ChatGPT

Topological Data Analysis & the “Donut of Attention” – Research Map

Persistent Homology in Quantum Gravity Simplicial Spacetimes

Nonperturbative **quantum gravity models** based on simplicial manifolds (like **CDT/DT**) have recently adopted **topological data analysis (TDA)** to probe the “quantum foam” structure of spacetime [1]. By computing **Betti numbers** of triangulated geometries at varying coarse-graining scales, researchers obtain a **“topological fingerprint”** of quantum spacetime [2] [1]. This reveals how **topological features** (connected components, loops, voids) emerge or vanish under scale changes, providing new observables to characterize discrete spacetime phases. Key findings include the detection of **“baby universe” bubbles** (small \$S^2\$ components) at Planckian scales in Euclidean 2D gravity [3] and differences between Euclidean and Lorentzian dynamics. Table 1 summarizes representative work in this area:

Citation (Domain)	**Summary of Findings**	**TDA Metrics**	**Phase/Topology Signatures**	**DonutOS Insight** (Torus-Scanner Fit)
van der Duin et al., 2025 (Euclidean 2D CDT/DT) [4] [5] *Quantum Gravity & Effective Topology*	Introduces *effective homology* as an observable in lattice quantum gravity. Coarse-graining 2D **Euclidean** triangulations produces many small *"bubbles"* (extra S^2 components) at fine scales, reflecting a **fractal quantum foam** structure [4]. These bubbles (Betti-2 holes) appear in large numbers at the smallest scale and then vanish as the resolution grows, leaving one large component (the mother universe) [4]. This nontrivial homology across all scales is tied to the well-known fractal nature of Euclidean quantum gravity (minimal-neck baby universes) [5].	**Betti numbers** $β_0, β_1, β_2$ measured on triangulations as a function of **coarse-graining scale** $δ$. Used persistent homology (via GUDHI library) to track creation/ merging of holes [6] [7]. Focus on $β_2$ (2D voids) as function of $δ$.	**"Quantum foam" regime:** High $β_2$ (many void bubbles) at small $δ$ [4]. **Extended smooth regime:** $β_2 \to 1$ at large $δ$ (single connected 2-sphere) [4]. *Betti-1 remained 0* (no persistent loops on a sphere) [8].	**Foam Detector:** Spike in $β_2$ at fine scales signals a *bubble-rich, foam-like* state. DonutOS can mirror this with a *"Quantum Foam" badge* when many voids are detected at high resolution. As $β_2$ drops toward 1 at coarser scales, the UI can transition the scene to a *smooth sphere-like* overlay, indicating consolidation of bubbles into one continuum.

Citation (Domain)	**Summary of Findings**	**TDA Metrics**	**Phase/Topology Signatures**	**DonutOS Insight** (Torus-Scanner Fit)
van der Duin et al., 2025 (Lorentzian 2D CDT) [4] [5] *Quantum Gravity & Effective Topology*	Applies effective homology to **Lorentzian** 2D CDT (causal dynamical triangulations). In contrast to the Euclidean case, **causal structure suppresses local topology change**: the coarse-grained Lorentzian spacetimes remain simply connected, with $\beta_2 \approx 1$ at all scales [5]. No explosion of baby-universe bubbles occurs (all nontrivial homology is “global” only) [5]. This indicates the **quantum geometry is topologically smoother** when causality constraints are imposed. The authors note this difference as evidence that Lorentzian quantum spacetime lacks the frothy foam of the Euclidean model.	**Betti numbers vs scale** (same methodology as Euclidean). Tracked $\beta_2(\delta)$ in 2D CDT simulations of fixed toroidal spatial topology [9].	**Causal phase:** *No foam bubbles:* β_2 stays ~1 (topology fixed) across scales [5]. Only one connected component ($\beta_0=1$) and no persistent loops ($\beta_1=0$) throughout. *(In CDT, phases A/B/C correspond to geometric properties, but topology here stays trivial.)*	**Topology Stability Indicator:** A flat β_2 curve implies a *“sphere-like” continuous phase*. DonutOS can flag this with a *“Simply Connected” (no foam)* status – e.g. a solid grid overlay – meaning the system’s structure has no toroidal or fragmented substructure. The stark contrast between Euclidean vs Lorentzian cases suggests a UI toggle: a *causality lock* could enforce sphere-like (single-component) visuals, suppressing bubble indicators.

Citation (Domain)	**Summary of Findings**	**TDA Metrics**	**Phase/Topology Signatures**	**DonutOS Insight** (Torus-Scanner Fit)
Loll et al., 2022 (CDT **Phase Diagram**) [10] *Topology & Phase Transitions*	Investigates how **spacetime topology influences phase transitions** in 4D CDT. Studied the CDT phase diagram (phases A, B, C, C_b) with spatial topology fixed to T^3 (torus) vs S^3 [11]. Found that the order of certain transitions (e.g. between branched-polymer "A" and extended "C" phase) can change with topology [10]. This work doesn't use persistent homology, but suggests that *different underlying topologies (sphere vs torus)* lead to measurably different geometric phases. It underscores the idea that **toroidal vs spherical topologies** might be distinguished by observables.	*Traditional observables (volume profiles, etc.)*, not TDA – but conceptually highlights **topology as an order parameter**.	**Phase A (collapsed)** vs **Phase C (extended)** geometries; studied for both S^3 and T^3 spatial topology [10]. Toroidal spatial slices allowed completing the phase diagram.	**Phase-Topology Badge:** Motivates DonutOS to incorporate *base-topology awareness*. For example, a *"Torus mode"* indicator if the underlying state-space has periodic boundaries. Different UI color themes could be triggered if the system is in a **toroidal phase** vs **spherical phase**, reflecting how behavior changes (analogous to phase shifts seen in CDT with different topologies).

Notes: These studies demonstrate **how persistent homology can quantify quantum spacetime structure**. In particular, the rise and fall of $β_2$ in 2D dynamical triangulations [4] directly corresponds to the emergence of *baby universes* – a phenomenon long qualitatively described in quantum gravity. The *effective Betti numbers* serve as **diagnostics for spacetime phases**: e.g. a **high $β_2$ count signals a crumpled, foam-like regime**, whereas constant low $β_2$ indicates a smooth extended phase. This approach is expected to extend to 4D CDT, where one could topologically distinguish the **"crumpled" (A) phase,** "branched polymer" (B) phase, and **"de Sitter" (C) phase** by their Betti fingerprints. For DonutOS, **torus-confidence scanning** can draw inspiration here: monitoring Betti numbers of user-attention "triangulations" might reveal **fragmentation vs integration** in a user's focus, analogous to baby universes splitting off or remerging in the quantum realm.

Topological Phase Transitions in Random Complexes & Graphs

Across random graphs and simplicial complexes, TDA has been used to **detect phase transitions** akin to percolation, but in higher dimensions. Classical Erdős–Rényi graphs $G(n,p)$ undergo a sharp change at the connectivity threshold $p_c\sim 1/n$, where the giant component forms and the first cycle appears [12]. In *higher-dimensional analogues* (Linial–Meshulam **random d-complexes** $Y_d(n,p)$), there are analogous thresholds for the emergence and disappearance of each homology group [13] [14]. Notably, **homology is not monotonic** in these models: as one increases connectivity, a given Betti number β_k *first rises then falls*, implying *two phase transitions* – one where k-dimensional holes *appear* and another where they *fill in* [14].

TDA provides tools to capture these transitions quantitatively. For example, **persistent homology barcodes** of random Vietoris–Rips or Čech complexes reveal that the **largest loop or void appears right at criticality** [15]. Concepts like **"homological percolation"** have been introduced to describe the formation of giant topological cycles, analogous to giant components in ordinary percolation [16]. Table 2 highlights research on topological transitions in random complexes and evolving networks:

Citation (Model)	**Summary of Results**	**Topological Metrics**	**Phase Transition Signatures**	**DonutOS Fit** (Scene/Badge Mapping)
Linial & Peled, 2016 (Linial–Meshulam *random complexes*) [12]	Proved a **phase transition for d-th homology** in the $Y_d(n,p)$ model of random d-simplicial complexes. As p increases, there is a sharp threshold where H_d (the highest-dimensional homology) becomes non-zero [13]. Below this threshold no d-cycles exist; just above it, a giant d-cycle appears *with high probability*. They determined the exact threshold p_c for homology emergence, and showed it coincides with formation of a “giant shadow” (the high-d analog of a giant component) [12]. For $d\ge2$, the birth of this giant cycle is a first-order transition (sudden jump in Betti rank).	**Betti numbers** $β_d$ (and Euler characteristic) as functions of p. Analytical thresholds via probabilistic combinatorics.	**Homology appear threshold:** at $p \sim c/n^{1/d}$, $β_d$ jumps from 0 to $\mathcal{O}(1)$ [13]. **Homology vanish threshold:** at higher p, $β_d$ returns to 0 as holes fill in (for clique complexes) [14]. **Giant cycle** (d-cycle spanning $\Omega(n)$ volume) emerges at p_c (analogous to percolation cluster).	**Critical Transition Alert:** DonutOS can monitor connectivity *and* loop counts as control parameters vary. A sudden rise in a higher-dimensional Betti (e.g. $β_1$ or $β_2$) would trigger a *“Topology Percolation”* alert – signaling the formation of a large-scale loop/void (e.g. an attention loop forming). Likewise, a drop to $β_k=0$ at high density can cue a *“hole collapse”* notification.

Citation (Model)	**Summary of Results**	**Topological Metrics**	**Phase Transition Signatures**	**DonutOS Fit** (Scene/Badge Mapping)
Kahle, 2017 (Survey) [14] *Random Graphs → Random Complexes*	Survey of **random topology results**. Emphasizes that random clique complexes exhibit *two* transitions for each homology dimension k [14] : one where H_k appears and one where it disappears as p grows. For example, in a random graph's clique complex (Rips complex), H_1 first appears when the first cycle forms (near the ER graph connectivity threshold), and later H_1 vanishes when enough triangles fill every cycle (at higher p) [14] . This non-monotonic $β_k(p)$ is analogous to **"donut-shaped"** windows of connectivity – loops exist only in an intermediate regime. Kahle also summarizes threshold scalings: e.g. H_k appears around $p \sim n^{-1/k}$ and vanishes around $p \sim ((k/2)\ln n)/n$.	**Persistent homology** in parameter p; theoretical **threshold estimates** for appearance/ vanishing of $β_k$.	**Loop phase:** intermediate p where $β_1,β_2,…$ maximal (complex is "like a swiss cheese"). **Sparse phase:** low p, $β_0$ large (fragmented components), higher $β_k=0$. **Dense phase:** high p, $β_0=1$ and all loops filled ($β_k=0$).	**Phase Badges:** DonutOS can classify network-like data by matching these regimes. E.g., a *"Fragmented" badge* when high $β_0$ (many pieces, no loops), a *"Loopy (Torus-like)" badge* when $β_1$ is appreciable (indicating one or more persistent cycles in the data structure), and a *"Fully Connected" label* when only $β_0=1$ remains (all holes filled). The **Donuscope** timeline can highlight the window of loopiness as a distinct phase.

Citation (Model)	**Summary of Results**	**Topological Metrics**	**Phase Transition Signatures**	**DonutOS Fit** (Scene/Badge Mapping)
Bobrowski, Kahle & Skraba, 2017 (Random *geometric* complexes) [15] [16]	Initiated study of **persistent homology in random geometric graphs** (points in unit cube). They examined the *maximally persistent cycle* in random Čech/Rips filtrations. The **longest-lived k-dimensional hole** appears near the critical radius for percolation [15] . As $n\to\infty$, the persistence (lifetime) of this largest hole scales up and its birth time approaches the continuum percolation threshold [17] . In essence, at the exact point where a giant component percolates, a giant loop (for $k=1$) or void ($k=2$) also forms with maximal persistence. They quantified this by proving that the expected **max persistence** of a k-cycle grows on the order of $((\log n)/\log\log n)^{1/k}$ [18] .	**Persistent homology** (barcodes) for random point cloud as radius grows. Metrics: **longest bar length** in H_k. Asymptotic probability analysis for persistence.	**Percolation-like transitions:** At critical radius r_c, β_0 drops (points connect) *and* a large 1-cycle appears (peak in β_1 persistence) [15] . For $k=2$, a large void appears at higher r. **Persistence plateau:** the largest cycle persists for $\Delta r \sim O((\log n/ \log\log n)^{1/k})$ before filling in [18] .	**Persistent Cycle Tracker:** DonutOS's torus-confidence scanner can use the *longest persistence* as a signal. A very long-lived 1D cycle in the data (e.g. in an attention graph) would raise a *"Stable Loop"* flag, analogous to a giant toroidal structure. The UI could visually highlight that loop (e.g. a ring overlay) during the interval it persists. This aligns with using persistence length as an indicator of robust *torus-like* structure in the user's state.

Citation (Model)	**Summary of Results**	**Topological Metrics**	**Phase Transition Signatures**	**DonutOS Fit** (Scene/Badge Mapping)
Speidel et al., 2018 (Continuum percolation *in 2D*) [15]	Applied PH to a 2D continuum percolation model (random disks in a plane). They found clear evidence that **topological invariants spike at the percolation threshold**. In simulations, the **longest-persisting loop** (H_1 feature) consistently was *born* at or extremely close to the critical disk radius for percolation [15]. This suggests that **persistent homology detects the percolation phase change**: as the system approaches criticality, a loop appears that spans the cluster structure and survives over a range of radius. Beyond criticality, that loop eventually gets filled by additional disks. In short, PH provided a new way to pinpoint r_c by looking for the most persistent cycle.	**Persistent H_0, H_1** as disk radius increases. Identified **longest bar** in H_1 and its birth r_b.	**Percolation critical point:** r_b for the longest H_1 bar $\approx r_c$ (giant loop forms at phase transition) [15]. **Subcritical:** many components ($β_0$ high), no loops. **Supercritical:** one component ($β_0 \to 1$), loop eventually filled ($β_1 \to 0$).	**Criticality Scanner:** DonutOS can use a surge in $β_1$ persistence as an *automatic detector of phase transition*. For instance, if monitoring how clustering of attention nodes changes with a threshold, the system can flash a *"Phase Shift – Loop Emergence"* icon when a long-lived cycle appears, indicating a tipping point between fragmented and unified states.

Citation (Model)	**Summary of Results**	**Topological Metrics**	**Phase Transition Signatures**	**DonutOS Fit** (Scene/Badge Mapping)
Courtney & Bianconi, 2020 (Higher-order network percolation) [16]	Defined **"homological percolation"** as the phase where giant cycles form in higher-dimensional networks. Using simplicial complexes with growing interactions, they showed that the **expected Euler characteristic** $\chi(p)$ crossing zero correlates with the emergence of a giant homological cluster [16]. In other words, when χ changes sign, the counts of holes vs. components balance out, signaling a **topological phase transition**. They demonstrated this in model and empirical simplicial complexes, identifying distinct HPTs (homological percolation transitions) separate from ordinary connectivity percolation.	**Euler characteristic** $\chi = \beta_0 - \beta_1 + \beta_2 - \cdots$ as a function of p. Critical points estimated by $\chi(p)=0$ [16]. Also tracked giant k-cycles (size scaling).	**Homological percolation transition (HPT):** p where a **giant k-cycle** first appears (for some $k\ge1$). Empirically near zeros of $\mathbb{E}[\chi]$ [16]. Multiple HPTs can occur (e.g. giant 1-cycle, giant 2-cycle at different p).	**Euler Gauge & Warnings:** DonutOS could display an **Euler meter** (showing χ of the current attention graph). Crossing $\chi=0$ would prompt a *"Topology Flip"* warning – meaning the system has as many loops/voids as components, a hallmark of a *critical transition*. This single metric combines Betti numbers to give users a quick sense of balance between fragmentation and connectivity (a zero indicating a sweet spot between order and chaos).

Pranav et al., 2017 (Cosmic Web structure) [19]	Used persistent Betti numbers to **classify cosmological structure** (galaxy distributions). They found that different topological features correspond to distinct cosmic environments: e.g. **$β_0$ counts clusters**, **$β_1$ captures loops in filaments**, and **$β_2$ tracks voids (bubbles)** in the large-scale structure. By analyzing persistence diagrams from simulations, they could identify the **“Cosmic Web” phases** – clusters (nodes), filaments (1D chains), walls (2D sheets), and vast voids – and how these appear across scales [19]. The presence of long-persistence $β_2$ features signaled large voids (foam-like structure), whereas a dominance of $β_0$ with low $β_{1,2}$ indicated a **fragmented galaxy distribution**. This is one of the first multiscale topological descriptions of a physical phase	**Multiscale filtration of density field** (alpha complexes). Metrics: persistence diagrams for H_0,H_1,H_2; **Betti curves** vs density threshold.	**Void percolation:** At a certain density cutoff, voids percolate (giant void appears, peak in $β_2$) [19]. **Filamentary phase:** persistent $β_1$ loops identify cosmic filaments connecting clusters [19]. **Cluster phase:** at high density, $β_0$ = number of disconnected clusters (no loops/voids).	**“Cosmic Foam” Overlay:** This work directly informs DonutOS’s visualization vocabulary. For instance, a **foam-like state** in DonutOS (many voids) can be labeled when $β_2$ is high (analogous to cosmic voids). A **“network/filament” state** could be declared when $β_1$ is significant (many loops connecting components, like a mesh). And a **“clustered” state** when $β_0$ dominates (many isolated peaks). The UI can use icons (◍ clusters, loops, voids) drawn from cosmic web analogies to indicate the structural makeup of the attention network at different thresholds.

Citation (Model)	**Summary of Results**	**Topological Metrics**	**Phase Transition Signatures**	**DonutOS Fit** (Scene/Badge Mapping)
	transition (matter going from a nearly uniform state to a web of voids and filaments).			

Notes: The above works illustrate that **topological invariants behave like order parameters** for phase transitions in random structures. In particular, **β_0 (connected components)** maps to the usual percolation cluster behavior, while **higher Betti numbers (β_1, β_2)** diagnose more complex transitions (emergence of loops, voids, “holes” in networks). A striking insight is that **toroidal or loop-like structures only exist in an intermediate regime** – e.g. a random graph must be neither too sparse nor too dense to support a nontrivial cycle [14]. This resonates with the **Donut of Attention** scenes (e.g. a “donut” arises only under certain balanced conditions). By borrowing measures like the **Euler characteristic** or persistent Betti counts, DonutOS could automatically detect when the user’s attention state enters a **“toroidal” phase** (e.g. a recurring loop in tasks or thoughts) versus when it fragments or consolidates.

Bridging Topological Metrics into DonutOS (Donuscope Integration)

Using the above research, we can outline how **Donuscope’s phase history and overlays** might leverage persistent homology:

- **Filtration Analogy:** In physics models, scale (edge length δ or bond probability p) is the filtration parameter. For DonutOS, one can similarly **filter the attention graph** by some relevance threshold or time window, constructing a simplicial complex of related focus points. By sweeping this threshold (from strict to loose connections), the system produces a *persistence barcode of the user’s attention state*. This would mirror how coarse-graining was varied in CDT [7] or radius in percolation [15].

- **Key Betti Features for DonutOS:** Based on the literature, a minimal set of **Betti-based features** can describe attention topology:

- **β_0 (Connected Components):** Signals fragmentation. A surge in β_0 indicates the user’s attention is split into many disconnected pieces (*“fractured” state)* [20] *. DonutOS could display a* “Fragmented Focus”* badge when β_0 exceeds a threshold, analogous to warning of a shattered network.
- **β_1 (1D Loops/Tori):** Signals cyclic or **toroidal structure**. Non-zero β_1 means at least one loop in the attention graph – e.g. the user cycling through tasks without linear progression (*torus-like attention*) [21]. A *“Looping Pattern”* indicator can light up when a persistent cycle is detected, with a confidence level proportional to its persistence length [15]. For instance, if two independent loops are present ($\beta_1=2$), DonutOS might even use a **torus icon** to denote a strong toroidal mode of operation.

- **β_2 (2D Voids/Bubbles):** Signals higher-order **"gaps"** or **foam-like structure**. A high β_2 could occur if the attention data has distinct voids – perhaps topics that are being circumnavigated but never filled in (akin to unanswered questions forming a hole in knowledge). Drawing from quantum gravity [3] and cosmic web analysis [19], DonutOS could issue a *"Attention Void"* or *"Cognitive Foam"* alert when multiple 2D holes are identified, indicating the user's mental map has big empty patches being maintained. Conversely, a **drop in β_2 to zero** could trigger a *"Void Collapse"* notification – analogous to bubbles coalescing or being resolved (the user has finally filled in previously open gaps, making their knowledge space simply connected).

- **UI Encodings and Badges:** We recommend the UI use **intuitive graphical encodings** for these topological events:

- A **torus-shaped ring** graphic ( ) can represent a torus-like phase (persistent cycle present). Its color or size could reflect persistence: e.g. a bold ring for a long-lived loop vs a faint ring for a short-lived one.
- A **shattered grid icon** (□) can denote a fragmented state (high β_0). This might appear when the user's focus is scattered – akin to multiple disconnected clusters.
- A **foam or bubble icon** ( ) could signify a foamy state (many voids, high β_2). This corresponds to an environment with lots of missing pieces or parallel independent scopes (comparable to the baby-universe "foam" in CDT [22]).

- A **sphere or solid disk** (●) can indicate a simply-connected "sphere-like" state (no loops or voids, only one component). This is a stable, unified attention state – all tasks connected with no uncharted gaps.

- **Phase-Label Vocabulary:** DonutOS scene presets can borrow terminology directly from these domains:

- **"Fragmented"** – attention in pieces ($\beta_0 \gg 1$).
- **"Integrated"** – one component ($\beta_0 = 1$) but possibly complex internally.
- **"Loopy"** or **"Toroidal"** – presence of cycles ($\beta_1 > 0$) indicating repetitive transitions in focus (the user revisits topics periodically, forming a loop).
- **"Spherical"** – no significant loops or voids ($\beta_1=\beta_2=0$), a smooth focus landscape.
- **"Foam"** – multiple voids (β_2 high) meaning the attention landscape has several **unconnected void regions** (like multiple topics being skirted around but not delved into).
- **"Phase Transition"** – a dynamic label when the system detects a topological change (e.g. when a loop first appears or disappears). For example, if β_1 jumps from 0 to 1, a label *"Entering Torus Phase"* could briefly highlight on the timeline.

The **Donuscope timeline** can plot these Betti numbers over time or as the user's context changes. This would create a live *"Betti curve"* for the user: spikes in these curves serve as **milestones** (just as in brain networks analysis, Betti curves identified critical thresholds [23]). By mapping academic concepts to UI cues – *fragmentation warnings*, *toroidal mode indicators*, *collapse alerts* – DonutOS can guide the user with a rich, mathematically-informed feedback of their attentional state.

Cross-Branch Connections and Speculative Bridges

Finally, we note intriguing connections between these topological methods in physics and other domains – from neuroscience to user interface design – that could inspire DonutOS's development:

- **Neural Manifolds & Cognitive Loops:** In neuroscience, **persistent homology has revealed toroidal and circular manifolds** underlying neural activity. For instance, **grid cell populations** in the entorhinal cortex form a 2-torus representation of 2D space [24] [21], and head direction cells map out an S^1 loop. These findings confirm that neural circuits naturally operate on donut-like topologies. DonutOS could leverage this by aligning its *attention torus* concept with cognitive models – e.g. detecting when a user's thought patterns form a **closed loop (torus)** similar to a grid-cell attractor. The system's "torus-confidence" readout thus bridges to real neural coding principles [25], potentially making DonutOS useful in monitoring cognitive states or even neurofeedback therapy (identifying when a user is "in the loop" versus exploring new territory).

- **Human-Computer Interaction (HCI) & UX:** Topological thinking is entering HCI as well – e.g. using TDA to analyze gaze patterns or gesture trajectories for underlying loops and voids [26]. A UI can be thought of as a **manifold of user states**; TDA helps find features like "attention cycles" or "dead-ends" in UI engagement. DonutOS's membrane-like UI (the "Donut") could be enriched by **interactive topology cues**: imagine the interface surface deforming to show a hole when the user is stuck in a loop, or fragmenting into pieces when multitasking becomes too disparate. By doing so, the system provides an **intuitive visual metaphor** for complex state – much like a map showing "you are here" on a torus or sphere depending on context.

- **Symbolic–Geometric Hybrids (Category Theory & Logic):** Category theory offers a unifying language that could connect symbolic reasoning with topological data. Persistent homology itself can be seen as a **functor from data to barcode**, and recent work in applied topology treats barcodes in categorical terms. This suggests DonutOS could eventually interpret topological patterns in a more **semantic or logical way** – for example, categorizing a "torus" pattern as a particular type of cognitive loop (perhaps corresponding to a known workflow or habit). Moreover, ideas from **loop quantum gravity** (which replaces spacetime with networks of discrete loops) and from **knowledge representation** hint that *loops = recurrent processes* and *voids = knowledge gaps*. DonutOS might incorporate a rule engine that recognizes, say, a torus in attention as a need for creative intervention (to break the loop), or a high-genus (multi-hole) state as the user juggling many contexts (thus maybe suggesting focus or grouping).

- **Information Geometry & Membranes:** There is a parallel between the **information spaces** in AI (e.g. high-dimensional embedding spaces) and the **phase spaces** in physics. Both can have nontrivial topology. Techniques like TDA are being used to study the shape of loss landscapes in neural networks or the structure of knowledge graphs [27]. DonutOS can be seen as an **information membrane** where user intents and knowledge are mapped. By inferring topology (holes, connectivity) of this membrane, DonutOS could adapt its interaction style – for example, if the membrane has a large void, the system might proactively introduce relevant information to fill that gap (analogous to gravity pulling matter into a cosmic void). If the membrane splits (two separate components), the UI might prompt the user to either bridge them (create connections) or consciously separate the concerns (open a new "workspace" donut). These are speculative, but grounded in the idea that **topology crosses disciplinary boundaries**: from quantum foam to brain activity to user behavior, *loops and holes* carry meaning.

In summary, the **Donut of Attention project** stands at a fertile intersection of quantum gravity, network science, and cognitive HCI. By assembling this research map, we highlight that **persistent homology and Betti numbers offer a powerful common language** – describing phases of spacetime, phases of random networks, and potentially phases of mind and interaction. Adopting these concepts, DonutOS can implement a *torus-confidence scanner* and phase badges that are not just whimsical visuals, but rooted in cutting-edge scientific understanding of loops, holes, and connectivity in complex systems. [21] [19]

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