Data Structure

AVL Tree

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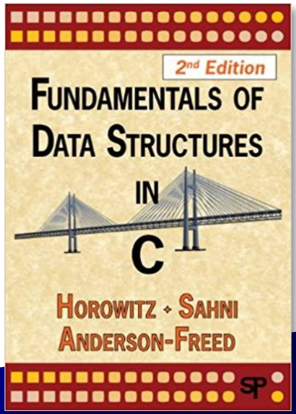
Ch. 10.2 AVL Trees



2nd Edition

FUNDAMENTALS OF DATA STRUCTURES IN C

HOROWITZ · SAHNI

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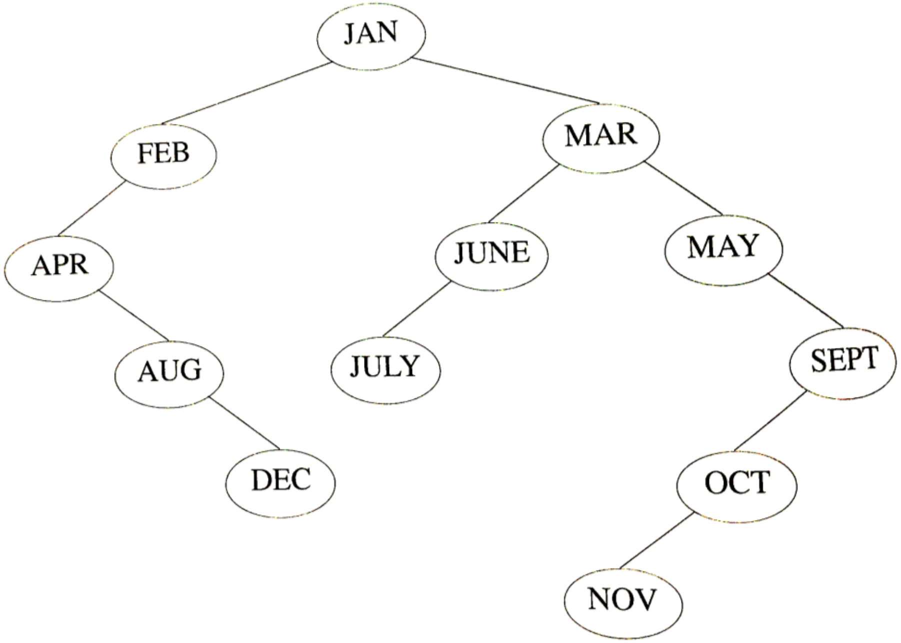
AVL Tree Definition

- A height-balanced tree is a binary tree where the difference of the heights of two subtrees of a node is more than 1
  - an empty tree is height-balanced
  - a tree is height-balanced where left and right subtrees are height-balanced and their height difference is no more than 1

- AVL tree is a height-balanced binary search tree

- The balanced factor of a node in a binary tree, BF(T) is defined as $h_L - h_R$ where h_L and h_R represent the heights of the left and the right subtrees correspondingly

Examples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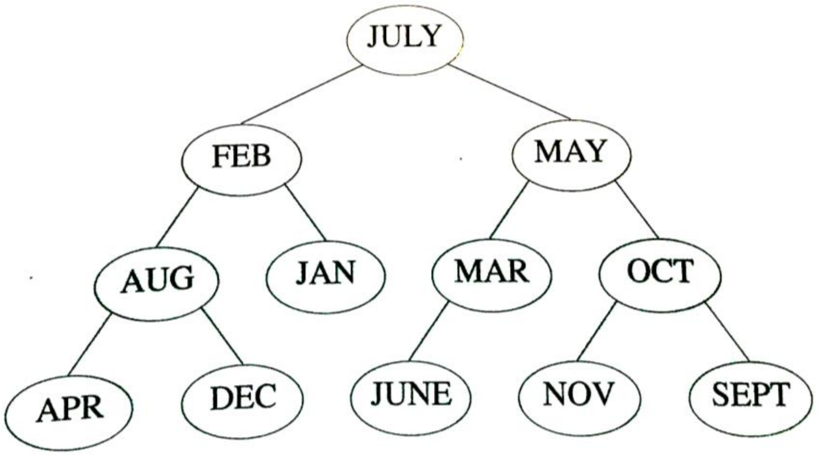
JULY
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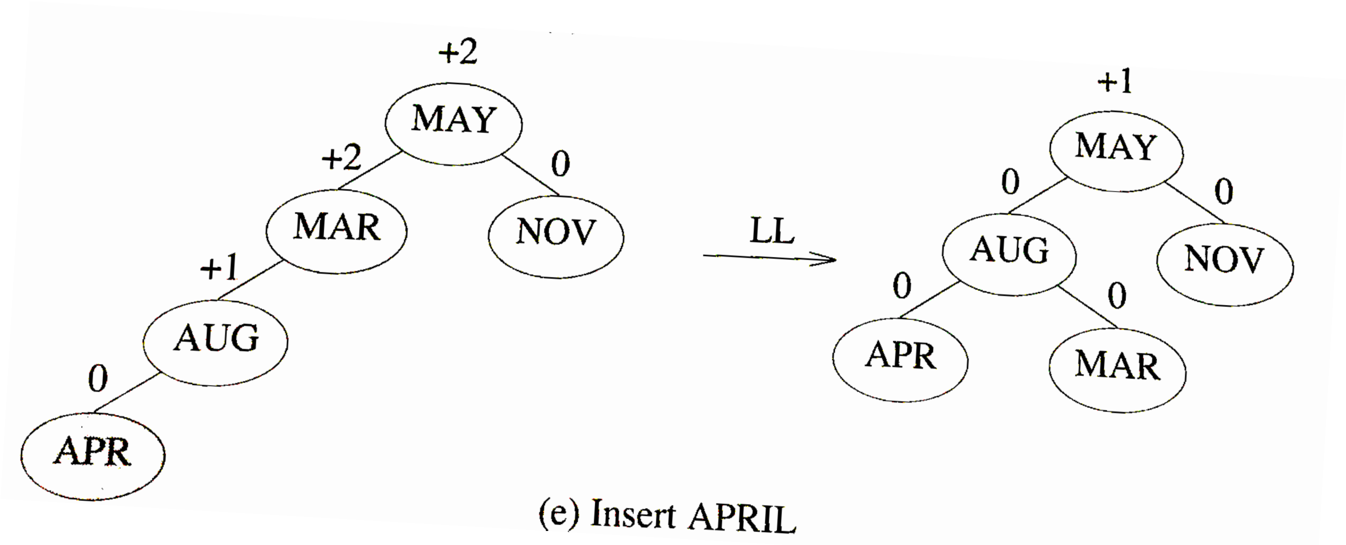
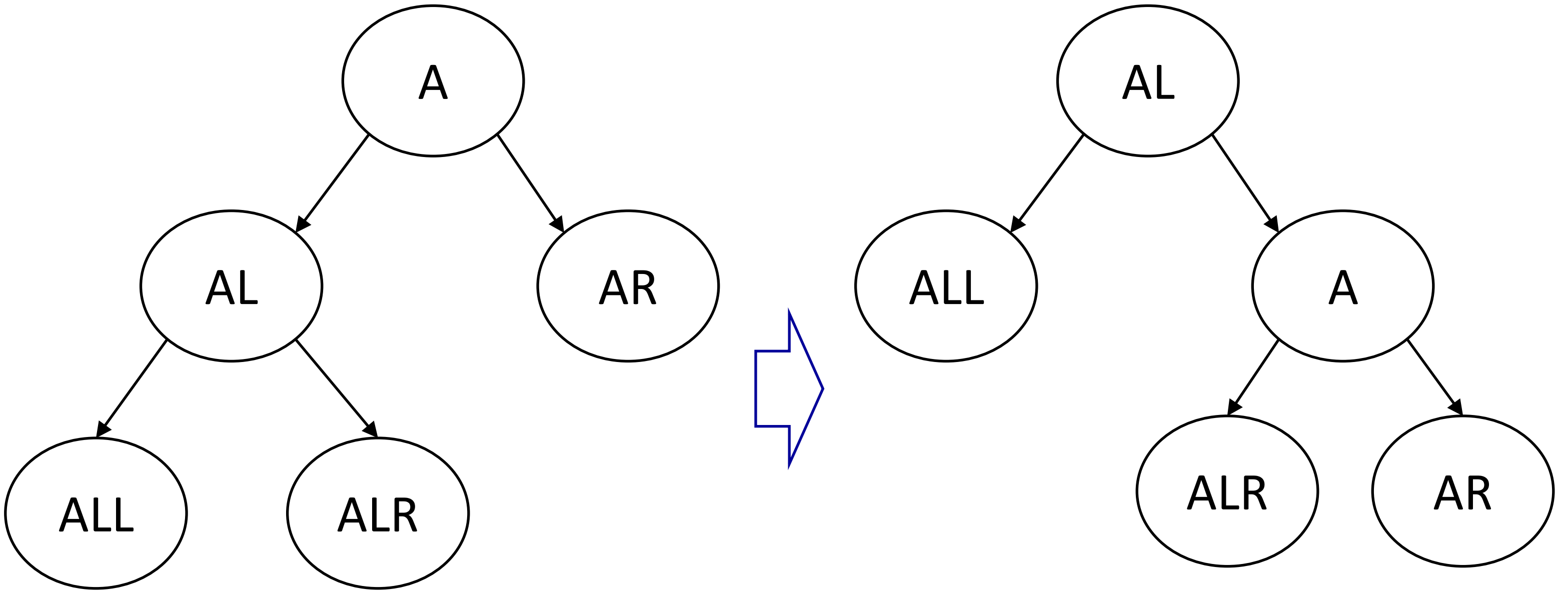
Insertion (1)

- Insert a new element *Y* as BST does
- Find first ancestor *A* whose BF is not -1, 0, and 1.
- Case 1. Y is in the left subtree of the left subtree of A
  - clockwise rotation regarding A



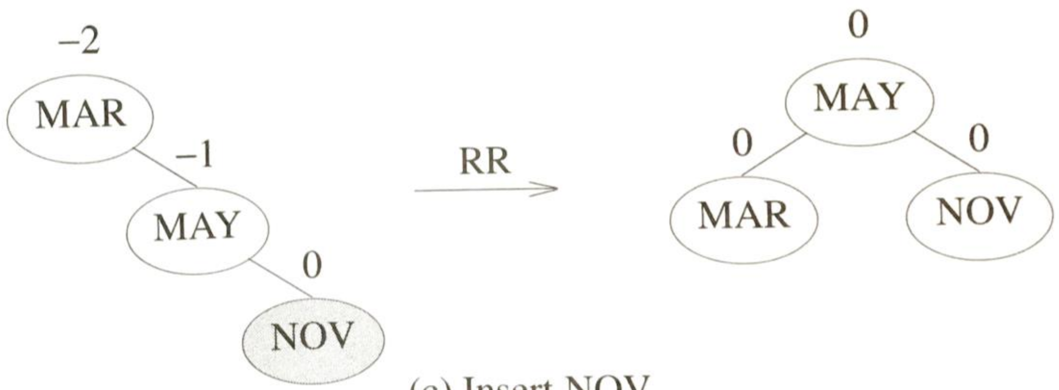
(e) Insert APRIL

Left Rotation (clockwise rotation)



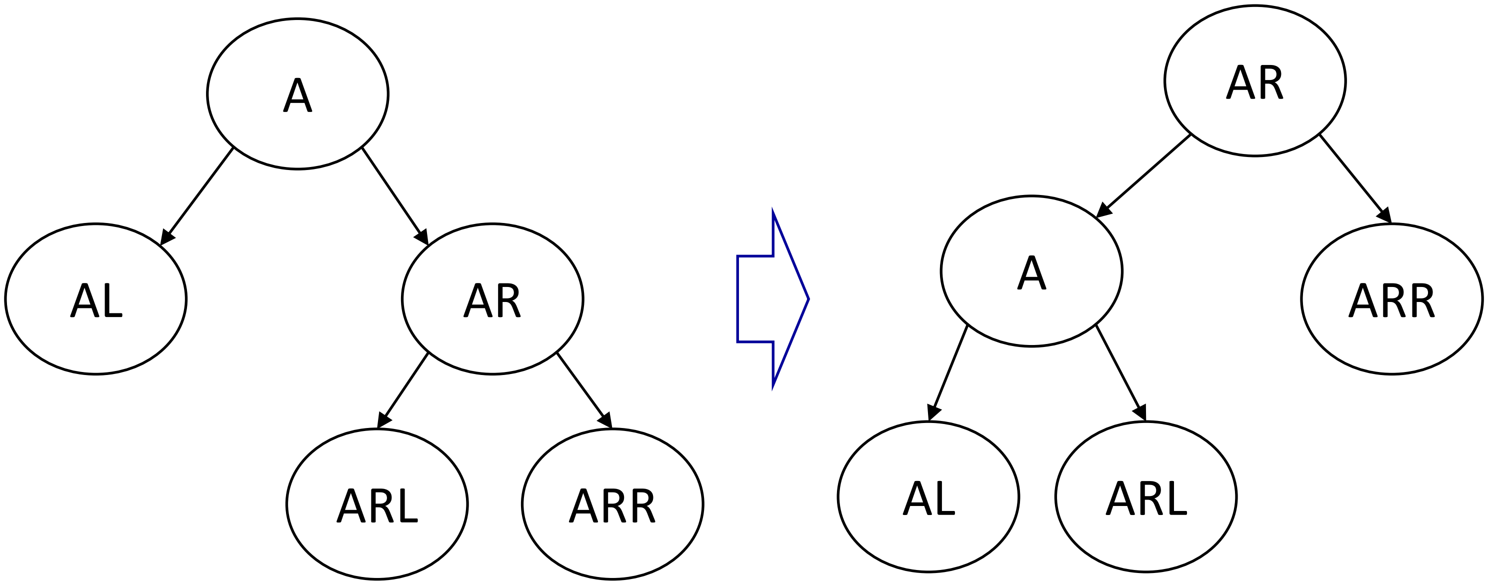
Insertion (2)

- Insert a new element *Y* as BST does
- Find first ancestor *A* whose BF is not -1, 0, and 1.
- Case 2. Y is in right subtree of the right subtree of A
  - Counter-clockwise rotation regarding A



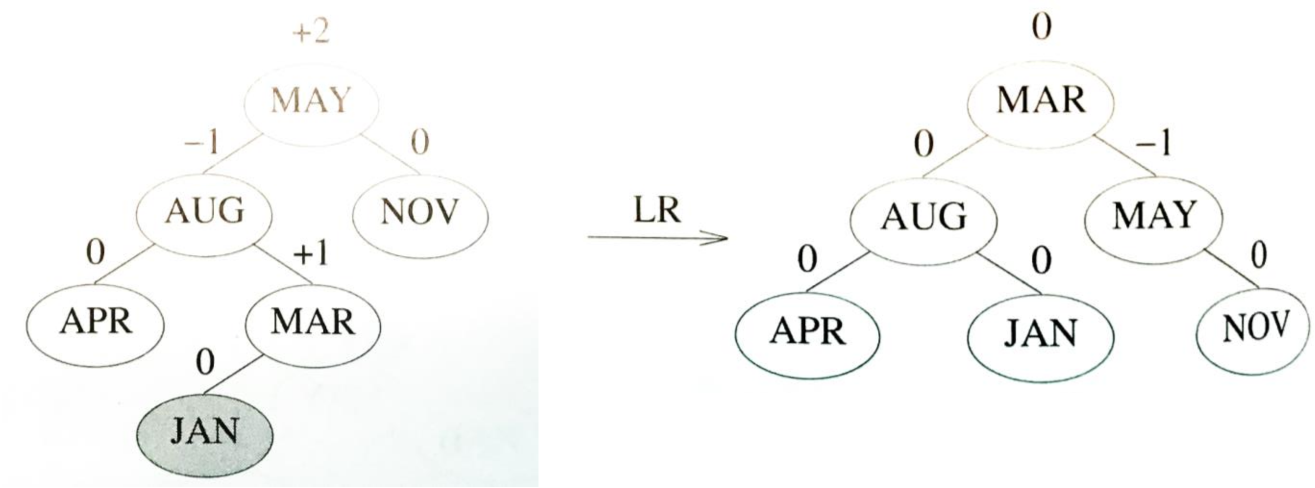
(c) Insert NOV

Right Rotation (counter-clockwise)

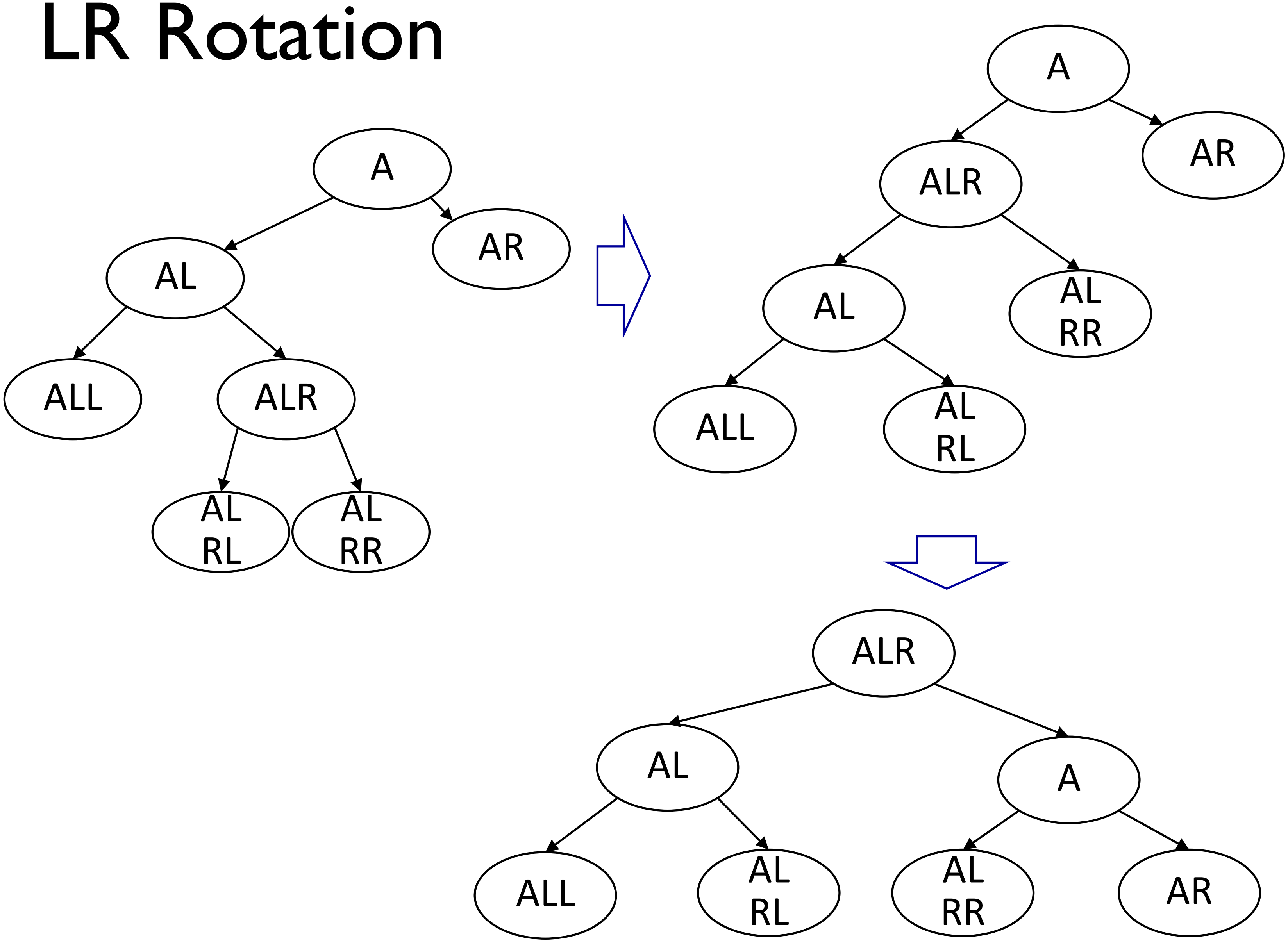


Insertion (2)

- Insert a new element *Y* as BST does
- Find first ancestor *A* whose BF is not -1, 0, and 1.
- Case 3. Y is in right subtree of the left subtree of A
  - Counter-clockwise rotation and then clockwise rotation



LR Rotation


A
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ALR
AL RL
AL RR
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AL RL
AL RR
AR

Delete Operation

1. Insert a specified node *X* as BST does
2. Find first ancestor *A* of *X* whose BF is not -1, 0, and 1.
3. Rebalance *A* as does for insertion
4. Set *X* as *A*
5. Repeat from 2 until no ancestors are remained as unblanced.