VCG-Based Voting Mechanism: Design and Analysis

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Abstract

Our study aims at social choice problem: how to make collective decisions in an efficient manner among self-interested individuals with diverse preferences. One of the most important goals of the mechanism is social optimal. To achieve this, we study how to encourage participants truthfully report their preferences.

A most the popular mechanism to achieve this is VCG mechanism. However, it has a significant drawback: it is not budget-balanced. Some research sacrifice social optimal to achieve budget balance.[Faltings 2004] Some studies in Bayesian game assumption and sacrifice Individual Rationality.[d'Aspremont et al. 2004]

We focus on Bayesian game assumption and proposed some new mechanisms. Although they still can't achieve Individual Rationality. As we shown in the simulation section, they are fairer and have lower probability of breaking IR compared with previous researches.

Keywords

Social Choice, Mechanism Design, VCG, Bayesian Game

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1 INTRODUCTION

In daily life, we often encounter situations where multiple rational agents are faced with making a collective decision among various choices, with each agent attempting to maximize their own utility in this decision-making game. Consider the following examples: a group of people deciding where to dine together; residents of different parts of a city choosing a common drop-off point when taking a taxi home from the airport.

In a simple voting decision mechanism, each individual ranks different restaurants based on their preferences. The social choice is then made using voting mechanisms such as Borda Count, majority rule, or runoff mechanism. However, voting mechanisms have numerous drawbacks. For instance, in some voting mechanisms, agents can easily manipulate the outcome by altering their rankings to achieve a better result. A more critical issue is that such mechanisms can lead to situations where some individuals gain significantly high benefits while others receive very low or even negative benefits, which appears to be unfair. Therefore, we consider using an auction mechanism as an alternative.

In an auction mechanism, agents are required to report their utility for each candidate option. Therefore, a straightforward idea is to design a mechanism that discourages manipulation — we want *truthfulness*, meaning we aim to create a system where it is the dominant strategy for each agent to honestly report their utility. This way, when making social choices, we can easily predict the impact of different social choice on each agent's utility. Additionally, we want this mechanism to achieve *Pareto optimality*, or *efficiency*, which means maximizing social welfare, which is the sum of each agent's utility.

The Groves mechanism achieves both goals by requiring each individual to make a payment to a non-agent center. A specific type of Groves mechanism, the Vickrey-Clarke-Groves (VCG) mechanism, also ensures *individual rationality*, meaning that each individual's final payoff, which is their utility minus the payment, is always positive.

In auction mechanisms, the VCG mechanism is known for its desirable properties, but it faces a significant challenge with budget imbalance due to the absence of a central agent for surplus taxes, leading to the so-called VCG surplus. To address this, various VCG-based mechanisms have been proposed to achieve budget balance while maintaining efficiency and incentive compatibility. [Faltings 2004] introduced a method involving random selection of one agent as a "center" to achieve exact budget balance, albeit at the cost of Pareto efficiency. Building on this, our paper proposes a new mechanism, Mechanism 1, which optimizes the distribution process without compromising on fairness.

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However, recognizing the potential unfairness to the chosen center in Mechanism 1, we introduce Mechanism 2, a decentralized approach that improves fairness through a refined payment system. Both mechanisms aim to preserve the core advantages of the VCG mechanism while enhancing budget balance and individual rationality.

2 BACKGROUND AND RELATED WORK

2.1 NOTATION

- Let I be the set of agents participating in a mechanism.
- Let $n = |I|$ denote the number of agents.
- Let S be the set of candidates, and $k = |S|$ the number of candidates.
- Let $\theta_{i,j}$ represent the true utility of agent i for candidate j.
- Let $m_{i,j}$ represent the reported utility of agent i for candidate j.

2.2 MECHANISM DESIGN BACKGROUND

Definition 2.1 (Mechanism). A mechanism is a 2-tuple (s, p) of functions:

- Social choice function $s : \mathbb{R}^{n\times m} \to S$ takes reported utilities of all the agents and make a social choice $s(m)$.
- Transaction function $p_i : \mathbb{R}^{n\times m} \to \mathbb{R}$ takes reported utilities of all the agents and outputs agent i's transaction $p_i(m)$. "Transaction" refers to the payment that agent i needs to make to an external entity (such as a center or something else)..

Definition 2.2 (Pareto Efficient, Strategyproof, IR, IC and BB).

A mechanism that maximizes social welfare is termed **Pareto Efficient** or **social optimal**.

When no agent can ever benefit from lying, i.e., when being truthful is a dominant strategy, a mechanism is called **Strategyproof**. A somewhat weaker property is **Incentive Compatibility (IC)**; in an IC mechanism truthfulness is a Nash equilibrium: no agent can benefit from being dishonest when all other agents are truthful.

Individual Rationality (IR) is a fundamental concept that ensures participation in a mechanism is beneficial from each participant's perspective. **IR** can be categorized into two main forms:

- **Ex-post IR**: A participant is not worse off after participating in the mechanism than if they had not participated. It ensures that no loss is incurred from participation after all outcomes are realized.
- **Ex-ante IR**: A participant expects to benefit from participating based on the average expected utility before the mechanism begins, considering all possible outcomes and probabilities. It guarantees that participation is beneficial in expectation.

A **budget-balanced(BB)** game or mechanism is one where the sum of transactions made by the agents equals 0.

Definition 2.3 (Groves Mechanism). The Groves Mechanism is a collection of mechanisms (s, p) such that

$$s(m) = \arg\max_{s'\in S} \sum_{i\in I} m_{i,s'}$$

$$p_i(m) = h_i(m_{-i}) - \sum_{j\neq i} m_{j,s(m)}$$

where $h_i(m_{-i})$ is a function independent of m_i.

Definition 2.4 (VCG Mechanism). The VCG Mechanism is a mechanism (s, V) such that

$$s(m) = \arg\max_{s'\in S} \sum_{i\in I} m_{i,s'}$$

$$V_i(m) = \max_{s^*} \sum_{j\neq i} m_{j,s^*} - \sum_{j\neq i} m_{j,s(m)}$$

Definition 2.5 (AGV Mechanism). The AGV Mechanism is a mechanism (s, A) such that

$$s(m) = \arg\max_{s'\in S} \sum_{i\in I} m_{i,s'}$$

$$A_i(m) = -\mathcal{E}_i(m_i) + \frac{1}{n-1} \sum_{j\neq i} \mathcal{E}_j(m_j)$$

where $\mathcal{E}_i(m_i) = E_{\theta_{-i}} \sum_{j\neq i} m_{j,s(\theta_{-i},m_i)}$.

2.3 RELATED WORK

Work in the area of designing budget-balanced mechanisms is relatively scarce, presumably due to the primacy of the goal of social efficiency in combination with the strong negative result of the Myerson-Satterthwaite impossibility theorem: that no mechanism is capable of achieving individual rationality, efficiency, and budget balance at the same time for general valuation functions, even if we loosen our solution concept to Bayes-Nash equilibrium [Myerson and Satterthwaite 1983]. This is an extension of the Hurwicz impossibility theorem [Hurwicz 1975], which proves the result for dominant-strategy equilibria.

To achieve the goal of budget balance, many VCG-based mechanisms have been proposed. [Faltings 2004] proposed a method where one or more individuals are randomly chosen as the center, and then VCG machanism is implemented using the payoffs of the other individuals. The VCG transactions of the other individuals are then allocated to these centers. This method also achieves exact budget balance but sacrifices Pareto efficiency, as the payoffs of the randomly chosen centers are not considered. [Cavallo 2006] and [Guo and

Conitzer 2007] attempted to redistribute the VCG surplus back to each agent in a specific case, but their methods do not return all the surplus, i.e., some of the VCG surplus remains unreturned, and we can't apply their method in our more general problem.

Another proposal for achieving budget-balance was that [d'Aspremont et al. 2004] proposed the d'AGVA (or AGV) mechanism, which, under the condition that the designer and each agent know the true distribution of all agents' payoffs, can achieve Bayes-Nash equilibrium (rather than dominant-strategy equilibrium), pareto efficiency, and exact budget balance. Additionally, this mechanism violates ex post individual rationality but satisfies ex ante individual rationality. One of our approach is close to this work, but we found that using our method significantly reduces the probability of agents experiencing negative returns compared to their work.

3 OUR NEW MECHANISM

3.1 ASSUMPTIONS

Our approach is based on two important assumptions:

- **Consensus on other agents' type**: Every agent i know the distribution of other agents' true utility θ_{-i}.
- **Quasilinear Utility:** The final total payoff $u_{i,s(m)}$ of each agent equals the utility corresponding to the social choice minus the transaction:

$$u_{i,s(m)} = \theta_{i,s(m)} - p_i(m)$$

3.2 ONE OF OUR METHOD

We designed a "pre-distribute and return" approach. In this method, we randomly select a center, which is required to pre-distribute (in the expected sense) everyone's VCG transaction, and then subsequently accept the return of the VCG transaction from others. The essence of this approach is to ensure that the expected transaction of the randomly selected center equals the VCG transaction.

Definition 3.1 (Mechanism 1). Mechanism 1 is a 5-step process, and it can solve our problem when the utility function is independent for every agent, which is as follows:

(1) Let n send a message m_n to others, that is the utility function of n that n claimed.
(2) n give C to others, every non-negative outgiving is acceptable.
(3) Other agents i send m_i, also this is the utility function of i that i claimed.
(4) i give $V_i(m)$ to n.
(5) According to m, we can get the social choise s that maximize $\sum_i m_{i,s}$.

Here C is:

$$\sum_{i=1}^{n} E_{\theta_{-n}}(V_i(\theta_{-n}, m_n))$$

Theorem 3.2. *Mechanism 1 achieves (Dominant Strategy) Incentive compatibility.*

Proof. Incentive compatibility for $1 \sim n-1$ is obvious, since their payment is strictly a VCG transaction minus a redistribution payment from agent n that does not depend on their own bid.

So we can always assume $m_i = \theta_i$ for $i \in [1, n)$, and the previous C can make n also incentive compatibility:

The expected income of n is:

$$\begin{aligned}
&E_{\theta_{-n}}(U_n)\\
=&E_{\theta_{-n}}\left(\theta_{n,s(\theta_{-n},m_n)} + \sum_{i=1}^{n-1} V_i(\theta_{-n}, m_n) - \sum_{i=1}^{n} V_i(\theta_{-n}, m_n)\right)\\
=&E_{\theta_{-n}}(\theta_{n,s(\theta_{-n},m_n)} - V_n(\theta_{-n}, m_n))
\end{aligned}$$

We can find, this is equal to $E_{\theta_{-n}}$ (Income of n in VCG mechanism), so n will always claim $m_i = \theta_i$. □

Theorem 3.3. *Mechanism 1 achieves ex ante Individual Rationality.*

Proof. We just need to show in Mechanism 1, at every moment, every agent will have non-negative expected income in next steps.

For every agent, we can only consider the step that it will pay some payment to others.

We can find, for n, when it pay C to others, the expected income is: $E_{\theta_{-n}}$(Income of n in VCG mechanism), we can find it is non-negative by Income of n in VCG mechanism ≥ 0.

For $i \in [1, n)$, when it pay $V_i(m)$ to n, the income is Income of i in VCG mechanism$+C_i \geq 0$, here $\sum_i C_i = C, C_i \geq 0$. □

3.3 ANOTHER METHOD

Mechanism 1 achieves many desirable properties, but it has one drawback: it is unfair to the selected center. On average, other agents receive an additional $1/n$ of the center's VCG transaction, while the center does not receive any extra benefit. This raises a question: can we reduce this unfairness by increasing the number of centers? The answer is yes.

Consider randomly choose two agents as centers. To extend the approach of Mechanism 1, now in stage (4), each non-center agent gives each center half of their VCG transaction. This way, both the centers only needs to distribute half of the expected total transaction.

To ensure that the centers' final expected transaction remain their own VCG transaction, a payment is made between the centers after all utilities are reported: one center pays the other half of their VCG transaction to the other center.

As a result, each center receives an additional payment from the other center, making it fairer.

Extending this idea, we can choose all n agents as centers, thus achieving the **decentralized** Mechanism 2.

Definition 3.4 (Mechanism 2). Mechanism 2 is a mechanism (s, p) such that

$$\begin{aligned} s(m) =& \arg\max_{s' \in S} \sum_{i \in I} m_{i,s'} \\ p_i(m) =& D_i(m_i) - \frac{1}{n-1} \sum_{j \neq i} D_j(m_j) \\ &+ \frac{n-1}{n} V_i(\theta_{-i}, m_i) - \frac{1}{n} \sum_{k \neq i} V_k(\theta_{-i}, m_i) \end{aligned}$$

where $D_i(m_i) = \frac{1}{n} \sum_{j=1}^{n} E_{\theta_{-i}}(V_j(\theta_{-i}, m_i))$.

Theorem 3.5. *Mechanism 2 achieves (Bayesian-Nash) Incentive compatibility and ex ante Individual Rationality.*

Proof. When all other agents truthfully report their utilities, then agent i's expected transaction is:

$$E_{\theta_{-i}}(p_i(m) = E_{\theta_{-i}}\left(V_i(\theta_{-i}, m_i) - C\right)$$

where $C = \frac{1}{n(n-1)} \sum_{j \neq i} D_j(m_{-j})$.

This value is strictly a VCG transaction plus a redistribution payment from other agents which does not depend on their own bid, from IC property of VCG mechanism, we know that Mechanism 2 also achieves IC.

Additionally, because each person is IR in the VCG mechanism and each person's transaction in Mechanism 2 is less than the VCG transaction, Mechanism 2 achieves IR. □

Theorem 3.6. *Mechanism 2 achieves Pareto efficiency and exact Budget balance.*

Proof. From the social choice scheme of Mechanism 2, it can be seen that Mechanism 2 always selects the outcome with the highest social welfare, thereby achieving Perato Efficiency.

Additionally, one can easily check that $\sum_{i=1}^{n} p_i(m) = 0$, so budget balance is also achieved. □

From the above, it can be seen that Mechanism 2 achieves decentralization thus being more fair, and it reduces the variance in each person's earnings. This is because the amount distributed to others is $1/n$ of the expected total transaction, rather than the full expected total transaction as in Mechanism 1. However, this mechanism loses the Perfect Bayesian Equilibrium compared to Mechanism 1 and strongly relies on everyone reporting their utilities simultaneously.

A mechanism very similar to Mechanism 2, Mechanism 3, does not require first distributing $1/n$ of the expected total transaction and then waiting for other agents to return the payment. Instead, this mechanism directly replaces the "pre-distribute and return" approach with direct distribution. In other words, Mechanism 3 distributes the expected return amount from other agents directly during the initial distributing phase.

Definition 3.7 (Mechanism 3). Mechanism 3 is a mechanism (s, p) such that

$$s(m) = \arg\max_{s' \in S} \sum_{i \in I} m_{i,s'}$$

$$p_i(m) = E_{\theta_{-i}}(V_i(\theta_{-i}, m_i)) - \frac{1}{n-1} \sum_{j \neq i} E_{\theta_{-j}}(V_j(\theta_{-j}, m_j))$$

The relationship between Mechanism 3 and Mechanism 2: Mechanism 3 distributes

$$\begin{aligned} &E_{\theta_{-i}}(D_i(m_i) + \frac{n-1}{n} V_i(\theta_{-i}, m_i) - \frac{1}{n} \sum_{k \neq i} V_k(\theta_{-i}, m_i)) \\ =& E_{\theta_{-i}}(V_i(\theta_{-i}, m_i)). \end{aligned}$$

And note the high similarity between Mechanism 3 and the AGV mechanism: Mechanism 3 distributes

$$\begin{aligned} &E_{\theta_{-i}}(V_i(\theta_{-i}, m_i)) \\ =& E_{\theta_{-i}}(\sum_{j \neq i} m_{j,s(m)} - \max_{s^*} \sum_{j \neq i} m_{j,s^*}) \end{aligned}$$

while the AGV mechanism actually distributes

$$E_{\theta_{-i}}(\sum_{j \neq i} m_{j,s(m)}).$$

In the following simulation, we can see that Mechanism 3 actually performs worse than Mechanism 2 (although it is still far superior to the AGV Mechanism). However, Mechanism 3 remains significant because it reveals the close relationship between our proposed mechanism and the AGV Mechanism.

4 EXAMPLE

For better understanding our approach, we give an example with 4 agents and 4 options, we assume that θ_i is i.i.d. from $[0, 1]$, the following table is an utility function instance:

Person \Option	Opt 1	Opt 2	Opt 3	Opt 4
P1	0.09	0.27	0.48	0.15
P2	0.25	0.15	0.45	0.32
P3	0.69	0.90	0.67	0.30
P4	0.46	0.82	0.14	0.26

Suppose the randomly chosen central agent is $P4$.

Using Falting's mechanism, we select $P4$ as the central agent, then the social choice will be $Opt3$ and the sum utility is $0.48 + 0.45 + 0.67 + 0.14 = 1.73$.

Using mechanism 1,2,3 and AGV mechanism, the social choice will be $Opt2$, the sum utility is $0.27+0.15+0.90+0.82 = 2.14$.

Using mechanism 1, their utilities are 0.29, 0.24, 0.98, 0.63.

Using mechanism 2, their utilities are 0.34, 0.23, 0.97, 0.60.

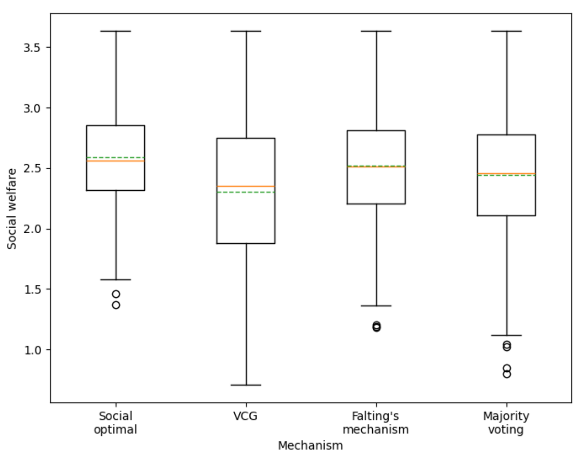
Using mechanism 3, their utilities are 0.28, 0.18, 0.90, 0.78.

Using AGV mechanism, their utilities are -0.41, -0.41, 1.88, 1.07.

5 SIMULATIONS

In this section, we also assume that there are 4 agents and 4 options with θ_i is i.i.d. from $[0, 1]$.

First, we run 1000 times simulation to evaluate social welfare of different mechanisms. The result is shown in the following figure:



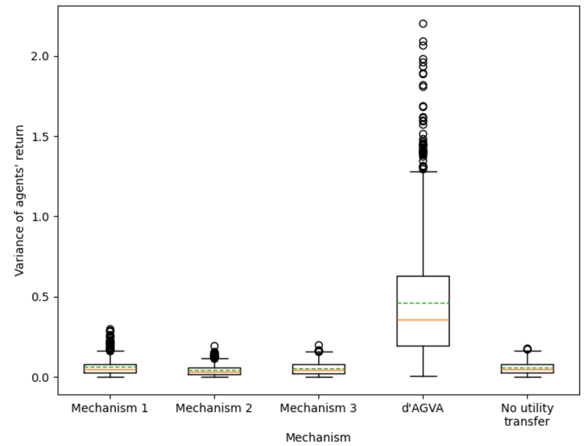
This shows that mechanisms that can achieve social welfare have significant advantages over majority voting mechanism: the majority voting mechanism has very bad performance in extreme situations.

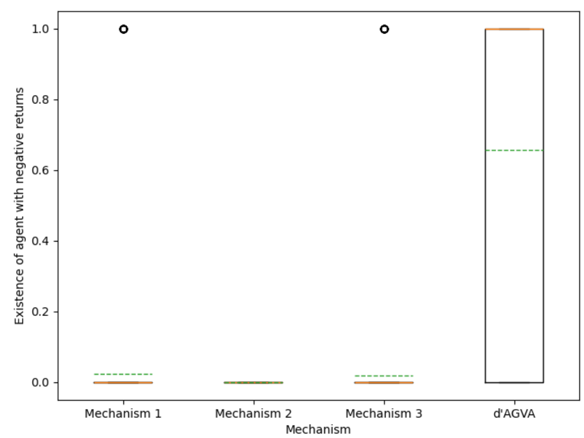
To compare with AGV mechanism, we run 1000 times simulation and count "the variance of agents' returns" and "the existence of agent with negative returns".

The two images in the right column show that, compared with AGV mechanism, our mechanisms are fairer and have lower probability of breaking IR.

6 CONCLUSION

To address the issue of budget balance that the VCG mechanism fails to achieve in decision-making games, we introduce the assumption that all agents have Consensus on other agents' utilities. Based on this, Mechanism 1 is proposed, which involves randomly selecting a center and performing a "pre-distribute and return" operation. This mechanism retains the favorable properties of the VCG mechanism and achieves budget balance.





However, Mechanism 1 suffers from unfairness due to the random selection of the center. To mitigate this, Mechanism 2 is proposed, which achieves fairness through derandomization while reducing the variance of total earnings for each individual. Mechanism 2 strongly relies on simultaneous bidding by all agents compared to Mechanism 1.

In the text, a variant similar to Mechanism 2, Mechanism 3, is mentioned, revealing the close resemblance of our proposed mechanisms to the AGV mechanism. Furthermore, we observe that Mechanism 2, under the assumption of consistency with AGV mechanisms, significantly outperforms the AGV mechanism in simulations.

Future research directions include combining the independence from simultaneous bidding of Mechanism 1 with the fairness and reduced variance advantages of Mechanism 2. Additionally, exploring why Mechanism 2 performs better than the AGV mechanism and using this as a basis to seek potentially superior solutions.

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