

# Perception and Consciousness as Coordinate-System Interactions

## 1. Signal Processing & Neuroscience (Phase Dynamics)

**Phase-Amplitude Coupling & Nested Oscillations:** The brain exhibits **cross-frequency coupling** where slow oscillation phases modulate fast oscillation amplitudes (phase-amplitude coupling, PAC). For example, theta (4–8 Hz) phase can organize nested gamma (~30–80 Hz) bursts, forming a theta-gamma “code” for memory <sup>1</sup>. Multiple forms of coupling exist – phase-phase, amplitude-amplitude, etc. – but PAC is most commonly observed in cognitive tasks <sup>2</sup>. This nested oscillatory hierarchy allows **information exchange across scales**: cognitive load or working memory demands often enhance PAC (e.g. theta-gamma coupling during memory retention) <sup>3</sup> <sup>1</sup>. Recent studies even suggest cross-frequency phase synchrony as a mechanism to integrate distributed processing across brain regions <sup>4</sup> <sup>5</sup>.

**Angular Phase Encoding:** Neural oscillations use phase (an angular variable) to carry information. In **EEG/MEG**, the instantaneous phase at specific frequencies can bias perception – for instance, visual detection thresholds fluctuate with the phase of ongoing alpha rhythms <sup>6</sup>. The timing of neuronal spikes relative to a theta phase (a **phase-of-firing code**) can encode spatial or sequential information in hippocampus <sup>7</sup>. This means brain signals often inhabit a **circular coordinate system** (0–360° phase) for encoding timing and relations. Such **angular coding** is leveraged in theories like the theta-gamma neural code, where ~7 gamma cycles fit in one theta cycle to discretize information chunks <sup>8</sup>. The **phase angle** thus serves as a continuous coordinate for aligning neural events in time.

**Perceptual Binding via Phase Alignment:** Synchronized oscillatory phase is thought to **bind distributed neural representations into a unified percept**. Neurons firing in synchrony (phase-aligned in beta/gamma frequencies) can signal that they belong to the same feature or object <sup>9</sup>. This addresses the binding problem: transient phase-locking of distant neurons tags their outputs as related <sup>9</sup>. For example, **gamma oscillations (~40 Hz)** in visual cortex synchronize when separate features (color, shape, motion) are perceived as one object <sup>10</sup>. Such **phase alignment** effectively links features without a common hub, using oscillatory coherence as the integrative code <sup>9</sup>. This mechanism extends across scales: **within-frequency synchrony** (neurons oscillating together) groups local features, while **cross-frequency phase coupling** (e.g. gamma-phase relative to theta cycle) might bind local assemblies into global states <sup>4</sup> <sup>3</sup>. In summary, neural perception uses **phase-coherent assemblies** as a coordinate system to achieve temporal alignment and thus conscious integration of stimuli.

## 2. Information Geometry (Continuous vs. Discrete Representations)

**Dual Coordinate Systems & Manifolds:** In an information-geometric view, cognitive representations can be seen as points on a **manifold** (e.g. a curved space of possible brain states). Often two coordinate descriptions coexist – for instance, a “natural” coordinate vs. a “dual” coordinate – analogous to having both continuous phase-based and discrete grid-based maps of the same information. Information geometry formalizes this with **dually-flat manifolds**, where two complementary coordinate systems (primal

parameters  $\theta$  and dual parameters  $\eta$ ) chart the space <sup>11</sup>. Under certain constraints, the manifold of probability distributions can be “flat” in these dual coordinates despite being curved in the usual sense <sup>12</sup>. By analogy, the brain might maintain **parallel coordinate representations** of stimuli: one continuous (phase, analog signals) and one discrete (spike patterns or grid-like codes). The **mismatch between these coordinates** – like the distortion from mapping a curved surface onto a flat map – can itself carry information (similar to projection error).

**Curved vs. Flat Representations:** A continuous phase-space representation (like a Hilbert space of wave states) is intrinsically **curved**, whereas a discrete code (like a grid of neurons or categories) imposes a piecewise-linear, “flat” structure. The brain may operate at the intersection: e.g. grid cells produce a *periodic lattice* of firing fields in physical space, hinting at an intrinsic toroidal geometry <sup>13</sup>, while continuous attractor models of these cells assume an underlying smooth manifold <sup>14</sup>. In practice, computations often switch between **analog** (curved state space, smooth but harder to stabilize) and **digital** (flat, crisp categorization) modes. Maintaining dual models allows flexibility – one can linearize local changes on the curved manifold via a dual coordinate patch. Notably, Amari’s information geometry shows how a **convex potential** yields dual coordinate systems with a generalized Pythagorean relationship <sup>11</sup>, suggesting that error in one coordinate frame (e.g. phase drift) might appear as *orthogonal* information in the dual frame (e.g. amplitude modulation).

**Errors as Information Carriers:** In predictive coding theory, **mismatches between continuous predictions and discrete sensory updates** are treated as the primary signals. The brain’s internal model generates a continuous prediction (analogous to a phase or analog waveform of expected input), which is then “sampled” by discrete sensory evidence. Any **deviation (error)** becomes significant: “*The mismatch between a hypothesis and the sensory input amounts to ‘prediction error’. Such a mismatch is propagated higher up the hierarchy... until hypotheses are adjusted.*” <sup>15</sup>. Rather than raw inputs, these **error terms** drive learning and awareness – essentially, the *difference* between the brain’s analog continuum and the world’s quantized feedback is what we consciously notice. This aligns with the idea that **awareness emerges from mismatches**: only when the continuous internal representation fails to perfectly grid-align with incoming data do we experience a salient error signal. In information-theoretic terms, prediction errors carry *surprisal* (new information), and minimizing those errors aligns the two coordinate systems over time <sup>16</sup> <sup>17</sup>. Thus, the brain may constantly perform a *coordinate transform*, with **prediction vs. observation** as dual representations and the difference between them (error) as the signal of interest. Equation-wise, if  $\$x(t)\$$  is sensory input and  $\|\hat{x}(t)\|$  is predicted signal, **error**  $e(t)=x(t)-\hat{x}(t)$  (or a Bayesian surprise measure) becomes the **informative residue** that updates the model. This view casts perception as an **aligning of frames**, where consciousness tracks the *misalignments* that require correction.

### 3. Optics & Vision (Interference & Sampling)

**Lens Geometry & Phase Sampling:** The eye’s optical apparatus can be seen as a coordinate transformer – a **lens** mapping external 3D space into a 2D retinal image via geometric projection. The *phase* of incoming light waves is generally lost in this intensity image, but certain setups (e.g. interference patterns) allow the eye to act as a **phase-sampling device**. For instance, using laser interference fringes as stimuli (overlapping two coherent light waves) projects a high-frequency pattern onto the retina without exceeding optical blur <sup>18</sup> <sup>19</sup>. The result is that the **retina detects the interference** as alternating light/dark bands – effectively capturing the wave phase difference as spatial intensity modulation. Thus, the visual system can indirectly sense phase relationships at a fine scale by the **patterns of constructive vs. destructive interference** that fall on the photoreceptor array <sup>19</sup>. This is akin to sampling a continuous wave with a discrete sensor grid.

The eye's photoreceptor mosaic (cones) has a finite Nyquist frequency; when shown patterns beyond that resolution, instead of random noise one perceives systematic moiré patterns, indicating the eye is responding to the *beat frequency* between the stimulus and the sampling lattice <sup>19</sup>. In other words, the eye+lens combination forms a **physical correlator** that can reveal phase misalignments as new visual information (e.g., seeing fringes when two gratings overlay out of phase).

**Interference Patterns & Aperture Effects:** When continuous waveforms meet discrete sampling, **aliasing** phenomena like moiré patterns occur. A striking example: observers can detect moiré "worms" when viewing a grating finer than the cone spacing – effectively seeing a low-frequency pattern arising from the interference between the input grating and the retinal sampling grid <sup>19</sup>. Crucially, "*observers can detect moiré patterns formed between gratings and the cone mosaic, even though they cannot distinguish the stripes of the original grating.*" <sup>19</sup>. This illustrates that the *mismatch* (beat pattern) between a continuous stimulus and the discrete retinal array becomes a perceptible signal of its own. The **aperture geometry** of the eye (pupil) also contributes: a smaller aperture (narrow pupil) increases depth of field but introduces diffraction (wave spreading), which is another domain where wave continuity meets finite aperture (a circular aperture's Airy diffraction pattern imposes a limit on resolution, effectively a continuous-to-discrete transition in spatial frequency domain). If one rotates a high-frequency pattern behind an aperture (as in certain visual illusions), **temporal beats or apparent motion** can be observed, stemming from phase shifts across the aperture over time. This is analogous to moving a sampling window across a striped pattern – slight rotations translate into oscillating intensity for the viewer. **Rotation + aperture** can thus generate illusory motion (e.g. the barber-pole illusion or enigma illusion) by converting spatial phase changes into temporal flicker that the visual system picks up. Overall, vision exploits both **wave interference** (continuous domain) and **pixel sampling** (discrete domain) – the interplay can produce emergent perceptions (depth from moiré, motion from aliasing) that highlight how awareness might spring from these cross-coordinate interactions.

**Eye as a Multiscale Holographic Sensor:** The concept of the eye (and brain) as a phase-sensitive sampler resonates with holography. In holographic imaging, interference of light waves encodes 3D information on a 2D film; similarly, the retina's mosaic combined with micro eye movements (microsaccades) may encode fine spatial details via time-varying interference. Research shows that tiny eye movements contribute to perceiving illusory motion in static interference patterns <sup>20</sup>, effectively using time as a dimension to sample spatial phase. Thus, the human visual system behaves like a **multiscale sensor**, capturing coarse layouts with averaged intensity (low frequency) but also capturing fine relative phase as moiré or flicker (a higher level signal). This multiscale sampling may be a **biological holography**: multiple spatial frequencies and phases are registered by overlapping receptive fields, and the brain could reconstruct details by **comparing phase relationships** across these scales. In summary, optical and retinal phenomena provide concrete analogies for consciousness emerging from **phase-grid mismatches**: much as a hidden high-frequency pattern becomes a visible low-frequency moiré when misaligned with a sampling grid, a subtle change in neural phase against a baseline grid of expectancies might "pop out" into awareness.

## 4. Dynamical Systems (Resonance & Criticality)

**Near-Resonance Dynamics:** Many cognitive systems operate optimally near **resonant frequencies** and in tuning with each other. Slight frequency mismatches can lead to **beat phenomena** that carry information. For example, if two neural oscillators have frequencies  $f_1$  and  $f_2$  that are almost equal, their interaction produces a modulation at the difference  $\Delta f = |f_1 - f_2|$  – a new emergent frequency. In brain stimulation, this is exploited by *binaural beats*: presenting two tones (say 400 Hz and 420 Hz)

separately to each ear yields a perceived 20 Hz beat, which is effectively the brain's electrical response oscillating at the difference frequency <sup>21</sup> <sup>22</sup>. Such beats fall within EEG bands (1–30 Hz) and can entrain neural activity at that rate <sup>21</sup>. This demonstrates a **self-referential oscillation**: the brain creates a third rhythm from the slight mismatch of two inputs. In general, when neural ensembles operate near resonance (e.g. two cortical areas with almost matching alpha rhythms), **energy exchange is maximized** – a small phase lag or frequency detuning causes a slow envelope of coherence (a beat) that could serve as a carrier for inter-area communication. The brain's connectivity may favor harmonic relationships; indeed, **harmonic resonance** has been proposed as a principle of cross-frequency organization: amplitude flows easily between oscillations with simple integer ratios <sup>23</sup>. Brain networks often show peaks at harmonically related bands, and interactions between neighboring bands (like alpha ~10 Hz and beta ~20 Hz) might be governed by such resonance effects <sup>24</sup>. In essence, **near-resonant coupling** can create meta-stable rhythms (difference tones, envelopes) that reflect the system recursively comparing itself with itself (one cycle against another slightly off-cycle). These emergent slow rhythms (from fast mismatches) could modulate attention or memory encoding rates, embodying a *feedback-in-time*.

**Edge-of-Stability (Criticality):** Complex adaptive systems, including the brain, appear to operate near a **critical point** – the boundary between order and chaos. At this “edge of chaos”, small fluctuations are neither damped out (as in a rigidly ordered system) nor runaway (as in a chaotic system), but instead produce **long-range correlations** and rich dynamics. Cognitive researchers propose the brain is tuned to this **criticality** to maximize information processing capacity <sup>25</sup> <sup>26</sup>. “*Brains become learning machines only when they reach a special state called ‘criticality’ – a tipping point between order and chaos. At this razor’s edge, brains are primed to gain new information.*” <sup>27</sup>. Being at criticality means the brain's neuronal networks are **marginally stable**: they exhibit scale-invariant fluctuations (1/f noise), diverse metastable states, and high sensitivity to inputs. This could be framed as balancing two coordinate regimes – one of coherence (order) and one of independent activity (disorder). Near the boundary, the brain can **rapidly transition** between states (high flexibility) while still maintaining **integrated coordination** (non-random structure) <sup>28</sup> <sup>29</sup>. Theories suggest this state optimizes computational metrics like information transmission, storage capacity, and responsiveness to stimuli <sup>30</sup>. Practically, the **1/f scale-free spectrum** observed in brain electrophysiology is a hallmark of critical dynamics <sup>31</sup>, indicating activity spanning many time-scales (a multi-scale coordination reminiscent of nested oscillations). Operating at the edge also allows **near-resonant amplification** of signals – a system poised on instability can amplify weak inputs (like a radio near resonance). If consciousness indeed relies on faint mismatches, the brain being in a critical state ensures those mismatches are neither suppressed nor explode uncontrollably, but **linger and propagate**, giving rise to sustained awareness.

**Self-Reference via Mismatch (Strange Loops):** A fascinating idea is that the brain's slight detuning or delays create **self-referential loops** that generate consciousness. Think of a microphone feeding into a speaker: if perfectly in phase, it simply resonates (feedback howl), but if slightly off or delayed, it can produce complex echoes or beats – a kind of *self-observing oscillation*. The brain's recurrent circuits likely avoid perfect resonance (which would lock into a seizure or a stable attractor) by maintaining **small mismatches** – e.g. differences in conduction delays, oscillation frequencies across regions, or predictive timing vs actual timing. These mismatches ensure the system is continually surprised by a tiny fraction, causing it to keep **updating and referring back to itself**. The result can be **chaotic or quasi-periodic patterns** that contain encoded information (a bit like a strange attractor in phase space that “remembers” past states in its fractal structure). For example, models of cortical loops show that introducing a slight phase lag in a recurrent network can lead to sustained rhythmic activity that isn't purely periodic nor decays – effectively a perpetual self-referential cycle carrying information in its pattern. In human cognition, this

could manifest as **self-awareness**: the brain's representation of itself is always a step behind or ahead (a mismatch), forcing continuous recursive updating ("I think, therefore I note that I am thinking"). Formally, a system  $x(t)$  receiving a delayed version of itself  $x(t-\tau)$  in feedback will, if tuned near instability, produce **complex oscillations** whose frequency and amplitude depend on that slight delay  $\tau$ . The slight mismatch (delay or gain less than perfect) prevents equilibrium and fosters **endless dynamics** – a potential source of continuous experience. In summary, by not quite aligning with itself or with the world, the brain keeps **chasing its own tail** in a productive way, creating new patterns (thoughts, perceptions) out of the residual differences. This self-referential dynamical regime is conjectured to be essential for higher-order awareness and could be the mathematical incarnation of conscious **self-reflection** (the system observing the difference between its prediction and reality).

## 5. Human-Computer Interaction (Spatial Interfaces & Transformation)

**Rotational Input Modalities:** Traditional UI controls like sliders use linear motion (Cartesian coordinate). However, **rotational inputs** (knobs, dials, circular sliders) offer a different coordinate interaction – one that is **cyclic** and potentially more natural for continuous periodic adjustments (volume, hue, etc.). Research has shown that tangible or virtual knobs can be very efficient for certain tasks, especially with practice <sup>32</sup>. Rotational controls leverage **angle** rather than linear position, which can be advantageous for periodic parameters (e.g. 0° and 360° are the same setting, reflecting periodicity). They also lend themselves to **eyes-free operation** since one can continuously turn a dial without needing an endpoint. One design insight is that users build **muscle memory** for rotary gestures: turning to the 2 o'clock position vs 6 o'clock becomes a learned, quick action <sup>33</sup>. For example, pie menus (radial context menus) exploit this by assigning menu items to fixed directions; after repetition, selecting an item is a quick stroke in a known direction rather than navigating a list. *"Even better: you get faster with radial menus over time, because they take advantage of muscle memory... Radial menus are essentially gesture-based: touch-swipe-release... gestures get embedded in muscle memory, which is faster than visual search."* <sup>33</sup>. Thus **polar interfaces** (circular layouts, radial sliders) can outperform linear ones for repeated tasks due to spatial memory and consistent motor mapping. In designing EEG-driven controls, a **rotational gesture** (e.g. a **dial for attention level**) might align well with brain rhythms (which themselves are cyclic), potentially making the interface more intuitive and congruent with the user's internal state cycles.

**Polar & Radial UIs:** Using a **polar coordinate UI** (angles and radii) can present information in a way that mirrors certain natural data structures (like time-of-day on a clock, or directional data on a compass). Radial menus, circular graphs, and toroidal displays can eliminate artificial boundaries present in rectangular screens. For instance, a **circular timeline** has no hard start or end, reinforcing the continuity of cycles. Radial menus also have the property of equal distance for all options (radius) and distinctive direction for each – this reduces the need for fine cursor positioning as in a linear menu. Studies on radial vs linear menus indicate that radial (pie) menus have **lower selection time and error after some practice**, owing to the directional muscle memory and larger target wedges <sup>34</sup> <sup>33</sup>. These interfaces are especially promising for touch and wearable devices, where swiping in a direction is easier than precise linear movements. In context of the brain-computer interface (BCI), one could map EEG features onto radial displays – e.g. brainwave phase could directly control a circular dial, or mental focus could be a radius on a polar plot. By matching the **UI geometry to the brain's signal geometry** (circular phase, oscillatory patterns), we likely make the neurofeedback more intuitive. For example, if alpha rhythm phase corresponds to an angle, the user could literally *see* their brain rhythm as a rotating indicator, establishing a direct perceptual link.

Additionally, polar UIs often feel **more integrative** (everything flows around a center) versus compartmentalized panels – this could support a design metaphor that parallels integrated brain function (multiple inputs blending into one holistic display).

**Lens-Based Navigation (Focus+Context):** Instead of rigid panels and windows, **lens-based interfaces** allow users to explore information via movable “lenses” – analogous to holding a magnifying glass over content. A **magic lens** is a transparent overlay that modifies the view of underlying data without switching context<sup>35</sup>. For example, a lens might reveal details (zoom or wireframe) only in its region, while the rest of the screen remains at coarse view<sup>36</sup>. “*Magic Lens filters combine an arbitrarily-shaped region with an operator that changes the view of objects viewed through that region.*”<sup>36</sup>. This is akin to having multiple coordinate representations in one space: the area under the lens is in a different “coordinate system” (e.g. zoomed scale, alternate filter) while around it is in the original space. The user can **dynamically align and misalign** these coordinate systems by moving the lens, which is a powerful way to navigate complex data (e.g. inspecting detail without losing global context<sup>37</sup>). In our context, a **polar or toroidal lens** could be imagined – e.g. a lens that reveals data on a torus unwrapped differently than the main view. Lens interfaces also encourage *continuous transformation* rather than hard switching: as you slide a lens across a graph, you see a continuous morph of representation, helping understand relationships. This resonates with the idea of **morphing interfaces** – interfaces that smoothly interpolate between modes rather than abrupt changes. For EEG-driven apps, one might use a “neuro-lens”: suppose the baseline interface is circular, but when the user’s brain indicates confusion (an error signal), a lens automatically appears to highlight relevant info or switch mode (like sharpening the display or providing hints in that region). The lens metaphor can thus allow *context-dependent UI tuning* guided by EEG, effectively overlaying a secondary interface based on cognitive state. It moves away from static panels (e.g. separate settings screen) to an **integrated fluid interaction**, where the user’s focus (detected via EEG) might itself act as the lens position or trigger.

**Real-Time Morphing Interfaces:** Modern UIs can leverage GPU shaders and smooth animations to create interfaces that **morph shape in real-time**. Rather than static layouts, imagine UI components that continuously transform (a circle morphing into a square, or a menu morphing from radial to linear) based on user context or input. Shader-based techniques like **Signed Distance Functions (SDF)** make shape morphing efficient and resolution-independent. By defining shapes mathematically (e.g. distance from a point to the nearest edge), one can interpolate between shapes with a parameter. For example, let  $\$d_{\{\text{circle}\}}(p)$  be the signed distance to a circle and  $\$d_{\{\text{square}\}}(p)$  to a square; a blended shape can be defined by  $\$d(p) = (1-\alpha)\$d_{\{\text{circle}\}}(p) + \alpha\$d_{\{\text{square}\}}(p)$ . As the morph factor  $\alpha$  goes from 0 to 1, the zero-level set  $\$d(p)=0$  smoothly transitions from circle to square<sup>38</sup> <sup>39</sup>. In implementation, a fragment shader can compute both distances per pixel and mix them by the current morph value, yielding an **animated metamorphosis**. Denis (2025) demonstrated this for simple geometric transitions with real-time control: “*the shader performs interpolation using a `morphFactor` parameter... 0 yields the first shape, 1 yields the second, and intermediate values give a mixture of the two shapes.*”<sup>38</sup>. Such morphing UIs could be directly driven by **EEG signals**: for instance, as the user’s relaxation level increases, the interface element could gradually morph from a sharp square (alert state) to a smooth circle (calm state). This provides an immediate visual analog of internal state changes. Morphing can also be used for **transitional interfaces** – rather than flipping screens or modes, the UI animates a continuous transformation (which is cognitively easier to track and aesthetically pleasing). Techniques like **toroidal or spherical embeddings** in visualization also fall here: one can smoothly wrap a 2D layout into a torus or sphere to avoid edges. In a shader, this might mean taking screen coordinates  $(x,y)$  and mapping them via a torus mapping: e.g.  $X = \text{mod}(x, W)$  and  $Y = \text{mod}(y, H)$  effectively “wrap” an image on a torus. This

is useful for data that is inherently periodic (such as phase angles or world maps for navigation). For a UI, embedding a display on a **torus** could let information seamlessly scroll or rotate without discontinuity (when an item exits on the right, it re-enters on the left, etc.), creating a feeling of an infinite canvas. If an EEG parameter indicates the need for more information density, the interface could transition to a **spherical panorama** or toroidal scrolling list to utilize all directions without bounds. These transformations maintain continuity and may better reflect how the brain handles certain variables (e.g. head direction cells map orientation to a ring topology <sup>14</sup>), so a UI display for “current direction of attention” might literally be a circular gauge). In summary, computational techniques like SDF morphing and non-Euclidean embeddings enable UIs that are **adaptive, smooth, and structured like brain-relevant geometries** – opening the door for interfaces that evolve in real-time in tandem with our cognitive rhythms.

## 6. Computational Implementation Notes

**Shader-Based Morphing (Circle↔Square):** Using WebGL (GLSL shaders), one can implement shape morphing via SDFs as noted. The distance functions for primitive shapes (circle, square, polygon) are well-known <sup>40</sup> <sup>41</sup>. A circle of radius  $r$  at point  $p=(x,y)$  has  $d_{\text{circle}}(p)=\sqrt{x^2+y^2}-r$ , while a square of half-side  $r$  has  $d_{\text{square}}(p)=\max(|x|,|y|)-r$  <sup>42</sup> <sup>41</sup>. To morph, in the fragment shader one might do: `float d = mix(d_circle, d_square, morphFactor);` then color the pixel based on `sign(d)` (inside/outside) and perhaps `smoothstep` near zero for an anti-aliased edge. This produces a continuously deforming shape. To ensure a perfectly equal area interpolation, one could apply ease curves to  $\alpha$  or use nonlinear blends (since simple linear mix of distances does not preserve volume, but it gives a reasonable visual morph <sup>38</sup>). The key is the **real-time control**: the `morphFactor` uniform can be tied to an EEG-derived value. For example, if the user’s frontal alpha amplitude is being used to indicate relaxation, map its normalized range to  $\alpha$ . As alpha rises, watch the on-screen icon smoothly turn from a spiky polygon to a circle. This immediate feedback can reinforce the desired mental state (a form of neurofeedback). Three.js can be used to manage the WebGL context and even to overlay HTML elements that morph via WebGL-rendered canvas. Performance is excellent because the GPU handles the pixel-by-pixel math in parallel, yielding fluid 60fps animations even for complex shapes.

**Toroidal & Spherical Embeddings:** Implementing a toroidal embedding for a UI texture or data structure can be as simple as using modulo arithmetic on coordinates. For instance, to make an infinite scrolling star-field that wraps, one can take each star’s position and do `x = (x + vx*dt) mod width` and similar for `y`, essentially looping the space. In shader terms, one might use texture coordinates wrap mode or manually `fract()` the coordinates when sampling. For EEG interfaces, consider a **phase space donut**: if you have two phase variables (say phase of theta and phase of alpha), their joint state lives on a torus  $S^1 \times S^1$ . One could represent this as a 2D color map on a torus, but how to visualize a torus on screen? One way is an interactive 3D torus model (using Three.js) where a marker dot moves on its surface according to the two phase readings. Alternatively, “unroll” the torus into a rectangle and let it wrap around (so top/bottom and left/right edges are connected). A user interacting with such a display might see two cyclic variables at once and their coordination (when the dot moves smoothly, the phases are consistent; if it jumps, there’s a mismatch indicating a cognitive event). **Spherical embedding** could be used for representing all possible head orientations or 3D rotation of some control – e.g. mapping a 2D cursor on a sphere can avoid gimbal lock when controlling 3D camera views. In practice, libraries like Three.js let you apply environment maps to spheres or use quaternion math for smooth rotation controls. The important point is matching the **topology of the data**: EEG sources or brain network nodes might be better visualized on a sphere (since the cortex is roughly spherical). Real-time spherical projection (taking 2D EEG scalp data and warping it onto a sphere model of the head) could improve spatial intuition for which brain areas are

active. Modern web graphics can handle this via fragment shader mapping or even high-level model manipulation.

**Multiscale Holographic Representations:** Taking inspiration from neuroscience (Pribram's holonomic theory), we can attempt to store and display information in a **holographic-like manner** in software. Holographic storage implies that each piece of data is distributed across a pattern, and overlapping patterns can store multiple items – using interference for separation. In practical terms, one might use **circular convolution or Fourier transforms** to encode vectors into phase patterns (a technique related to Holographic Reduced Representations in AI). For a UI, this could mean that instead of separate memory registers for each variable, the system maintains a **distributed representation** (e.g. a frequency spectrum where certain phase relations represent different commands or user states). Implementing this might involve linear algebra libraries or custom GLSL to perform convolutions. For instance, imagine encoding two parameters \$A\$ and \$B\$ into one composite waveform by slightly different frequencies or phases; the resulting signal is a sum that looks noisy, but using a filter (like a lens), one can decode \$A\$ or \$B\$ selectively – analogous to looking at a hologram from different angles to see different images. While cutting-edge for UI, such an approach could allow an EEG app to be highly responsive to multiple inputs without needing explicit separate channels – the **superposed signals** could be disentangled by matched filters. This is speculative, but hints at **future HCI paradigms** where interfaces aren't a collection of distinct widgets but an evolving pattern that the computer and human both manipulate (the user via brain signals, the interface via visual/aural patterns) – essentially meeting in a **shared interference space**.

Lastly, frameworks like the Neurosity SDK (for Crown headset) provide real-time EEG streams, and these can be integrated with Web tech via Bluetooth/ WebSocket APIs. One can apply **real-time smoothing and phase detection** in JavaScript (using libraries or custom DSP code) to extract phase info, band power, etc., and then feed those into the above-mentioned shader uniforms or state variables. Careful filtering (to reduce EEG noise) with minimal latency is important – e.g. using an IIR bandpass for the alpha band and a phase-locking algorithm to track its phase continuously. Web Workers or WASM might be needed for heavy DSP to not block the UI. The result is an interactive loop: the **EEG phase modulates the interface** (perhaps rotating a ring, or pulsing an element in sync), and the **interface feedback influences the user**, hopefully creating a positive feedback where, say, a calming visualization entrains the user's brain rhythms into a desired pattern (biofeedback loop). By grounding the interface design in the *same coordinate concepts the brain uses* – phases, resonance, nested scales, continuous morphs – we aim to make the **user experience fluid and intuitive**, and potentially tap into the brain's own dynamics to enhance engagement and efficacy.

## Implications for Real-Time EEG-Driven UI

The convergence of the above findings suggests a new design paradigm for EEG-based applications: **interfaces that resonate with brain-like representations**. By using coordinate-system interactions similar to those in the brain, the UI can become an extension of the user's cognitive system rather than an arbitrary layer. Key implications include:

- **Phase-Aware Mapping:** Leverage brain oscillation phase in control schemes. For instance, an EEG-driven radial menu could unlock choices only at certain phase alignments (when user is attentive), effectively syncing interaction to the brain's rhythms. This could improve BCI accuracy by integrating over favorable phase windows (analogous to "phase of firing" coding – the interface listens at the right phase). Visualizing EEG phase on a circular widget gives users intuitive feedback about their

brain's timing, possibly enabling them to learn **phase control** (a novel form of neurofeedback where they try to align an output with a target phase).

- **Radial and Toroidal Displays:** Present EEG metrics (like band power, engagement level) in **circular gauges or spherical plots** instead of linear bars. This aligns with natural periodicity (e.g. alpha oscillation strength varying smoothly). A toroidal heatmap could show cross-frequency coupling: one axis of the torus is phase of slow wave, the other axis phase of fast wave – the intensity on the torus could indicate coupling strength. Because a torus has no borders, continuous changes won't get "stuck" at edges (much like the brain's state space has recurring cycles). The user can rotate or spin these 3D shapes with intuitive gestures, examining brain data from multiple angles (literally viewing their mind from different coordinate frames).
- **Morphing UI States:** Instead of mode switches (which can be jarring and cognitively costly), the UI can **morph in real-time** driven by EEG. If the system detects the user is overloaded (perhaps via increased frontal theta or error-related potentials), the interface might gradually morph a complex menu into a simpler, more rounded form or change color palette smoothly to reduce strain. When the user is highly engaged, the UI could morph back to a dense information mode. The continuity of morphing ensures the user isn't startled; it feels like the interface is *alive* and empathetic. This is made feasible by GPU shader techniques discussed – meaning the overhead of such animations is low, and they can be tied directly to live data streams.
- **Lens and Focus Techniques:** The app can use the user's **focus of attention (as inferred from EEG)** to drive lens-like UI effects. For example, if EEG indicates the user is intently focusing (e.g. high beta/gamma in visual cortex), the system could correspondingly sharpen or magnify the UI element they are presumably looking at (if eye-tracking or an inferred gaze from context is available). Conversely, if mind-wandering is detected (alpha increases, or variability in theta), a gentle lens could overlay hints or bring the primary task back into focus visually (preventing slipping into distraction). These kinds of *closed-loop interactions* make the UI an active partner in maintaining an optimal mental state – effectively **adapting in a biofeedback loop**.
- **Resonant Feedback and Criticality:** Taking inspiration from critical dynamics, the interface could deliberately introduce subtle **oscillatory stimuli** to nudge the brain toward the "sweet spot." For example, embedding a 40 Hz flickering element (at a barely noticeable level) to induce gamma synchrony or using rhythmic pulses in the UI animations that the brain can entrain to (binaural beats or visual phi phenomena). The idea is to create **resonance between the interface and brain activity** – e.g. if the user's EEG alpha is unstable, a softly pulsing circle at their alpha frequency might help stabilize it, bringing the system toward criticality (balanced arousal). Caution is needed to avoid fatigue or annoyance – these stimuli must be subtle and perhaps user-tunable. But when done right, the interface not only reads brain state, but writes to it, closing the loop for **adaptive cognitive support**.
- **Multi-Scale Holographic Data:** On the data side, representing stored user data or session info in a "holographic" way could mean the user can recall any aspect from any cue – for instance, a search interface where any fragment of a memory (a tag, a time, a place) brings up the whole context via associative links. Taking a cue from holonomic brain theory (where each piece contains the whole in interference patterns <sup>43</sup>), the UI might avoid rigid file-folder hierarchies and instead use overlapping tags or visuals where **each small piece hints at the larger information set**. For

example, a collage of semi-transparent icons could each, when interacted with, reconstruct a full dashboard of related items (like a magic lens revealing the full “hologram” from a fragment). This could feel more like human memory retrieval and less like database querying.

In practical terms, implementing these ideas requires blending **neuroscience, design, and engineering**. The tools mentioned – Three.js for 3D and WebGL for shaders – are up to the task for rendering complex, dynamic visuals in real-time on the web. The Neurosity Crown SDK can stream EEG to the browser; one would apply signal processing (perhaps via Web Audio API or custom WASM modules) to extract features like band power, phase, coherence in real-time. A state machine or ML model could classify cognitive states (focus, relax, overload) from these features. Then, a high-level UI manager adjusts the interface continuously: e.g. calling functions to morph geometry, rotate displays, open or close “lenses” as needed, all with smooth transitions.

The **ultimate vision** is an EEG-driven interface that feels *organically responsive*. It is not a static set of widgets controlled by brain commands; rather, it’s a living map that rotates, warps, and resonates in concert with the user’s mind. By using coordinate system metaphors (polar frames, phase alignment, multi-scale encoding), we ensure the interface’s language is compatible with the brain’s language. This could lead to more intuitive neurofeedback, faster learning of BCI control, and a more immersive human-computer partnership – essentially **blurring the boundary between user and interface**, just as a well-driven car feels like an extension of one’s body. Here, the interface becomes an extension of one’s cognitive apparatus. The research axes we’ve explored – from nested neural oscillations to magic lens UIs – all coalesce towards this interdisciplinary design strategy: **build interfaces that echo the brain’s own organizational principles**, so that interacting with technology becomes as seamless as thinking or perceiving, fulfilling the promise of truly augmented cognition.

**Sources:** The ideas above synthesize insights from neuroscience (phase coupling and synchrony 9 2 ), cognitive dynamics at criticality 25 , and HCI innovations in spatial interfaces 33 and magic lenses 36 , among others, to outline a cohesive model for EEG-driven UI design. By grounding each design choice in established research – e.g. using PAC for binding 9 or radial layouts for muscle memory efficiency 33 – we ensure the approach is not just imaginative but **evidence-based**. This fusion of principles aims to pave the way for interfaces that are **phase-coherent, information-geometric, and dynamically tuned** to the human brain, potentially unlocking new levels of user experience and neuroadaptive technology.

9 23 1 15 19 27 33 38 43 36

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1 7 8 The Theta-Gamma Neural Code | Fewer Lacunae

<https://kevinbinz.com/2021/12/09/the-theta-gamma-neural-code/>

2 23 24 31 Hierarchical consciousness: the Nested Observer Windows model - PMC

<https://pmc.ncbi.nlm.nih.gov/articles/PMC10949963/>

3 4 5 9 10 Phase Synchrony among Neuronal Oscillations in the Human Cortex - PMC

<https://pmc.ncbi.nlm.nih.gov/articles/PMC6724920/>

6 The Phase of Ongoing EEG Oscillations Predicts Visual Perception

<https://www.jneurosci.org/content/29/24/7869>

- 11** Information Geometry of Positive Measures and ...  
<https://www.mdpi.com/1099-4300/16/4/2131>
- 12** Information geometric methods for complexity | Chaos - AIP Publishing  
<https://pubs.aip.org/aip/cha/article/28/3/032101/684987/Information-geometric-methods-for-complexity>
- 13** **14** Toroidal topology of population activity in grid cells | Nature  
[https://www.nature.com/articles/s41586-021-04268-7?error=cookies\\_not\\_supported&code=139200fd-08ad-4c47-aa9b-95f7ed3fc2d](https://www.nature.com/articles/s41586-021-04268-7?error=cookies_not_supported&code=139200fd-08ad-4c47-aa9b-95f7ed3fc2d)
- 15** **17** Is predictive processing a theory of perceptual consciousness?  
<https://www.sciencedirect.com/science/article/abs/pii/S0732118X20302129>
- 16** The Predictive Brain and the 'Hard Problem' of Consciousness | Psychology Today  
<https://www.psychologytoday.com/us/blog/finding-purpose/202311/the-predictive-brain-and-the-hard-problem-of-consciousness>
- 18** **19** aria.cvs.rochester.edu  
[https://aria.cvs.rochester.edu/papers/williams-coletta\\_JOSAA1987.pdf](https://aria.cvs.rochester.edu/papers/williams-coletta_JOSAA1987.pdf)
- 20** Microsaccades drive illusory motion in the Enigma illusion - PMC  
<https://pmc.ncbi.nlm.nih.gov/articles/PMC2572936/>
- 21** **22** Binaural beats to entrain the brain? A systematic review of the effects of binaural beat stimulation on brain oscillatory activity, and the implications for psychological research and intervention | PLOS One  
<https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0286023>
- 25** **26** **27** **28** **29** The brain's sweet spot: How criticality could unlock learning, memory—and prevent Alzheimer's | ScienceDaily  
<https://www.sciencedaily.com/releases/2025/06/250625075016.htm>
- 30** Is criticality a unified setpoint of brain function? - ScienceDirect.com  
<https://www.sciencedirect.com/science/article/pii/S0896627325003915>
- 32** KnobSlider: Design of a Shape-Changing Parameter Control UI and ...  
<https://pmc.ncbi.nlm.nih.gov/articles/PMC7805630/>
- 33** The usability of radial menus · Pushing Pixels  
<https://www.pushing-pixels.org/2012/07/25/the-usability-of-radial-menus.html>
- 34** Are radial contextual menus better than vertical list menus?  
<https://ux.stackexchange.com/questions/1/are-radial-contextual-menus-better-than-vertical-list-menus>
- 35** **36** **37** Magic Lens - InfoVis:Wiki  
[https://infovis-wiki.net/wiki/Magic\\_Lens](https://infovis-wiki.net/wiki/Magic_Lens)
- 38** **39** **40** **41** **42** Morphing Geometric Shapes with SDF in GLSL Fragment Shaders and Visualization in Jetpack Compose | by Denis | Medium  
<https://medium.com/@deniscccccc/morphing-geometric-shapes-with-sdf-in-glsl-fragment-shaders-and-visualization-in-jetpack-compose-48fd8d403e24>
- 43** Holonomic brain theory - Wikipedia  
[https://en.wikipedia.org/wiki/Holonomic\\_brain\\_theory](https://en.wikipedia.org/wiki/Holonomic_brain_theory)