Lecture 10: Model-Based Reinforcement Learning

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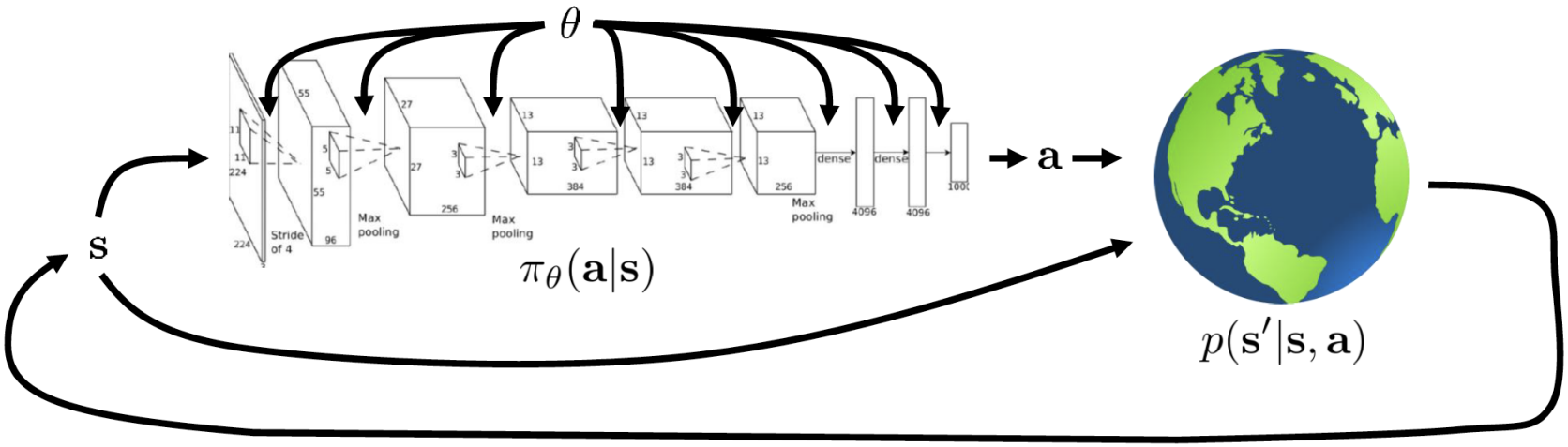
Dec. 3rd, 2020

Table of Contents

Recall: The RL objective



$$\underbrace{p_\theta(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)}_{\pi_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\theta^\star = \arg\max_\theta E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

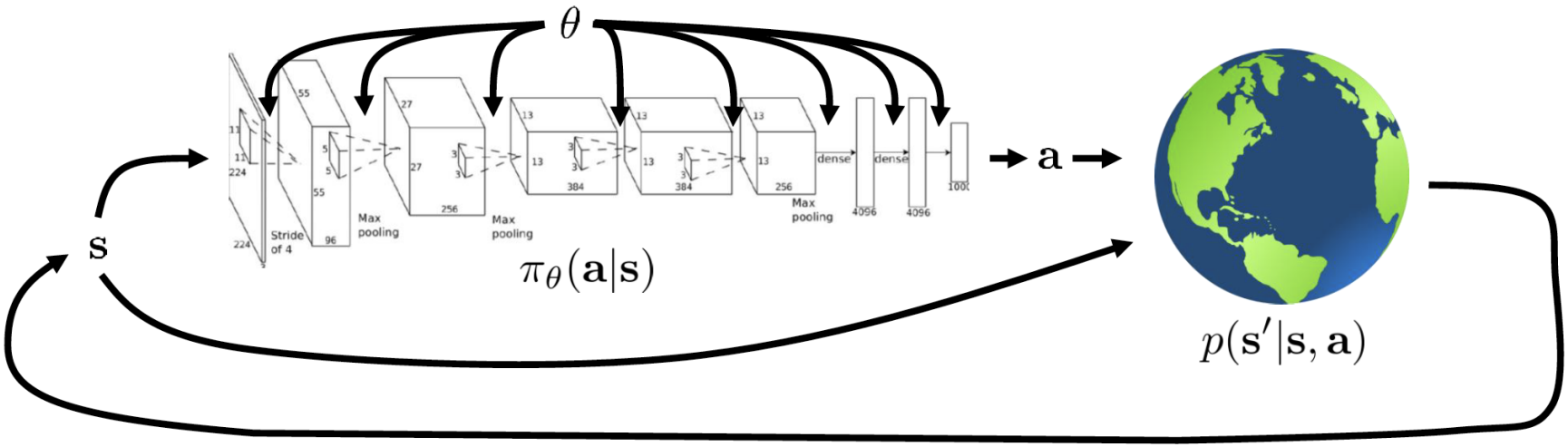
Dynamics of the MDP $p : \mathcal{S} \times \mathcal{R} \times \mathcal{S} \times \mathcal{A} \to [0, 1]$

$$p(s', r|s, a) = Pr\{S_t = s', R_t = r|S_{t-1} = s, A_{t-1} = a\}$$

$$\sum_{s' \in \mathcal{S}} \sum_{r \in \mathcal{R}} p(s', r|s, a) = 1, \quad \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$$

- The probabilities given by p completely characterize the environment's dynamics

Recall: Model-free RL



$$\underbrace{p_\theta(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)}_{\pi_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}$$

assume this is unknown

don't even attempt to learn it

$$\theta^\star = \arg\max_\theta E_{\tau \sim p_\theta(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

What if we knew the transition dynamics?

- Often we do know the dynamics
  - Games (e.g., Atari games, chess, Go)
  - Easily modeled systems (e.g., navigating a car)
  - simulated environments (e.g., simulated robots, video games)
- Often we learn the dynamics
  - System identification - fit unknown parameters of a known model
  - Learning - fit a general-purpose model to observed transition data

Does knowing the dynamics make things easier?

Often, yes!

How can we make decisions if we know the dynamics?

- How can we choose actions under perfect knowledge of the system dynamics?
- Optimal control, trajectory optimization, planning

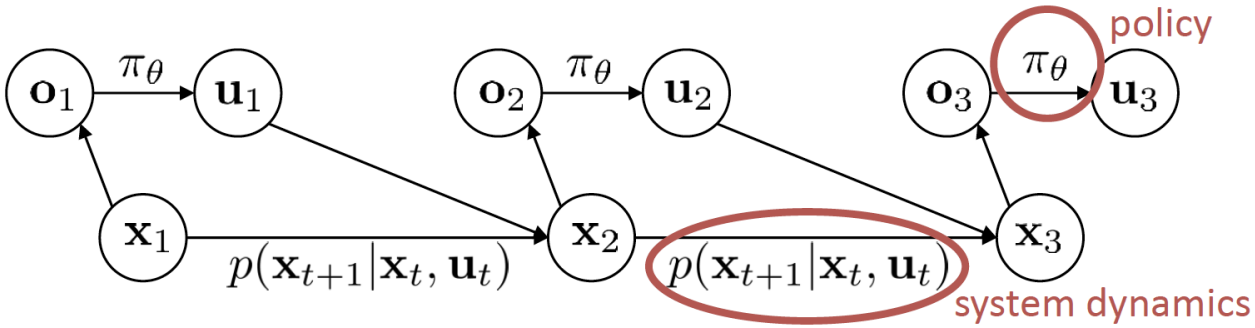
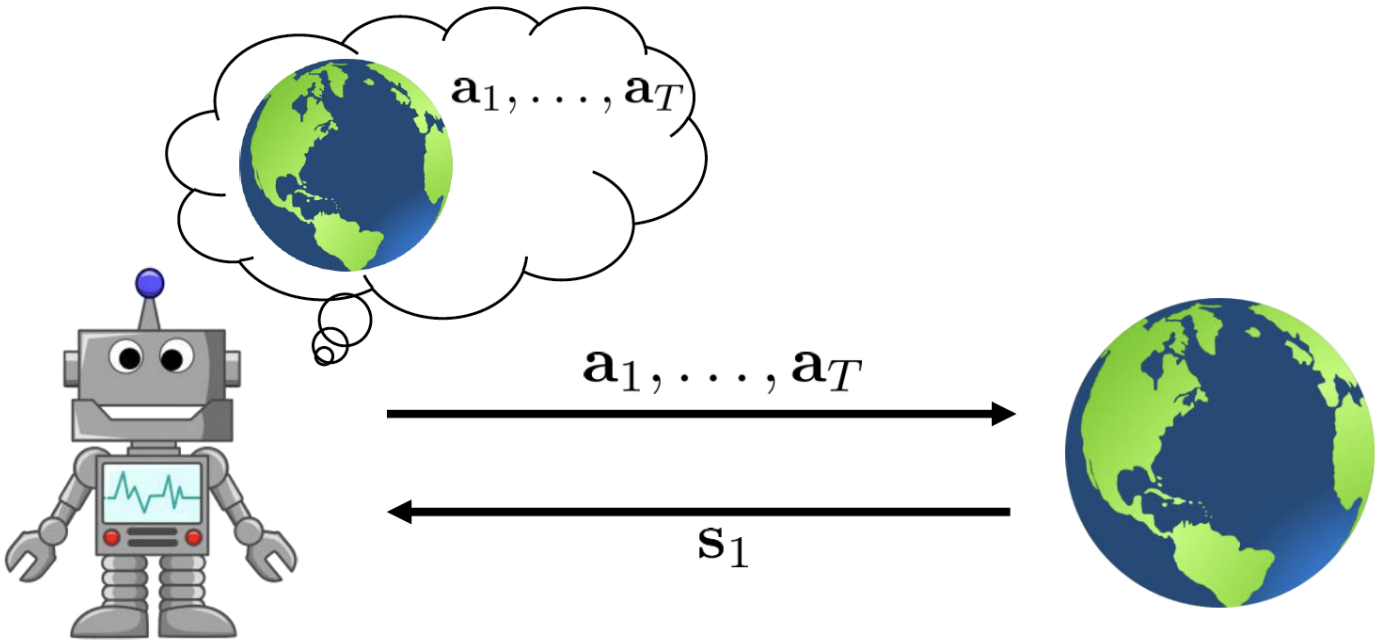


Table of Contents

The deterministic case



$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t) \text{ s.t. } \mathbf{a}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$$

Method 1: Stochastic optimization

abstract away optimal control/planning:

$$\mathbf{a}_1, \dots, \mathbf{a}_T = \arg\max_{\mathbf{a}_1,\dots,\mathbf{a}_T} \underbrace{J(\mathbf{a}_1, \dots, \mathbf{a}_T)}_{\text{don't care what this is}} \qquad \mathbf{A} = \arg\max_{\mathbf{A}} J(\mathbf{A})$$

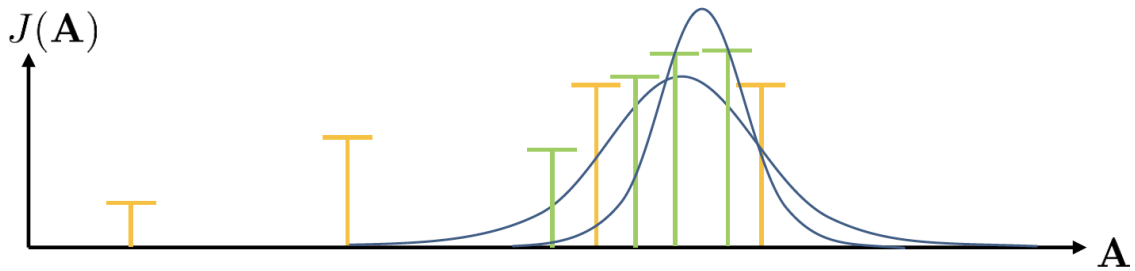
simplest method: guess & check "random shooting method"

1. pick $\mathbf{A}_1, \dots, \mathbf{A}_N$ from some distribution (e.g., uniform)
2. choose $\mathbf{A}_i$ based on $\arg\max_i J(\mathbf{A}_i)$

Method 2: Cross-entropy method (CEM)

1. pick $\mathbf{A}_1, \ldots, \mathbf{A}_N$ from some distribution (e.g., uniform)
2. choose $\mathbf{A}_i$ based on $\arg\max_i J(\mathbf{A}_i)$

can we do better?



typically use Gaussian distribution

see also: CMA-ES (sort of like CEM with momentum)

cross-entropy method with continuous-valued inputs:

1. sample $\mathbf{A}_1, \ldots, \mathbf{A}_N$ from $p(\mathbf{A})$
2. evaluate $J(\mathbf{A}_1), \ldots, J(\mathbf{A}_N)$
3. pick the *elites* $\mathbf{A}_{i_1}, \ldots, \mathbf{A}_{i_M}$ with the highest value, where $M < N$
4. refit $p(\mathbf{A})$ to the elites $\mathbf{A}_{i_1}, \ldots, \mathbf{A}_{i_M}$

Analysis of Methods 1 & 2

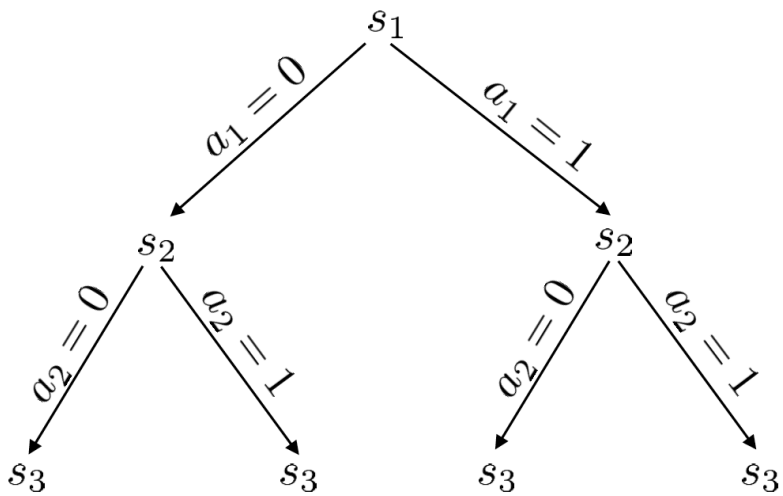
- What's the upside?
  - Very fast if parallelized
  - Extremely simple
- What's the problem?
  - Very harsh dimensionality limit
  - Only open-loop planning

Table of Contents

Discrete case: Monte Carlo tree search (MCTS)

s_t a_t

discrete planning as a search problem



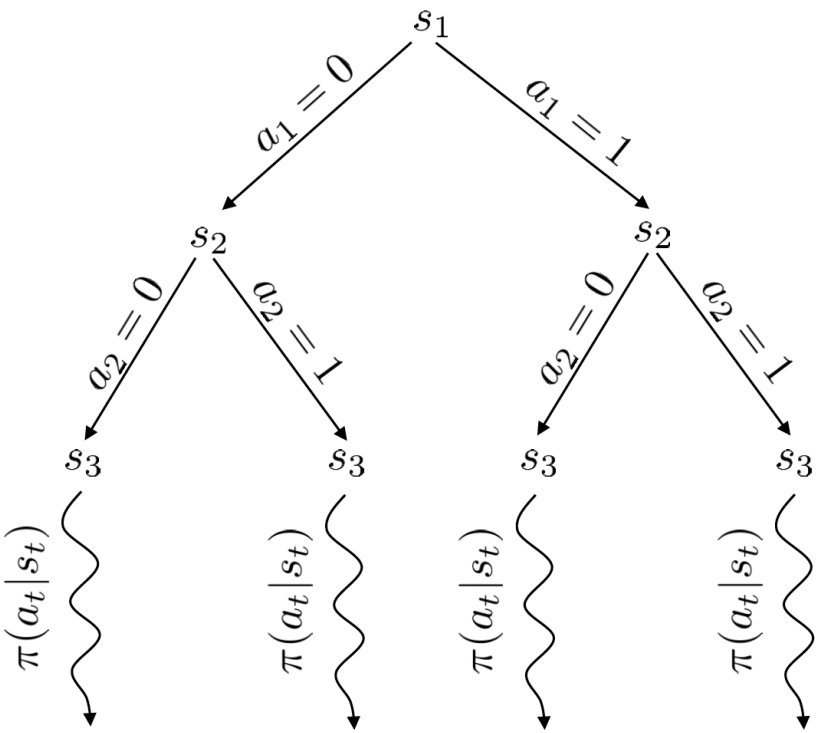
Discrete case: Monte Carlo tree search (MCTS)

how to approximate value without full tree?

s_t

a_t

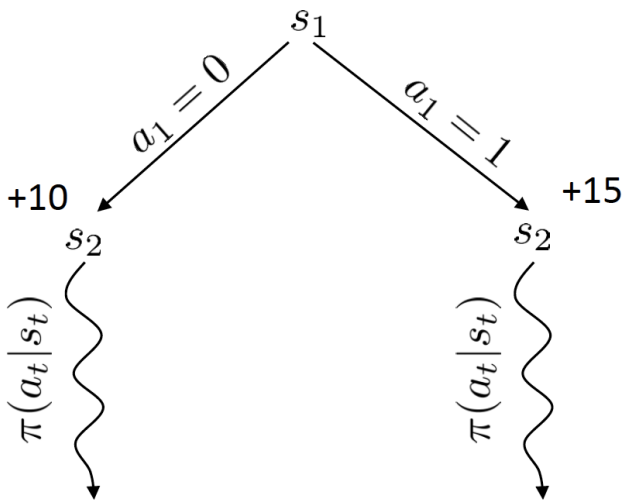
e.g., random policy



Discrete case: Monte Carlo tree search (MCTS)

can't search all paths – where to search first?

s_t a_t



intuition: choose nodes with best reward, but also prefer rarely visited nodes

Discrete case: Monte Carlo tree search (MCTS)

generic MCTS sketch

1. find a leaf s_l using TreePolicy(s_1)
2. evaluate the leaf using DefaultPolicy(s_l)
3. update all values in tree between s_1 and s_l

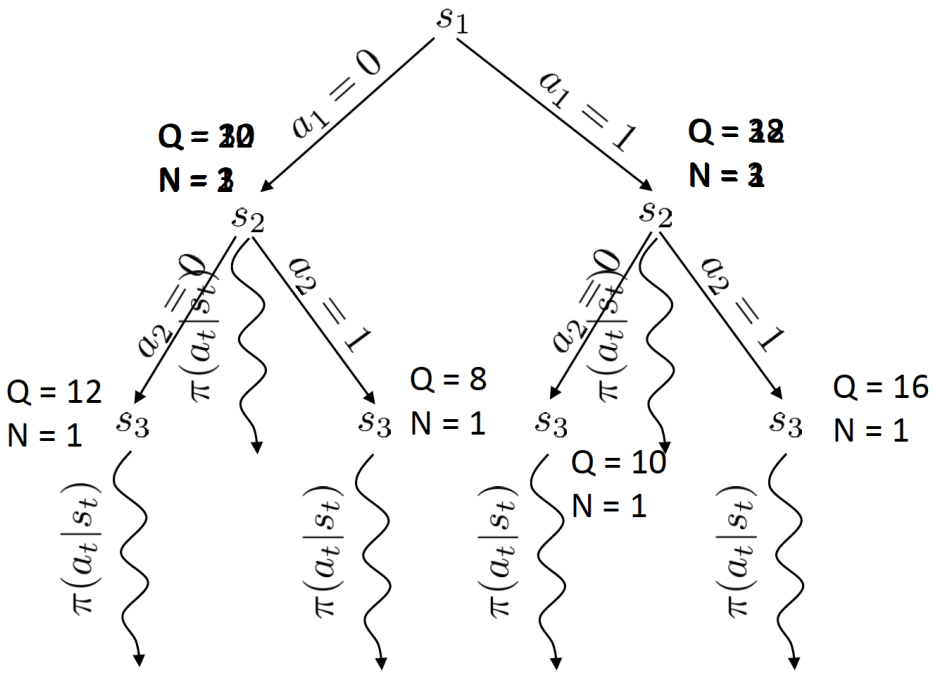
take best action from s_1

UCT TreePolicy(s_t)

if s_t not fully expanded, choose new a_t

else choose child with best Score(s_{t+1})

$$\text{Score}(s_t) = \frac{Q(s_t)}{N(s_t)} + 2C\sqrt{\frac{2\ln N(s_{t-1})}{N(s_t)}}$$



Additional reading

- Browne, Powley, Whitehouse, Lucas, Cowling, Rohlfshagen, Tavener, Perez, Samothrakis, Colton. (2012). **A Survey of Monte Carlo Tree Search Methods**.
  - Survey of MCTS methods and basic summary

Table of Contents

Can we use derivatives?

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \text{ s.t. } \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\ldots)\ldots), \mathbf{u}_T)$$

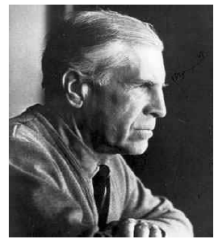
usual story: differentiate via backpropagation and optimize!

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

in practice, it really helps to use a 2^{nd} order method!

$\mathbf{s}_t$ – state
$\mathbf{a}_t$ – action

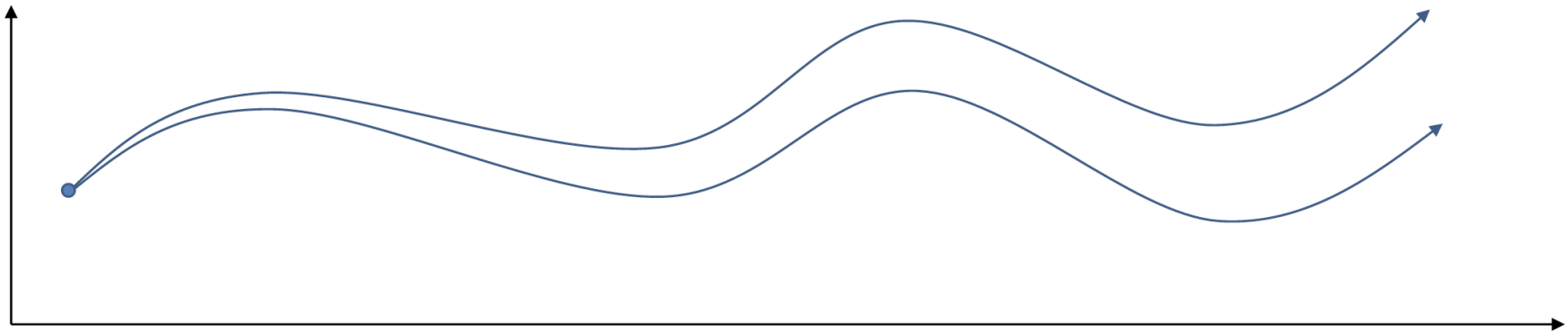
$\mathbf{x}_t$ – state
$\mathbf{u}_t$ – action



Shooting methods vs. collocation

shooting method: optimize over actions only

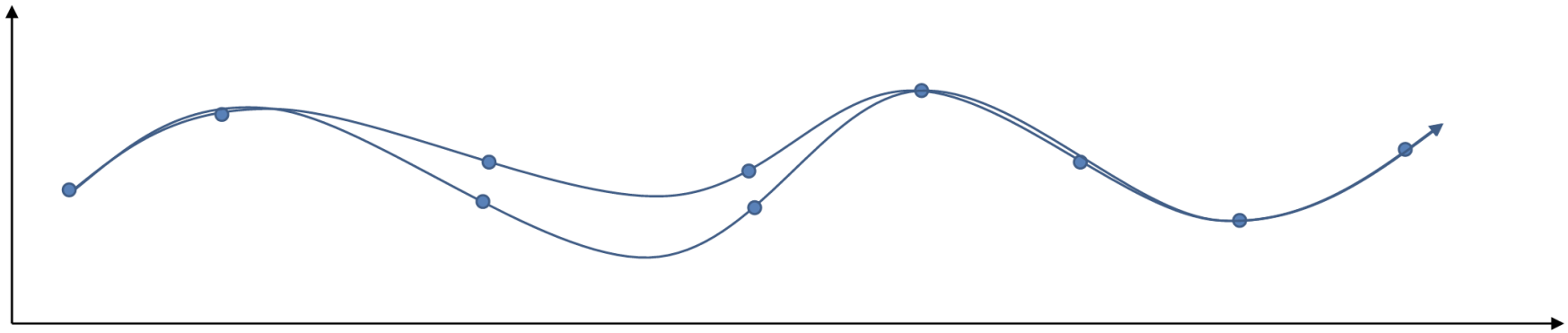
$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} c(\mathbf{x}_1,\mathbf{u}_1) + c(f(\mathbf{x}_1,\mathbf{u}_1),\mathbf{u}_2) + \cdots + c(f(f(\dots)\dots),\mathbf{u}_T)$$



Shooting methods vs. collocation

collocation method: optimize over actions and states, with constraints

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T,\mathbf{x}_1,\ldots,\mathbf{x}_T} \sum_{t=1}^{T} c(\mathbf{x}_t,\mathbf{u}_t) \text{ s.t. } \mathbf{x}_t = f(\mathbf{x}_{t-1},\mathbf{u}_{t-1})$$



Linear case: Linear-quadratic regulator (LQR)

- The theory of optimal control is concerned with operating a dynamic system at minimum cost
- The LQ problem: the system dynamics are described by a set of linear differential equations, the cost is described by a quadratic function
- LQR: a feedback controller, one of the most fundamental problems in control theory

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} c(\mathbf{x}_1,\mathbf{u}_1) + c(f(\mathbf{x}_1,\mathbf{u}_1),\mathbf{u}_2) + \cdots + c(f(f(\dots)\dots),\mathbf{u}_T)$$

$$f(\mathbf{x}_t,\mathbf{u}_t) = \mathbf{F}_t \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix} + \mathbf{f}_t$$

linear

$$c(\mathbf{x}_t,\mathbf{u}_t) = \frac{1}{2} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{C}_t \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix} + \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{c}_t$$

quadratic

The LQR algorithm

Backward recursion

for $t = T$ to 1:

$$\mathbf{Q}_t = \mathbf{C}_t + \mathbf{F}_t^T \mathbf{V}_{t+1} \mathbf{F}_t$$

$$\mathbf{q}_t = \mathbf{c}_t + \mathbf{F}_t^T \mathbf{V}_{t+1} \mathbf{f}_t + \mathbf{F}_t^T \mathbf{v}_{t+1}$$

$$Q(\mathbf{x}_t, \mathbf{u}_t) = \text{const} + \frac{1}{2} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{Q}_t \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix} + \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{q}_t$$

$$\mathbf{u}_t \leftarrow \arg\min_{\mathbf{u}_t} Q(\mathbf{x}_t, \mathbf{u}_t) = \mathbf{K}_t \mathbf{x}_t + \mathbf{k}_t$$

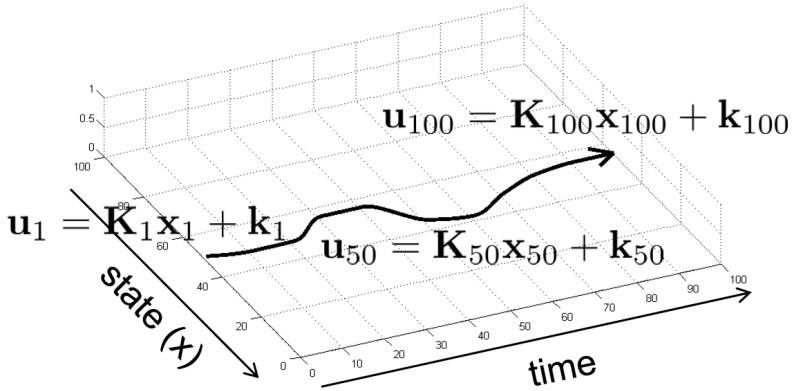
$$\mathbf{K}_t = -\mathbf{Q}_{\mathbf{u}_t, \mathbf{u}_t}^{-1} \mathbf{Q}_{\mathbf{u}_t, \mathbf{x}_t}$$

$$\mathbf{k}_t = -\mathbf{Q}_{\mathbf{u}_t, \mathbf{u}_t}^{-1} \mathbf{q}_{\mathbf{u}_t}$$

$$\mathbf{V}_t = \mathbf{Q}_{\mathbf{x}_t, \mathbf{x}_t} + \mathbf{Q}_{\mathbf{x}_t, \mathbf{u}_t} \mathbf{K}_t + \mathbf{K}_t^T \mathbf{Q}_{\mathbf{u}_t, \mathbf{x}_t} + \mathbf{K}_t^T \mathbf{Q}_{\mathbf{u}_t, \mathbf{u}_t} \mathbf{K}_t$$

$$\mathbf{v}_t = \mathbf{q}_{\mathbf{x}_t} + \mathbf{Q}_{\mathbf{x}_t, \mathbf{u}_t} \mathbf{k}_t + \mathbf{K}_t^T \mathbf{Q}_{\mathbf{u}_t} + \mathbf{K}_t^T \mathbf{Q}_{\mathbf{u}_t, \mathbf{u}_t} \mathbf{k}_t$$

$$V(\mathbf{x}_t) = \text{const} + \frac{1}{2} \mathbf{x}_t^T \mathbf{V}_t \mathbf{x}_t + \mathbf{x}_t^T \mathbf{v}_t$$



we know $\mathbf{x}_1$!

Forward recursion

for $t = 1$ to T:

$$\mathbf{u}_t = \mathbf{K}_t \mathbf{x}_t + \mathbf{k}_t$$

$$\mathbf{x}_{t+1} = f(\mathbf{x}_t, \mathbf{u}_t)$$

The LQR algorithm

Backward recursion

for $t = T$ to 1:

total cost from now until end if we take $\mathbf{u}_t$ from state $\mathbf{x}_t$

$$\mathbf{Q}_t = \mathbf{C}_t + \mathbf{F}_t^T \mathbf{V}_{t+1} \mathbf{F}_t$$

$$\mathbf{q}_t = \mathbf{c}_t + \mathbf{F}_t^T \mathbf{V}_{t+1} \mathbf{f}_t + \mathbf{F}_t^T \mathbf{v}_{t+1}$$

$$Q(\mathbf{x}_t, \mathbf{u}_t) = \text{const} + \frac{1}{2} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{Q}_t \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix} + \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}^T \mathbf{q}_t$$

$$\mathbf{u}_t \leftarrow \arg\min_{\mathbf{u}_t} Q(\mathbf{x}_t, \mathbf{u}_t) = \mathbf{K}_t \mathbf{x}_t + \mathbf{k}_t$$

$$\mathbf{K}_t = -\mathbf{Q}_{\mathbf{u}_t, \mathbf{u}_t}^{-1} \mathbf{Q}_{\mathbf{u}_t, \mathbf{x}_t}$$

$$\mathbf{k}_t = -\mathbf{Q}_{\mathbf{u}_t, \mathbf{u}_t}^{-1} \mathbf{q}_{\mathbf{u}_t}$$

$$\mathbf{V}_t = \mathbf{Q}_{\mathbf{x}_t, \mathbf{x}_t} + \mathbf{Q}_{\mathbf{x}_t, \mathbf{u}_t} \mathbf{K}_t + \mathbf{K}_t^T \mathbf{Q}_{\mathbf{u}_t, \mathbf{x}_t} + \mathbf{K}_t^T \mathbf{Q}_{\mathbf{u}_t, \mathbf{u}_t} \mathbf{K}_t$$

$$\mathbf{v}_t = \mathbf{q}_{\mathbf{x}_t} + \mathbf{Q}_{\mathbf{x}_t, \mathbf{u}_t} \mathbf{k}_t + \mathbf{K}_t^T \mathbf{Q}_{\mathbf{u}_t} + \mathbf{K}_t^T \mathbf{Q}_{\mathbf{u}_t, \mathbf{u}_t} \mathbf{k}_t$$

$$V(\mathbf{x}_t) = \text{const} + \frac{1}{2} \mathbf{x}_t^T \mathbf{V}_t \mathbf{x}_t + \mathbf{x}_t^T \mathbf{v}_t$$

total cost from now until end from state $\mathbf{x}_t$

$V(\mathbf{x}_t) = \min_{\mathbf{u}_t} Q(\mathbf{x}_t, \mathbf{u}_t)$

Table of Contents

Model-based RL

- Often the system's dynamics are **unknown**
- Model-based RL: learn the transition dynamics, then figure out how to choose actions

Why learn the model?

- If we knew $f(s_t, a_t) = s_{t+1}$, we could use tools from "Optimal Control and Planning"
  - Or $p(s_{t+1}|s_t, a_t)$ in the stochastic case
- So let's learn $f(s_t, a_t)$ from data, and then **plan** through it

Model-based RL version 0.5

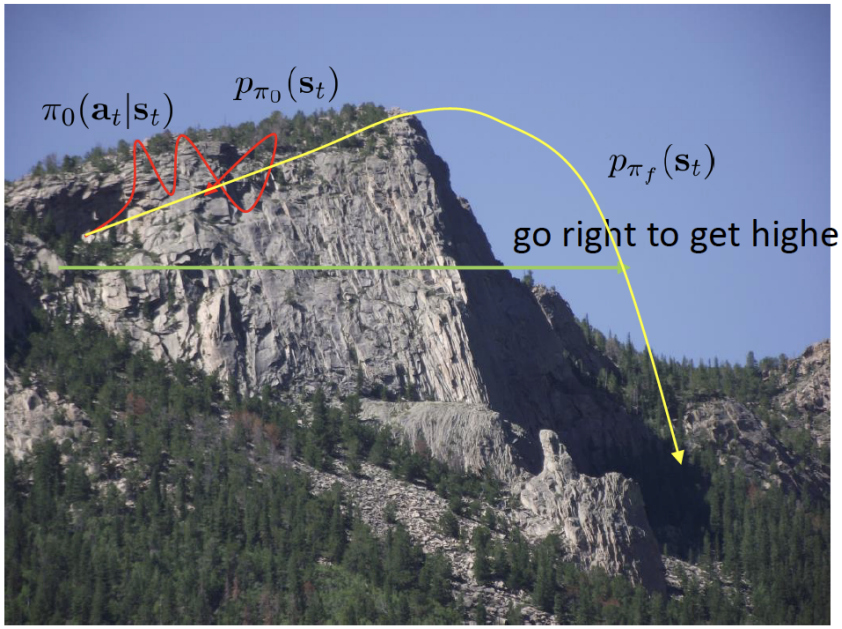
1. run base policy $\pi_0(a_t|s_t)$, e.g., random policy, to collect $\mathcal{D} = \{(s, a, s')_i\}$
2. learn dynamics model $f(s, a)$ to minimize $\sum_i ||f(s_i, a_i) - s'_i||^2$
3. plan through $f(s, a)$ to choose actions

Does it work? Yes!

- Essentially how system identification works in classical robotics
- Some care should be taken to design a good base policy
- Particularly effective if we can hand engineer a dynamics representation using our knowledge of physics, and fit just a few parameters

Does it work? No!

Does it work?



No!

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions

$$p_{\pi_f}(\mathbf{s}_t) \neq p_{\pi_0}(\mathbf{s}_t)$$

- Distribution mismatch problem becomes exacerbated as we use more expressive model classes

Model-based RL version 1.0

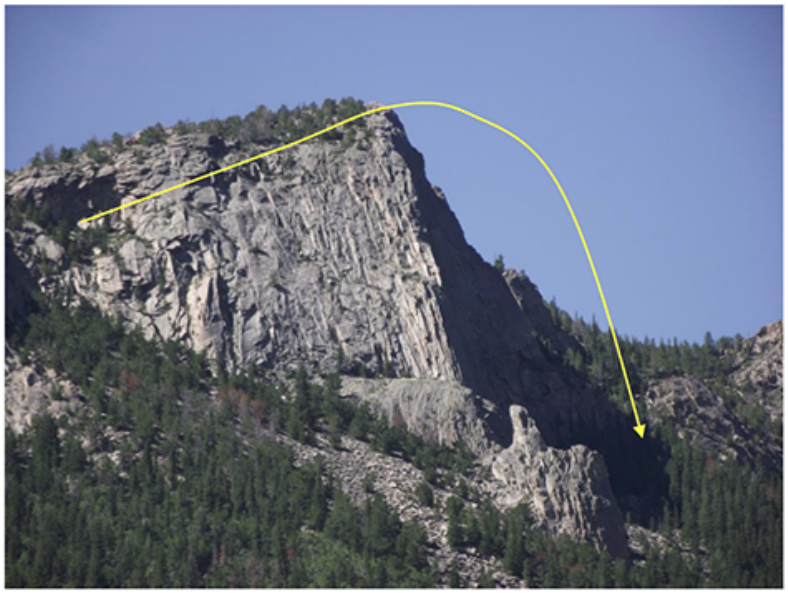
can we make $p_{\pi_0}(\mathbf{s}_t) = p_{\pi_f}(\mathbf{s}_t)$?

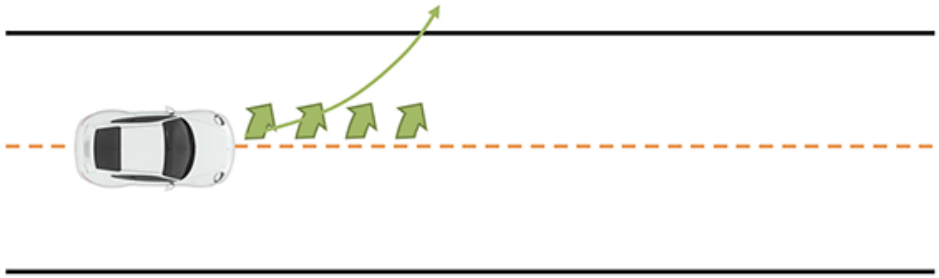
where have we seen that before? need to collect data from $p_{\pi_f}(\mathbf{s}_t)$

model-based reinforcement learning version 1.0:

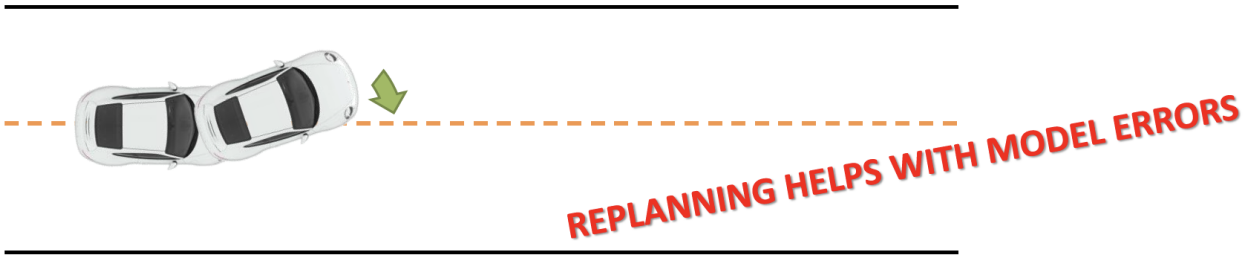
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute those actions and add the resulting data $\{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_j\}$ to $\mathcal{D}$

What if we make a mistake? - Error cumulation





Model-based RL version 1.5

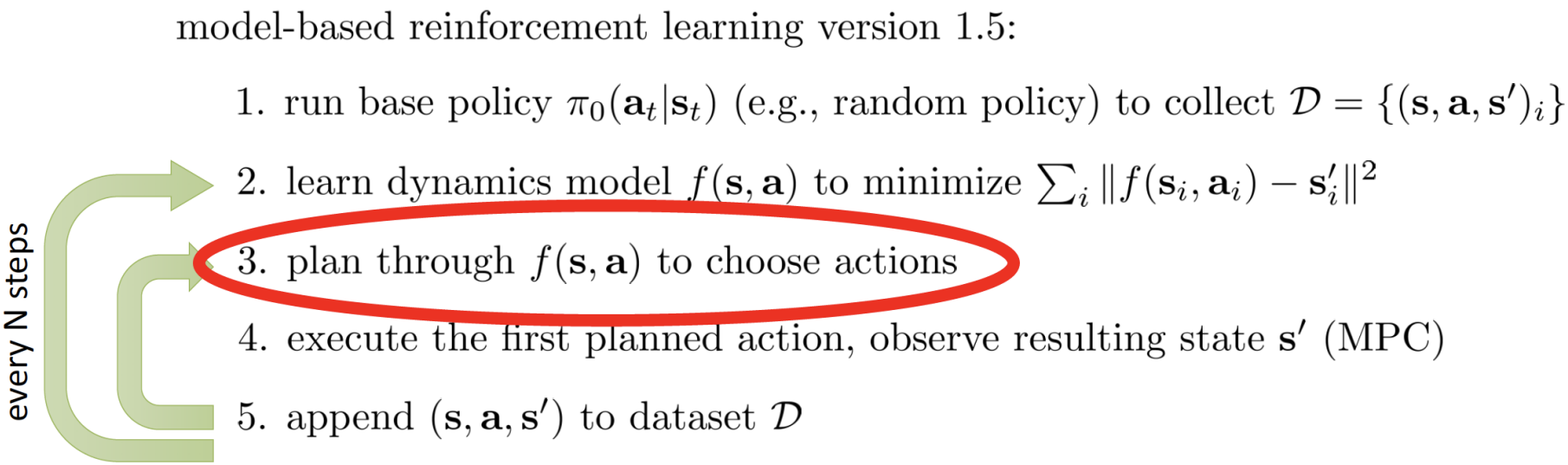


model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

every N steps

How to replan? - Refer to "Optimal Control and Planning"

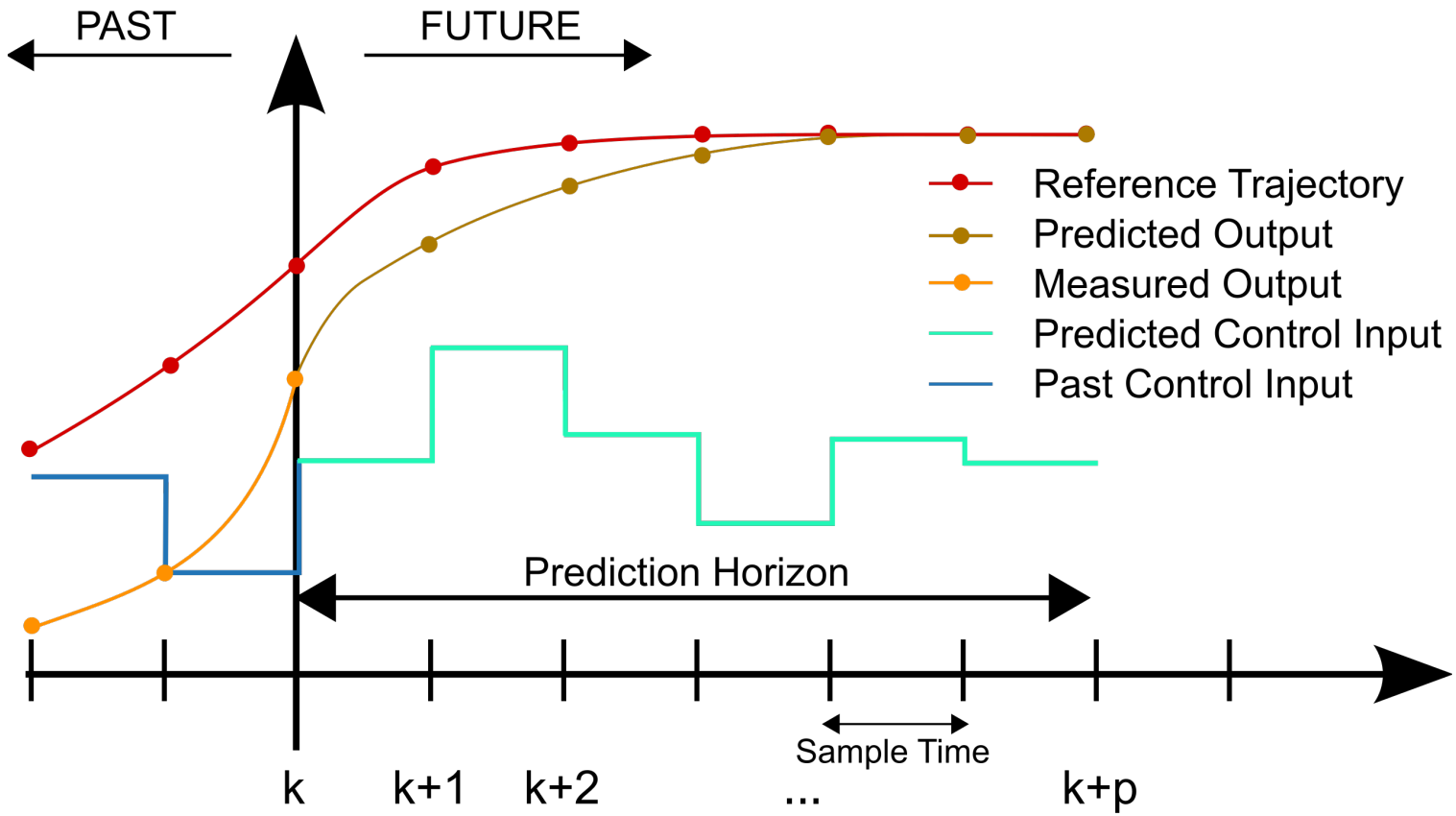


- The more you replan, the less perfect each individual plan needs to be
- Can use shorter horizons
- Even random sampling can often work well here!

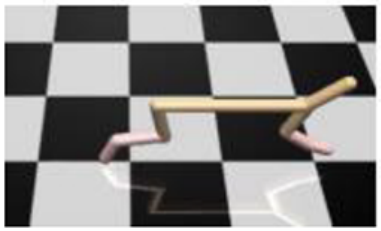
Alternative: Model predictive control (MPC)

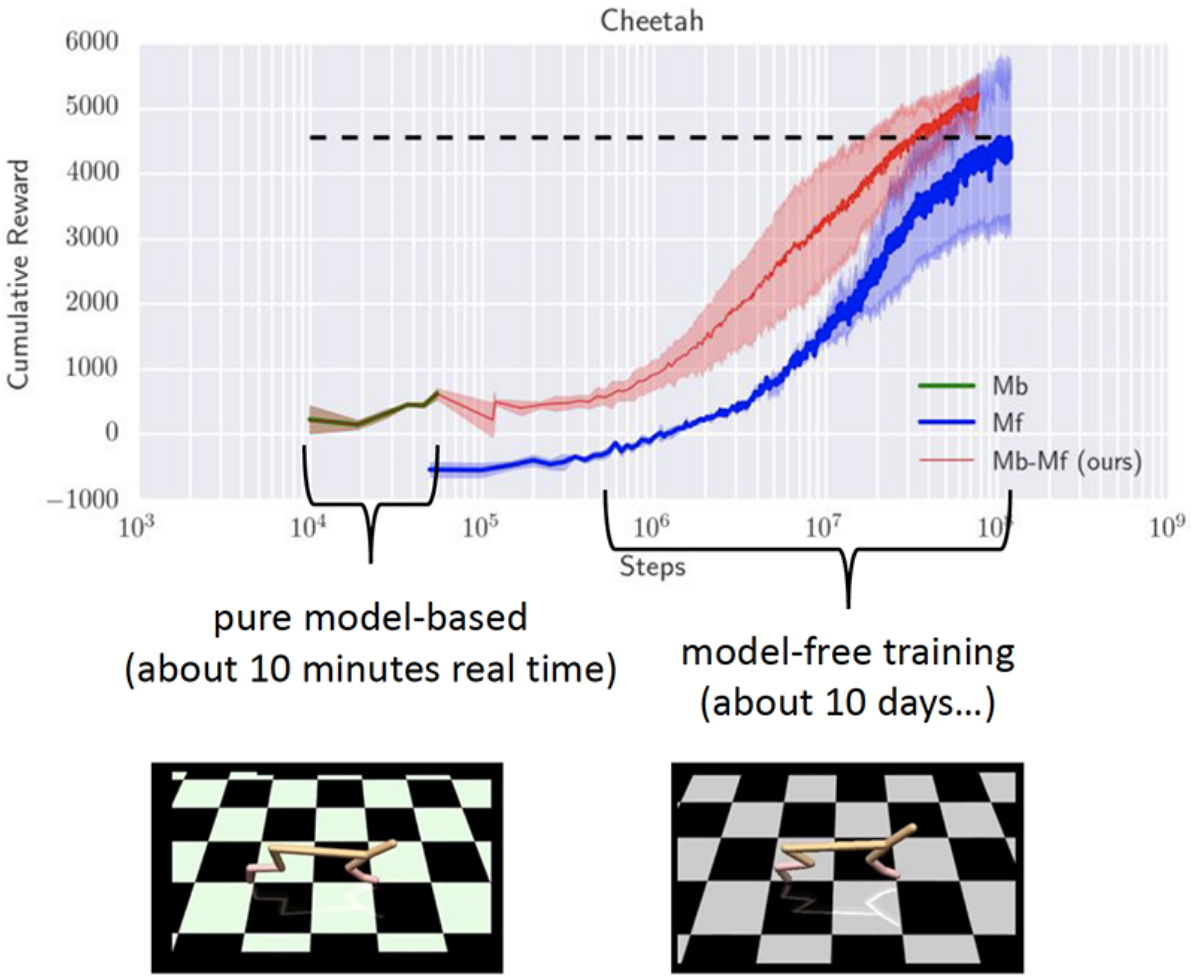
- An advanced method of process control that is used to control a process while satisfying a set of constraints
- Allows the current timeslot to be optimized, while keeping future timeslots in account
- Optimizes a finite time-horizon, but only implementing the current timeslot and then optimizing again, repeatedly

Model predictive control (MPC)



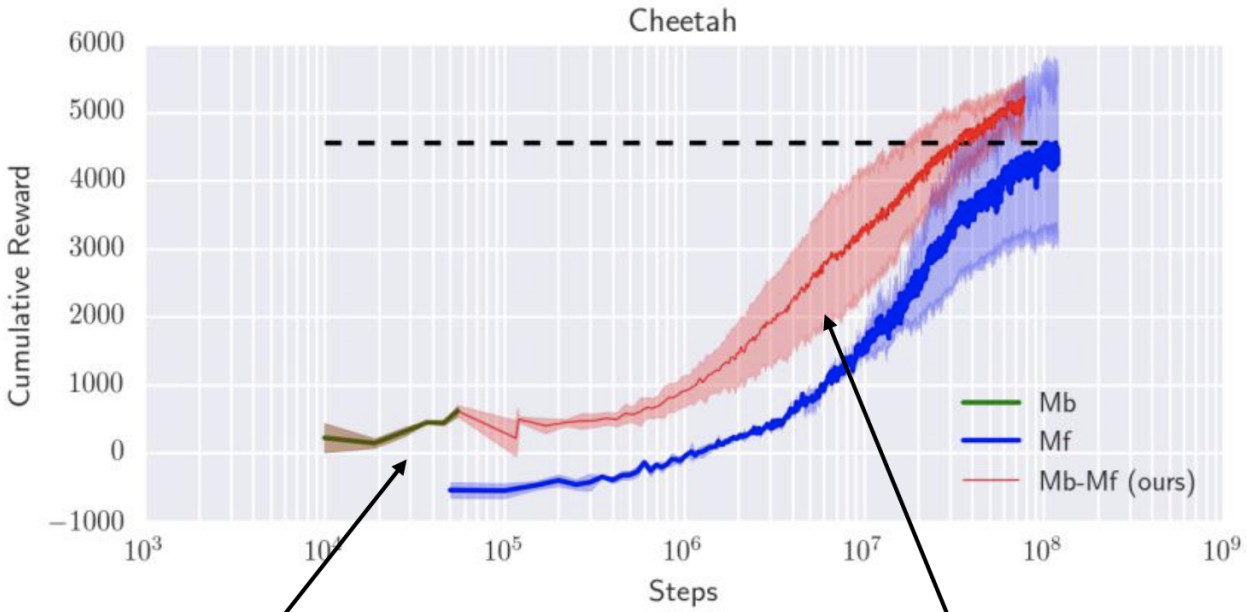
A performance gap in model-based RL





- Anusha Nagabandi, et al., "Neural Network Dynamics for Model-Based Deep Reinforcement Learning with Model-Free Fine-Tuning," *IEEE International Conference on Robotics and Automation (ICRA)*.

Why the performance gap?

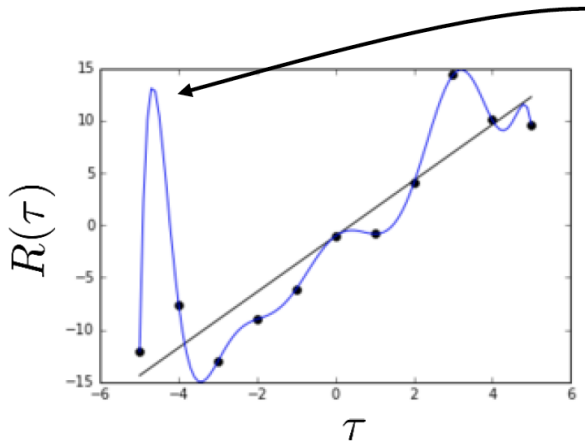


Why the performance gap?

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

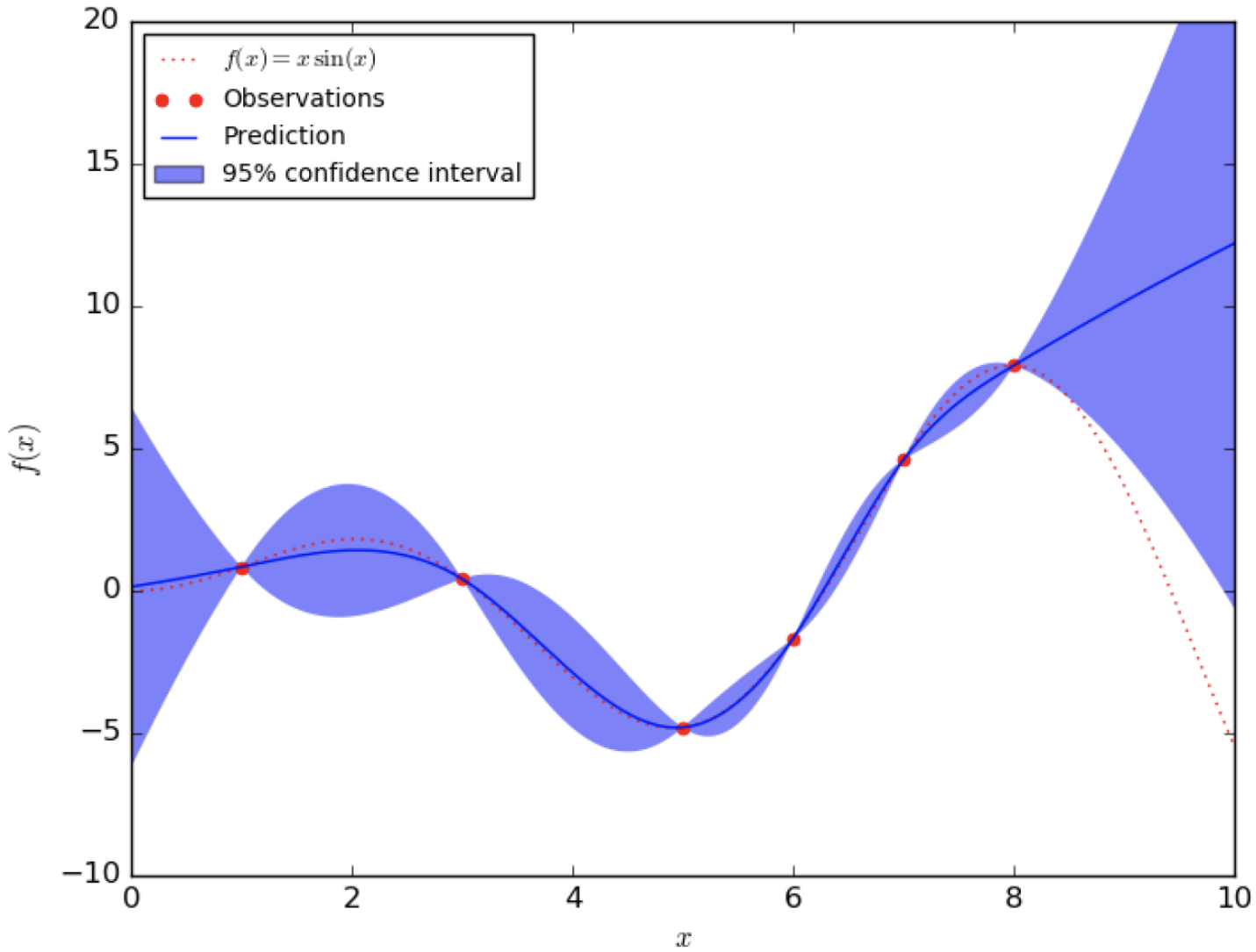
every N steps



very tempting to go here...

Table of Contents

Uncertainty - Bayesian models



Intuition behind uncertainty-aware RL

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

every N steps

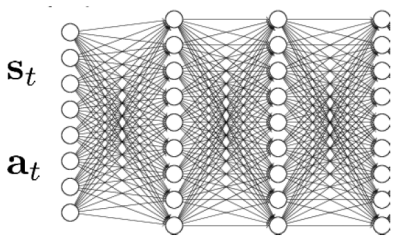
only take actions for which we think we'll get high reward in expectation (w.r.t. uncertain dynamics)

This avoids "exploiting" the model

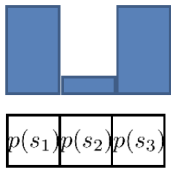
The model will then adapt and get better

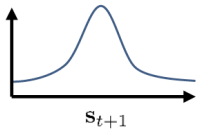
How can we have uncertainty-aware models?

Idea 1: use output entropy

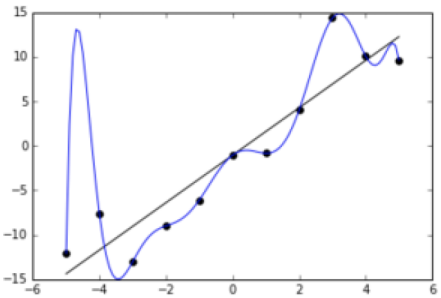


$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$





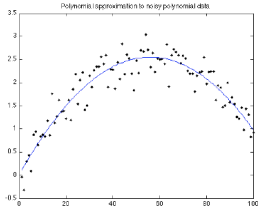
why is this not enough?



what is the variance here?

Two types of uncertainty:

aleatoric or *statistical* uncertainty →



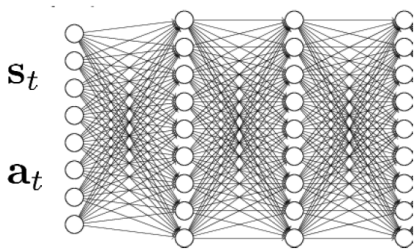
← *epistemic* or *model* uncertainty

"the model is certain about the data, but we are not certain about the model"

How can we have uncertainty-aware models?

Idea 2: estimate model uncertainty

"the model is certain about the data, but we are not certain about the model"



$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$

parameters θ

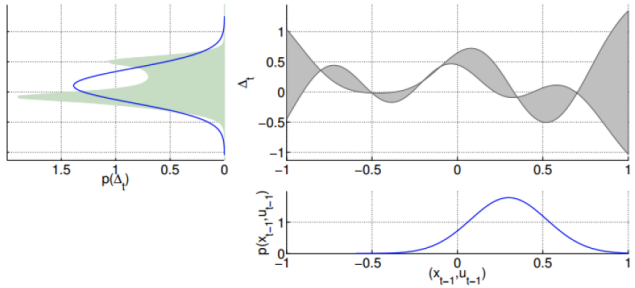
usually, we estimate

$$\arg\max_\theta \log p(\theta|\mathcal{D}) = \arg\max_\theta \log p(\mathcal{D}|\theta)$$

can we instead estimate $p(\theta|\mathcal{D})$?

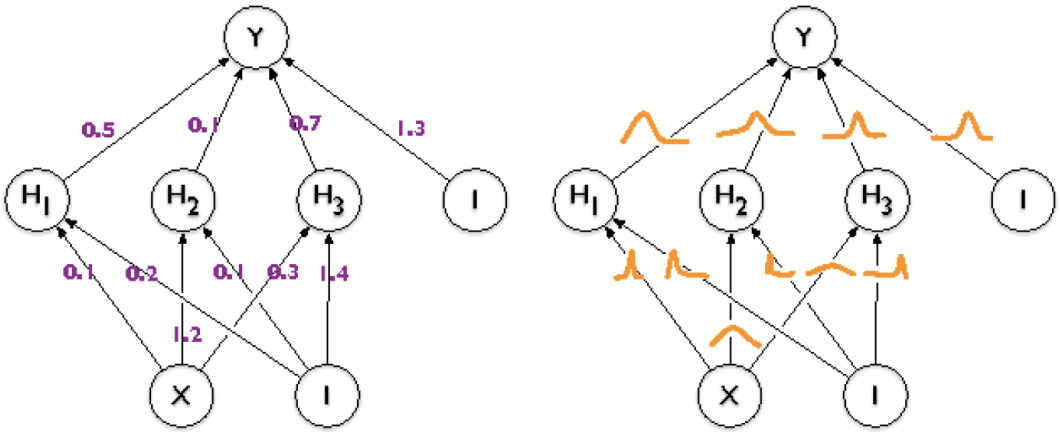
predict according to:

$$\int p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t, \theta) p(\theta|\mathcal{D}) d\theta$$



the entropy of this tells us the model uncertainty!

Quick overview of Bayesian neural networks



common approximation:

$$p(\theta|\mathcal{D}) = \prod_i p(\theta_i|\mathcal{D})$$

$$p(\theta_i|\mathcal{D}) = \mathcal{N}(\mu_i, \sigma_i)$$

expected weight

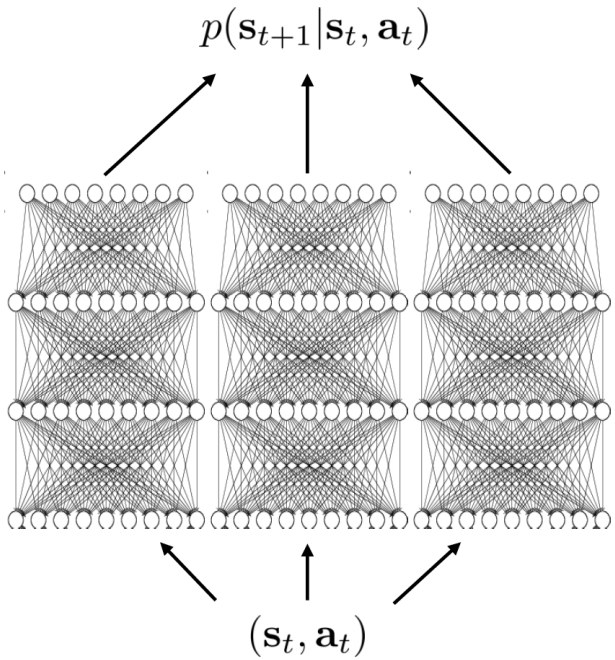
uncertainty about the weight

For more, see:

Blundell et al., Weight Uncertainty in Neural Networks

Gal et al., Concrete Dropout

Bootstrap ensembles



Train multiple models and see if they agree!

$$\text{formally:} \quad p(\theta|\mathcal{D}) \approx \frac{1}{N}\sum_i \delta(\theta_i)$$

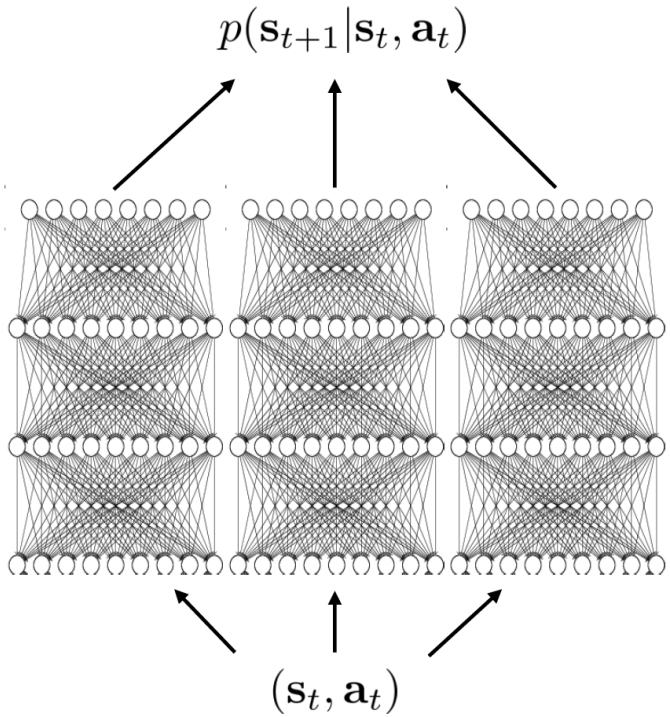
$$\int p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t, \theta)p(\theta|\mathcal{D})d\theta \approx \frac{1}{N}\sum_i p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t, \theta_i)$$

How to train?

Main idea: need to generate “independent” datasets to get “independent” models

θ_i is trained on $\mathcal{D}_i$, sampled *with replacement* from $\mathcal{D}$

Bootstrap ensembles in deep learning



This basically works

Very crude approximation, because the number of models is usually small (< 10)

Resampling with replacement is usually unnecessary, because SGD and random initialization usually makes the models sufficiently independent

How to plan with uncertainty

Before: $J(\mathbf{a}_1, \ldots, \mathbf{a}_H) = \sum_{t=1}^{H} r(\mathbf{s}_t, \mathbf{a}_t)$, where $\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$

Now: $J(\mathbf{a}_1, \ldots, \mathbf{a}_H) = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{H} r(\mathbf{s}_{t,i}, \mathbf{a}_t)$, where $\mathbf{s}_{t+1,i} = f_i(\mathbf{s}_{t,i}, \mathbf{a}_t)$

distribution over deterministic models

In general, for candidate action sequence $\mathbf{a}_1, \ldots, \mathbf{a}_H$:

Step 1: sample $\theta \sim p(\theta|\mathcal{D})$

Step 2: at each time step t, sample $\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t, \theta)$

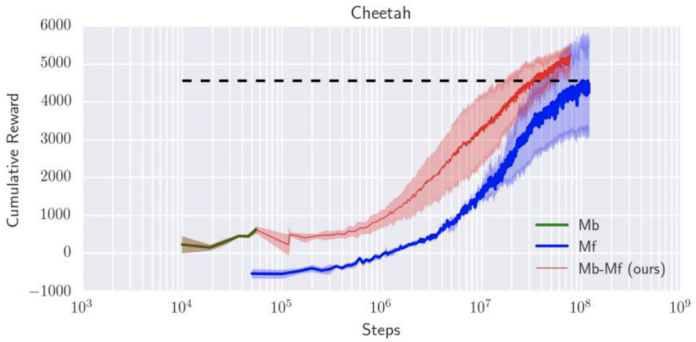
Step 3: calculate $R = \sum_t r(\mathbf{s}_t, \mathbf{a}_t)$

Step 4: repeat steps 1 to 3 and accumulate the average reward

Other options: moment matching, more complex posterior estimation with BNNs, etc.

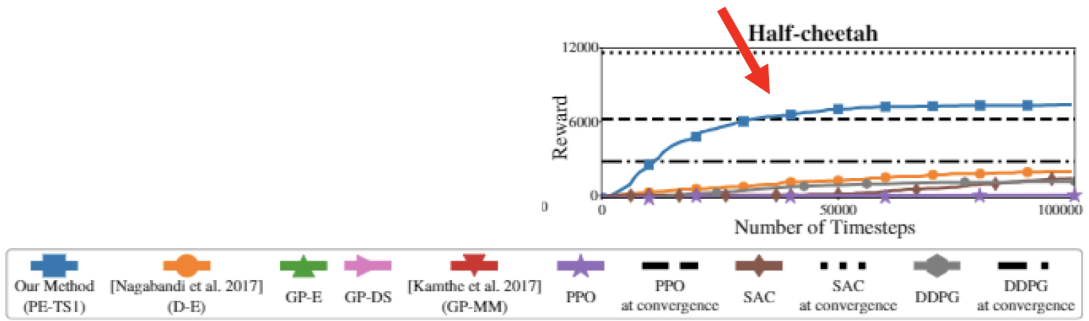
Example: model-based RL with ensembles

Deep Reinforcement Learning in a Handful of Trials using Probabilistic Dynamics Models



before

exceeds performance of model-free after 40k steps (about 10 minutes of real time)



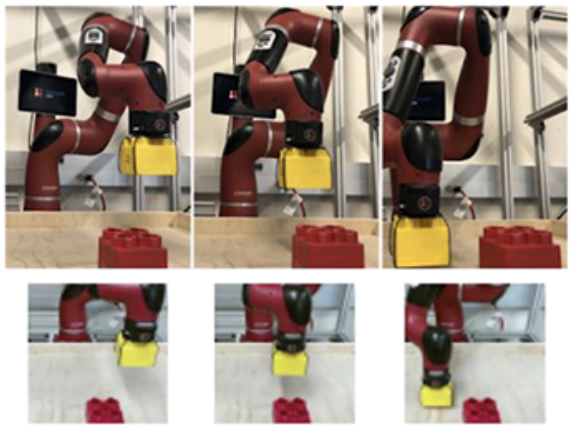
after

Further readings

- Nagabandi, et al., Deep dynamics models for learning dexterous manipulation, 2019.
- Nagabandi, et al., Neural network dynamics for model-based deep reinforcement learning with model-free fine-tuning
- Chua, et al., Deep reinforcement learning in a handful of trials using probabilistic dynamics models
- Feinberg, et al., Model-based value expansion for efficient model-free reinforcement learning
- Buckman, et al., Sample-efficient reinforcement learning with stochastic ensemble value expansion

Table of Contents

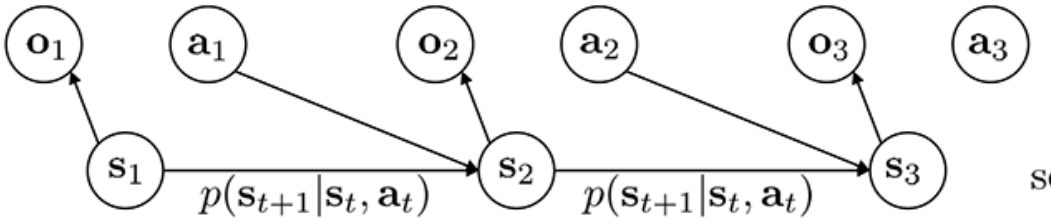
What about complex observations?



$$f(\mathbf{s}_t, \mathbf{a}_t) = \mathbf{s}_{t+1}$$

What is hard about this?

- High dimensionality
- Redundancy
- Partial observability

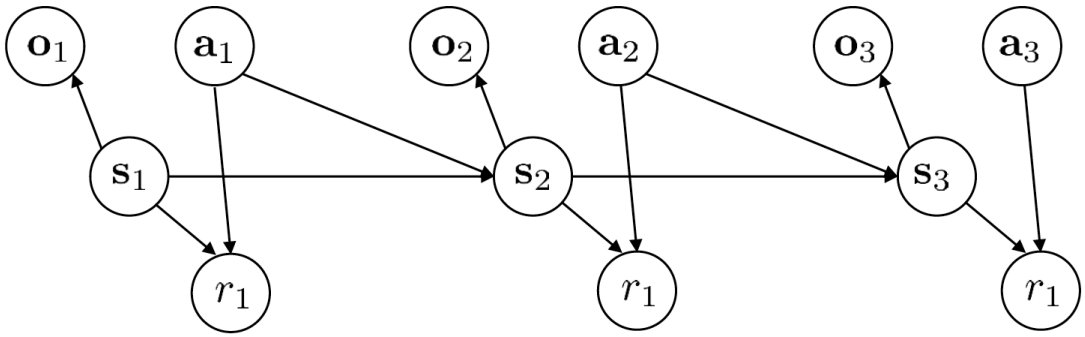


high-dimensional but not dynamic

low-dimension but dynamic

separately learn $p(\mathbf{o}_t|\mathbf{s}_t)$ and $p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$?

State space (latent space) models



$p(\mathbf{o}_t|\mathbf{s}_t)$ observation model

$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$ dynamics model

$p(r_t|\mathbf{s}_t, \mathbf{a}_t)$ reward model

How to train?

standard (fully observed) model: $\max_\phi \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \log p_\phi(\mathbf{s}_{t+1,i}|\mathbf{s}_{t,i}, \mathbf{a}_{t,i})$

latent space model: $\max_\phi \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} E\left[\log p_\phi(\mathbf{s}_{t+1,i}|\mathbf{s}_{t,i}, \mathbf{a}_{t,i}) + \log p_\phi(\mathbf{o}_{t,i}|\mathbf{s}_{t,i})\right]$

expectation w.r.t. $(\mathbf{s}_t, \mathbf{s}_{t+1}) \sim p(\mathbf{s}_t, \mathbf{s}_{t+1}|\mathbf{o}_{1:T}, \mathbf{a}_{1:T})$

Model-based RL with latent space models

$$\max_{\phi} \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} E\left[\log p_\phi(\mathbf{s}_{t+1,i}|\mathbf{s}_{t,i}, \mathbf{a}_{t,i}) + \log p_\phi(\mathbf{o}_{t,i}|\mathbf{s}_{t,i})\right]$$

expectation w.r.t. $(\mathbf{s}_t, \mathbf{s}_{t+1}) \sim p(\mathbf{s}_t, \mathbf{s}_{t+1}|\mathbf{o}_{1:T}, \mathbf{a}_{1:T})$

learn *approximate* posterior $q_\psi(\mathbf{s}_t|\mathbf{o}_{1:t}, \mathbf{a}_{1:t})$ "encoder"

many other choices for approximate posterior:

$q_\psi(\mathbf{s}_t, \mathbf{s}_{t+1}|\mathbf{o}_{1:T}, \mathbf{a}_{1:T})$ full smoothing posterior

+ most accurate
- most complicated

$q_\psi(\mathbf{s}_t|\mathbf{o}_t)$ single-step encoder

+ simplest
- least accurate

we'll talk about this one for now

Model-based RL with latent space models

$$\max_{\phi} \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} E\left[\log p_\phi(\mathbf{s}_{t+1,i}|\mathbf{s}_{t,i}, \mathbf{a}_{t,i}) + \log p_\phi(\mathbf{o}_{t,i}|\mathbf{s}_{t,i})\right]$$

expectation w.r.t. $\mathbf{s}_t \sim q_\psi(\mathbf{s}_t|\mathbf{o}_t), \mathbf{s}_{t+1} \sim q_\psi(\mathbf{s}_{t+1}|\mathbf{o}_{t+1})$

$q_\psi(\mathbf{s}_t|\mathbf{o}_t)$

simple special case: $q(\mathbf{s}_t|\mathbf{o}_t)$ is *deterministic*

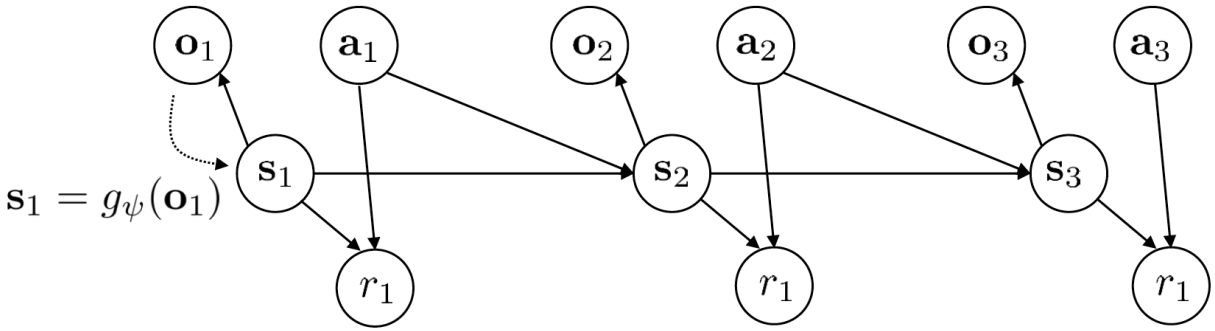
stochastic case requires variational inference

$q_\psi(\mathbf{s}_t|\mathbf{o}_t) = \delta(\mathbf{s}_t = g_\psi(\mathbf{o}_t)) \Rightarrow \mathbf{s}_t = g_\psi(\mathbf{o}_t)$ deterministic encoder

$$\max_{\phi,\psi} \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \log p_\phi(g_\psi(\mathbf{o}_{t+1,i})|g_\psi(\mathbf{o}_{t,i}), \mathbf{a}_{t,i}) + \log p_\phi(\mathbf{o}_{t,i}|g_\psi(\mathbf{o}_{t,i}))$$

Everything is differentiable, can train with backprop

Model-based RL with latent space models

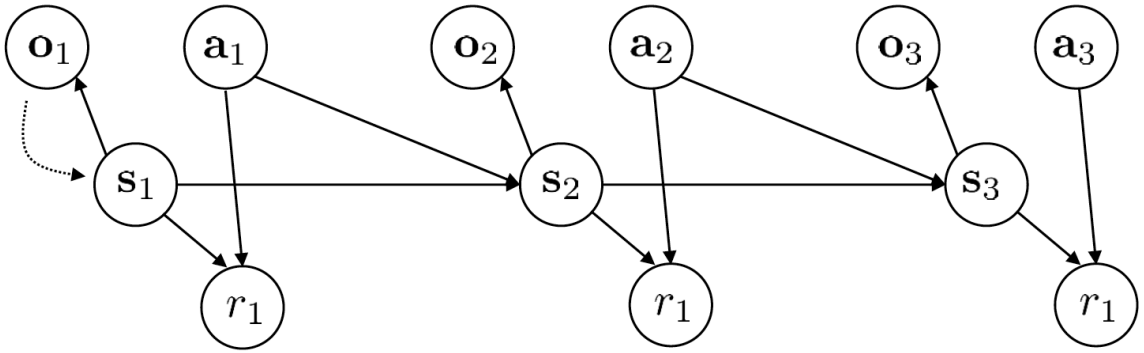


$$\max_{\phi,\psi} \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \log p_\phi(g_\psi(\mathbf{o}_{t+1,i}) | g_\psi(\mathbf{o}_{t,i}), \mathbf{a}_{t,i}) + \log p_\phi(\mathbf{o}_{t,i} | g_\psi(\mathbf{o}_{t,i})) + \log p_\phi(r_{t,i} | g_\psi(\mathbf{o}_{t,i}))$$

latent space dynamics | image reconstruction | reward model

Many practical methods use a stochastic encoder to model uncertainty

Model-based RL with latent space models



model-based reinforcement learning with latent state:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{o}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{o}, \mathbf{a}, \mathbf{o}')_i\}$
2. learn $p_\phi(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$, $p_\phi(r_t|\mathbf{s}_t)$, $p(\mathbf{o}_t|\mathbf{s}_t)$, $g_\psi(\mathbf{o}_t)$
3. plan through the model to choose actions
4. execute the first planned action, observe resulting $\mathbf{o}'$ (MPC)
5. append $(\mathbf{o}, \mathbf{a}, \mathbf{o}')$ to dataset $\mathcal{D}$

every N steps

Example: Latent dynamics model

Embed to Control: A Locally Linear Latent Dynamics Model for Control from Raw Images

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Martin Riedmiller
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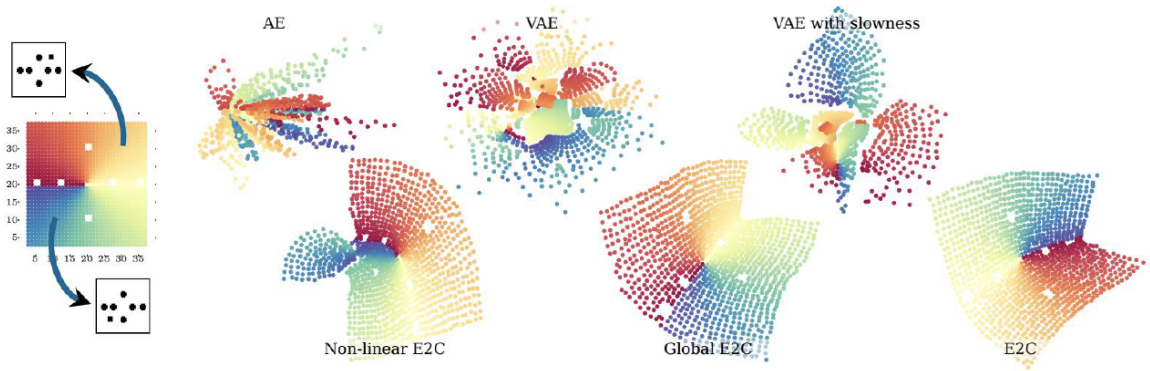
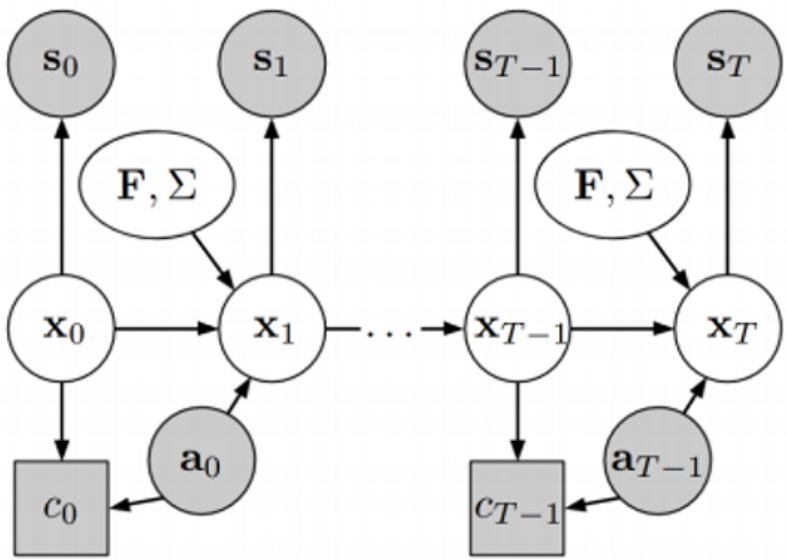


Example: Deep structured latent representations

SOLAR: Deep Structured Latent Representations for Model-Based Reinforcement Learning



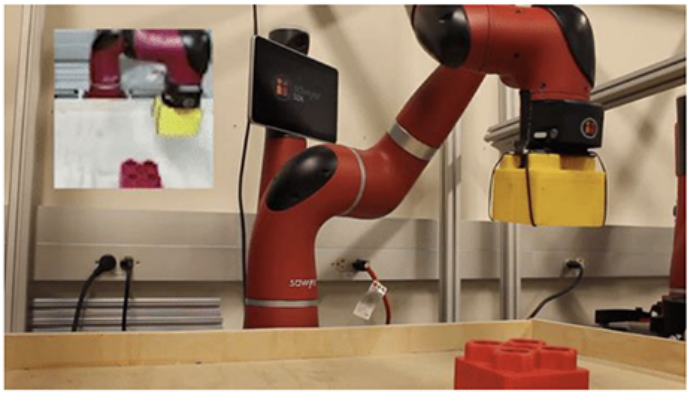


Table of Contents

Backpropagate into the policy with a model

- Policy gradient:

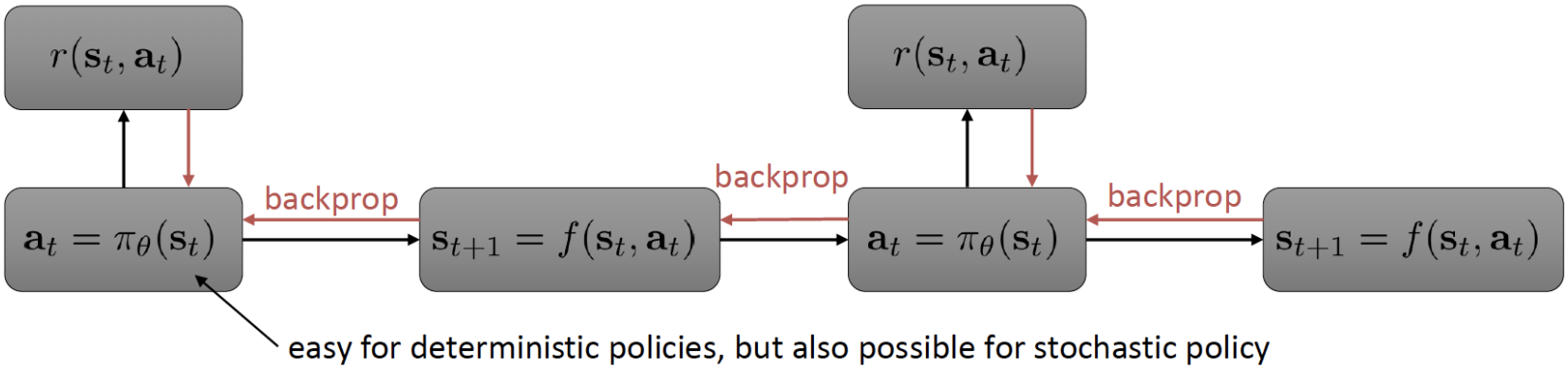
$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(a_{i,t}|s_{i,t}) \hat{Q}^\pi_{i,t}$$

- Backpropagate with a model:

$$\nabla_\theta J(\theta) \approx \mathbb{E}\left[\sum_{t=1}^{T} \frac{\mathrm{d}r_t}{\mathrm{d}s_t} \prod_{t'=2}^{t} \frac{\mathrm{d}s_{t'}}{\mathrm{d}a_{t'-1}} \frac{\mathrm{d}a_{t'-1}}{\mathrm{d}s_{t'-1}}\right]$$

- Policy gradient might be more stable
  - if enough samples are used
  - because it does not require multiplying many Jacobians

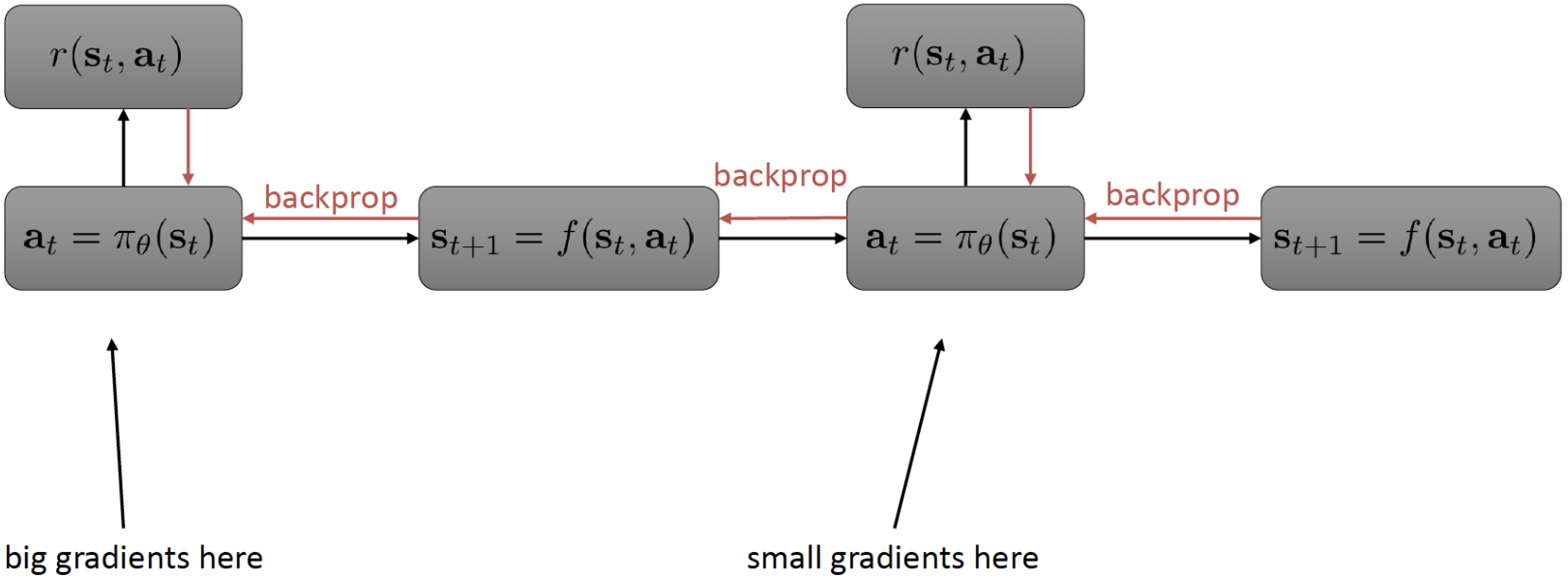
Backpropagate into the policy with a model



model-based reinforcement learning version 2.0:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. backpropagate through $f(\mathbf{s}, \mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
4. run $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to $\mathcal{D}$

What is the problem?



- Parameter sensitivity problem: policy parameters couple all time steps
- Vanishing and exploding gradient

What is the solution?

- Use model-free RL algorithms, with the model used to generate synthetic samples
  - Seems weirdly backwards
  - Actually works very well
  - Essentially "model-based acceleration" for model-free RL
- Use simpler policies than neural networks
  - LQR with learned models (LQR-FLM - **F**itted **L**ocal **M**odels)
  - Train **local** policies to solve simple tasks
  - Combine them into **global** policies via supervised learning

Table of Contents

Model-free optimization with a model

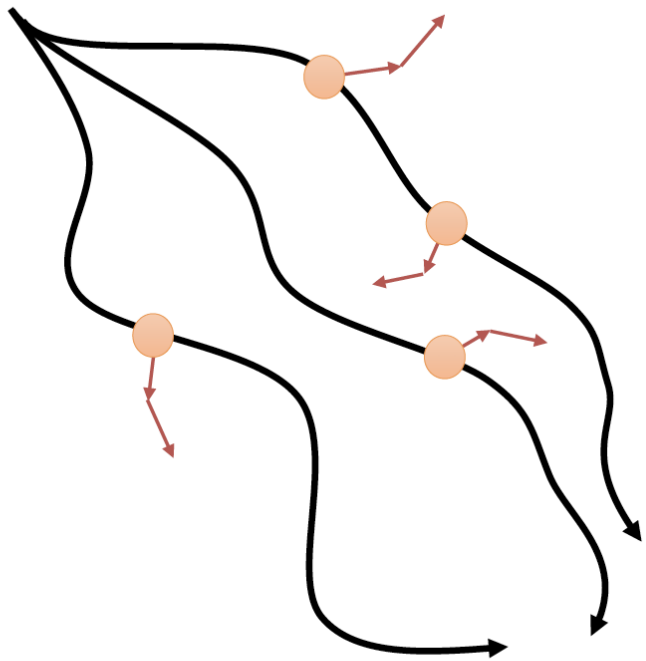
- **Dyna**: online Q-learning that performs model-free RL with a model

1. given state s, pick action a using exploration policy
2. observe s' and r, to get transition (s, a, s', r)
3. update model $\hat{p}(s'|s, a)$ and $\hat{r}(s, a)$ using (s, a, s')
4. Q-update: $Q(s, a) \leftarrow Q(s, a) + \alpha E_{s',r}[r + \max_{a'} Q(s', a') - Q(s, a)]$
5. repeat K times:
   6. sample $(s, a) \sim \mathcal{B}$ from buffer of past states and actions
   7. Q-update: $Q(s, a) \leftarrow Q(s, a) + \alpha E_{s',r}[r + \max_{a'} Q(s', a') - Q(s, a)]$

Richard S. Sutton, "Integrated architectures for learning, planning, and reacting based on approximating dynamic programming," *ICML*, 1990.

General "Dyna-style" model-based RL recipe

1. collect some data, consisting of transitions (s, a, s', r)
2. learn model $\hat{p}(s'|s, a)$ (and optionally, $\hat{r}(s, a)$)
3. repeat K times:
   4. sample $s \sim \mathcal{B}$ from buffer
   5. choose action a (from $\mathcal{B}$, from π, or random)
   6. simulate $s' \sim \hat{p}(s'|s, a)$ (and $r = \hat{r}(s, a)$)
   7. train on (s, a, s', r) with model-free RL
   8. (optional) take N more model-based steps



+ only requires short (as few as one step) rollouts from model

+ still sees diverse states

General "Dyna-style" model-based RL recipe

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly
3. use $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j\}$ to update model $\hat{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a})$
4. sample $\{\mathbf{s}_j\}$ from $\mathcal{B}$
5. for each $\mathbf{s}_j$, perform model-based rollout with $\mathbf{a} = \pi(\mathbf{s})$
6. use all transitions $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$ along rollout to update Q-function

- Model-based acceleration (MBA)
  - Gu et al., "Continuous deep Q-learning with model-based acceleration," 2016.
- Model-based value-expansion (MVE)
  - Feinberg et al., "Model-based value-expansion," 2018.
- Model-based policy optimization (MBPO)
  - Janner et al., "When to trust your model: Model-based policy optimizatio," 2019.

Table of Contents

Local models

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \text{ s.t. } \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\ldots)\ldots), \mathbf{u}_T)$$

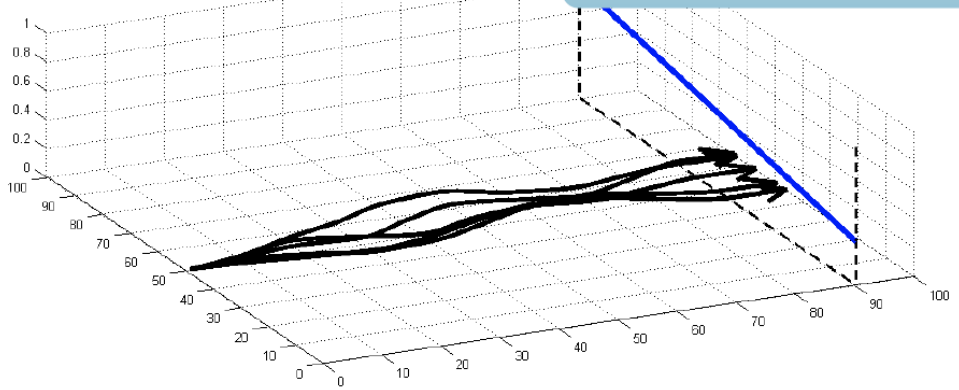
usual story: differentiate via backpropagation and optimize!

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

Local models

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

idea: just fit $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}$ around current trajectory or policy!



LQR gives us a linear feedback controller

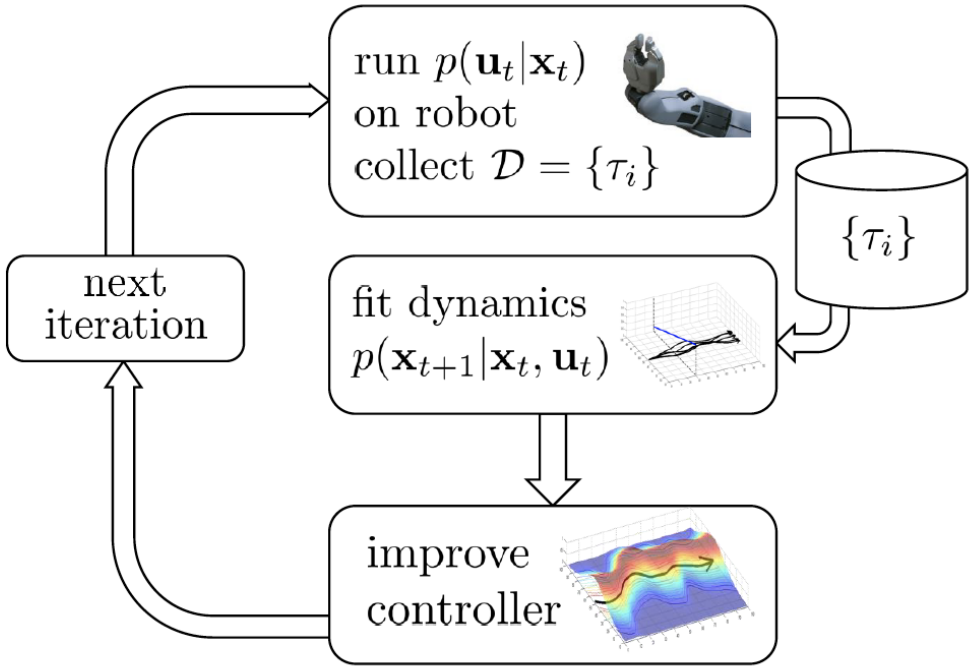
can **execute** in the real world!

Local models

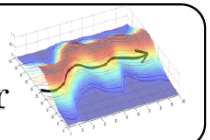
$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(f(\mathbf{x}_t, \mathbf{u}_t), \Sigma)$$

$$f(\mathbf{x}_t, \mathbf{u}_t) \approx \mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t$$

$$\mathbf{A}_t = \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t = \frac{df}{d\mathbf{u}_t}$$

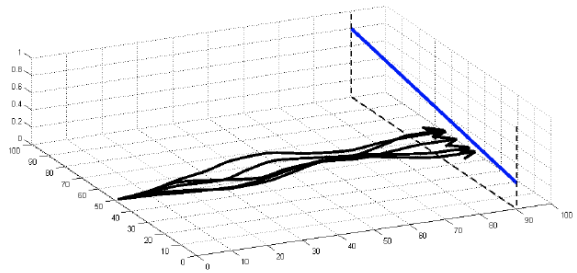


What controller to execute?

improve controller 

iLQR produces: $\hat{\mathbf{x}}_t$, $\hat{\mathbf{u}}_t$, $\mathbf{K}_t$, $\mathbf{k}_t$

$$\mathbf{u}_t = \mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t$$



Version 0.5: $p(\mathbf{u}_t|\mathbf{x}_t) = \delta(\mathbf{u}_t = \hat{\mathbf{u}}_t)$

Doesn't correct deviations or drift

Version 1.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \delta(\mathbf{u}_t = \mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t)$

Better, but maybe a little too good?

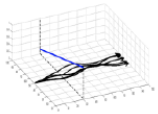
Version 2.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$

Add noise so that all samples don't look the same!

Set $\Sigma_t = \mathbf{Q}^{-1}_{\mathbf{u}_t,\mathbf{u}_t}$

How to fit the dynamics?

fit dynamics
$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$



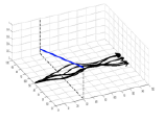
$\{(\mathbf{x}_t, \mathbf{u}_t, \mathbf{x}_{t+1})_i\}$

fit $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$ at each time step using linear regression

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(\mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t + \mathbf{c}, \mathbf{N}_t) \qquad \mathbf{A}_t \approx \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t \approx \frac{df}{d\mathbf{u}_t}$$

How to fit the dynamics?

fit dynamics
$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$

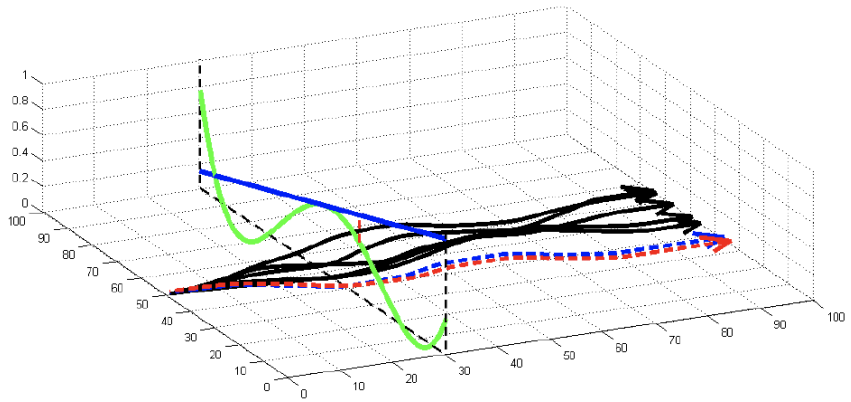


$\{(\mathbf{x}_t, \mathbf{u}_t, \mathbf{x}_{t+1})_i\}$

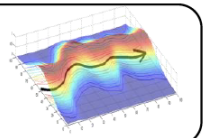
fit $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$ at each time step using linear regression

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(\mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t + \mathbf{c}, \mathbf{N}_t) \qquad \mathbf{A}_t \approx \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t \approx \frac{df}{d\mathbf{u}_t}$$

How to stay close to the old controller?



improve $p(\mathbf{u}_t|\mathbf{x}_t)$



$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$$

$$p(\tau) = p(\mathbf{x}_1)\prod_{t=1}^{T} p(\mathbf{u}_t|\mathbf{x}_t)p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$$

What if the new $p(\tau)$ is "close" to the old one $\bar{p}(\tau)$?

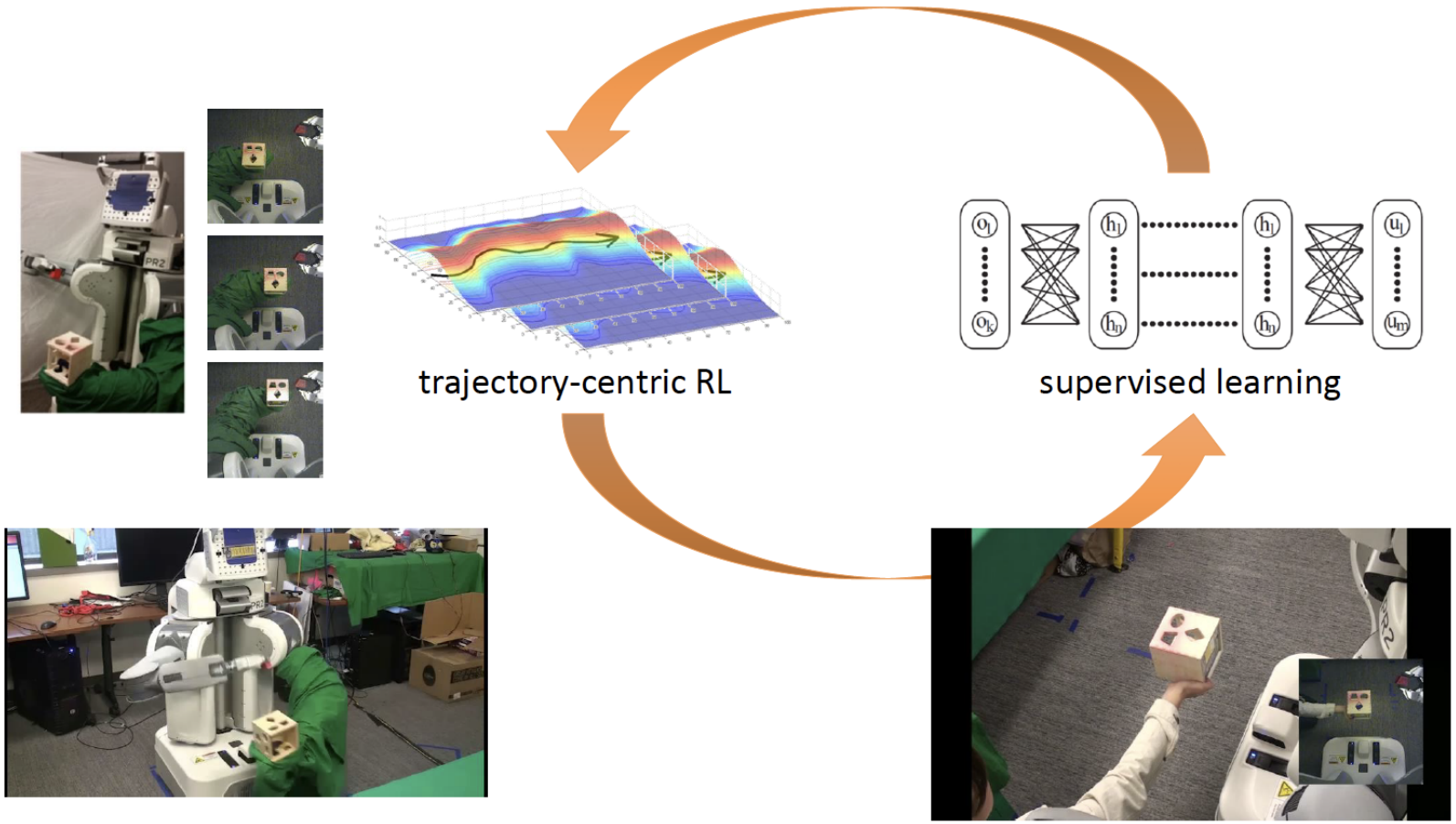
If trajectory distribution is close, then dynamics will be close too!

What does "close" mean? $D_{\mathrm{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$

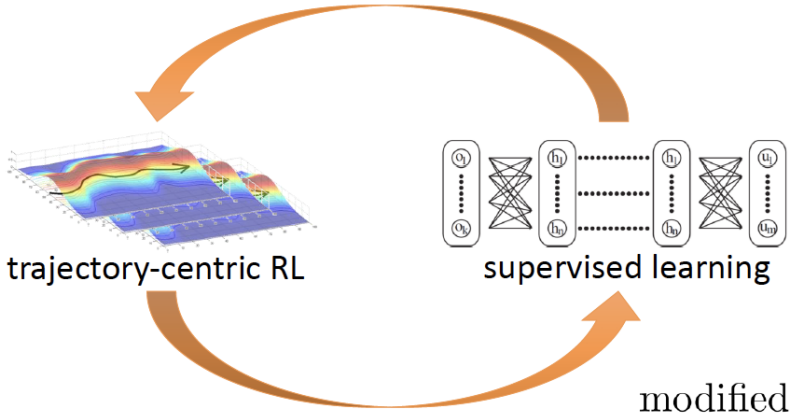
This is easy to do if $\bar{p}(\tau)$ also came from linear controller!

Global policies from local models

- Guided policy search: High-level idea



Guided policy search: Algorithm sketch



modified cost to keep $\pi_{\mathrm{LQR},i}$ close to π_θ

1. optimize each local policy $\pi_{\mathrm{LQR},i}(\mathbf{u}_t|\mathbf{x}_t)$ on initial state $\mathbf{x}_{0,i}$ w.r.t. $\tilde{c}_{k,i}(\mathbf{x}_t, \mathbf{u}_t)$
2. use samples from step (1) to train $\pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$ to mimic each $\pi_{\mathrm{LQR},i}(\mathbf{u}_t|\mathbf{x}_t)$
3. update cost function $\tilde{c}_{k+1,i}(\mathbf{x}_t, \mathbf{u}_t) = c(\mathbf{x}_t, \mathbf{u}_t) + \lambda_{k+1,i} \log \pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$

Lagrange multiplier

- Levine et al., "End-to-end training of deep visuomotor policies," JMLR 2016.

Underlying principle: Distillation

Ensemble models: single models are often not the most robust – instead train many models and average their predictions

this is how most ML competitions (e.g., Kaggle) are won

this is very expensive at test time

Can we make a single model that is as good as an ensemble?

Distillation: train on the ensemble's predictions as "soft" targets

logit

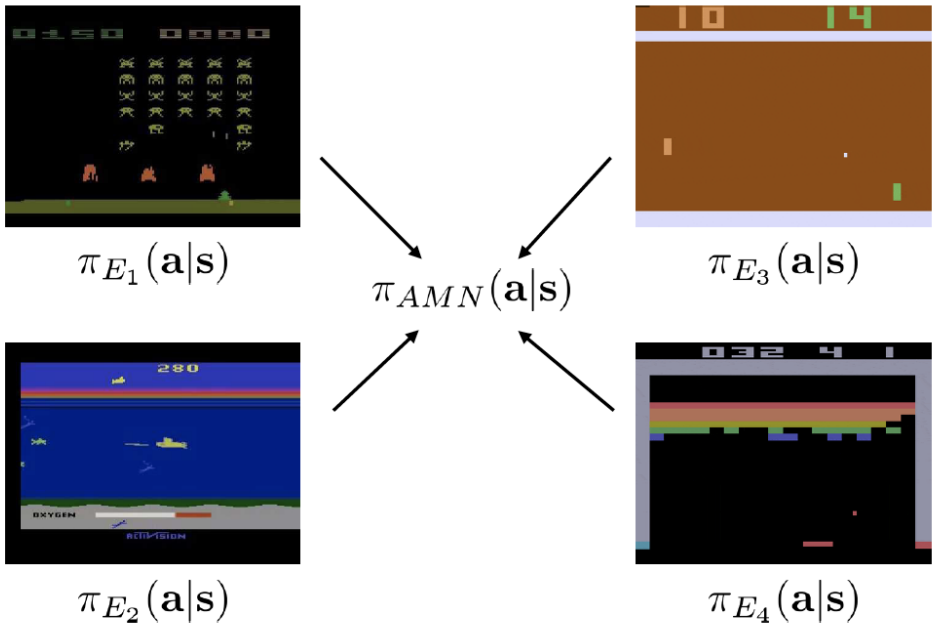
$$p_i = \frac{\exp(z_i/T)}{\sum_j \exp(z_j/T)}$$

temperature

Intuition: more knowledge in soft targets than hard labels!

- Hinton et al., "Distilling the knowledge in a neural network," 2015.

Distillation for multi-task transfer



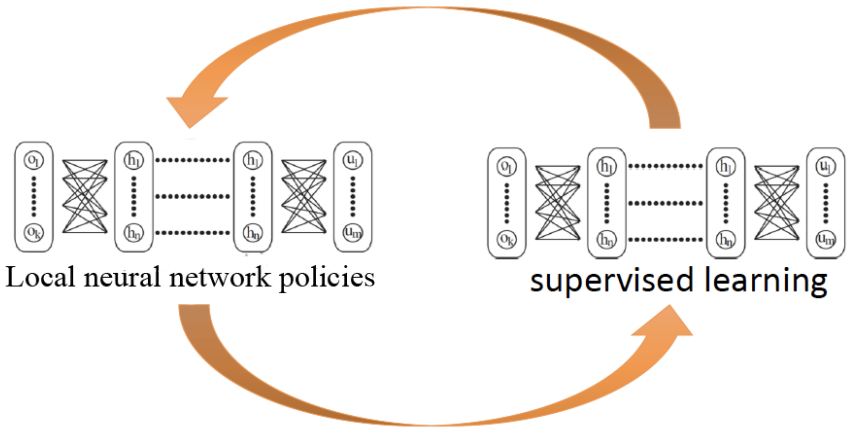
$$\mathcal{L} = \sum_{\mathbf{a}} \pi_{E_i}(\mathbf{a}|\mathbf{s}) \log \pi_{AMN}(\mathbf{a}|\mathbf{s})$$

(just supervised learning/distillation)

analogous to guided policy search, but for multi-task learning

- Parisotto et al., "Actor-mimic: Deep multitask and transfer reinforcement learning," ICLR 2016.

Combining weak policies into a strong policy



Divide and Conquer Reinforcement Learning

Divide and conquer reinforcement learning algorithm sketch:

1. optimize each local policy $\pi_{\theta_i}(\mathbf{a}_t|\mathbf{s}_t)$ on initial state $\mathbf{s}_{0,i}$ w.r.t. $\tilde{r}_{k,i}(\mathbf{s}_t, \mathbf{a}_t)$
2. use samples from step (1) to train $\pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$ to mimic each $\pi_{\theta_i}(\mathbf{u}_t|\mathbf{x}_t)$
3. update reward function $\tilde{r}_{k+1,i}(\mathbf{x}_t, \mathbf{u}_t) = r(\mathbf{x}_t, \mathbf{u}_t) + \lambda_{k+1,i} \log \pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$

- Ghosh et al., "Divide and conquer reinforcement learning," ICLR 2018.

Further readings: Guided policy search & Distillation

- Levine et al., "End-to-end training of deep visuomotor policies," JMLR 2016.
- Rusu et al., "Policy distillation," 2015.
- Parisotto et al., "Actor-mimic: Deep multitask and transfer reinforcement learning," ICLR 2016.
- Ghosh et al., "Divide and conquer reinforcement learning," ICLR 2018.
- Teh et al., "Distral: Robust multitask reinforcement learning," 2017.

Learning objectives of this lecture

- You should be able to...
  - Know the common methods for optimal control and planning when the system's dynamics are known
    - Stochastic optimization, cross-entropy method, MCTS, LQR
  - Know how to learn the model, the different versions, practical considerations
    - Uncertainty estimation, latent models...
  - Know how to backpropagate directly into the policy with a model
    - Model-based acceleration for model-free learning
    - Using local policies, combing them into a global one

Model-based RL suggested readings

- Lecture 10 & 11 & 12 of CS285 at UC Berkeley, **Deep Reinforcement Learning, Decision Making, and Control**
  - `http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-10.pdf`
  - `http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-11.pdf`
  - `http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-12.pdf`
- Recent papers
  - Nagabandi et al., **Neural Network Dynamics for Model-Based Deep Reinforcement Learning with Model-Free Fine-Tuning**, IEEE ICRA 2017.
  - Grady Williams et al., **Aggressive Driving with Model Predictive Path Integral Control**, IEEE ICRA 2016.
  - Buckman et al., **Sample Efficient Reinforcement Learning with Stochastic Ensemble Value Expansion**, NIPS 2018.

THE END