Chapter 9: Hypothesis Tests for One Population Mean

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Section 9.1

Null and Alternative Hypotheses; Hypothesis Test

Null hypothesis: A hypothesis to be tested. We use the symbol H_0 to represent the null hypothesis.

Alternative hypothesis: A hypothesis to be considered as an alternative to the null hypothesis. We use the symbol H_a to represent the alternative hypothesis.

Hypothesis test: The problem in a hypothesis test is to decide whether the null hypothesis should be rejected.

Basic Logic of Hypothesis Testing

Take a random sample from the population. If the sample data are consistent with the null hypothesis, do not reject the null hypothesis; if the sample data are inconsistent with the null Hypothesis and supportive of the alternative hypothesis, reject the null hypothesis in favor of the alternative hypothesis.

Type I and Type II Errors

Type I error: Rejecting the null hypothesis when it is in fact true.

Type II error: Not rejecting the null hypothesis when it is in fact false.

Significance Level

The probability of making a Type I error, that is, of rejecting the null hypothesis, is called the significance level, α, of a hypothesis test.

Relation between Type I and Type II Error Probabilities

For a fixed sample size, the smaller we specify the significance level, α, the larger will be the probability, β, of not rejecting a false hypothesis.

Possible Conclusions for a Hypothesis Test

Suppose that a hypothesis test is conducted at a small significance level.

- If the hypothesis is rejected, we could conclude that the data provided sufficient evidence to support the alternative hypothesis.
- If the null hypothesis is not rejected, we conclude that the data do not provide sufficient evidence to support the alternative hypothesis.

Section 9.2

Rejection Region, Nonrejection Region, and Critical Values

Rejection region: The set of values for the test statistic that leads to nonrejection of the null hypothesis.

Nonrejection region: The set of values for the test statistic that leads to nonrejection of the null hypothesis.

Critical Value(s): The value or values of the test statistic that separate the rejection and nonrejection regions. A critical value is considered part of the rejection region.

If the value of the test statistic falls in the rejection region, reject the null hypothesis; otherwise, do not reject the null hypothesis.

Obtaining Critical Values

Suppose that a hypothesis test is to be performed at the significance level α. Then the critical value(s) must be chosen so that, if the null hypothesis is true, the probability is α that the test statistic will fall in the rejection region.

Critical-Value Approach to Hypothesis Testing

Step 1 State the null and alternative hypothesis.

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic.

Step 4 Determine the critical value(s).

Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the result of the hypothesis test.

Section 9.3

P-Value

The P-value of a hypothesis test is the the probability of getting sample data at least as inconsistent with the null hypothesis (and supportive of the alternative) hypothesis) as the sample data actually obtained. We use the letter P to denote the P-value. Small P-values provide evidence against the null hypothesis; large P-values do not.

Decision Criterion for Hypothesis Test Using the P-Values

If the P-value is less than or equal to the specified significance level, reject the null hypothesis; otherwise, do not reject the null hypothesis. In other words, if $P \leq \alpha$, reject H_0; otherwise, do not reject H_α.

P-Value as the Observed Significance Level

The P-value of a hypothesis test equals the smallest significance level at which the null hypothesis can be rejected, that is, the smallest significance level for which the observed sample data result in rejection of H_0.

Determining a P-Value

To determine the P-Value of a hypothesis test, we assume that the null hypothesis is true and compute the probability of observing a value of the test statistic as extreme as or more extreme that that observed. By extreme we mean "far from what we would expect to observe if the null hypothesis is true."

P-Value Approach to Hypothesis Testing

Step 1 State the null and alternative hypotheses.

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic.

Step 4 Determine the P-value, P.

Step 5 If $P \leq \alpha$, reject H_a; otherwise, do not reject H_0.

Step 6 Interpret the result of the hypothesis test.

Hypothesis Tests Without Significance Levels:

Many researchers do do not explicitly refer to significance levels. Instead, they simply obtain the P-value and use it (or let the reader us it) to assess the strength of the evidence against the null hypothesis.

Section 9.4

One-Mean z-Test

Purpose To perform a hypothesis test for a population mean, μ.

Assumptions
1. Simple random sample.
2. Normal population or large sample.
3. σ known

Step 1 The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

$$H_a : \mu ! = \mu_0 \quad or \quad H_a : \mu < \mu_0 \quad or \quad H_a : \mu > \mu_0$$

$$(Two \quad tailed) \qquad (Left \quad Tailed) \qquad (Right \quad tailed)$$

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

and denote that value z_0.

Critical-Value Approach

Step 4 The critical value(s) are

$$\pm z_{\alpha/2} \qquad or \qquad -z_{\alpha} \qquad or \qquad z_{\alpha}$$
$$(Two \quad tailed) \qquad (Left \quad tailed) \qquad (Right \quad tailed)$$

Use a z-table to find the critical values.

Step 5 If the value of the test statistic fall in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

P-Value Approach

Step 4 Use a z-table to obtain the P-value.

Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Note: The hypothesis test is exact for normal populations and is approximate for large samples from nonnormal populations.

When to Use the One-Mean z-Test

- For small samples, less than 15 roughly, the z-test should be used only when the variable under consideration is normally distributed or very close to being so.
- For samples of moderate size, between 15 and 30 roughly, the z-test can be used unless the data contain outliers or the variable under consideration is far from being normally distributed.
- For large samples, around 30 or more, the z-test can be used essentially without restriction. However, if outliers are present and their removal is not justified , you should perform the hypothesis test once with outliers and once without them to see what effect the outliers have. If the conclusion is affected, use a different procedure or take another sample, if possible.
- If outliers are present but their removal is justified and result in a data set for which the z-test us appropriate (as previously stated), the procedure can be used.

Section 9.5

One-Mean t-Test

Purpose To perform a hypothesis test for a population mean, μ.

Assumptions 1. Simple random sample

2. Normal Population or large sample

3. σ unknown

Step 1 The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

$$H_0 : \mu ! = \mu_0 \quad or \quad H_0 : \mu < \mu_0 \quad or \quad H_0 : \mu > \mu_0$$

$$(Two \quad tailed) \qquad (Left \quad tailed) \qquad (Right \quad tailed)$$

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

and denote that value t_0.

Critical-Value Approach

Step 4 The critical value(s) are

$$\pm t_{\alpha/2} \qquad or \qquad -t_\alpha \qquad or \qquad t_\alpha$$

$$(Two \quad tailed) \qquad (Left \quad tailed) \qquad (Right \quad tailed)$$

Step 5 If the value of the test statistic fall in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

P-Value Approach

Step 4 The t statistic had $df = n - 1$. Use a t-table to estimate the P-value; or obtain it exactly by using technology.

Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Section 9.6

Section 9.7

Section 9.8
