

# Lecture 6: Advanced Policy Gradients

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- Solve the constrained optimization problem
- Natural gradient

## 2 Trust region policy optimization (TRPO)

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# Problems of vanilla policy gradient (REINFORCE)

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(a_{i,t} | s_{i,t}) Q(s_{i,t}, a_{i,t})$$

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

- Hard to select the step size  $\alpha$ 
  - Too big step: Bad policy  $\rightarrow$  data collected under bad policy  $\rightarrow$  we cannot recover (in Supervised Learning, data does not depend on neural network weights)
  - Too small step: Not efficient use of experience (in Supervised Learning, data can be trivially re-used)

# Problems of vanilla policy gradient (REINFORCE)



- Small changes in the policy parameters can unexpectedly lead to big **changes** in the policy

# Gradient descent in parameter space

- The step size in gradient descent results from solving the following optimization problem, e.g., using line search

$$d^* = \arg \max_{\|d\| \leq \epsilon} J(\theta + d)$$

- Euclidean distance in parameter space
- Stochastic gradient descent (SGD)

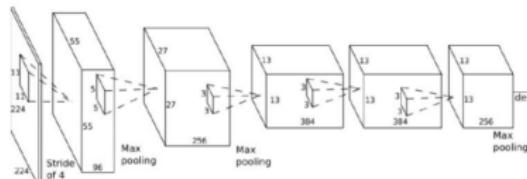
$$\theta \leftarrow \theta + d^*$$

# Hard to pick the threshold $\epsilon$

- It is hard to predict the result on the parameterized distribution
  - Especially for nonlinear function approximators, e.g., neural networks



$s_t$



$\mathbf{a}_t$



# Gradient descent in distribution space

- Gradient descent in parameter space

$$d^* = \arg \max_{\|d\| \leq \epsilon} J(\theta + d)$$

- **Natural gradient descent**: the step size in parameter space is determined by considering the KL divergence in the distributions before and after the update

$$d^* = \arg \max_d J(\theta + d), \quad s.t. D_{KL}(\pi_\theta || \pi_{\theta+d}) \leq \epsilon$$

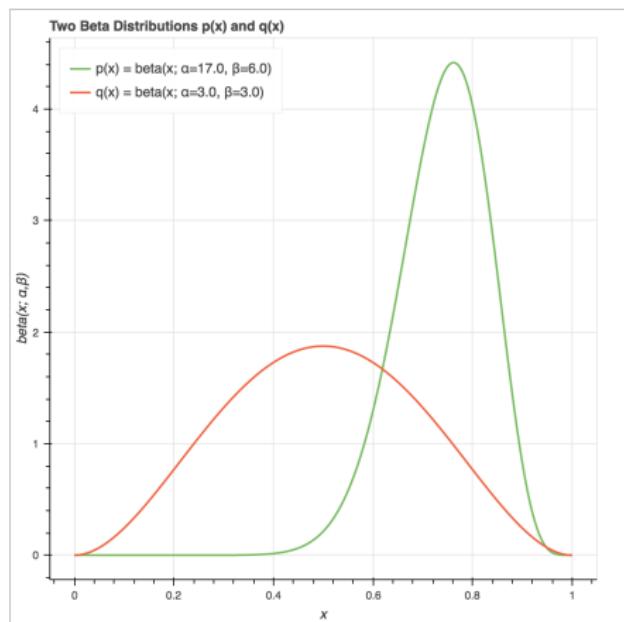
- **KL divergence** in distribution space
- Easier to pick the distance threshold!!!

# Distance for probability distributions

- How to calculate the distance between two points in a 2D coordinate?

$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- Euclidean distance



- How to calculate the distance between two **probability distributions**,  $p(x)$  and  $q(x)$ ?

# Kullback-Leibler (KL) divergence

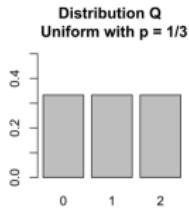
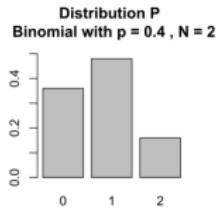
- A measure of how one probability distribution,  $p(x)$ , is different from a second, reference probability distribution,  $q(x)$

$$D_{\text{KL}}(p(x)||q(x)) = \sum_i p(x_i) \log \frac{p(x_i)}{q(x_i)}$$

$$D_{\text{KL}}(p(x)||q(x)) = \int_x p(x) \log \frac{p(x)}{q(x)} dx$$

- A KL divergence of 0 indicates that the two distributions are identical

# KL divergence: An example



$$\begin{aligned} D_{\text{KL}}(P \parallel Q) &= \sum_{x \in \mathcal{X}} P(x) \ln \left( \frac{P(x)}{Q(x)} \right) \\ &= 0.36 \ln \left( \frac{0.36}{0.333} \right) + 0.48 \ln \left( \frac{0.48}{0.333} \right) + 0.16 \ln \left( \frac{0.16}{0.333} \right) \\ &= 0.0852996 \end{aligned}$$

$$\begin{aligned} D_{\text{KL}}(Q \parallel P) &= \sum_{x \in \mathcal{X}} Q(x) \ln \left( \frac{Q(x)}{P(x)} \right) \\ &= 0.333 \ln \left( \frac{0.333}{0.36} \right) + 0.333 \ln \left( \frac{0.333}{0.48} \right) + 0.333 \ln \left( \frac{0.333}{0.16} \right) \\ &= 0.097455 \end{aligned}$$

x	0	1	2
Distribution P(x)	0.36	0.48	0.16
Distribution Q(x)	0.333	0.333	0.333

# KL divergence: A test

- Suppose two Gaussian distributions:

$$p(x) \sim \mathcal{N}(\mu_1, \sigma_1^2), \quad q(x) \sim \mathcal{N}(\mu_2, \sigma_2^2)$$

- What is  $D_{\text{KL}}(p(x)||q(x))$ ?

$$\log \frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1 - \mu_2)^2}{2\sigma_2^2} - \frac{1}{2}$$

# KL divergence between two Gaussians

$$p(x) \sim \mathcal{N}(\mu_1, \sigma_1^2), \quad \mathbb{E}_{p(x)}[x] = \mu_1, \quad \mathbb{V}_{p(x)}[x] = \sigma_1^2$$

$$\mathbb{V}_{p(x)}[x] = \mathbb{E}[x^2] - \mathbb{E}[x]^2$$

$$\begin{aligned}\text{D}_{\text{KL}}(p(x)||q(x)) &= \mathbb{E}_{p(x)}[\log p(x) - \log q(x)] \\ &= \mathbb{E}_{p(x)} \left[ -\log(\sqrt{2\pi\sigma_1}) - \frac{(x - \mu_1)^2}{2\sigma_1^2} + \log(\sqrt{2\pi\sigma_2}) + \frac{(x - \mu_2)^2}{2\sigma_2^2} \right] \\ &= \log \frac{\sigma_2}{\sigma_1} - \frac{\mathbb{E}_{p(x)}[(x - \mu_1)^2]}{2\sigma_1^2} + \frac{\mathbb{E}_{p(x)}[(x - \mu_1)^2 + 2(\mu_1 - \mu_2)x + u_2^2 - \mu_1^2]}{2\sigma_2^2} \\ &= \log \frac{\sigma_2}{\sigma_1} - \frac{\sigma_1^2}{2\sigma_1^2} + \frac{\sigma_1^2 + 2(\mu_1 - \mu_2)\mu_1 + u_2^2 - \mu_1^2}{2\sigma_2^2} \\ &= \log \frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1 - \mu_2)^2}{2\sigma_2^2} - \frac{1}{2}\end{aligned}$$

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# Back to natural gradient descent

- How to solve this **constrained optimization problem**?

$$d^* = \arg \max_d J(\theta + d), \quad s.t. \quad D_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) \leq \epsilon$$

- What tool to use?
  - Turn the constrained optimization problem to an unconstrained one?

# Lagrangian multiplier

- How to solve this constrained optimization problem?

$$d^* = \arg \max_d J(\theta + d), \quad s.t. D_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) \leq \epsilon$$

- Use the **Lagrangian multiplier**  $\lambda$ , turn to the **unconstrained penalized objective**

$$d^* = \arg \max_d J(\theta + d) - \lambda(D_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) - \epsilon)$$

# Taylor expansion for the unconstrained penalized objective

$$d^* = \arg \max_d J(\theta + d) - \lambda(D_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) - \epsilon)$$

- First-order Taylor expansion for the loss

$$J(\theta + d) \approx J(\theta) + \nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d$$

- Second-order Taylor expansion for the KL

$$D_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) \approx \frac{1}{2} d^T \cdot \nabla_{\theta'}^2 D_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} \cdot d$$

# Taylor series/expansion

- A representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point

$$\begin{aligned}f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \\&= f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots\end{aligned}$$

- Examples

$$e^x = ?$$

$$\frac{1}{1-x} = ?$$

# Taylor series/expansion

- A representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point

$$\begin{aligned}f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \\&= f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots\end{aligned}$$

- Examples

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

## Taylor expansion of $J(\theta + d)$

- Let  $\theta' = \theta + d$  is the independent variable
- That is,  $x = \theta'$ ,  $a = \theta$ ,  $x - a = d$
- **What is the Taylor expansion of  $J(\theta + d)$ ?**

$$\begin{aligned}f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \\&= f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots\end{aligned}$$

# Taylor expansion of $J(\theta + d)$

- Let  $\theta' = \theta + d$  is the independent variable
- That is,  $x = \theta'$ ,  $a = \theta$ ,  $x - a = d$
- **What is the Taylor expansion of  $J(\theta + d)$ ?**
- First-order Taylor expansion for the loss:

$$J(\theta + d) \approx J(\theta) + \nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d$$

# Taylor expansion of KL

- Let  $\theta' = \theta + d$  is the independent variable
- That is,  $x = \theta'$ ,  $a = \theta$ ,  $x - a = d$
- **What is the Taylor expansion of  $D_{KL}(\pi_\theta || \pi_{\theta+d})$ ?**

$$\begin{aligned}f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \\&= f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots\end{aligned}$$

# Taylor expansion of KL

- Let  $\theta' = \theta + d$  is the independent variable
- That is,  $x = \theta'$ ,  $a = \theta$ ,  $x - a = d$
- **What is the Taylor expansion of  $D_{KL}(\pi_\theta || \pi_{\theta+d})$ ?**
- Second-order Taylor expansion for  $D_{KL}(\pi_\theta || \pi_{\theta'})$ :

$$\begin{aligned} D_{KL}(\pi_\theta || \pi_{\theta'}) &\approx D_{KL}(\pi_\theta || \pi_\theta) + d^T \nabla_{\theta'} D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} \\ &\quad + \frac{1}{2} d^T \nabla_{\theta'}^2 D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} d \end{aligned}$$

# Taylor expansion of KL

$$D_{KL}(\pi_\theta || \pi_{\theta'}) \approx D_{KL}(\pi_\theta || \pi_\theta) + d^T \nabla_{\theta'} D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} + \frac{1}{2} d^T \nabla_{\theta'}^2 D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} d$$

$$D_{KL}(\pi_\theta || \pi_{\theta'}) = \int \pi_\theta(x) \log \frac{\pi_\theta(x)}{\pi_{\theta'}(x)} dx = \underbrace{\int \pi_\theta(x) \log \pi_\theta(x) dx}_{\text{independent of } \theta'} - \int \pi_\theta(x) \log \pi_{\theta'}(x) dx$$

$$\begin{aligned}\nabla_{\theta'} D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} &= -\nabla_{\theta'} \int \pi_\theta(x) \log \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= -\int \pi_\theta(x) \nabla_{\theta'} \log \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= -\int \frac{\pi_\theta(x)}{\pi_{\theta'}(x)} \nabla_{\theta'} \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= -\nabla_{\theta'} \int \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= 0\end{aligned}$$

# Taylor expansion of KL

$$D_{KL}(\pi_\theta || \pi_{\theta'}) \approx D_{KL}(\pi_\theta || \pi_\theta) + d^T \nabla_{\theta'} D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} + \frac{1}{2} d^T \nabla_{\theta'}^2 D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} d$$

$$\begin{aligned}\nabla_{\theta'}^2 D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} &= - \int \pi_\theta(x) \nabla_{\theta'}^2 \log \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= - \int \pi_\theta(x) \frac{\pi_{\theta'}(x) \nabla_{\theta'}^2 \pi_{\theta'}(x) - \nabla_{\theta'} \pi_{\theta'}(x) \nabla_{\theta'} \pi_{\theta'}(x)^T}{\pi_{\theta'}(x)^2} dx|_{\theta'=\theta} \\ &= - \underbrace{\nabla_{\theta'}^2 \int \pi_{\theta'}(x) dx|_{\theta'=\theta}}_0 + \int \pi_\theta(x) \nabla_{\theta'} \log \pi_{\theta'}(x) \nabla_{\theta'} \log \pi_{\theta'}(x)^T dx|_{\theta'=\theta} \\ &= \mathbb{E}_{x \sim \pi_\theta} [\nabla_{\theta'} \log \pi_{\theta'}(x) \nabla_{\theta'} \log \pi_{\theta'}(x)^T|_{\theta'=\theta}]\end{aligned}$$

# Hessian of KL = Fisher information matrix (FIM)

- **Hessian:** A square matrix of second-order partial derivatives of a scalar-valued function, which describes the local curvature of a function of many variables

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

- **Fisher information:** a way of measuring the amount of information that an observable random variable  $X$  carries about an unknown parameter  $\theta$  upon which the probability of  $X$  depends

$$\mathbf{F}(\theta) = \mathbb{E}_{x \sim \pi_\theta} [\nabla_\theta \log \pi_\theta(x) \nabla_\theta \log \pi_\theta(x)^T]$$

# Hessian of KL = Fisher information matrix (FIM)

- The FIM is exactly the **Hessian matrix** of KL divergence

$$\underbrace{\nabla_{\theta'}^2 D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta}}_{\text{Hessian of KL}} = \underbrace{\mathbb{E}_{x \sim \pi_\theta} [\nabla_{\theta'} \log \pi_{\theta'}(x) \nabla_{\theta'} \log \pi_{\theta'}(x)^T |_{\theta'=\theta}]}_{\text{FIM}}$$

$$\begin{aligned} D_{KL}(\pi_\theta || \pi_{\theta'}) &\approx \underbrace{D_{KL}(\pi_\theta || \pi_\theta)}_0 + d^T \underbrace{\nabla_{\theta'} D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta}}_0 + \frac{1}{2} d^T \underbrace{\nabla_{\theta'}^2 D_{KL}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta}}_{\mathbf{F}(\theta)} d \\ &= \frac{1}{2} d^T \mathbf{F}(\theta) d \\ &= \frac{1}{2} (\theta' - \theta)^T \mathbf{F}(\theta) (\theta' - \theta) \end{aligned}$$

## Back to Taylor expansion of KL

$$D_{KL}(\pi_\theta || \pi_\theta + d) \approx \frac{1}{2} d^T \mathbf{F}(\theta) d$$

- KL divergence is roughly analogous to a distance measure between distributions
- Fisher information serves as a local distance metric between distributions: how much you change the distribution if you move the parameters a little bit in a given direction

# Back to solving the KL constrained problem

$$\begin{aligned} d^* &= \arg \max_d J(\theta + d) - \lambda(\mathbf{D}_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) - \epsilon) \\ &\approx \arg \max_d J(\theta) + \nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d - \lambda\left(\frac{1}{2}d^T \nabla_{\theta'}^2 \mathbf{D}_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} d - \epsilon\right) \\ &= \arg \max_d \nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d - \frac{1}{2}\lambda d^T \mathbf{F}(\theta) d \end{aligned}$$

- Set the gradient to 0:

$$\begin{aligned} 0 &= \frac{\partial}{\partial d} \left( \nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d - \frac{1}{2}\lambda d^T \mathbf{F}(\theta) d \right) \\ &= \nabla_{\theta'} J(\theta')|_{\theta'=\theta} - \lambda \mathbf{F}(\theta) d \end{aligned}$$

$$d^* = \frac{1}{\lambda} \mathbf{F}^{-1}(\theta) \nabla_{\theta'} J(\theta')|_{\theta'=\theta} = \frac{1}{\lambda} \mathbf{F}^{-1}(\theta) \nabla_{\theta} J(\theta)$$

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# Natural gradient descent

- The natural gradient:

$$\tilde{\nabla}_{\theta} J(\theta) = \mathbf{F}^{-1}(\theta) \nabla_{\theta} J(\theta)$$

- Natural gradient descent:

$$\theta' = \theta + \alpha \cdot \mathbf{F}^{-1}(\theta) \hat{g}$$

- How to determine the learning rate  $\alpha$ :

$$D_{KL}(\pi_{\theta} || \pi_{\theta} + d) \approx \frac{1}{2} (\theta' - \theta)^T \mathbf{F}(\theta) (\theta' - \theta) \leq \epsilon$$

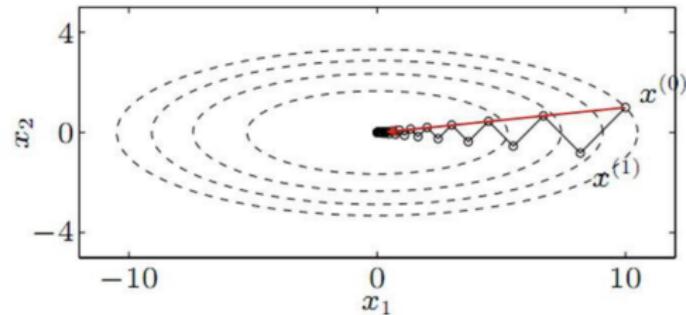
$$\frac{1}{2} (\alpha \hat{g})^T \mathbf{F}(\alpha \hat{g}) = \epsilon$$

$$\boxed{\alpha = \sqrt{\frac{2\epsilon}{\hat{g}^T \mathbf{F} \hat{g}}}}$$

# Geometric interpretation of natural policy gradient

- Find **the steepest direction** for parameter updating

Essentially the same problem as this:



# Natural gradient descent → Natural policy gradient (NPG)

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**Algorithm 1** Natural Policy Gradient

---

Input: initial policy parameters  $\theta_0$

**for**  $k = 0, 1, 2, \dots$  **do**

    Collect set of trajectories  $\mathcal{D}_k$  on policy  $\pi_k = \pi(\theta_k)$

    Estimate advantages  $\hat{A}_t^{\pi_k}$  using any advantage estimation algorithm

    Form sample estimates for

- policy gradient  $\hat{g}_k$  (using advantage estimates)
- and KL-divergence Hessian / Fisher Information Matrix  $\hat{H}_k$

    Compute Natural Policy Gradient update:

$$\theta_{k+1} = \theta_k + \sqrt{\frac{2\epsilon}{\hat{g}_k^T \hat{H}_k \hat{g}_k}} \hat{H}_k^{-1} \hat{g}_k$$

**end for**

---

- Originated from **natural gradient descent** in supervised learning
- Very **expensive** to compute the **inverse of Hessian matrix** for a large number of parameters

# Review of natural policy gradient

- The gradient
  - Constrain parameter update in parameter space (using Euclidean distance)
- The natural gradient
  - Constrain parameter update in distribution space (using KL divergence)
  - The meaning of “natural”: the distance metric is **invariant** to function parameterization
- Fisher information matrix (FIM)
  - Second-order information: a local distance metric between distributions
  - The FIM is exactly the Hessian matrix of KL divergence
  - Expensive to compute for a large number of parameters

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# Trust region policy optimization (TRPO)

- John Schulman, Sergey Levine, Philipp Moritz, Michael Jordan, and Pieter Abbeel, **Trust Region Policy Optimization**, ICML, 2015.
- The family of **statistical learning**
  - John Schulman → Pieter Abbeel → Andrew Ng → Michael Jordan

## John Schulman's Homepage

I'm a research scientist at [OpenAI](#). I co-lead the reinforcement learning (RL) team, where we work on (1) designing better RL algorithms that enable agents to learn much faster in novel situations; (2) designing better training environments that teach agents transferrable skills. We mostly use [games](#) as a [testbed](#).



Previously, I received my [PhD](#) in Computer Science from UC Berkeley, where I had the good fortune of being advised by [Pieter Abbeel](#). Prior to my recent work in RL, I spent some time working on robotics, enabling robots to [tie knots](#) and [stitches](#) and plan movement using [trajectory optimization](#).

- [Publications](#)
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# Trust region policy optimization (TRPO)



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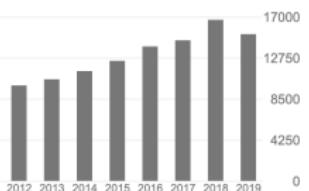
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TITLE	CITED BY	YEAR
<a href="#">Latent dirichlet allocation</a> DM Blei, AY Ng, MI Jordan <i>Journal of machine Learning research</i> 3 (Jan), 993-1022	29247	2003
<a href="#">On spectral clustering: Analysis and an algorithm</a> AY Ng, MI Jordan, Y Weiss <i>Advances in neural information processing systems</i> , 849-856	7927	2002
<a href="#">Adaptive mixtures of local experts.</a> RA Jacobs, MI Jordan, SJ Nowlan, GE Hinton <i>Neural computation</i> 3 (1), 79-87	4089	1991

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# TRPO - The KL constrained problem

$$\begin{aligned} & \underset{\theta}{\text{maximize}} && \hat{\mathbb{E}}_t \left[ \frac{\pi_{\theta}(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right] \\ & \text{subject to} && \hat{\mathbb{E}}_t [\text{KL}[\pi_{\theta_{\text{old}}}(\cdot | s_t), \pi_{\theta}(\cdot | s_t)]] \leq \delta. \end{aligned}$$

- ▶ Also worth considering using a penalty instead of a constraint

$$\underset{\theta}{\text{maximize}} \quad \hat{\mathbb{E}}_t \left[ \frac{\pi_{\theta}(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right] - \beta \hat{\mathbb{E}}_t [\text{KL}[\pi_{\theta_{\text{old}}}(\cdot | s_t), \pi_{\theta}(\cdot | s_t)]]$$

Again the KL penalized problem!

---

**Algorithm 3** Trust Region Policy Optimization

---

Input: initial policy parameters  $\theta_0$

**for**  $k = 0, 1, 2, \dots$  **do**

    Collect set of trajectories  $\mathcal{D}_k$  on policy  $\pi_k = \pi(\theta_k)$

    Estimate advantages  $\hat{A}_t^{\pi_k}$  using any advantage estimation algorithm

    Form sample estimates for

- policy gradient  $\hat{g}_k$  (using advantage estimates)
- and KL-divergence Hessian-vector product function  $f(v) = \hat{H}_k v$

    Use CG with  $n_{cg}$  iterations to obtain  $x_k \approx \hat{H}_k^{-1} \hat{g}_k$

    Estimate proposed step  $\Delta_k \approx \sqrt{\frac{2\delta}{x_k^T \hat{H}_k x_k}} x_k$

    Perform backtracking line search with exponential decay to obtain final update

$$\theta_{k+1} = \theta_k + \alpha^j \Delta_k$$

**end for**

---

# Line search with monotonic policy improvement

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**Algorithm 2** Line Search for TRPO

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Compute proposed policy step  $\Delta_k = \sqrt{\frac{2\delta}{\hat{g}_k^T \hat{H}_k^{-1} \hat{g}_k}} \hat{H}_k^{-1} \hat{g}_k$

**for**  $j = 0, 1, 2, \dots, L$  **do**

    Compute proposed update  $\theta = \theta_k + \alpha^j \Delta_k$

**if**  $\mathcal{L}_{\theta_k}(\theta) \geq 0$  and  $\bar{D}_{KL}(\theta || \theta_k) \leq \delta$  **then**

        accept the update and set  $\theta_{k+1} = \theta_k + \alpha^j \Delta_k$

        break

**end if**

**end for**

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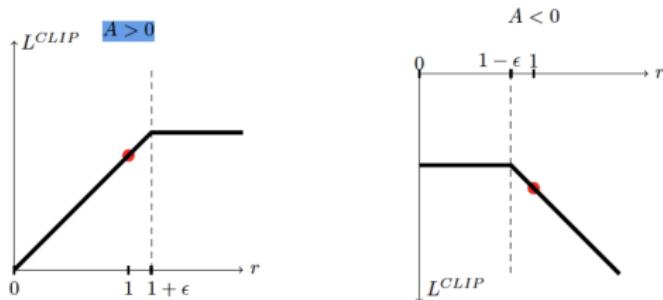
# Proximal policy optimization (PPO): Clipped objective

- Recall the surrogate objective

$$L^{IS}(\theta) = \hat{\mathbb{E}}_t \left[ \frac{\pi_\theta(a_t | s_t)}{\pi_{\theta_{old}}(a_t | s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t [r_t(\theta) \hat{A}_t].$$

- Form a lower bound via clipped importance ratios

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t \left[ \min(r_t(\theta) \hat{A}_t, \text{clip}(r_t(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_t) \right]$$



- Prevent large changes of policies, constrain the policy update
- Achieve similar performance to TRPO without second-order information (no Fisher matrix!)

# Proximal policy optimization (PPO): Adaptive KL penalty

Input: initial policy parameters  $\theta_0$ , initial KL penalty  $\beta_0$ , target KL-divergence  $\delta$

**for**  $k = 0, 1, 2, \dots$  **do**

    Collect set of partial trajectories  $\mathcal{D}_k$  on policy  $\pi_k = \pi(\theta_k)$

    Estimate advantages  $\hat{A}_t^{\pi_k}$  using any advantage estimation algorithm

    Compute policy update

$$\theta_{k+1} = \arg \max_{\theta} \mathcal{L}_{\theta_k}(\theta) - \beta_k \bar{D}_{KL}(\theta || \theta_k)$$

        by taking  $K$  steps of minibatch SGD (via Adam)

**if**  $\bar{D}_{KL}(\theta_{k+1} || \theta_k) \geq 1.5\delta$  **then**

$$\beta_{k+1} = 2\beta_k$$

**else if**  $\bar{D}_{KL}(\theta_{k+1} || \theta_k) \leq \delta/1.5$  **then**

$$\beta_{k+1} = \beta_k/2$$

**end if**

**end for**

Don't use second order approximation for KL which is expensive, use standard gradient descent

- Penalty coefficient  $\beta$  changes between iterations to approximately enforce KL-divergence constraint
- Achieve similar performance to TRPO without second-order information (no Fisher matrix!)

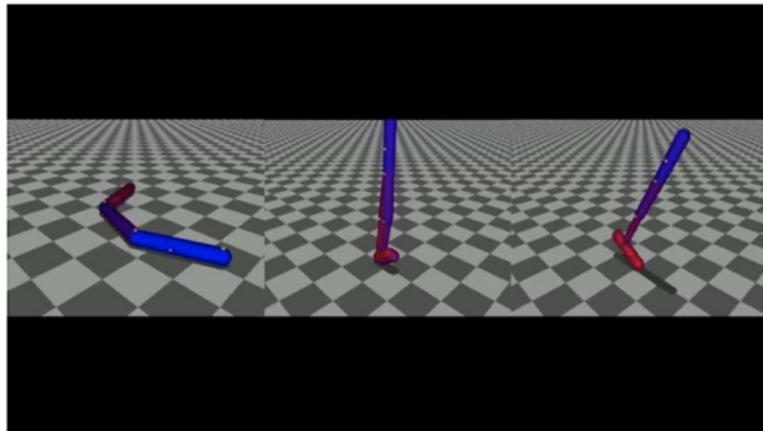
# Review

- TRPO: again the KL penalty problem
  - Natural policy gradient + Monotonic policy improvement + Line search
  - Still need to compute the natural gradient with Hessian matrix
- PPO
  - Achieve TRPO-like performance without second-order computation
  - Clipped objective, adaptive KL penalty

$$\begin{aligned} & \underset{\theta}{\text{maximize}} \quad \hat{\mathbb{E}}_t \left[ \frac{\pi_{\theta}(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right] \\ & \text{subject to} \quad \hat{\mathbb{E}}_t [\text{KL}[\pi_{\theta_{\text{old}}}(\cdot | s_t), \pi_{\theta}(\cdot | s_t)]] \leq \delta. \end{aligned}$$

## Example: trust region policy optimization

- Natural gradient with automatic step adjustment
- Discrete and continuous actions
- Code available (see Duan et al. '16)



# Learning objectives of this lecture

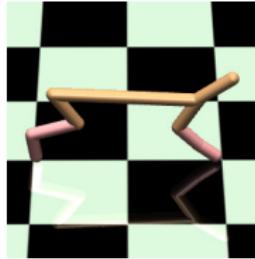
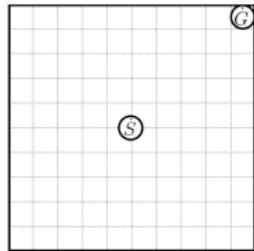
- You should be able to...
  - Know how to derive the natural policy gradient
  - Be aware of several advanced algorithms, e.g., TRPO, PPO
  - Enhance your mathematical skills

# References

- Lecture 9 of CS285 at UC Berkeley, *Deep Reinforcement Learning, Decision Making, and Control*
  - <http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-9.pdf>
- Classic papers
  - Peters & Schaal (2008). **Reinforcement learning of motor skills with policy gradients**: very accessible overview of optimal baselines and natural gradient.
- DRL policy gradient papers
  - Schulman, L., Moritz, Jordan, Abbeel (2015). **Trust region policy optimization**: deep RL with natural policy gradient and adaptive step size.
  - Schulman, Wolski , Dhariwal , Radford, Klimov (2017). **Proximal policy optimization algorithms**: deep RL with importance sampled policy gradient.
  - Y. Duan, et al., **Benchmarking Deep Reinforcement Learning for Continuous Control**, ICML, 2016.

# Homework 3

- Study the policy gradient algorithm in detail
- Implement the series of policy gradient algorithms on problems 1 & 2
  - Problem 1: the point maze navigation, continuous state-action space
  - Problem 2: the MuJoCo HalfCheetah, make the robot run forward
  - Must use vanilla policy gradient and natural policy gradient, encourage to use TRPO and PPO
- Write a report introducing the algorithms and your experimentation
  - Explanations, steps, evaluation results, visualizations...
  - Submit the code and the report to [yuanyang@smail.nju.edu.cn](mailto:yuanyang@smail.nju.edu.cn)



# THE END