

# Lecture 4: Deep Reinforcement Learning

## Actor-Critic Algorithms

Chunlin Chen & Zhi Wang

Department of Control and Systems Engineering  
Nanjing University

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- 1 Improving the policy gradient with a critic
- 2 Policy evaluation – fit the value function
- 3 The actor-critic algorithm
- 4 Actor-critics with  $n$ -step returns and eligibility traces

# Today's lecture

- Improving the policy gradient with a critic
- The policy evaluation problem
- Discount factors
- The actor-critic algorithm
- Goals
  - Understand how policy evaluation fits into policy gradients
  - Understand how actor-critic algorithms work

# Recall: policy gradients

REINFORCE algorithm:

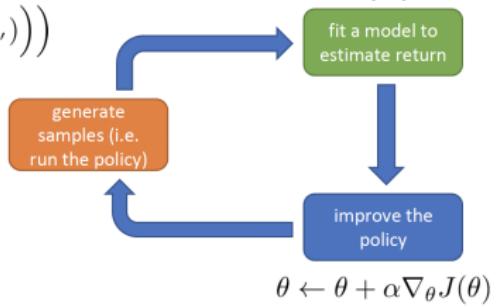
1. sample  $\{\tau^i\}$  from  $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$  (run the policy)
2.  $\nabla_\theta J(\theta) \approx \sum_i \left( \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i) \left( \sum_{t'=t}^T r(\mathbf{s}_{t'}^i, \mathbf{a}_{t'}^i) \right) \right)$
3.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \hat{Q}_{i,t}^\pi$$



“reward to go”

$$\hat{Q}^\pi(\mathbf{x}_t, \mathbf{u}_t) = \sum_{t'=t}^T r(\mathbf{x}_{t'}, \mathbf{u}_{t'})$$



# Improving the policy gradient

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( \underbrace{\sum_{t'=t}^T r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})}_{\text{"reward to go"}} \right)$$

“reward to go”

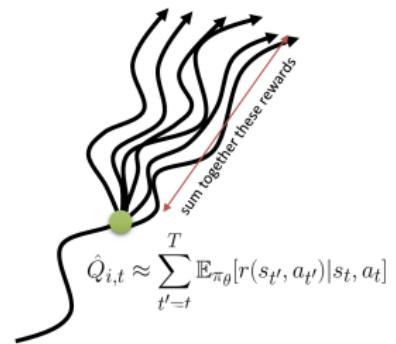
$$\hat{Q}_{i,t}$$

$\hat{Q}_{i,t}$ : estimate of expected reward if we take action  $\mathbf{a}_{i,t}$  in state  $\mathbf{s}_{i,t}$

can we get a better estimate?

$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_{\theta}} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : true *expected* reward-to-go

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$



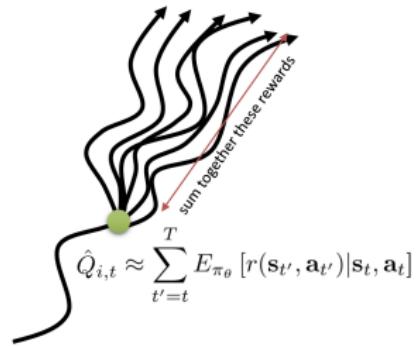
# What about the baseline?

$$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]: \text{true expected reward-to-go}$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) (Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) - V(\mathbf{s}_{i,t}))$$

$$b_t = \frac{1}{N} \sum_i Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \quad \text{average what?}$$

$$V(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q(\mathbf{s}_t, \mathbf{a}_t)]$$



# State value function & Action value function

- Action value function  $Q(s, a)$ : total reward from taking  $a$  in  $s$

$$\begin{aligned} Q_\pi(s, a) &= \mathbb{E}[R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s, A_t = a] \\ &= \sum_{s', r} p(s', r | s, a) [r + \gamma v_\pi(s')] \end{aligned}$$

- State value function  $v(s)$ : total reward from  $s$

$$v_\pi(s) = \mathbb{E}_{a \sim \pi(a|s)}[Q_\pi(s, a)]$$

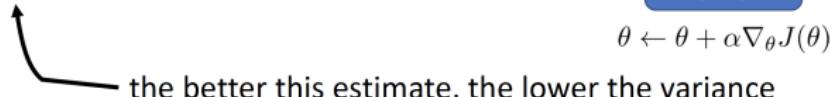
# State value function & Action value function

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta}[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]: \text{total reward from taking } \mathbf{a}_t \text{ in } \mathbf{s}_t$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]: \text{total reward from } \mathbf{s}_t$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t): \text{how much better } \mathbf{a}_t \text{ is}$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$



the better this estimate, the lower the variance

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( \underbrace{\sum_{t'=t}^T r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) - b}_{\text{unbiased, but high variance single-sample estimate}} \right)$$

unbiased, but high variance single-sample estimate

# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta}[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

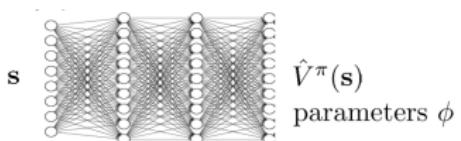
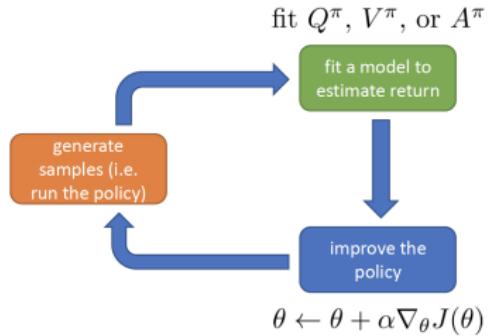
fit what to what?

$Q^\pi, V^\pi, A^\pi$ ?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)}[V^\pi(\mathbf{s}_{t+1})]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + \underline{V^\pi(\mathbf{s}_{t+1})} - V^\pi(\mathbf{s}_t)$$

let's just fit  $V^\pi(\mathbf{s})$ !



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# Policy evaluation (the prediction problem)

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^T E_{\pi_\theta}[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t]$$

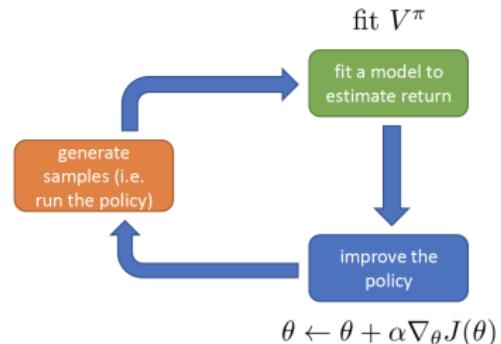
$$J(\theta) = E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)}[V^\pi(\mathbf{s}_1)]$$

how can we perform policy evaluation?

Monte Carlo policy evaluation (this is what policy gradient does)

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^T r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

$$V^\pi(\mathbf{s}_t) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t'=t}^T r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \quad (\text{requires us to reset the simulator})$$



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$



# Monte-Carlo evaluation with function approximation

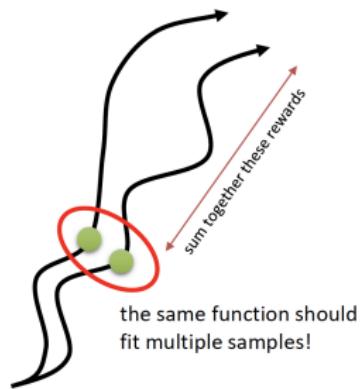
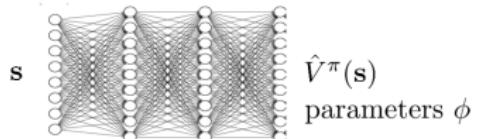
$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^T r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

not as good as this:  $V^\pi(\mathbf{s}_t) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t'=t}^T r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$

but still pretty good!

training data:  $\left\{ \left( \mathbf{s}_{i,t}, \underbrace{\sum_{t'=t}^T r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})}_{y_{i,t}} \right) \right\}$

supervised regression:  $\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$



## Can we do better? – From MC to TD

$$\begin{aligned} v_\pi(s) &= \mathbb{E}_\pi[G_t | S_t = s] \\ &= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} | S_t = s] \\ &= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s] \\ &= \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a) [r + \gamma v_\pi(s')] \end{aligned}$$

- **MC**: The expected  $G_t$  is not known, a sample return is used in place of the real expected return
- **DP**: The true  $v_\pi$  is not known, and the current estimate  $V(S_{t+1})$  is used instead
- **TD**: It samples the expected values  $R_{t+1}$ , and it uses the current estimate  $V(S_{t+1})$  instead of the true  $v_\pi$ 
  - Combine the sampling of MC with the bootstrapping of DP

## Temporal-difference evaluation with function approximation

$$\text{ideal target: } y_{i,t} = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_{i,t}] \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + V^\pi(\mathbf{s}_{i,t+1}) \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \underbrace{\hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})}$$

Monte Carlo target:  $y_{i,t} = \sum_{t'=t}^T r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$  directly use previous fitted value function!

$$\text{training data: } \left\{ \underbrace{\left( \mathbf{s}_{i,t}, r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1}) \right)}_{y_{i,t}} \right\}$$

$$\text{supervised regression: } \mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$$

sometimes referred to as a “bootstrapped” estimate

# Policy evaluation examples

TD-Gammon, Gerald Tesauro 1992

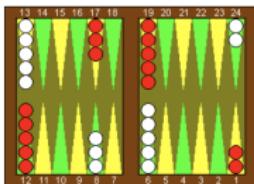


Figure 2. An illustration of the normal opening position in backgammon. TD-Gammon has sparked a near-universal conversion in the way experts play certain opening rolls. For example, with an opening roll of 4-1, most players have now switched from the traditional move of 13-9, 6-5, to TD-Gammon's preference, 13-9, 24-23. TD-Gammon's analysis is given in Table 2.

AlphaGo, Silver et al. 2016



reward: game outcome

value function  $\hat{V}_\phi^\pi(\mathbf{s}_t)$ :

expected outcome given board state

reward: game outcome

value function  $\hat{V}_\phi^\pi(\mathbf{s}_t)$ :

expected outcome given board state

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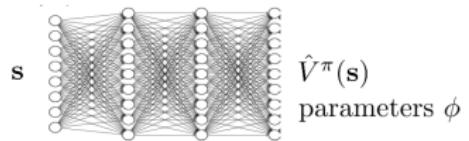
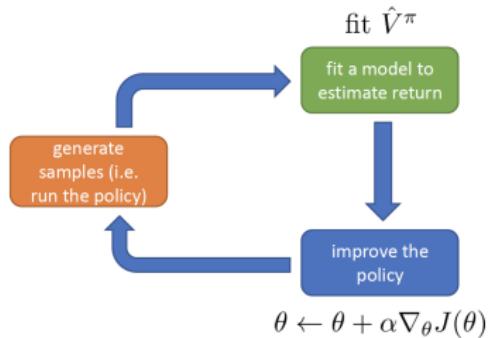
# An actor-critic algorithm

batch actor-critic algorithm:

1. sample  $\{\mathbf{s}_i, \mathbf{a}_i\}$  from  $\pi_\theta(\mathbf{a}|\mathbf{s})$  (run it on the robot)
2. fit  $\hat{V}_\phi^\pi(\mathbf{s})$  to sampled reward sums
3. evaluate  $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \hat{V}_\phi^\pi(\mathbf{s}'_i) - \hat{V}_\phi^\pi(\mathbf{s}_i)$
4.  $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$$



$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t]$$

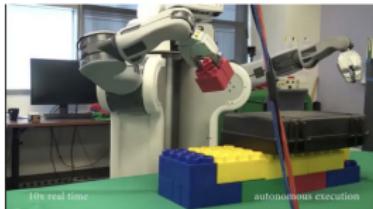
# Aside: discount factors

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$$

what if  $T$  (episode length) is  $\infty$ ?

$\hat{V}_\phi^\pi$  can get infinitely large in many cases



episodic tasks



continuous/cyclical tasks

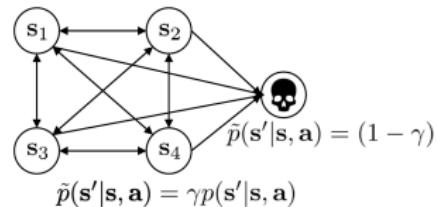
simple trick: better to get rewards sooner than later

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$$



discount factor  $\gamma \in [0, 1]$  (0.99 works well)

$\gamma$  changes the MDP:



## Aside: discount factors

- Assume that:  $0 \leq r_{min} \leq r \leq r_{max} \leq \infty$
- Without discount factor: unbounded

$$\begin{aligned} V(s_t) &= \mathbb{E}[r_t + r_{t+1} + r_{t+2} + \dots] \\ &\geq r_{min} + r_{min} + r_{min} + \dots \\ &= \infty \end{aligned}$$

- With discount factor: bounded

$$\begin{aligned} V(s_t) &= \mathbb{E}[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots] \\ &\leq r_{max} + \gamma r_{max} + \gamma^2 r_{max} + \dots \\ &= \frac{r_{max}}{1 - \gamma} \end{aligned}$$

# Discount factors for policy gradients

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$$

with critic:

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \underbrace{\left( r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1}) - \hat{V}_\phi^\pi(\mathbf{s}_{i,t}) \right)}_{\hat{A}^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})}$$

what about (Monte Carlo) policy gradients?

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( \sum_{t'=t}^T \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

# Actor-critic algorithms (with discount)

batch actor-critic algorithm:

- 
1. sample  $\{\mathbf{s}_i, \mathbf{a}_i\}$  from  $\pi_\theta(\mathbf{a}|\mathbf{s})$  (run it on the robot)
  2. fit  $\hat{V}_\phi^\pi(\mathbf{s})$  to sampled reward sums
  3. evaluate  $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}_\phi^\pi(\mathbf{s}'_i) - \hat{V}_\phi^\pi(\mathbf{s}_i)$
  4.  $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
  5.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

online actor-critic algorithm:

- 
1. take action  $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$ , get  $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
  2. update  $\hat{V}_\phi^\pi$  using target  $r + \gamma \hat{V}_\phi^\pi(\mathbf{s}')$
  3. evaluate  $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}') - \hat{V}_\phi^\pi(\mathbf{s})$
  4.  $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
  5.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

# Batch-mode (offline) vs. online

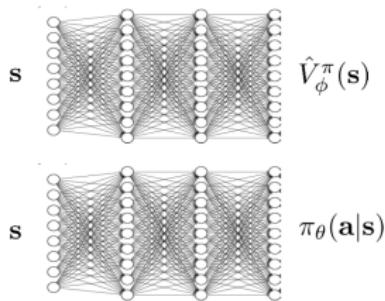
- Batch-model (offline) algorithms
  - Collect a batch of samples using some policy
  - Fit the state- or action-value function iteratively
- Online algorithms
  - Take some action to collect one sample
  - Fit the value function
  - Iteratively alternate the above two steps

# Architecture design

online actor-critic algorithm:

1. take action  $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$ , get  $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update  $\hat{V}_\phi^\pi$  using target  $r + \gamma \hat{V}_\phi^\pi(\mathbf{s}')$
3. evaluate  $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}') - \hat{V}_\phi^\pi(\mathbf{s})$
4.  $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

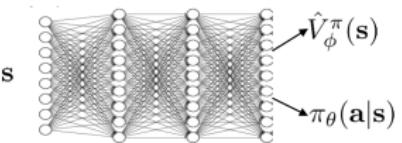
two network design



+ simple & stable

- no shared features between actor & critic

shared network design

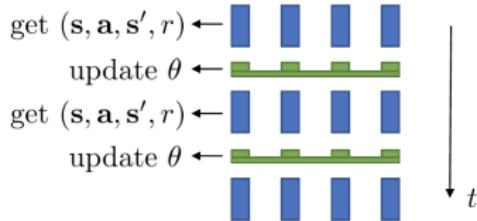


# Online actor-critic in practice

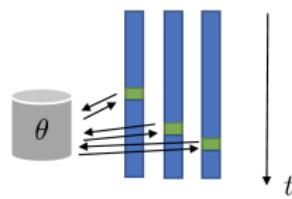
online actor-critic algorithm:

1. take action  $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$ , get  $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update  $\hat{V}_\phi^\pi$  using target  $r + \gamma \hat{V}_\phi^\pi(\mathbf{s}') \leftarrow$  works best with a batch (e.g., parallel workers)
3. evaluate  $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}') - \hat{V}_\phi^\pi(\mathbf{s})$
4.  $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a}) \leftarrow$
5.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

synchronized parallel actor-critic



asynchronous parallel actor-critic



# Critics as state-dependent baselines

Actor-critic:

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}_{\phi}^{\pi}(\mathbf{s}_{i,t+1}) - \hat{V}_{\phi}^{\pi}(\mathbf{s}_{i,t}) \right)$$

+ lower variance (due to critic)  
- not unbiased (if the critic is not perfect)

Policy gradient:

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( \left( \sum_{t'=t}^T \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right) - b \right)$$

+ no bias  
- higher variance (because single-sample estimate)

can we use  $\hat{V}_{\phi}^{\pi}$  and still keep the estimator unbiased?

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( \left( \sum_{t'=t}^T \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right) - \hat{V}_{\phi}^{\pi}(\mathbf{s}_{i,t}) \right)$$

+ no bias  
+ lower variance (baseline is closer to rewards)

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- 4 Actor-critics with  $n$ -step returns and eligibility traces

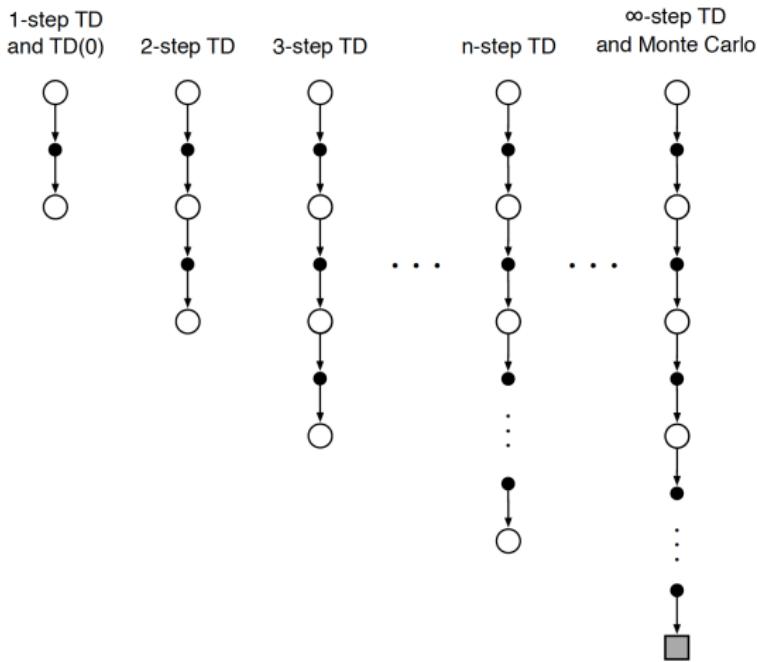
## $n$ -step bootstrapping: Combine MC and one-step TD

- Neither MC or one-step TD is always the best, we generalize both methods so that one can shift from one to the other smoothly as needed to meet the demands of a particular task
- One-step TD: In many applications, one wants to be able to update the action very fast to take into account anything that has changed
- However, bootstrapping works best if it is over a length of time in which a significant and recognizable state change has occurred

$n = 1$	$n$ -step TD	$n = \infty$
$\text{TD}(0)$	$\leftrightarrow$	$MC$

# $n$ -step TD prediction

- Perform an update based on an intermediate number of rewards, more than one, but less than all of them until termination



## Recall MC and TD(0) updates

- In MC updates, the target is the **complete return**

$$G_t = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{T-t+1} R_T$$

$$\begin{aligned} V(S_t) &\leftarrow V(S_t) + \alpha[\textcolor{red}{G_t} - V(S_t)] \\ &= V(S_t) + \alpha[\textcolor{red}{R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{T-t+1} R_T} - V(S_t)] \end{aligned}$$

- In TD(0) updates, the target is the **one-step return**

$$G_{t:t+1} = R_{t+1} + \gamma V(S_{t+1})$$

$$\begin{aligned} V(S_t) &\leftarrow V(S_t) + \alpha[\textcolor{red}{G_{t:t+1}} - V(S_t)] \\ &= V(S_t) + \alpha[\textcolor{red}{R_{t+1} + \gamma V(S_{t+1})} - V(S_t)] \end{aligned}$$

## $n$ -step TD update rule

- For  $n$ -step TD, set the target as the  **$n$ -step return**

$$G_{t:t+n} = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n})$$

- All  $n$ -step returns can be considered approximations to the complete return, truncated after  $n$  steps and then corrected for the remaining missing terms by  $V(S_{t+n})$

$$V(S_t) \leftarrow V(S_t) + \alpha[G_{t:t+n} - V(S_t)]$$

$$= V(S_t) + \alpha[R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n}) - V(S_t)]$$

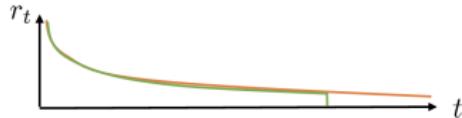
# Actor-critics with $n$ -step returns

$$\hat{A}_C^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{t+1}) - \hat{V}_\phi^\pi(\mathbf{s}_t)$$

- + lower variance
- higher bias if value is wrong (it always is)
- + no bias
- higher variance (because single-sample estimate)

$$\hat{A}_{MC}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}_\phi^\pi(\mathbf{s}_t)$$

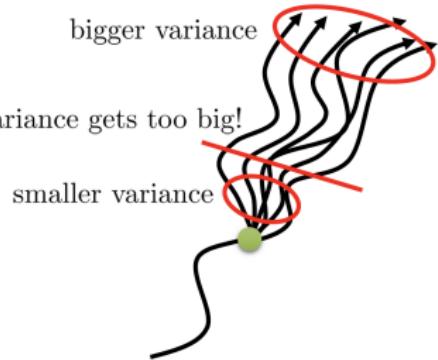
Can we combine these two, to control bias/variance tradeoff?



cut here before variance gets too big!

$$\hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}_\phi^\pi(\mathbf{s}_t) + \gamma^n \hat{V}_\phi^\pi(\mathbf{s}_{t+n})$$

choosing  $n > 1$  often works better!



# Eligibility traces: unify/generalize TD and MC

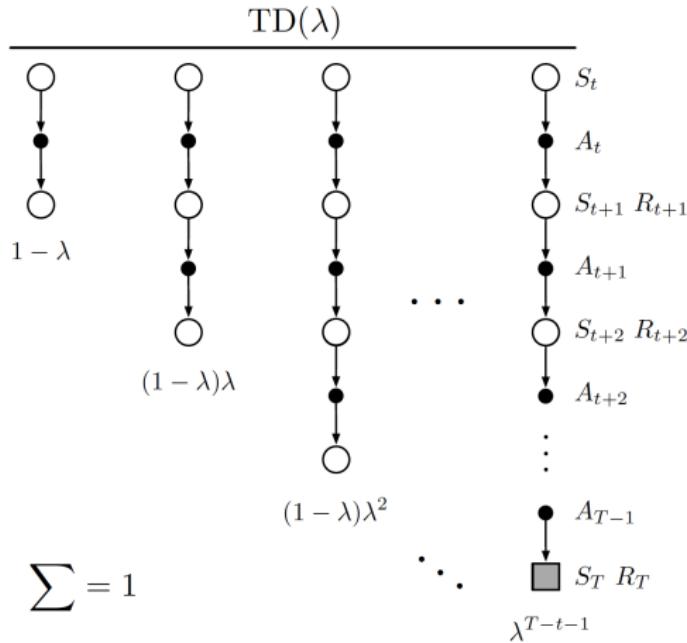
- Almost any TD method can be combined with eligibility traces to obtain a more general method that may learn more efficiently
  - e.g., the popular  $\text{TD}(\lambda)$  algorithm,  $\lambda$  refers the use of an eligibility trace
  - Produce a family of methods spanning a spectrum that has MC methods at one end ( $\lambda = 1$ ) and one-step TD methods at the other ( $\lambda = 0$ )
- Eligibility traces offer an elegant algorithmic mechanism with significant computational advantages (compared to  $n$ -step TD)
  - Only a single trace vector is required rather than a store of the last  $n$  feature vectors
  - Learning also occurs continually and uniformly in time rather than being delayed and then catching up at the end of the episode
  - Learning can occur and effect behavior immediately after a state is encountered rather than being delayed  $n$ -steps

# The $\lambda$ -return

- How to interrelate TD and MC?
  - e.g., average one-step and infinite-step returns,  $G = (G_t + G_{t:t+1})/2$
  - An update that averages simpler component updates is called a **compound update**
- The TD( $\lambda$ ) algorithm can be understood as one particular way of averaging  $n$ -step updates

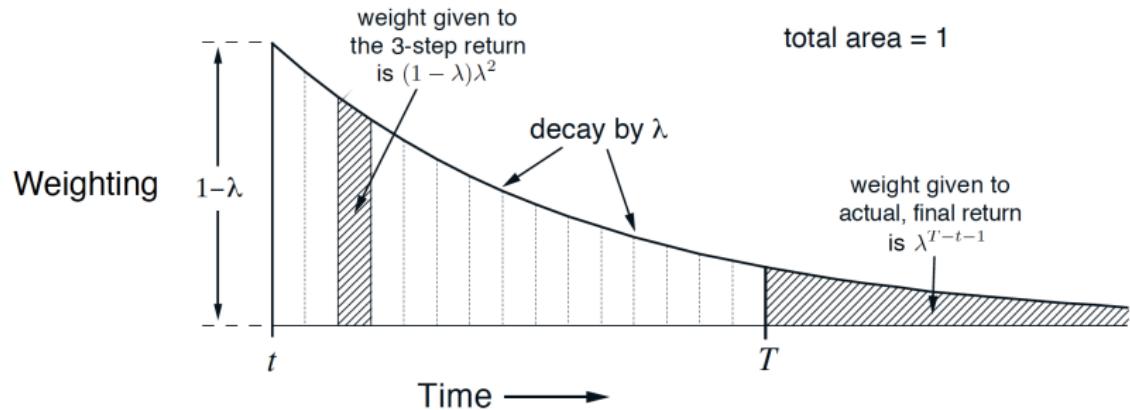
$$\begin{aligned} G_t^\lambda &= (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1} G_{t:t+n} \\ &= (1 - \lambda) \sum_{n=1}^{T-t-1} \lambda^{n-1} G_{t:t+n} + \lambda^{T-t-1} G_t \end{aligned}$$

# Backup diagram for TD( $\lambda$ )



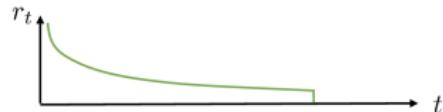
**Figure 12.1:** The backup diagram for TD( $\lambda$ ). If  $\lambda = 0$ , then the overall update reduces to its first component, the one-step TD update, whereas if  $\lambda = 1$ , then the overall update reduces to its last component, the Monte Carlo update.

# The weight distribution



**Figure 12.2:** Weighting given in the  $\lambda$ -return to each of the  $n$ -step returns.

# Generalized advantage estimation: Actor-critics with eligibility traces



Do we have to choose just one n?

Cut everywhere all at once!

$$\hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}_\phi^\pi(\mathbf{s}_t) + \gamma^n \hat{V}_\phi^\pi(\mathbf{s}_{t+n})$$

$$\hat{A}_{\text{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{n=1}^{\infty} w_n \hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t)$$

Weighted combination of n-step returns

How to weight?

Mostly prefer cutting earlier (less variance)

$$w_n \propto \lambda^{n-1}$$

exponential falloff

$$\hat{A}_{\text{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma((1-\lambda)\hat{V}_\phi^\pi(\mathbf{s}_{t+1}) + \lambda(r(\mathbf{s}_{t+1}, \mathbf{a}_{t+1}) + \gamma((1-\lambda)\hat{V}_\phi^\pi(\mathbf{s}_{t+2}) + \lambda r(\mathbf{s}_{t+2}, \mathbf{a}_{t+2}) + \dots))$$

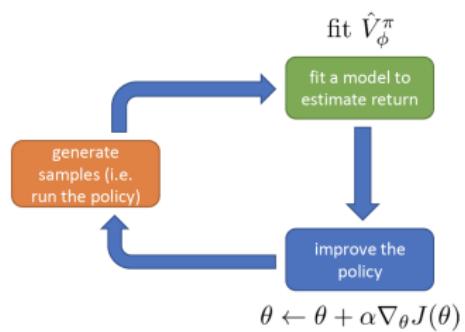
$$\hat{A}_{\text{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} (\gamma \lambda)^{t'-t} \delta_{t'}$$

$$\delta_{t'} = r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{t'+1}) - \hat{V}_\phi^\pi(\mathbf{s}_{t'})$$

similar effect as discount!

# Review

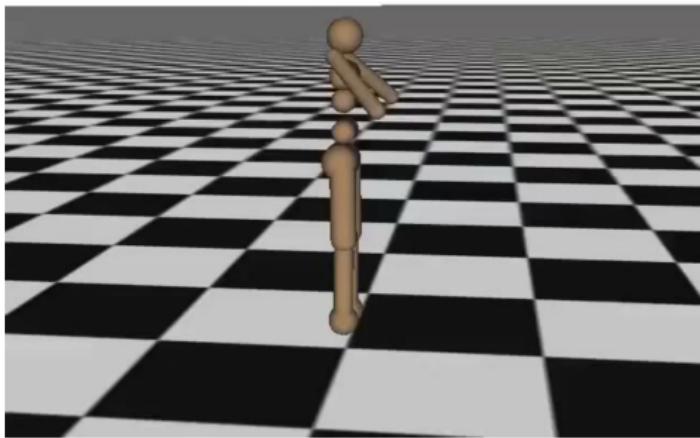
- Actor critic algorithms
  - Actor: the policy
  - Critic: value function
  - Reduce variance of policy gradient
- Policy evaluation
  - Fitting value function to policy
- Discount factors
  - Bound the value function
  - Also a variance reduction trick
- Actor critic algorithm design
  - One network (with two heads) or two networks
  - Batch mode, or online (+ parallel)
- State-dependent baselines
  - Another way to use the critic
  - Can combine:  $n$ -step returns or eligibility traces



# Actor-critic examples

- High dimensional continuous control with generalized advantage estimation (Schulman, Moritz, L., Jordan, Abbeel '16)
- Batch-mode actor-critic
- Blends Monte Carlo and function approximator estimators (GAE)

Iteration 0



# Actor-critic examples

- Asynchronous methods for deep reinforcement learning (Mnih, Badia, Mirza, Graves, Lillicrap, Harley, Silver, Kavukcuoglu, 2016)
- Online actor critic, parallelized batch
- $n$ -step returns with  $n = 4$
- Single network for actor and critic

# Learning objectives of this lecture

- You should be able to...
  - Extend policy gradient methods to actor-critic algorithms
  - Use policy evaluation to fit the critic, i.e., the value function
  - Be able to implement the basic actor-critic algorithm
  - Know the actor-critics with  $n$ -step returns
  - Know the actor-critics with eligibility traces, i.e., generalized advantage estimation

# Actor-critic suggested readings

- Lecture 6 of CS285 at UC Berkeley, **Deep Reinforcement Learning, Decision Making, and Control**
  - <http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-6.pdf>
- Classic papers
  - Sutton, McAllester , Singh, Mansour (1999). **Policy gradient methods for reinforcement learning with function approximation:** actor critic algorithms with value function approximation
- DRL actor-critic papers
  - Mnih , Badia , Mirza, Graves, Lillicrap , Harley, Silver, Kavukcuoglu (2016). **Asynchronous methods for deep reinforcement learning:** A3C parallel online actor-critic.
  - Schulman, Moritz, L., Jordan, Abbeel (2016). **High dimensional continuous control using generalized advantage estimation:** batch mode actor-critic with blended Monte Carlo and function approximator returns
  - Gu, Lillicrap , Ghahramani , Turner, L. (2017). **Q-Prop: sample efficient policy gradient with an off-policy critic:** policy gradient with Q-function control variate
  - Tuomas Haarnoja, et al. (2018). **Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor.**

# THE END