

Lecture 0: Preliminaries of Machine Learning

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1 Overview

2 Machine Learning

- Supervised Learning, Linear Regression
- Unsupervised Learning, K-means Clustering

What do you think machine learning is?

Machine Learning



what society thinks I
do



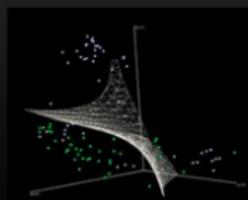
what my friends think
I do



what my parents think
I do

$$\begin{aligned} J_w &= \frac{1}{2} \|w\|^2 + \sum_{i=1}^n \ell(x_i, w^\top b) + \sum_{i=1}^n \alpha_i \\ \alpha_i &\geq 0, \forall i \\ w &= \sum_{i=1}^n \alpha_i x_i, \sum_{i=1}^n \alpha_i = 0 \\ \nabla \hat{\ell}(\theta_t) &= \frac{1}{n} \sum_{i=1}^n \nabla \ell(x_i, y_i; \theta_t) + \nabla r(\theta_t), \\ \theta_{t+1} &= \theta_t - \eta_t \nabla \ell(x_{t(1)}, y_{t(1)}; \theta_t) - \eta_t \cdot \nabla r(\theta_t), \\ \mathbb{E}_{i \sim Q_t} [\ell(x_{t(i)}, y_{t(i)}; \theta_t)] &= \frac{1}{n} \sum_{i=1}^n \ell(x_i, y_i; \theta_t), \end{aligned}$$

what other programmers
think I do



what I think I do

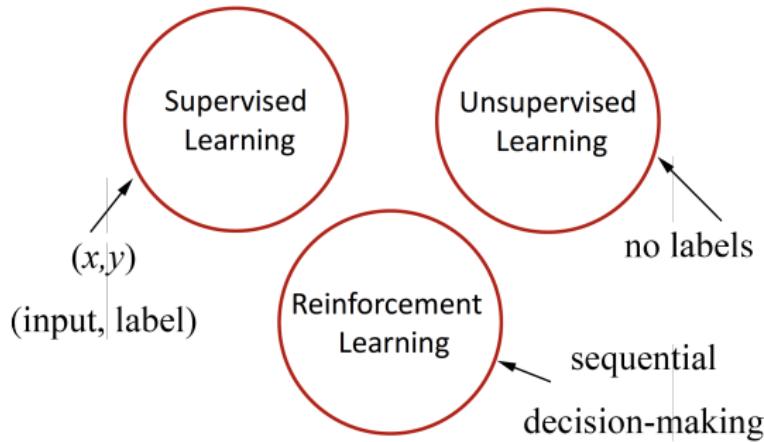
```
>>> from sklearn import svm
```

what I really do

What do you think deep learning is?



Taxonomy of Machine Learning



What we'll cover

- Preliminaries of machine learning
 - Supervised learning: regression, classification
 - Unsupervised learning: clustering, dimensionality reduction
- Finite Markov decision processes (MDPs)
- Tabular RL algorithms:
 - Dynamic programming
 - Monte-Carlo methods
 - Temporal difference learning
- Deep RL algorithms:
 - Policy gradients
 - Actor-critics
 - Value function-based methods
- State-of-the-arts:
 - Transfer RL, meta-RL
 - Evolution strategies for RL
 - Model-based RL
 - Hierarchical RL

Grading

- Homework: 50% (10% per HW \times 5 HWs)
 - Preliminaries of machine learning
 - Tabular RL algorithms
 - Policy gradient
 - Actor-critic
 - Deep Q-learning
- Final Project: 50%
 - Make a presentation (with slides in English) of one paper selected from a given list
- Teaching assistant: Yuanyang Zhu
 - All homeworks should be submitted to yuanyang@smail.nju.edu.cn

Useful links and references

- Course website
 - http://heyuanmingong.github.io/nju_drl/index.html
- Book
 - Richard S. Sutton, and Andrew G. Barto, *Reinforcement Learning: An Introduction*. 2nd Edition,
<http://202.119.32.195/cache/2/03/incompleteideas.net/5b682cef88335157bc5542267cf3f442/RLbook2018.pdf>
- Famous open classes
 - CS 285 at UC Berkeley, Deep Reinforcement Learning,
<http://rail.eecs.berkeley.edu/deeprlcourse/>
 - CS 234 at Stanford, Reinforcement Learning,
<http://web.stanford.edu/class/cs234/>

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Pre-requisite

- Probability
 - distribution, random variable, expectation, conditional probability, variance, density
- Linear algebra
 - matrix multiplication
 - eigenvector
- Basic programming (in Python)

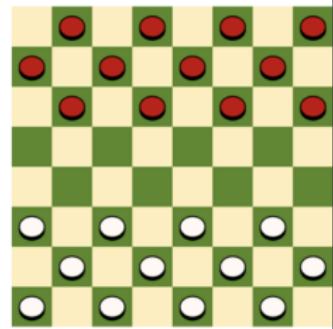
Definition of Machine Learning

Arthur Samuel (1959): Machine Learning is the field of study that gives the computer the ability to learn without being explicitly programmed.



A. L. Samuel*

**Some Studies in Machine Learning
Using the Game of Checkers. II—Recent Progress**

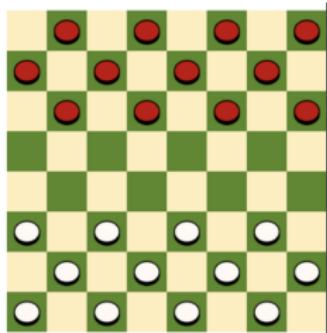


Definition of Machine Learning

Tom Mitchell (1998): a computer program is said to learn from experience E with respect to some class of tasks T and performance measure P , if its performance at tasks in T , as measured by P , improves with experience E .

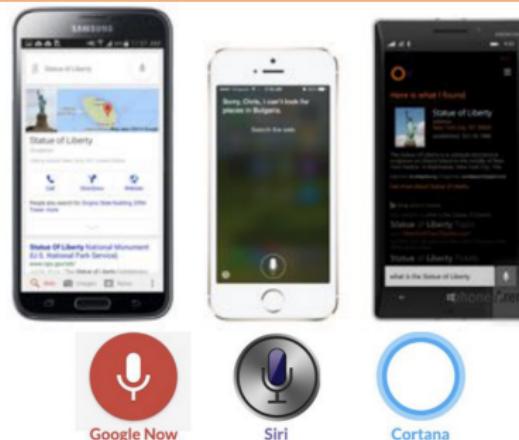
Experience (data): games played by the program
(with itself)

Performance measure: winning rate



Example: Speech Recognition

1. Learning to recognize spoken words

THEN	NOW
<p>“...the SPHINX system (e.g. Lee 1989) learns speaker-specific strategies for recognizing the primitive sounds (phonemes) and words from the observed speech signal...neural network methods...hidden Markov models...”</p> <p>(Mitchell, 1997)</p> 	 <p>Source: https://www.stonetemple.com/great-knowledge-box-showdown/#VoiceStudyResults</p>

Example: Robotics

2. Learning to drive an autonomous vehicle

THEN	NOW
<p>“...the ALVINN system (Pomerleau 1989) has used its learned strategies to drive unassisted at 70 miles per hour for 90 miles on public highways among other cars...”</p> <p>(Mitchell, 1997)</p> 	 <p>https://www.geek.com/wp-content/uploads/2016/03/uber.jpg</p>

Example: Games / Reasoning

3. Learning to beat the masters at board games

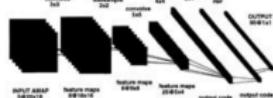
THEN	NOW
<p>“...the world’s top computer program for backgammon, TD-GAMMON (Tesauro, 1992, 1995), learned its strategy by playing over one million practice games against itself...”</p> <p>(Mitchell, 1997)</p> 	

Example: Computer Vision

4. Learning to recognize images

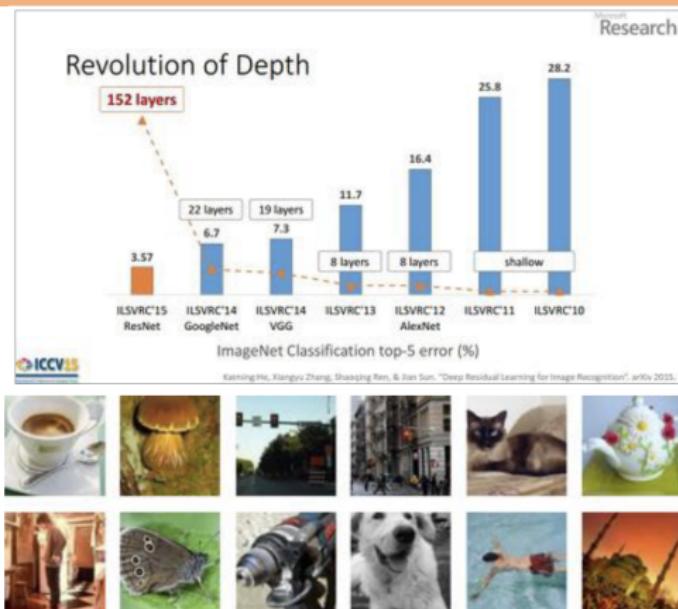
THEN

“...The recognizer is a convolution network that can be spatially replicated. From the network output, a hidden Markov model produces word scores. The entire system is globally trained to minimize word-level errors....”

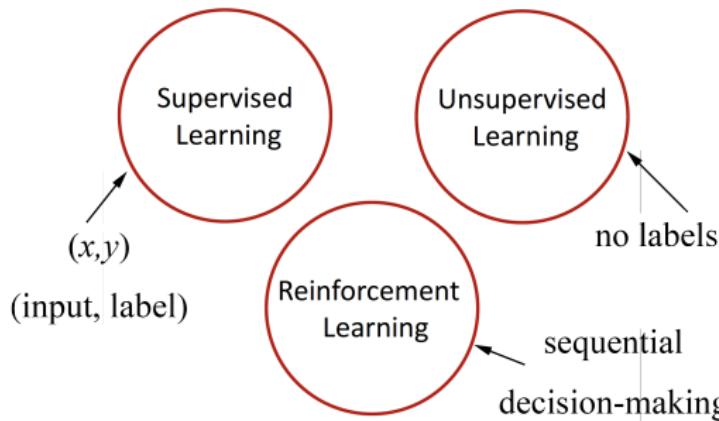


(LeCun et al., 1995)

NOW



Taxonomy of Machine Learning



- a simplistic view based on tasks
- can also be viewed as tools/methods

- Yann LeCun's cake analogy at NIPS 2016:
 - If intelligence is a cake, the bulk of the cake is unsupervised learning, the icing on the cake is supervised learning, and the cherry on the cake is reinforcement learning.

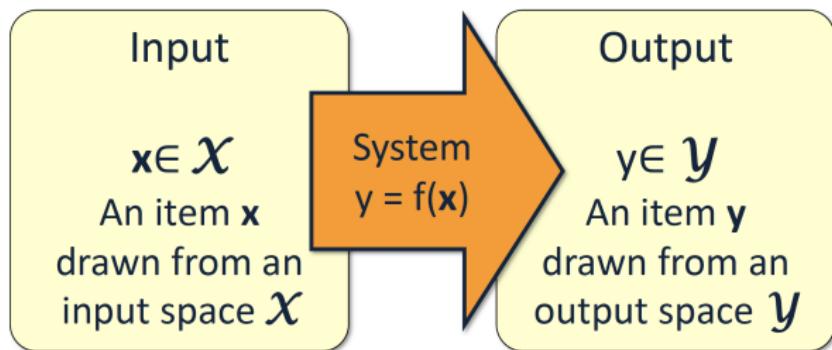
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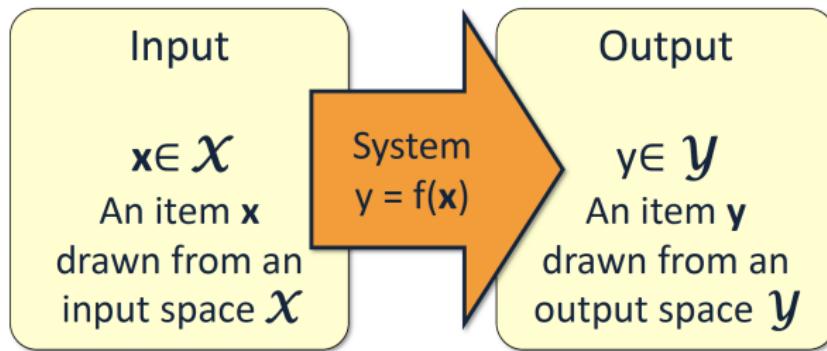
- Supervised Learning, Linear Regression
- Unsupervised Learning, K-means Clustering

Supervised Learning



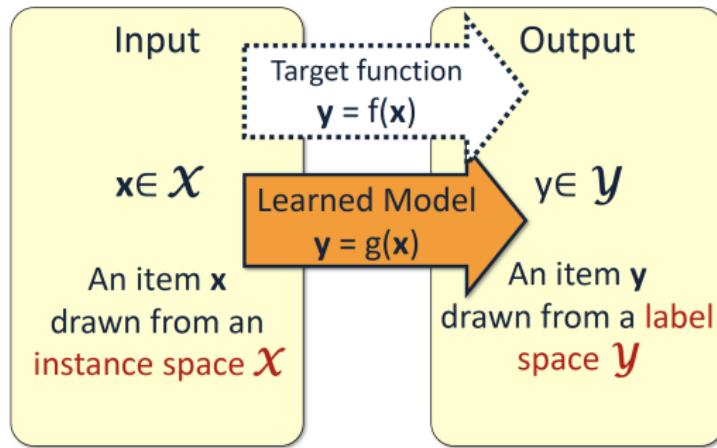
- We consider systems that apply a function $f(\cdot)$ to input items x and return an output $y = f(x)$
- In supervised learning, we deal with systems whose $f(\cdot)$ is learned from samples (x, y)

Why Use Learning?



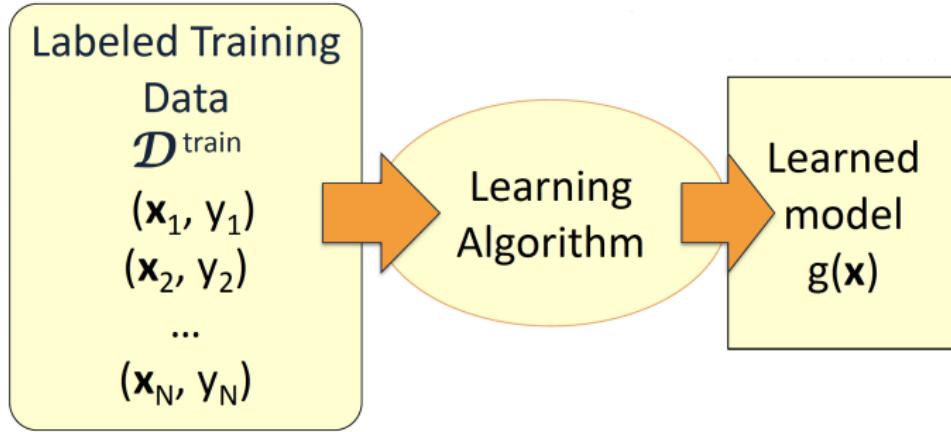
- We typically use machine learning when the function $f(\cdot)$ we want the system to apply is **unknown** to us, and we cannot “think” about it.
- The function could actually be simple.

Learned Model - Hypothesis



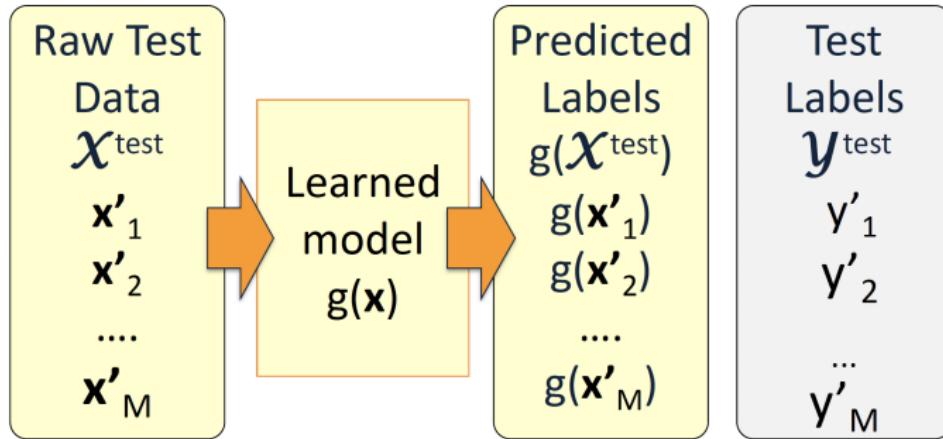
- We need to choose what kind of model we want to learn
 - Linear model, nonlinear model...
 - Parametric model, nonparametric model...
 - Decision trees, neural networks, Gaussian processes...

Training



- Given the learning samples in $\mathcal{D}^{\text{train}}$
- The learner returns a model $g(\cdot)$

Testing



- Apply the model to raw test data
- Evaluate by comparing predicted labels against the test labels

Goal of Supervised Learning

- Given: Samples (\mathbf{x}, \mathbf{y}) of some unknown function
 $f(\cdot) : \mathbf{x} \rightarrow \mathbf{y}$
- Find: A good approximation of f

- \mathbf{x} provides some representation of the input
 - $\mathbf{x} \in \{0, 1\}^d$ or $\mathbf{x} \in \mathbb{R}^d$
- The target function (label)
 - $f(\mathbf{x}) \in \{-1, +1\}$ Binary Classification
 - $f(\mathbf{x}) \in \{1, 2, \dots, k\}$ Multi-class Classification
 - $f(\mathbf{x}) \in \mathbb{R}^k$ Regression

Linear Regression

The output variable y as a linear combination of the input variables x_1, x_2, \dots, x_d plus a noise term ϵ :

$$y = \underbrace{\theta_0 + \theta_1 x_1 + \dots + \theta_d x_d}_{g(\mathbf{x}) = \boldsymbol{\theta}^T \mathbf{x}} + \epsilon$$

- $\boldsymbol{\theta} = [\theta_0, \dots, \theta_d]^T$: model parameters, **to be learned**
- Use linear regression for at least two purposes:
 - **describe** relationships in the data by interpreting $\boldsymbol{\theta}$
 - **predict** future outputs for inputs that we have not yet seen

Example: House Price Prediction

x_1	x_2	y
Living area (feet ²)	#bedrooms	Price (1000\$)
2104	3	400
1600	3	330
2400	3	369
1416	2	232
3000	4	540
:	:	:

$$g(\mathbf{x}) = \hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

Cost Function

Given a training set, how do we pick, or learn, the parameters θ ?



Make $g(\mathbf{x})$ close to y , at least for the training examples we have



Mean squared error



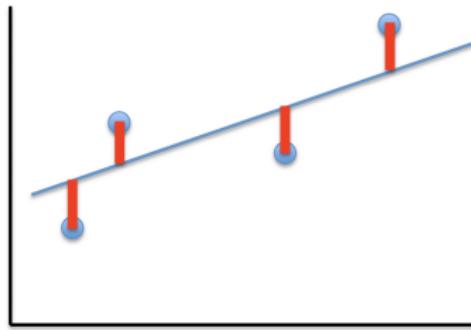
$$J(\boldsymbol{\theta}) = \frac{1}{2} \sum_{i=1}^n (g(\mathbf{x}^i) - y^i)^2$$



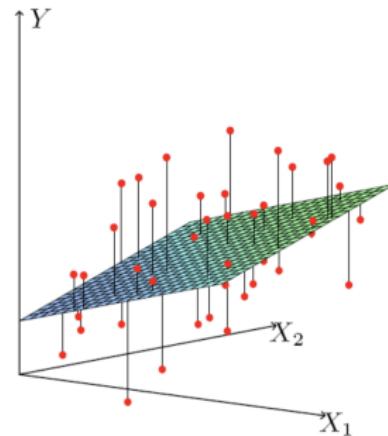
Ordinary least-squares regression model

Cost Function

$$\boldsymbol{\theta}^* = \min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{2} \sum_{i=1}^n (g(\mathbf{x}^i) - y^i)^2$$



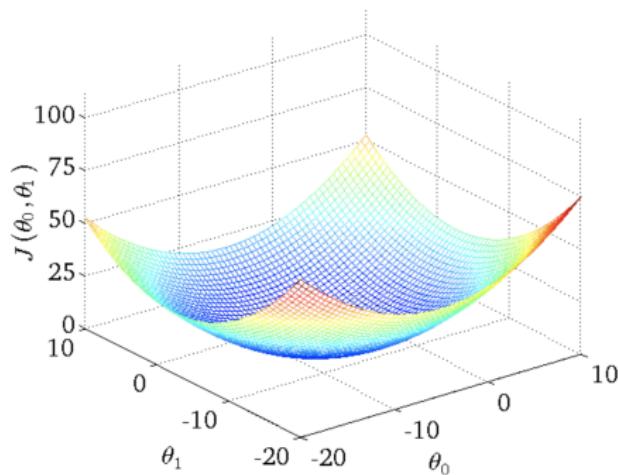
$$\hat{y} = \theta_0 + \theta_1 x$$



$$\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

Cost Function

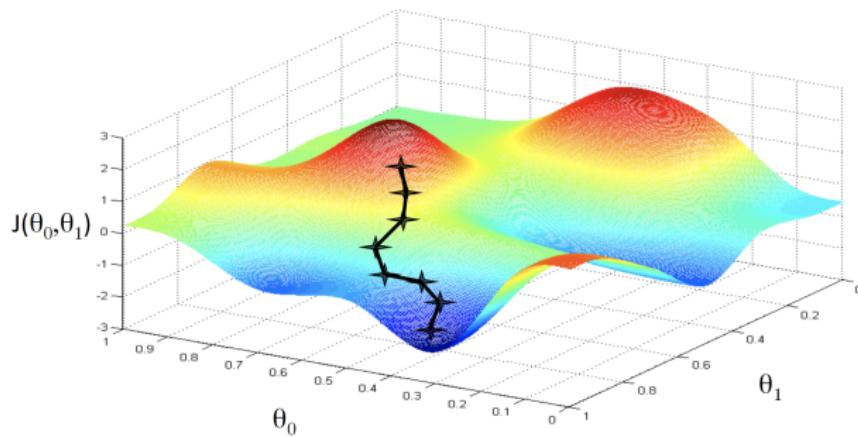
$$\theta^* = \min_{\theta} J(\theta) = \frac{1}{2} \sum_{i=1}^n (g(x^i) - y^i)^2$$



Least-squares cost function is convex!

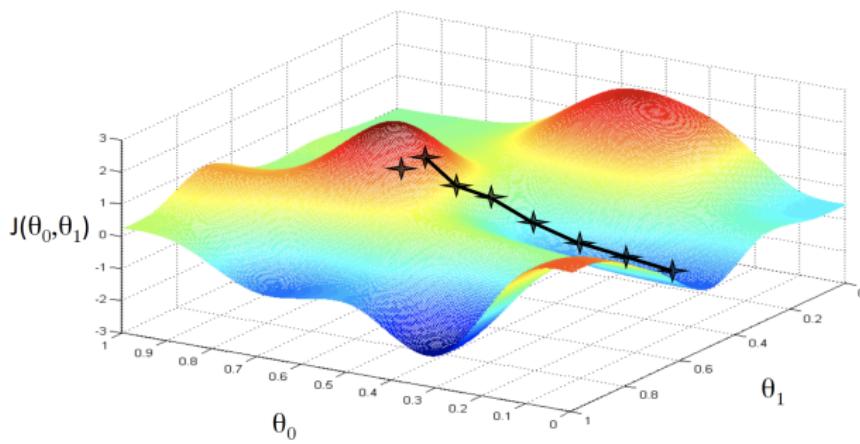
Gradient Descent

- A search procedure:
 - starts with some “initial guess” for θ
 - repeatedly changes θ to make $J(\theta)$ smaller
 - until converging to a minimum



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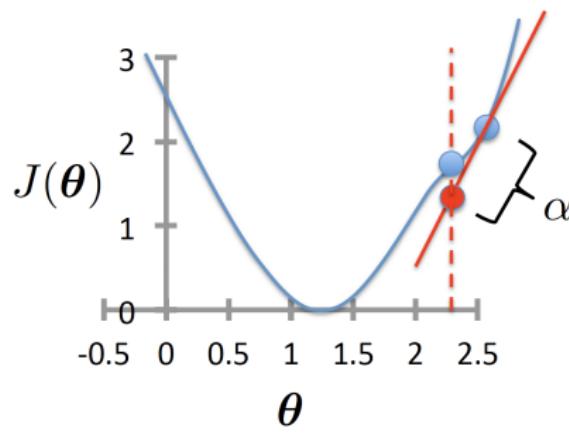
Gradient Descent

- Initialize θ
- Repeat until convergence

$$\theta_j \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

simultaneous update
for $j = 0 \dots d$

learning rate (small)
e.g., $\alpha = 0.05$



Gradient Descent for Linear Regression

$$\begin{aligned}\frac{\partial}{\partial \theta_j} J(\boldsymbol{\theta}) &= \frac{\partial}{\partial \theta_j} \frac{1}{2} (g(\mathbf{x}) - y)^2 \\&= (g(\mathbf{x}) - y) \cdot \frac{\partial}{\partial \theta_j} (g(\mathbf{x}) - y) \cdot \\&= (g(\mathbf{x}) - y) \cdot \frac{\partial}{\partial \theta_j} \left(\sum_{i=0}^d \theta_i x_i - y \right) \\&= (g(\mathbf{x}) - y) x_j\end{aligned}$$

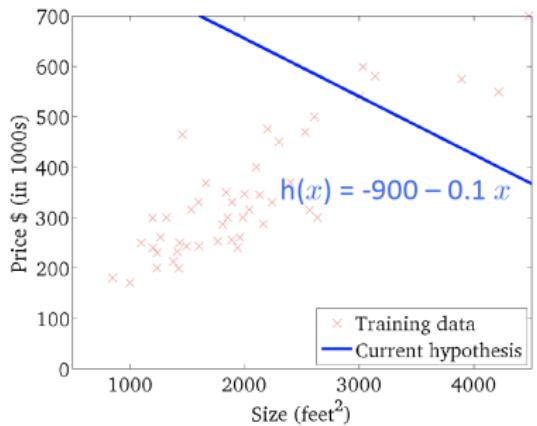
Least Mean Squares (LMS) update rule

- $\theta_j \leftarrow \theta_j - \alpha \cdot (g(\mathbf{x}^i) - y^i) x_j^i$
- Assume convergence when $\|\boldsymbol{\theta}_{new} - \boldsymbol{\theta}_{old}\|_2 < \epsilon$

Visualization of Gradient Descent

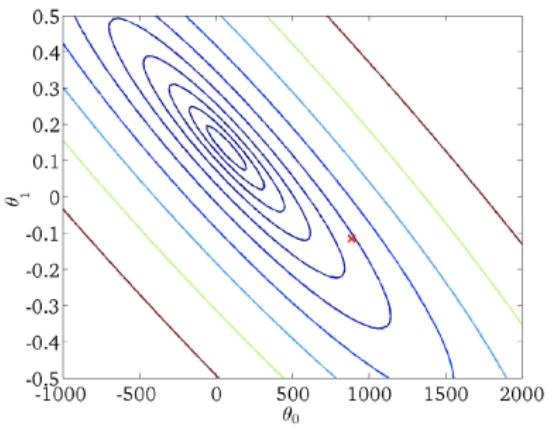
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



$$J(\theta_0, \theta_1)$$

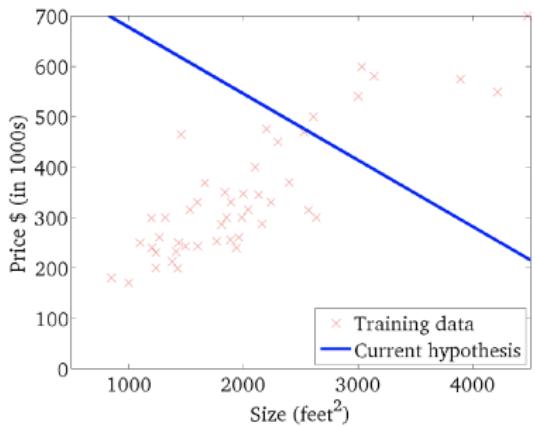
(function of the parameters θ_0, θ_1)



Visualization of Gradient Descent

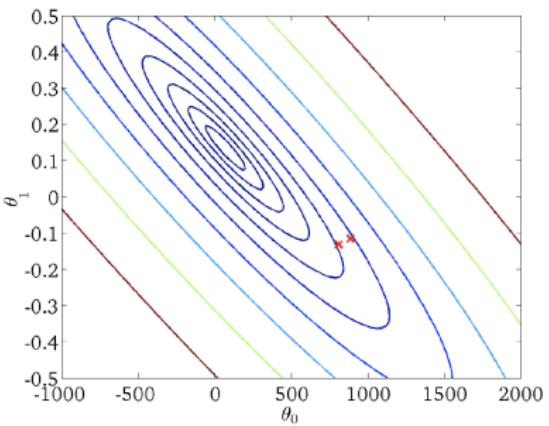
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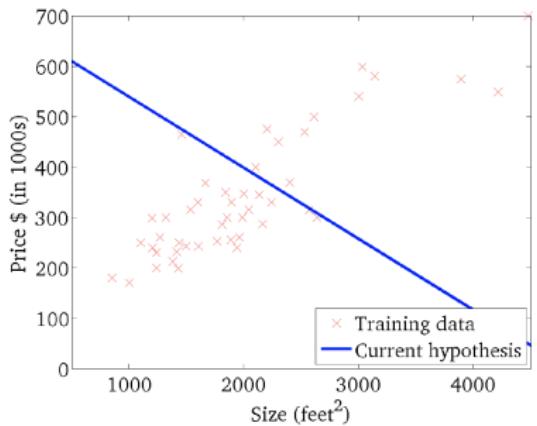
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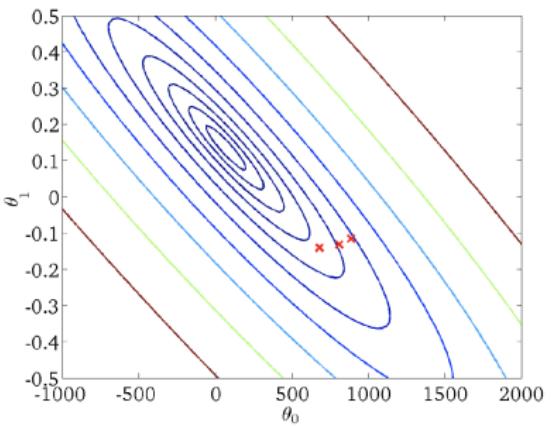
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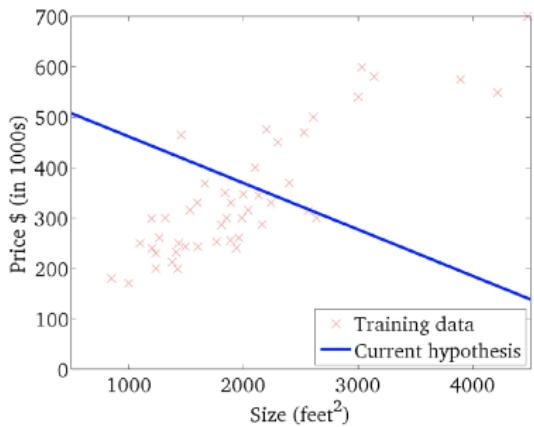
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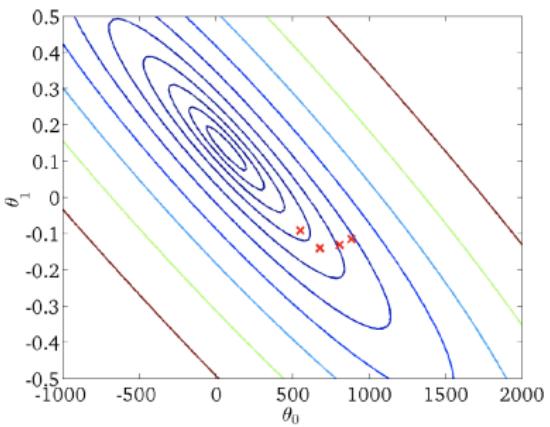
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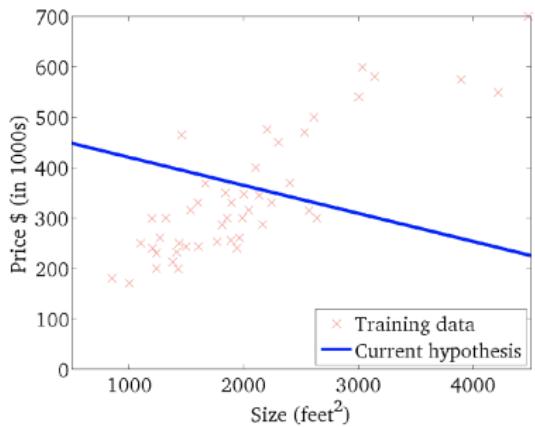
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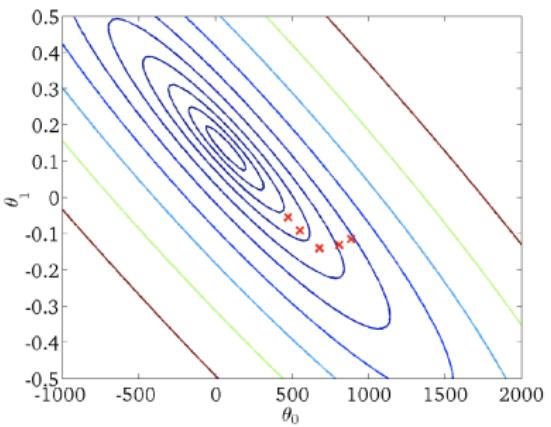
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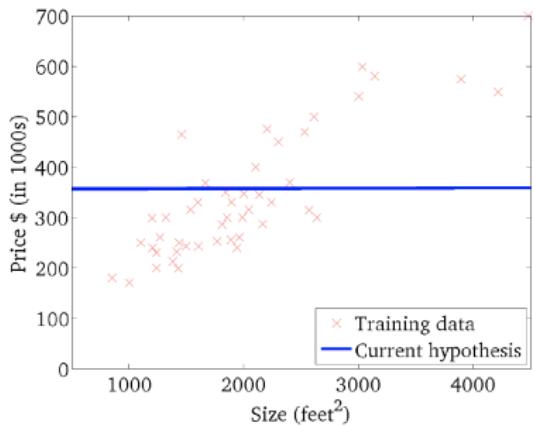
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Visualization of Gradient Descent

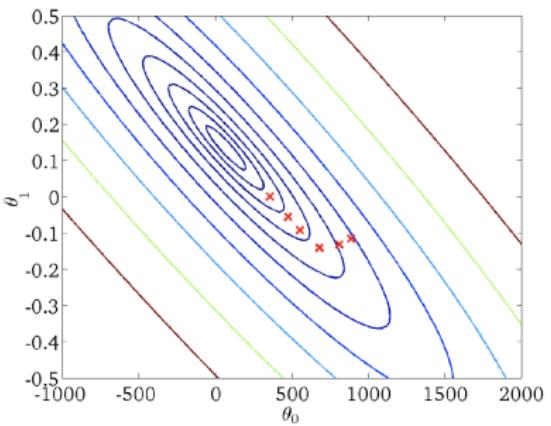
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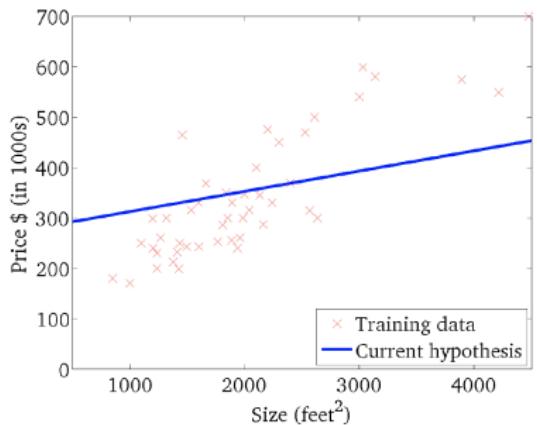
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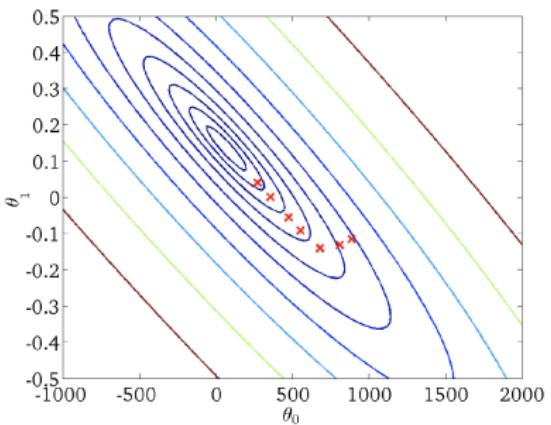
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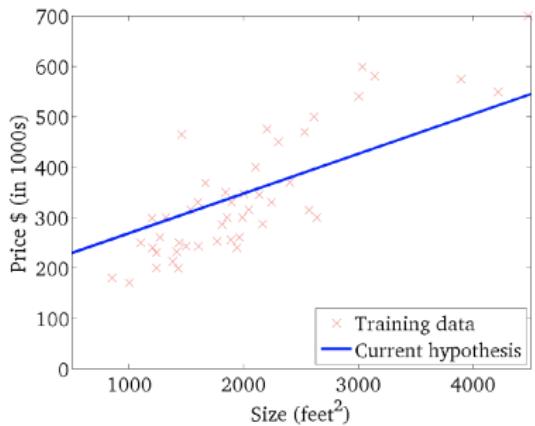
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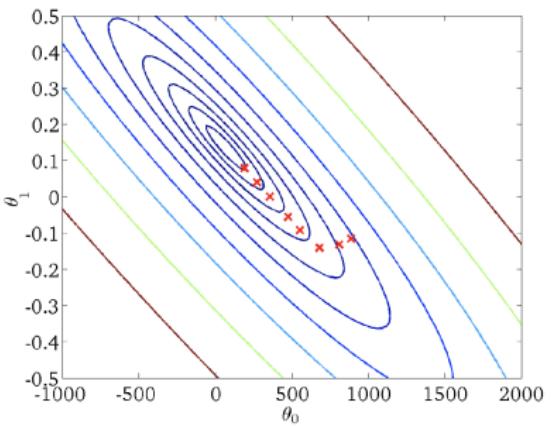
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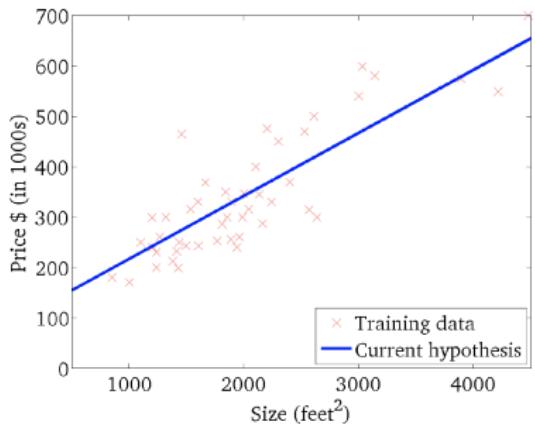
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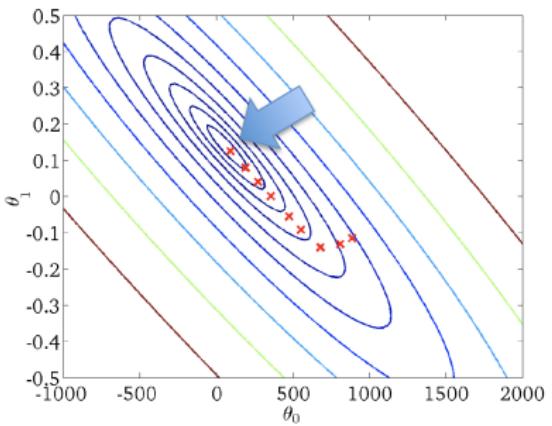
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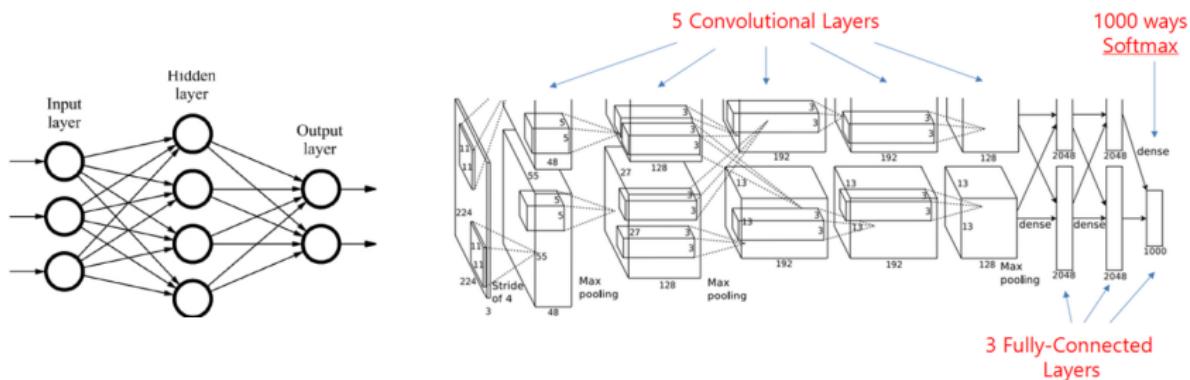
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



Neural Networks? Learn more after class...

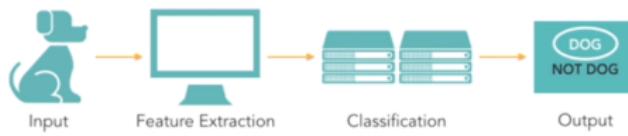
- Input-output pairs $\{(x_i, y_i)\}$
- Function approximator $y \approx g(x)$, using a neural network?
- Cost function, e.g., mean-squared error, cross-entropy loss
- Optimization - gradient descent



Deep Learning? Learn more after class...

- is part of a broader family of machine learning methods based on artificial neural networks with representation learning
 - deep neural networks, deep belief networks, recurrent neural networks, convolutional neural networks, etc
 - learning can be supervised, semi-supervised or unsupervised
 - in an **end-to-end** manner

TRADITIONAL MACHINE LEARNING



DEEP LEARNING



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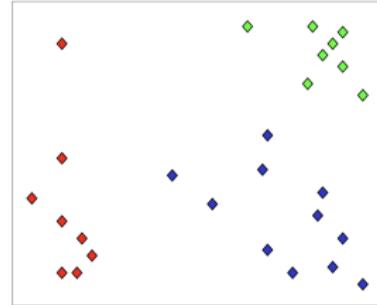
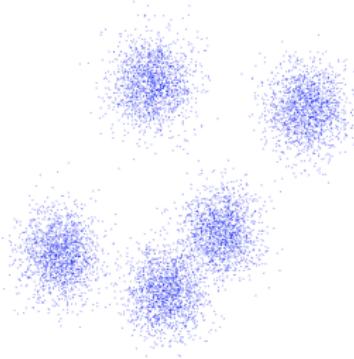
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Unsupervised Learning

- Unsupervised learning is a type of machine learning that **looks for previously undetected patterns** in a data set with **no pre-existing labels** and with a minimum of human supervision.
- Main methods: **Clustering** and **dimensionality reduction**



Clustering

- **Goal:** Automatically partition **unlabeled** data into groups of similar datapoints.
- **Question:** When and why would we want to do this?
- **Useful for:**
 - Automatically organizing data
 - Understanding hidden structure in data
 - Preprocessing for further analysis, representing high-dimensional data in a low-dimensional space (e.g., for visualization purposes)
- Typical algorithms: K-means, Gaussian mixture models...

Applications (Clustering comes up everywhere...)

- Cluster news articles or web pages or search results by topic
- Cluster protein sequences by function or genes according to expression profile
- Cluster users of social networks by interest (community detection)
- Cluster customers according to purchase history
- Cluster galaxies or nearby stars (e.g. Sloan Digital Sky Survey)
- And many many more applications...

K-means Clustering

- **Input:** A set S of n points, also a distance/dissimilarity measure specifying the distance $d(x_i, x_j)$ between pairs (x_i, x_j) .
- **Goal:** Output a partition of the data.
- **K-means:** Find center points c_1, c_2, \dots, c_k to minimize

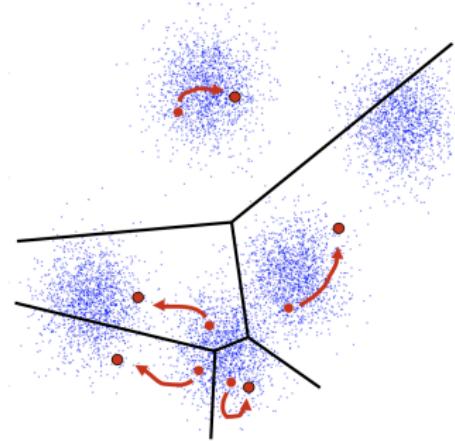
$$\text{minimize} \sum_{i=1}^n \min_{j \in \{1, \dots, k\}} d(x_i, c_j)$$



Euclidean K-means Clustering

- **Input:** a set of n datapoints,
 $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ in \mathbb{R}^d , target number of clusters k .
- **Output:** k representatives,
 $\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_k \in \mathbb{R}^d$
- **Objective:** choose $\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_k \in \mathbb{R}^d$ to minimize

$$\text{minimize} \sum_{i=1}^n \min_{j \in \{1, \dots, k\}} \|\mathbf{x}_i - \mathbf{c}_j\|^2$$



The Iterative Method

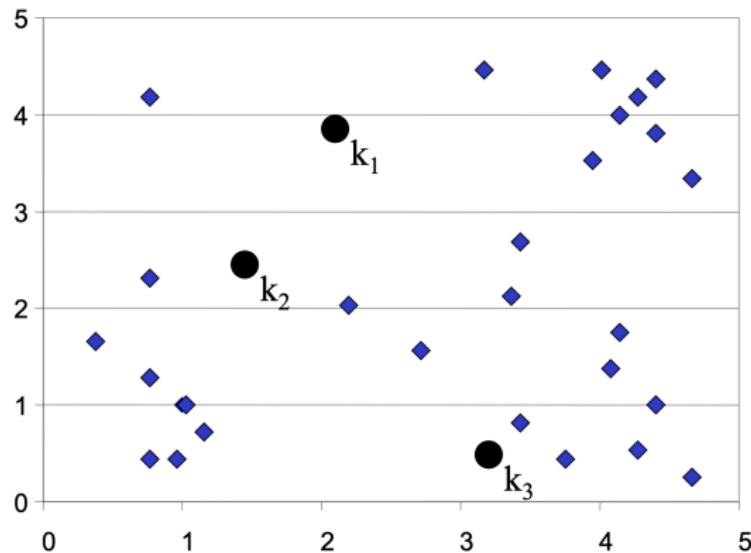
- **Input:** a set of n datapoints, $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ in \mathbb{R}^d , target number of clusters k .
- **Initialize** centers $\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_k \in \mathbb{R}^d$, and clusters $C_1, C_2, \dots, C_k \in \mathbb{R}^d$ in any way
- **Repeat** until there is no further change in the cost
 - for each j : $C_j \leftarrow \{ \text{whose closest center is } \mathbf{c}_j \}$
 - for each j : $\mathbf{c}_j \leftarrow \text{mean of } C_j$

Holding $\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_k$ fixed,
pick optimal C_1, C_2, \dots, C_k

Holding C_1, C_2, \dots, C_k fixed,
pick optimal $\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_k$

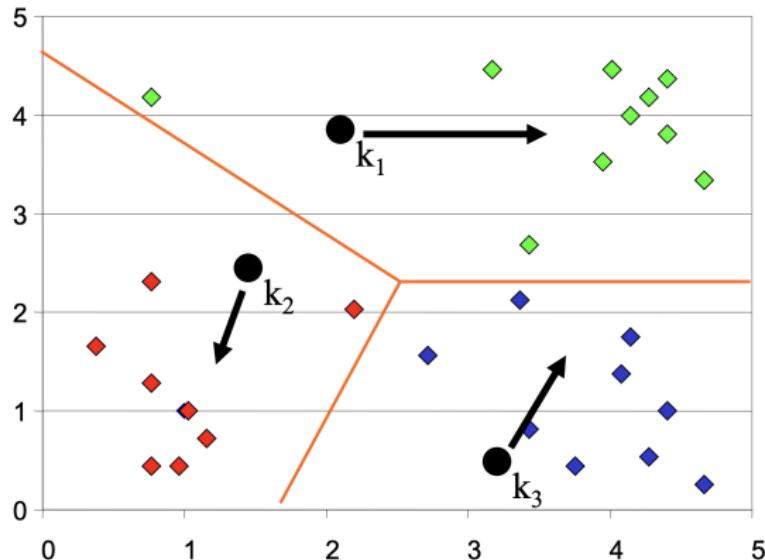
K-means Clustering: Initialization

- Decide k , and initialize k centers (randomly)



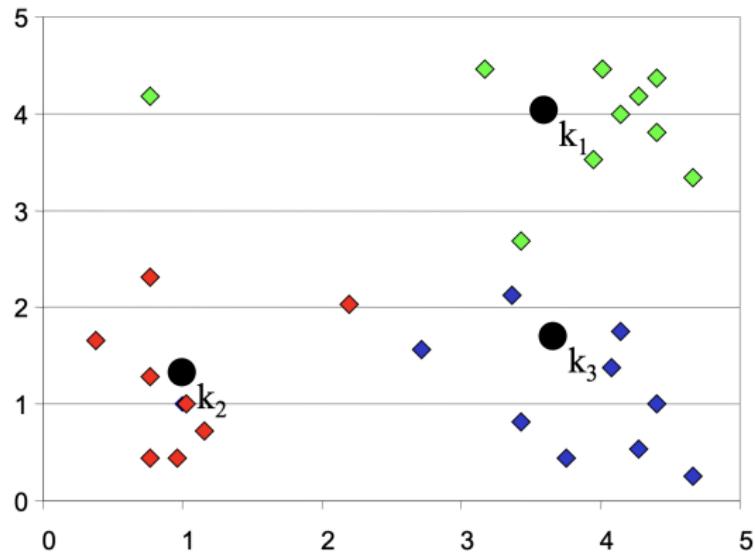
K-means Clustering: Iteration 1

- Assign all objects to the nearest center. Move a center to the mean of its members.



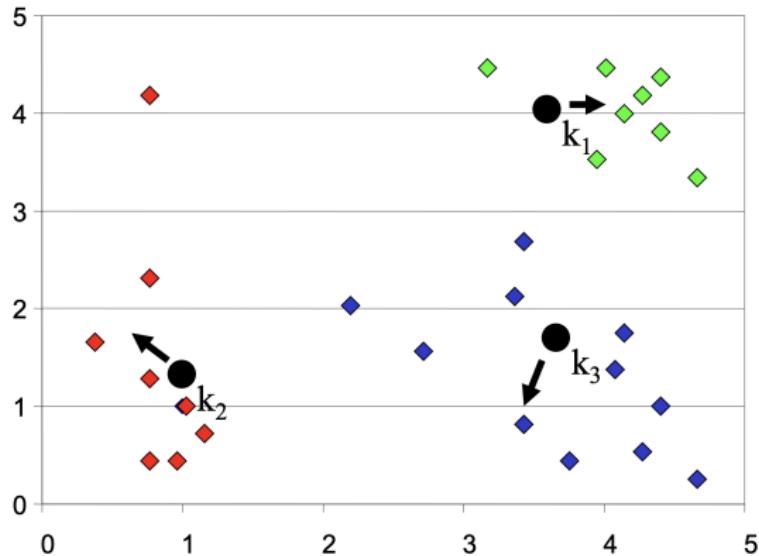
K-means Clustering: Iteration 2

- After moving centers, re-assign the objects...



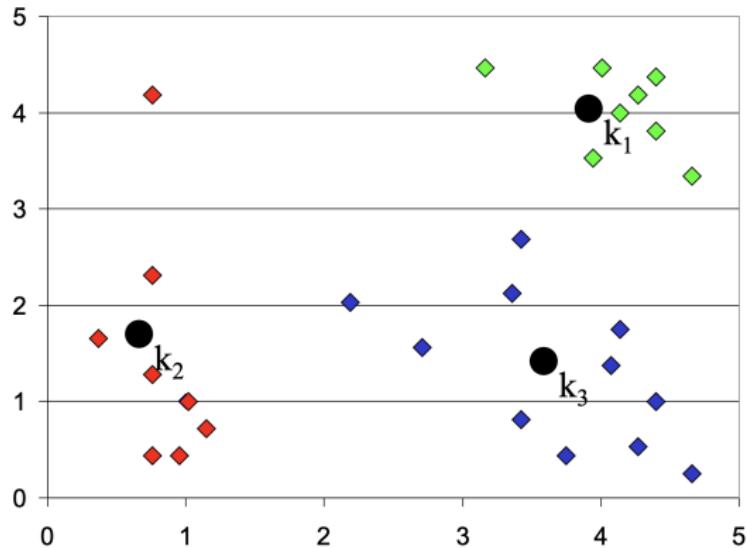
K-means Clustering: Iteration 2

- After moving centers, re-assign the objects to nearest centers. Move a center to the mean of its new members.

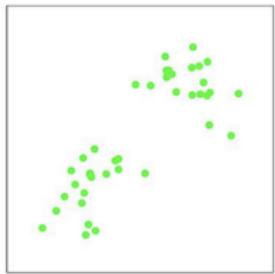


K-means Clustering: Finished!

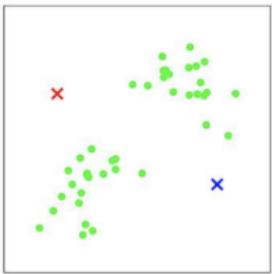
- Re-assign and move centers, until no objects changed membership.



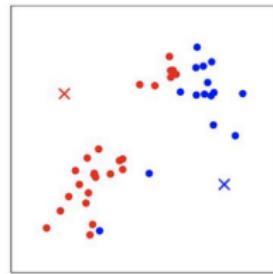
Example for K-means



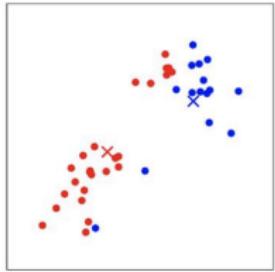
(a)



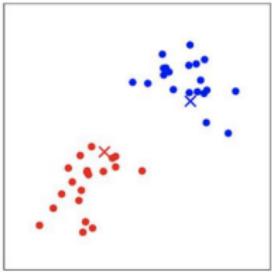
(b)



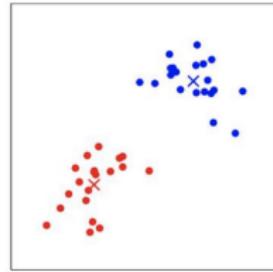
(c)



(d)



(e)



(f)

Learning objectives of this lecture

You should be able to...

- Know elements of supervised learning and unsupervised learning
- Be familiar with basic algorithms, e.g., linear regression and k-means
- Learn neural networks after class

References

- Stanford University, *Machine Learning*
 - <http://cs229.stanford.edu/>
- Carnegie Mellon University, *Introduction to Machine Learning*
 - <https://www.cs.cmu.edu/~ninanmf/courses/401sp18/lectures.shtml>

Homework 1

- Study the following two algorithms after class:
 - Logistic regression for two-class classification, **supervised learning**
 - Gaussian mixture model with expectation-maximization for clustering, **unsupervised learning**
- Write a report (in English) that introduces the two algorithms
 - Theories, explanations, formula derivations, potential applications...
 - Submit the report to yuanyang@smail.nju.edu.cn

THE END