Chapter 10: Notes

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Section 10.1

[

The sampling Distribution of the Difference between Two Sample Means for Independent Samples] Suppose that x is a normally distributed variable on each of two populations. Then, for independent samples of size n_1 and n_2 from two populations,

- $\mu_{\bar{x_1}-\bar{x_2}} = \mu_1 - \mu_2$,
- $\sigma_{\bar{x_1}-\bar{x_2}} = \sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}$, and
- $\bar{x_1} - \bar{x_2}$ is normally distributed.

Section 10.2

Distribution of the Pooled t-Statistic

Suppose that x is normally distriuted variable on each of two populations and that the population standard deviations are equal. Then, for independent samples od sizes n_1 and n_2 from the two populations, the variable

$$t = \frac{(\bar{x_1} - \bar{x_2}) - (\mu_1 - \mu_2)}{s_p\sqrt{(1/n_1) + (1/n_2)}}$$

has the t-Distribution with $df = n_1 + n_2 - 2$.

Pooled t Test

Purpose To perform a hypothesis test to compare two population means, μ_1 and μ_2.

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples
4. Equal population standard deviation, σ

Step 1 The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is

$$H_a : \mu ! = \mu_0 \quad or \quad H_a : \mu < \mu_0 \quad or \quad H_a : \mu > \mu_0$$

$$(Two \quad tailed) \qquad (Left \quad Tailed) \qquad (Right \quad tailed)$$

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$t = \frac{\bar{x_1} - \bar{x_2}}{s_p\sqrt{(1/n_2) + (1/n_2)}},$$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$

Denote the value of the test statistic t_0.

Critical-Value Approach

Step 4 The critical value(s) are

$$\pm z_{\alpha/2} \qquad or \qquad - z_\alpha \qquad or \qquad z_\alpha$$

$$(Two \quad tailed) \qquad (Left \quad tailed) \qquad (Right \quad tailed)$$

Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

P-Value Approach

Step 4 The t-statistic has $df = n_1 + n_2 - 2$. Use a t-table to estimate the P-value, or obtain it exactly using technology.

Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Note: The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Pooled t-Interval Procedure

Purpose To find a confidence interval for the difference between two population means, μ_1 and μ_2.

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations or large samples
4. Equal population standard deviations

Step 1 For a confidence level of $1 - \alpha$, use a t-table to find $t_{\alpha/2}$ with $df = n_1 + n_2 - 2$.

Step 2 The endpoints of the confidence interval for $\mu_1 - \mu_2$ are

$$(\bar{x_1} - \bar{x_2}) \pm t_{\alpha/2} * s_p \sqrt{(1/n_1) + (1/n_2)},$$

where s_p is the pooled sample standard deviation.

Step 3 Interpret the confidence interval.

Note: The confidence interval is exact for normal populations and is approximately correct for larger samples from nonnormal populations.

Section 10.3

Distribution of the Nonpooled t-Statistic

Suppose that x is a normally distributed variable on each of two populations. Then, for independent samples of sizes n_1 and n_2 from the two populations, the variable

$$t = \frac{(\bar{x_1} - \bar{x_2} - (\mu_1 - \mu_2))}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}$$

has approximately a t-distribution. The degrees of freedom used is obtained from the sample data. It is denoted Δ and given by

$$\Delta = \frac{[(s_1^2/n_1) + (s_2^2/n_2)]^2}{\frac{(s_1^2/n_1)^2}{n_1-1)} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

rounded down to the nearest integer.

Nonpooled t-Test

Purpose To perform a hypothesis test to compare population means, μ_1 and μ_2.

Assumptions 1. Simple random sample

2. Independent samples
3. Normal populations or large samples

Step 1 The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

$$H_a : \mu ! = \mu_0 \quad or \quad H_a : \mu < \mu_0 \quad or \quad H_a : \mu > \mu_0$$

$$(Two \quad tailed) \qquad (Left \quad Tailed) \qquad (Right \quad tailed)$$

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$t = \frac{\bar{x_1} - \bar{x_2}}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}},$$

Denote the value of the test statistic t_0.

0.0.1 Critical-Value Approach

Step 4 The critical value(s) are

$$\begin{array}{ccccc} \pm z_{\alpha/2} & or & -z_{\alpha} & or & z_{\alpha} \\ (Two \quad tailed) & & (Left \quad tailed) & & (Right \quad tailed) \end{array}$$

with $df = \Delta$, where

$$\Delta = \frac{[(s_1^2/n_1) + (s_2^2/n_2)]^2}{\frac{(s_1^2/n_1)^2}{n_1-1)} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

rounded down to the nearest integer. Use a t-table to find the critical value(s).

Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

0.0.2 P-Value Approach

Step 4 The t-statistic has $df = \Delta$, where

$$\Delta = \frac{[(s_1^2/n_1) + (s_2^2/n_2)]^2}{\frac{(s_1^2/n_1)^2}{n_1-1)} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

rounded down to the nearest integer. Use a t-table to estimate the P-value, or obtain it exactly with using technology.

Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Nonpooled t-Interval Procedure

Purpose To find the confidence interval for the difference between two population means, μ_1 and μ_2.

Assumptions
1. Simple random sample
2. Independent samples
3. Normal populations or large samples

Step 1 For a confidence level of $1 - \alpha$, use a t-table to find $t_{\alpha/2}$ with $df = \Delta$, where

$$\Delta = \frac{[(s_1^2/n_1) + (s_2^2/n_2)]^2}{\frac{(s_1^2/n_1)^2}{n_1-1)} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

rounded down to the nearest integer.

Step 2 The endpoints of the confidence interval for $\mu_1 - \mu_2$ are

$$(\bar{x_1} - \bar{x_2}) \pm t_{\alpha/2} * \sqrt{(s_1^2/n_1) + (s_2^2/n_2)}.$$

Step 3 Interpret the confidence interval.

Choosing between a Pooled and Nonpooled t-Procedure

Suppose you want to use independent simple random samples to compare the means of two populations. To decide between a pooled t-procedure and a nonpooled t-procedure, follow these guidelines: If you are reasonably sure that the populations have nearly equal standard deviations, use a pooled t-procedure; otherwise, use a nonpooled t-procedure.

Section 10.4

Section 10.5

Section 10.6

Section 10.7
