Lecture 6: Deep Reinforcement Learning

Value Function Methods

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Dec. 10th, 2019

Table of Contents

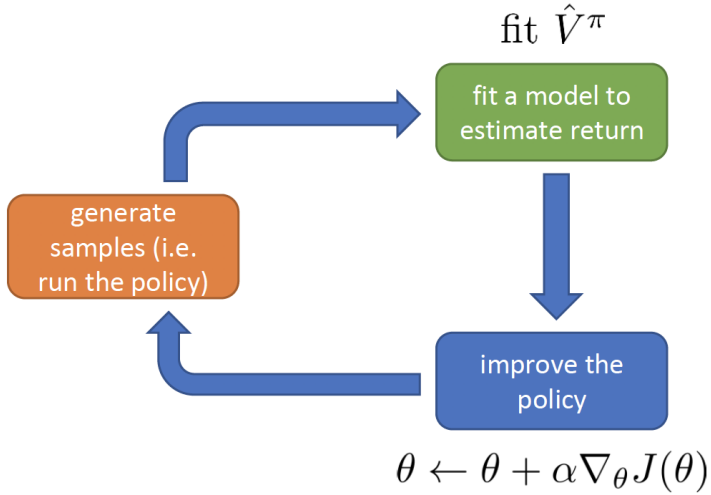
Contents and Goals

- What if we just use a critic, without an actor?
- Extracting a policy from a value function
- The fitted value iteration, fitted Q-iteration algorithms
- Goals
  - Understand how value functions give rise to policies
  - Understand the Q-learning with function approximation algorithm
  - Understand practical considerations for Q-learning

Recall: actor-critic

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i)\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



Can we omit policy gradient completely?

$A^\pi(\mathbf{s}_t, \mathbf{a}_t)$: how much better is $\mathbf{a}_t$ than the average action according to π

$\arg\max_{\mathbf{a}_t} A^\pi(\mathbf{s}_t, \mathbf{a}_t)$: best action from $\mathbf{s}_t$, if we then follow π

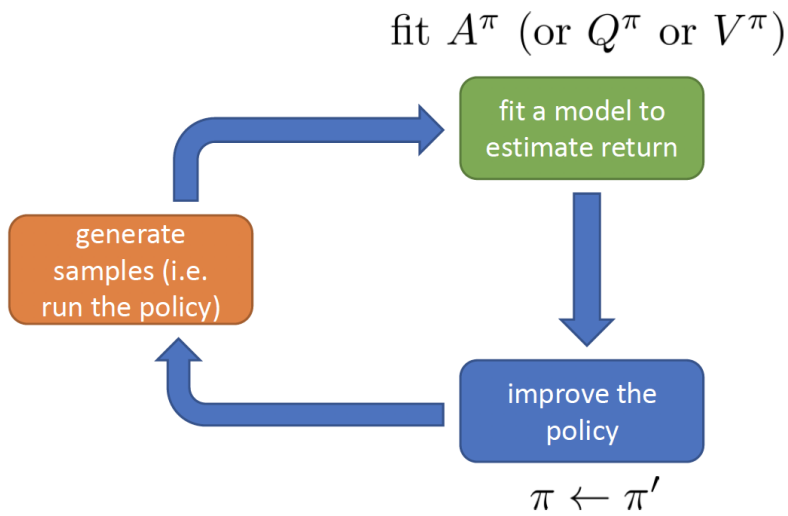
at *least* as good as any $\mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)$

regardless of what $\pi(\mathbf{a}_t|\mathbf{s}_t)$ is!

forget policies, let's just do this!

$$\pi'(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} A^\pi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

as good as π
(probably better)



Recall the Policy Improvement Theorem

- Let π and π' be any pair of deterministic policies such that,

$$Q_\pi(s, \pi'(s)) \geq v_\pi(s), \quad \forall s \in \mathcal{S}.$$

Then the policy π' must be as good as, or better than, π.

Policy improvement theorem

$$
\begin{aligned}
v_\pi(s) &\leq Q_\pi(s, \pi'(s)) \\
&= \mathbb{E}[R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s, A_t = \pi'(s)] \\
&= \mathbb{E}_{\pi'}[R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s] \\
&\leq \mathbb{E}_{\pi'}[R_{t+1} + \gamma Q_\pi(S_{t+1}, \pi'(S_{t+1})) | S_t = s] \\
&= \mathbb{E}_{\pi'}[R_{t+1} + \gamma \mathbb{E}_{\pi'}[R_{t+2} + \gamma v_\pi(S_{t+2}) | S_{t+1}, A_{t+1} = \pi'(S_{t+1})] | S_t = s] \\
&= \mathbb{E}_{\pi'}[R_{t+1} + \gamma R_{t+2} + \gamma^2 v_\pi(S_{t+2}) | S_t = s] \\
&\leq \mathbb{E}_{\pi'}[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 v_\pi(S_{t+3}) | S_t = s] \\
&\leq ... \\
&\leq \mathbb{E}_{\pi'}[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 R_{t+4} + ... | S_t = s] \\
&= v_{\pi'}(s)
\end{aligned}
$$

Policy improvement

- Consider the new **greedy** policy, π', selecting at each state the action that appears best according to $Q_\pi(s, a)$

$$\begin{aligned}\pi'(s) &= \arg\max_a Q_\pi(s, a) \\ &= \arg\max_a \mathbb{E}[R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s, A_t = a] \\ &= \arg\max_a \sum_{s', a} p(s', r | s, a)[r + \gamma v_\pi(s')]\end{aligned}$$

- The process of making a new policy that improves on an original policy, by making greedy w.r.t. the value function of the original policy, is called **policy improvement**
  - The greedy policy meets the conditions of the policy improvement theorem

The Generalized Policy Iteration framework

High level idea:

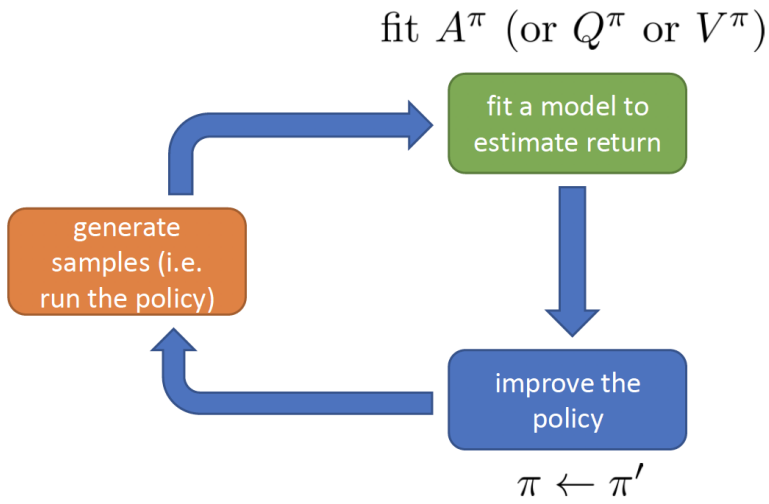
policy iteration algorithm:

1. evaluate $A^\pi(\mathbf{s}, \mathbf{a})$ ⟵ how to do this?
2. set $\pi \leftarrow \pi'$

$$\pi'(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} A^\pi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

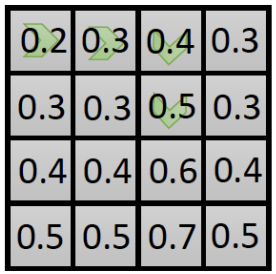
as before: $A^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma E[V^\pi(\mathbf{s}')] - V^\pi(\mathbf{s})$

let's evaluate $V^\pi(\mathbf{s})$!



Dynamic programming (model-based)

Let's assume we know $p(\mathbf{s}'|\mathbf{s},\mathbf{a})$, and $\mathbf{s}$ and $\mathbf{a}$ are both discrete (and small)



16 states, 4 actions per state

can store full $V^\pi(\mathbf{s})$ in a table!

$\mathcal{T}$ is $16 \times 16 \times 4$ tensor

bootstrapped update: $V^\pi(\mathbf{s}) \leftarrow E_{\mathbf{a}\sim\pi(\mathbf{a}|\mathbf{s})}[r(\mathbf{s},\mathbf{a}) + \gamma E_{\mathbf{s}'\sim p(\mathbf{s}'|\mathbf{s},\mathbf{a})}[V^\pi(\mathbf{s}')]]$

just use the current estimate here

$$\pi'(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} A^\pi(\mathbf{s}_t,\mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases} \longrightarrow \text{deterministic policy } \pi(\mathbf{s}) = \mathbf{a}$$

simplified: $V^\pi(\mathbf{s}) \leftarrow r(\mathbf{s},\pi(\mathbf{s})) + \gamma E_{\mathbf{s}'\sim p(\mathbf{s}'|\mathbf{s},\pi(\mathbf{s}))}[V^\pi(\mathbf{s}')]$

Recall policy iteration

- Using policy improvement theorem, we can obtain a sequence of monotonically improving policies and value functions

$$\pi_0 \xrightarrow{E} v_{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} v_{\pi_1} \xrightarrow{I} \pi_2 \xrightarrow{E} \ldots \xrightarrow{I} \pi_* \xrightarrow{E} v_*$$

- This process is guaranteed to converge to an optimal policy and optimal value function in a finite number of iterations
  - Each policy is guaranteed to be a strictly improvement over the previous one unless it is already optimal
  - A finite MDP has only a finite number of policies

Policy iteration algorithm

Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$

2. Policy Evaluation
   Loop:
   $\Delta \leftarrow 0$
   Loop for each $s \in \mathcal{S}$:
   $v \leftarrow V(s)$
   $V(s) \leftarrow \sum_{s',r} p(s', r \,|\, s, \pi(s)) \big[r + \gamma V(s')\big]$
   $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

3. Policy Improvement
   policy-stable $\leftarrow$ *true*
   For each $s \in \mathcal{S}$:
   old-action $\leftarrow \pi(s)$
   $\pi(s) \leftarrow \arg\max_a \sum_{s',r} p(s', r \,|\, s, a) \big[r + \gamma V(s')\big]$
   If *old-action* $\neq \pi(s)$, then *policy-stable* $\leftarrow$ *false*
   If *policy-stable*, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2

Dynamic programming with policy iteration

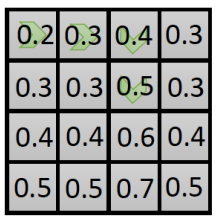
policy iteration:

1. evaluate $V^\pi(\mathbf{s})$
2. set $\pi \leftarrow \pi'$

$$\pi'(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} A^\pi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

policy evaluation:

$$V^\pi(\mathbf{s}) \leftarrow r(\mathbf{s}, \pi(\mathbf{s})) + \gamma E_{\mathbf{s}' \sim p(\mathbf{s}'|\mathbf{s}, \pi(\mathbf{s}))}[V^\pi(\mathbf{s}')]$$



16 states, 4 actions per state

can store full $V^\pi(\mathbf{s})$ in a table!

$\mathcal{T}$ is $16 \times 16 \times 4$ tensor

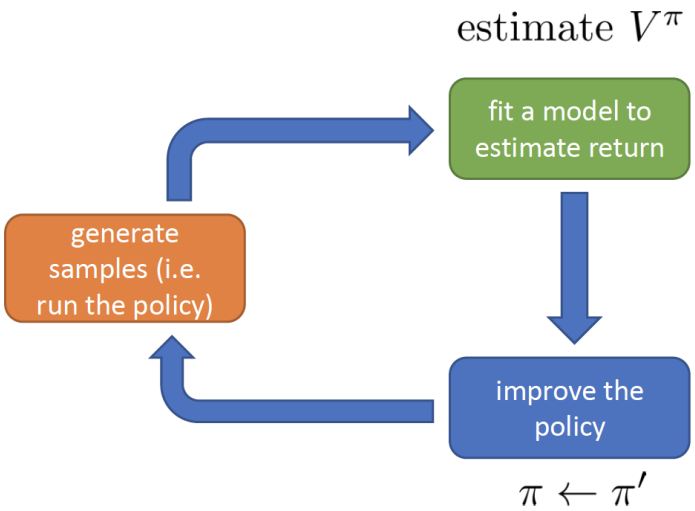


Table of Contents

Recall: Value iteration = Truncate policy evaluation for one sweep

- In policy iteration, stop policy evaluation after just one sweep

$$v_{k+1}(s) = \sum_{s',r} p(s', r|s, \pi_k(s))[r + \gamma v_k(s')]$$

$$\pi_{k+1}(s) = \arg\max_a \sum_{s',r} p(s', r|s, a)[r + \gamma v_{k+1}(s')]$$

- Combine into one operation, called **value iteration** algorithm

$$v_{k+1}(s) = \max_a \sum_{s',r} p(s', r|s, a)[r + \gamma v_k(s')]$$

  - For arbitrary v_0, the sequence $\{v_k\}$ converges to v_* under the same conditions that guarantee the existence of v_*

Value iteration

- Bellman optimality equation

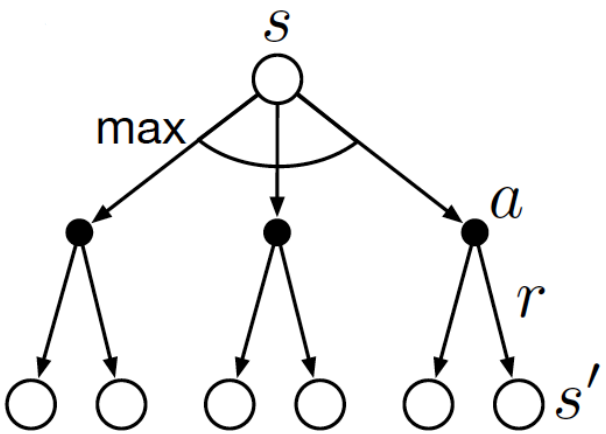
$$v_*(s) = \max_a \sum_{s',r} p(s', r|s, a)[r + \gamma v_*(s')]$$

- Value iteration

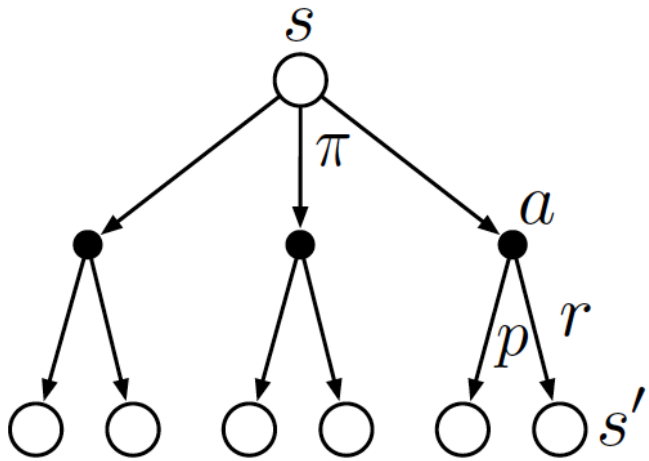
$$v_{k+1}(s) = \max_a \sum_{s',r} p(s', r|s, a)[r + \gamma v_k(s')]$$

  - Turn Bellman optimality equation into an update rule
  - Directly approximate the optimal state-value function, v_*

Value iteration vs. policy evaluation



Backup diagram for value iteration



Backup diagram for policy evaluation

Value iteration algorithm

Value Iteration, for estimating $\pi \approx \pi_*$

Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop:
| $\Delta \leftarrow 0$
| Loop for each $s \in \mathcal{S}$:
| $\quad v \leftarrow V(s)$
| $\quad V(s) \leftarrow \max_a \sum_{s',r} p(s', r|s, a)\big[r + \gamma V(s')\big]$
| $\quad \Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

Output a deterministic policy, $\pi \approx \pi_*$, such that
$\pi(s) = \text{argmax}_a \sum_{s',r} p(s', r|s, a)\big[r + \gamma V(s')\big]$

- One sweep = one sweep of policy evaluation + one sweep of policy improvement

Dynamic programming with value iteration

$$\pi'(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} A^\pi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

$$A^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma E[V^\pi(\mathbf{s}')] - V^\pi(\mathbf{s})$$

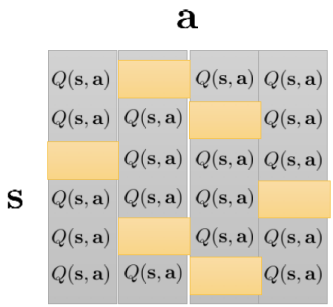
$$\arg\max_{\mathbf{a}_t} A^\pi(\mathbf{s}_t, \mathbf{a}_t) = \arg\max_{\mathbf{a}_t} Q^\pi(\mathbf{s}_t, \mathbf{a}_t)$$

$$Q^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma E[V^\pi(\mathbf{s}')] \text{ (a bit simpler)}$$

skip the policy and compute values directly!

value iteration algorithm:

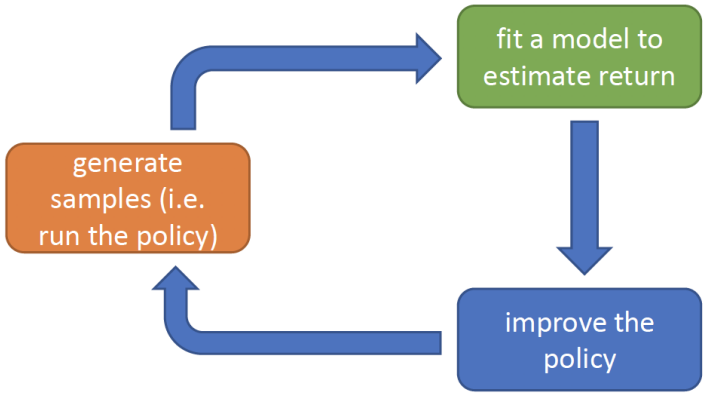
1. set $Q(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma E[V(\mathbf{s}')]$
2. set $V(\mathbf{s}) \leftarrow \max_{\mathbf{a}} Q(\mathbf{s}, \mathbf{a})$



$\arg\max_{\mathbf{a}} Q(\mathbf{s}, \mathbf{a}) \rightarrow$ policy

approximates the new value!

$$Q^\pi(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma E_{\mathbf{s}' \sim p(\mathbf{s}'|\mathbf{s},\mathbf{a})}[V^\pi(\mathbf{s}')]$$



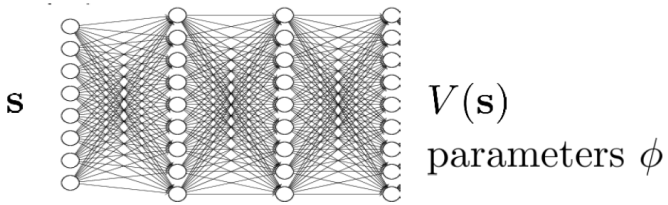
$$V^\pi(\mathbf{s}) \leftarrow \max_{\mathbf{a}} Q^\pi(\mathbf{s}, \mathbf{a})$$

Fitted value iteration with function approximation

how do we represent $V(\mathbf{s})$?

big table, one entry for each discrete $\mathbf{s}$

neural net function $V : \mathcal{S} \rightarrow \mathbb{R}$



$$\mathbf{s} = 0 : V(\mathbf{s}) = 0.2$$
$$\mathbf{s} = 1 : V(\mathbf{s}) = 0.3$$
$$\mathbf{s} = 2 : V(\mathbf{s}) = 0.5$$

curse of dimensionality

$$|\mathcal{S}| = (255^3)^{200 \times 200}$$

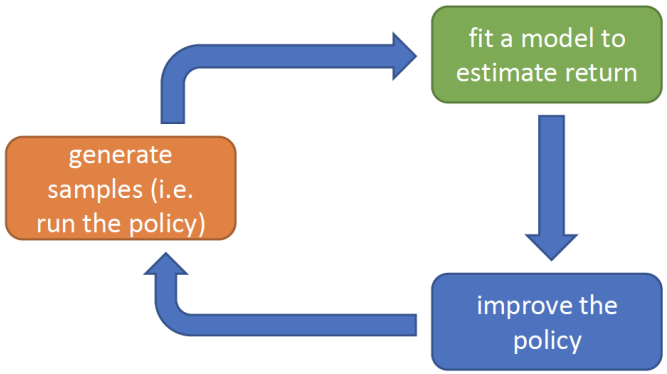
(more than atoms in the universe)

$$\mathcal{L}(\phi) = \frac{1}{2} \left\| V_\phi(\mathbf{s}) - \max_{\mathbf{a}} Q^\pi(\mathbf{s}, \mathbf{a}) \right\|^2$$

$$Q^\pi(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma E_{\mathbf{s}' \sim p(\mathbf{s}'|\mathbf{s},\mathbf{a})}[V^\pi(\mathbf{s}')]$$

fitted value iteration algorithm:

1. set $\mathbf{y}_i \leftarrow \max_{\mathbf{a}_i}(r(\mathbf{s}_i, \mathbf{a}_i) + \gamma E[V_\phi(\mathbf{s}'_i)])$
2. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \left\| V_\phi(\mathbf{s}_i) - \mathbf{y}_i \right\|^2$



$$V^\pi(\mathbf{s}) \leftarrow \max_{\mathbf{a}} Q^\pi(\mathbf{s}, \mathbf{a})$$

Determine optimal policies from optimal value functions

- For v_*: a one-step search
  - Actions that appear best after one-step search will be optimal actions
- For Q_*: no need to do a one-step-ahead search
  - $a_* = \arg\max_a Q_*(s, a)$
  - The optimal action-value function allows optimal actions to be selected without having to know anything about possible successor states and their values, i.e., without having to know anything about the environment's dynamics

What if we don't know the transition dynamics?

- From prediction problems to control problems
- from state-value functions to action-value functions

fitted value iteration algorithm:

1. set $\mathbf{y}_i \leftarrow \max_{\mathbf{a}_i}(r(\mathbf{s}_i, \mathbf{a}_i) + \gamma E[V_\phi(\mathbf{s}'_i)])$
2. set $\phi \leftarrow \arg\min_\phi \frac{1}{2}\sum_i \|V_\phi(\mathbf{s}_i) - \mathbf{y}_i\|^2$

need to know outcomes for different actions!

Back to policy iteration…

policy iteration:

1. evaluate $Q^\pi(\mathbf{s}, \mathbf{a})$
2. set $\pi \leftarrow \pi'$

$$\pi'(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} Q^\pi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

policy evaluation:

$$V^\pi(\mathbf{s}) \leftarrow r(\mathbf{s}, \pi(\mathbf{s})) + \gamma E_{\mathbf{s}' \sim p(\mathbf{s}'|\mathbf{s}, \pi(\mathbf{s}))}[V^\pi(\mathbf{s}')]$$

$$Q^\pi(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma E_{\mathbf{s}' \sim p(\mathbf{s}'|\mathbf{s}, \mathbf{a})}[Q^\pi(\mathbf{s}', \pi(\mathbf{s}'))]$$

can fit this using samples

Can we do the "max" trick again?

policy iteration:

1. evaluate $V^\pi(\mathbf{s})$
2. set $\pi \leftarrow \pi'$

fitted value iteration algorithm:

1. set $\mathbf{y}_i \leftarrow \max_{\mathbf{a}_i}(r(\mathbf{s}_i, \mathbf{a}_i) + \gamma E[V_\phi(\mathbf{s}'_i)])$
2. set $\phi \leftarrow \arg\min_\phi \frac{1}{2}\sum_i \|V_\phi(\mathbf{s}_i) - \mathbf{y}_i\|^2$

forget policy, compute value directly

can we do this with Q-values **also**, without knowing the transitions?

fitted Q iteration algorithm:

1. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma E[V_\phi(\mathbf{s}'_i)]$ ← approxiate $E[V(\mathbf{s}'_i)] \approx \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
2. set $\phi \leftarrow \arg\min_\phi \frac{1}{2}\sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

doesn't require simulation of actions!

+ works even for off-policy samples (unlike actor-critic)
+ only one network, no high-variance policy gradient
- no convergence guarantees for non-linear function approximation (more on this later)

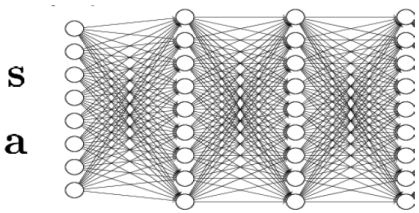
Fitted Q-iteration (FQI)

full fitted Q-iteration algorithm:

parameters

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy — dataset size N, collection policy
2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ — iterations K
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$ — gradient steps S

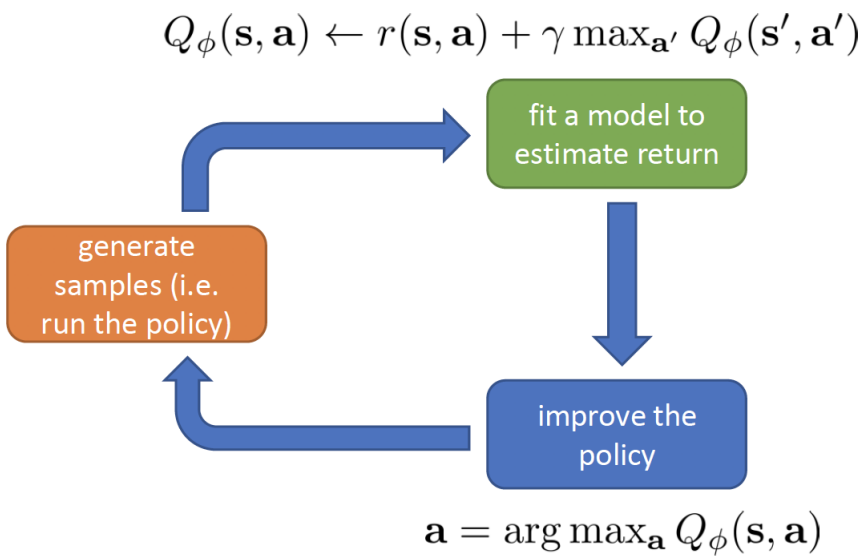
$K\times$



$Q_\phi(\mathbf{s}, \mathbf{a})$
parameters ϕ

Review

- Value based methods
  - Don't learn a policy explicitly
  - Just learn state- or action-value function
- If we have value function, we have a policy
- Fitted Q-iteration



On-policy vs. Off-policy

Target policy $\pi(a\|s)$	Behavior policy $b(a\|s)$
To be evaluated or improved	To explore to generate data
Make decisions finally	Make decisions in training phase

- **On-policy** methods: $\pi(a|s) = b(a|s)$
  - Evaluate or improve the policy that is used to make decisions during training
  - e.g., SARSA
- **Off-policy** methods: $\pi(a|s) \neq b(a|s)$
  - Evaluate or improve a policy different from that used to generate the data
  - Separate exploration from control
  - e.g., Q-learning

Q-learning vs. SARSA

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)]$$

- Q-learning approximates the optimal action-value function for an optimal policy, $Q \approx Q_* = Q_{\pi_*}$
  - The target policy is greedy w.r.t Q, $\pi(a|s) = \arg\max_a Q(s, a)$
  - The behavior policy can be others, e.g., $b(a|s) = \varepsilon$-greedy

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$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)]$$

- SARSA approximates the action-value function for the behavior policy, $Q \approx Q_\pi = Q_b$
  - The target and the behavior policy are the same, e.g., $\pi(a|s) = b(a|s) = \varepsilon$-greedy

Why FQI is off-policy?

full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy

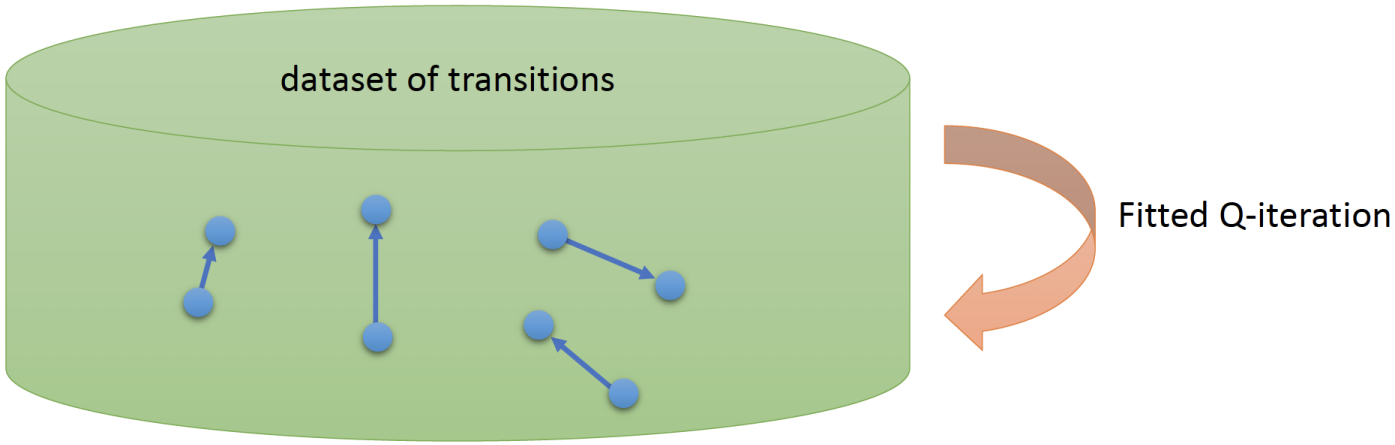
$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

given $\mathbf{s}$ *and* $\mathbf{a}$, transition is independent of π

this approximates the value of π' at $\mathbf{s}'_i$

$$\pi'(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} Q^\pi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$



What is FQI optimizing?

full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy

$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ ← this max improves the policy (tabular case)
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

↑ error $\mathcal{E}$

$$\mathcal{E} = \frac{1}{2} E_{(\mathbf{s},\mathbf{a})\sim\beta} \left[\left(Q_\phi(\mathbf{s}, \mathbf{a}) - [r(\mathbf{s}, \mathbf{a}) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}', \mathbf{a}')] \right)^2 \right]$$

if $\mathcal{E} = 0$, then $Q_\phi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}', \mathbf{a}')$

this is an *optimal* Q-function, corresponding to optimal policy π':

$$\pi'(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} Q_\phi(\mathbf{s}_t, \mathbf{a}_t) & \text{maximizes reward} \\ 0 \text{ otherwise} & \text{sometimes written } Q^\star \text{ and } \pi^\star \end{cases}$$

most guarantees are lost when we leave the tabular case (e.g., when we use neural network function approximation)

Batch-mode (offline) vs. online

- Batch-model (offline) algorithms
  - Collect a batch of samples using some policy
  - Fit the state- or action-value function iteratively
- Online algorithms
  - Take some action to collect one sample
  - Fit the value function
  - Iteratively alternate the above two steps

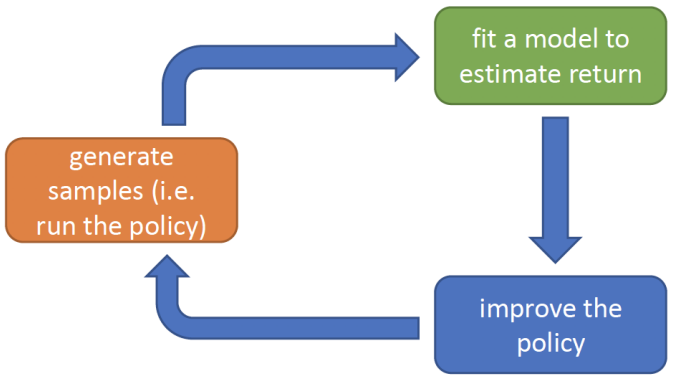
Online Q-learning algorithms

full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy
2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

($K\times$)

$$Q_\phi(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}', \mathbf{a}')$$



$$\mathbf{a} = \arg\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a})$$

off policy, so many choices here!

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

Exploration strategies

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

final policy:

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} Q_\phi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

why is this a bad idea for step 1?

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 - \epsilon \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} Q_\phi(\mathbf{s}_t, \mathbf{a}_t) \\ \epsilon/(|\mathcal{A}| - 1) \text{ otherwise} \end{cases}$$

"epsilon-greedy"

$$\pi(\mathbf{a}_t|\mathbf{s}_t) \propto \exp(Q_\phi(\mathbf{s}_t, \mathbf{a}_t))$$

"Boltzmann exploration"

Looking back from DRL

- Tabular RL algorithms
  - R. S. Sutton and A. G. Barto, **Reinforcement Learning: An Introduction**, 2nd Edition.
- RL with linear function approximation
  - M. G. Lagoudakis and R. Parr, **Least-Squares Policy Iteration**, *Journal of Machine Learning Research*, 2003.
- RL with nonlinear function approximation, e.g., neural network
  - J. A. Boyan, et al., **Generalization in reinforcement learning: Safely approximating the value function**, NIPS 1995.
  - R. S. Sutton, et al., **Policy gradient methods for reinforcement learning with function approximation**, NIPS 2000.
  - Martin Riedmiller, **Neural Fitted Q Iteration – First Experiences with a Data Efficient Neural Reinforcement Learning Method**, ECML 2005.
- RL with deep neural networks
  - **Human-level performance, the milestone of artificial intelligence**

Review

- Value-based methods
  - Don't learn a policy explicitly
  - Just learn state- or action-value function
- If we have value function, we have a policy
- Fitted Q-iteration
  - Batch mode, off-policy method
- Q-learning
  - Online analogue of fitted Q-iteration

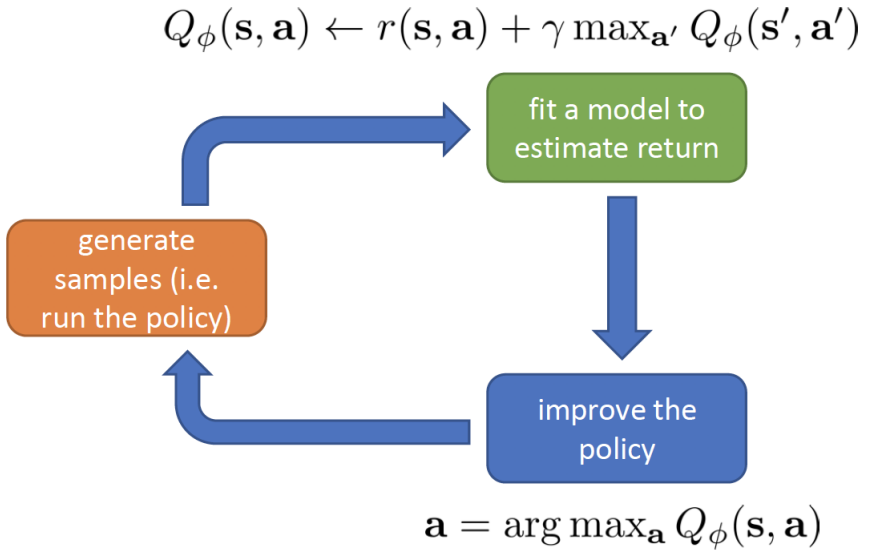


Table of Contents

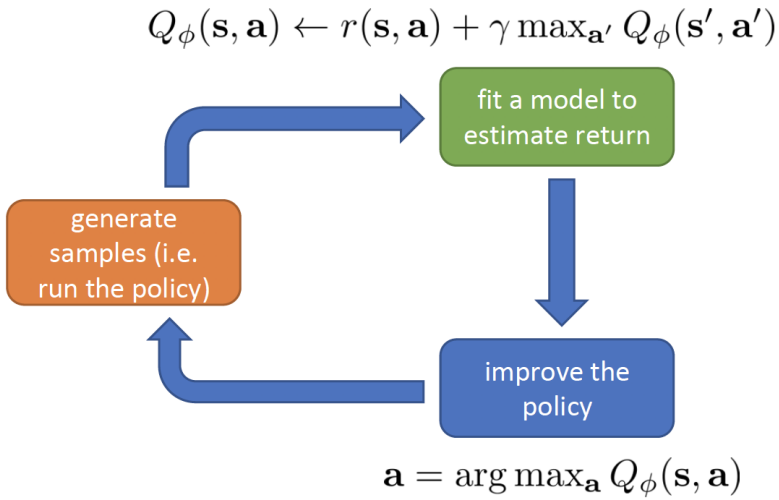
Contents and Goals

- How we can make Q-learning work with deep networks
  - Use replay buffers, separate target networks
- Tricks for improving Q-learning in practice
  - Double Q-learning, multi-step Q-learning
- Continuous Q-learning methods
- Goals
  - Understand how to implement Q-learning so that it can be used with complex function approximators
  - Understand how to extend Q-learning to continuous actions

Recall: Q-learning

full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy
2. $K\times$ set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $K\times$ set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$



online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

Problem 1: Correlated samples in MDPs

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

these are correlated!

isn't this just gradient descent? that converges, right?

Q-learning is *not* gradient descent!

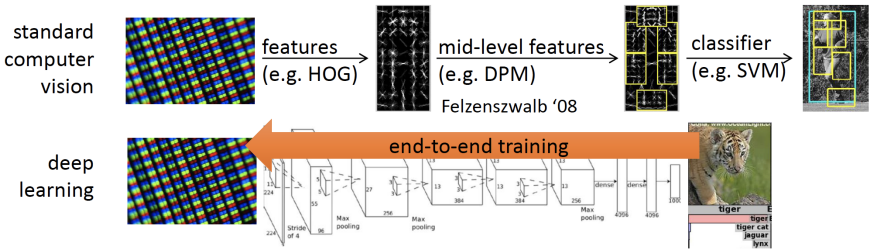
$$\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$$

no gradient through target value

Recall: Supervised learning vs. Sequential decision making

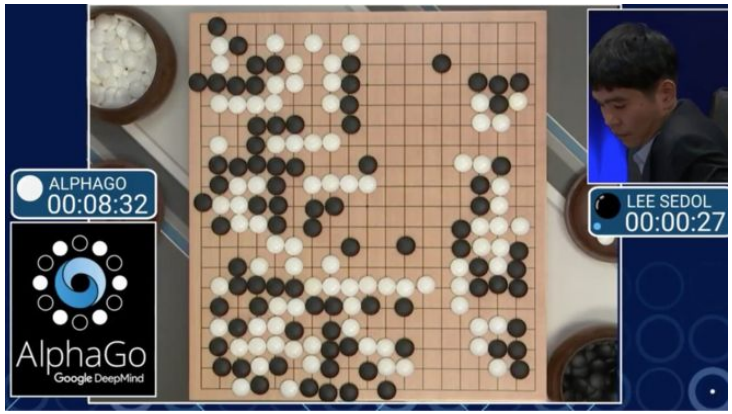
Supervised learning

- Samples are independent and identically distributed (i.i.d.)
- Given an input, map an optimal output



Reinforcement learning

- Samples are not i.i.d., temporally co-related
- Given an initial state, find a sequence of optimal actions

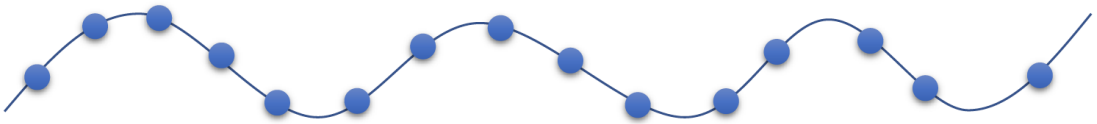


Correlated samples in online Q-learning

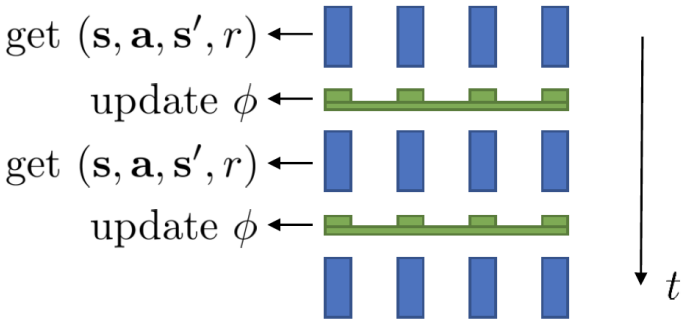
online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

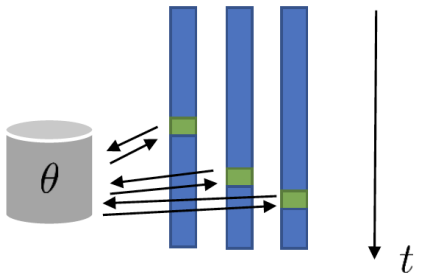
- sequential states are strongly correlated
- target value is always changing



synchronized parallel Q-learning



asynchronous parallel Q-learning



Another solution: replay buffers

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

special case with K = 1, and one gradient step

full fitted Q-iteration algorithm:

~~1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy~~

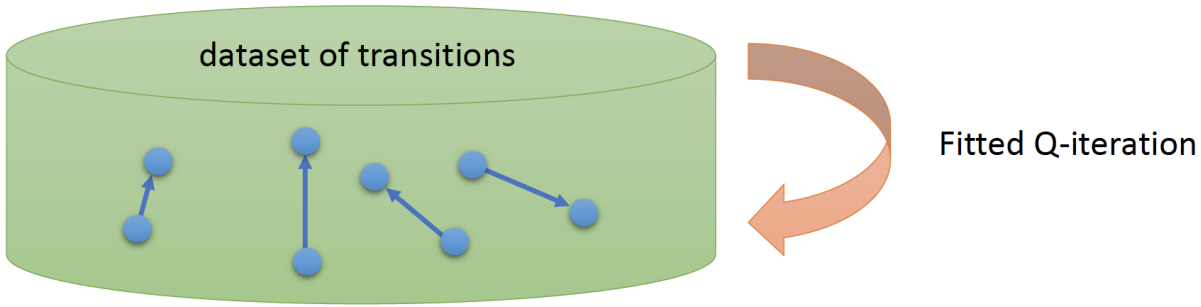
$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

any policy will work! (with broad support)

just load data from a buffer here

still use one gradient step



Another solution: replay buffers

Q-learning with a replay buffer:

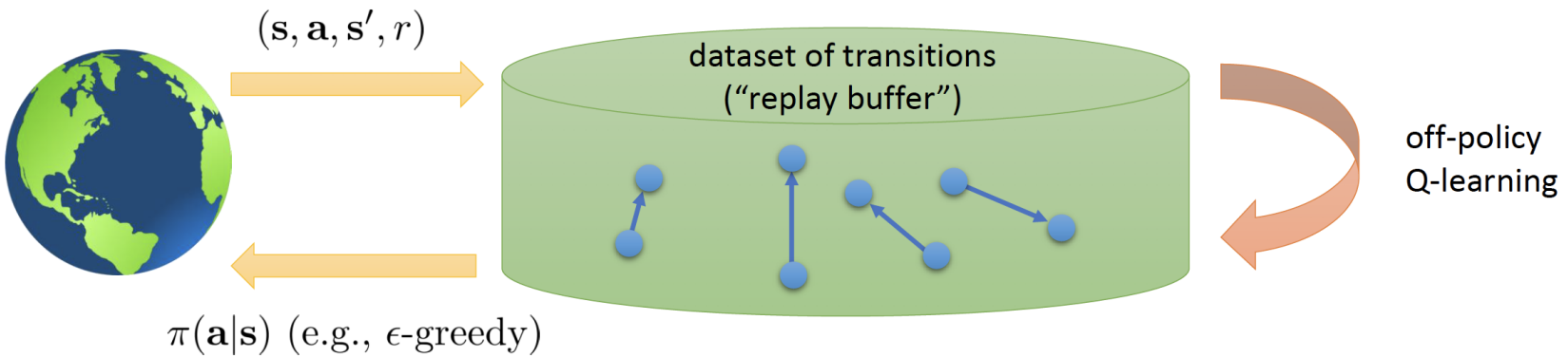
1. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
2. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

+ samples are no longer correlated

+ multiple samples in the batch (low-variance gradient)

but where does the data come from?

need to periodically feed the replay buffer...



Putting it together

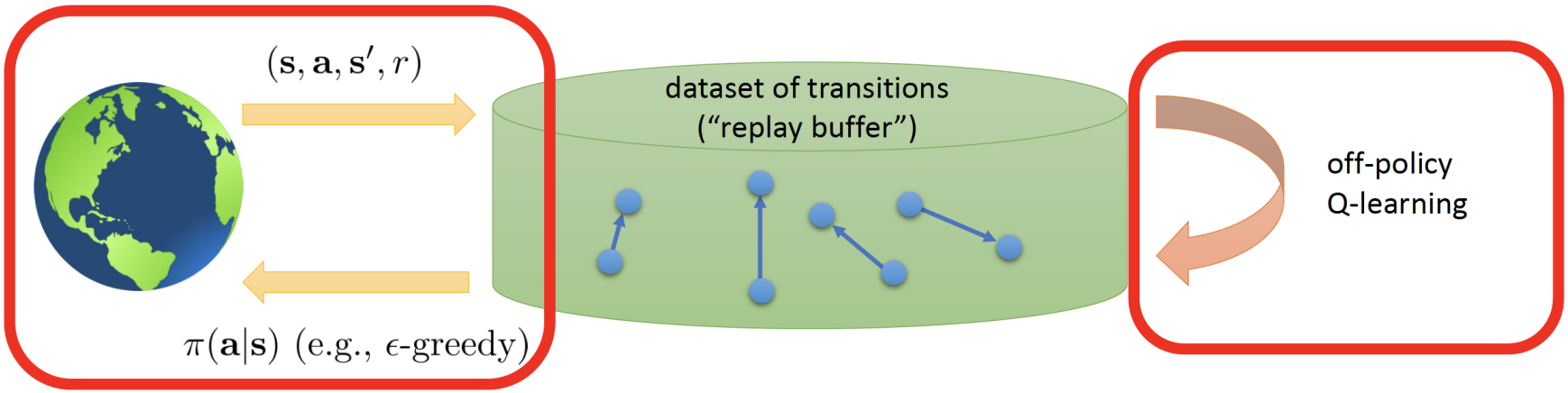
full Q-learning with replay buffer:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$

$K\times$

2. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
3. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

K = 1 is common, though larger K more efficient



Problem 2: Moving target in the Bellman equation

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

~~these are correlated!~~

use replay buffer

Q-learning is *not* gradient descent!

$$\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$$

no gradient through target value

This is still a problem!

Q-learning and regression

full Q-learning with replay buffer:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$

$K\times$

2. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
3. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

one gradient step, moving target

full fitted Q-iteration algorithm:

1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy

$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

perfectly well-defined, stable regression

Q-learning with target networks

Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$
   - $N\times$
     - 2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$
     - $K\times$
       - 3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
       - 4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

targets don't change in inner loop!

supervised regression

"Classic" deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$
2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to $\mathcal{B}$ — $N\times$
3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$ — $K\times$
4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

"classic" deep Q-learning algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly
3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$ using *target* network $Q_{\phi'}$
4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$

$K = 1$

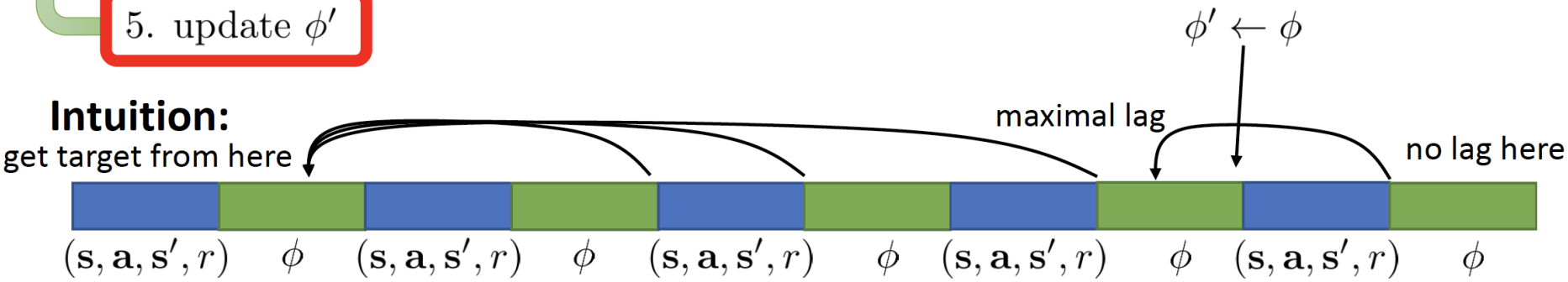
5. update ϕ': copy ϕ every N steps

Alternative target network

"classic" deep Q-learning algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly
3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$ using *target* network $Q_{\phi'}$
4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$
5. update ϕ'

Intuition:



Feels weirdly uneven, can we always have the same lag?

Popular alternative (similar to Polyak averaging):

5. update ϕ': $\phi' \leftarrow \tau\phi' + (1-\tau)\phi$ $\quad\quad$ $\tau = 0.999$ works well

Fitted Q-iteration and Q-learning

Q-learning with replay buffer and target network:

DQN: $N = 1$, $K = 1$

1. save target network parameters: $\phi' \leftarrow \phi$
   - $N\times$
     - 2. collect M datapoints $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to $\mathcal{B}$
     - $K\times$
       - 3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
       - 4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

Fitted Q-learning (written similarly as above):

1. collect M datapoints $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to $\mathcal{B}$
   - $N\times$
     - 2. save target network parameters: $\phi' \leftarrow \phi$
     - $K\times$
       - 3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
       - 4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

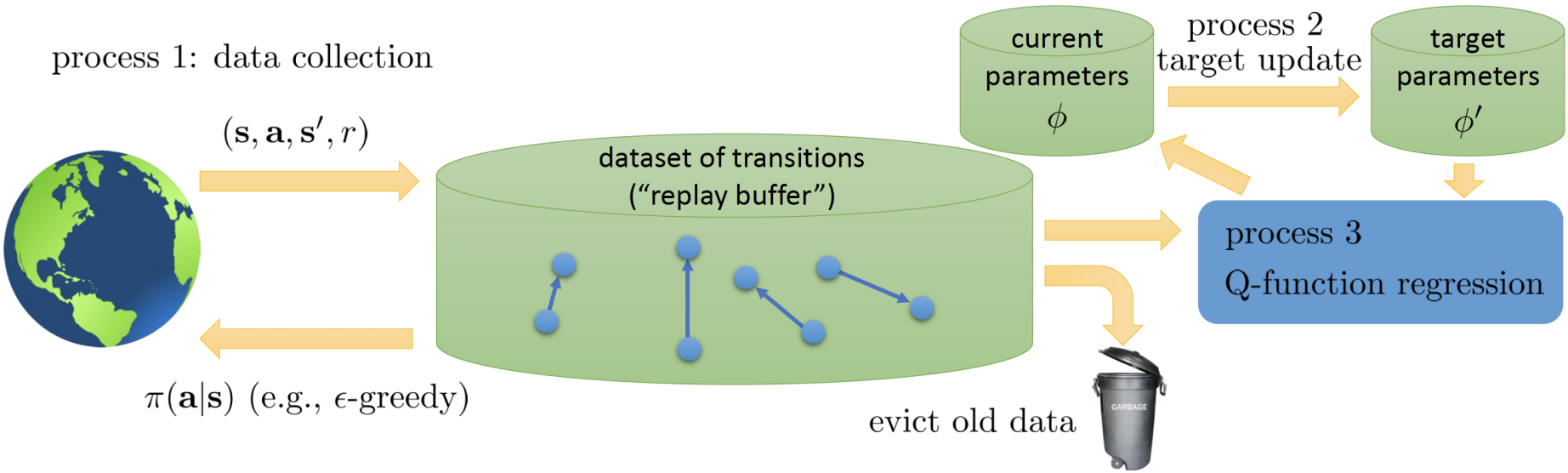
} **just SGD**

A more general view

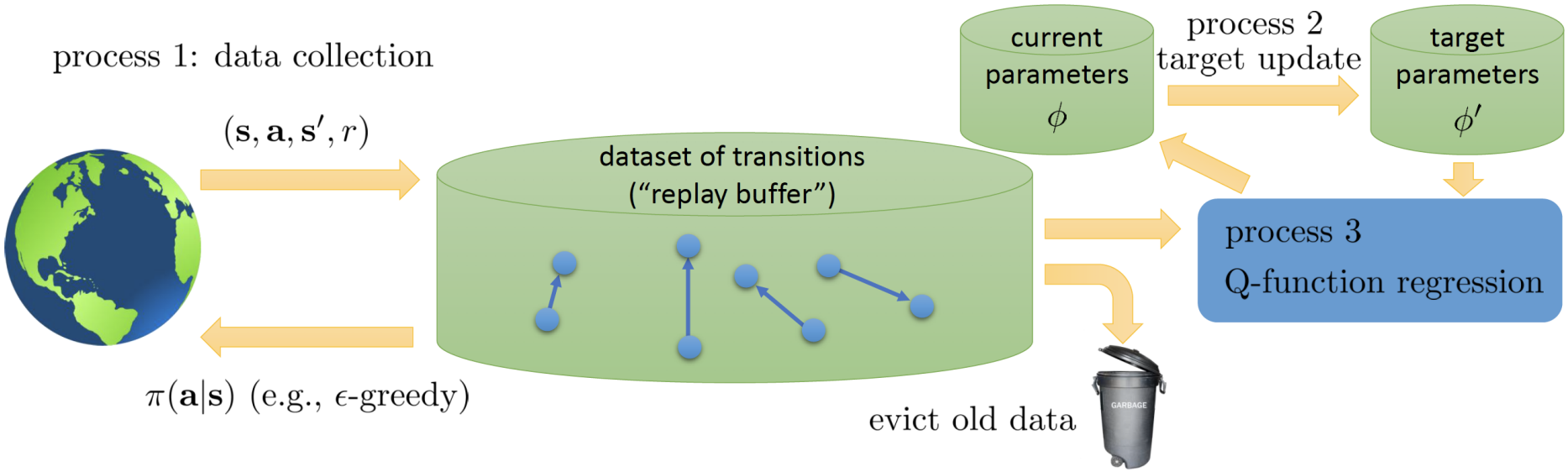
Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$
2. collect M datapoints $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to $\mathcal{B}$
3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from $\mathcal{B}$
4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

($K\times$ over steps 3–4, $N\times$ over steps 2–4)



A more general view



- Online Q-learning: evict immediately, process 1, process 2, and process 3 all run at the same speed
- DQN: process 1 & 3 run at the same speed, process 2 is slow
- Fitted Q-iteration: process 3 in the inner loop of process 2, which is in the inner loop of process 1

Table of Contents

Overestimation in Q-learning

target value $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$

this last term is the problem

imagine we have two random variables: X_1 and X_2

$E[\max(X_1, X_2)] \geq \max(E[X_1], E[X_2])$

$Q_{\phi'}(\mathbf{s}', \mathbf{a}')$ is not perfect – it looks "noisy"

hence $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}')$ *overestimates* the next value!

note that $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}') = \underline{Q_{\phi'}}(\mathbf{s}', \underline{\arg\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}')})$

value *also* comes from $Q_{\phi'}$ action selected according to $Q_{\phi'}$

Double Q-learning

$$E[\max(X_1, X_2)] \geq \max(E[X_1], E[X_2])$$

note that $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}') = \underline{Q_{\phi'}}(\mathbf{s}', \underline{\arg\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}')})$

value *also* comes from $Q_{\phi'}$ action selected according to $Q_{\phi'}$

if the noise in these is decorrelated, the problem goes away!

idea: don't use the same network to choose the action and evaluate value!

"double" Q-learning: use two networks:

$$Q_{\phi_A}(\mathbf{s}, \mathbf{a}) \leftarrow r + \gamma Q_{\phi_B}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi_A}(\mathbf{s}', \mathbf{a}'))$$

$$Q_{\phi_B}(\mathbf{s}, \mathbf{a}) \leftarrow r + \gamma Q_{\phi_A}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi_B}(\mathbf{s}', \mathbf{a}'))$$

if the two Q's are noisy in *different* ways, there is no problem

Double Q-learning in practice

where to get two Q-functions?

just use the current and target networks!

standard Q-learning: $y = r + \gamma Q_{\phi'}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}'))$

double Q-learning: $y = r + \gamma Q_{\phi'}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi}(\mathbf{s}', \mathbf{a}'))$

just use current network (not target network) to evaluate action

still use target network to evaluate value!

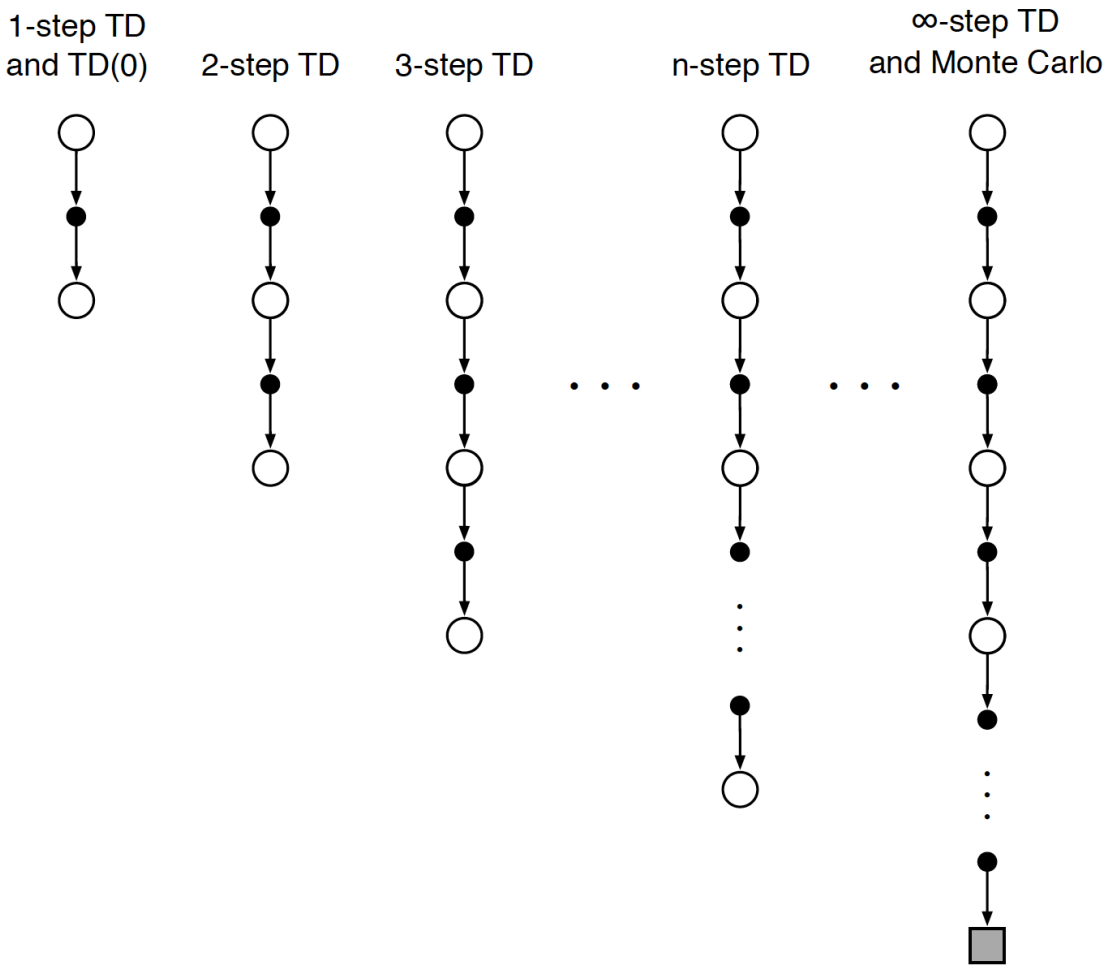
n-step bootstrapping: Combine MC and one-step TD

- Neither MC or one-step TD is always the best, we generalize both methods so that one can shift from one to the other smoothly as needed to meet the demands of a particular task
- One-step TD: In many applications, one wants to be able to update the action very fast to take into account anything that has changed
- However, bootstrapping works best if it is over a length of time in which a significant and recognizable state change has occurred

$n = 1$	n-step TD	$n = \infty$
TD(0)	$\longleftrightarrow$	MC

n-step TD prediction

- Perform an update based on an intermediate number of rewards, more than one, but less than all of them until termination



Recall MC and TD(0) updates

- In MC updates, the target is the **complete return**

$$G_t = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-t+1} R_T$$

$$\begin{aligned} V(S_t) &\leftarrow V(S_t) + \alpha[G_t - V(S_t)] \\ &= V(S_t) + \alpha[R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-t+1} R_T - V(S_t)] \end{aligned}$$

- In TD(0) updates, the target is the **one-step return**

$$G_{t:t+1} = R_{t+1} + \gamma V(S_{t+1})$$

$$\begin{aligned} V(S_t) &\leftarrow V(S_t) + \alpha[G_{t:t+1} - V(S_t)] \\ &= V(S_t) + \alpha[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)] \end{aligned}$$

n-step TD update rule

- For n-step TD, set the target as the n-**step return**

$$G_{t:t+n} = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n})$$

- All n-step returns can be considered approximations to the complete return, truncated after n steps and then corrected for the remaining missing terms by $V(S_{t+n})$

$$\begin{aligned} V(S_t) &\leftarrow V(S_t) + \alpha[G_{t:t+n} - V(S_t)] \\ &= V(S_t) + \alpha[R_{t+1} + \gamma R_{t+2} + ... + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n}) - V(S_t)] \end{aligned}$$

Deep Q-learning with n-step bootstrapping

Q-learning target: $y_{j,t} = r_{j,t} + \gamma \max_{\mathbf{a}_{j,t+1}} Q_{\phi'}(\mathbf{s}_{j,t+1}, \mathbf{a}_{j,t+1})$

these are the only values that matter if $Q_{\phi'}$ is bad!

these values are important if $Q_{\phi'}$ is good

where does the signal come from?

Q-learning does this: max bias, min variance

remember this?

Actor-critic: $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}_{i,t+1}) - \hat{V}^\pi_\phi(\mathbf{s}_{i,t}) \right)$

+ lower variance (due to critic)
- not unbiased (if the critic is not perfect)

Policy gradient: $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\left(\sum_{t'=t}^{T} \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right) - b \right)$

+ no bias
- higher variance (because single-sample estimate)

can we construct multi-step targets, like in actor-critic?

$y_{j,t} = \sum_{t'=t}^{t+N-1} \gamma^{t-t'} r_{j,t'} + \gamma^N \max_{\mathbf{a}_{j,t+N}} Q_{\phi'}(\mathbf{s}_{j,t+N}, \mathbf{a}_{j,t+N})$

N-step return estimator

Deep Q-learning with n-step bootstrapping

$$y_{j,t} = \sum_{t'=t}^{t+N-1} \gamma^{t-t'} r_{j,t'} + \gamma^N \max_{\mathbf{a}_{j,t+N}} Q_{\phi'}(\mathbf{s}_{j,t+N}, \mathbf{a}_{j,t+N})$$

this is supposed to estimate $Q^\pi(\mathbf{s}_{j,t}, \mathbf{a}_{j,t})$ for π

+ less biased target values when Q-values are inaccurate

+ typically faster learning, especially early on

- only actually correct when learning on-policy

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} Q_\phi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

why?

we need transitions $\mathbf{s}_{j,t'}, \mathbf{a}_{j,t'}, \mathbf{s}_{j,t'+1}$ to come from π for $t' - t < N - 1$

(not an issue when $N = 1$)

how to fix?

- ignore the problem
  - often works very well
- cut the trace – dynamically choose N to get only on-policy data
  - works well when data mostly on-policy, and action space is small
- importance sampling

For more details, see: “Safe and efficient off-policy reinforcement learning.” Munos et al. ‘16

Q-learning with continuous actions

What's the problem with continuous actions?

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \begin{cases} 1 \text{ if } \mathbf{a}_t = \arg\max_{\mathbf{a}_t} Q_\phi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 \text{ otherwise} \end{cases}$$

this max

$$\text{target value } y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$$

this max
particularly problematic (inner loop of training)

How do we perform the max?

Option 1: optimization

- gradient based optimization (e.g., SGD) a bit slow in the inner loop
- action space typically low-dimensional – what about stochastic optimization?

Q-learning with stochastic optimization

Simple solution:

$$\max_{\mathbf{a}} Q(\mathbf{s}, \mathbf{a}) \approx \max \{Q(\mathbf{s}, \mathbf{a}_1), \ldots, Q(\mathbf{s}, \mathbf{a}_N)\}$$

$(\mathbf{a}_1, \ldots, \mathbf{a}_N)$ sampled from some distribution (e.g., uniform)

+ dead simple

+ efficiently parallelizable

- not very accurate

but… do we care? How good does the target need to be anyway?

More accurate solution:

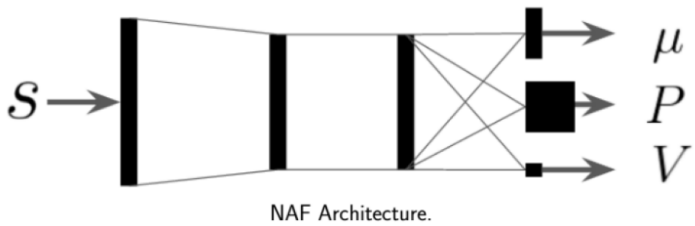
- cross-entropy method (CEM)
  - simple iterative stochastic optimization
- CMA-ES
  - substantially less simple iterative stochastic optimization

works OK, for up to about 40 dimensions

Easily maximizable Q-functions

Option 2: use function class that is easy to optimize

$$Q_\phi(\mathbf{s}, \mathbf{a}) = -\frac{1}{2}(\mathbf{a} - \mu_\phi(\mathbf{s}))^T P_\phi(\mathbf{s})(\mathbf{a} - \mu_\phi(\mathbf{s})) + V_\phi(\mathbf{s})$$



NAF Architecture.

NAF: **N**ormalized **A**dvantage **F**unctions

$$\arg\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a}) = \mu_\phi(\mathbf{s}) \qquad \max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a}) = V_\phi(\mathbf{s})$$

+ no change to algorithm

+ just as efficient as Q-learning

- loses representational power

Q-learning with continuous actions

Option 3: learn an approximate maximizer

DDPG (Lillicrap et al., ICLR 2016)

"deterministic" actor-critic
(really approximate Q-learning)

$\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a}) = Q_\phi(\mathbf{s}, \arg\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a}))$

idea: train another network $\mu_\theta(\mathbf{s})$ such that $\mu_\theta(\mathbf{s}) \approx \arg\max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a})$

how? just solve $\theta \leftarrow \arg\max_\theta Q_\phi(\mathbf{s}, \mu_\theta(\mathbf{s}))$ $\qquad \frac{dQ_\phi}{d\theta} = \frac{d\mathbf{a}}{d\theta}\frac{dQ_\phi}{d\mathbf{a}}$

new target $y_j = r_j + \gamma Q_{\phi'}(\mathbf{s}'_j, \mu_\theta(\mathbf{s}'_j)) \approx r_j + \gamma Q_{\phi'}(\mathbf{s}'_j, \arg\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j))$

Deep deterministic policy gradient (DDPG)

Option 3: learn an approximate maximizer

DDPG:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly
3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mu_{\theta'}(\mathbf{s}'_j))$ using *target* nets $Q_{\phi'}$ and $\mu_{\theta'}$
4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$
5. $\theta \leftarrow \theta + \beta \sum_j \frac{d\mu}{d\theta}(\mathbf{s}_j) \frac{dQ_\phi}{d\mathbf{a}}(\mathbf{s}_j, \mathbf{a})$
6. update ϕ' and θ' (e.g., Polyak averaging)

Recall: Actor-critic algorithms

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

DDPG vs. Actor-critic

- DDPG
  - The actor: approximate the optimal policy $a = \arg\max Q_\phi(s, a)$
  - The critic: approximate the optimal action-value function Q^*_ϕ
  - Off-policy, more sample efficient
- Actor-critic
  - The actor: approximate the current policy $a \sim \pi(a|s)$
  - The critic: approximate the state-value function V^π_ϕ
  - On-policy

Review

- Q-learning in practice
  - Replay buffers
  - Target networks
- Generalized fitted Q-iteration
- Multi-step Q-learning
- Q-learning with continuous actions
  - Random sampling
  - Analytic optimization
  - Second "actor" network

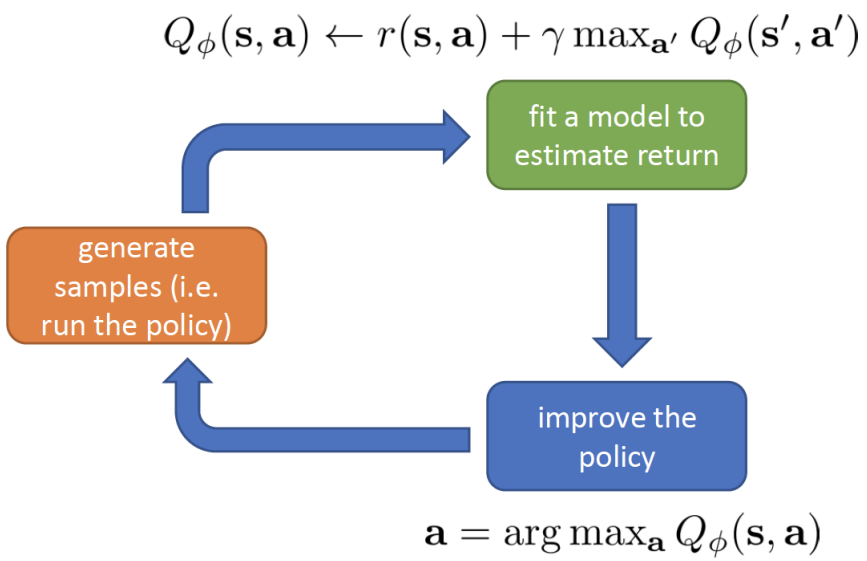


Table of Contents

Simple practical tips for Q-learning

- Q-learning takes some care to stabilize
  - Test on easy, reliable tasks first, make sure your implementation is correct

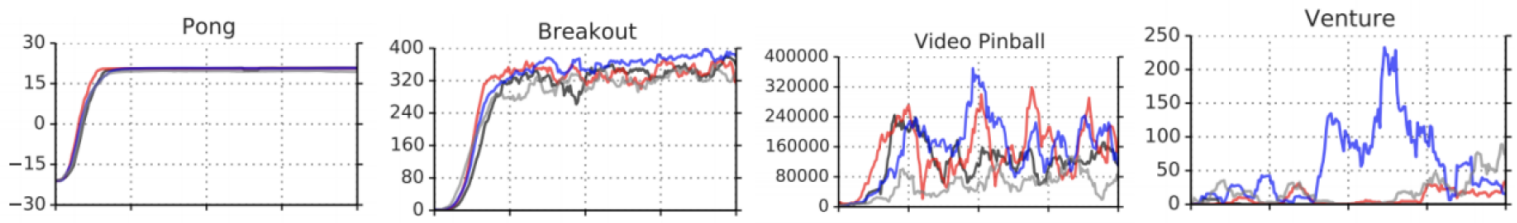


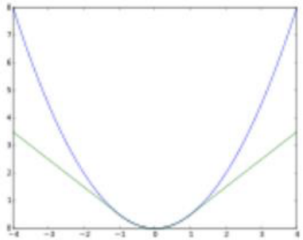
Figure: From T. Schaul, J. Quan, I. Antonoglou, and D. Silver. "Prioritized experience replay". *arXiv preprint arXiv:1511.05952* (2015), Figure 7

- Large replay buffers help improve stability
  - Looks more like fitted Q-iteration
- It takes time, be patient – might be no better than random for a while
- Start with high exploration (epsilon) and gradually reduce

Advanced tips for Q-learning

- Bellman error gradients can be big; clip gradients or user Huber loss

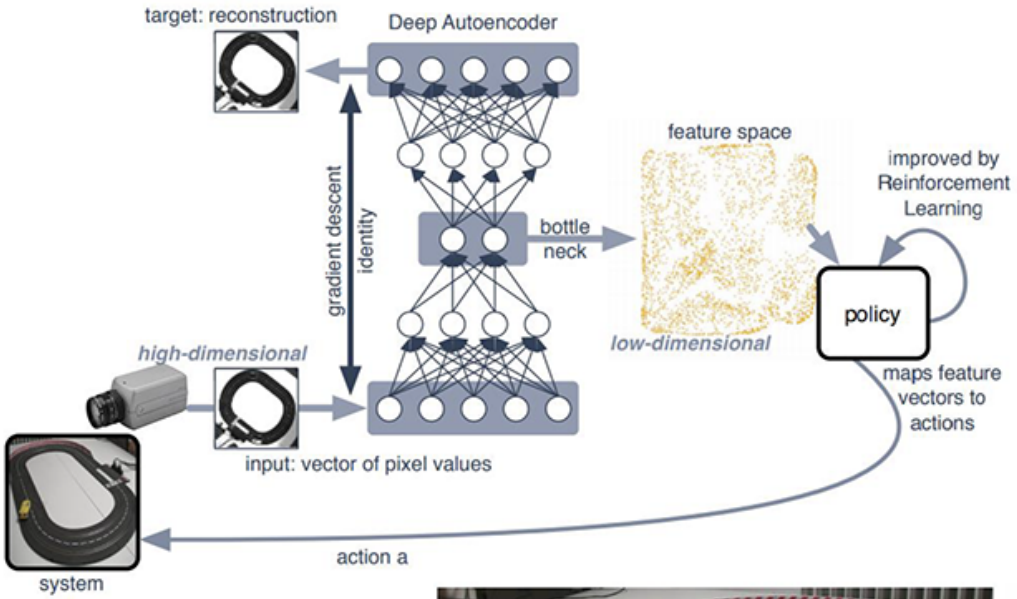
$$L(x) = \begin{cases} x^2/2 & \text{if } |x| \leq \delta \\ \delta|x| - \delta^2/2 & \text{otherwise} \end{cases}$$

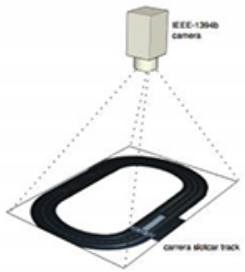


- Double Q-learning helps *a lot* in practice, simple and no downsides
- N-step returns also help a lot, but have some downsides
- Schedule exploration (high to low) and learning rates (high to low), Adam optimizer can help too
- Run multiple random seeds, it's very inconsistent between runs

Fitted Q-iteration in a latent space

- "Autonomous reinforcement learning from raw visual data," Lange & Riedmiller '12
- Q-learning on top of latent space learned with autoencoder
- Uses fitted Q-iteration
- Extra random trees for function approximation (but neural net for embedding)





Q-learning with convolutional networks

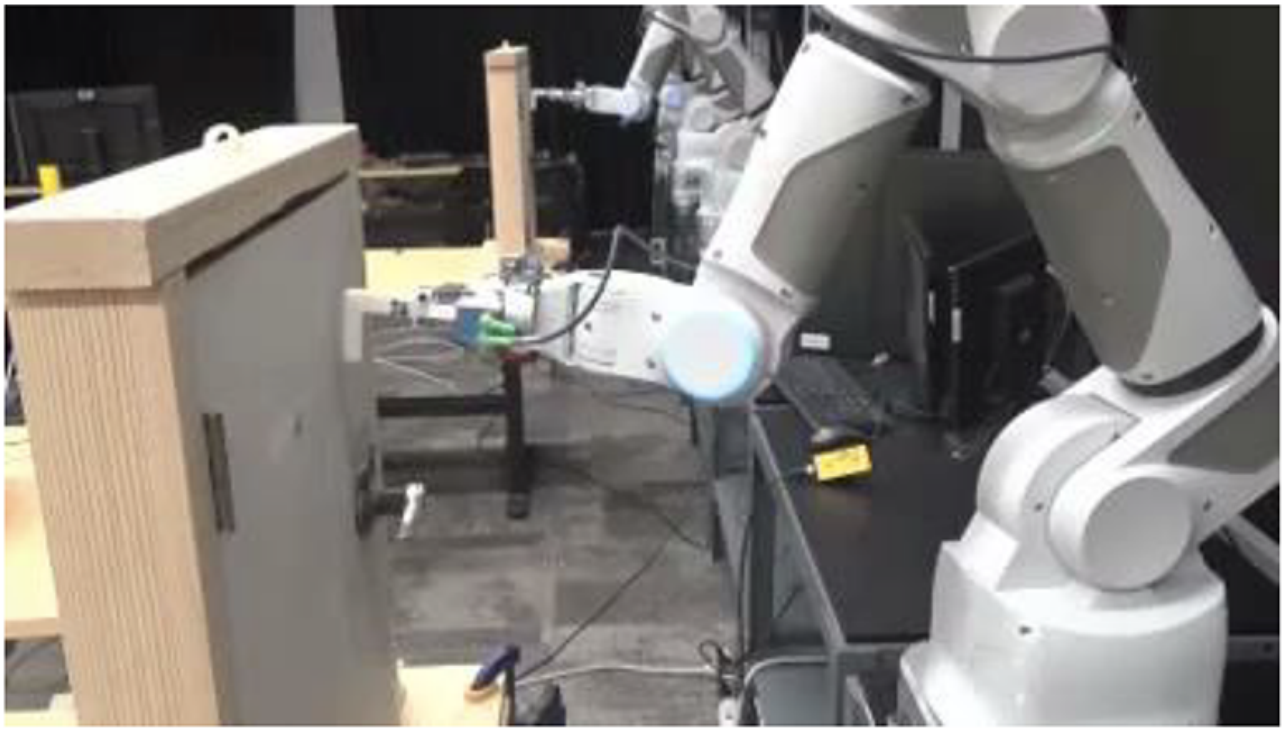
- "Human-level control through deep reinforcement learning," Mnih et al. '13
- Q-learning with convolutional networks
- Uses replay buffer and target network
- One-step backup
- One gradient step
- Can be improved a lot with double Q-learning (and other tricks)

Q-learning with continuous actions

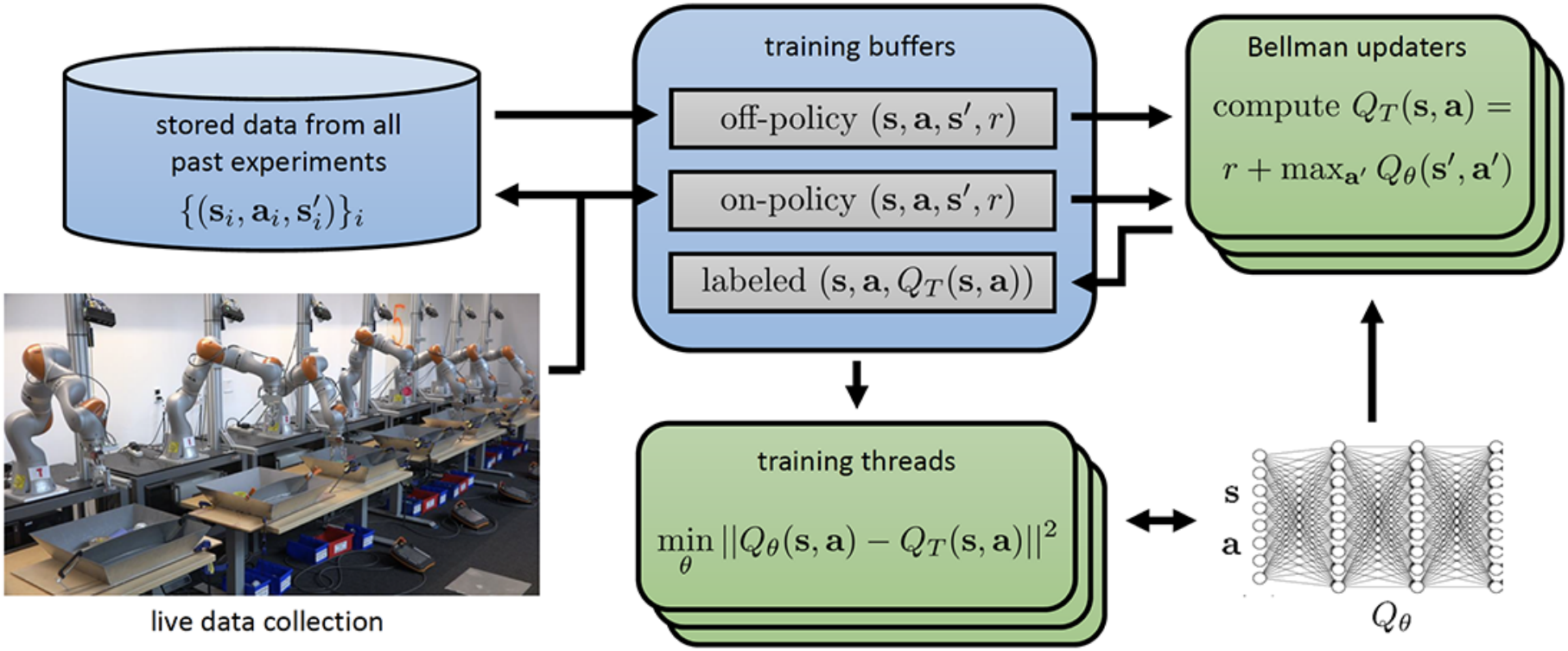
- "Continuous control with deep reinforcement learning," Lillicrap et al. 2015.
- Continuous actions with maximizer network
- Uses replay buffer and target network (with Polyak averaging)
- One-step backup
- One gradient step per simulator step

Q-learning on a real robot

- "Robotic manipulation with deep reinforcement learning and ...," Gu*, Holly*, et al. '17
- Continuous actions with NAF (quadratic in actions)
- Uses replay buffer and target network
- One-step backup
- Four gradient steps per simulator step for efficiency
- Parallelized across multiple robots



Large-scale Q-learning with continuous actions (QT Opt)



Kalashnikov, Irpan, Pastor, Ibarz, Herzong, Jang, Quillen, Holly, Kalakrishnan, Vanhoucke, Levine. **QT-Opt: Scalable Deep Reinforcement Learning of Vision-Based Robotic Manipulation Skills**

$$\text{minimize} \sum_i (Q(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \max_{\mathbf{a}'_i} Q(\mathbf{s}'_i, \mathbf{a}'_i)])^2$$

Table of Contents

Value iteration theory in tabular cases

- Bellman optimality equation

$$v_*(s) = \max_a \sum_{s',r} p(s', r|s, a)[r + \gamma v_*(s')]$$

- Value iteration

$$v_{k+1}(s) = \max_a \sum_{s',r} p(s', r|s, a)[r + \gamma v_k(s')]$$

  - Turn Bellman optimality equation into an update rule
  - Directly approximate the optimal state-value function, v_*
  - For arbitrary v_0, the sequence $\{v_k\}$ converges to v_* under the same conditions that guarantee the existence of v_*

Value function learning theory

value iteration algorithm:

1. set $Q(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma E[V(\mathbf{s}')]$
2. set $V(\mathbf{s}) \leftarrow \max_{\mathbf{a}} Q(\mathbf{s}, \mathbf{a})$

0.2	0.3	0.4	0.3
0.3	0.3	0.5	0.3
0.4	0.4	0.6	0.4
0.5	0.5	0.7	0.5

does it converge? and if so, to what?

define an operator $\mathcal{B}$: $\mathcal{B}V = \max_{\mathbf{a}} r_{\mathbf{a}} + \gamma \mathcal{T}_{\mathbf{a}} V$

$r_{\mathbf{a}}$: stacked vector of rewards at all states for action $\mathbf{a}$

$\mathcal{T}_{\mathbf{a}}$: matrix of transitions for action $\mathbf{a}$ such that $\mathcal{T}_{\mathbf{a},i,j} = p(\mathbf{s}' = i | \mathbf{s} = j, \mathbf{a})$

$V^\star$ is a *fixed point* of $\mathcal{B}$ $V^\star(\mathbf{s}) = \max_{\mathbf{a}} r(\mathbf{s}, \mathbf{a}) + \gamma E[V^\star(\mathbf{s}')]$, so $V^\star = \mathcal{B}V^\star$

always exists, is always unique, always corresponds to the optimal policy

...but will we reach it?

Value function learning theory

value iteration algorithm:

1. set $Q(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \gamma E[V(\mathbf{s}')]$
2. set $V(\mathbf{s}) \leftarrow \max_{\mathbf{a}} Q(\mathbf{s}, \mathbf{a})$

0.2	0.3	0.4	0.3
0.3	0.3	0.5	0.3
0.4	0.4	0.6	0.4
0.5	0.5	0.7	0.5

$V^\star$ is a *fixed point* of $\mathcal{B}$ $\quad V^\star(\mathbf{s}) = \max_{\mathbf{a}} r(\mathbf{s}, \mathbf{a}) + \gamma E[V^\star(\mathbf{s}')]$, so $V^\star = \mathcal{B}V^\star$

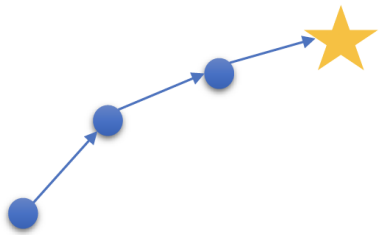
we can prove that value iteration reaches $V^\star$ because $\mathcal{B}$ is a *contraction*

contraction: for any V and $\bar{V}$, we have $\|\mathcal{B}V - \mathcal{B}\bar{V}\|_\infty \leq \gamma \|V - \bar{V}\|_\infty$

gap always gets smaller by γ!
(with respect to ∞-norm)

what if we choose $V^\star$ as $\bar{V}$? $\mathcal{B}V^\star = V^\star$!

$\|\mathcal{B}V - V^\star\|_\infty \leq \gamma \|V - V^\star\|_\infty$



Non-tabular value function learning

value iteration algorithm (using $\mathcal{B}$):

1. $V \leftarrow \mathcal{B}V$

fitted value iteration algorithm (using $\mathcal{B}$ and Π):

1. $V \leftarrow \Pi\mathcal{B}V$

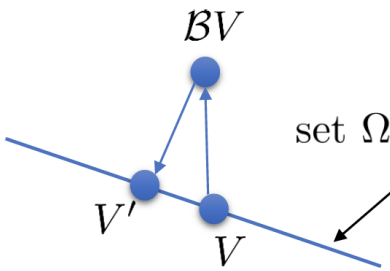
fitted value iteration algorithm:

1. set $\mathbf{y}_i \leftarrow \max_{\mathbf{a}_i}(r(\mathbf{s}_i, \mathbf{a}_i) + \gamma E[V_\phi(\mathbf{s}'_i)])$
2. set $\phi \leftarrow \arg\min_\phi \frac{1}{2}\sum_i \|V_\phi(\mathbf{s}_i) - \mathbf{y}_i\|^2$

what does this do?

define new operator Π: $\Pi V = \arg\min_{V' \in \Omega} \frac{1}{2}\sum \|V'(\mathbf{s}) - V(\mathbf{s})\|^2$

Π is a *projection* onto Ω (in terms of ℓ_2 norm)



set Ω (e.g., neural nets)

updated value function

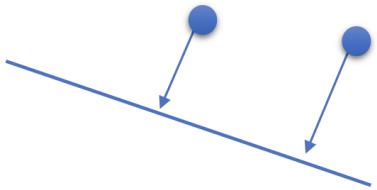
$V' \leftarrow \arg\min_{V' \in \Omega} \frac{1}{2}\sum \|V'(\mathbf{s}) - (\mathcal{B}V)(\mathbf{s})\|^2$

all value functions represented by, e.g., neural nets

Non-tabular value function learning

fitted value iteration algorithm (using $\mathcal{B}$ and Π):

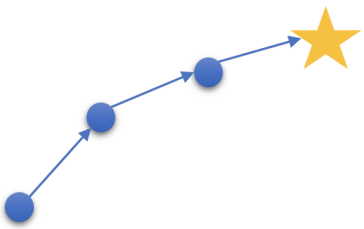
1. $V \leftarrow \Pi\mathcal{B}V$

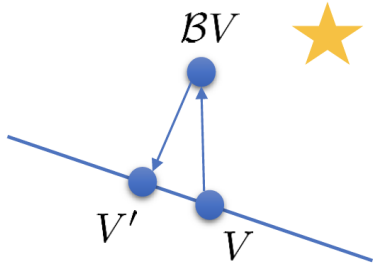


$\mathcal{B}$ is a contraction w.r.t. ∞-norm ("max" norm) $\quad \|\mathcal{B}V - \mathcal{B}\bar{V}\|_\infty \leq \gamma\|V - \bar{V}\|_\infty$

Π is a contraction w.r.t. ℓ_2-norm (Euclidean distance) $\quad \|\Pi V - \Pi\bar{V}\|^2 \leq \|V - \bar{V}\|^2$

but... $\Pi\mathcal{B}$ is not a contraction of any kind





Conclusions:
value iteration converges (tabular case)
fitted value iteration does **not** converge
not in general
often not in practice

What about fitted Q-iteration?

fitted Q iteration algorithm:

1. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma E[V_\phi(\mathbf{s}'_i)]$
2. set $\phi \leftarrow \arg\min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

define an operator $\mathcal{B}$: $\mathcal{B}Q = r + \gamma \mathcal{T} \max_{\mathbf{a}} Q$

max now after the transition operator

define an operator Π: $\Pi Q = \arg\min_{Q' \in \Omega} \frac{1}{2} \sum \|Q'(\mathbf{s}, \mathbf{a}) - Q(\mathbf{s}, \mathbf{a})\|^2$

fitted Q-iteration algorithm (using $\mathcal{B}$ and Π):

1. $Q \leftarrow \Pi \mathcal{B} Q$

Applies also to online Q-learning

$\mathcal{B}$ is a contraction w.r.t. ∞-norm ("max" norm)

Π is a contraction w.r.t. ℓ_2-norm (Euclidean distance)

$\Pi\mathcal{B}$ is not a contraction of any kind

Fitted Q-iteration – Regression, but not gradient descent

online Q iteration algorithm:

1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$

isn't this just gradient descent? that converges, right?

Q-learning is *not* gradient descent!

$$\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$$

no gradient through target value

A sad corollary

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

An aside regarding terminology

V^π: value function for policy π
this is what the critic does

$V^\star$: value function for optimal policy $\pi^\star$
this is what value iteration does

ℓ_∞ contraction $\mathcal{B}$ (but without max)

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}^\pi_\phi(\mathbf{s}_i) - y_i \right\|^2$$

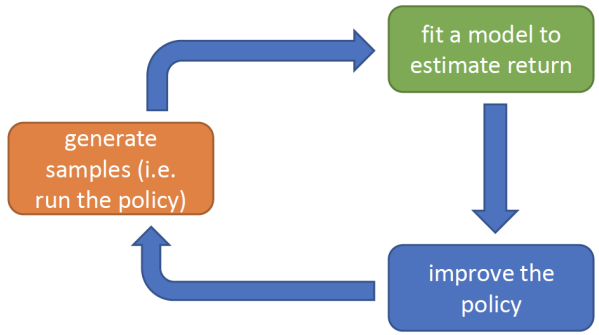
ℓ_2 contraction Π

fitted bootstrapped policy evaluation doesn't converge!

Review

- Value iteration theory
  - Linear operator for backup
  - Linear operator for projection
  - Backup is contraction
  - Value iteration converges
- Convergence with function approximation
  - Projection is also a contraction
  - Projection + backup is not a contraction
  - Fitted value iteration does not in general converge
- Implications for Q-learning
  - Q-learning, fitted Q-iteration, etc. does not converge with function approximation
- But we can make it work in practice!

$$Q_\phi(\mathbf{s},\mathbf{a}) \leftarrow r(\mathbf{s},\mathbf{a}) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}',\mathbf{a}')$$



$$\mathbf{a} = \arg\max_{\mathbf{a}} Q_\phi(\mathbf{s},\mathbf{a})$$

Learning objectives of this lecture

- You should be able to...
  - Extend discrete value iteration to fitted value iteration with function approximation
  - Understand fitted Q-iteration (FQI)
  - Use deep neural networks to approximate Q-functions, be able to implement deep Q-learning with replay buffers and target networks
  - Know deep Q-learning with n-step returns
  - Know how to make deep Q-learning work with continuous actions
  - Be aware of some theories of value function methods

Deep Q-learning suggested readings

- Lecture 7 & 8 of CS285 at UC Berkeley, **Deep Reinforcement Learning, Decision Making, and Control**
  - `http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-7.pdf`
  - `http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-8.pdf`
- Classic papers
  - Watkins. (1989). **Learning from delayed rewards**: introduces Q-learning
  - Lagoudakis. 2003. **Least-squares policy iteration**: linear function approximation
  - Riedmiller. (2005). **Neural fitted Q-iteration**: batch mode Q-learning with neural networks
- DRL Q-learning papers
  - Mnih et al. (2013). **Human level control through deep reinforcement learning**: Q-learning with convolutional networks for playing Atari.
  - Van Hasselt, Guez , Silver. (2015). **Deep reinforcement learning with double Q-learning**: a very effective trick to improve performance of deep Q-learning.
  - Lillicrap et al. (2016). **Continuous control with deep reinforcement learning**: continuous Q-learning with actor network for approximate maximization.
  - Wang, Schaul , Hessel, van Hasselt, Lanctot , de Freitas (2016). **Dueling network architectures for deep reinforcement learning**: separates value and advantage estimation in Q-function.
  - Z. Ren, et al., **Self-Paced Prioritized Curriculum Learning With Coverage Penalty in Deep Reinforcement Learning**, *TNNLS*, 2018.

THE END