

Basic linear algebra for deep Learning

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ABSTRACT

Like the artificial intelligence (AI) technology that's lurking everywhere, the researchers who work on AI operate behind the scenes. For every new robot, there are hundreds of computer scientists and researchers who put in thousands of hours to make sure it works. Computer scientists are attracted to AI for different reasons - computers are a lifelong passion, AI could hold the answers to our worst problems, or because AI could make their favorite science fiction books real.

AI is the new electricity and it will transform industries after industries. Just like electricity did at the time of our forefathers. Through this course, we will provide a comprehensive introduction to basics of artificial intelligence with programming concepts and codes that you can implement.

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CHAPTER 1

Introduction

In this document, we will be covering the basic mathematical knowledge required to learn deep learning. Linear algebra is a branch of mathematics that is concerned with mathematical structures closed under the operations of addition and scalar multiplication and that includes the theory of systems of linear equations, matrices, determinants, vector spaces, and linear transformations. In multiple cases, Linear algebra is so helpful in our implementation of models, It reduces the computation time and increases the efficiency of our algorithms.

CHAPTER 2

Mathematical objects

(11)

SCALAR

5	3	7
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Row Vector
(shape 1x3)

5
1.5
2

Column Vector
(shape 3x1)

4	19	8
16	3	5

MATRIX

			A	B	C
1	a	b	c		
2				c	G
3				g	J
4				j	
7	8	9			

TENSOR

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2.1 Scalar

A scalar is a single real number that is used to measure magnitude (size). Scalars have magnitude or a numerical value, and not direction. They are the opposite of vectors, which have both a magnitude (or numerical value) and a direction. Real numbers like -0.1, .001, 1, 1.435, 13 are always scalars in matrix and linear algebra^{**}; they cannot be vectors without a direction.

Scalars are used in matrix multiplication. When a matrix is multiplied by a number (a scalar), each element in the matrix is multiplied by that number to create a new matrix. In the following image, the matrix $\begin{bmatrix} 9 & 3 \\ 5 & 7 \end{bmatrix}$ is multiplied by the scalar 2. The new matrix is called a scalar multiple. In this case, it's a scalar multiple of 2. In linear algebra, scalars can be negative.

2.2 Vector

A vector is a list of numbers. There are (at least) two ways to interpret what this list of numbers mean: One way to think of the vector as being a point in a space. Then this list of numbers is a way of identifying that point in space, where each number represents the vector's component that dimension. Another way to think of a vector is a magnitude and a direction, e.g. a quantity like velocity (the fighter jets velocity is 250 mph north - by - northwest). In this way of thinking of it, a vector is a directed arrow pointing from the origin to the end point given by the list of numbers.

2.3 Matrix

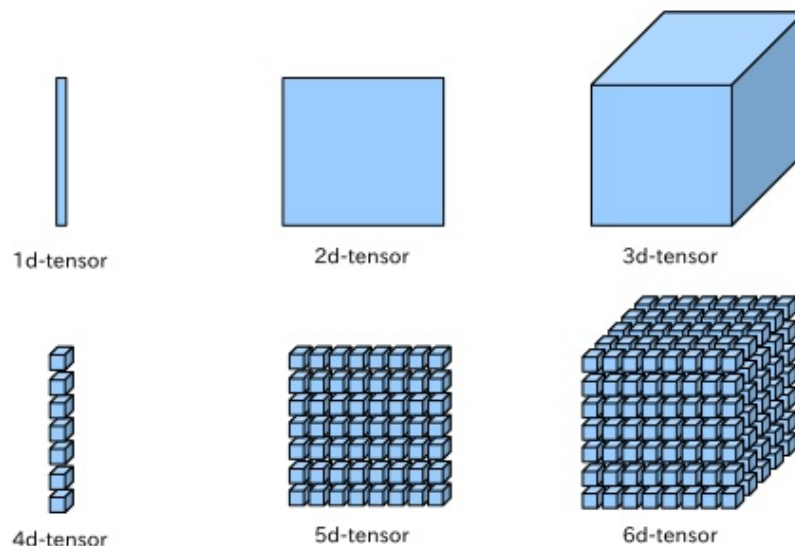
Matrix notation was invented primarily to express linear algebra relations in compact form. Compactness enhances visualization and understanding of essentials. A Matrix is an ordered 2D array of numbers and it has two indices. The first one points to the row and the second one to the column. For example, M_{23} refers to the value in the second row and the third column, which is 8 in the yellow graphic above. A Matrix can have multiple numbers of rows and columns. Note that a Vector is also a Matrix, but with only one row or one column.

2.4 Tensor

A Tensor is an array of numbers, arranged on a regular grid, with a variable number of axes. A Tensor has three indices, where the first one points to the row, the second to the column and the third one to the axis. For example, T232 points to the second row, the third column, and the second axis. This refers to the value 0 in the right Tensor in the graphic below:

What's Tensor

Tensor is a general name of multi-way array data. For example, 1d-tensor is a vector, 2d-tensor is a matrix and 3d-tensor is a cube. We can image 4d-tensor as a vector of cubes. In similar way, 5d-tensor is a matrix of cubes, and 6d-tensor is a cube of cubes.



CHAPTER 3

Computational Rules

3.1 Matrix-scalar Operations

If you multiply, divide, subtract, or add a Scalar to a Matrix, you do so with every element of the Matrix.

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

3.2 Matrix-Vector Multiplication

Multiplying a Matrix by a Vector can be thought of as multiplying each row of the Matrix by the column of the Vector. The output will be a Vector that has the same number of rows as the Matrix. The image below shows how this works:

$$\begin{bmatrix} A & B \\ C & D \\ E & F \end{bmatrix} \times \begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} A \times G + B \times H \\ C \times G + D \times H \\ E \times G + F \times H \end{bmatrix}$$

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