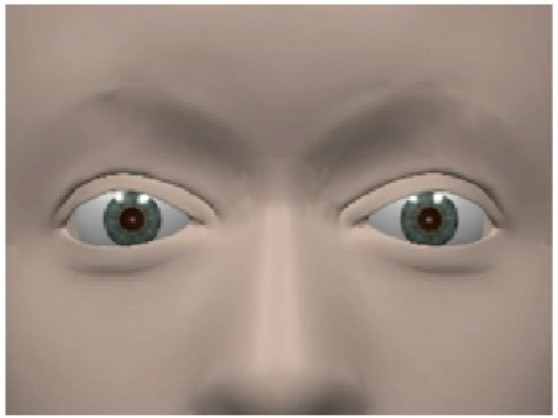
1. Introduction to 3D vision
2. Model fitting (line fitting)
    1. Least squares
    2. M-estimation
    3. RANSAC
    4. Hough transform
3. Image formation (review): pinhole camera model
4. Camera projection matrix and single view geometry
5. Camera calibration
6. Resectioning and camera pose

Introduction to 3D Vision

3D Vision: Eight Human Tricks

binocular stereo

accomodation



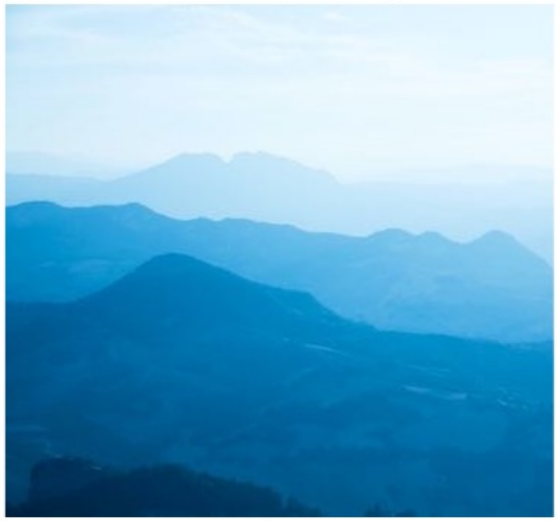
vergence

motion perspective

occlusion

apparent height

texture density



aerial perspective

Image Credit: Peter Corke

3D Vision: ~~Eight~~ Nine Human Tricks

Perspective Distortion

Image Credit: Pikist

3D Vision: Eight Human Tricks

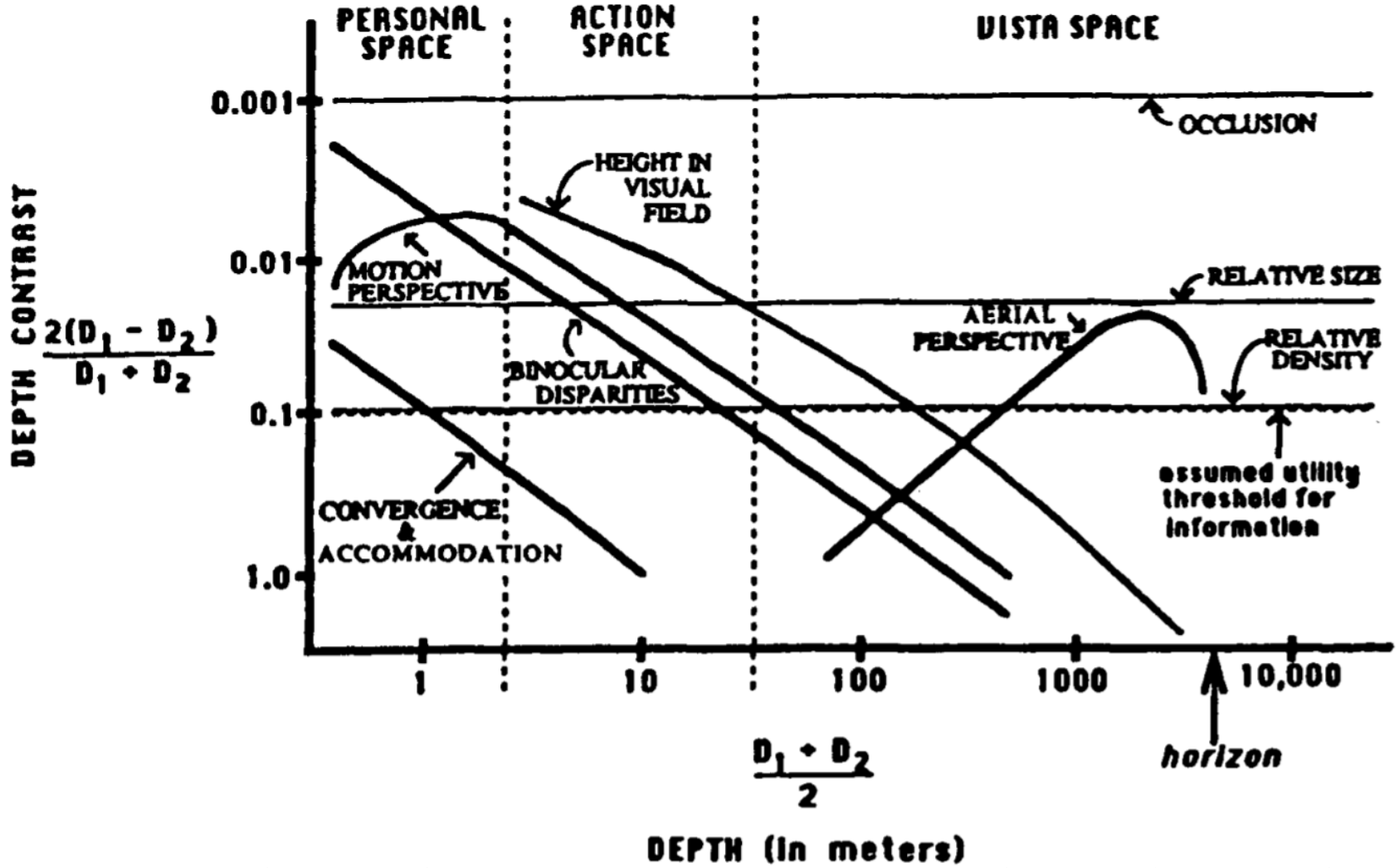
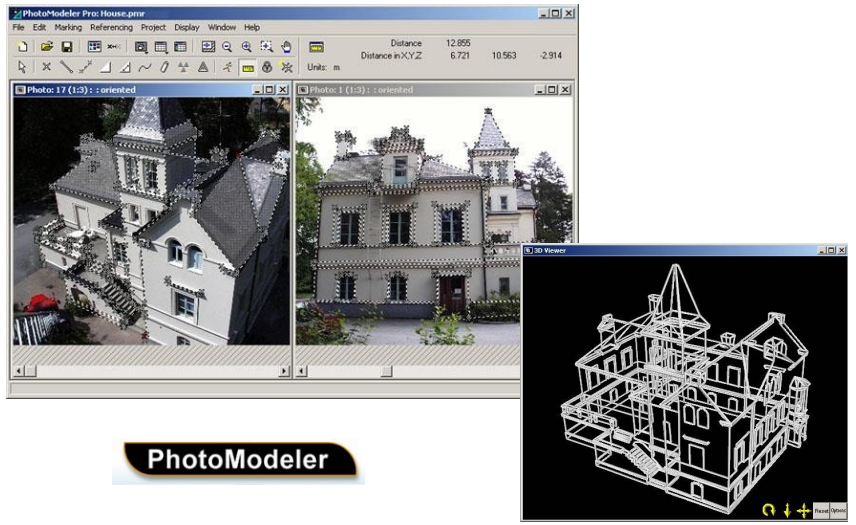


Image Credit: J. Cutting, “How the eye measures reality and virtual reality”, 1997

Why 3D Vision?

Structure-from-Motion (SfM):

- Input: set or sequence of images
- Output: camera poses, 3D model
- Applications:
  - Architecture (survey, stability analysis, renovation planning)
  - City modelling
  - Archaeology
  - Inspection (from drones)
  - Surgery



Building Rome in a Day

Credit: S. Agarwal et al., “Building Rome in a day”, 2009

NeRF

Credit: J. Barron et al., “Zip-NeRF”, ICCV 2023

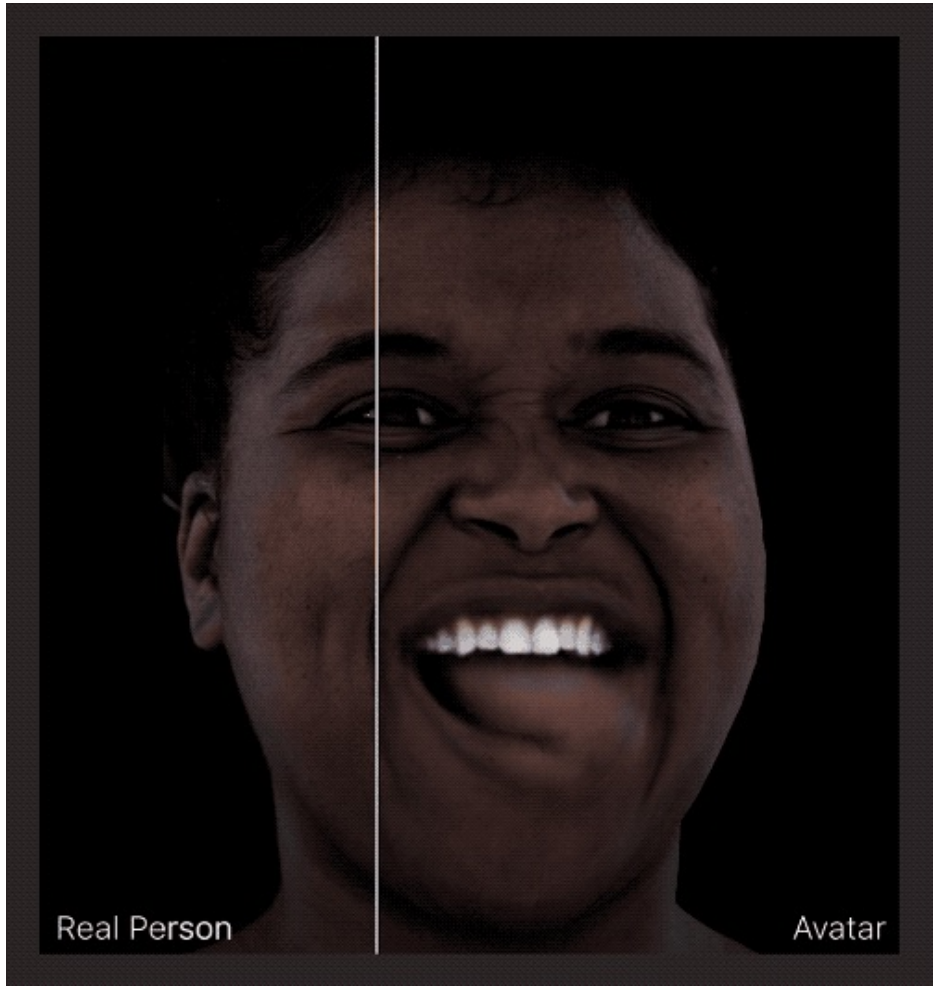
DUSt3R

- https://dust3r.europe.naverlabs.com/

Telepresence

Credit: Microsoft HoloLens

Telepresence

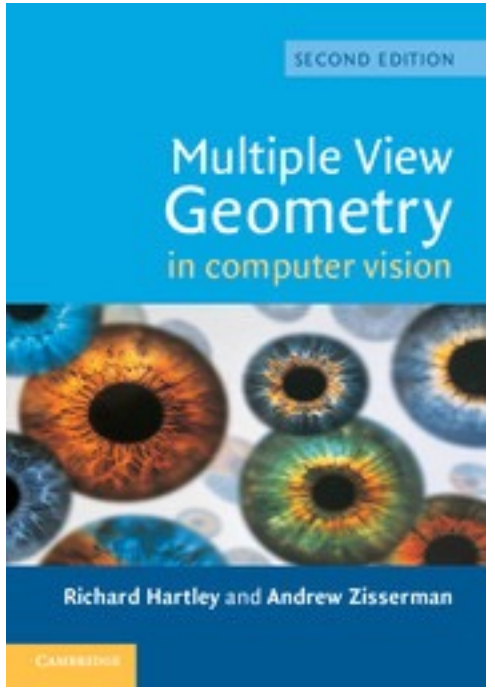


Credit: Facebook (Meta)

Reference Textbook for 3D Vision

- Multiple View Geometry in Computer Vision

Richard Hartley (ANU) | Andrew Zisserman (Oxford)



Recommended Reading

- Ch 6: Camera models
- Ch 7: Computation of camera matrix
- Ch 4: DLT algorithm

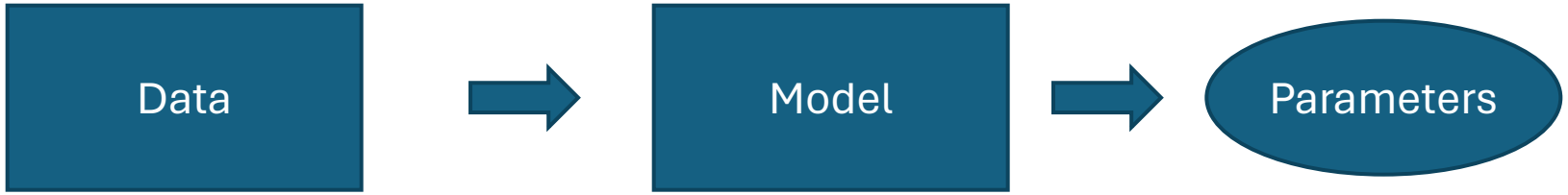
- More in-depth than the lectures
- Goes beyond what is required for this course

Model Fitting

Line Fitting

Model Fitting in Computer Vision

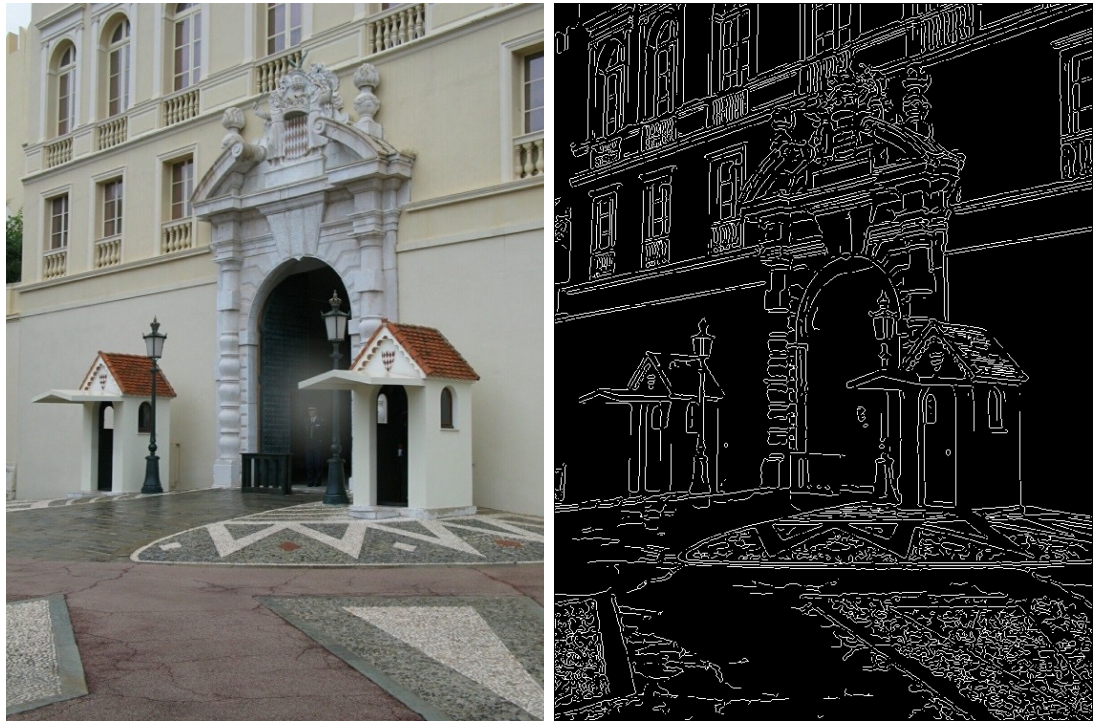
- Model fitting is a fundamental approach in computer vision



- Find model parameters that best fit the data
  - → Optimisation problem
- Model can be as simple as 2D lines, or more complex such as camera transform or 3D articulated object (e.g., skeleton model)
- We will introduce using 2D lines now, cameras later

A Binary Edge Map is Often Insufficient

- Fragmented edges
- Many broken lines
- Noise in edge orientations
- Membership not clear: which points on which line?

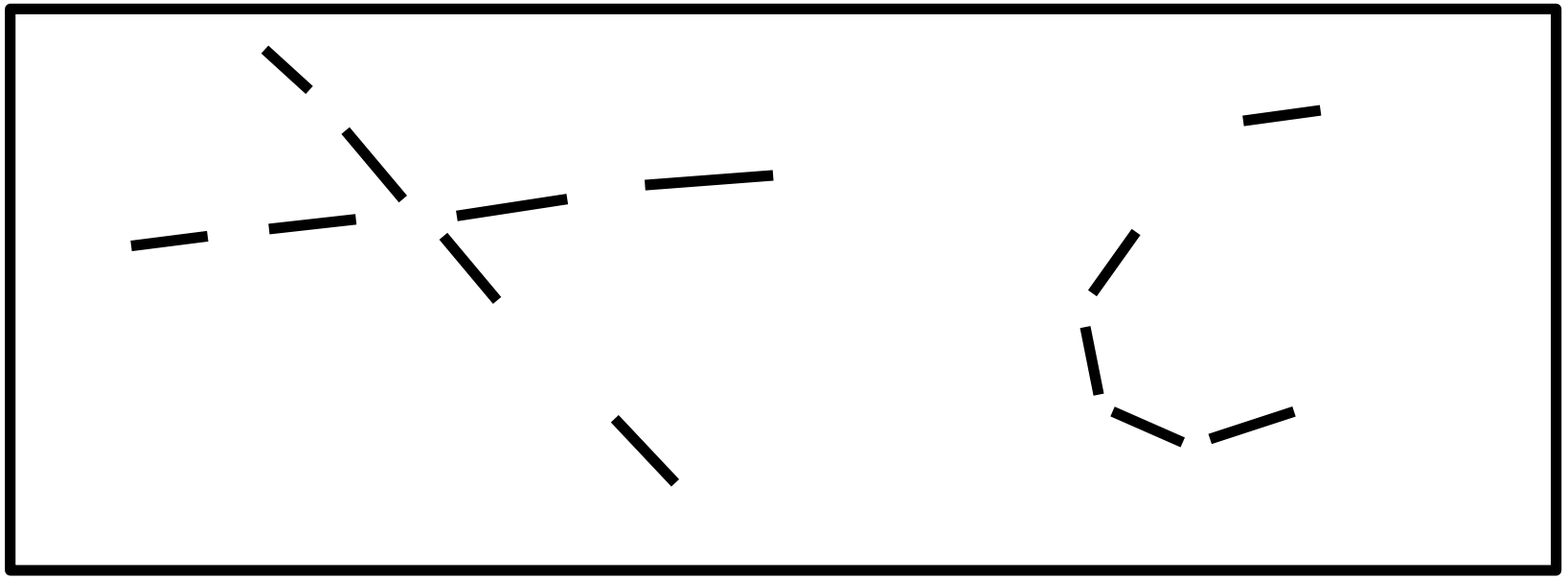


Any Feature Here?

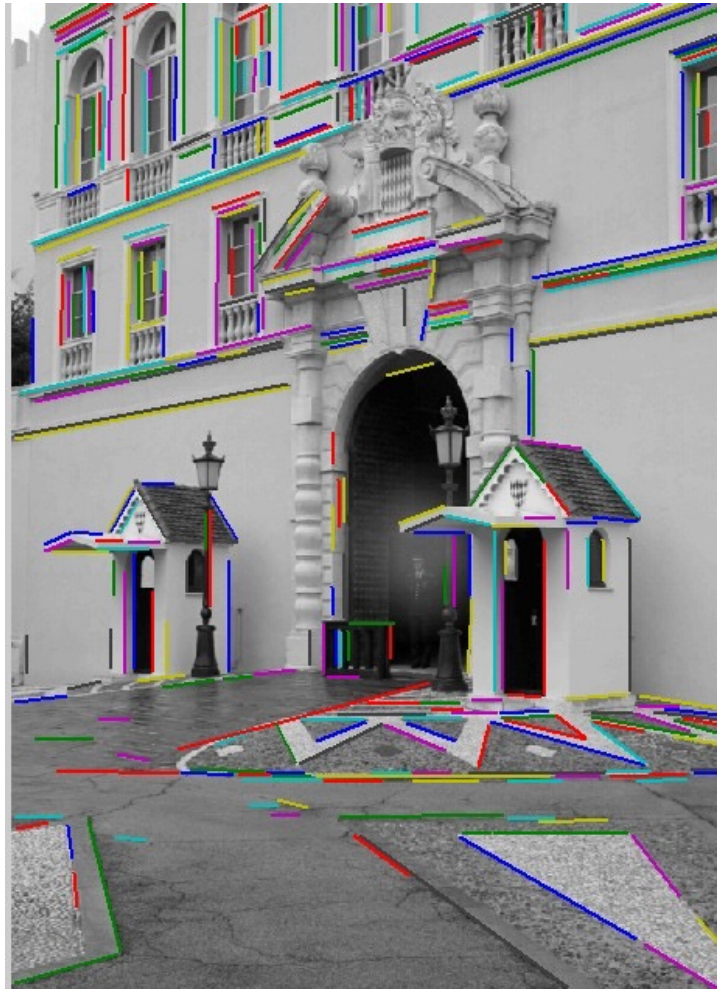
Vanishing Points and Lines

Goal of Line Fitting

- From fragmented edges to straight lines or curves



Goal of Line Fitting

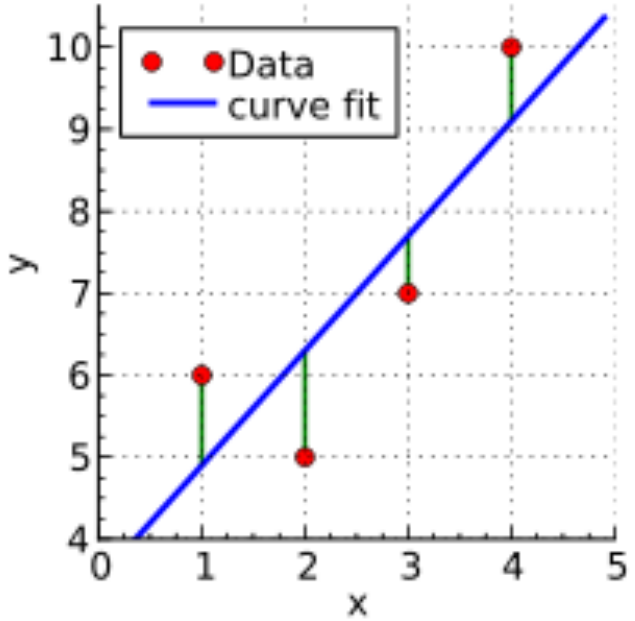


“Fitting” in general context

- Choose a parametric model to represent a set of features
- Membership determination
  - Decide which feature belongs to which model
  - Not a local decision; must look at the full image globally
- Key questions:
  - What is the best parametric model?
  - How many models (e.g., lines) are there?
  - Which point belongs to which line?
  - Remove noise points, wrong edge points

Basic Line Fitting: Least Squares

- Given a set of points that belong to a straight line, find the equation of the line
- Find the line parameters that yield minimal error in a least-square sense



Basic Line Fitting: Least Squares

- Given data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line model: $y_i = mx_i + b$
- Find $\boldsymbol{p} = (m, b)$ to minimise

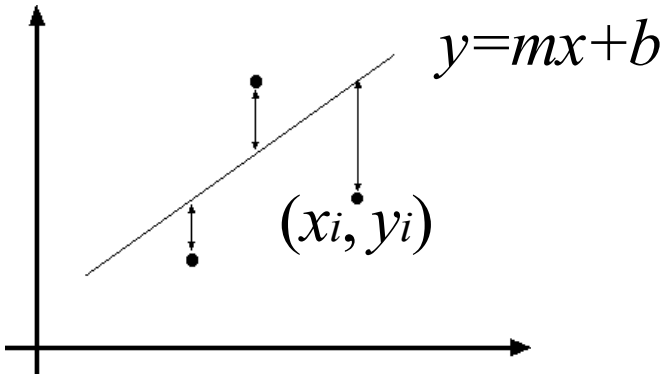
$$e = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

$$e = \sum_{i=1}^{n} \left(y_i - \begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} \right)^2$$

$$e = \left\| \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} - \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} \right\|^2$$

$$e = \|\boldsymbol{y} - A\boldsymbol{p}\|^2$$

$$e = \boldsymbol{y}^T\boldsymbol{y} - 2(A\boldsymbol{p})^T\boldsymbol{y} + (A\boldsymbol{p})^T(A\boldsymbol{p})$$



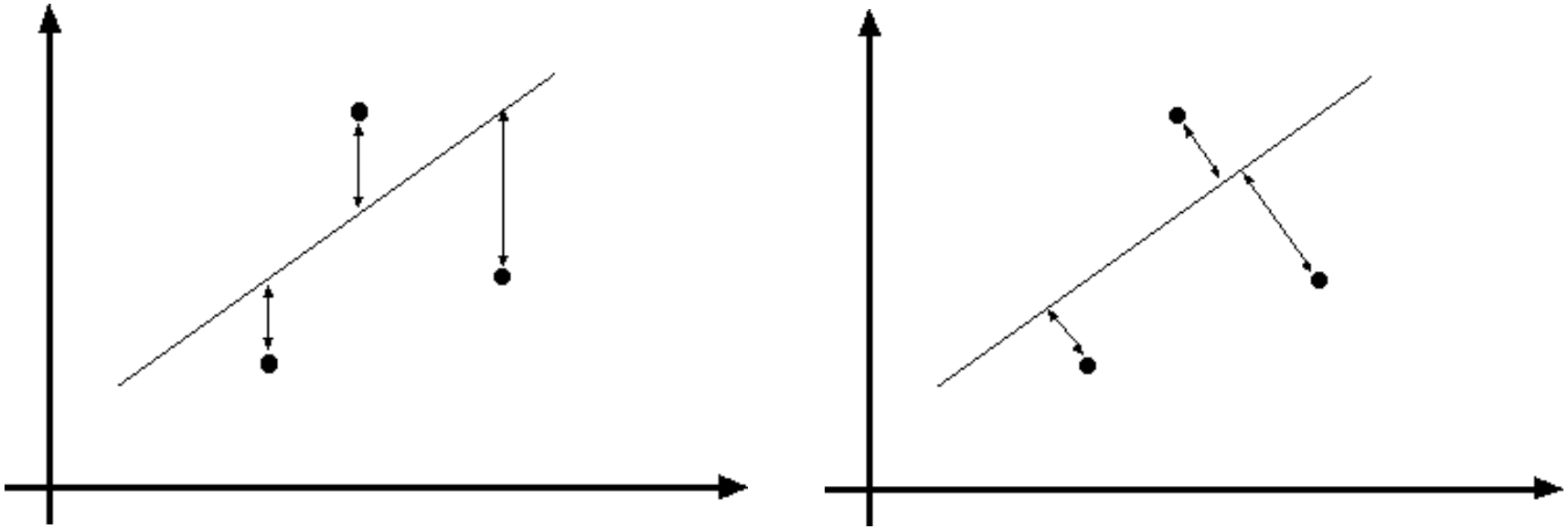
$$\frac{de}{d\boldsymbol{p}} = 2A^TA\boldsymbol{p} - 2A^T\boldsymbol{y} = 0$$

$$\therefore \boldsymbol{p} = (A^TA)^{-1}A^T\boldsymbol{y}$$

Modified from S. Lazebnik

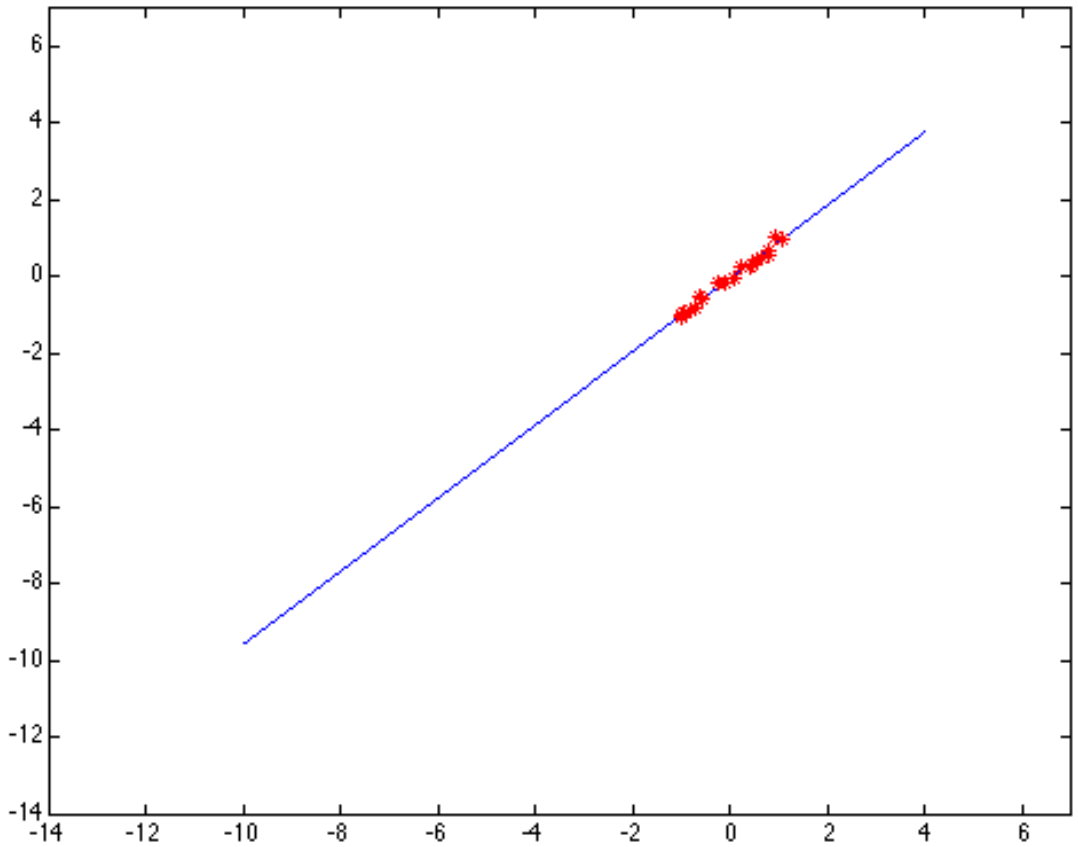
Basic Line Fitting: Issues

- Fitting performance is affected by the slope
- Fails completely for vertical lines
- Modified least squares using a perpendicular distance



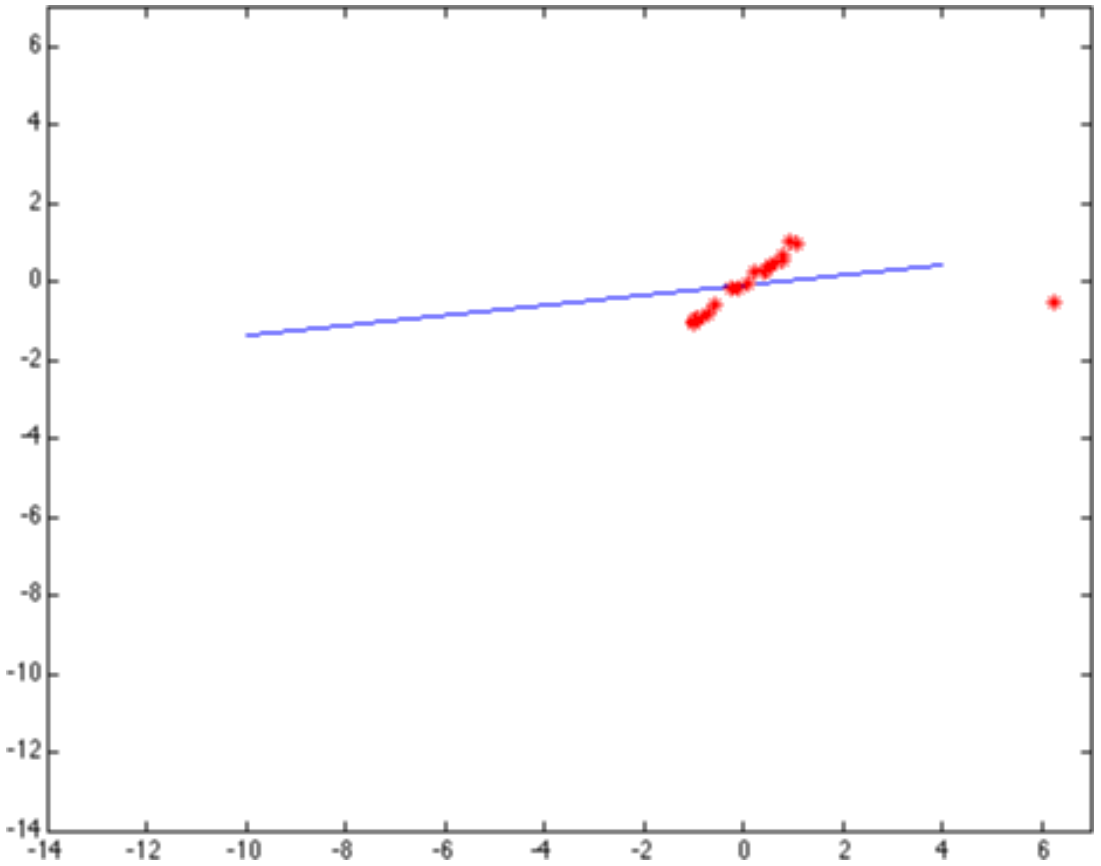
Least Squares: Robustness to Noise

- Least squares fit to the red points



Least Squares: Robustness to Noise

- Least squares fit with an outlier
- Problem: squared error heavily penalises outliers



Robust M-estimators

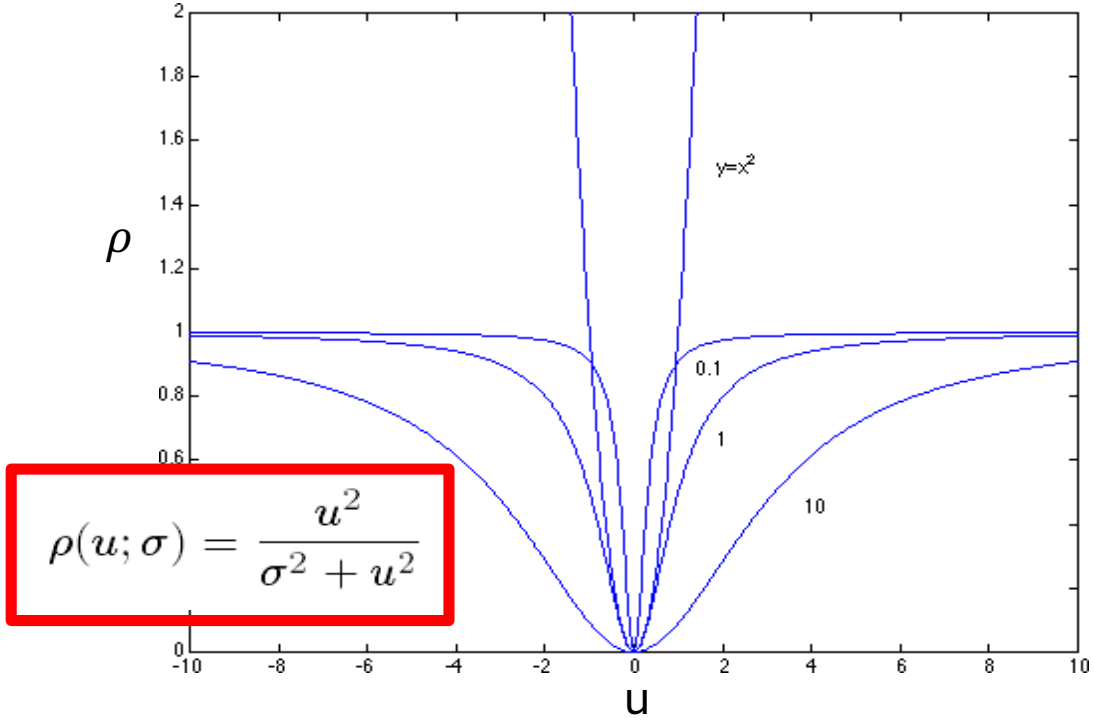
- General approach: find model parameters θ that minimise

$$\sum_i \rho(r_i(x_i, \theta); \sigma)$$

where:

  - r_i is the residual/loss of the ith point w.r.t. model parameters θ
  - ρ is a robust function with scale parameter σ

- Robust function behaves like squared distance for small values of the residual but saturates for larger values



$$\rho(u; \sigma) = \frac{u^2}{\sigma^2 + u^2}$$

Robust M-estimators

- General approach: find model parameters θ that minimise

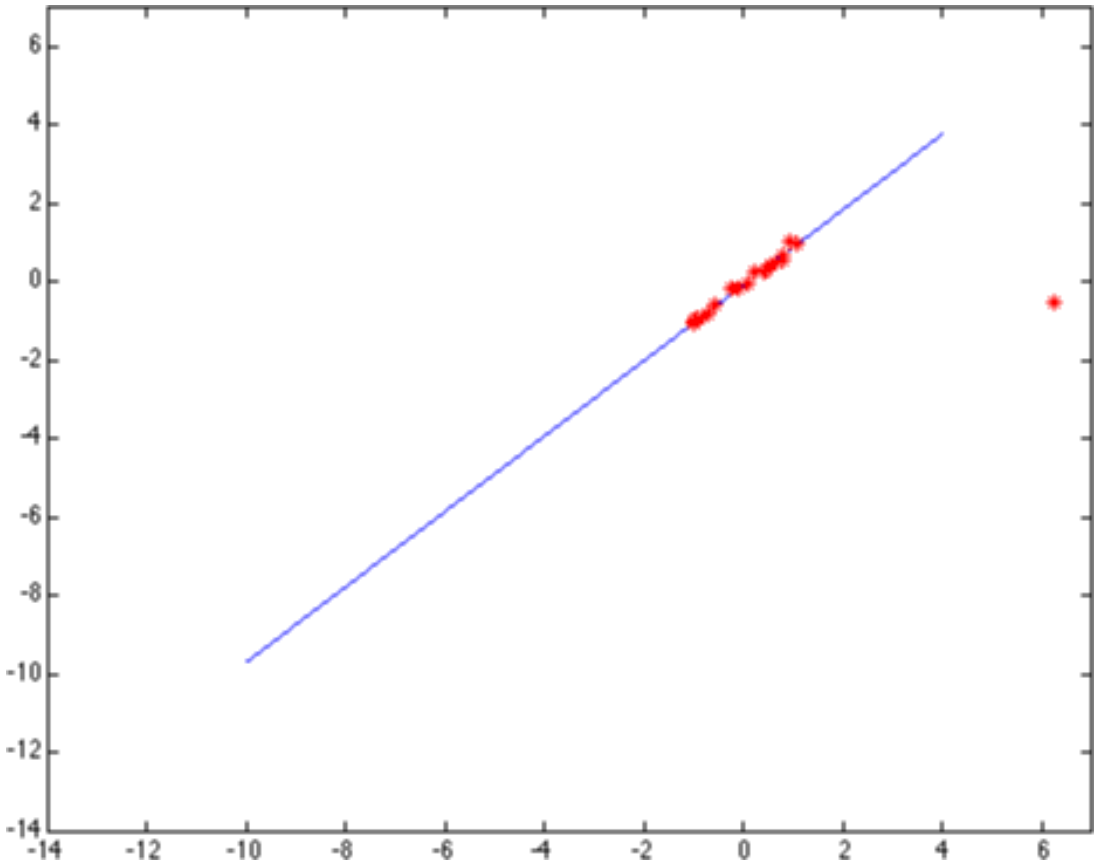
$$\sum_i \rho(r_i(x_i, \theta); \sigma)$$

  where:
  - r_i is the residual/loss of the ith point w.r.t. model parameters θ
  - ρ is a robust function with scale parameter σ

- Robust fitting is a nonlinear optimisation problem that must be solved iteratively
- Least squares solution can be used for initialisation
- Scale of the robust function should be chosen carefully
  - You need to know something about the noise!

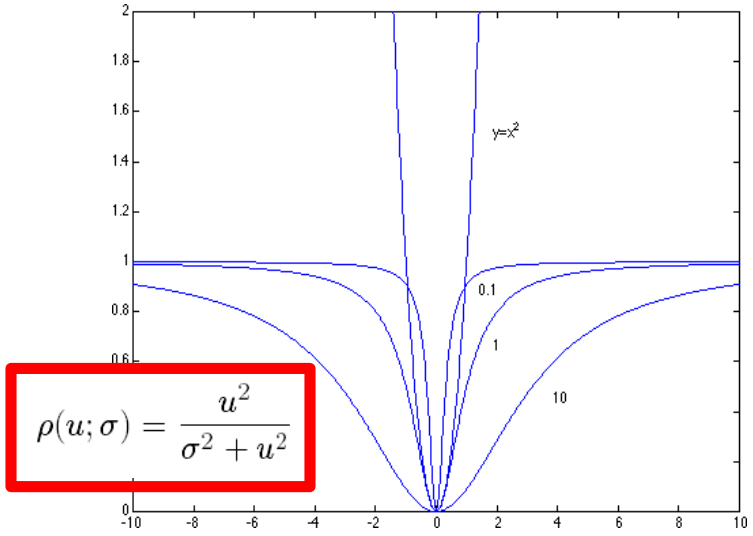
Choosing the Scale: Just Right

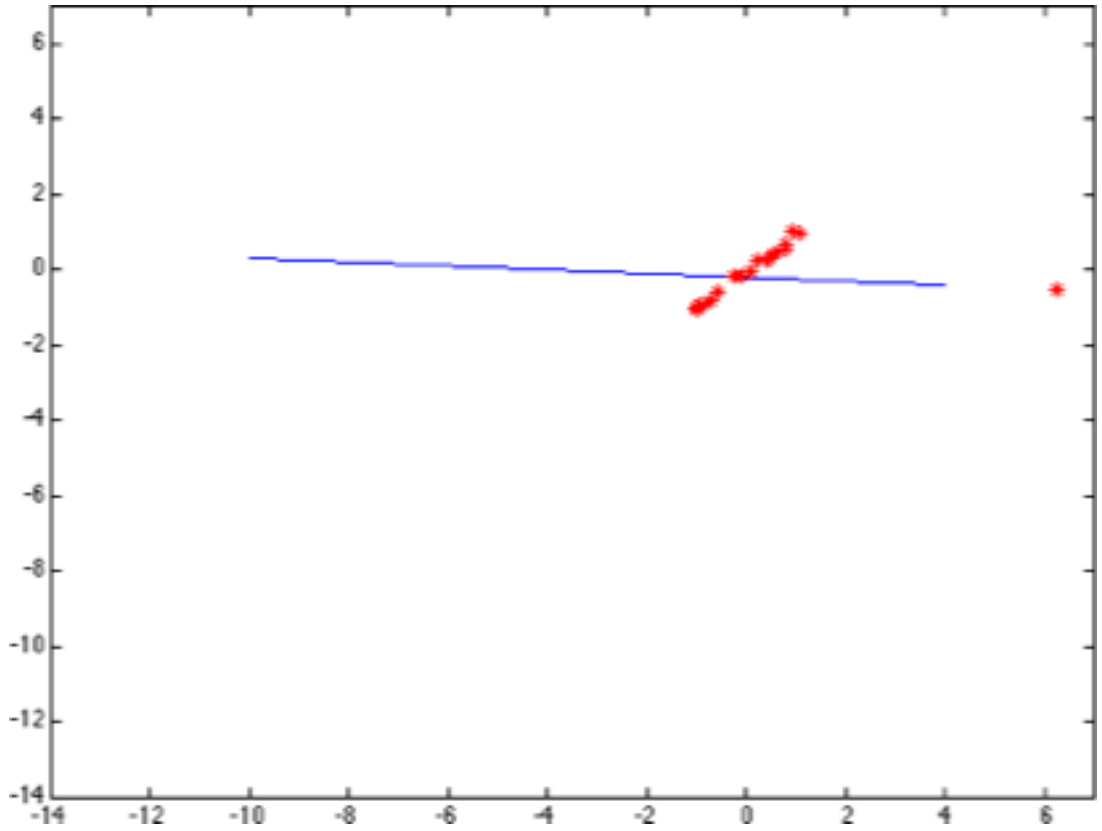
- The effect of the outlier is minimised



Choosing the Scale: Too Small

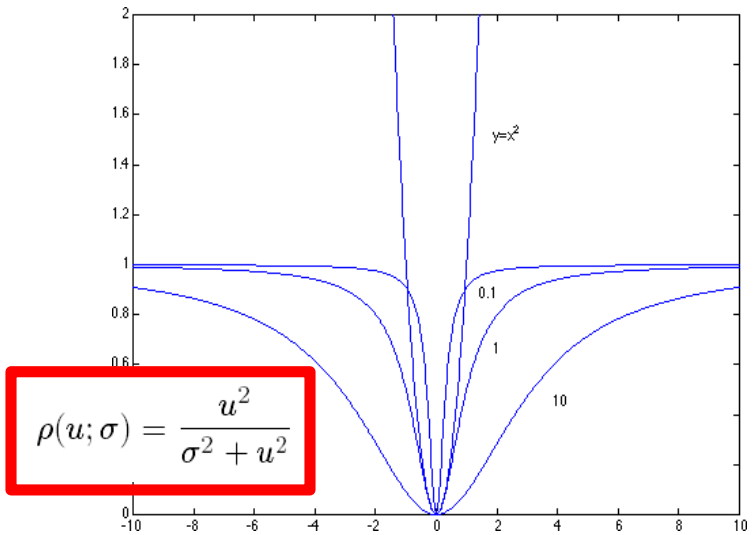
- The error value is almost the same for every point and the fit is very poor



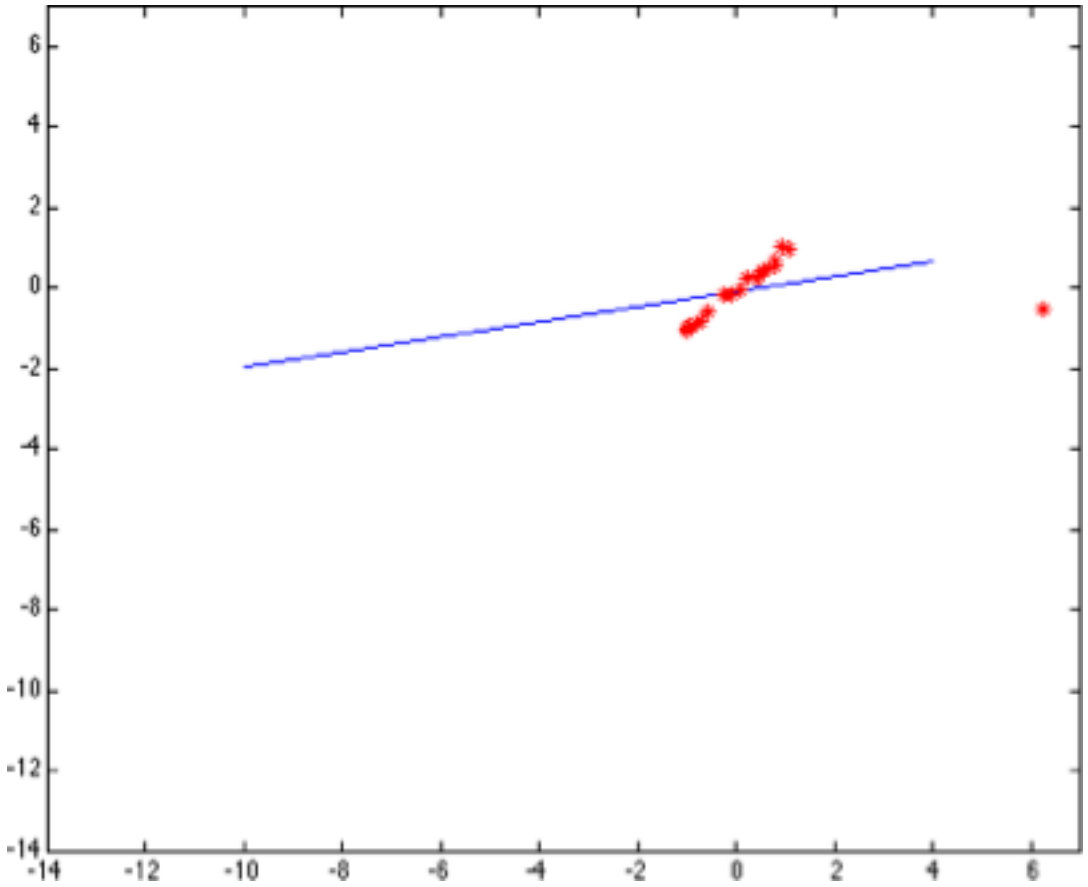


Choosing the Scale: Too Large

- Behaves much the same as least squares



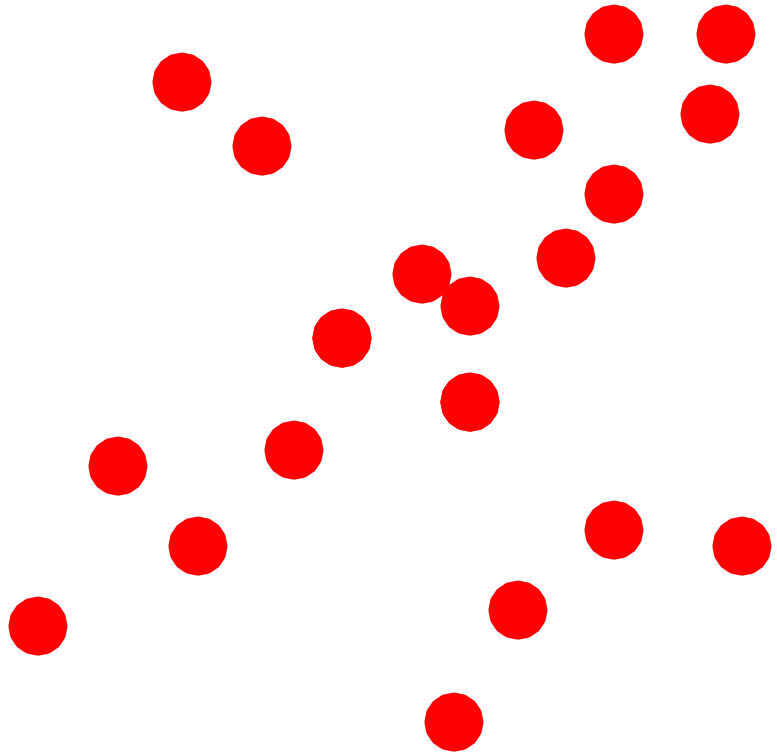
$$\rho(u;\sigma) = \frac{u^2}{\sigma^2 + u^2}$$



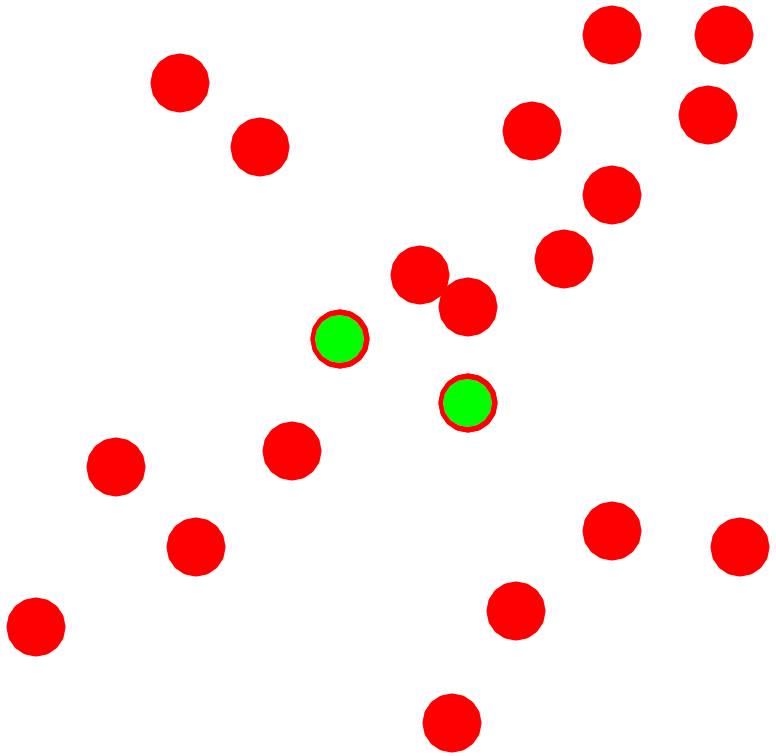
RANSAC

RANSAC: RANdom SAmple Consensus

- Fischler & Bolles 1981
- Basic idea (for line fitting):
  1. Randomly select two samples (minimal number for a line)
  2. Check how many other samples fall close to the line (within a distance threshold)
  3. Choose the line which has the largest number (“consensus”)



RANSAC



Algorithm:

1. **Sample** (randomly) the minimum number of points required to fit the model (n=2)
2. **Solve** for model parameters
3. **Score** by the fraction of inliers within a preset threshold

Repeat 1–3 until the best model is found

Illustration by Savarese

RANSAC

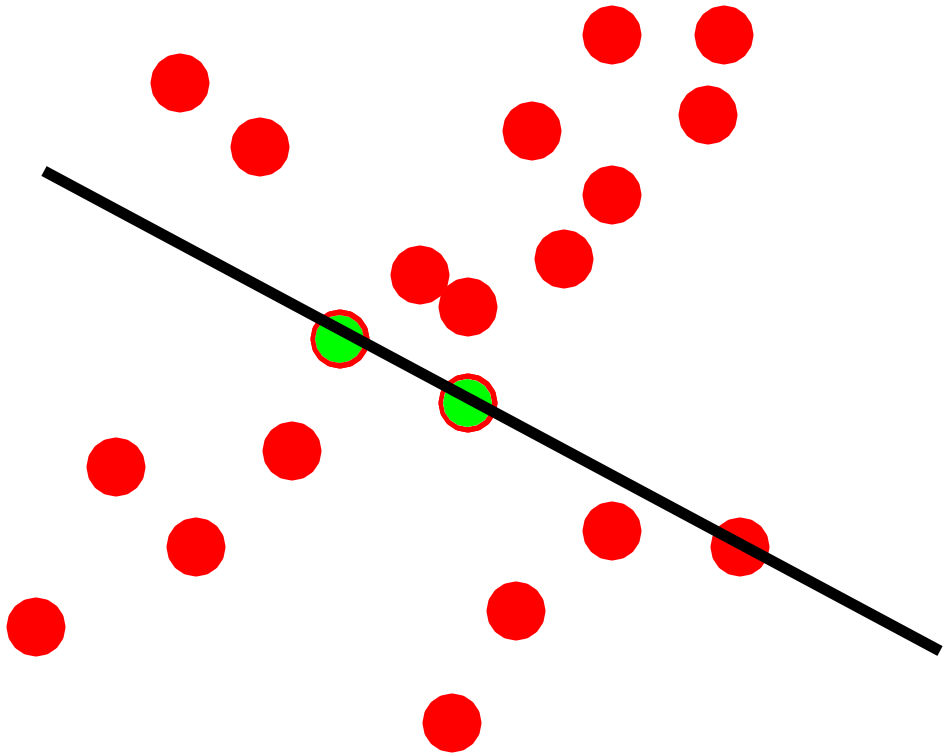


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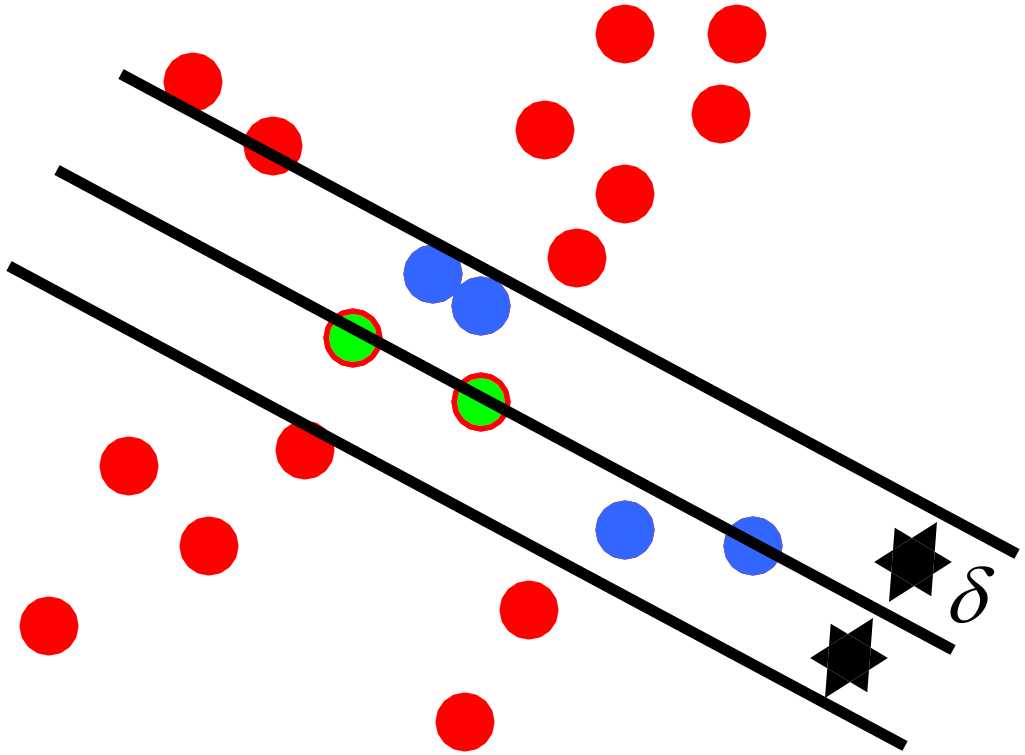
Illustration by Savarese

RANSAC

$N_I = 6$



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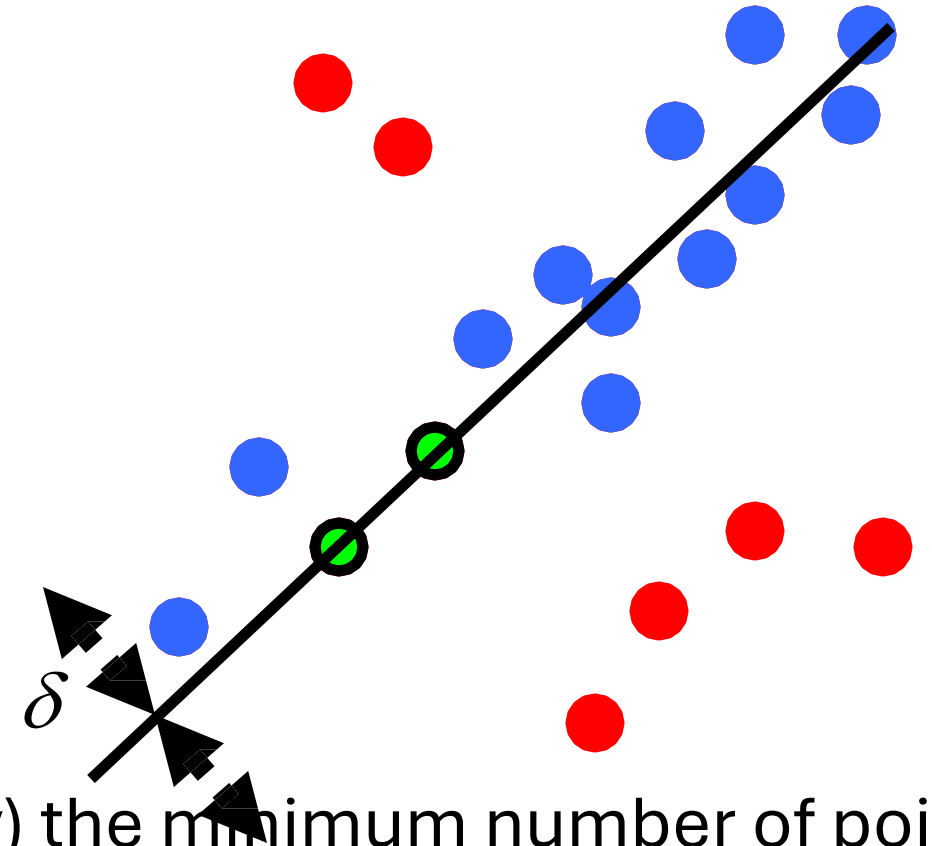
Repeat 1–3 until the best model is found

RANSAC

$N_I = 14$



Algorithm:

1. **Sample** (randomly) the minimum number of points required to fit the model (n=2)
2. **Solve** for model parameters
3. **Score** by the fraction of inliers within a preset threshold

Repeat 1–3 until the best model is found

RANSAC: How Many Samples to Choose?

- e: probability that a point is an outlier
  s: number of points in a sample
  N: number of samples (we want to compute this)
  p: desired probability that we get a good sample

$$1 - \left(1 - (1 - \mathrm{e})^{\mathrm{s}}\right)^{N} = p$$

- Probability of choosing s inliers (sample only contains inliers)

RANSAC: How Many Samples to Choose?

- e: probability that a point is an outlier
  s: number of points in a sample
  N: number of samples (we want to compute this)
  p: desired probability that we get a good sample

$$1 - \left(1 - (1 - \mathrm{e})^{\mathrm{s}}\right)^{N} = p$$

- Probability that one or more points in the sample were outliers (sample is contaminated)

RANSAC: How Many Samples to Choose?

- e: probability that a point is an outlier
  s: number of points in a sample
  N: number of samples (we want to compute this)
  p: desired probability that we get a good sample

$$1 - \left(1 - (1 - \mathrm{e})^{s}\right)^{N} = p$$

- Probability that all the *N* samples were contaminated

RANSAC: How Many Samples to Choose?

- e: probability that a point is an outlier
  s: number of points in a sample
  N: number of samples (we want to compute this)
  p: desired probability that we get a good sample

$$1 - \left(1 - (1 - \mathrm{e})^{\mathrm{s}}\right)^{N} = p$$

- Probability that at least one sample is good

RANSAC: How Many Samples to Choose?

- e: probability that a point is an outlier
  s: number of points in a sample
  N: number of samples (we want to compute this)
  p: desired probability that we get a good sample

$$1 - (1 - (1 - e)^s)^N = p$$
$$\therefore (1 - (1 - e)^s)^N = 1 - p$$
$$\therefore N \log(1 - (1 - e)^s) = \log(1 - p)$$
$$\therefore N = \frac{\log(1 - p)}{\log(1 - (1 - e)^s)} = 16.008 \rightarrow 17$$

- For a 99% probability of obtaining a good model, with a 50% outlier probability and s=2 samples per model

Sample size	Proportion of outliers						
N	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Figure Credit: Hartley & Zisserman

RANSAC: Pros and Cons

- Pros:
  - Robust to outliers
  - Applicable for large number of parameters
- Cons:
  - Computational time grows quickly with the fraction of outliers and the number of parameters
  - Not good for getting multiple line fits
  - Can rapidly find a good model, not necessarily a great model
- Common applications:
  - Computing a rotation and translation between two images (will be used later for fundamental matrix in 2- view geometry)

50

Hough Transform

51

Fitting Multiple Lines Using a Hough Transform

- Given a binary edge image, find the lines (or curves) that explain the data points best in the parameter space
- This parameter space is called a Hough space

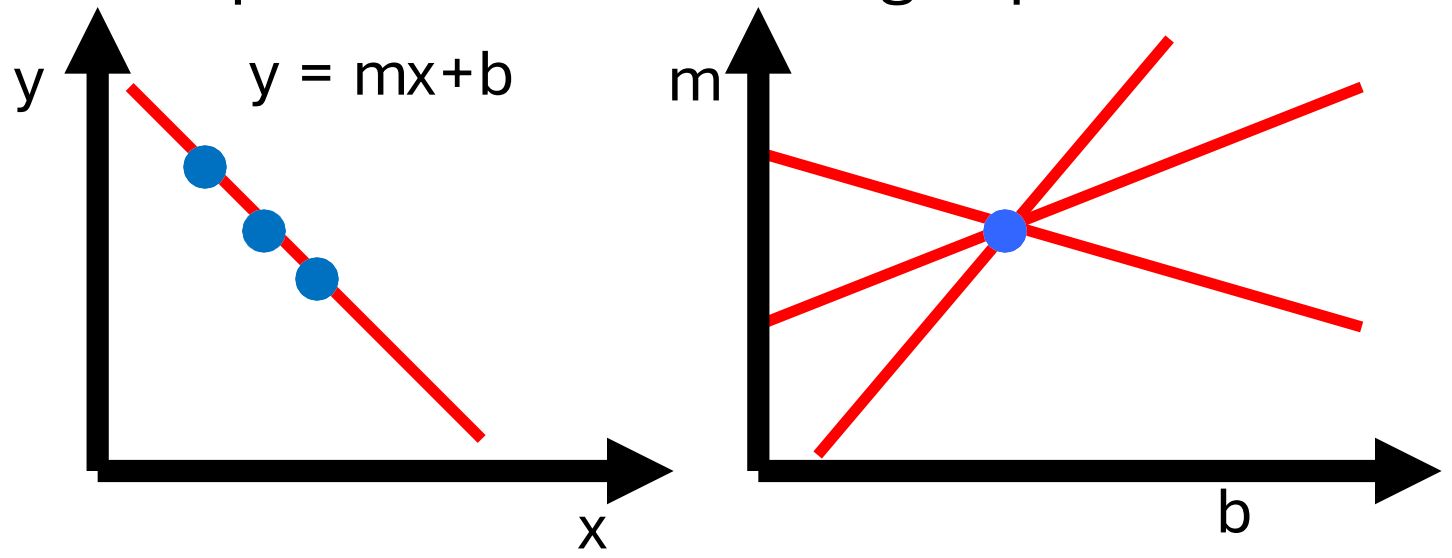


Image space Parameter (Hough) space

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959. (Patented)

A point (x_1, y_1) is mapped to a line in Hough space

$y_1 = m x_1 + b$

$b = -x_1 m + y_1$ (rewrite the equation in terms of m and b)

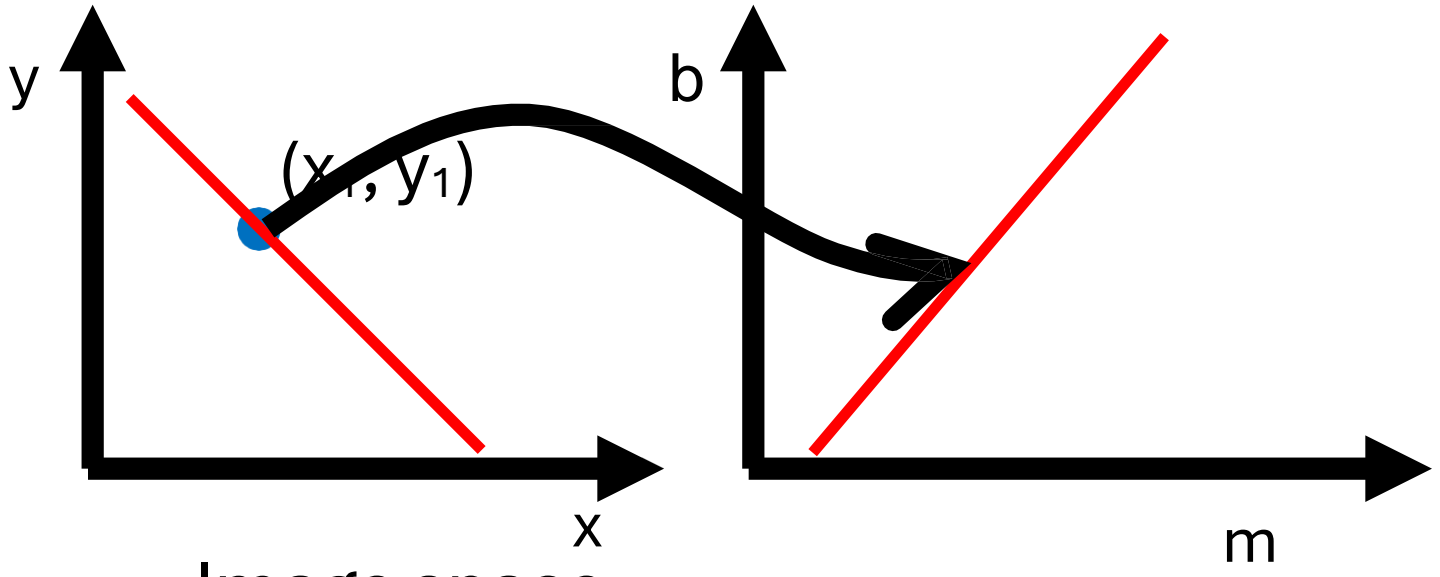


Image space

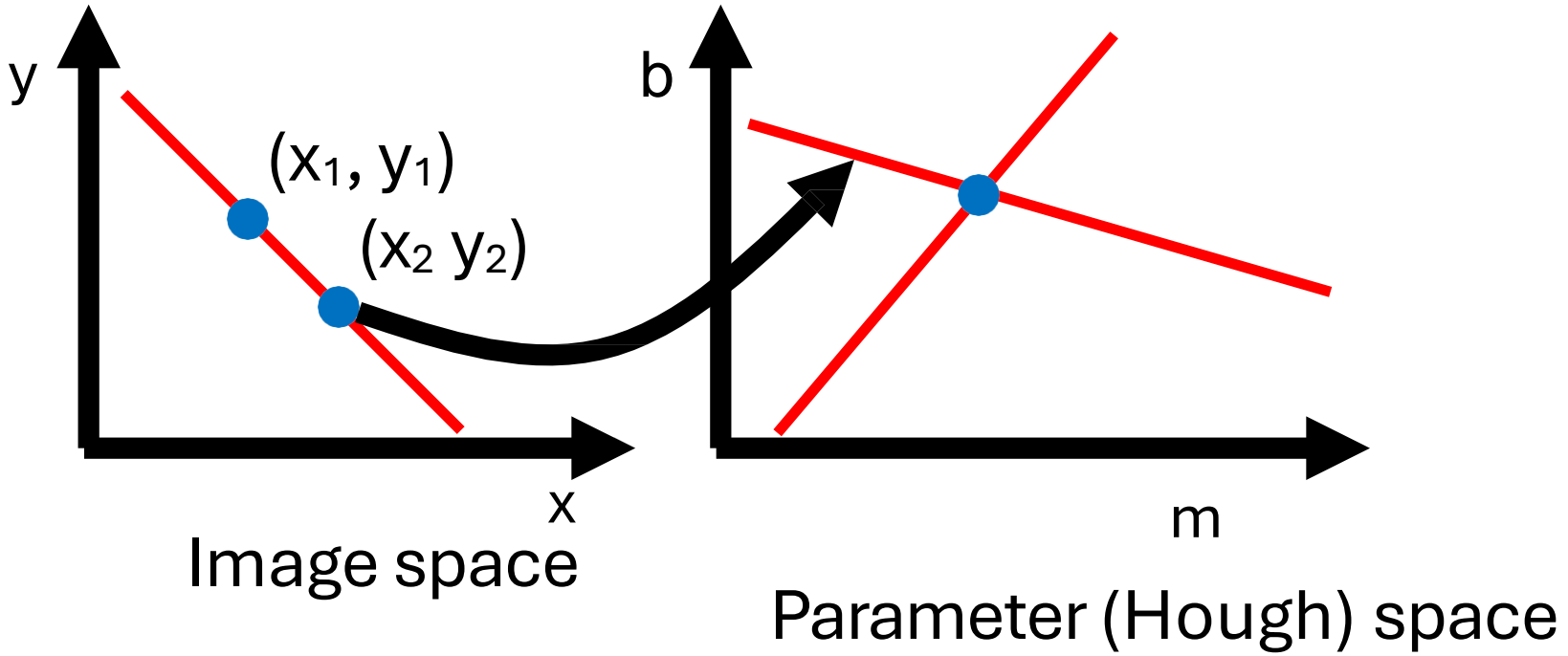
Parameter (Hough) space

A point (x_2, y_2) is mapped to a line in Hough space

$y_2 = m x_2 + b$

$b = -x_2 m + y_2$

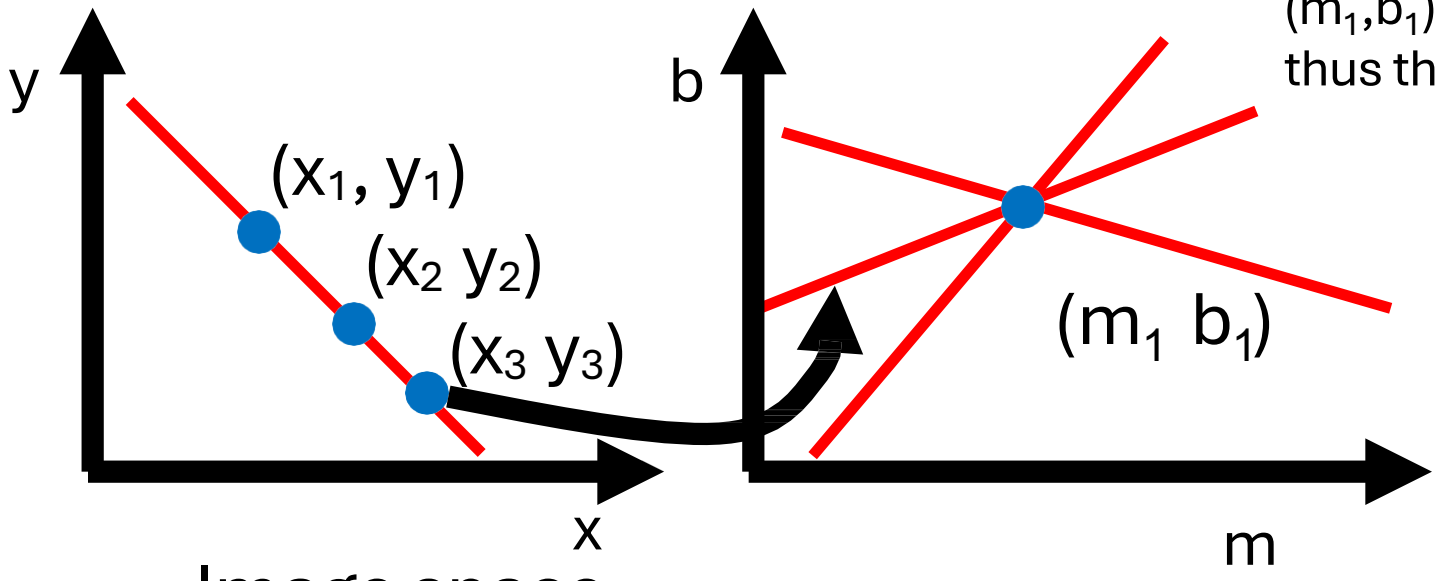
The intersection point satisfies both (x_1, y_1) and (x_2, y_2), thus the line



A point (x_3, y_3) is mapped to a line in Hough space

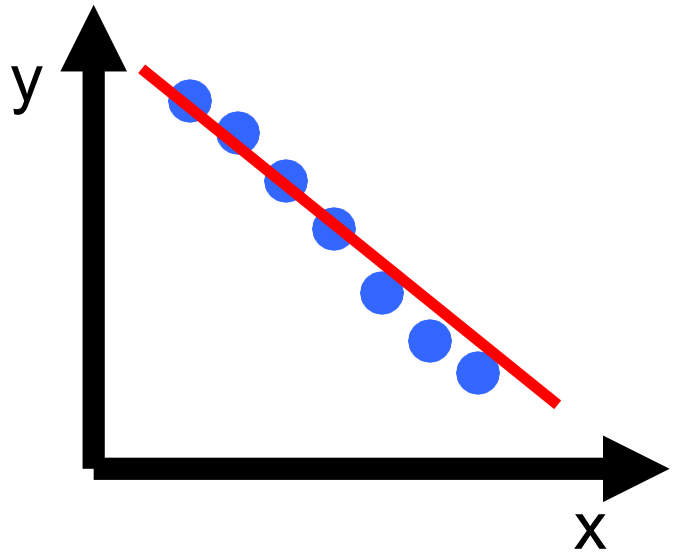
$y_3 = m x_3 + b$

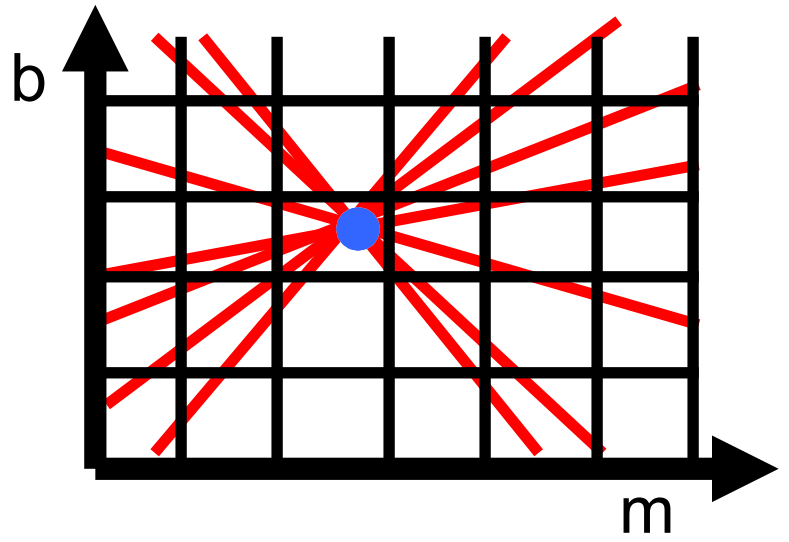
$b = -x_3 m + y_3$



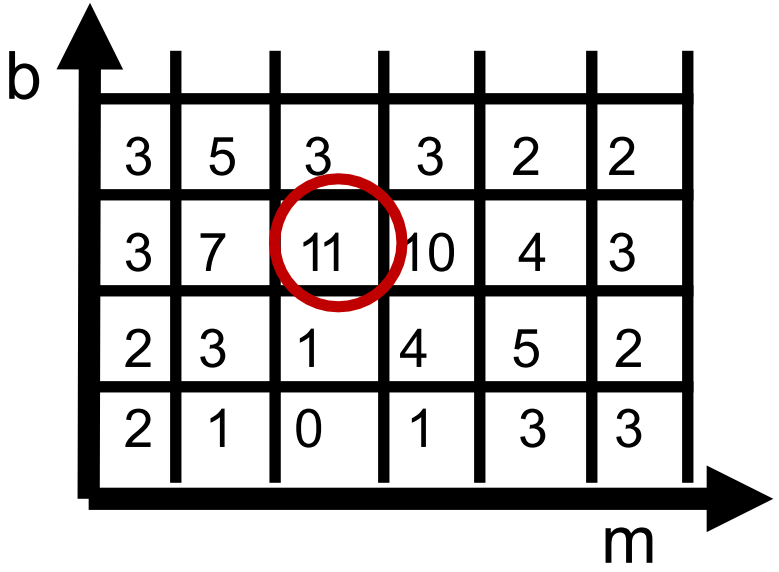
The intersection point (m_1, b_1) satisfies 3 points, thus the line

Hough Transform: Accumulator





- For each pixel, draw a line in the discretised Hough space, assigning a value of one to all the discretised positions it passes through
- Process all image pixels

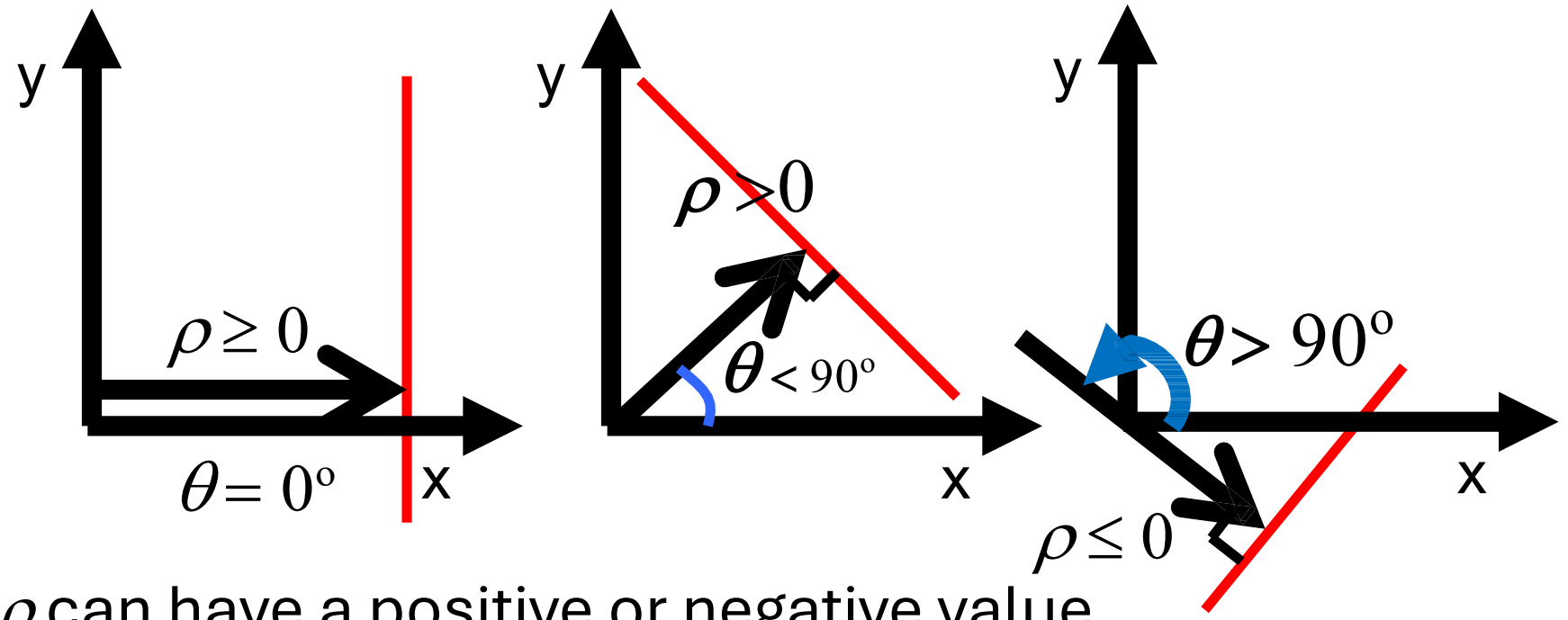


Hough Transform

- Can detect multiple lines in an image
  - from multiple local maxima
- Can easily be extended for circles and ellipses
- Computationally efficient
- Problem: *(m, b)* are unbounded
  - E.g., the slope parameter *m* can have an infinite value

Hough Transform: Polar Form

- Use a polar representation for the parameter space
- $\mathrm{x}\cos\theta + \mathrm{y}\sin\theta = \rho \rightarrow y = -\frac{\cos\theta}{\sin\theta}x + \frac{\rho}{\sin\theta}$ $\quad (0 \le \theta \le 180^o)$



- Note: ρ can have a positive or negative value

A point (x_1, y_1) is mapped to a sinusoid in the polar parameter space

$$x_1 \cos\theta + y_1 \sin\theta = \rho$$

$$\rho = x_1 \cos\theta + y_1 \sin\theta$$
$$= \alpha_1 \sin(\theta + \beta_1)$$

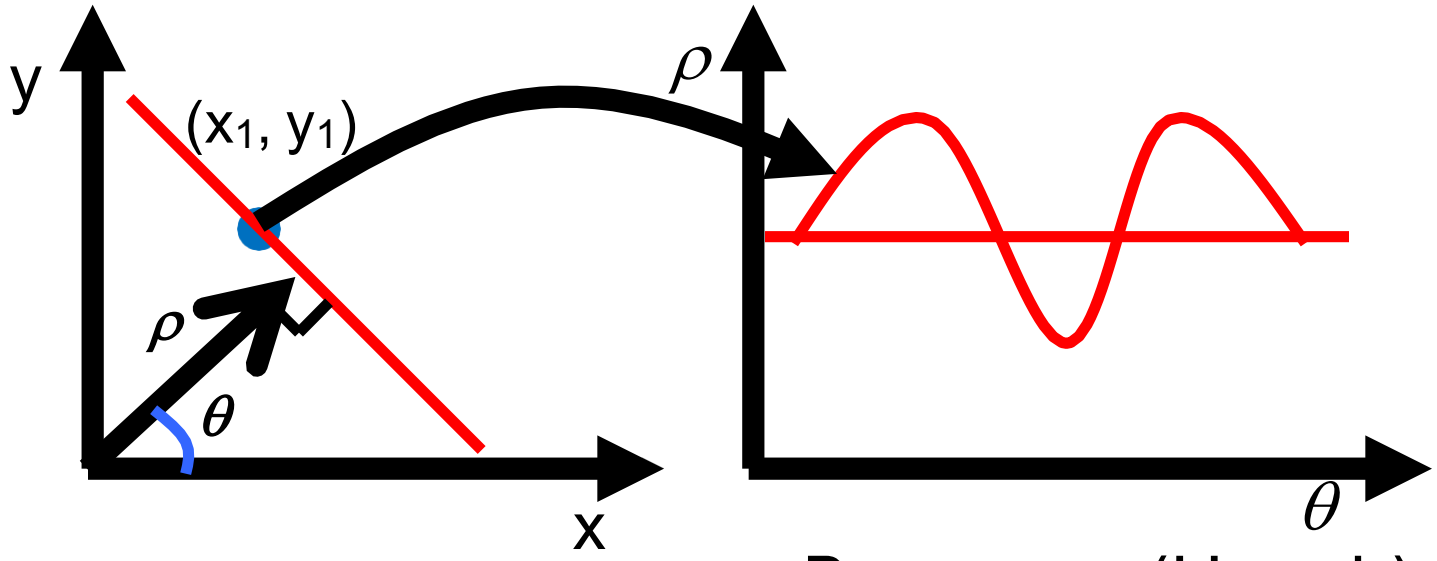


Image space

Parameter (Hough) space

$0 \le \theta \le 180^o$

How to map: just divide the right-hand side by $\sqrt{x_1^2 + y_1^2}$ and multiply by the same

A point (x_2, y_2) is mapped to a sinusoid in the polar parameter space

$$x_2 \cos\theta + y_2 \sin\theta = \rho$$

$$\rho = x_2 \cos\theta + y_2 \sin\theta$$
$$= \alpha_2 \sin(\theta + \beta_2)$$

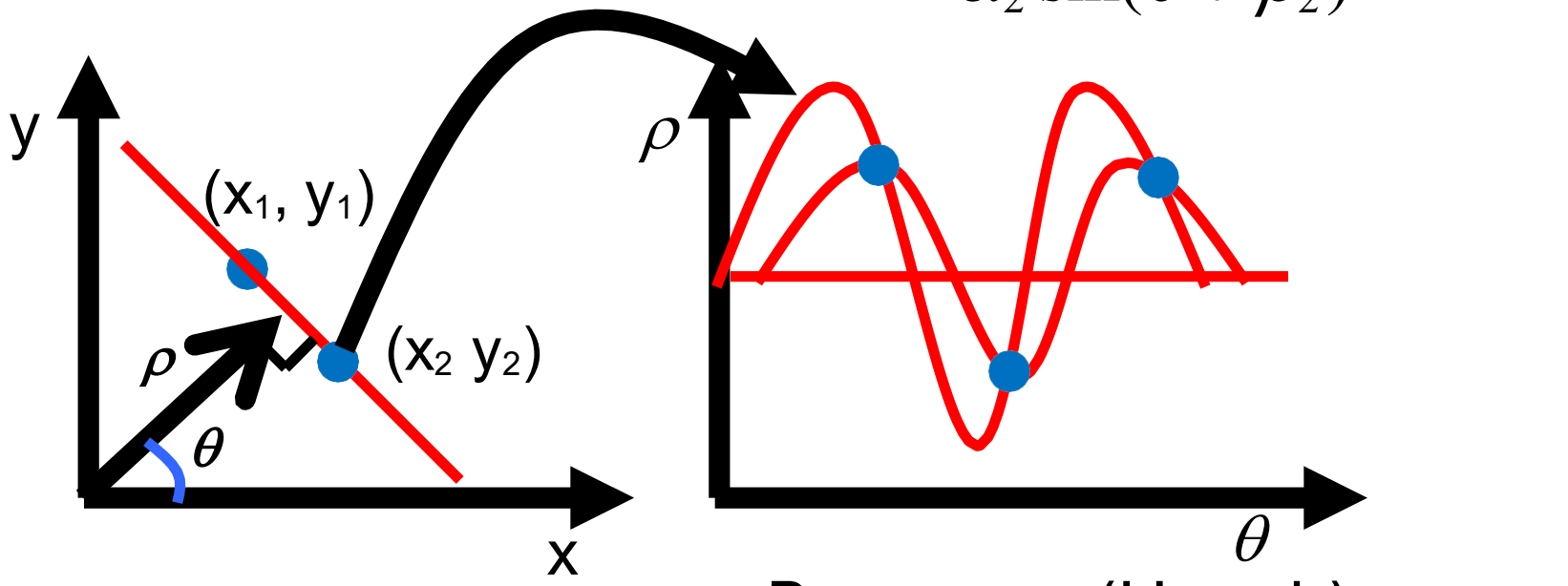


Image space

Parameter (Hough) space

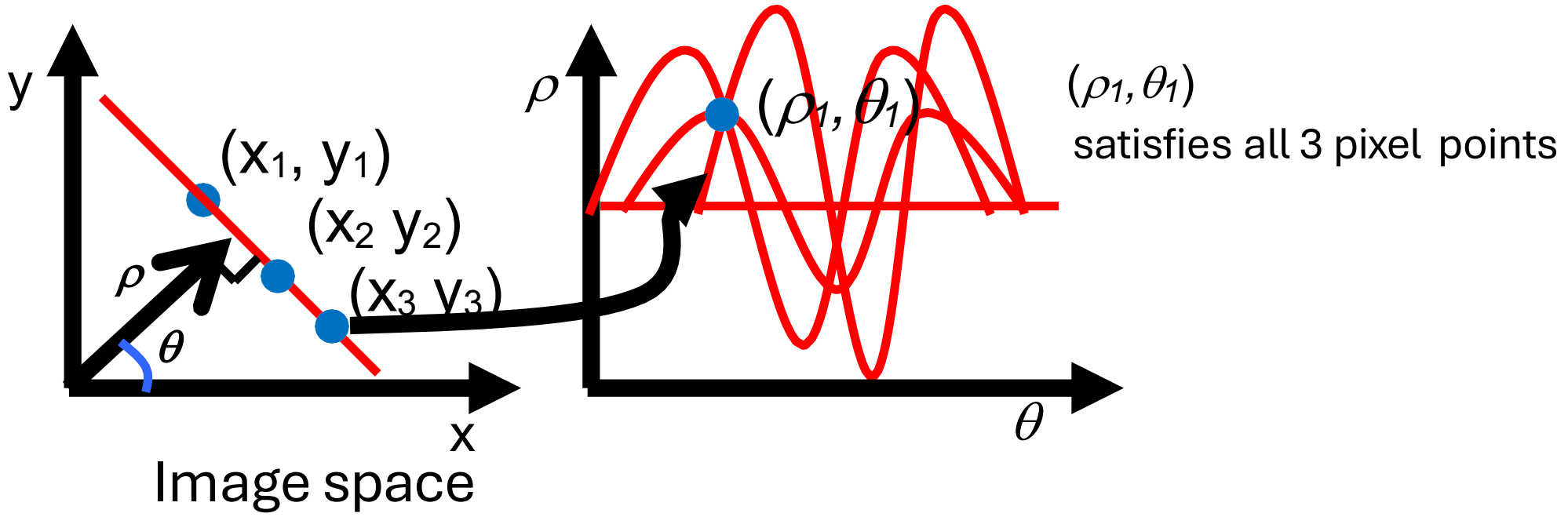
$$0 \le \theta \le 180^o$$

A point (x_3, y_3) is mapped to a sinusoid in the polar parameter space

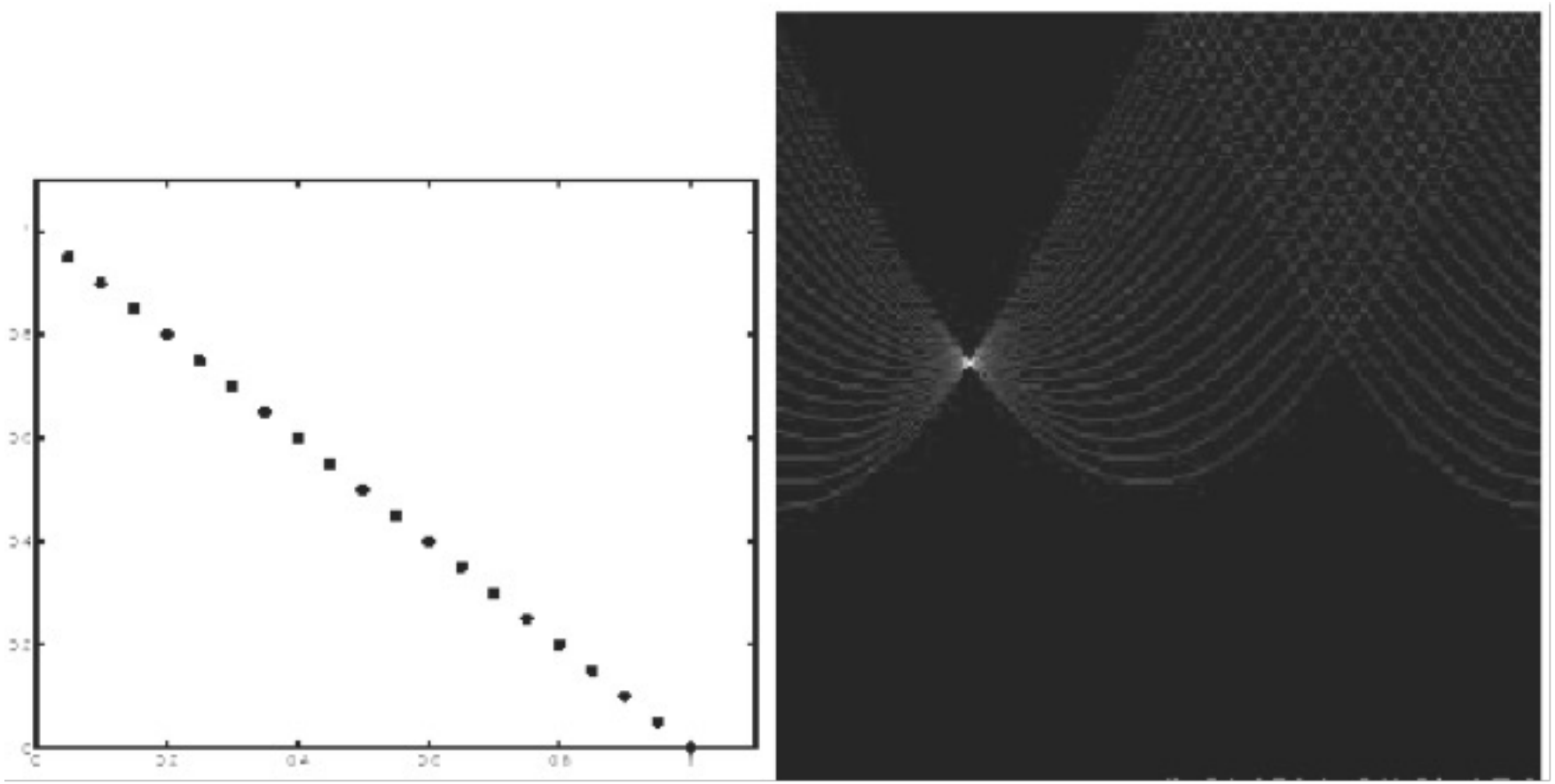
$$x_3 \cos\theta + y_3 \sin\theta = \rho$$

$$\rho = x_3 \cos\theta + y_3 \sin\theta$$

$$= \alpha_3 \sin(\theta + \beta_3)$$



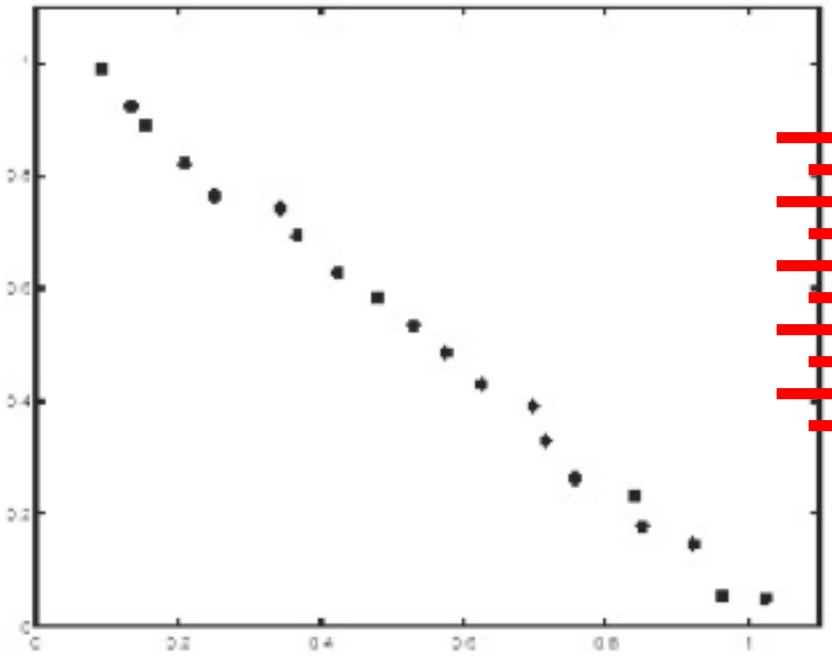
Hough Transform: Example

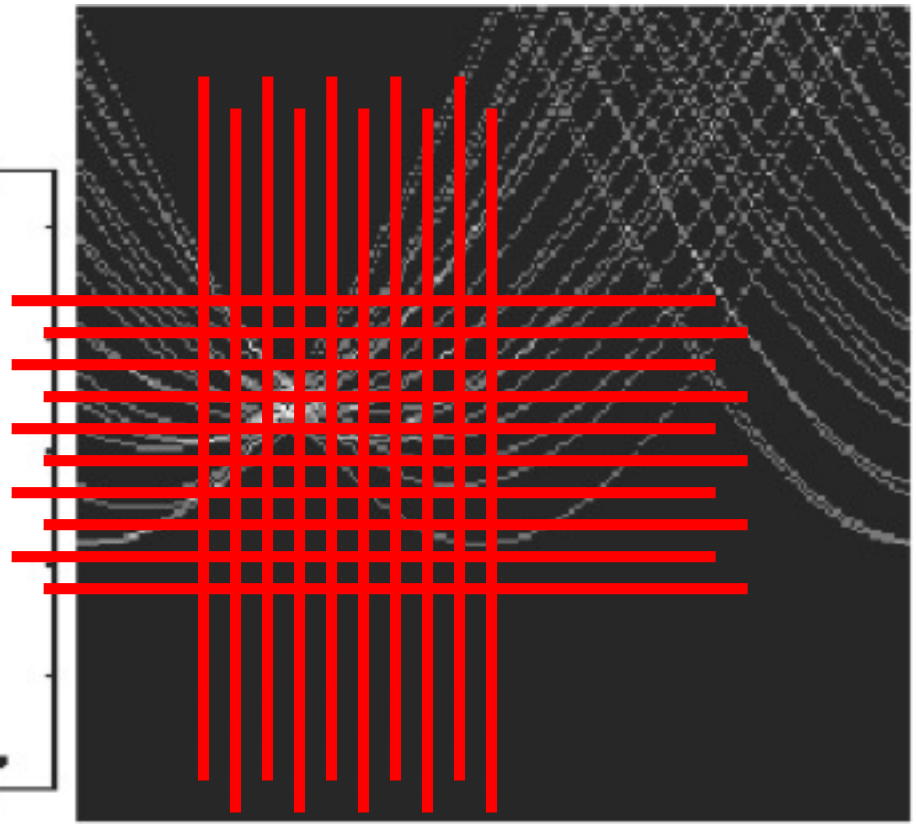


features votes

Hough Transform: Example

Noisy data



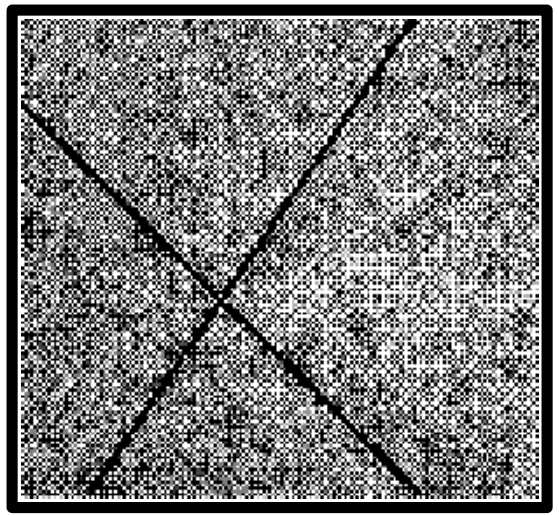


features

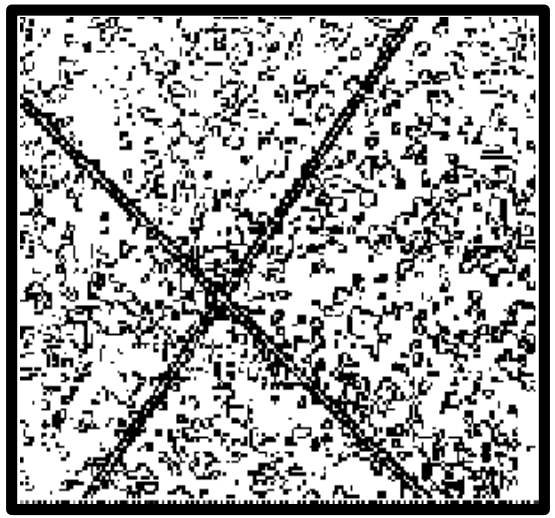
votes

Issue: the grid size needs to be adjusted…

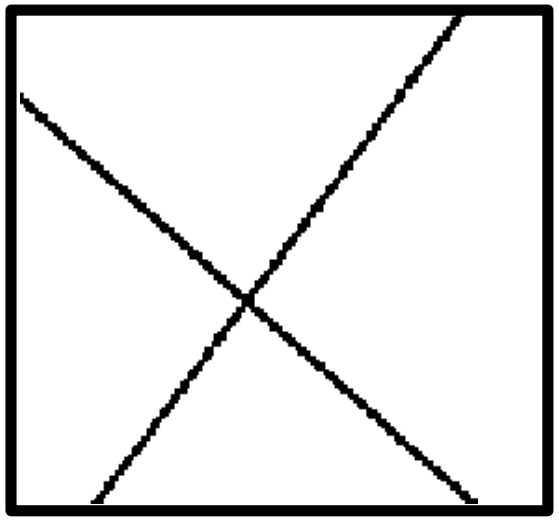
Hough Transform: Example



Image



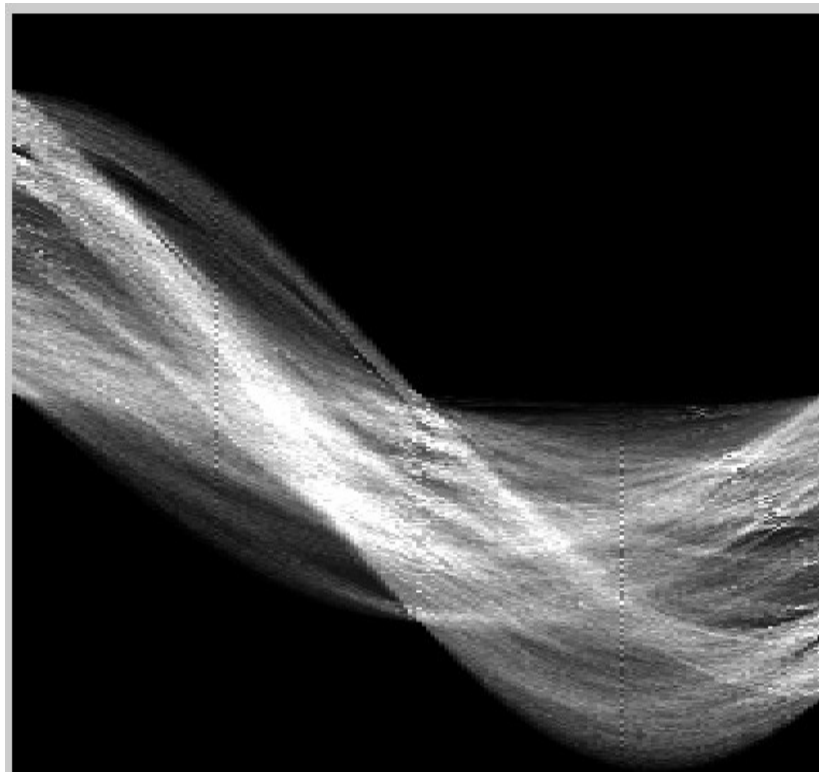
Edge Detection



Hough Transform

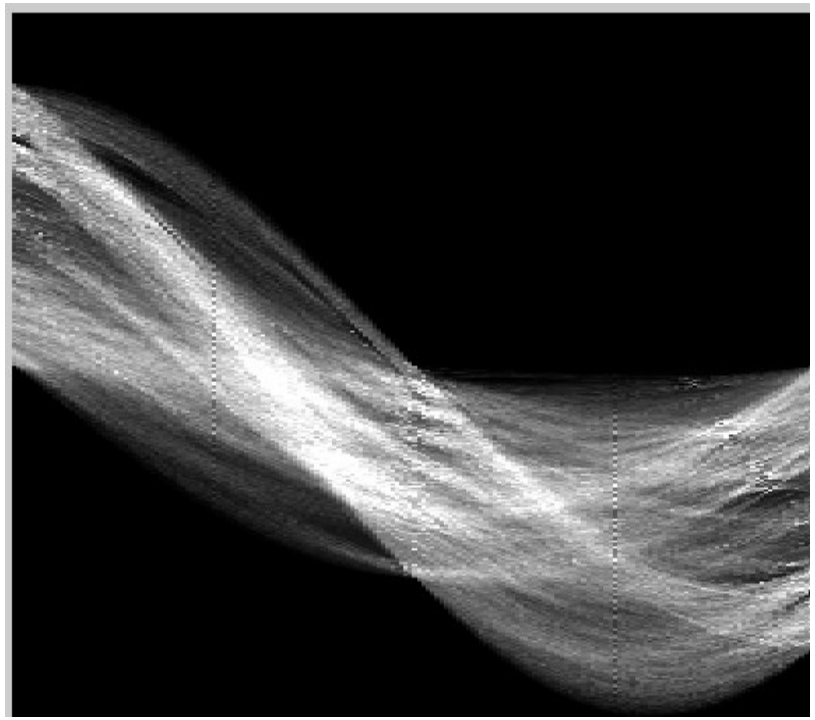
Hough Transform: 1) Image → Canny Edges

Hough Transform: 2) Canny Edges → Votes

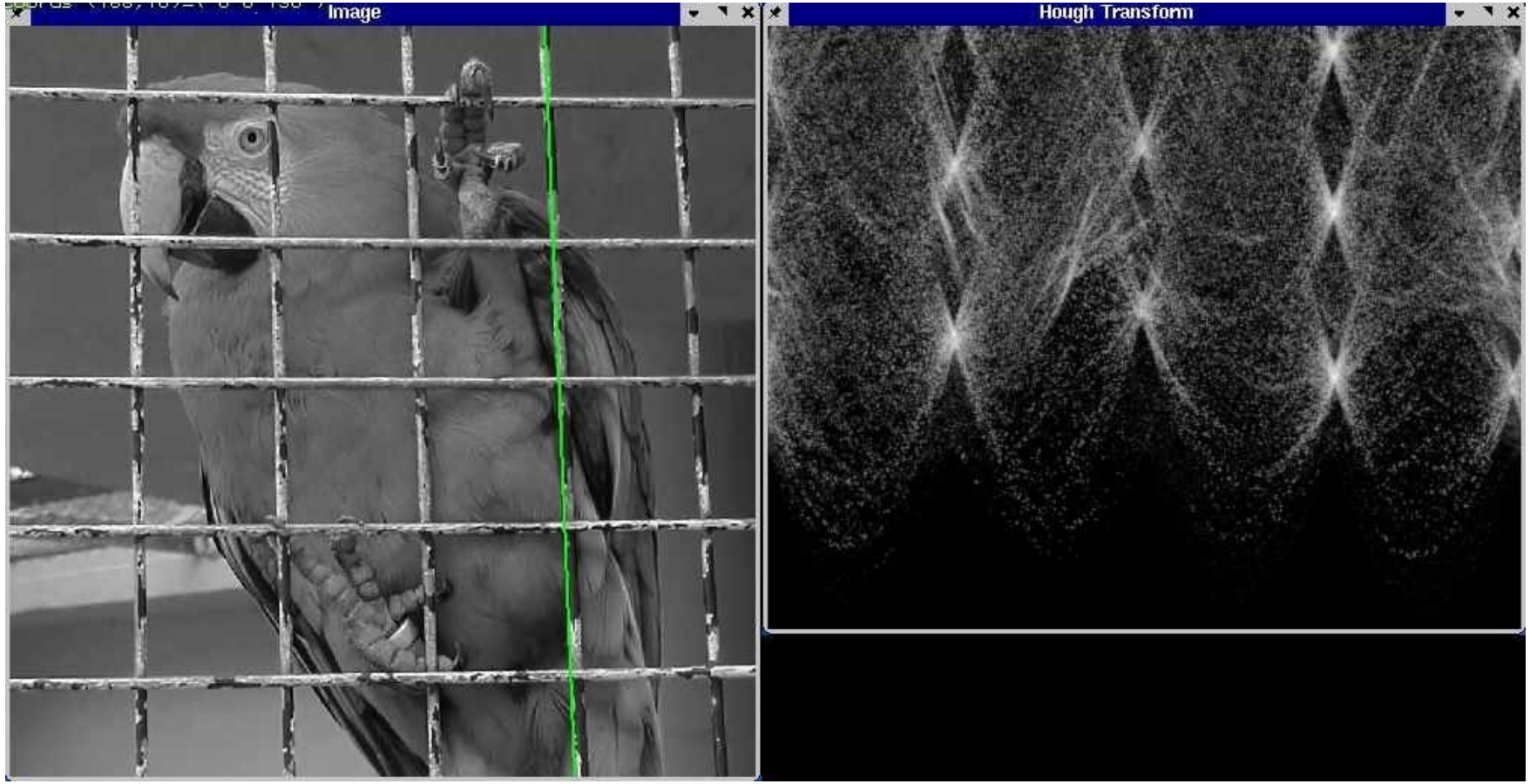


Hough Transform: 3) Votes → Lines

- Find peaks and post-process

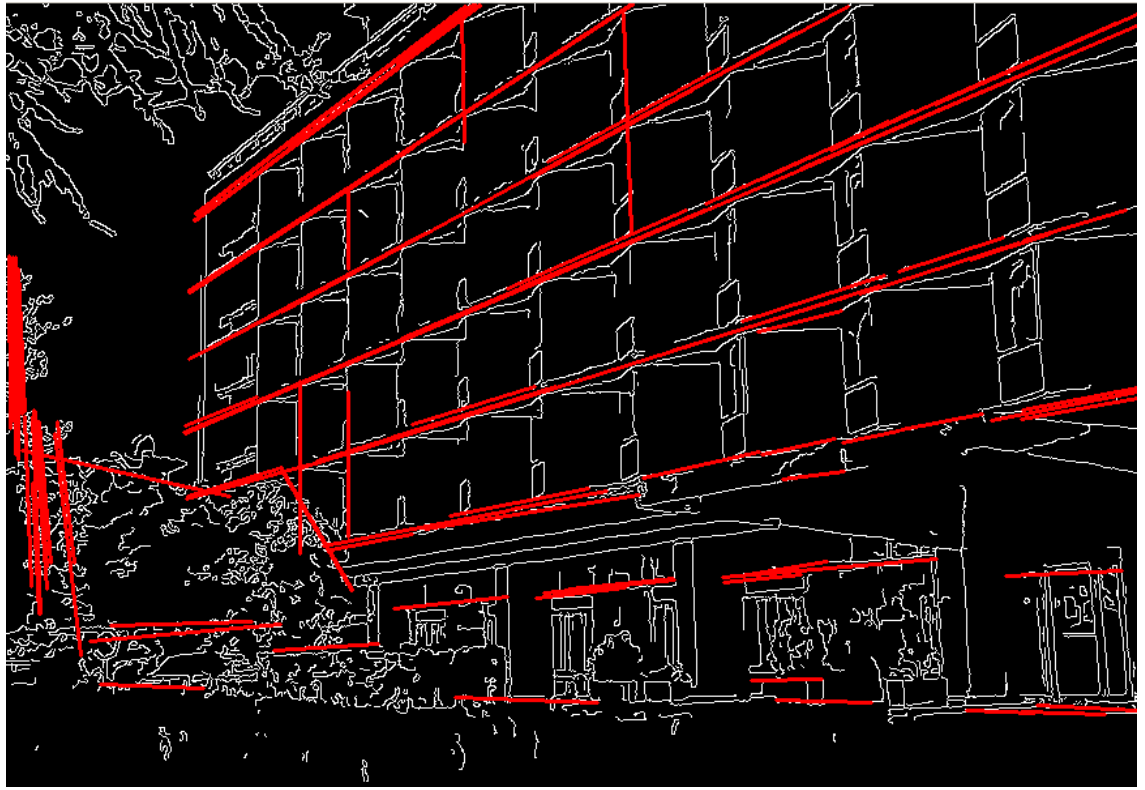


Hough Transform: Example



http://ostatic.com/files/images/ss_hough.jpg

Hough Transform: Example



Hough Transform: Pros and Cons

Pros:

- All points processed independently, so can cope with occlusion
- Some robustness to noise: noisy points are unlikely to contribute consistently to any single bin
- Can detect multiple instances of line/circle/ellipse in a single pass

Cons:

- Complexity of search time increases exponentially with the number of model parameters
- Non-target shapes can produce spurious peaks in parameter space
- Quantization: hard to pick a grid size

Summary

- Line features can be extracted using least squares or RANSAC fitting methods
- Both have difficulties in fitting multiple lines
- The Hough transform can handle multiple lines and the polar form is efficient due to the bounded search region
- Further reading: Chapter 4.3 in the book
- We will apply RANSAC to 3D Vision in future lectures