

Chapter 7: Notes

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Section 7.1

Sampling Error

Sampling error is the error resulting from using a sample to estimate a population characteristic.

Sampling Distribution of the Sample Mean

For a variable x and a given sample size, the distribution of the variable \bar{x} is called the sampling distribution of the sample mean. The sampling distribution of the sample mean is the distribution of all possible sample means for the samples of a given size.

Sample Size and Sampling Error

The larger the sample size, the smaller the sampling error tends to be in estimating a population mean, μ , by a sample mean, \bar{x} . The possible sample means cluster more closely around the population mean as the sample size increases.

Section 7.2

Mean of the Sample Mean

For samples of size n , the mean of the variable \bar{x} equals the mean of the variable under consideration. In symbols,

$$\mu_{\bar{x}} = \mu$$

For any sample size, the mean of all possible samples equals the population mean.

Standard Deviation of the Sample Mean

For samples of size n , the standard deviation of the variable \bar{x} equals the standard deviation of the variable under consideration divided by the square roots of the sample size. In symbols,

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

For each sample size, the standard deviation of all possible sample means equals the standard deviation divided by the square root of the sample size.

Note:

In the formula for the standard deviation of \bar{x} , the sample size, n appears in the denominator. This explains mathematically why the standard deviation of \bar{x} decreases as the sample size increases.

Section 7.3

Sampling Distribution of the Sample Mean for a Normally Distributed Variable

For a normally distributed variable, the possible sample means for samples of given size are also normally distributed. Suppose that a variable x of a population is normally distributed with mean μ and standard deviation σ . Then, for samples of size n , the variable \bar{x} is also normally distributed and has mean μ and standard deviation σ/\sqrt{n} .

The Central Limit Theorem (CLT)

For a relatively large sample size, the variable \bar{x} is approximately normally distributed, regardless of the distribution of the variable under consideration. The approximation becomes better with increasing sample size.

Sampling Distribution of the Sample Mean

Suppose that a variable x of a population has mean μ and standard deviation σ . Then, for samples of size n .

- the mean of \bar{x} equals the population mean, or $\mu_{\bar{x}} = \mu$;
- the standard deviation of \bar{x} equals the population standard deviation divided by the square root of the sample size, or $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$;
- if x is normally distributed, so is \bar{x} , regardless of sample size; and
- if the sample size is large, \bar{x} is approximately normally distributed, regardless of the distribution of x .

If a variable is normally distributed or the sample size is large, then the possible sample means have, at least approximately, a normal distribution with mean μ and standard deviation σ/\sqrt{n} .