Lecture 9: Deep Q-learning

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Contents and Goals

- How we can make Q-learning work with deep networks
  - Use replay buffers, separate target networks
- Tricks for improving Q-learning in practice
  - Double Q-learning, multi-step Q-learning
- Continuous Q-learning methods
- Goals
  - Understand how to implement Q-learning so that it can be used with complex function approximators
  - Understand how to extend Q-learning to continuous actions

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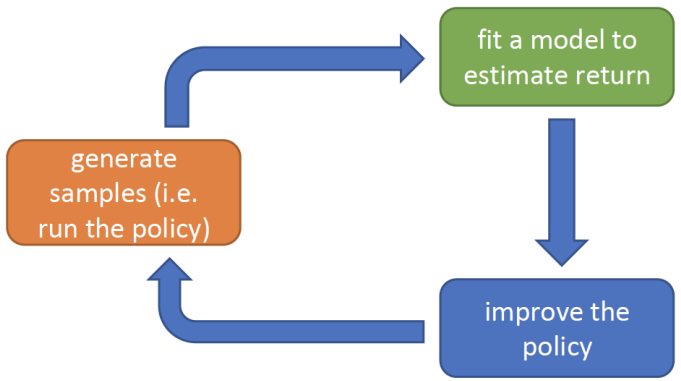
Recall: Q-learning

full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy
2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

($K\times$: steps 2–3 repeated K times)

$$Q_\phi(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}', \mathbf{a}')$$



$$\mathbf{a} = \arg\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a})$$

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

Problem 1: Correlated samples in MDPs

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

these are correlated!

isn't this just gradient descent? that converges, right?

Q-learning is *not* gradient descent!

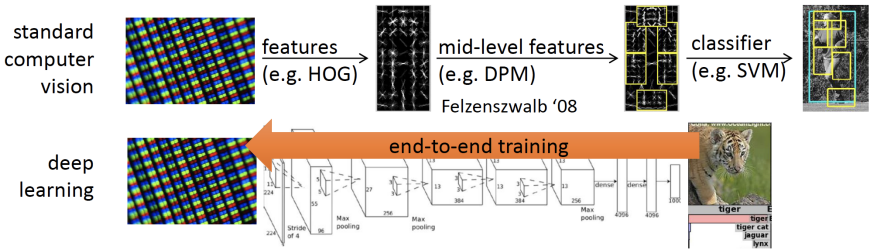
$$\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$$

no gradient through target value

Recall: Supervised learning vs. Sequential decision making

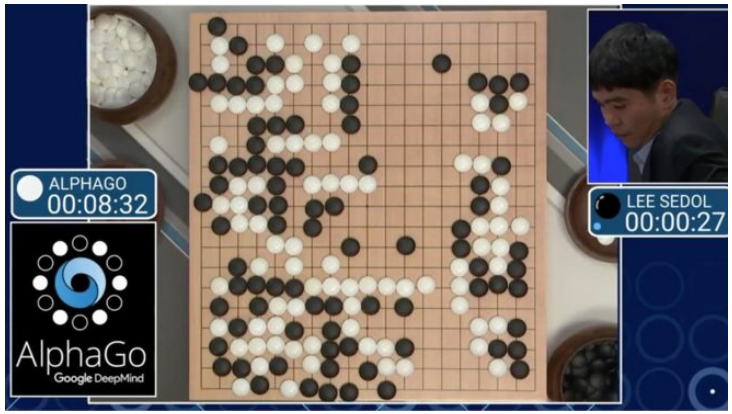
Supervised learning

- Samples are independent and identically distributed (i.i.d.)
- Given an input, map an optimal output



Reinforcement learning

- Samples are not i.i.d., temporally co-related
- Given an initial state, find a sequence of optimal actions

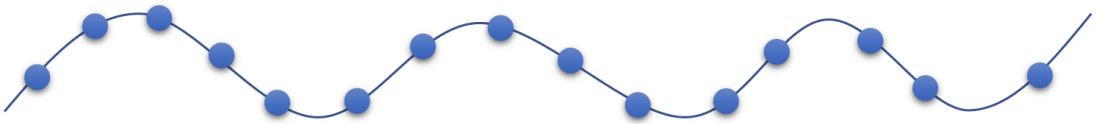


Correlated samples in online Q-learning

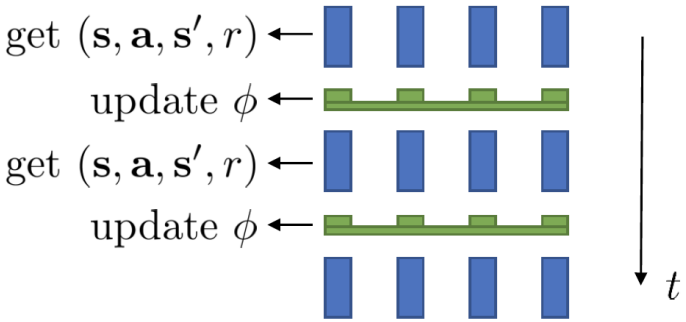
online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

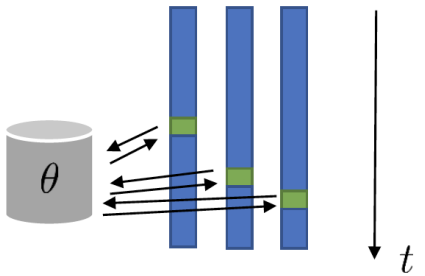
- sequential states are strongly correlated
- target value is always changing



synchronized parallel Q-learning



asynchronous parallel Q-learning



Solution 1: replay buffers

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

special case with K = 1, and one gradient step

full fitted Q-iteration algorithm:

~~1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy~~

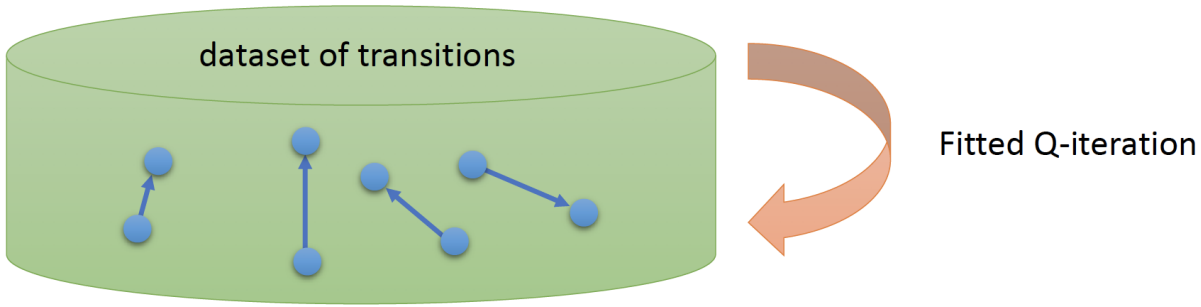
$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

any policy will work! (with broad support)

just load data from a buffer here

still use one gradient step



Replay buffers

Q-learning with a replay buffer:

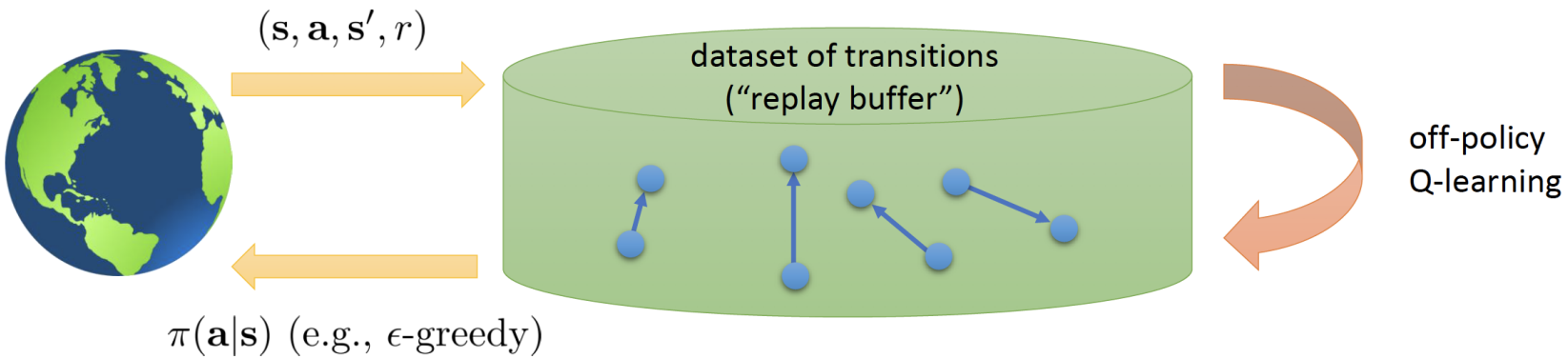
1. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
2. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

+ samples are no longer correlated

+ multiple samples in the batch (low-variance gradient)

but where does the data come from?

need to periodically feed the replay buffer...

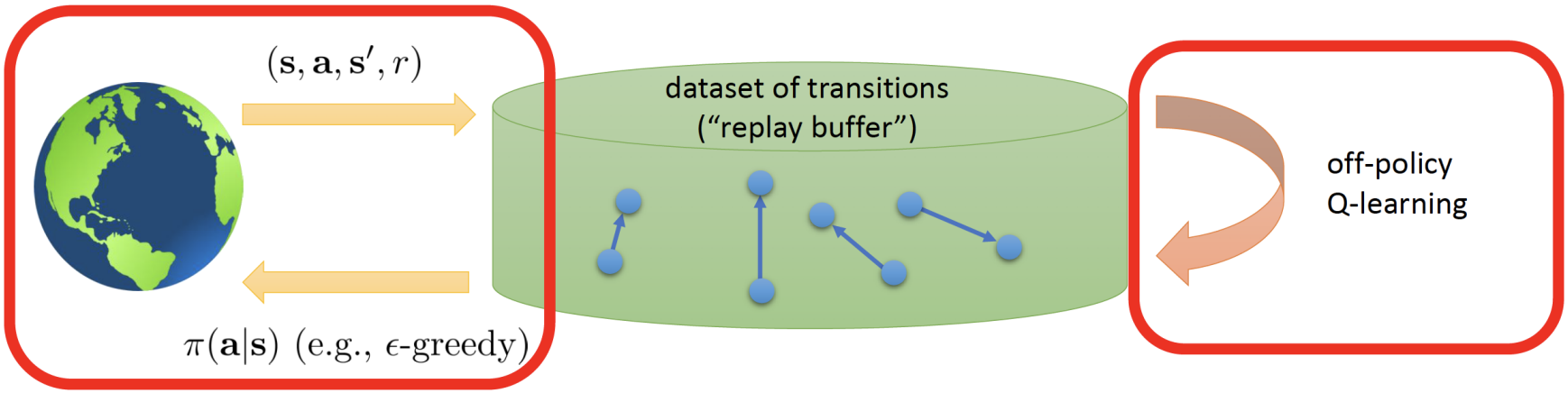


Putting it together

full Q-learning with replay buffer:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$
2. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$ — $K\times$
3. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

K = 1 is common, though larger K more efficient



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Problem 2: Moving target in the Bellman equation

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

~~these are correlated!~~

use replay buffer

Q-learning is *not* gradient descent!

$$\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$$

no gradient through target value

This is still a problem!

Q-learning and regression

full Q-learning with replay buffer:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$
2. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
3. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

($K\times$ repeat steps 2–3)

one gradient step, moving target

full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy
2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

($K\times$ repeat steps 2–3)

perfectly well-defined, stable regression

Solution 2: Target networks

Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$

$N\times$

2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$

$K\times$

3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$

4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

targets don't change in inner loop!

supervised regression

"Classic" deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$
2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$ — $N\times$
3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$ — $K\times$
4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

"classic" deep Q-learning algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly
3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$ using *target* network $Q_{\phi'}$
4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$

$K = 1$

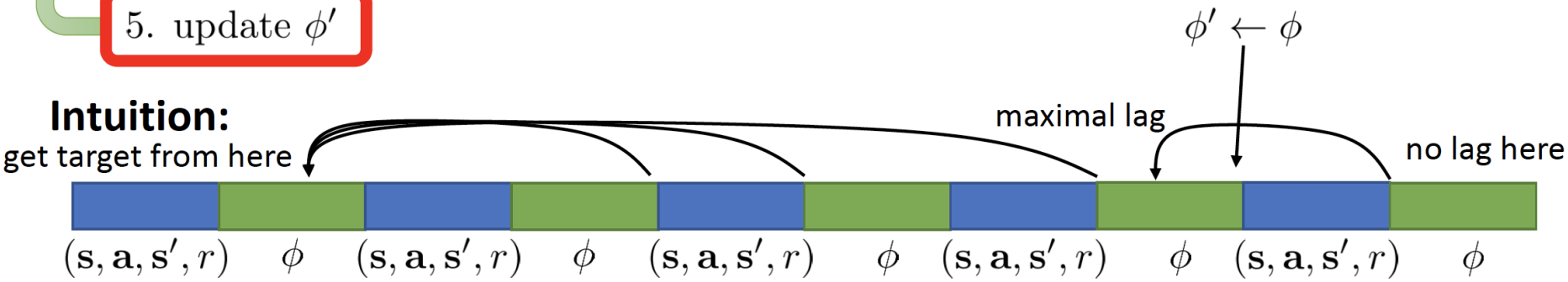
5. update ϕ': copy ϕ every N steps

Alternative target network

"classic" deep Q-learning algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly
3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$ using *target* network $Q_{\phi'}$
4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$
5. update ϕ'

Intuition:



Feels weirdly uneven, can we always have the same lag?

Popular alternative (similar to Polyak averaging):

5. update ϕ': $\phi' \leftarrow \tau\phi' + (1-\tau)\phi$ $\quad\quad$ $\tau = 0.999$ works well

Fitted Q-iteration and Q-learning

Q-learning with replay buffer and target network:

DQN: $N = 1$, $K = 1$

1. save target network parameters: $\phi' \leftarrow \phi$
   - $N\times$
     - 2. collect M datapoints $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to $\mathcal{B}$
     - $K\times$
       - 3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
       - 4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

Fitted Q-learning (written similarly as above):

1. collect M datapoints $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to $\mathcal{B}$
   - $N\times$
     - 2. save target network parameters: $\phi' \leftarrow \phi$
     - $K\times$
       - 3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
       - 4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

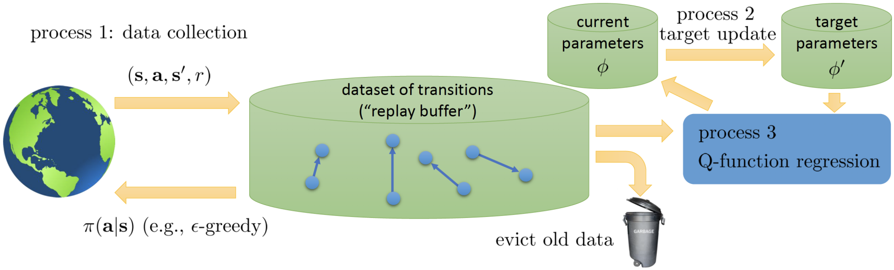
} **just SGD**

A more general view

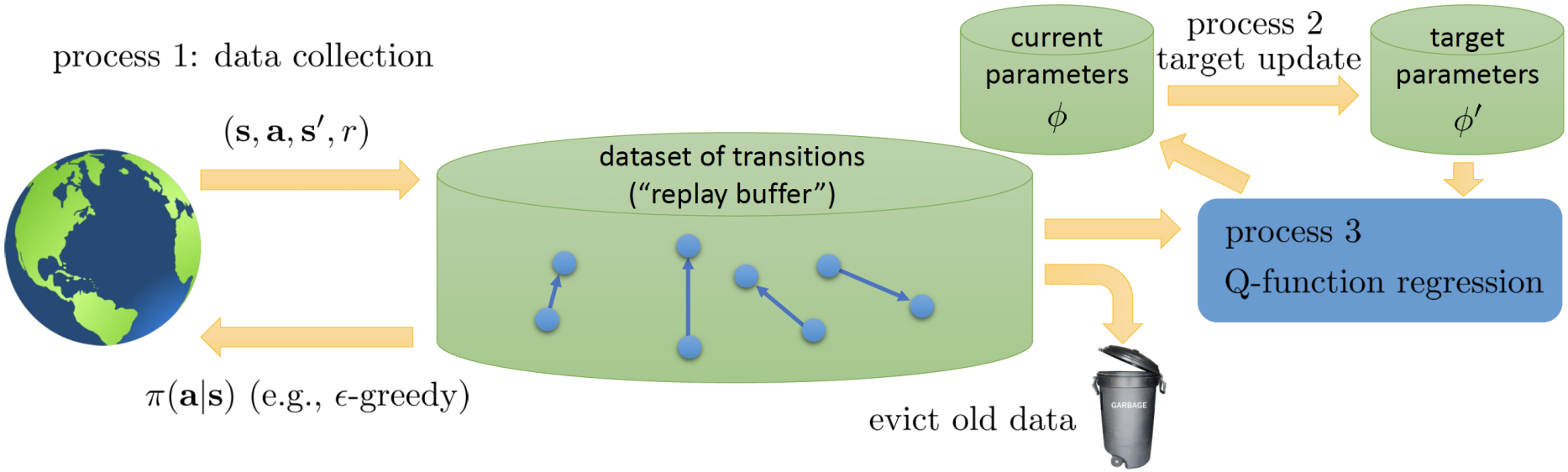
Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$
2. collect M datapoints $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to $\mathcal{B}$
3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

$N\times$ $K\times$



A more general view



- Online Q-learning: evict immediately, process 1, process 2, and process 3 all run at the same speed
- DQN: process 1 & 3 run at the same speed, process 2 is slow
- Fitted Q-iteration: process 3 in the inner loop of process 2, which is in the inner loop of process 1

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Q-learning with continuous actions

What's the problem with continuous actions?

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} Q_\phi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

this max

$$\text{target value } y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$$

this max
particularly problematic (inner loop of training)

How do we perform the max?

Option 1: optimization

- gradient based optimization (e.g., SGD) a bit slow in the inner loop
- action space typically low-dimensional – what about stochastic optimization?

Option 1: stochastic optimization

Simple solution:

$$\max_{\mathbf{a}} Q(\mathbf{s}, \mathbf{a}) \approx \max\{Q(\mathbf{s}, \mathbf{a}_1), \ldots, Q(\mathbf{s}, \mathbf{a}_N)\}$$

$(\mathbf{a}_1, \ldots, \mathbf{a}_N)$ sampled from some distribution (e.g., uniform)

+ dead simple

+ efficiently parallelizable

- not very accurate

but… do we care? How good does the target need to be anyway?

More accurate solution:

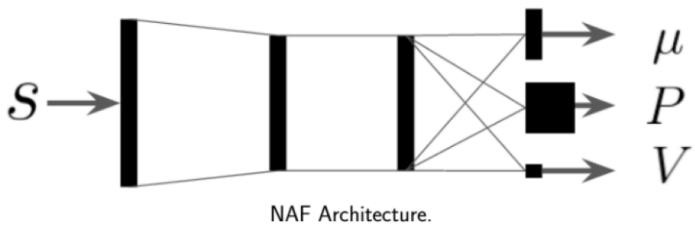
- cross-entropy method (CEM)
  - simple iterative stochastic optimization
- CMA-ES
  - substantially less simple iterative stochastic optimization

works OK, for up to about 40 dimensions

Option 2: easily maximizable Q-functions

Option 2: use function class that is easy to optimize

$$Q_\phi(\mathbf{s}, \mathbf{a}) = -\frac{1}{2}(\mathbf{a} - \mu_\phi(\mathbf{s}))^T P_\phi(\mathbf{s})(\mathbf{a} - \mu_\phi(\mathbf{s})) + V_\phi(\mathbf{s})$$



NAF Architecture.

NAF: **N**ormalized **A**dvantage **F**unctions

$$\arg\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a}) = \mu_\phi(\mathbf{s}) \qquad \max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a}) = V_\phi(\mathbf{s})$$

+ no change to algorithm

+ just as efficient as Q-learning

- loses representational power

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Q-learning with continuous actions

Option 3: learn an approximate maximizer

DDPG (Lillicrap et al., ICLR 2016)

"deterministic" actor-critic
(really approximate Q-learning)

$\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a}) = Q_\phi(\mathbf{s}, \arg\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a}))$

idea: train another network $\mu_\theta(\mathbf{s})$ such that $\mu_\theta(\mathbf{s}) \approx \arg\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a})$

how? just solve $\theta \leftarrow \arg\max_\theta Q_\phi(\mathbf{s}, \mu_\theta(\mathbf{s}))$ $\qquad \frac{dQ_\phi}{d\theta} = \frac{d\mathbf{a}}{d\theta}\frac{dQ_\phi}{d\mathbf{a}}$

new target $y_j = r_j + \gamma Q_{\phi'}(\mathbf{s}'_j, \mu_\theta(\mathbf{s}'_j)) \approx r_j + \gamma Q_{\phi'}(\mathbf{s}'_j, \arg\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j))$

Deep deterministic policy gradient (DDPG)

Option 3: learn an approximate maximizer

DDPG:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly
3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mu_{\theta'}(\mathbf{s}'_j))$ using *target* nets $Q_{\phi'}$ and $\mu_{\theta'}$
4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$
5. $\theta \leftarrow \theta + \beta \sum_j \frac{d\mu}{d\theta}(\mathbf{s}_j) \frac{dQ_\phi}{d\mathbf{a}}(\mathbf{s}_j, \mathbf{a})$
6. update ϕ' and θ' (e.g., Polyak averaging)

Recall: Actor-critic algorithms

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

DDPG vs. Actor-critic

- DDPG
  - The actor: approximate the optimal policy $a = \arg\max Q_\phi(s, a)$
  - The critic: approximate the optimal action-value function Q^*_ϕ
  - Off-policy, more sample efficient
- Actor-critic
  - The actor: approximate the current policy $a \sim \pi(a|s)$
  - The critic: approximate the state-value function V^π_ϕ
  - On-policy

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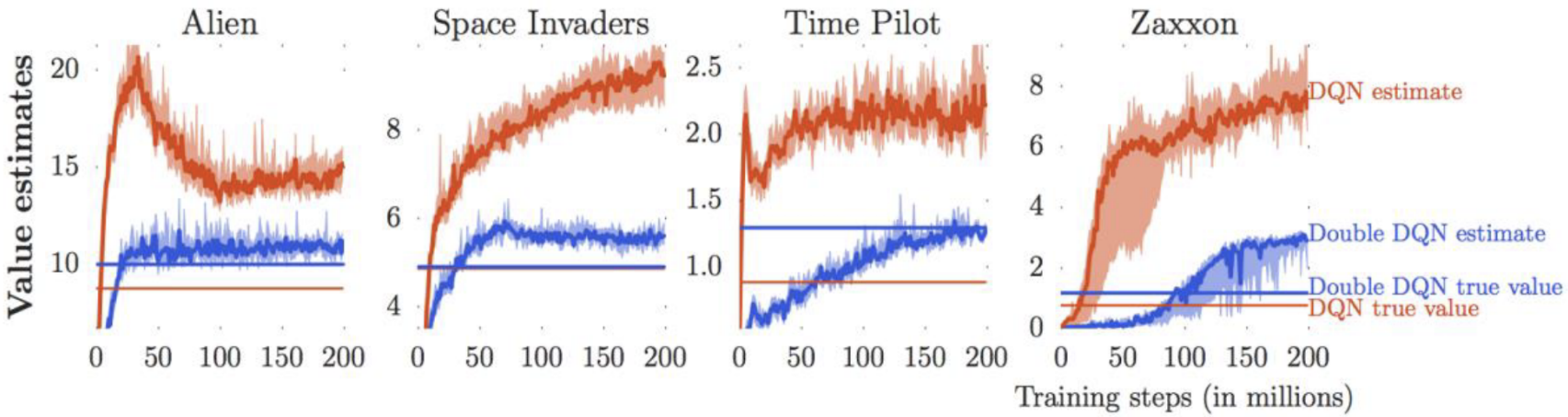
Are the Q-values accurate?



Overestimation in Q-learning

target value $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$

this last term is the problem

imagine we have two random variables: X_1 and X_2

$E[\max(X_1, X_2)] \geq \max(E[X_1], E[X_2])$

$Q_{\phi'}(\mathbf{s}', \mathbf{a}')$ is not perfect – it looks "noisy"

hence $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}')$ *overestimates* the next value!

note that $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}') = \underline{Q_{\phi'}}(\mathbf{s}', \underline{\arg\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}')})$

value *also* comes from $Q_{\phi'}$ action selected according to $Q_{\phi'}$

Double Q-learning

$$E[\max(X_1, X_2)] \geq \max(E[X_1], E[X_2])$$

note that $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}') = \underline{Q_{\phi'}}(\mathbf{s}', \underline{\arg\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}')})$

value *also* comes from $Q_{\phi'}$ action selected according to $Q_{\phi'}$

if the noise in these is decorrelated, the problem goes away!

idea: don't use the same network to choose the action and evaluate value!

"double" Q-learning: use two networks:

$$Q_{\phi_A}(\mathbf{s}, \mathbf{a}) \leftarrow r + \gamma Q_{\phi_B}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi_A}(\mathbf{s}', \mathbf{a}'))$$

$$Q_{\phi_B}(\mathbf{s}, \mathbf{a}) \leftarrow r + \gamma Q_{\phi_A}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi_B}(\mathbf{s}', \mathbf{a}'))$$

if the two Q's are noisy in *different* ways, there is no problem

Double Q-learning in practice

where to get two Q-functions?

just use the current and target networks!

standard Q-learning: $y = r + \gamma Q_{\phi'}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}'))$

double Q-learning: $y = r + \gamma Q_{\phi'}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi}(\mathbf{s}', \mathbf{a}'))$

just use current network (not target network) to evaluate action

still use target network to evaluate value!

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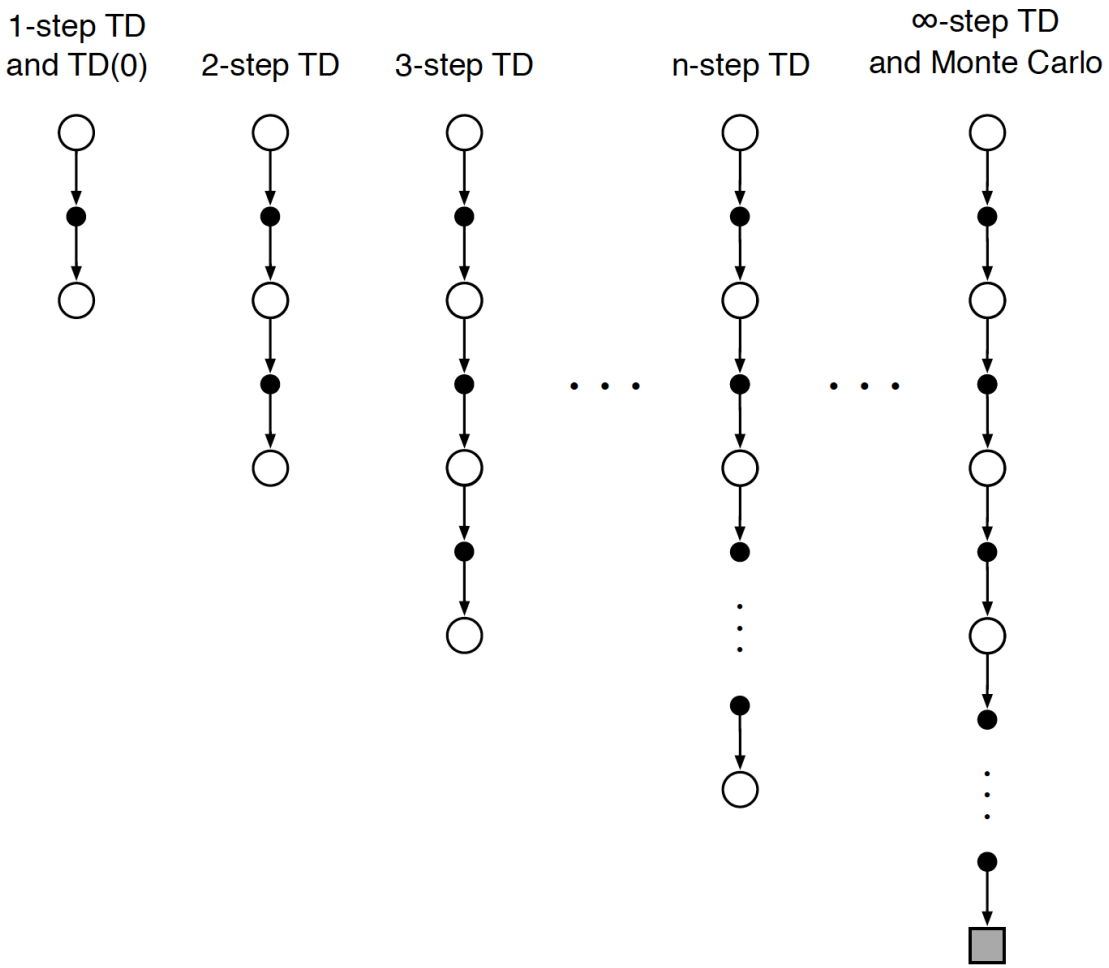
n-step bootstrapping: Combine MC and one-step TD

- Neither MC or one-step TD is always the best, we generalize both methods so that one can shift from one to the other smoothly as needed to meet the demands of a particular task
- One-step TD: In many applications, one wants to be able to update the action very fast to take into account anything that has changed
- However, bootstrapping works best if it is over a length of time in which a significant and recognizable state change has occurred

$n = 1$	n-step TD	$n = \infty$
TD(0)	$\longleftrightarrow$	MC

n-step TD prediction

- Perform an update based on an intermediate number of rewards, more than one, but less than all of them until termination



Recall MC and TD(0) updates

- In MC updates, the target is the **complete return**

$$G_t = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-t+1} R_T$$

$$\begin{aligned} V(S_t) &\leftarrow V(S_t) + \alpha[G_t - V(S_t)] \\ &= V(S_t) + \alpha[R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-t+1} R_T - V(S_t)] \end{aligned}$$

- In TD(0) updates, the target is the **one-step return**

$$G_{t:t+1} = R_{t+1} + \gamma V(S_{t+1})$$

$$\begin{aligned} V(S_t) &\leftarrow V(S_t) + \alpha[G_{t:t+1} - V(S_t)] \\ &= V(S_t) + \alpha[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)] \end{aligned}$$

n-step TD update rule

- For n-step TD, set the target as the n-**step return**

$$G_{t:t+n} = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n})$$

- All n-step returns can be considered approximations to the complete return, truncated after n steps and then corrected for the remaining missing terms by $V(S_{t+n})$

$$\begin{aligned} V(S_t) &\leftarrow V(S_t) + \alpha[G_{t:t+n} - V(S_t)] \\ &= V(S_t) + \alpha[R_{t+1} + \gamma R_{t+2} + ... + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n}) - V(S_t)] \end{aligned}$$

Deep Q-learning with n-step bootstrapping

Q-learning target: $y_{j,t} = r_{j,t} + \gamma \max_{\mathbf{a}_{j,t+1}} Q_{\phi'}(\mathbf{s}_{j,t+1}, \mathbf{a}_{j,t+1})$

these are the only values that matter if $Q_{\phi'}$ is bad!

these values are important if $Q_{\phi'}$ is good

where does the signal come from?

Q-learning does this: max bias, min variance

remember this?

Actor-critic: $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}_{i,t+1}) - \hat{V}^\pi_\phi(\mathbf{s}_{i,t}) \right)$

+ lower variance (due to critic)
- not unbiased (if the critic is not perfect)

Policy gradient: $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\left(\sum_{t'=t}^{T} \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right) - b \right)$

+ no bias
- higher variance (because single-sample estimate)

can we construct multi-step targets, like in actor-critic?

$y_{j,t} = \sum_{t'=t}^{t+N-1} \gamma^{t-t'} r_{j,t'} + \gamma^N \max_{\mathbf{a}_{j,t+N}} Q_{\phi'}(\mathbf{s}_{j,t+N}, \mathbf{a}_{j,t+N})$

N-step return estimator

Deep Q-learning with n-step bootstrapping

$$y_{j,t} = \sum_{t'=t}^{t+N-1} \gamma^{t-t'} r_{j,t'} + \gamma^N \max_{\mathbf{a}_{j,t+N}} Q_{\phi'}(\mathbf{s}_{j,t+N}, \mathbf{a}_{j,t+N})$$

this is supposed to estimate $Q^\pi(\mathbf{s}_{j,t}, \mathbf{a}_{j,t})$ for π

+ less biased target values when Q-values are inaccurate

+ typically faster learning, especially early on

- only actually correct when learning on-policy

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} Q_\phi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

why?

we need transitions $\mathbf{s}_{j,t'}, \mathbf{a}_{j,t'}, \mathbf{s}_{j,t'+1}$ to come from π for $t' - t < N - 1$

(not an issue when $N = 1$)

how to fix?

- ignore the problem
  - often works very well
- cut the trace – dynamically choose N to get only on-policy data
  - works well when data mostly on-policy, and action space is small
- importance sampling

For more details, see: “Safe and efficient off-policy reinforcement learning.” Munos et al. ‘16

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Simple practical tips for Q-learning

- Q-learning takes some care to stabilize
  - Test on easy, reliable tasks first, make sure your implementation is correct

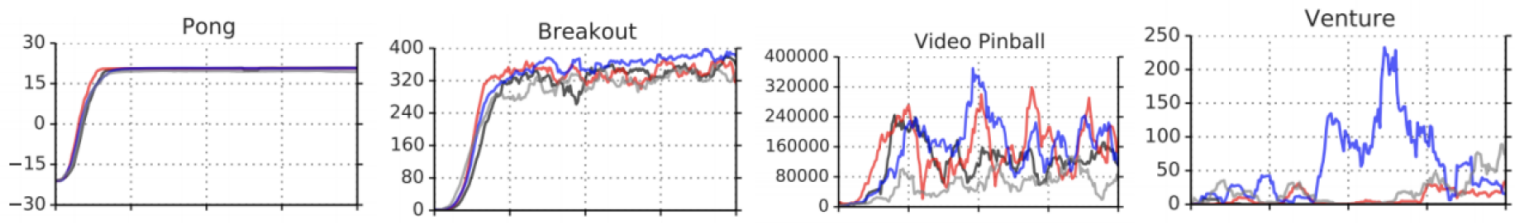


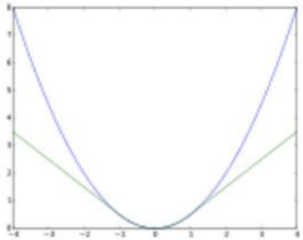
Figure: From T. Schaul, J. Quan, I. Antonoglou, and D. Silver. "Prioritized experience replay". *arXiv preprint arXiv:1511.05952* (2015), Figure 7

- Large replay buffers help improve stability
  - Looks more like fitted Q-iteration
- It takes time, be patient – might be no better than random for a while
- Start with high exploration (epsilon) and gradually reduce

Advanced tips for Q-learning

- Bellman error gradients can be big; clip gradients or user Huber loss

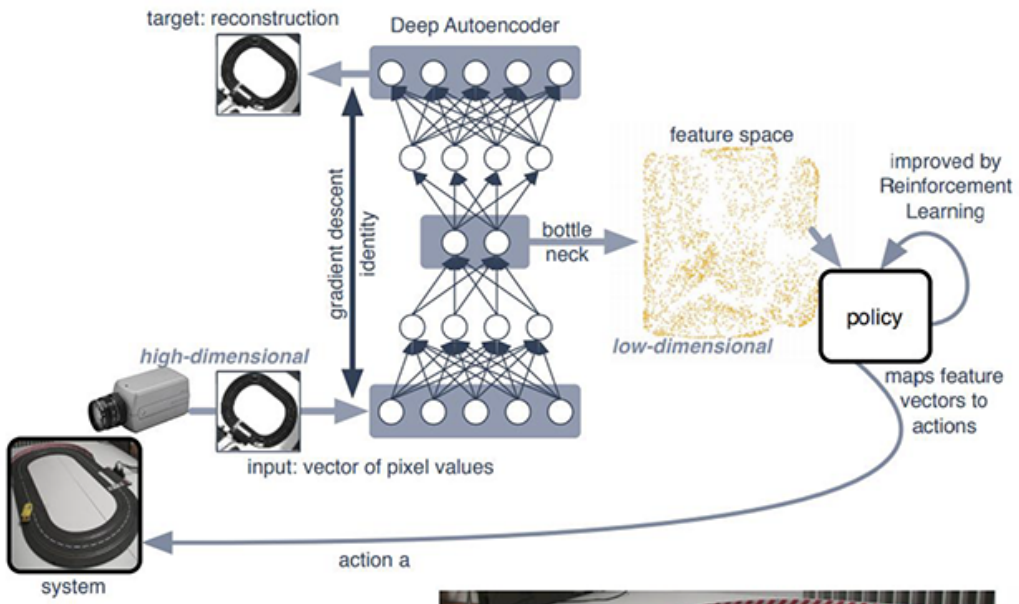
$$L(x) = \begin{cases} x^2/2 & \text{if } |x| \leq \delta \\ \delta|x| - \delta^2/2 & \text{otherwise} \end{cases}$$

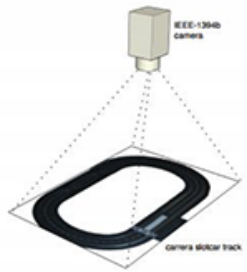


- Double Q-learning helps *a lot* in practice, simple and no downsides
- N-step returns also help a lot, but have some downsides
- Schedule exploration (high to low) and learning rates (high to low), Adam optimizer can help too
- Run multiple random seeds, it's very inconsistent between runs

Fitted Q-iteration in a latent space

- "Autonomous reinforcement learning from raw visual data," Lange & Riedmiller '12
- Q-learning on top of latent space learned with autoencoder
- Uses fitted Q-iteration
- Extra random trees for function approximation (but neural net for embedding)





Q-learning with convolutional networks

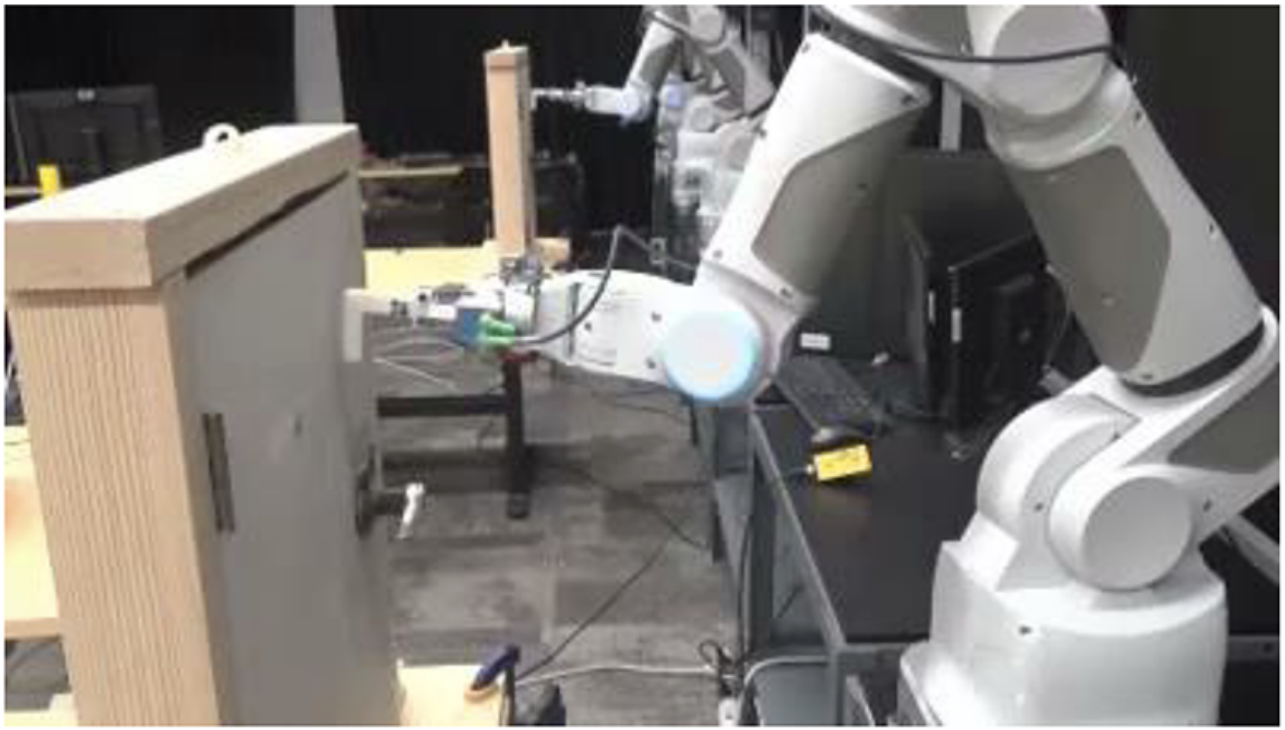
- "Human-level control through deep reinforcement learning," Mnih et al. '13
- Q-learning with convolutional networks
- Uses replay buffer and target network
- One-step backup
- One gradient step
- Can be improved a lot with double Q-learning (and other tricks)

Q-learning with continuous actions

- "Continuous control with deep reinforcement learning," Lillicrap et al. 2015.
- Continuous actions with maximizer network
- Uses replay buffer and target network (with Polyak averaging)
- One-step backup
- One gradient step per simulator step

Q-learning on a real robot

- "Robotic manipulation with deep reinforcement learning and ...," Gu*, Holly*, et al. '17
- Continuous actions with NAF (quadratic in actions)
- Uses replay buffer and target network
- One-step backup
- Four gradient steps per simulator step for efficiency
- Parallelized across multiple robots



Large-scale Q-learning with continuous actions (QT Opt)

live data collection

stored data from all past experiments

$\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i)\}_i$

training buffers

off-policy $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$

on-policy $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$

labeled $(\mathbf{s}, \mathbf{a}, Q_T(\mathbf{s}, \mathbf{a}))$

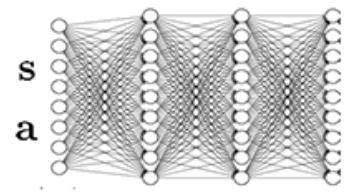
Bellman updaters

compute $Q_T(\mathbf{s}, \mathbf{a}) =$

$r + \max_{\mathbf{a}'} Q_\theta(\mathbf{s}', \mathbf{a}')$

training threads

$\min_\theta ||Q_\theta(\mathbf{s}, \mathbf{a}) - Q_T(\mathbf{s}, \mathbf{a})||^2$



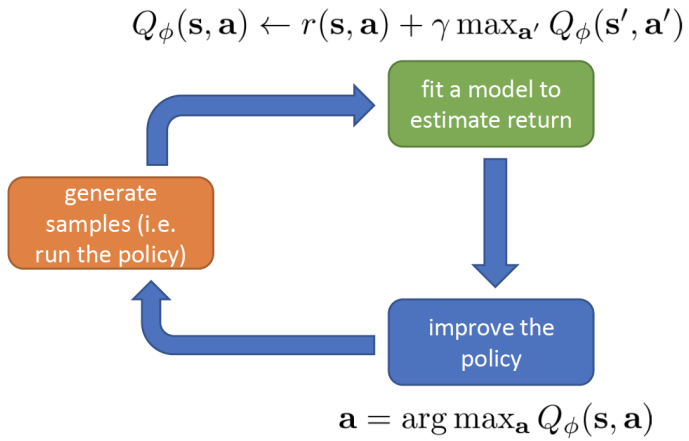
Q_θ

Kalashnikov, Irpan, Pastor, Ibarz, Herzong, Jang, Quillen, Holly, Kalakrishnan, Vanhoucke, Levine. **QT-Opt: Scalable Deep Reinforcement Learning of Vision-Based Robotic Manipulation Skills**

minimize $\sum_i (Q(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \max_{\mathbf{a}'_i} Q(\mathbf{s}'_i, \mathbf{a}'_i)])^2$

Review

- Q-learning with deep neural networks
  - Replay buffers
  - Target networks
- Generalized fitted Q-iteration
- Deep deterministic policy network
  - Deep Q-learning for continuous action space
  - Another network for approximating optimal policy
  - Off-policy
- Extensions
  - Double Q-learning
  - Multi-step Q-learning



Learning objectives of this lecture

- You should be able to...
  - Use deep neural networks to approximate Q-functions, be able to implement deep Q-learning with replay buffers and target networks
  - Use deep deterministic policy gradient for continuous actions
  - Know double Q-learning for addressing the overestimation problem
  - Know deep Q-learning with n-step returns

Deep Q-learning suggested readings

- Lecture 8 of CS285 at UC Berkeley, **Deep Reinforcement Learning, Decision Making, and Control**
  - `http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-8.pdf`

- DRL Q-learning papers
  - Mnih et al. (2013). **Human level control through deep reinforcement learning**: Q-learning with convolutional networks for playing Atari.
  - Van Hasselt, Guez , Silver. (2015). **Deep reinforcement learning with double Q-learning**: a very effective trick to improve performance of deep Q-learning.
  - Lillicrap et al. (2016). **Continuous control with deep reinforcement learning**: continuous Q-learning with actor network for approximate maximization.
  - Wang, Schaul , Hessel, van Hasselt, Lanctot , de Freitas (2016). **Dueling network architectures for deep reinforcement learning**: separates value and advantage estimation in Q-function.
  - Z. Ren, et al., **Self-Paced Prioritized Curriculum Learning With Coverage Penalty in Deep Reinforcement Learning**, *TNNLS*, 2018.

THE END