Chapter 6: Notes

Evan Hunt

November 28, 2019

Section 6.1

Basic Properties of Density Curves

Property 1: A density curve is always on or above the horizontal axis

Property 2: The total area under a density curve (and above the horizontal axis) equals 1.

Variables and Their Density Curves

For a variable with a density curve, the percentage of all possible observations of the variable that lie within any specified range equals (at least approximately) the corresponding area under the density curve, expressed as a percentage.

Normally Distributed Variable

A variable is said to be a normally distributed variable or to have a normal distribution if its distribution has the shape of a normal curve.

Normally Distributed Variables and the Normal-Curve Areas

For a normally distributed variable, the percentage of all possible observations that lie within any specified range equals the corresponding area under its associated normal curve,m expressed as a percentage. This result holds approximately for a variable that is approximately normally distributed.

Standard Normal Distribution; Standard Normal Curve

A normally distributed variable having mean 0 and standard deviation 1 is said to have the standard normal distribution. Its associated normal curve is called the standard normal curve.

Standardized Normally Distributed Variable

The standardized version of a normally distributed variable x,

$$z = \frac{x - \mu}{\sigma},$$

has the standard normal distribution. Subtracting from a normally distributed variable its mean and then dividing by its standard deviation results in a variable with the standard normal distribution.

Section 6.2

Basic Properties of the Standard Normal Curve

Property 1: The total area under the standard normal curve is 1.

Property 2: The standard normal curve extends indefinitely in both directions, approaching, but never touching, the horizontal axis.

Property 3: The standard normal curve is symmetric about 0; that is, curve to the right of the mean is a mirror image of the curve to the left of the mean.

Property 4: Almost all the area under the standard normal curve lies between -3 and 3.

The z_α Notation

The symbol z_α is used to denote the z-score that has an area of α (alpha) to its right under the standard normal curve. Read "z_α" as "z sub α" or as "z α".

Section 6.3

Procedure to Determine a Percentage or Probability for a Normally Distributed Variable

Step 1 Sketch the normal curve associated with the variable.

Step 2 Shade the region of interest and mark its delimiting x-value(s).

Step 3 Find the z-score(s) for the delimiting x value(s) found in Step 2.

Step 4 Use a $z$table to find the area under the standard normal curve delimiter by the z-score(s) found in Step 3.

Empirical Rule for Variables

For any variable whose distribution is bell-shaped (in particular, for any normally distributed variable), the following three properties hold.

Property 1: Approximately 68% of all possible observations lie within one standard deviation to either side of the mean, $[\mu - \sigma, \mu + \sigma]$.

Property 2: Approximately 95% of all possible observations lie within two standard deviations to either side of the mean, $[\mu - 2\sigma, \mu + 2\sigma]$.

Property 3: Approximately 99.7% of all possible observations lie within three standard deviations to either side of the mean, $[\mu - 3\sigma, \mu + 3\sigma]$.

Procedure to Determine the Observations Corresponding to Specified percentage or Probability for a Normally Distributed Variable

Step 1 Sketch the normal curve associated with the variable.

Step 2 Shade the region of interest.

Step 3 Use a z-table to determine the z-score(s) delimiting the region found in Step 2.

Step 4 Find the x-value(s) having the z-score(s) found in Step 3.

$$x = \mu + z * \sigma$$

Section 6.4

Guidelines for Assessing Normality Using a Normal Probability Plot

A normal probability plot that falls nearly in a straight line indicates a normal variable, and one that does not indicates a nonnormal variable. To assess the normality of a variable using sample data, construct a normal probability plot.

- If the plot is roughly linear, you can assume that the variable is approximately normally distributed.
- If the plot is not roughly linear, you can assume that the variable is not approximately distributed.

These guidelines should be interpreted loosely for small samples but usually interpreted strictly for large samples.

Section 6.5

Procedure to Approximate Binomial Probabilities by Normal Curve Areas

Step 1 Find n, the number of trials, and p, the success probability.

Step 2 Continue only if both np and $n(1-p)$ are 5 or greater.

Step 3 Find μ and σ, using the formulas $\mu = np$ and $\sigma = \sqrt{np(1-p)}$..

Step 4 Make the correction for continuity, and find the required area under the normal curve with parameters μ and σ.