3D Vision 4

Week 10

Multiple-view Geometry: Structure-from-Motion

Multiple-view Geometry: Novel View Synthesis

Announcements

- **Assignment 3** due 11:59pm Friday 17 May
  - **Zero** marks if either report or code submitted late (unless extension)
    - Submit early; you can always resubmit an updated version later
    - Depending on your internet connection and load on the TurnItIn servers, uploading can sometimes be slow, so please factor this into your submission schedule
  - Submit your report (PDF) and code (ZIP file) **separately under the correct tab** in the submission box
  - Follow the instructions under Submission Requirements

Weekly Study Plan: Overview

Wk	Starting	Lecture	Lab	Assessment
1	19 Feb	Introduction	✗	
2	26 Feb	Low-level Vision 1	1	
3	4 Mar	Low-level Vision 2	1	
		Mid-level Vision 1		
4	11 Mar	Mid-level Vision 2	1	CLab1 report due Friday
		High-level Vision 1		
5	18 Mar	High-level Vision 2	2	
6	25 Mar	High-level Vision 3[1]	2	
	1 Apr	Teaching break	✗	
	8 Apr	Teaching break	✗	
7	15 Apr	3D Vision 1	2	CLab2 report due Friday
8	22 Apr	3D Vision 2	3	
9	29 Apr	3D Vision 3	3	
10	6 May	3D Vision 4	3	
		Mid-level Vision 3		
11	13 May	High-level Vision 4	✗	CLab3 report due Friday
12	20 May	Course Review	✗	

Weekly Study Plan: Part B

Wk	Starting	Lecture	By
7	15 Apr	3D vision: introduction, camera model, single-view geometry	Dylan
8	22 Apr	3D vision: camera calibration, two-view geometry (homography)	Dylan
9	29 Apr	3D vision: two-view geometry (epipolar geometry, triangulation, stereo)	Dylan
10	6 May	3D vision: multiple-view geometry	Weijian
		Mid-level vision: optical flow, shape-from-X	Dylan
11	13 May	High-level vision: self-supervised learning, detection, segmentation	Dylan
12	20 May	Course review	Dylan

Outline

1. Multiple-view Geometry: Structure-from-Motion
2. Multiple-view Geometry: Novel View Synthesis

Multi-View Geometry

Structure From Motion (SfM)

Extending two-view epipolar geometry to multiple views

Neural Radiance Fields (NeRFs), ECCV 2020

Solves correspondences + triangulation implicitly

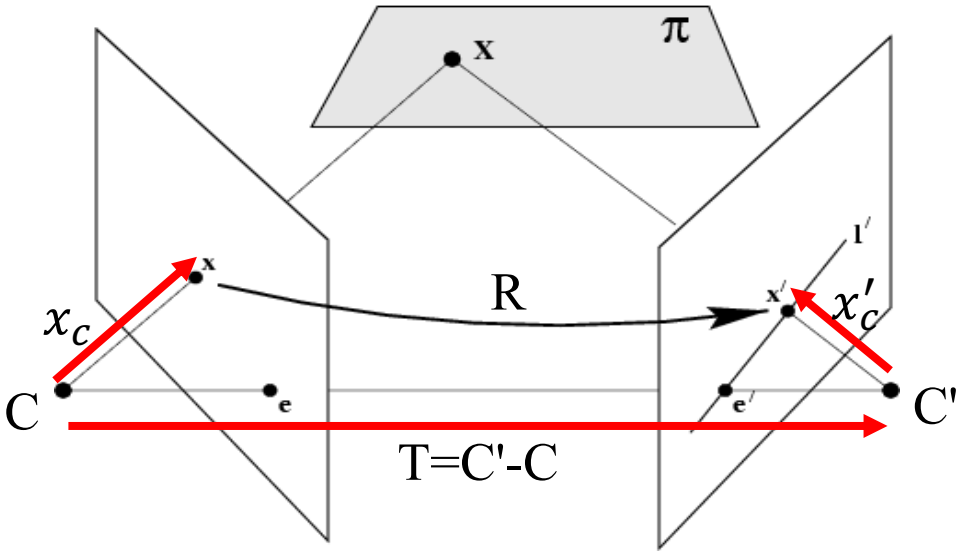
DUSt3R: Geometric 3D Vision Made Easy, CVPR 2024

Deep-learning based method, unify all 3D vision tasks

Structure From Motion

- Epipolar Geometry

  Geometric relationship between **two views** of the same scene captured from different viewpoints.



Epipolar Geometry

i) Fundamental Matrix

$$x_{img}'^{T} F x_{img} = 0$$

$$F = K'^{-T} E K^{-1}$$

ii) Essential Matrix

$$x_c'^{T} E x_c = 0$$

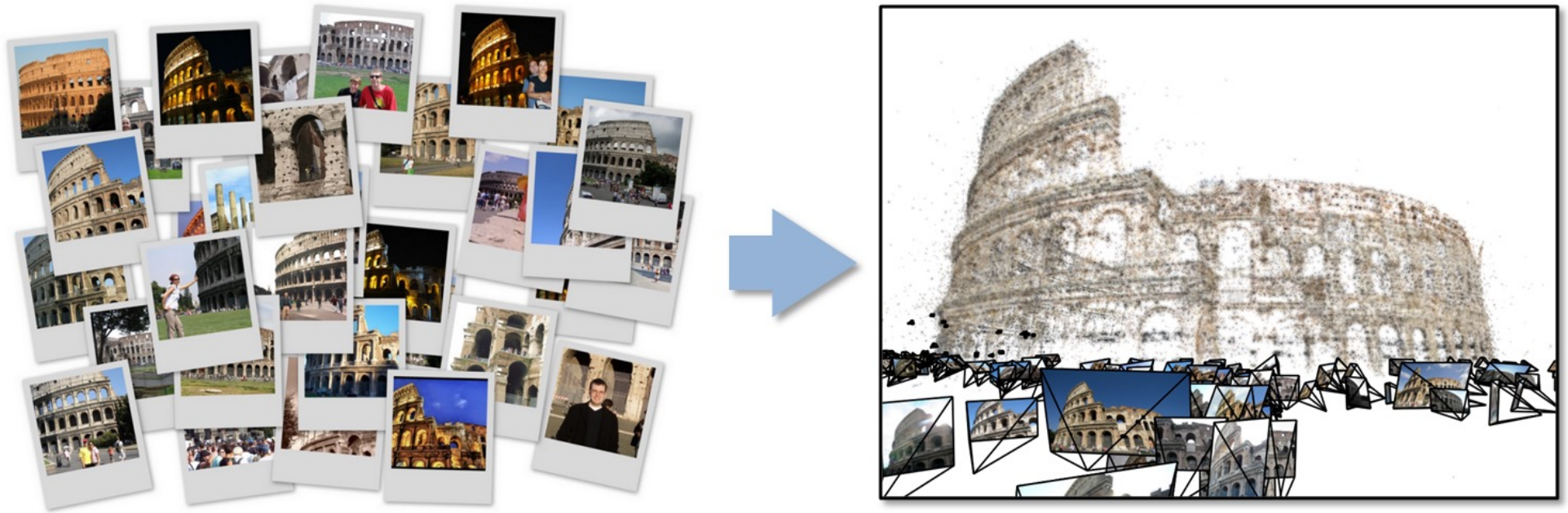
$$E = [t]_{\times} R$$

iii) Triangulation: Initial 3D points

Structure From Motion

- What is structure from motion?

  Structure from Motion is a 3D vision technique that **builds 3D structures from multiple 2D images**.



Structure From Motion

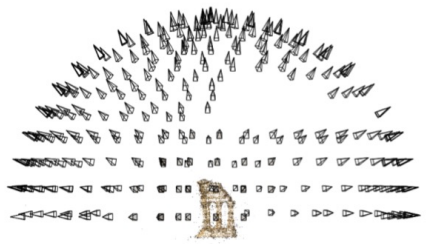
- What is structure from motion?

Input: multiple camera views (or video)

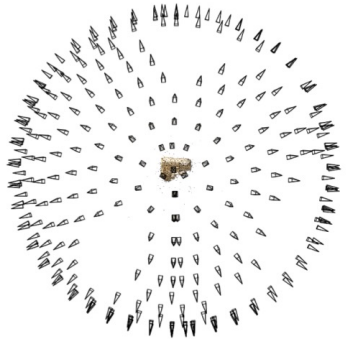
Output:
1) 3D structure: captured scene (3D location for 2D pixel)
2) camera "motion": camera parameters (intrinsic and extrinsic matrix)

Objective: minimises reprojection error (3D→2D)

$$x_{img} = PX_{world}$$
$$P = K[R|t]$$



Reconstruction (side)



Structure From Motion

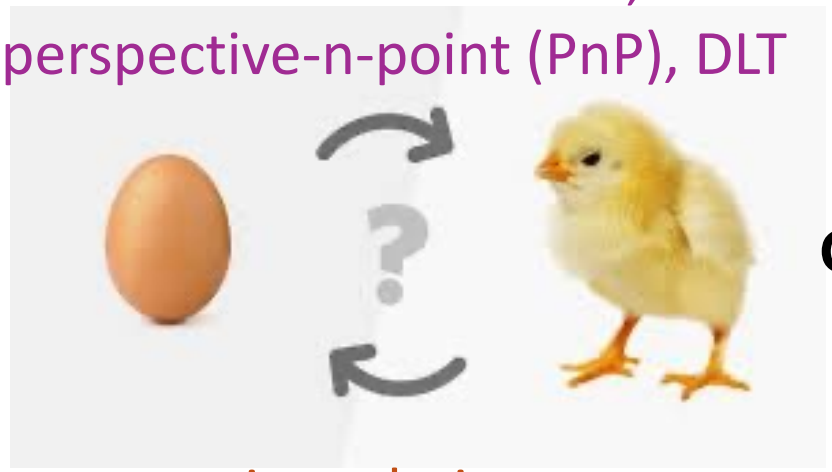
- What is structure from motion?

Output:

1) 3D structure: captured scene (3D location for 2D pixel)
2) camera motion: camera parameters (intrinsic and extrinsic matrix)

3D structure

camera calibration,
perspective-n-point (PnP), DLT



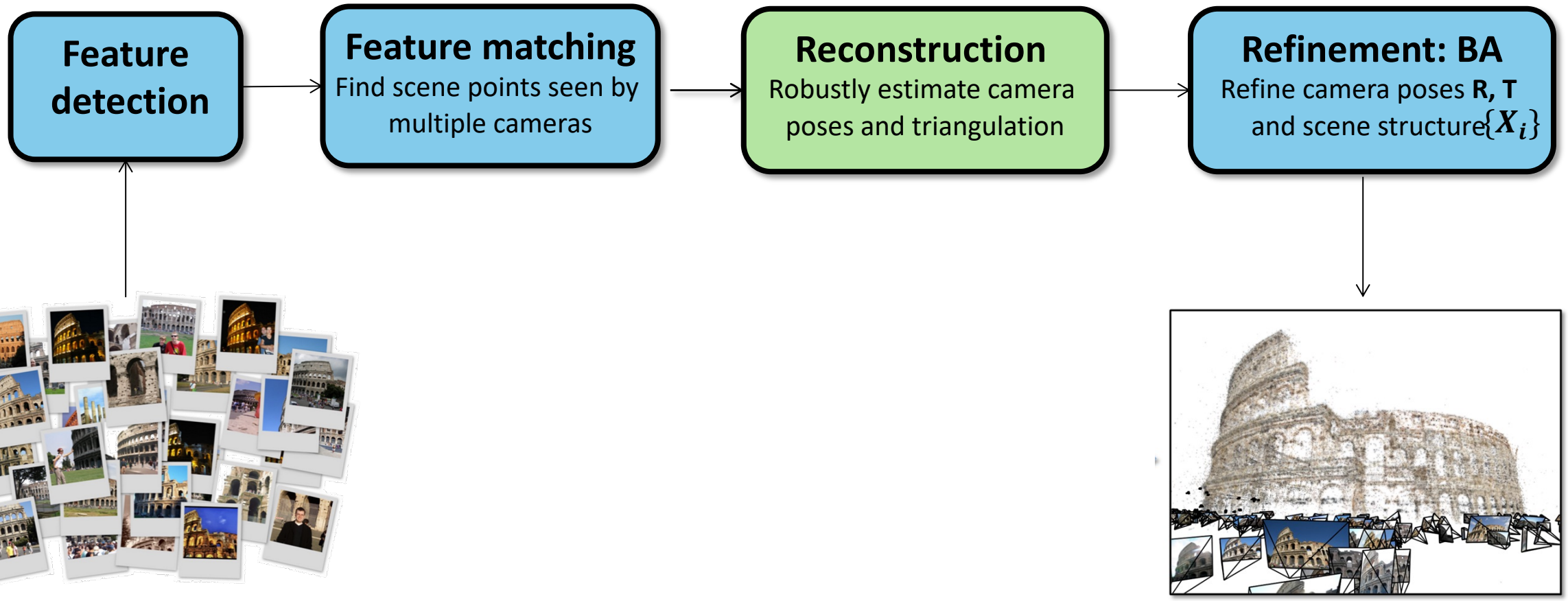
triangulation

$$x_{img} = PX_{world}$$
$$P = K[R|t]$$

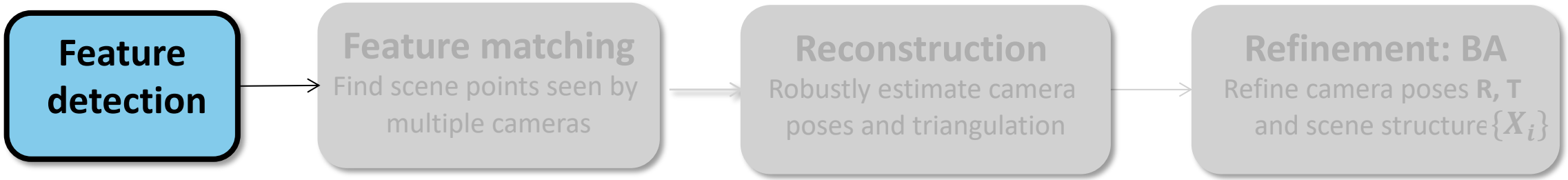
Camera parameters

Structure From Motion

- Core Steps of SfM



Structure From Motion: 1) Feature Detection



- Identifying points of interest within images that can be tracked across a series

- SIFT (Scale-Invariant Feature Transform)
- SURF (Speeded Up Robust Features): to scale and rotation changes
- ORB (Oriented FAST and Rotated BRIEF)

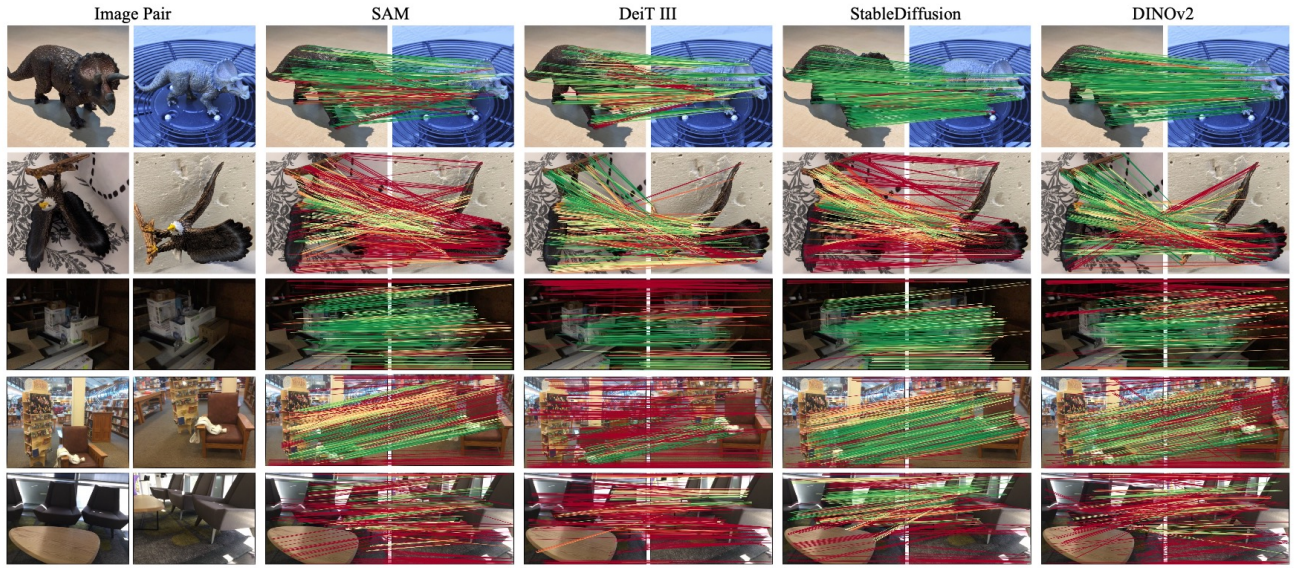
Structure From Motion: 1) Feature Detection

Feature detection → **Feature matching**: Find scene points seen by multiple cameras → **Reconstruction**: Robustly estimate camera poses and triangulation → **Refinement: BA**: Refine camera poses **R, T** and scene structure $\{X_i\}$

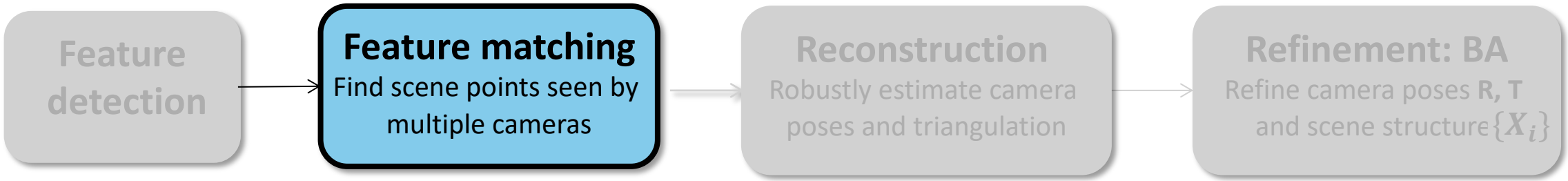
- Identifying points of interest within images that can be tracked across a series

Probing the 3D Awareness of Visual Foundation Models. In CVPR 2024

Deep features

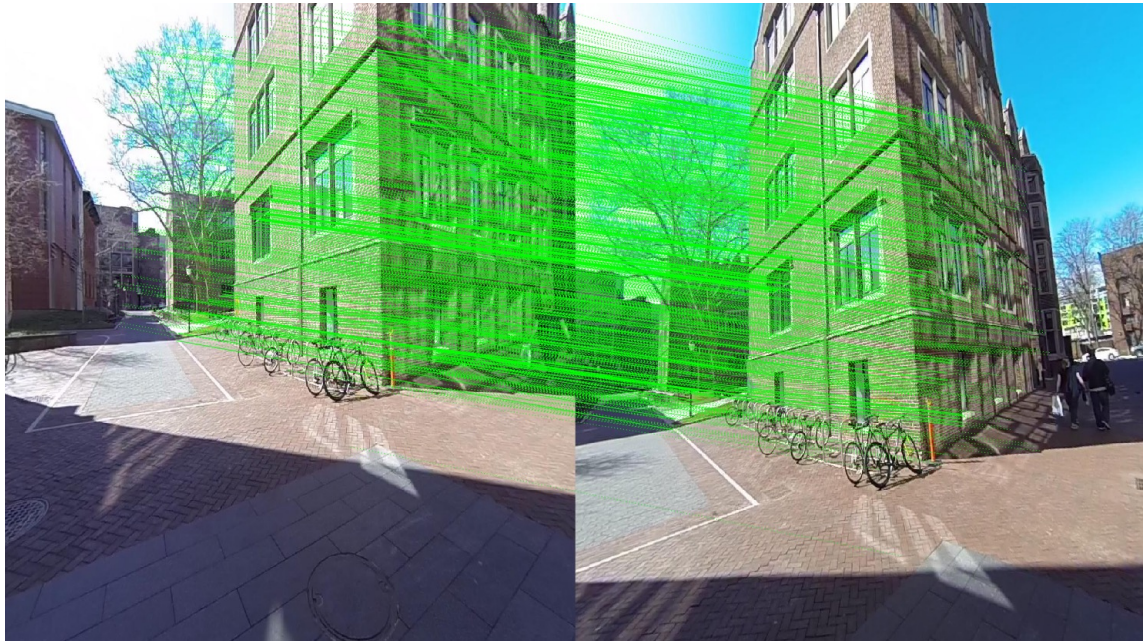


Structure From Motion: 2) Feature Matching



- Establishing correspondences between detected features across different images

Euclidean Distance



Structure From Motion: 2) Feature Matching

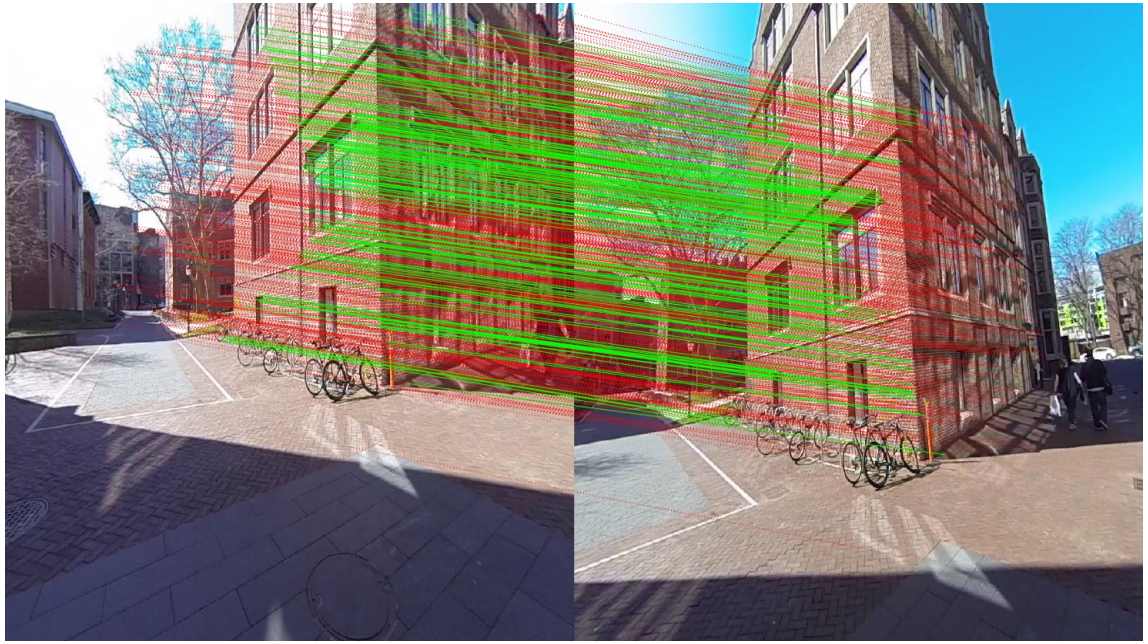
Feature detection → **Feature matching** Find scene points seen by multiple cameras → Reconstruction Robustly estimate camera poses and triangulation → Refinement: BA Refine camera poses **R, T** and scene structure $\{X_i\}$

- RANSAC algorithm to remove outliers

<u>Fundamental Matrix</u>

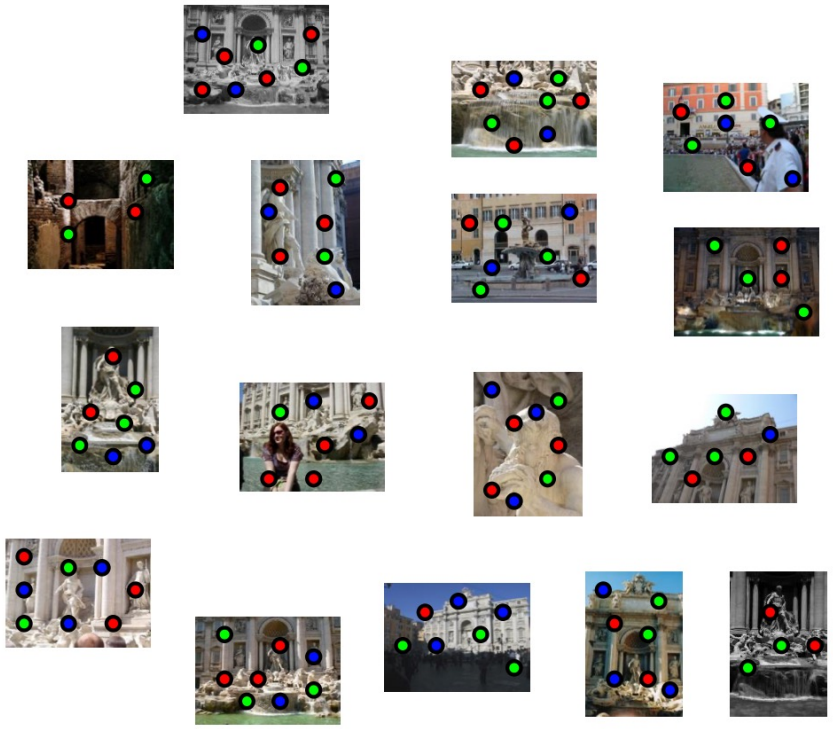
iteratively selecting a subset of the data

$$x_{img}'^{T} F x_{img} = 0$$

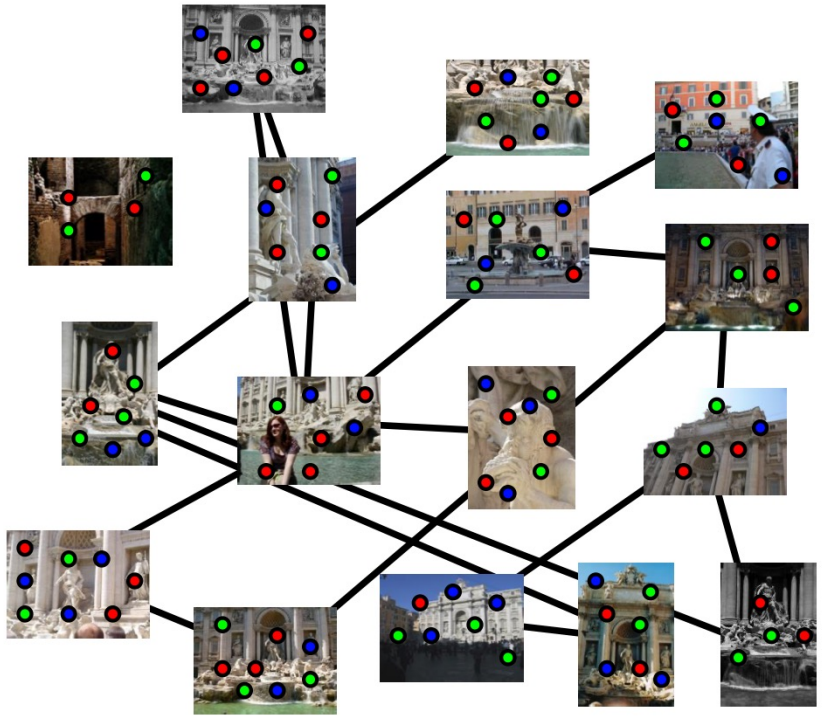


Structure From Motion: 2) Feature Matching

Feature detection → **Feature matching** Find scene points seen by multiple cameras → Reconstruction Robustly estimate camera poses and triangulation → Refinement: BA Refine camera poses **R, T** and scene structure $\{X_i\}$



matching



Detect features using SIFT [Lowe, IJCV 2004]

Structure From Motion: 3) Reconstruction

Feature detection → Feature matching: Find scene points seen by multiple cameras → **Reconstruction**: Robustly estimate camera poses and triangulation → Refinement: BA: Refine camera poses **R, T** and scene structure $\{X_i\}$

- Incremental SfM

Start with a Pair

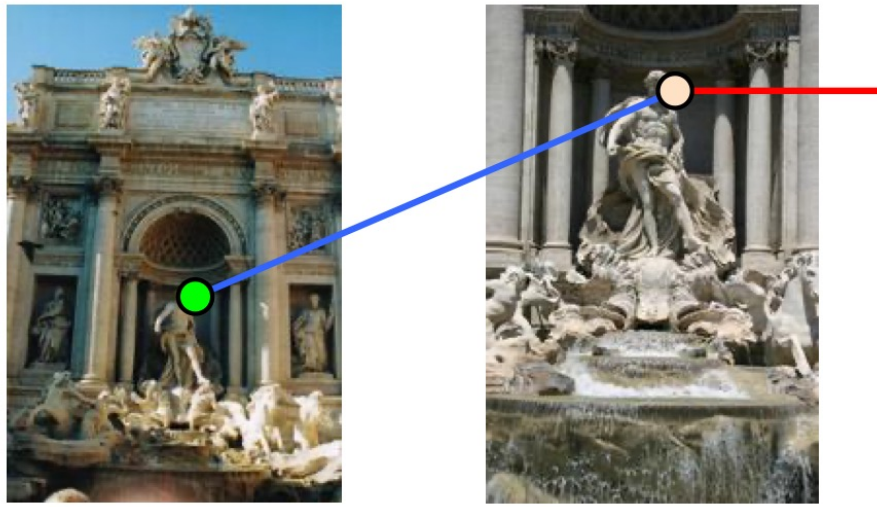


Image 1 Image 2

Epipolar Geometry

i) Fundamental Matrix

$$x_{img}'^{T} F x_{img} = 0$$

$$F = K'^{-T} E K^{-1}$$

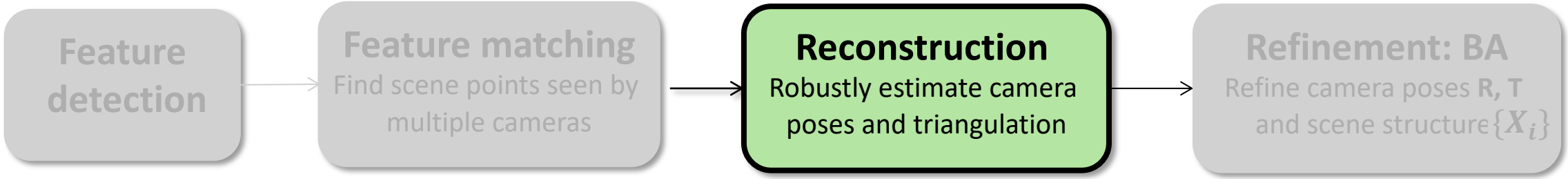
ii) Essential Matrix

$$x_c'^{T} E x_c = 0$$

$$E = [t]_{\times} R$$

iii) Triangulation: Initial 3D points

Structure From Motion: 3) Reconstruction



- Incremental SfM: adding a new image

Reference

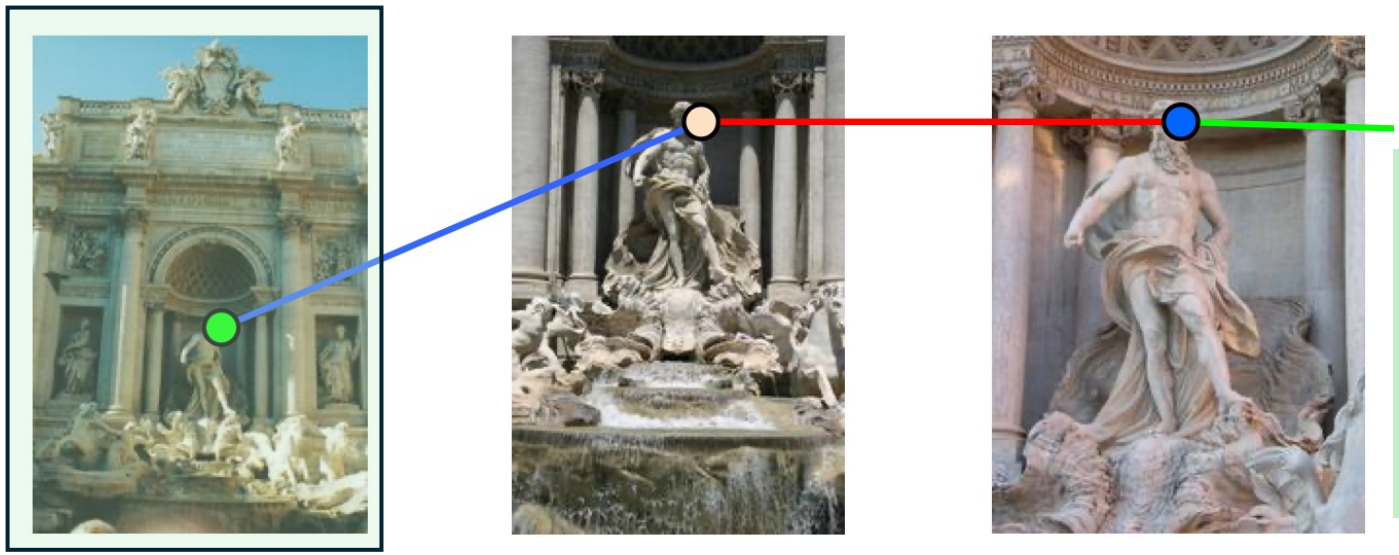


Image 1 Image 2 Image 3

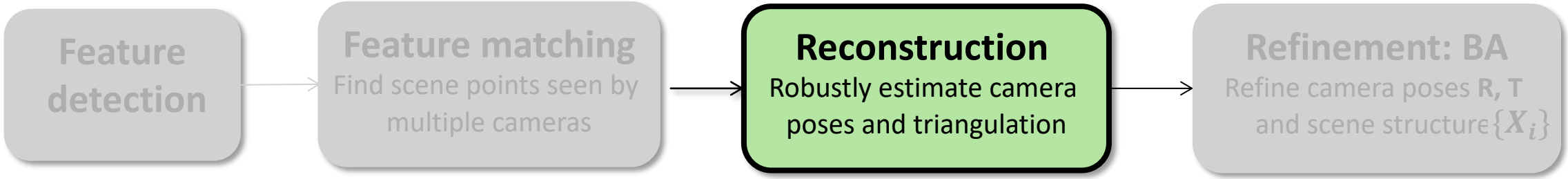
Add new image into existing pair

$$x_{img} = PX_{world}$$
$$P = K[R|t]$$

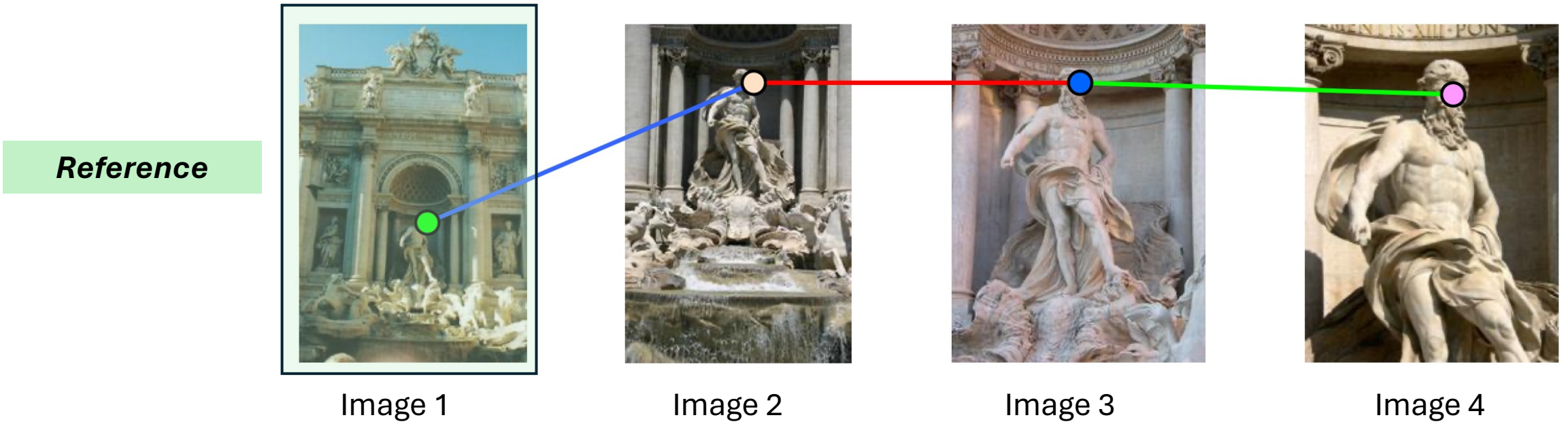
Perspective-n-Point (PnP)

Camera pose for new image

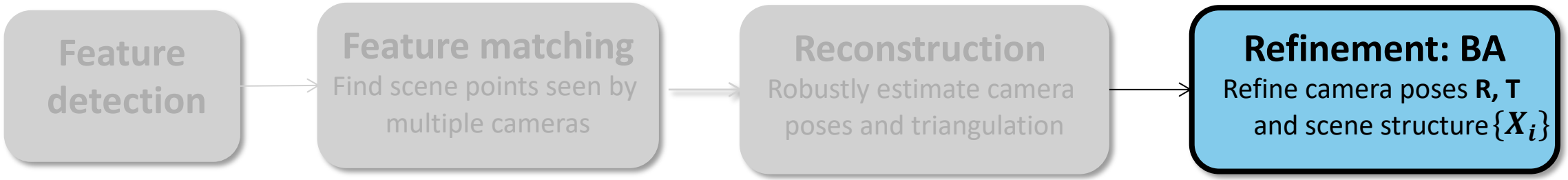
Structure From Motion: 3) Reconstruction



- Incremental SfM: adding a new image



Structure From Motion: 4) Refinement



- Bundle Adjustment

Input: Initial 3D structure and camera poses

Goal: Refine both properties

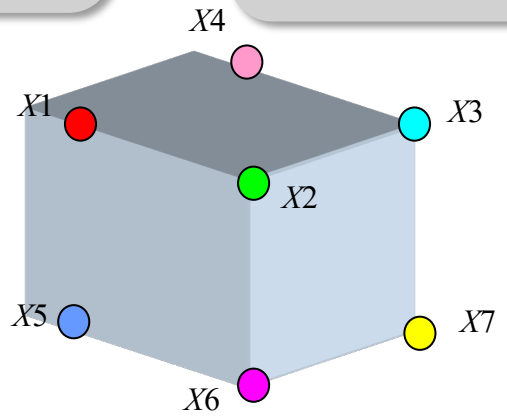
Objective: minimises reprojection error (3D→2D)

$$x_{img} = PX_{world}$$
$$P = K[R|t]$$

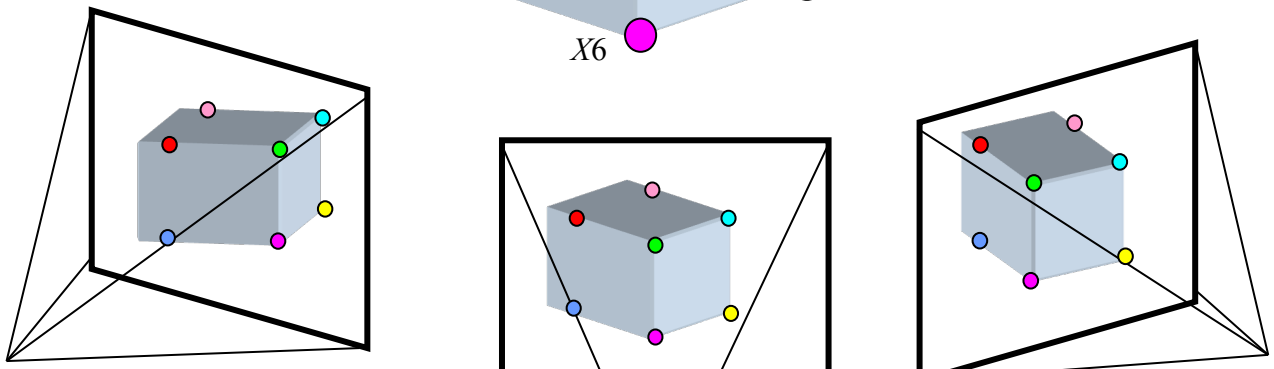
Structure From Motion: 4) Refinement

Feature detection → Feature matching (Find scene points seen by multiple cameras) → Reconstruction (Robustly estimate camera poses and triangulation) → **Refinement: BA** (Refine camera poses **R, T** and scene structure $\{X_i\}$)

- Bundle Adjustment



$$\textit{minimise} \quad \mathrm{f}(\mathbf{K}, \mathbf{R}, \mathbf{T}, \{\boldsymbol{X_i}\})$$



Camera 1
R_1, t_1

Camera 2
R_2, t_2

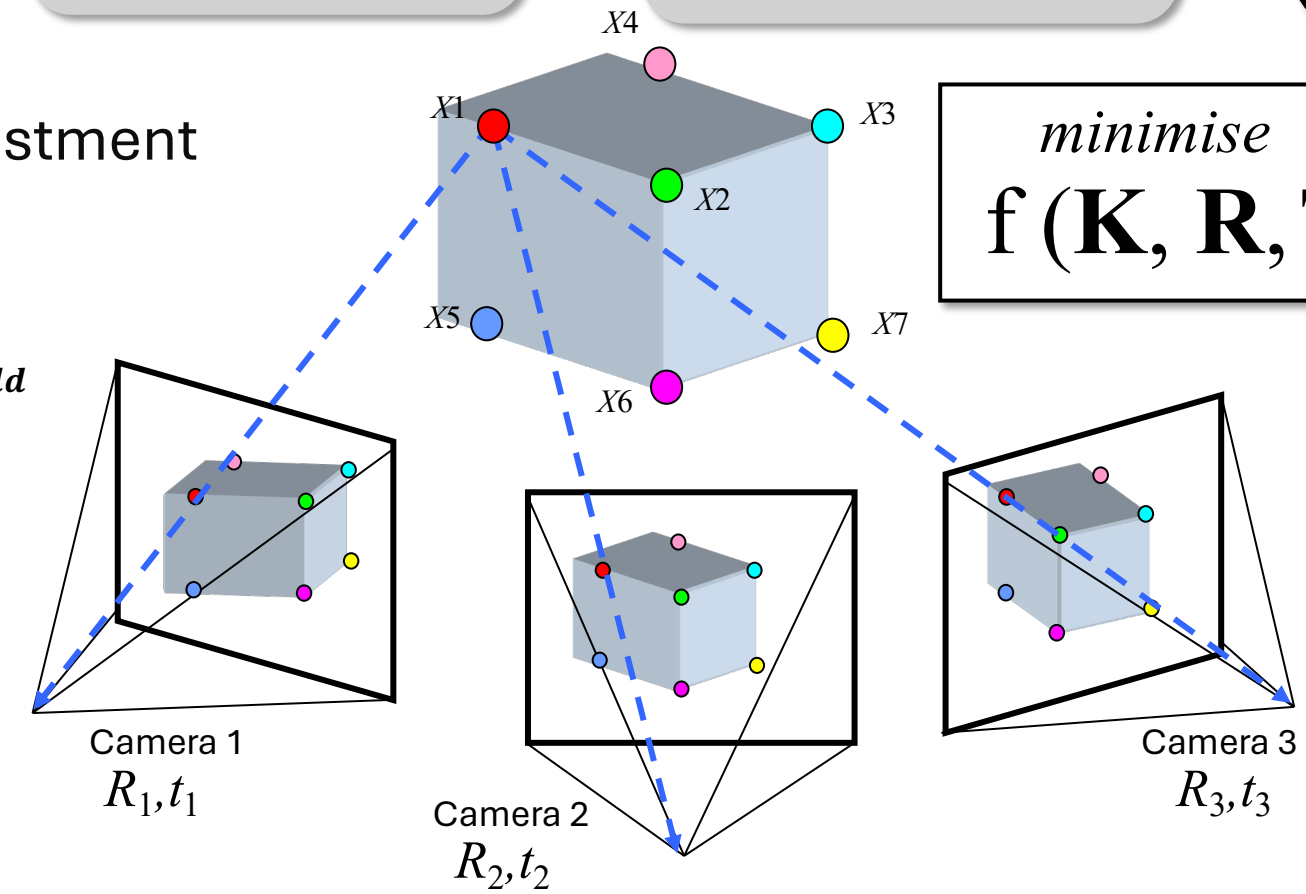
Camera 3
R_3, t_3

Structure From Motion: 4) Refinement

Feature detection → Feature matching (Find scene points seen by multiple cameras) → Reconstruction (Robustly estimate camera poses and triangulation) → **Refinement: BA** (Refine camera poses **R, T** and scene structure $\{X_i\}$)

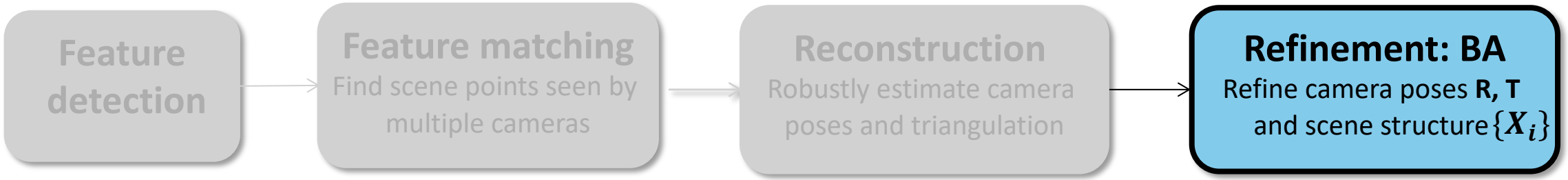
- Bundle Adjustment

$$x_{img} = PX_{world}$$
$$P = K[R|t]$$



$$\textit{minimise}\ \mathrm{f}(\mathbf{K}, \mathbf{R}, \mathbf{T}, \{\boldsymbol{X_i}\})$$

Structure From Motion: 4) Refinement



- Bundle Adjustment

Given: *m* images of *n* fixed 3D points

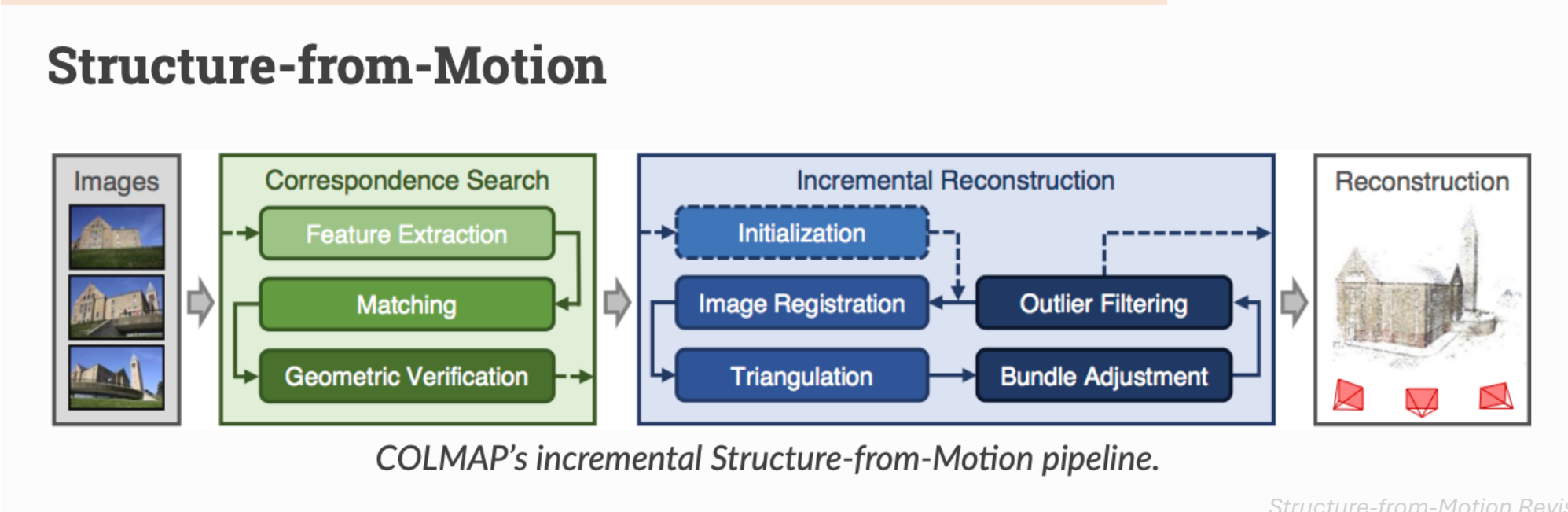
$$\mathbf{x}_{ij} = \mathbf{P}_i \mathbf{X}_j, \qquad i = 1,\dots, m, \quad j = 1, \dots, n$$

Problem: estimate *m* projection matrices $\mathbf{P}_i$ and *n* 3D points $\mathbf{X}_j$ from the *mn* corresponding points $\mathbf{x}_{ij}$

Structure From Motion: COLAMP

Feature detection → **Feature matching**
Find scene points seen by multiple cameras → **Reconstruction**
Robustly estimate camera poses and triangulation → **Refinement: BA**
Refine camera poses **R, T** and scene structure $\{X_i\}$

https://colmap.github.io/tutorial.html#structure-from-motion

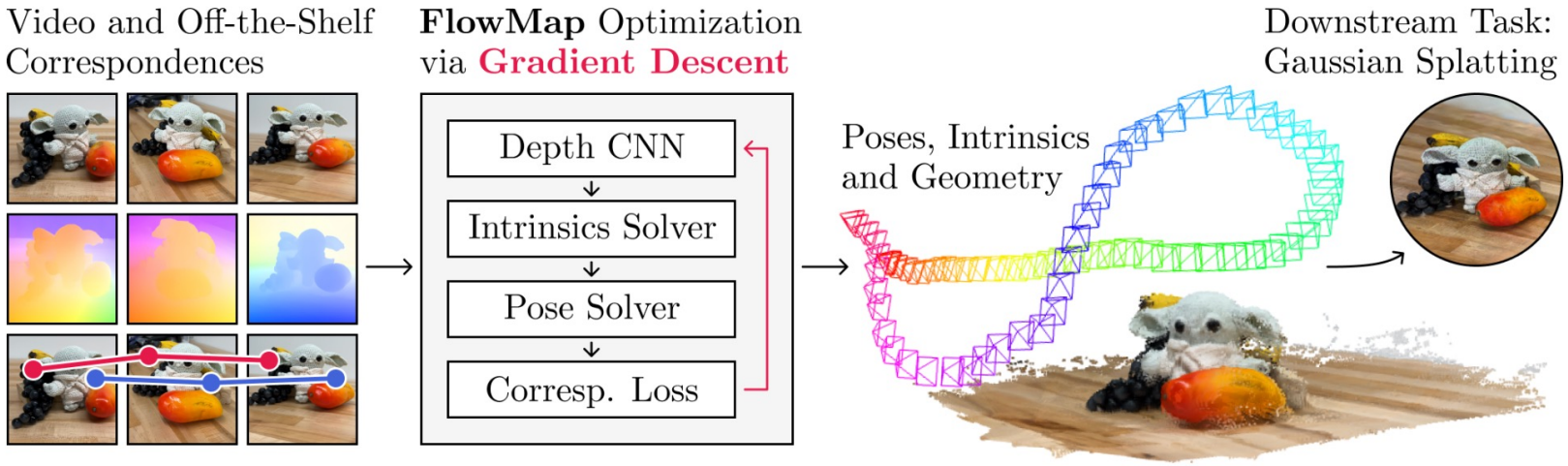


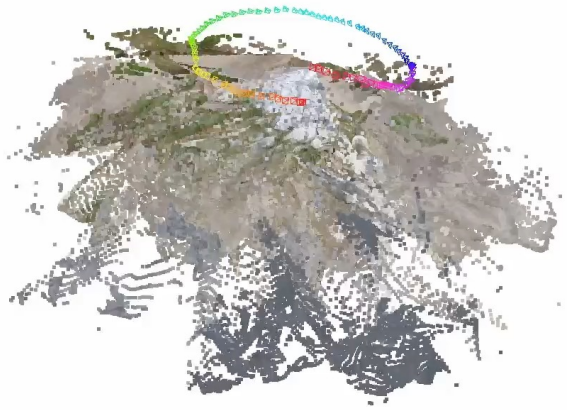
COLMAP's incremental Structure-from-Motion pipeline.

Structure-from-Motion Revisited. In CVPR 2016
Pixelwise View Selection for Unstructured Multi-View Stereo. In ECCV 2016

Structure From Motion: FlowMap

Feature detection → **Feature matching**
Find scene points seen by multiple cameras → **Reconstruction**
Robustly estimate camera poses and triangulation → **Refinement: BA**
Refine camera poses **R, T** and scene structure $\{X_i\}$



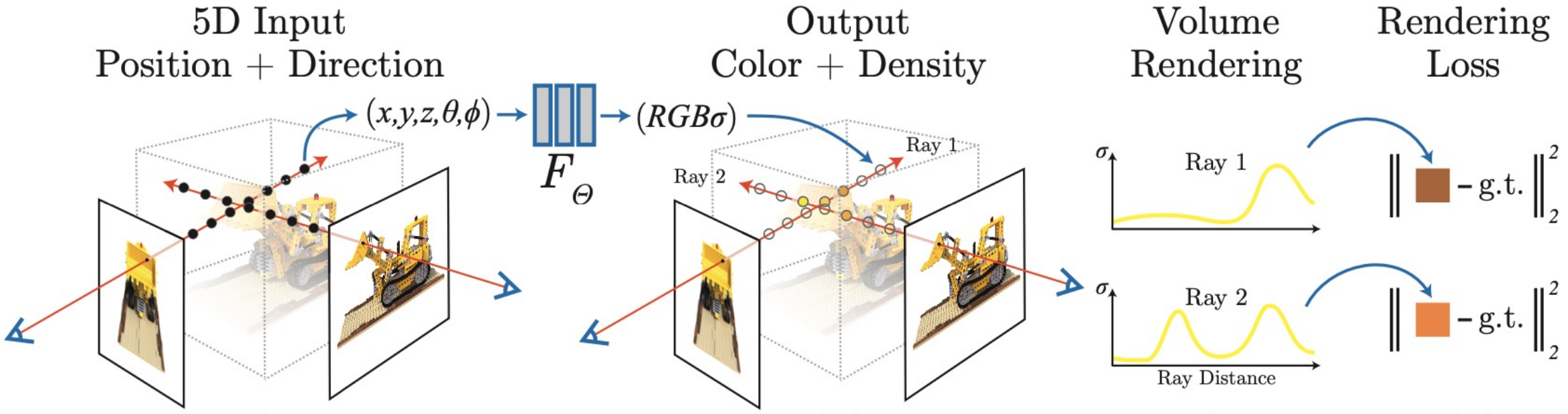


FlowMap: High-Quality Camera Poses, Intrinsics, and Depth via Gradient Descent. Arxiv 2024

Neural Radiance Fields (NeRFs)

- NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis

  (***representing 3D scenes using neural networks/MLPs***)



Neural Radiance Fields (NeRFs)

- NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis

  (***representing 3D scenes using neural networks/MLPs***)

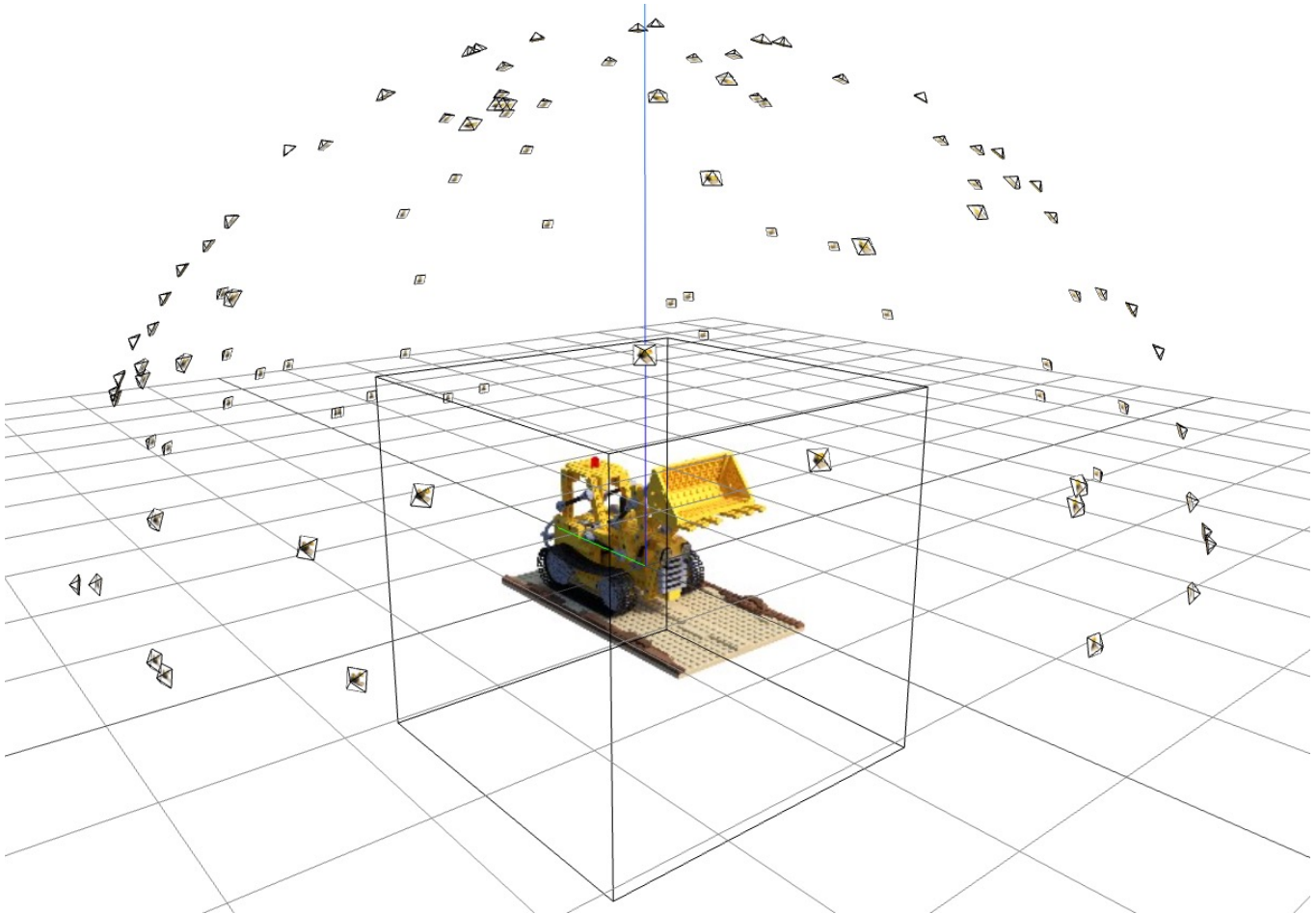
Input: multiple camera images (or video) + associated camera poses

Output: Neural Radiance Field (MLP weights)

Objective: minimises photometric error
(rendered image vs ground-truth image)

Neural Radiance Fields (NeRFs)

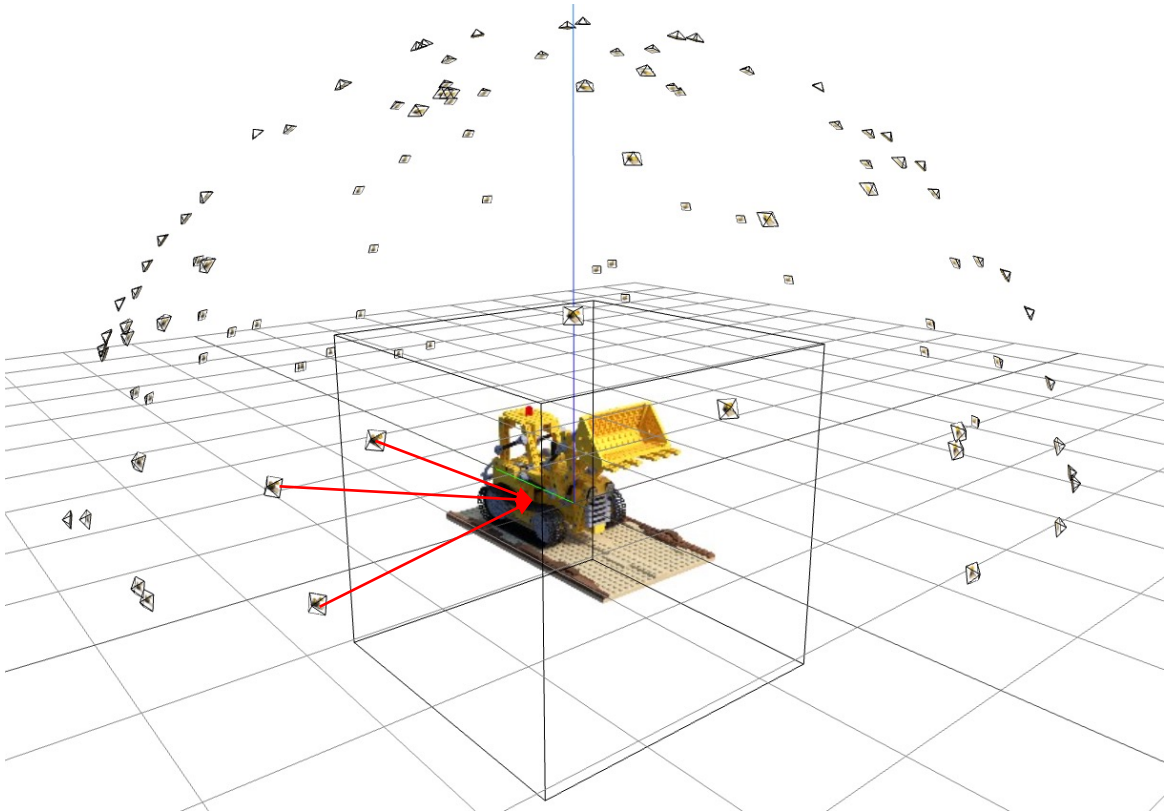
- Step 1: let take photos



Neural Radiance Fields (NeRFs)

- Step 2: using Colmap (SFMs) to calculate camera poses

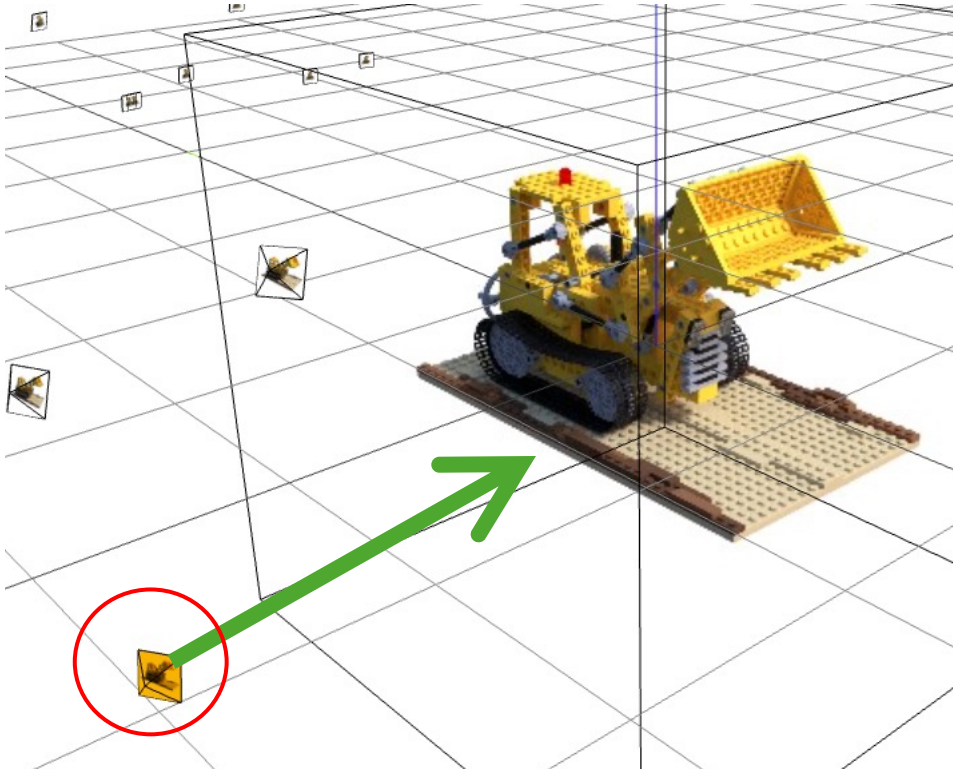
(detecting and matching features across the images)



Neural Radiance Fields (NeRFs)

- Step 3: Volumetric formulation of NeRF

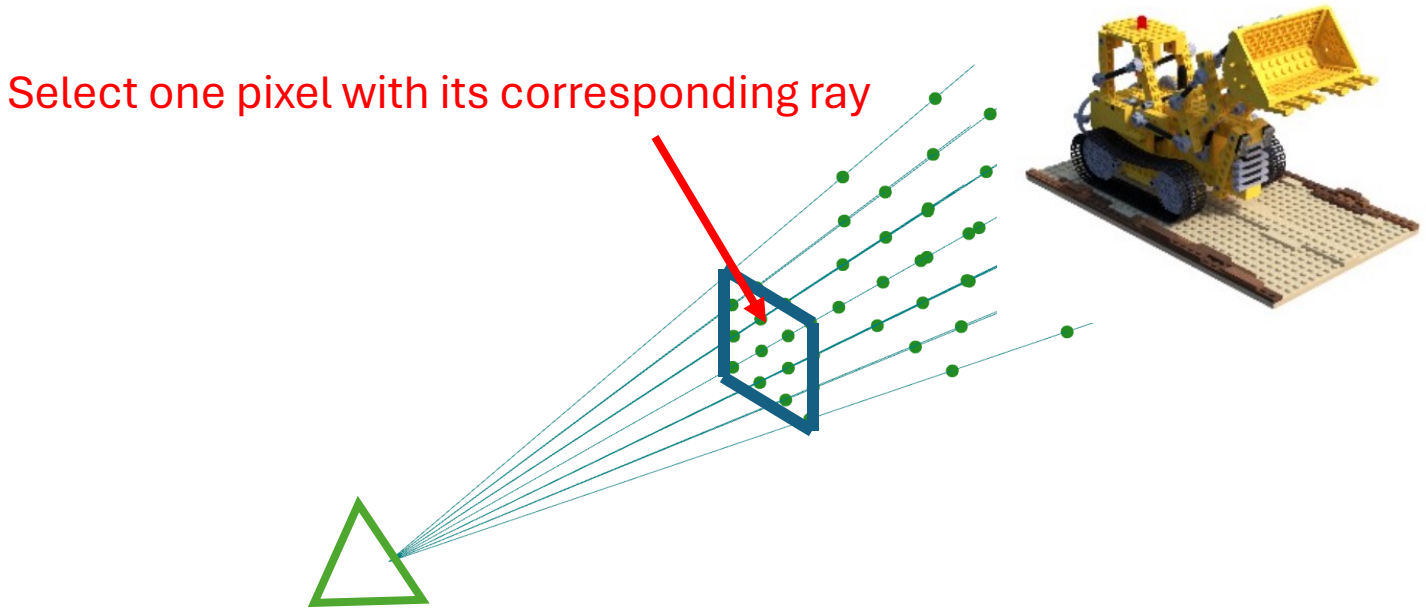
What will the image/ pixels captured by this camera look like?



Neural Radiance Fields (NeRFs)

- Step 3: Volumetric formulation of NeRF

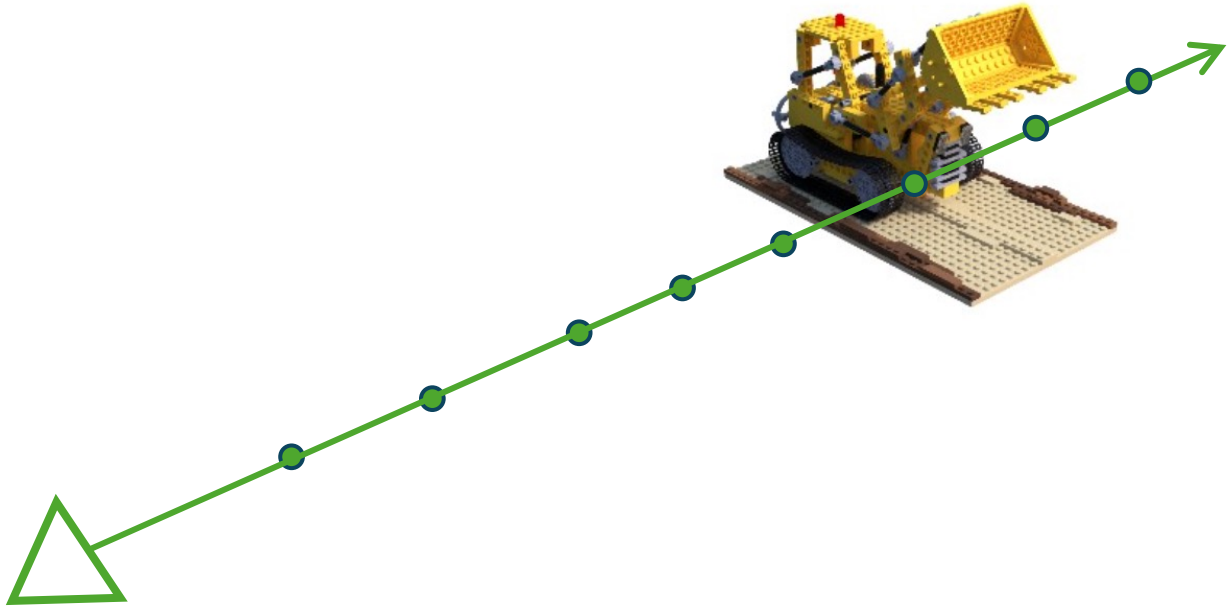
What will the image/ pixels captured by this camera look like?



Neural Radiance Fields (NeRFs)

- Step 3: Volumetric formulation of NeRF

Ray tracing for this pixel (straight ray)



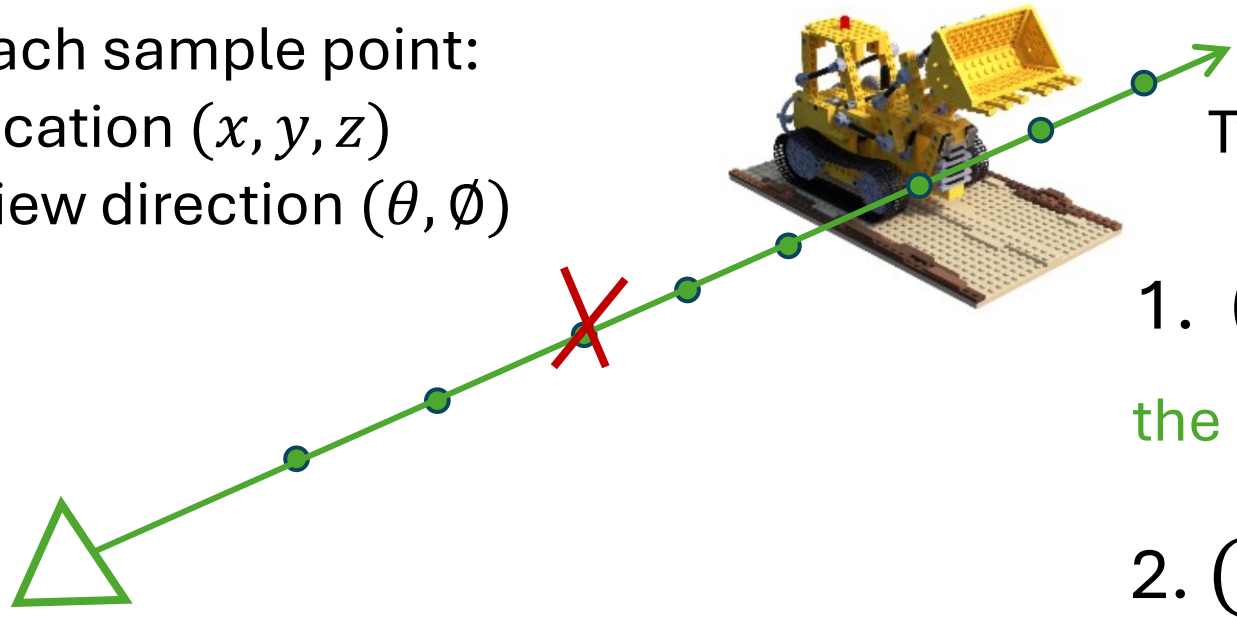
Each sample point, we know its location (x, y, z) and view direction $(\theta, \emptyset)$

Neural Radiance Fields (NeRFs)

- Step 3: Volumetric formulation of NeRF

Ray tracing for this pixel (straight ray)

Each sample point:
location (x, y, z)
view direction $(\theta, \emptyset)$



Two MLPs:

1. $(x, y, z) \xrightarrow{f} \sigma$

the chance the ray hits a particle

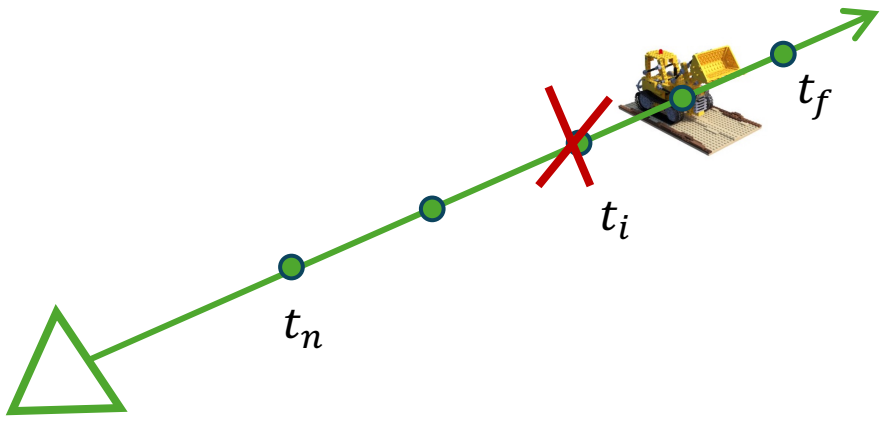
2. $(x, y, z, \theta, \emptyset) \xrightarrow{g} color$

color is view dependent

Neural Radiance Fields (NeRFs)

- Step 3: Volumetric formulation of NeRF

Accumulation for this ray and get its rendered pixel



$$C(\mathbf{r}) = \int_{t_n}^{t_f} T(t)\sigma(\mathbf{r}(t))\mathbf{c}(\mathbf{r}(t), \mathbf{d})dt\,,$$

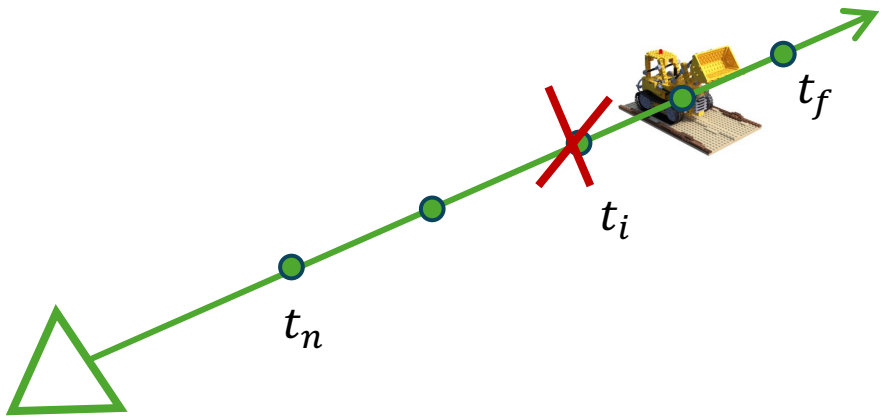
$$T(t) = \exp\left(-\int_{t_n}^{t} \sigma(\mathbf{r}(s))ds\right)$$

T: how much the light is blocked earlier along the ray

Neural Radiance Fields (NeRFs)

- Step 3: Volumetric formulation of NeRF

Accumulation for this ray and get its rendered pixel



$$\hat{C}(\mathbf{r}) = \sum_{i=1}^{N} T_i (1 - \exp(-\sigma_i \delta_i)) \mathbf{c}_i$$

$$T_i = \exp\left(- \sum_{j=1}^{i-1} \sigma_j \delta_j \right)$$

T: how much the light is blocked earlier along the ray

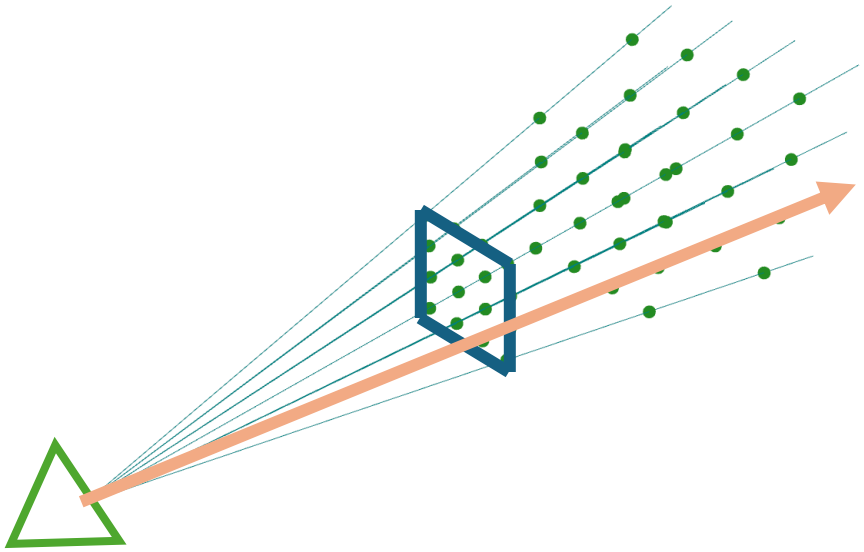
$$\delta_i = t_{i+1} - t_i$$

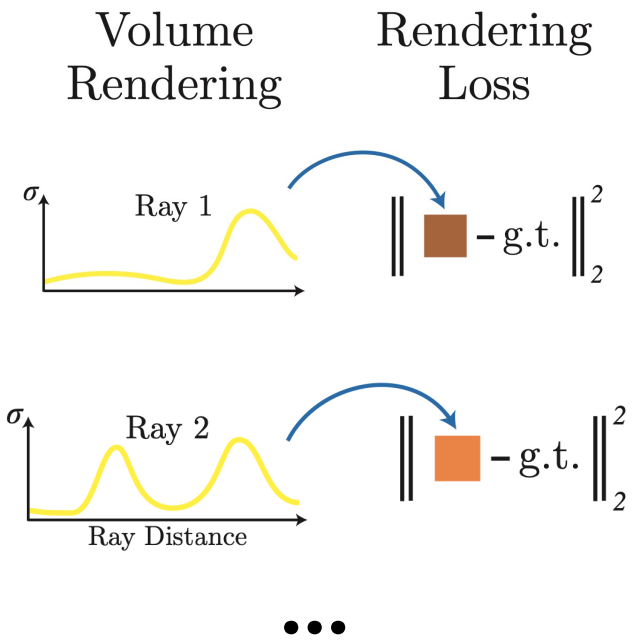
distance between adjacent point

Neural Radiance Fields (NeRFs)

- Step 3: Volumetric formulation of NeRF

Accumulation for this ray and get its rendered pixel



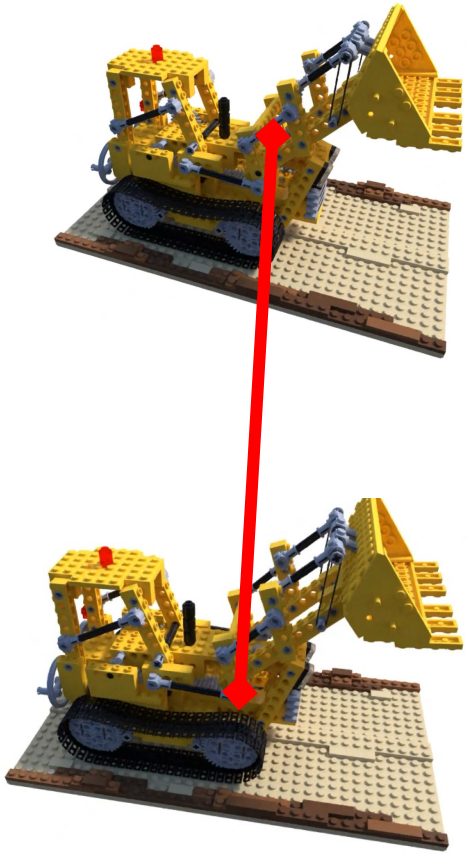


...

Neural Radiance Fields (NeRFs)

- Solves correspondences + triangulation implicitly

Camera view A



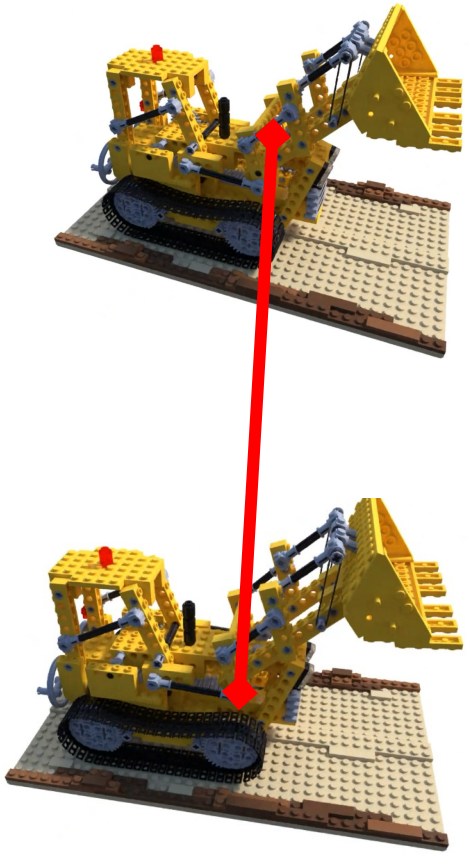
Camera view B

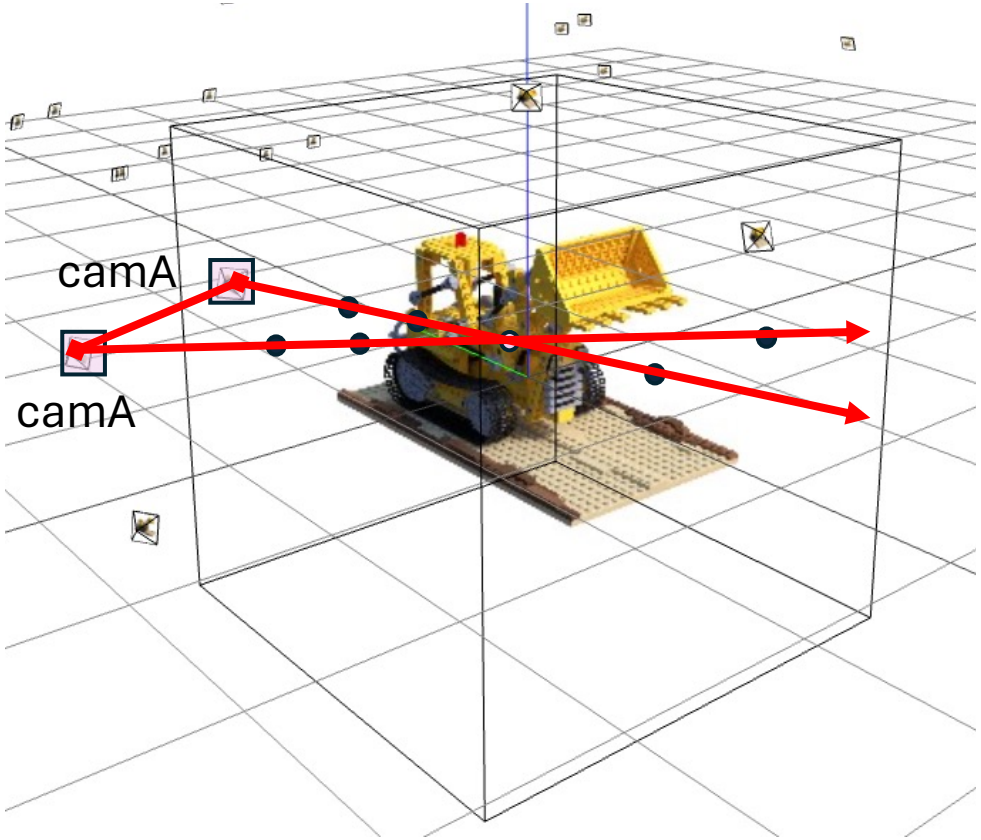
Neural Radiance Fields (NeRFs)

- Solves correspondences + triangulation implicitly

Camera view A

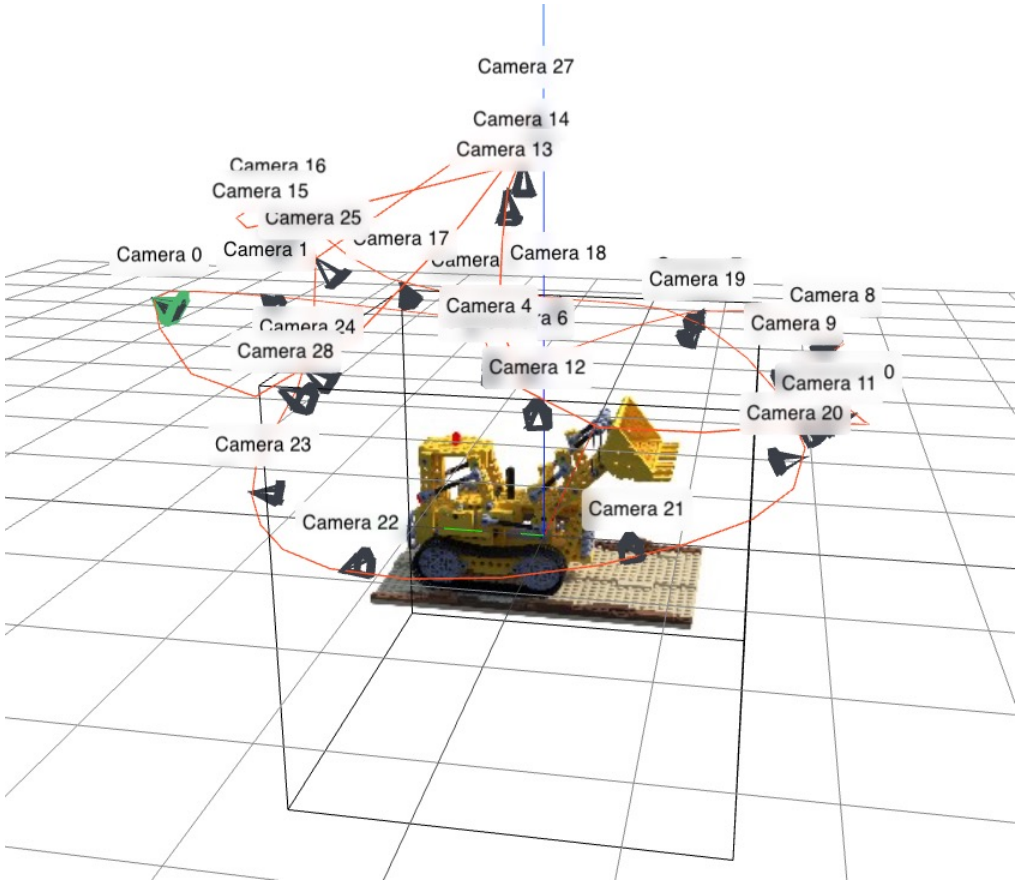
Camera view B

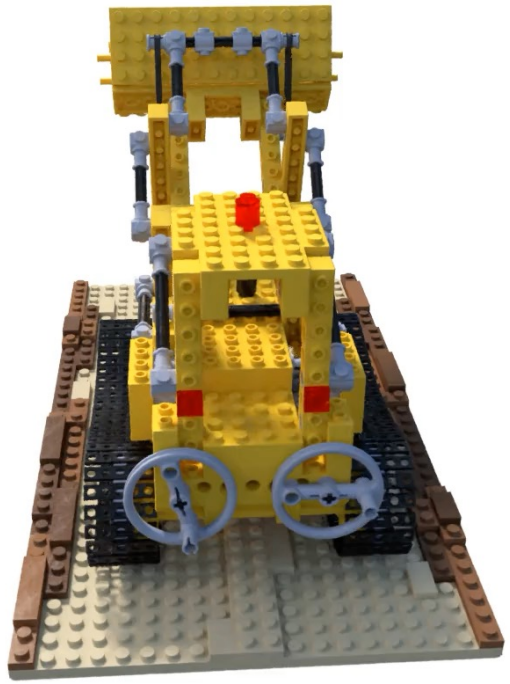




Neural Radiance Fields (NeRFs)

- Novel view synthesis (unseen camera views)

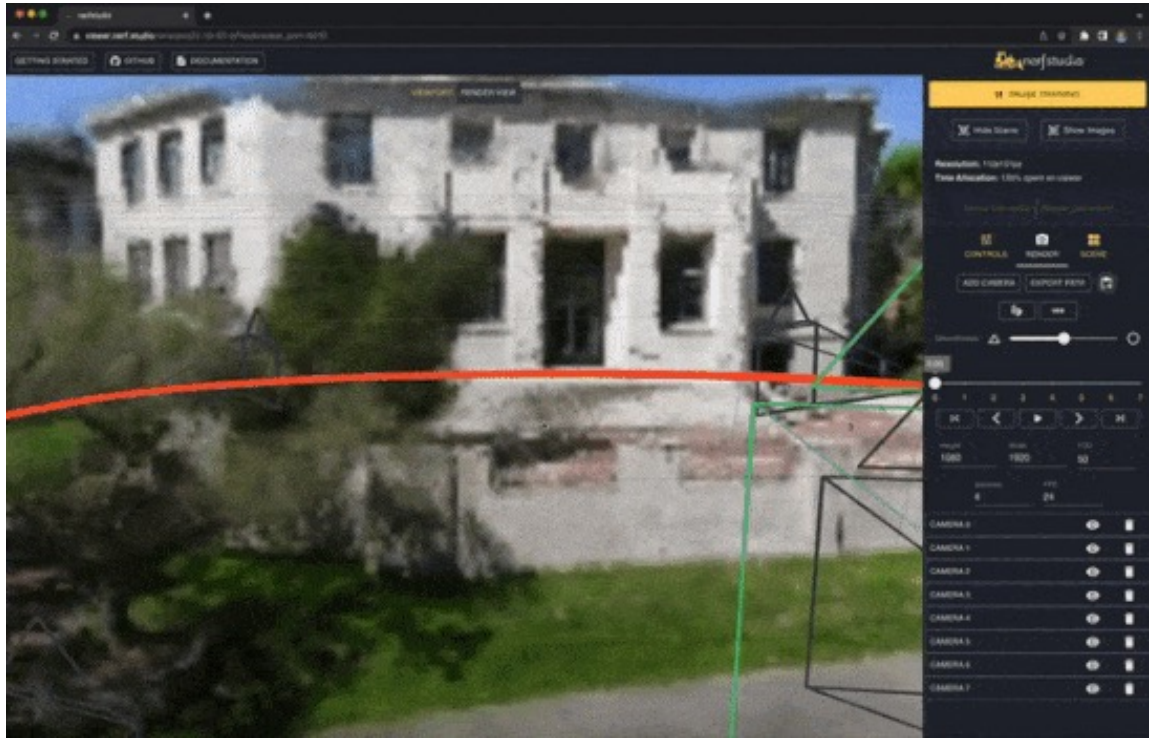




Neural Radiance Fields (NeRFs)

- More Examples

Building



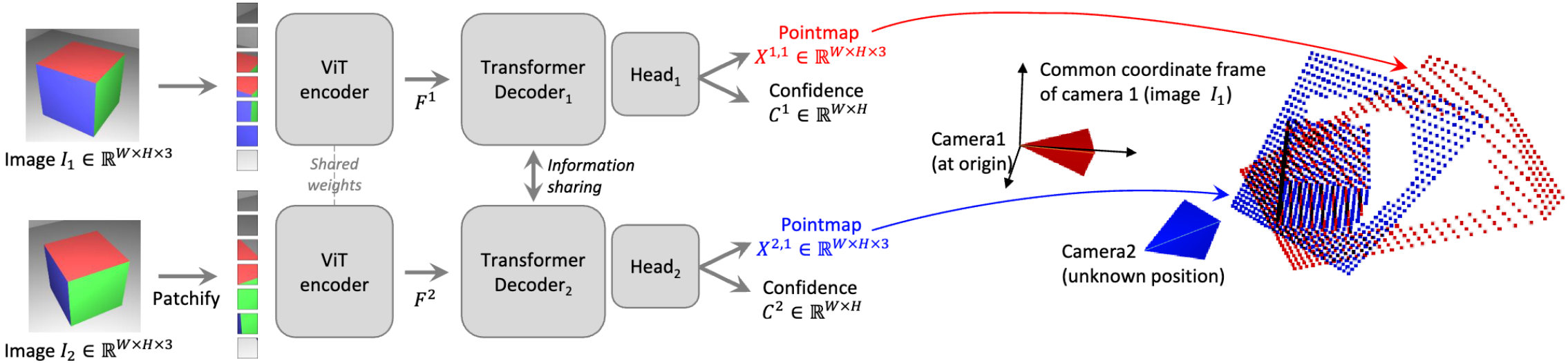
https://docs.nerf.studio

Neural Radiance Fields (NeRFs)

- More Examples

Human

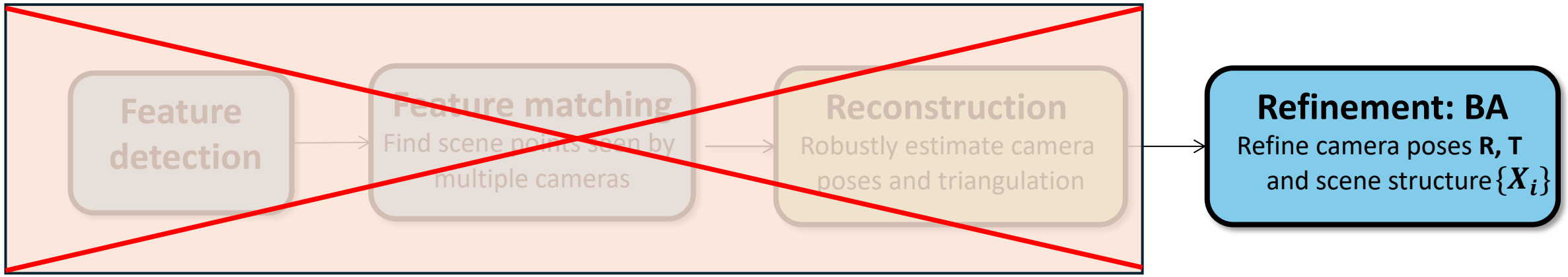
https://docs.nerf.studio



Wang et.al., DUSt3R: Geometric 3D Vision Made Easy. In CVPR 2024

DUSt3R: Geometric 3D Vision Made Easy

"Unify all 3D vision tasks"



DUSt3R: Geometric 3D Vision Made Easy

"Unify all 3D vision tasks"

Input: multiple camera views (without intrinsic)

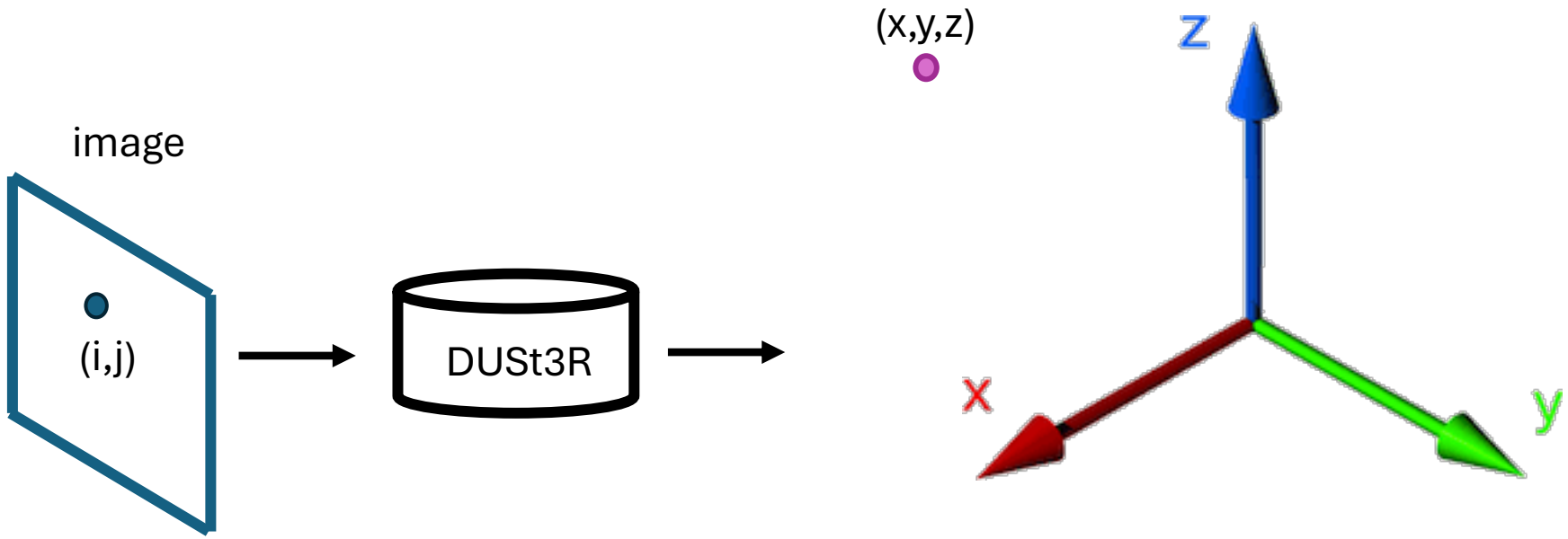
Output: 3D reconstruction + camera poses

Objective: minimises 3D matching loss

DUSt3R: Geometric 3D Vision Made Easy

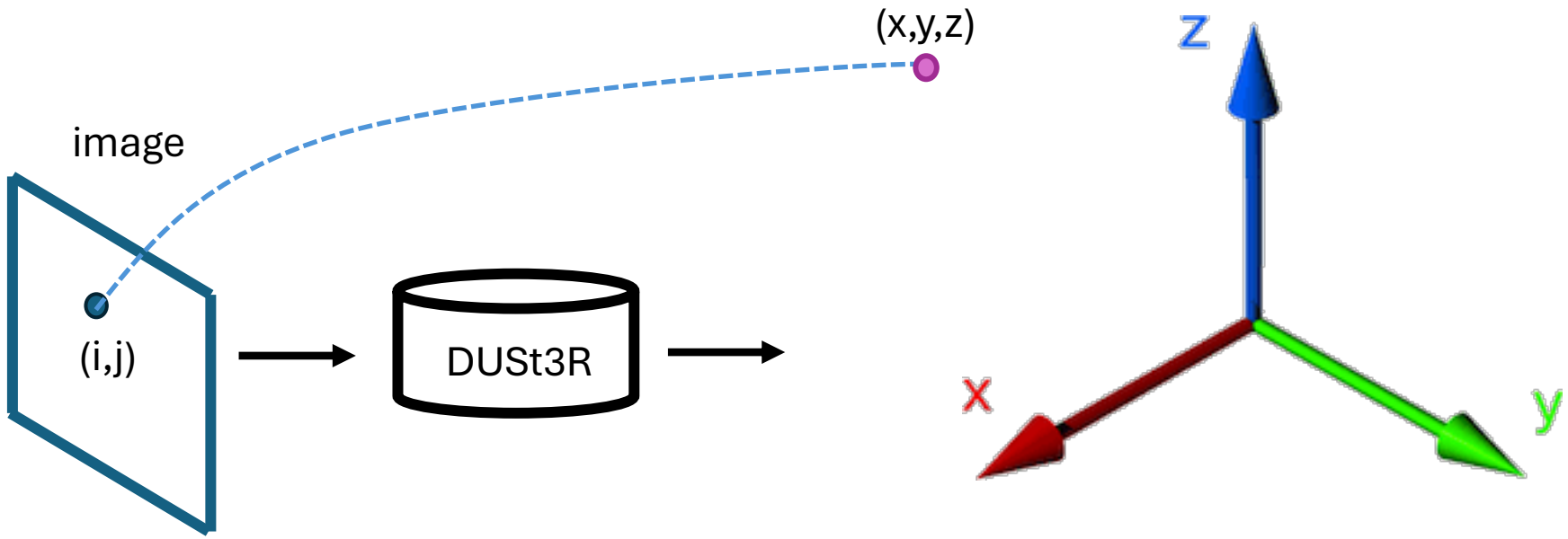
"Unify all 3D vision tasks"

DUSt3R: Geometric 3D Vision Made Easy



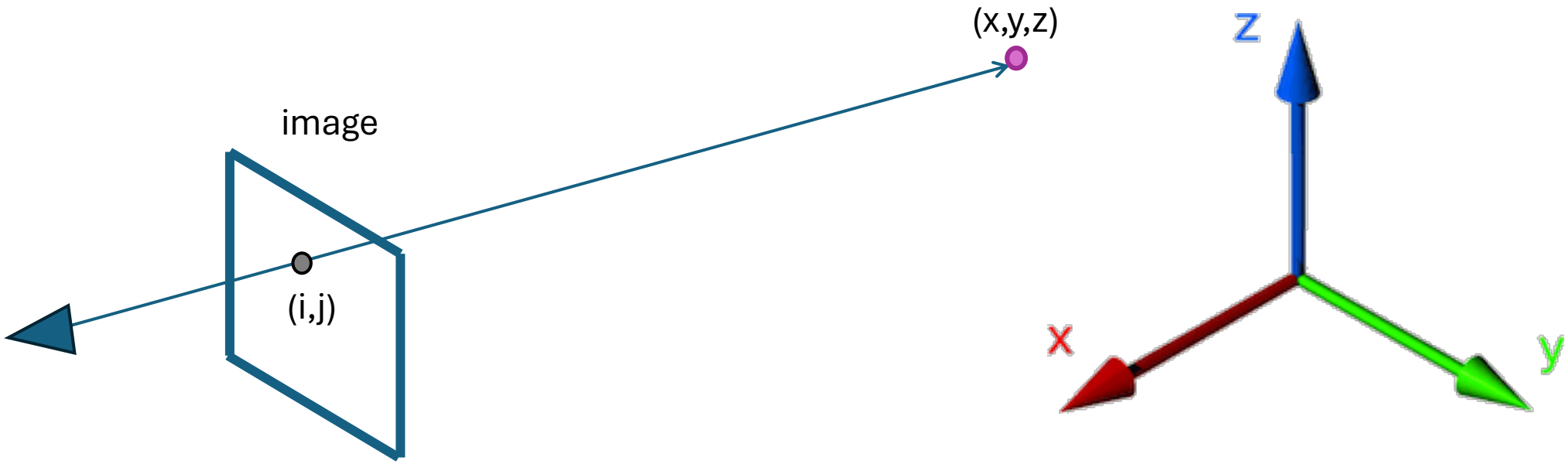
Pointmap: 2D to 3D

DUSt3R: Geometric 3D Vision Made Easy

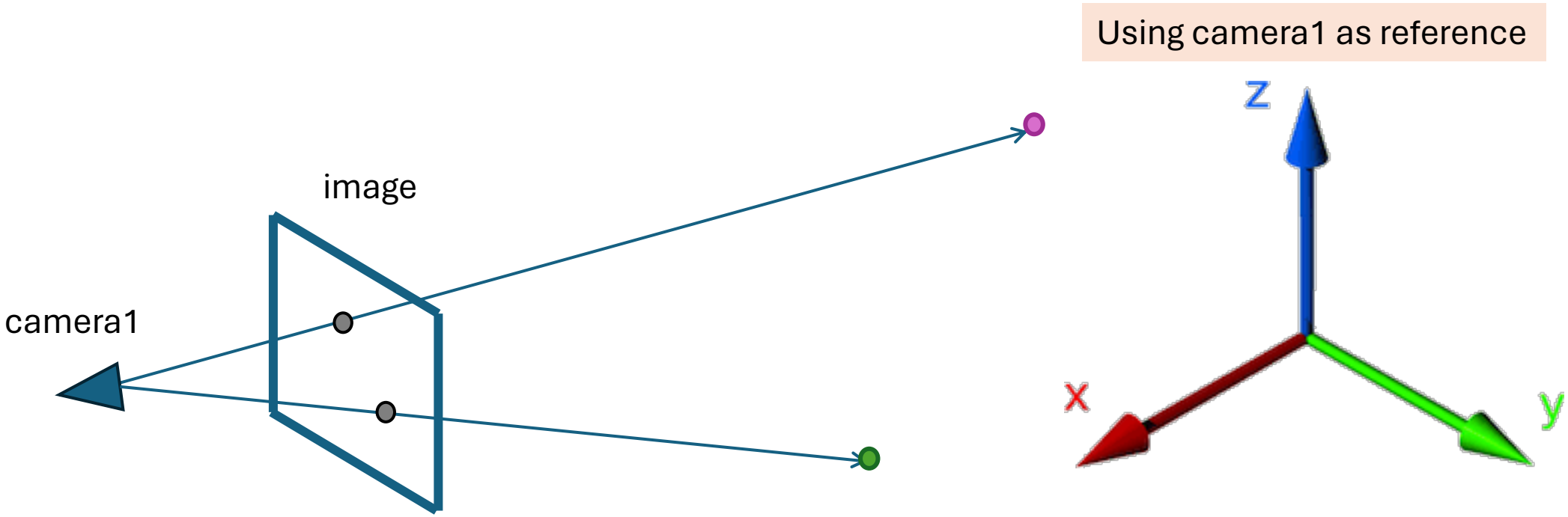


Pointmap: 2D to 3D

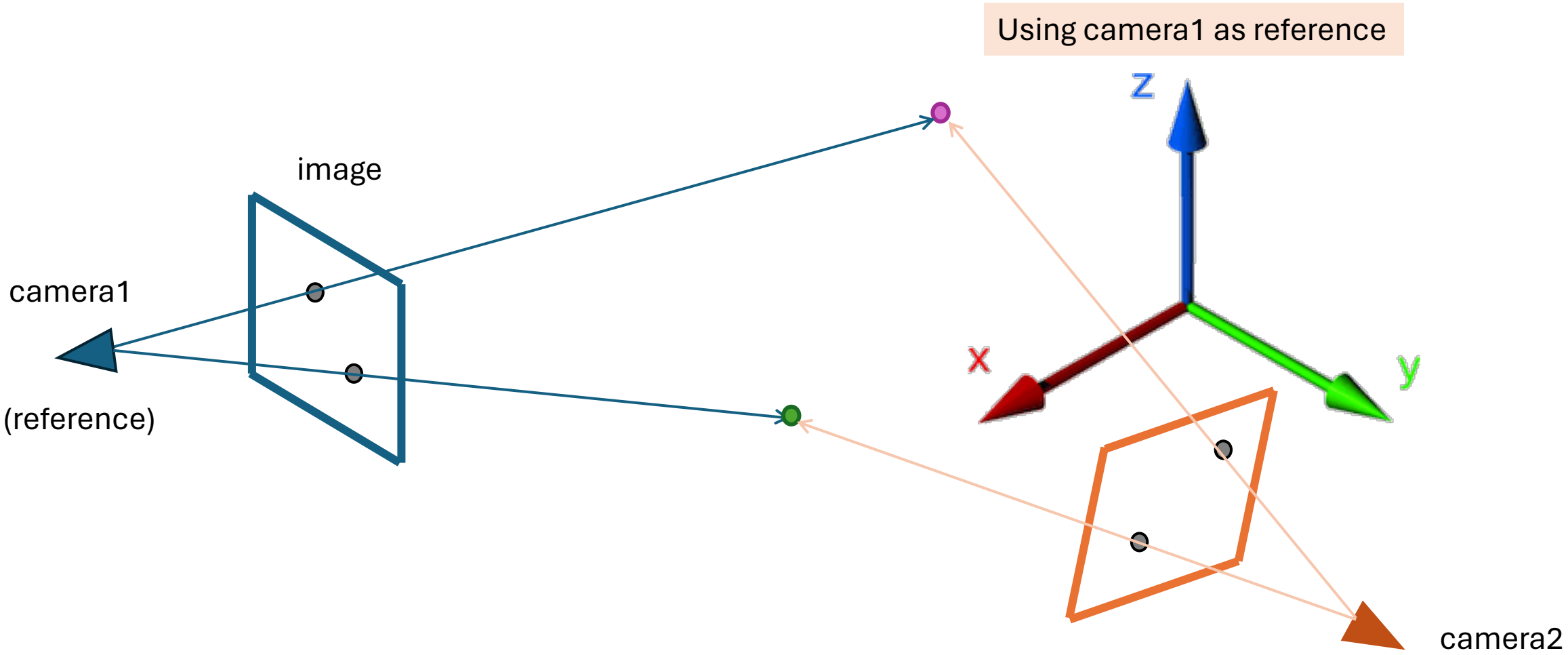
DUSt3R: Geometric 3D Vision Made Easy



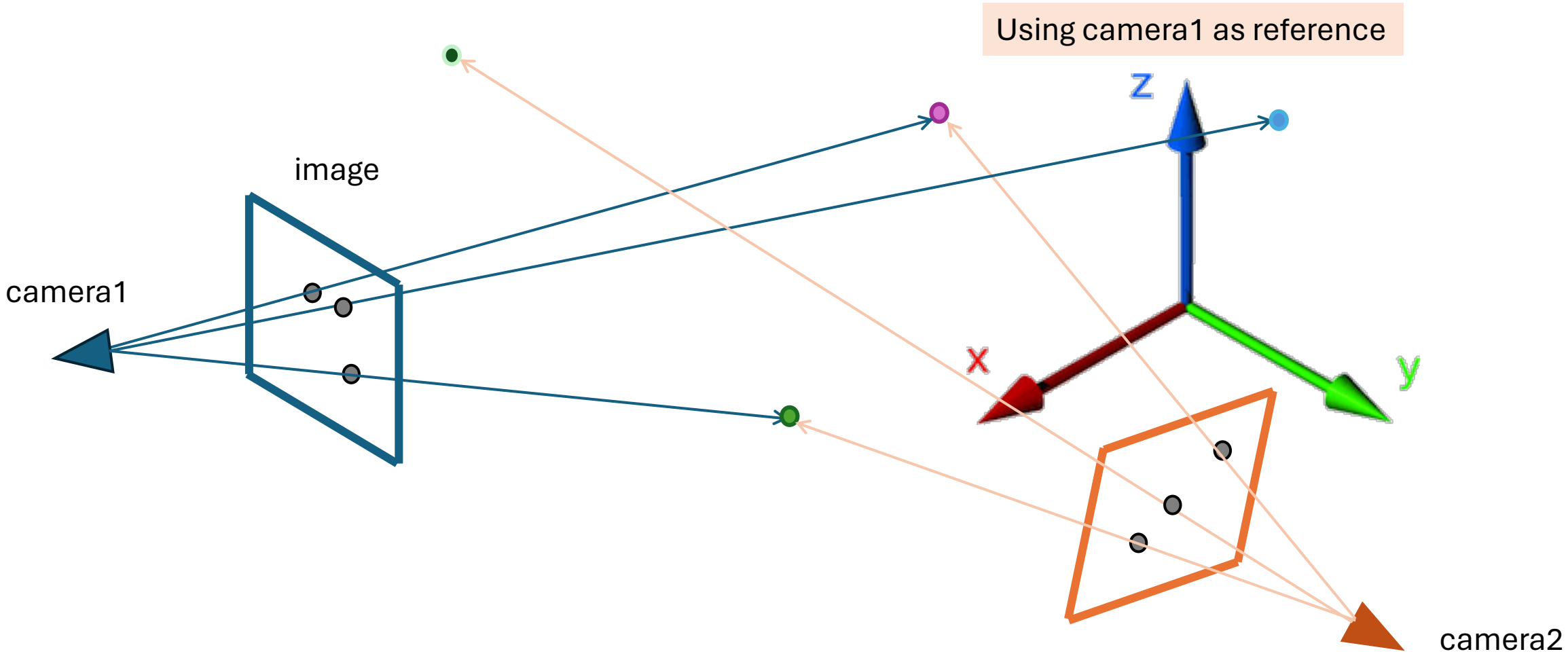
DUSt3R: Geometric 3D Vision Made Easy



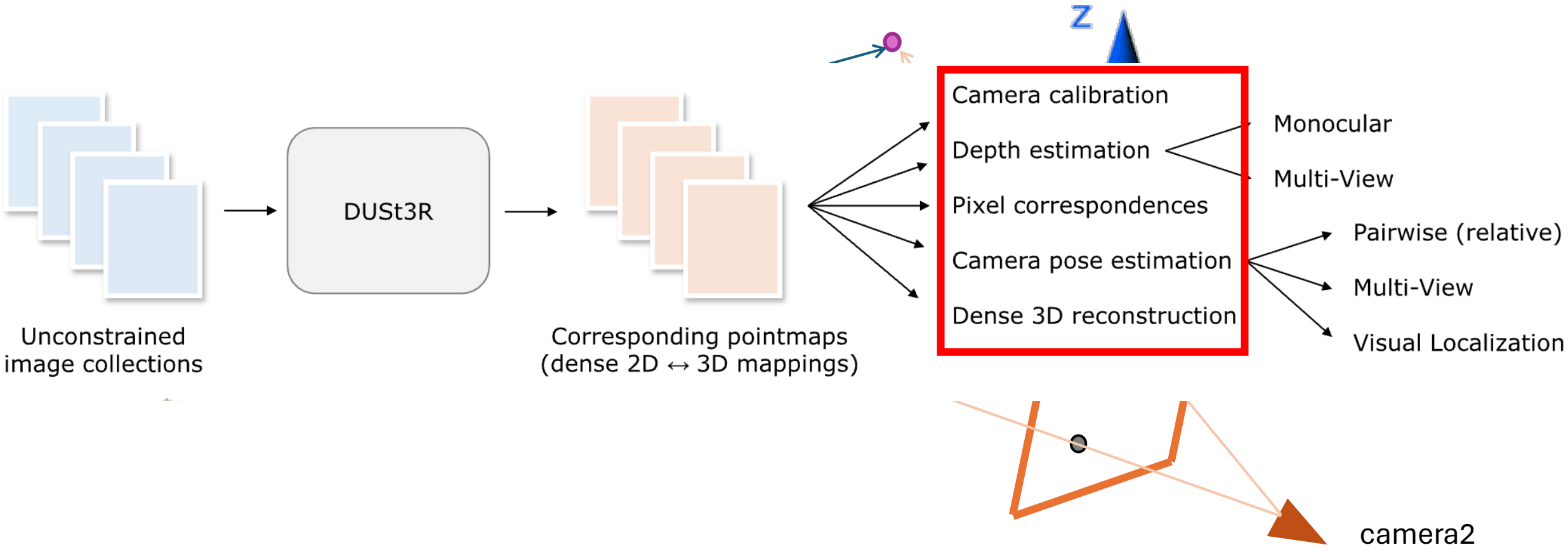
DUSt3R: Geometric 3D Vision Made Easy



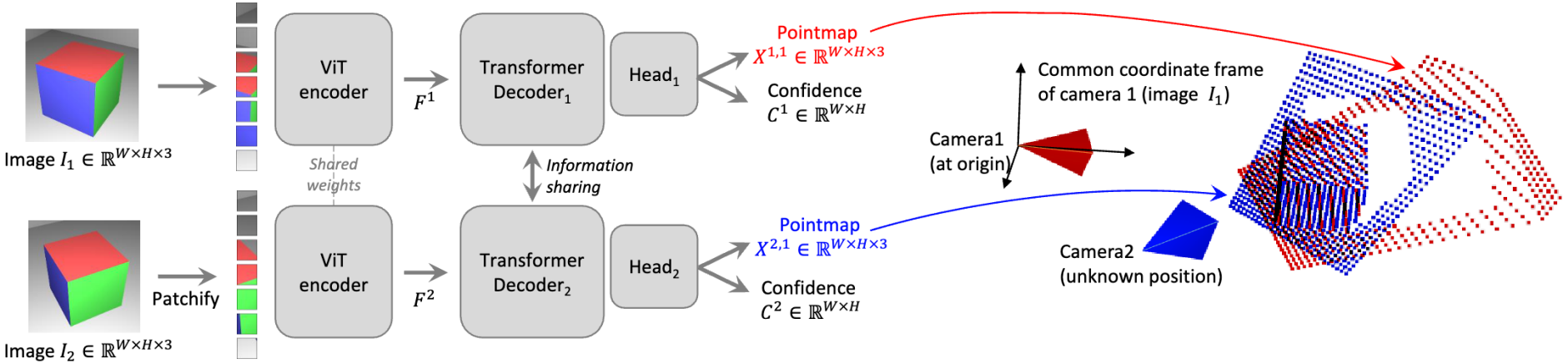
DUSt3R: Geometric 3D Vision Made Easy



DUSt3R: Geometric 3D Vision Made Easy



DUSt3R: Training Objective



The regression loss is defined as the Euclidean distance:

$$\ell_{\text{regr}}(v,i) = \left\| \frac{1}{z} X_i^{v,1} - \frac{1}{\bar{z}} \bar{X}_i^{v,1} \right\|$$

$v \in \{1,2\}$

With $z = \text{norm}(X^{1,1}, X^{2,1})$ and $\bar{z} = \text{norm}(\bar{X}^{1,1}, \bar{X}^{2,1})$ to handle the scale ambiguity between prediction and ground-truth.

DUSt3R: Applications

1. Point Matching

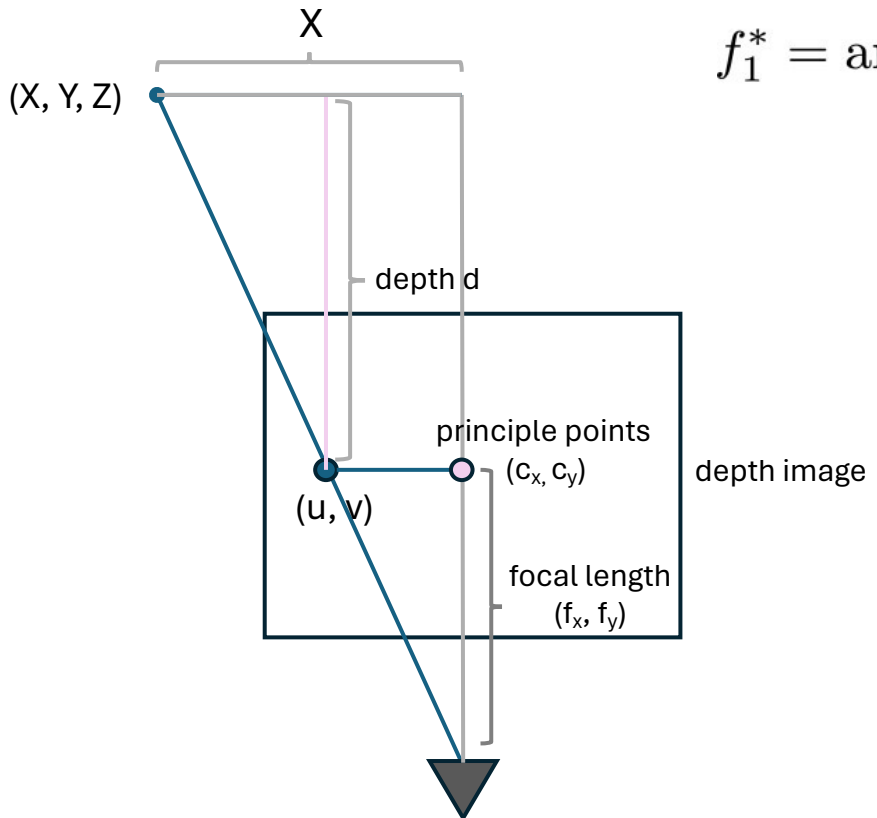
Achieved by mutual nearest neighbour (MNN) search in the **3D space**

$$\mathcal{M}_{1,2} = \{(i,j) \mid i = \mathrm{NN}_1^{1,2}(j) \text{ and } j = \mathrm{NN}_1^{2,1}(i)\}$$
$$\text{with } \mathrm{NN}_k^{n,m}(i) = \underset{j \in \{0,\ldots,WH\}}{\arg\min} \left\| X_j^{n,k} - X_i^{m,k} \right\|.$$

DUSt3R: Applications

2. Recovering intrinsic

Estimate the focal lengths by minimize:



$$f_1^* = \arg\min_{f_1} \sum_{i=0}^{W} \sum_{j=0}^{H} C_{i,j}^{1,1} \left\| (i', j') - f_1 \frac{(X_{i,j,0}^{1,1}, X_{i,j,1}^{1,1})}{X_{i,j,2}^{1,1}} \right\|$$

$$\mathrm{Z} = \frac{d}{depth\_scale}$$

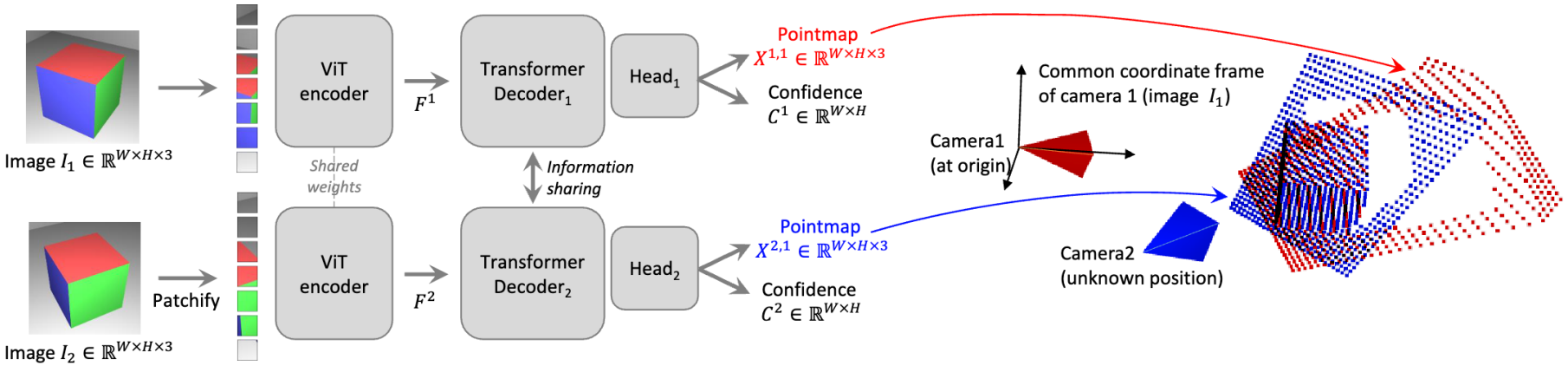
$$\mathrm{X} = \frac{(u - c_x) \cdot \mathrm{Z}}{f_x}$$

$$\mathrm{Y} = \frac{(u - c_y) \cdot \mathrm{Z}}{f_y}$$

DUSt3R: Applications

3. Relative Camera Pose

Essential Matrix



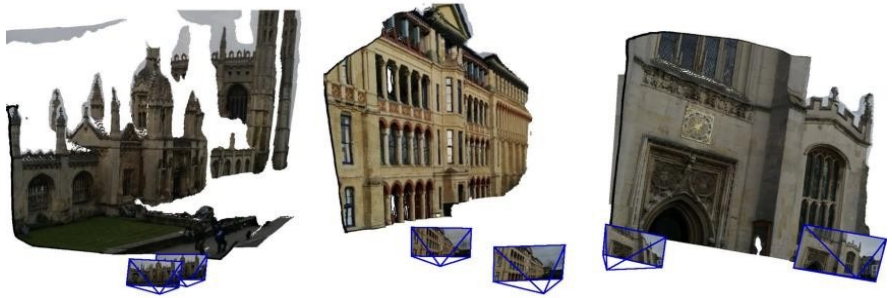
$$x_c'^T E x_c = 0$$

$$E = [t]_\times R$$

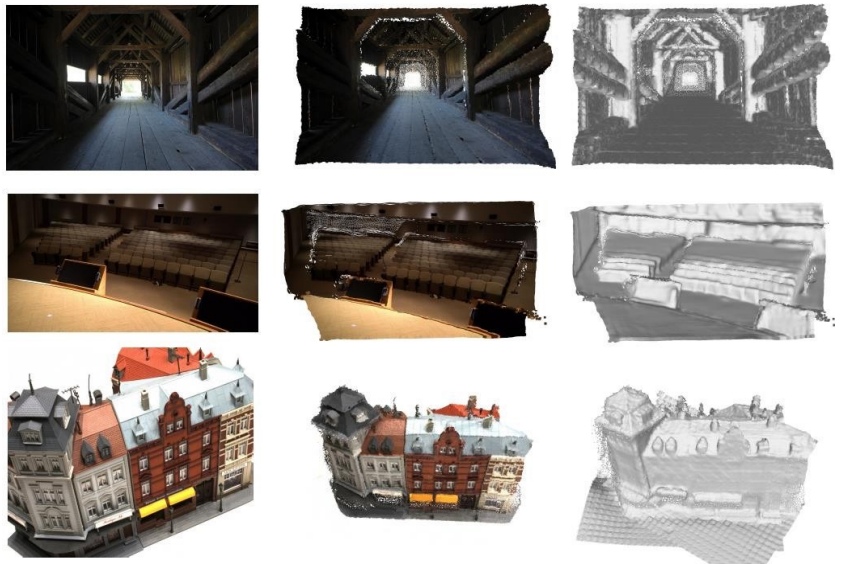
DUSt3R: Applications

4. Reconstruction

Two views



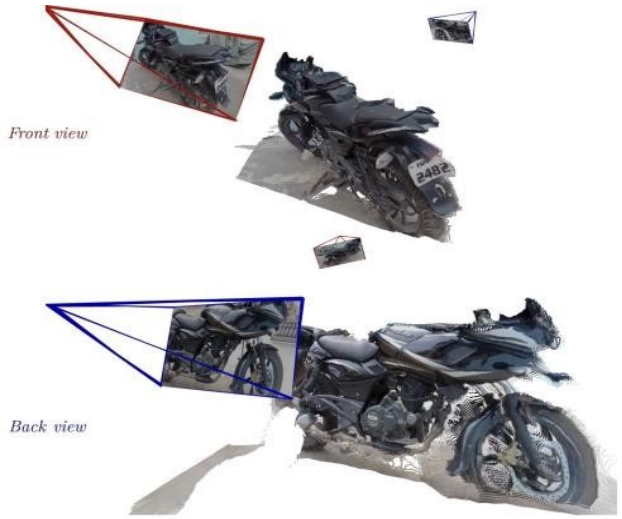
Dense Reconstruction



DUSt3R: Applications

4. Reconstruction

Opposite views



No overlap

Summary

Structure From Motion (SfM)

Extending two-view epipolar geometry to multiple views

Neural Radiance Fields (NeRFs), ECCV 2020

Solves correspondences + triangulation implicitly

DUSt3R: Geometric 3D Vision Made Easy, CVPR 2024

Deep-learning based method, unify all 3D vision tasks