m basis fields

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The transformation from x-basis to μ-basis is

$$\phi^j_\mu = \int \mathrm{d}x\, \mathrm{e}^{-\mu x} \phi^j_x. \tag{0.1}$$

Consider the D series representation $\hat{D}^{j,+}$. The lowest weight state $|j,-j\rangle$ corresponds to μ-basis field

$$\phi^j_\mu\big|_{\mu=0} = \int \mathrm{d}x\, \phi^j_x. \tag{0.2}$$

This can be examined in μ-basis:

$$J^-_0\, \phi^j_0 = \mu\phi^j_\mu\big|_{\mu=0} = 0, \tag{0.3}$$

or in x-basis:

$$J^-_0 \int \mathrm{d}x\, \phi^j_x = \int \mathrm{d}x\, -\partial_x \phi^j_x = 0. \tag{0.4}$$

However, this will lead to a problem. The eigenvalue of J^0_0 on $|j,-j\rangle$ shoule be $-j$. On the other hand, we have

$$\begin{aligned} J^0_0 \phi^j_{\mu=0} &= \int \mathrm{d}x\, J^0_0 \phi^j_x && (0.5)\\ &= \int \mathrm{d}x\, (x\partial_x - j)\phi^j_x && (0.6)\\ &= (1-j)\phi^j_{\mu=0} \neq -j\phi^j_{\mu=0}. && (0.7) \end{aligned}$$

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Since the fusion of two $\hat{\mathcal{D}}^+$ will not give $\hat{\mathcal{D}}^-$, the 3-point function involving three ϕ^j_0 should be equal to 0. However, we have

$$\begin{aligned} \left\langle \phi^j_0(z_1)\phi^j_0(z_2)\phi^j_0(z_3) \right\rangle &= \int \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3\, \langle \phi^j_{x_1}(z_1)\phi^j_{x_2}(z_2)\phi^j_{x_3}(z_3)\rangle && (0.8)\\ &= \int \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3\, C^{j_1,j_2,j_3}(z_1,z_2,z_3) x_{12}^{j_1+j_2-j_3} x_{23}^{j_2+j_3-j_1} x_{31}^{j_3+j_1-j_2}. && (0.9)\\ & && (0.10) \end{aligned}$$

There's no constraint on this integral such that the 3-point function should be 0.