NYU FRE 7773 - Week 4

Machine Learning in Financial Engineering

Ethan Rosenthal

Linear Models for Classification

Machine Learning in Financial Engineering

Ethan Rosenthal

Classification


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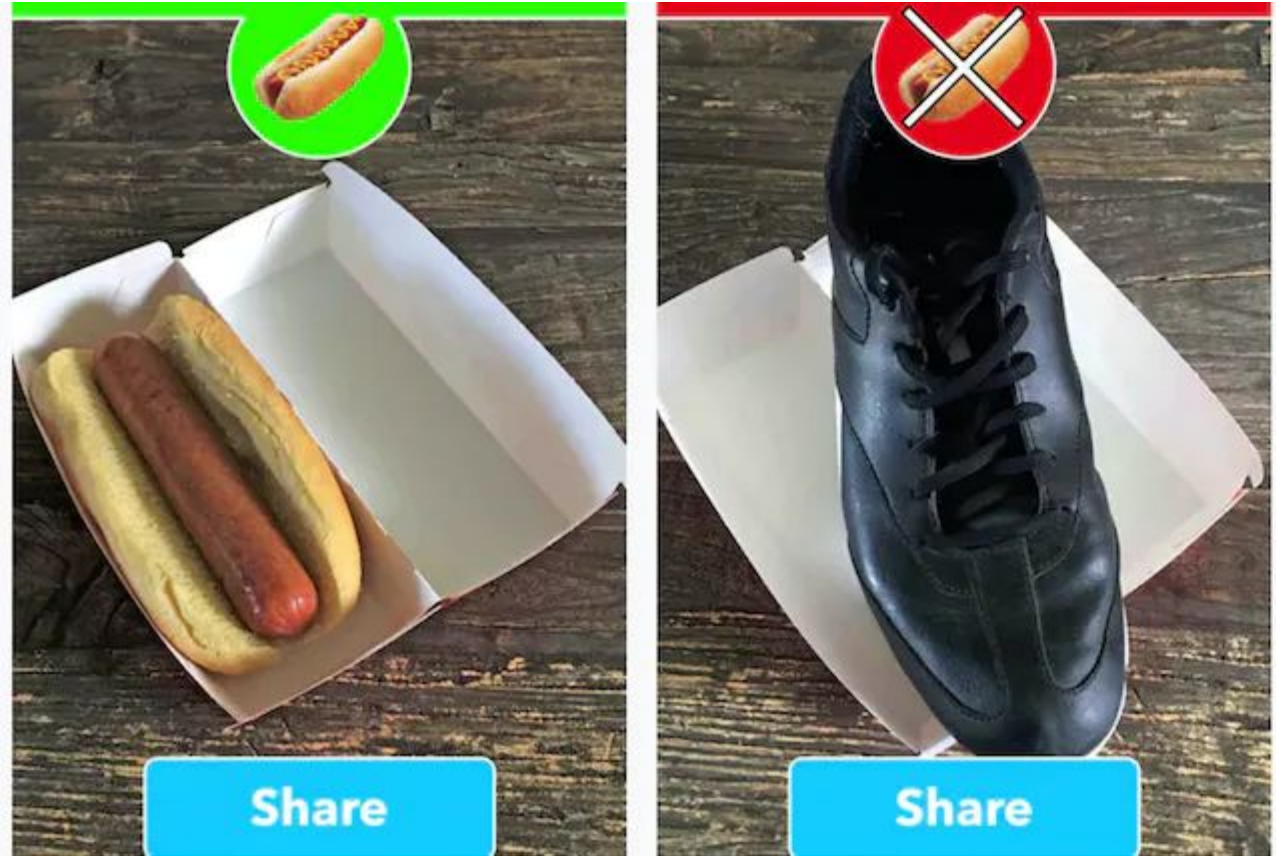

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Logistic Regression

The ML Recipe

1. Think up some model
2. Feed **data** into the model and make predictions.
3. Calculate the loss between predictions and true values.
4. Determine the model parameters that produce the minimum loss.

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Logistic Regression – The Model

- Recall our linear model for regression

$$\hat{y}_i = \sum_{j}^{p} x_{ij} \cdot \beta_j$$

- We can't use this for classification because we must predict 0 or 1.
- So, let's squash this between 0 and 1.

Logistic Regression – The Model

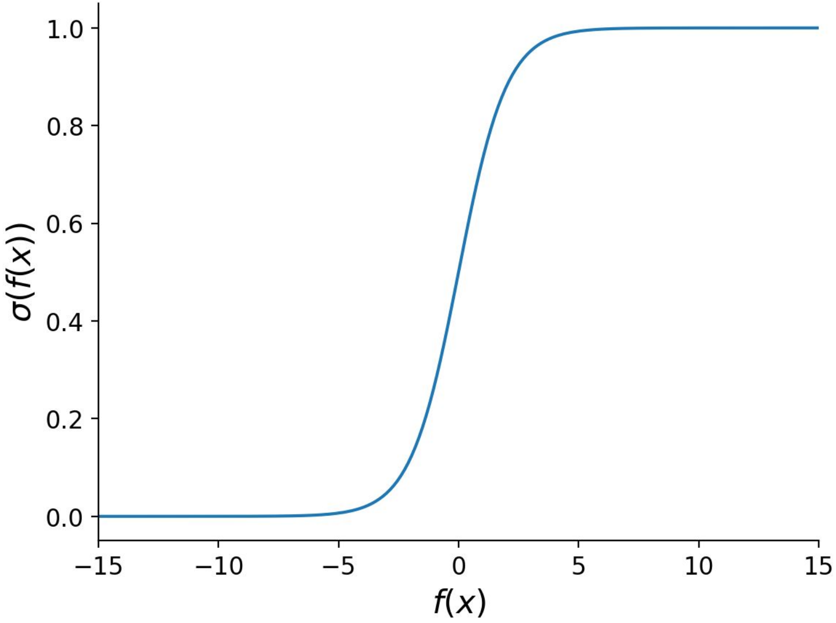
- Squash the linear model between 0 and 1.

$$\hat{y}_i = \sum_{j}^{p} x_{ij} \cdot \beta_j$$

$$f(x_i) = \sum_{j}^{p} x_{ij} \cdot \beta_j$$

$$\hat{y}_i = \sigma(f(x_i))$$

$$\sigma(f(x_i)) = \frac{1}{1 + e^{-f(x_i)}}$$



The ML Recipe

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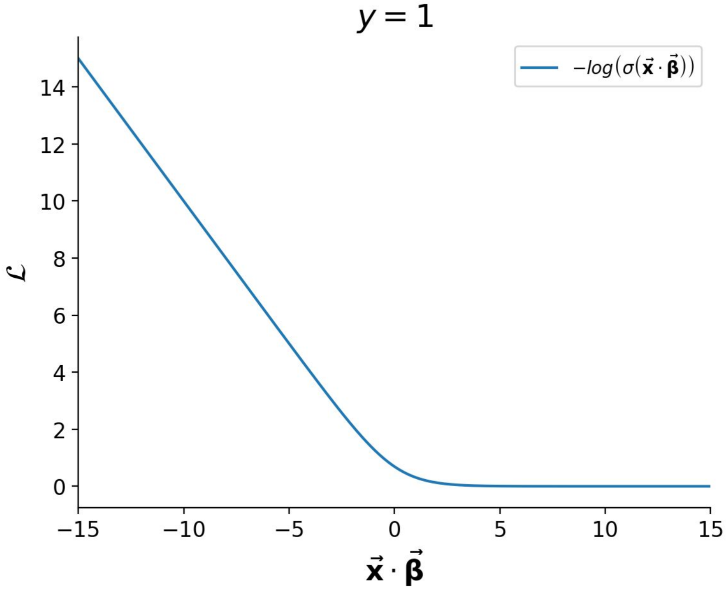
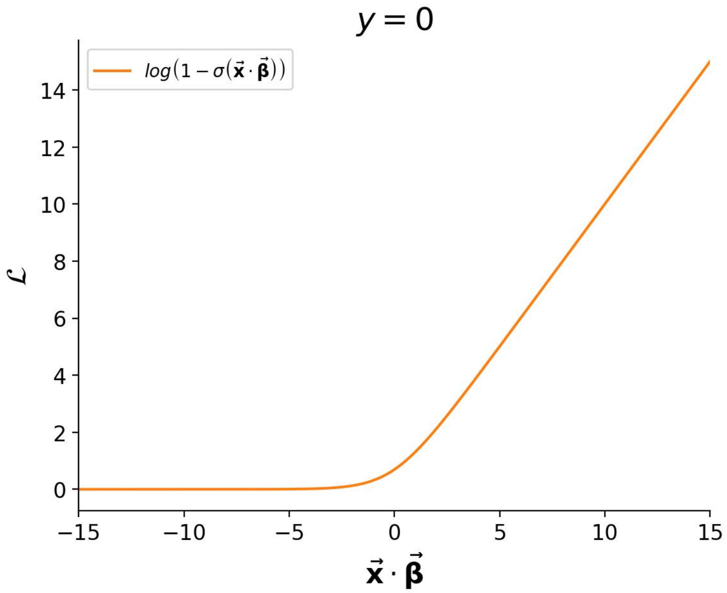
Logistic Regression – The Loss

$$\mathcal{L}\left(\vec{\beta}\right) = -\sum_{i=1}^{N} y_i \, log\left(\sigma\left(\overrightarrow{\mathbf{x_i}} \cdot \vec{\beta}\right)\right)$$



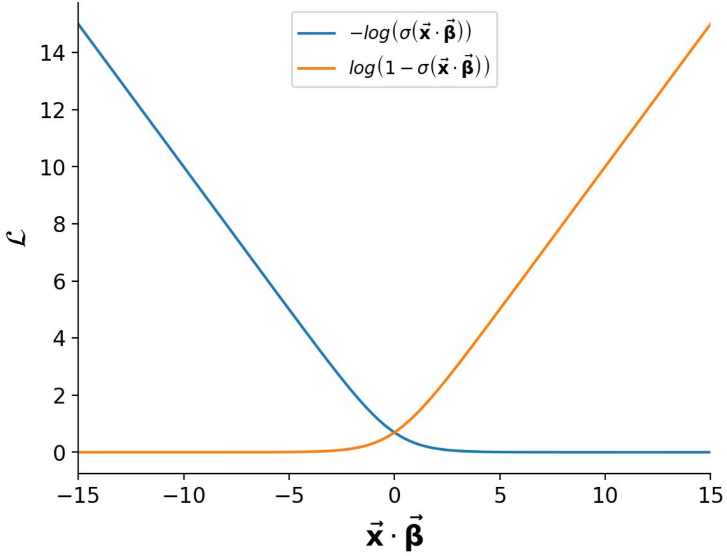
Logistic Regression – The Loss

$$\mathcal{L}\left(\vec{\beta}\right) = -\sum_{i=1}^{N} \qquad + (1 - y_i)\, log\left(1 - \sigma\left(\overrightarrow{\mathbf{x_i}} \cdot \vec{\beta}\right)\right)$$



Logistic Regression – The Loss

$$\mathcal{L}\left(\vec{\beta}\right) = -\sum_{i=1}^{N} y_i \, log\left(\sigma\left(\overrightarrow{\mathbf{x_i}} \cdot \vec{\beta}\right)\right) + (1 - y_i)\, log\left(1 - \sigma\left(\overrightarrow{\mathbf{x_i}} \cdot \vec{\beta}\right)\right)$$



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The ML Recipe

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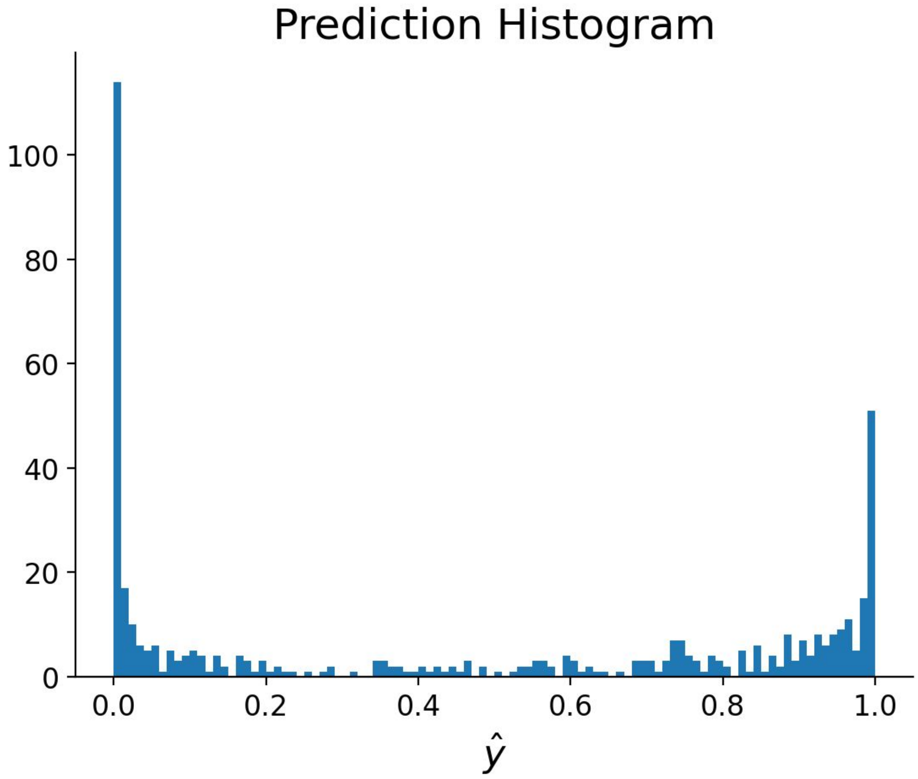
$$\frac{\partial \mathcal{L}}{\partial \vec{\beta}} = \sum_{i=1}^{N} \left(y_i - \frac{1}{1 + e^{\left(\vec{\mathbf{x}}_{\mathbf{i}} \cdot \vec{\beta}\right)}} \right) \vec{\mathbf{x}}_{\mathbf{i}}$$

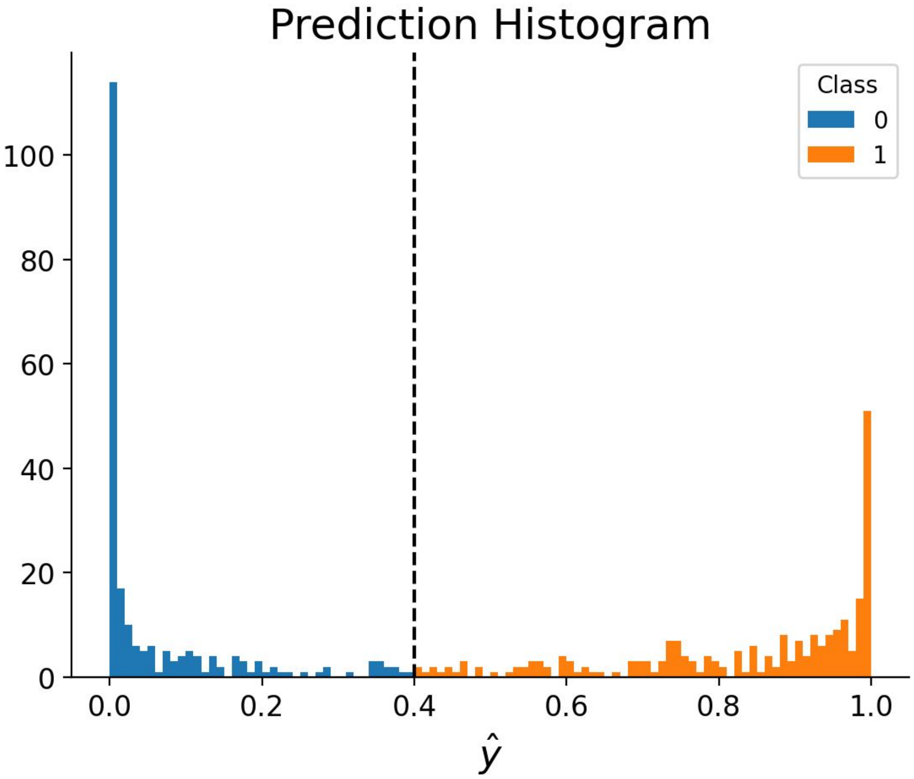
Logistic Regression – The Predictions

- Predictions lie between 0 and 1.
- They can be interpreted as probabilities.



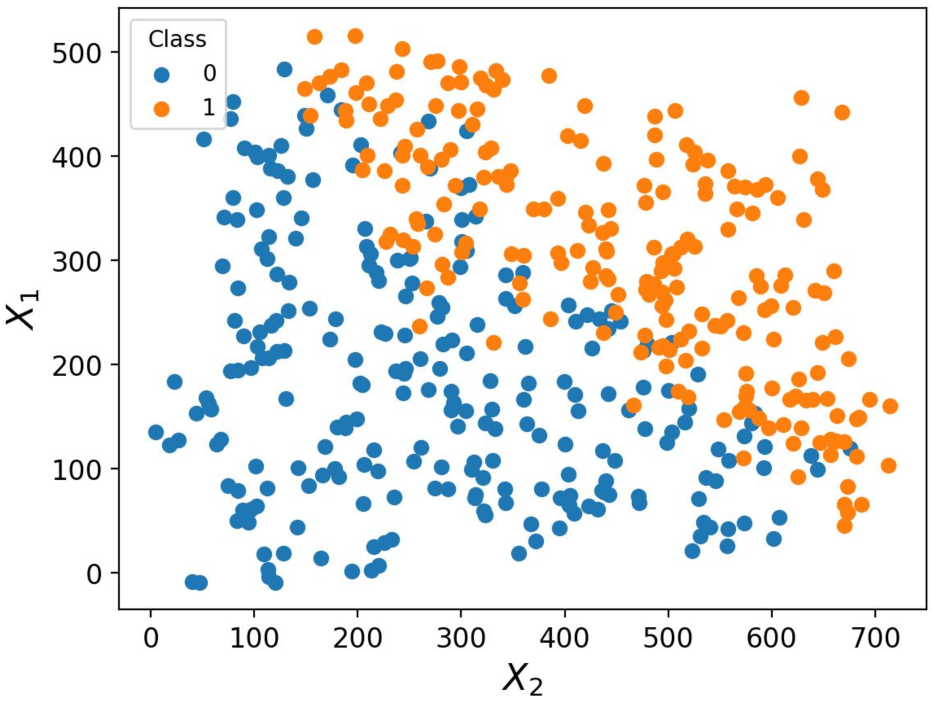
Logistic Regression – The Predictions

- Predictions lie between 0 and 1.
- They can be interpreted as probabilities.
- We pick a **threshold** to classify probabilities into classes.



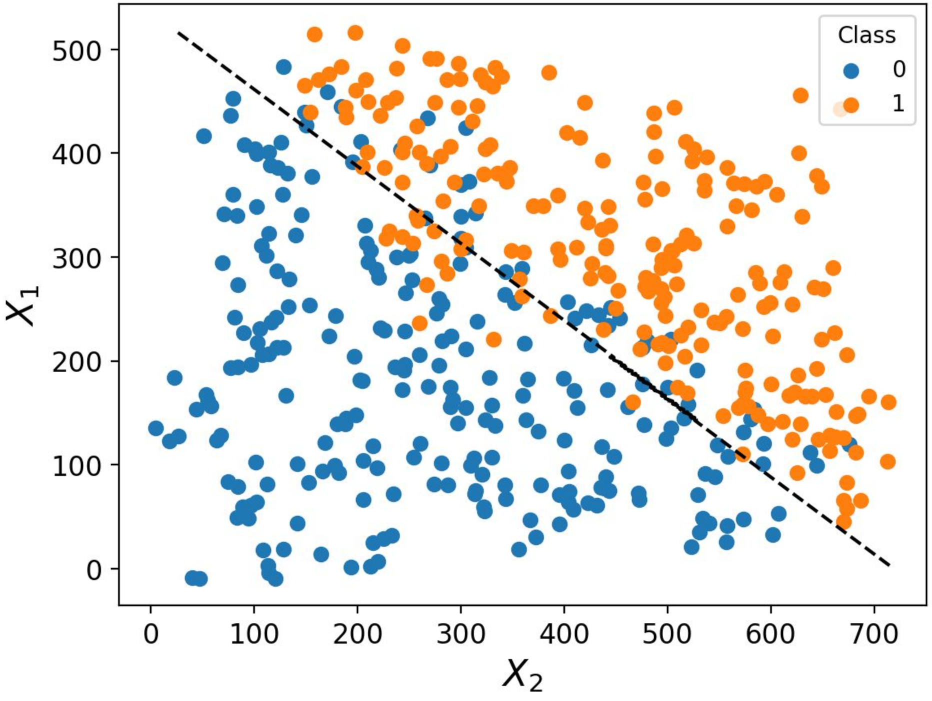
Logistic Regression – The Decision Boundary

- Imagine a classification problem with 2 features.
- For any value of X1 and X2, our model predicts a probability between 0 and 1.
- Predictions above (below) the threshold are predicted as 1 (0).



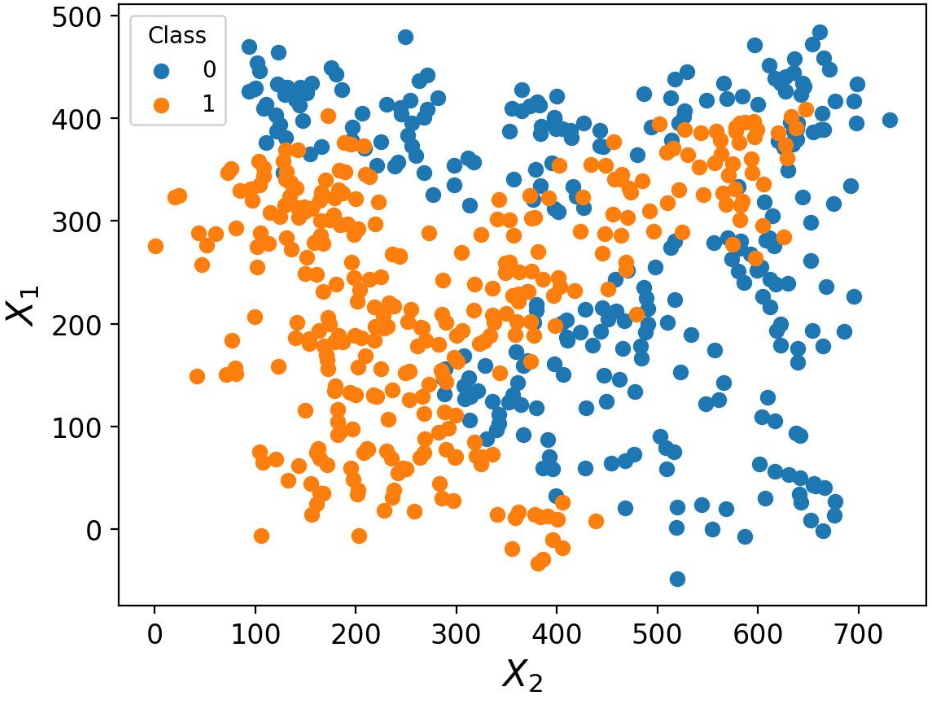
Logistic Regression – The Decision Boundary

- Imagine a classification problem with 2 features.
- For any value of X1 and X2, our model predicts a probability between 0 and 1.
- Predictions above (below) the threshold are predicted as 1 (0).
- The line that runs along the region of predictions where the prediction == threshold is the **decision boundary.**



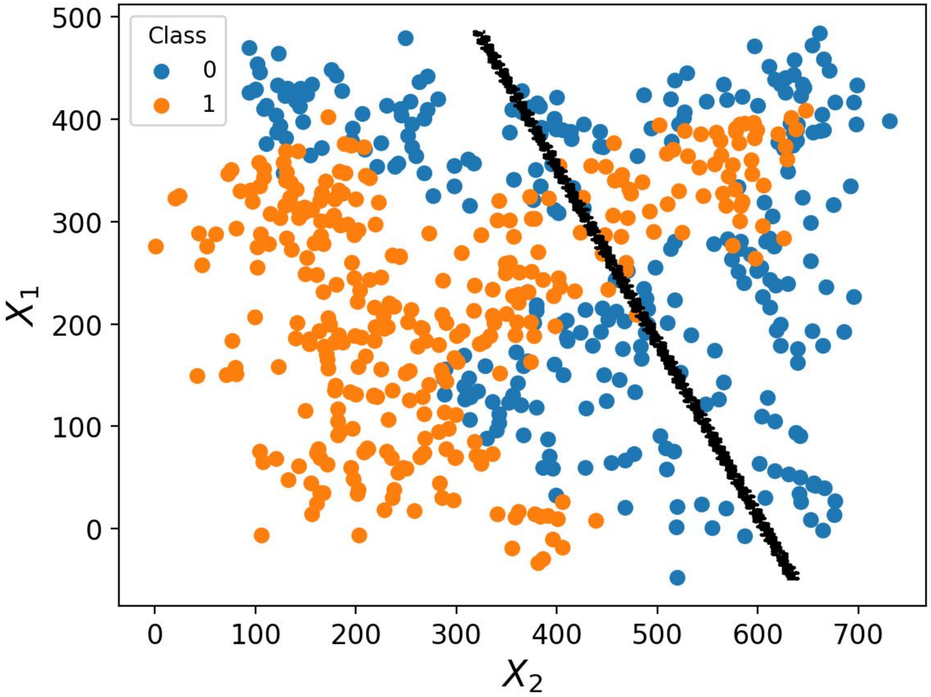
Logistic Regression – The Decision Boundary

- Problems are often nonlinear.



Logistic Regression – The Decision Boundary

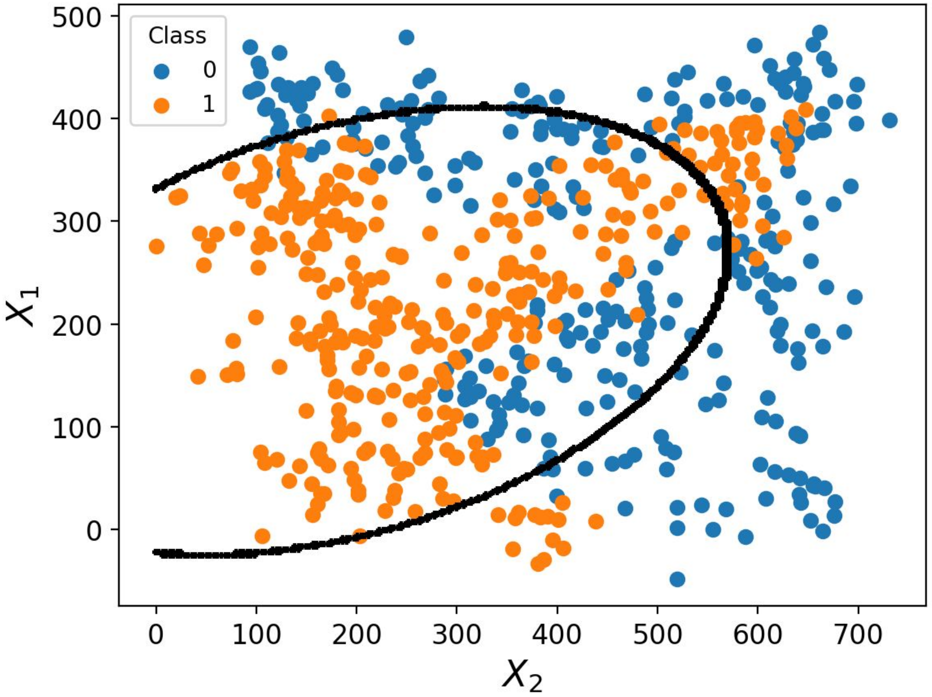
- Problems are often nonlinear.
- And linear decision boundaries don’t work well on them!



Logistic Regression – The Decision Boundary

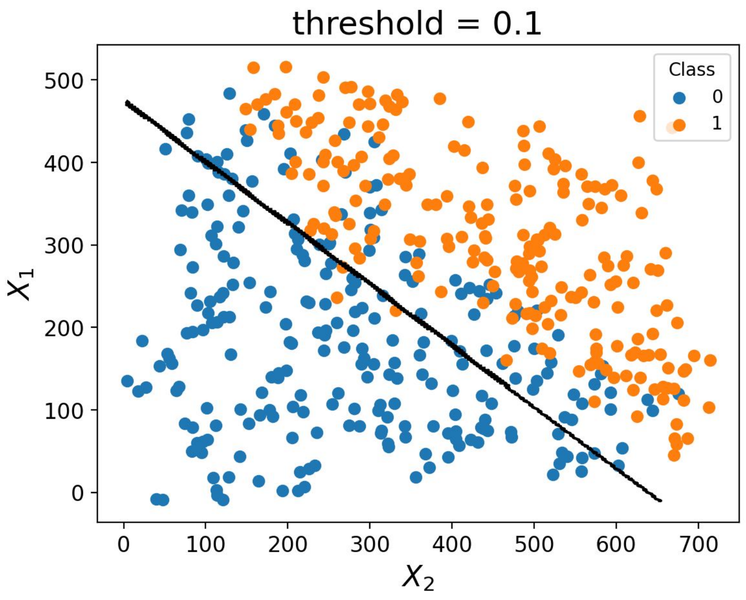
- Problems are often nonlinear.
- And linear decision boundaries don't work well on them!
- Can try creating nonlinear features:

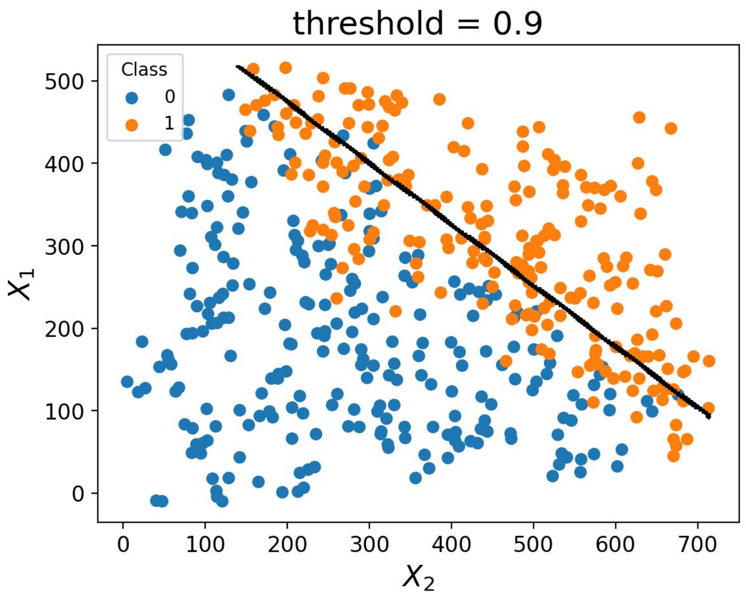
$$\hat{y}_i = \beta_0 + \beta_1 \cdot X_{i1} + \beta_2 \cdot X_{i2} + \beta_3 \cdot X_{i1} \cdot X_{i2} + \beta_4 \cdot X_{i1}^2 + \beta_5 \cdot X_{i2}^2$$



Logistic Regression – The Decision Boundary

Your threshold / decision boundary is a choice!



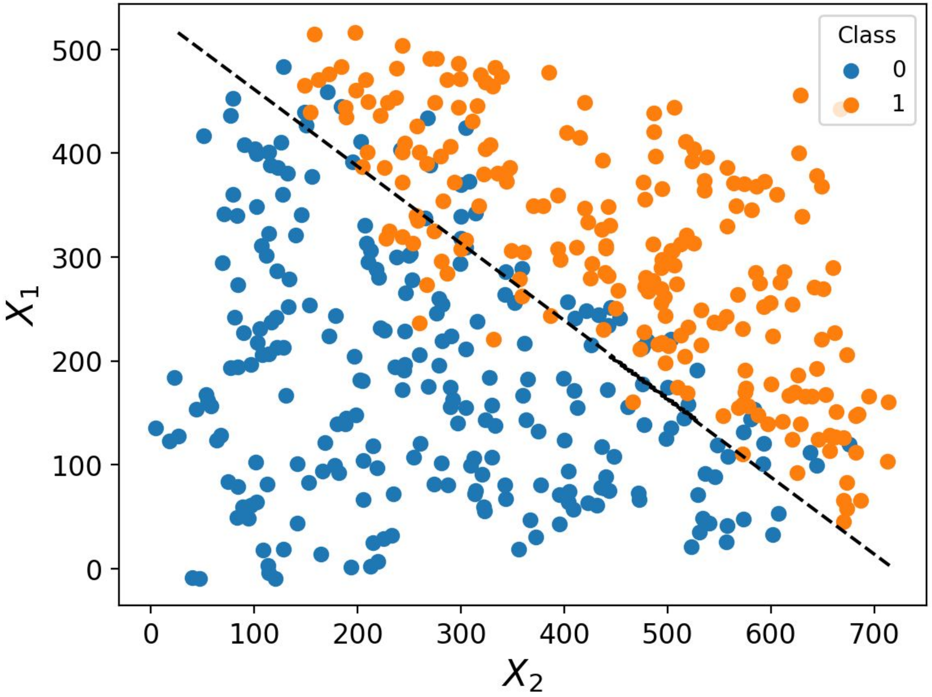


Classification Evaluation

Classification Evaluation

Confusion Matrix

Predicted Labels / True Labels	False	True
False	204	41
True	16	198



Classification Evaluation

True Positives
All orange dots on this side

Confusion Matrix

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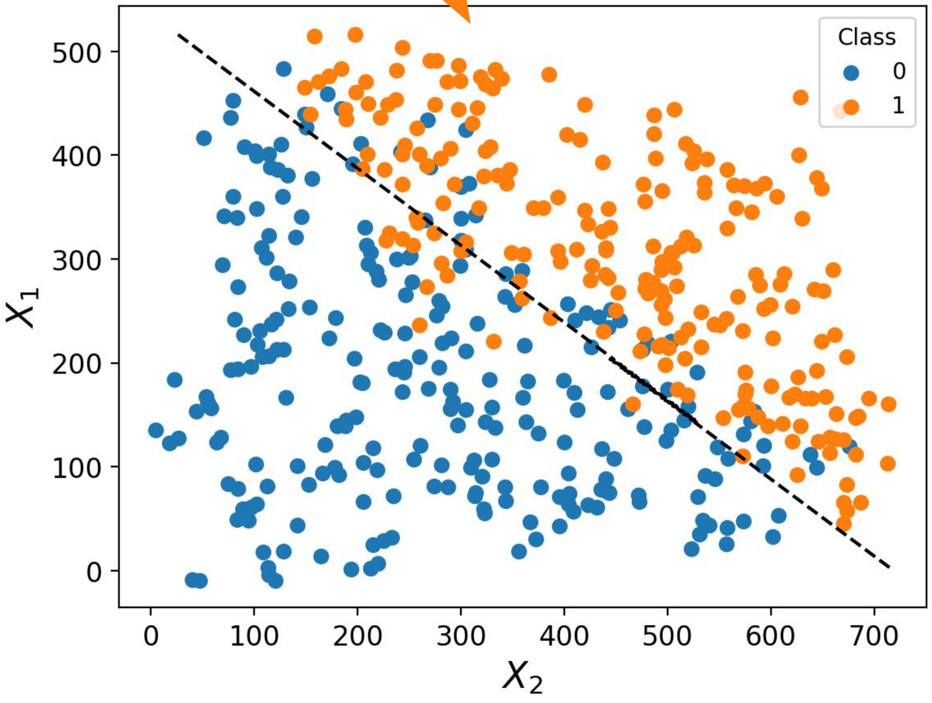

Classification Evaluation

Confusion Matrix

True Labels \ Predicted Labels	False	True
False	204	41
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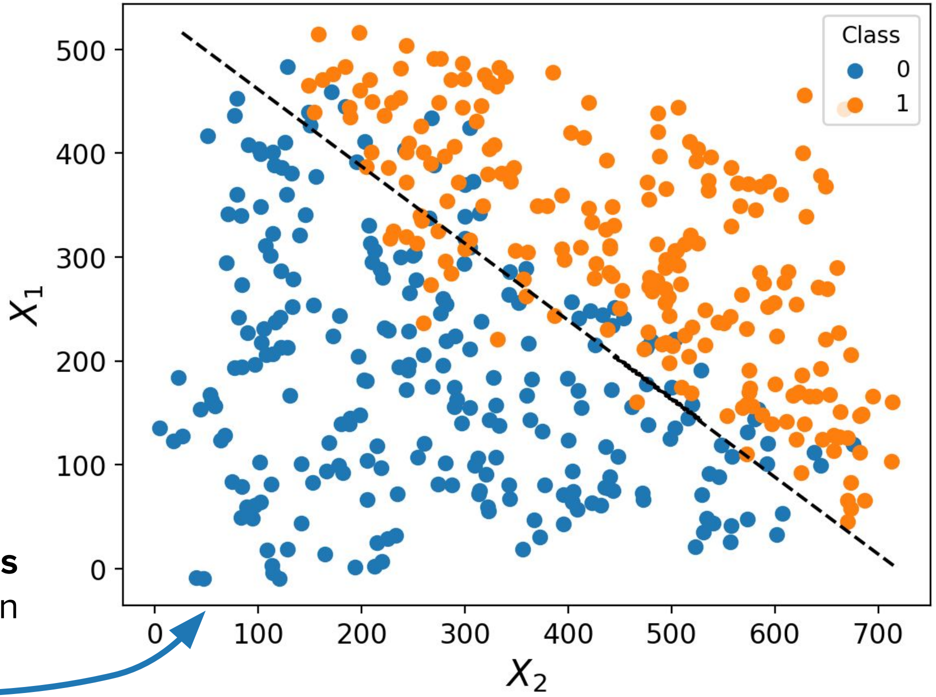
True Negatives
All blue dots on this side

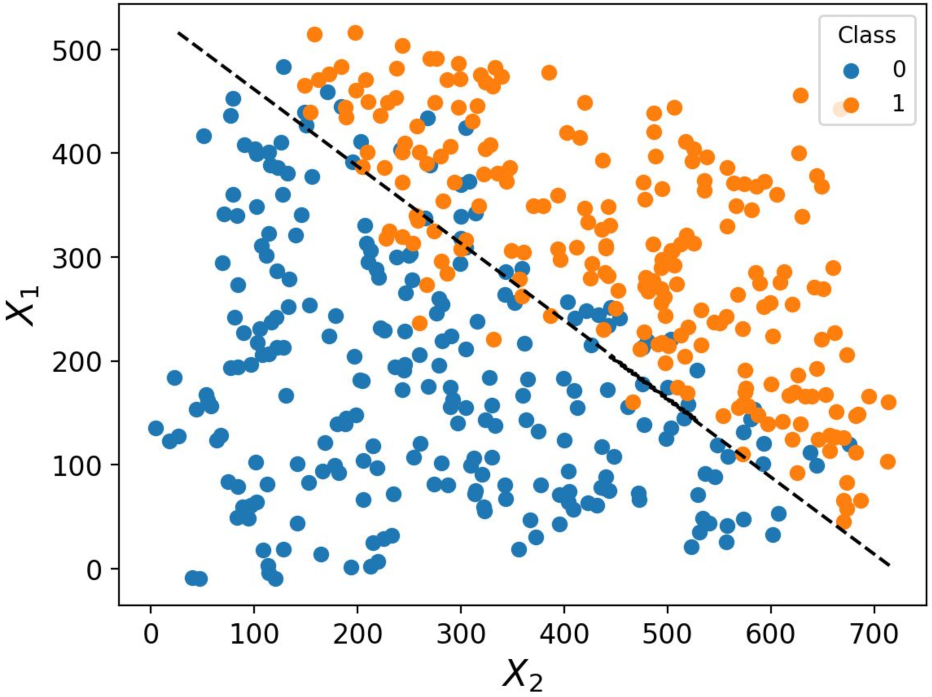


Classification Evaluation

False Positives
All blue dots on
this side

Confusion Matrix

Predicted Labels / True Labels	False	True
False	204	41
True	16	198



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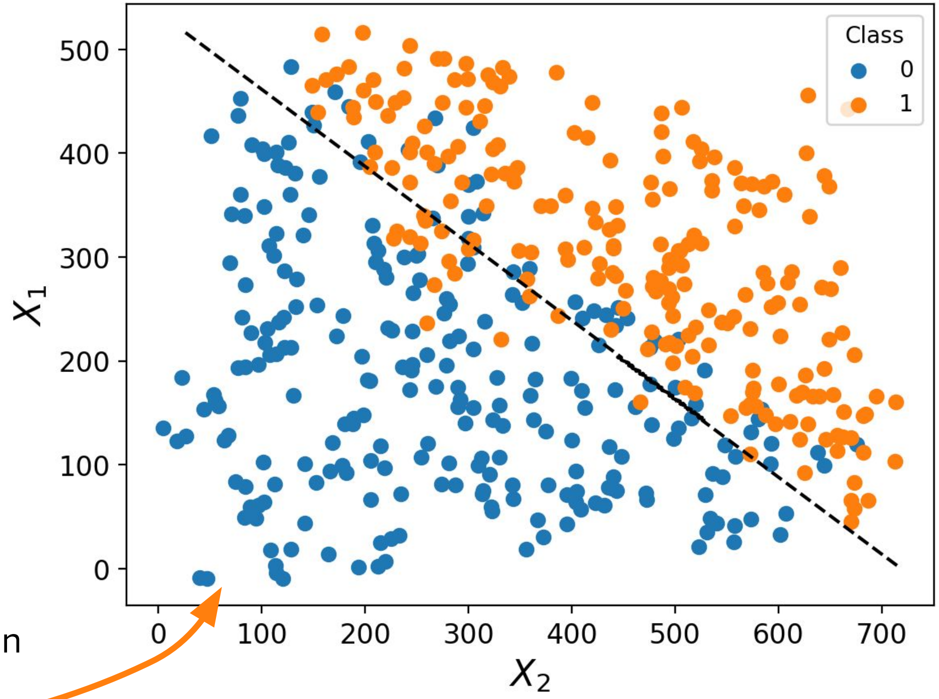

Classification Evaluation

Confusion Matrix

True Labels \ Predicted Labels	False	True
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$$accuracy = \frac{TP + TN}{N}$$

Total samples → N

$$precision = \frac{TP}{TP + FP}$$

$$recall = \frac{TP}{TP + FN}$$

Classification Evaluation

Confusion Matrix

True Labels \ Predicted Labels	False	True
False	204	41
True	16	198

$$precision = \frac{TP}{TP + FP}$$

$$recall = \frac{TP}{TP + FN}$$

Precision: For all of the samples my model predicted positive, what % of them were actually positive?

Recall: Out of all positive examples in my data, what % did my model predict positive?

Classification Evaluation

Confusion Matrix

True Labels \ Predicted Labels	False	True
False	204	41
True	16	198

$$precision = \frac{TP}{TP + FP}$$

$$recall = \frac{TP}{TP + FN}$$

F1 Score: Harmonic mean of precision and recall

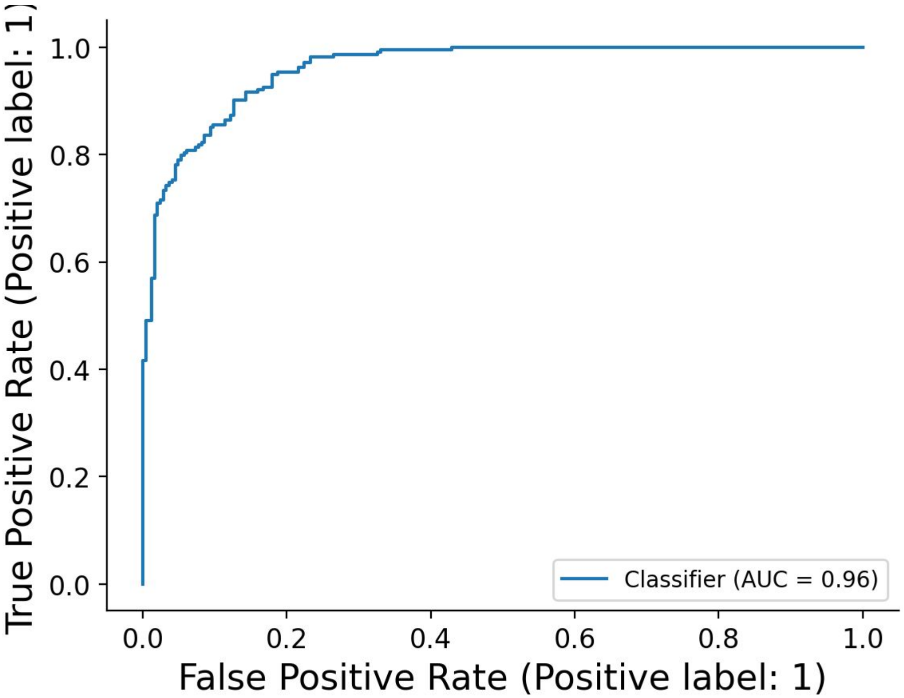
$$F_1 = \frac{2}{\frac{1}{precision} + \frac{1}{recall}} = 2\frac{precision \cdot recall}{precision + recall}$$

The ML Recipe

1. Think up some model
2. Feed **data** into the model and make predictions.
3. Calculate the loss between predictions and true values.
4. Determine the model parameters that produce the minimum loss.
5. Pick a threshold if it's a classification model.
    a. The threshold is a choice!
    b. This choice is part of your model.
    c. The confusion matrix is based on a single threshold value.

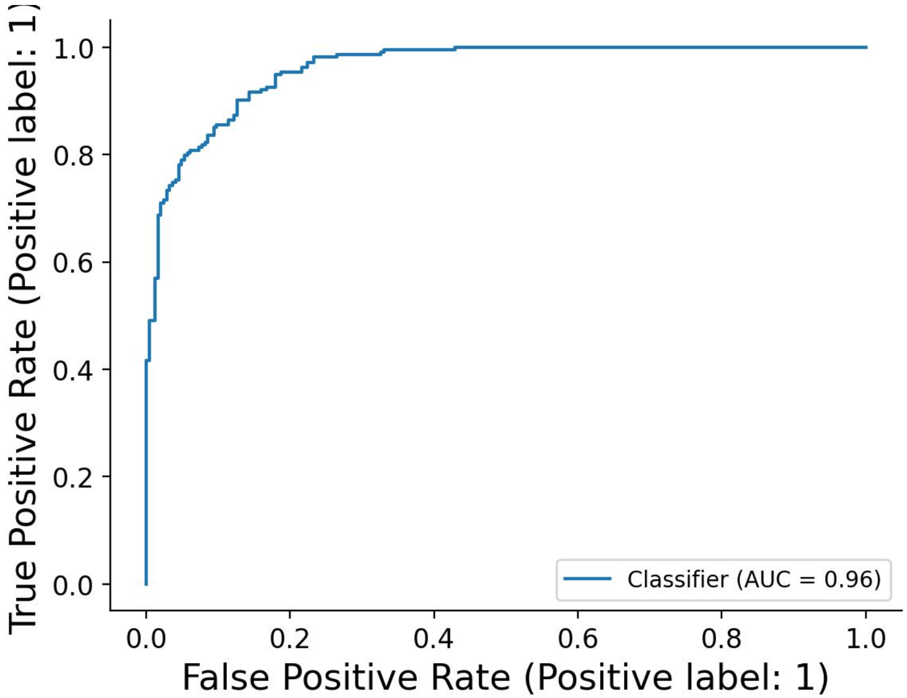
Classification Evaluation - Non-thresholded Measures

- Area Under the Receiver Operating Characteristic (ROC) Curve
  - (Sometimes just called AUC)
- TPR = Recall = TP / (TP + FN)
- FPR = TP / (FP + TN)
- Up and to the left is good
- Each point corresponds to a different threshold
- Typically, roughly concave.



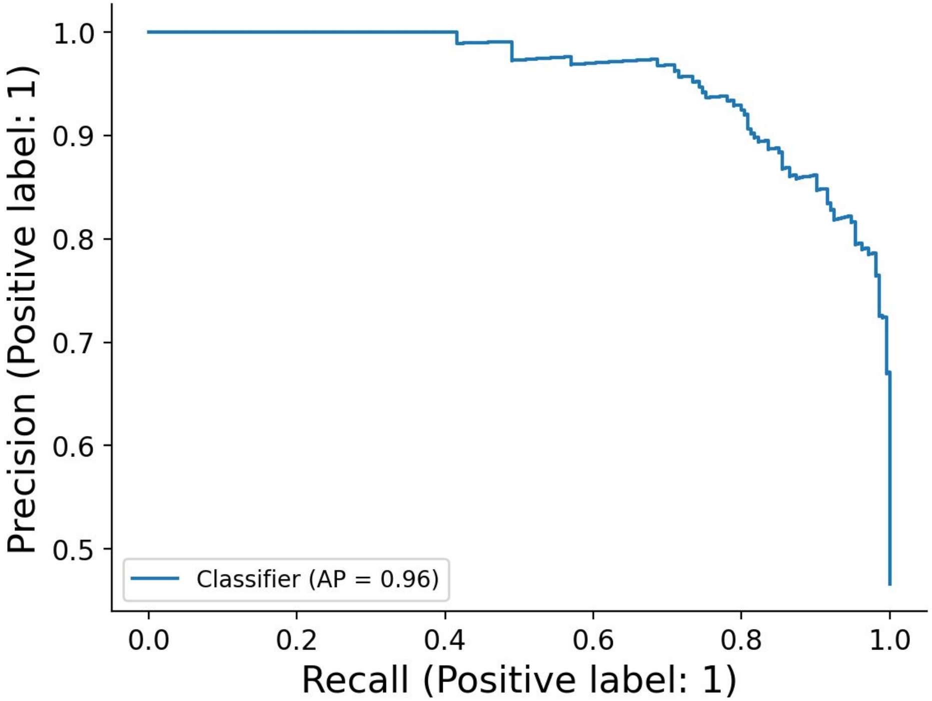
Classification Evaluation - Non-thresholded Measures

- Baseline AUROC is 0.5.
  - i.e. a random classifier
- Good, overall measure of model performance.
- Mitigates much of the impact from imbalance classes.
- Can interpret as “If you randomly draw 1 pos and 1 neg sample, what’s the probability you rank them correctly?”



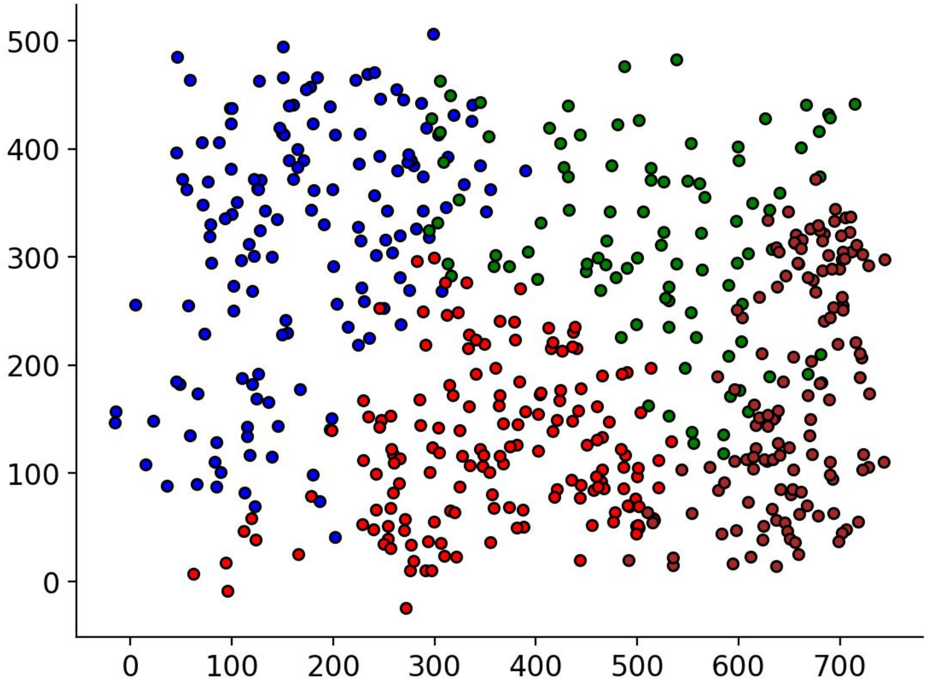
Classification Evaluation - Non-thresholded Measures

- Area Under the Precision Recall Curve (PR curve)
  - “Average Precision”
- Up and to the right is good.
- Poor models exhibit very non-concave PR curves
- Helpful for picking a threshold.

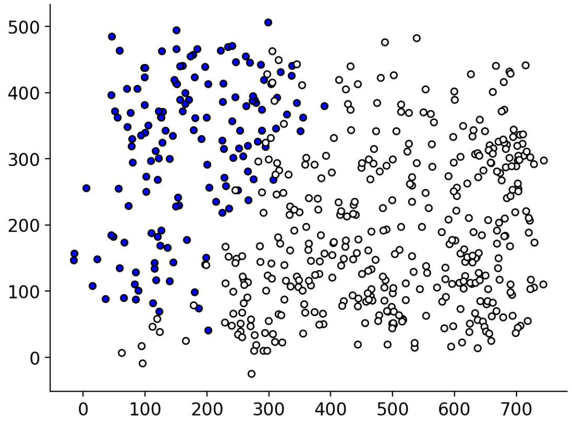


Multiclass Classification

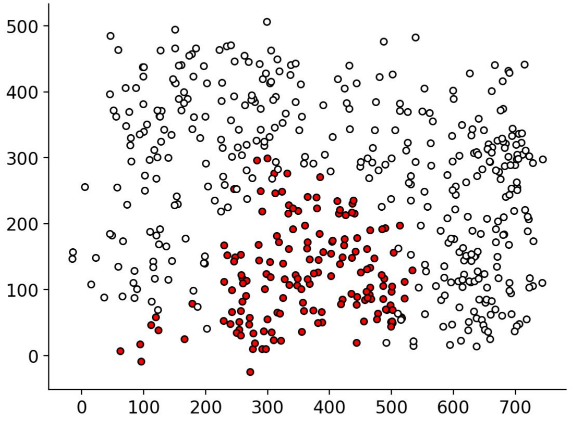
Multiclass Classification

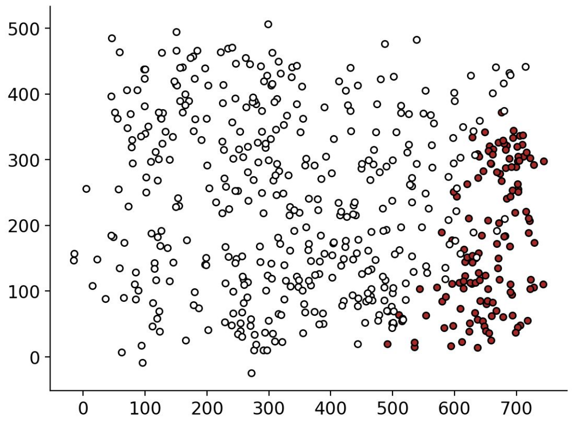


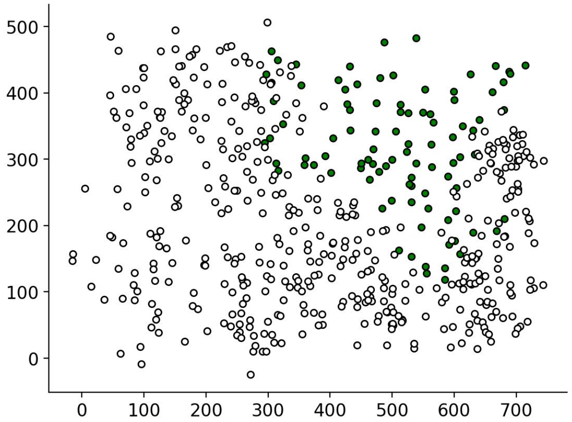
Multiclass Classification - One vs. All



- For N classes, train N binary classifiers.
- Treat each a Nth class as pos label, all other classes as negative label.

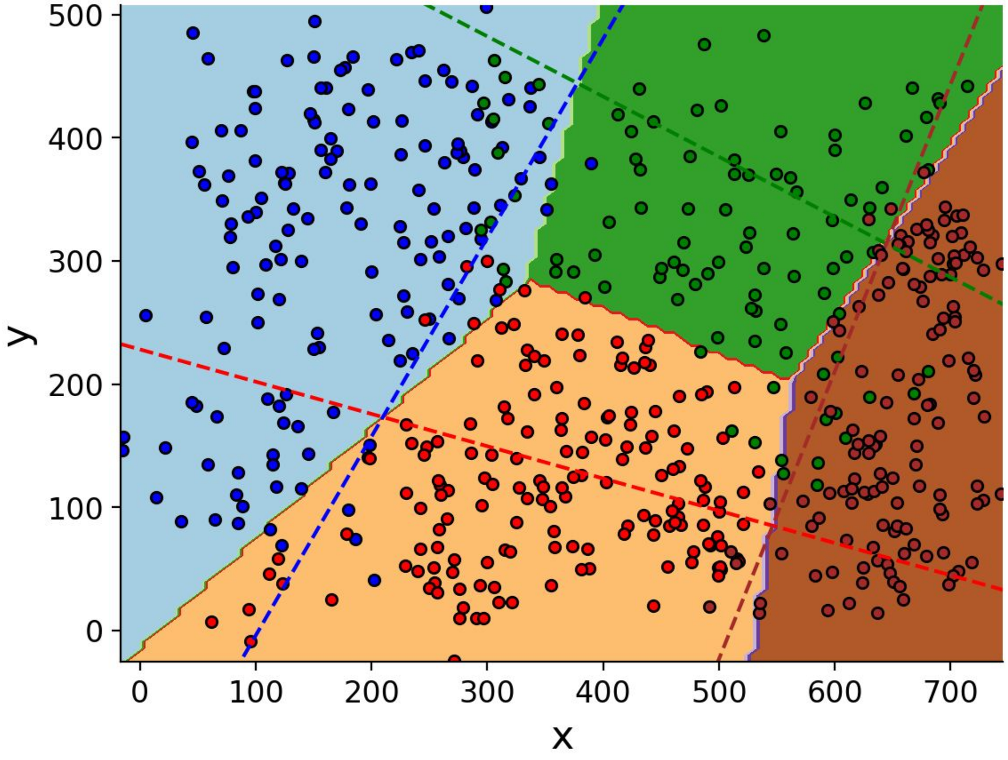






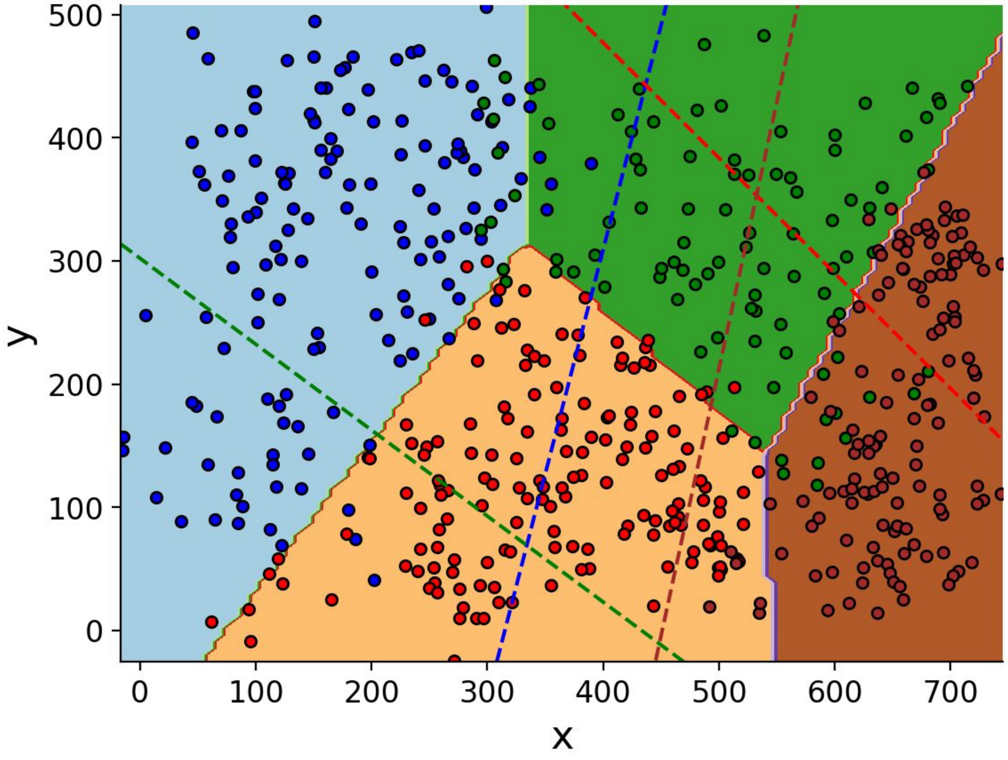
Multiclass Classification - One vs. Rest

- Max classifier score gives the predicted label.



Multiclass Classification - Multinomial

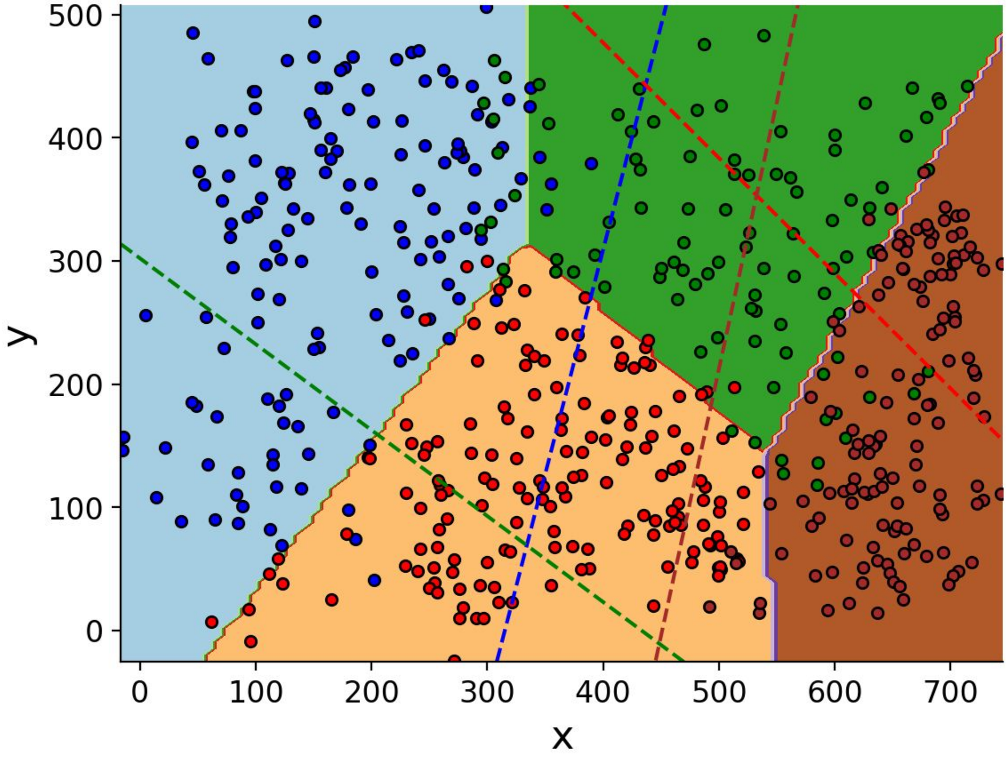
- Also called softmax or cross entropy



Multiclass Classification - Multinomial / Cross Entropy

$$\mathcal{L}_{C\mathcal{E}} = \sum_{i=1}^{N} \left(- \sum_{c=1}^{C} y_{ic} \, log \left(s \left(\overrightarrow{\mathbf{x_i}} \cdot \overrightarrow{\beta_\mathbf{c}} \right) \right) \right)$$

$$s \left(\overrightarrow{\mathbf{x_i}} \cdot \overrightarrow{\beta_{\mathbf{c}'}} \right) = \frac{e^{\overrightarrow{\mathbf{x_i}} \cdot \overrightarrow{\beta_{\mathbf{c}'}}}}{\sum_{c=1}^{C} e^{\overrightarrow{\mathbf{x_i}} \cdot \overrightarrow{\beta_\mathbf{c}}}}$$



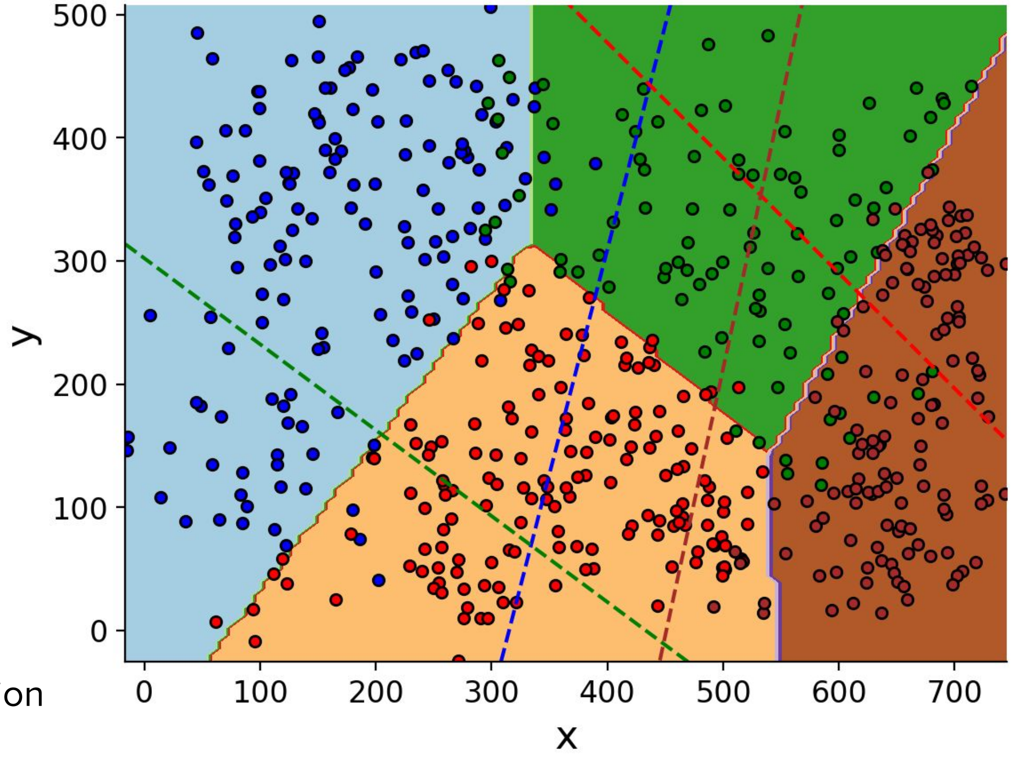
Multiclass Classification - Multinomial / Cross Entropy

$$\mathcal{L}_{C\mathcal{E}} = \sum_{i=1}^{N} \left(- \sum_{c=1}^{C} y_{ic} \, log \left(s \left(\overrightarrow{\mathbf{x_i}} \cdot \overrightarrow{\beta_{\mathbf{c}}} \right) \right) \right)$$

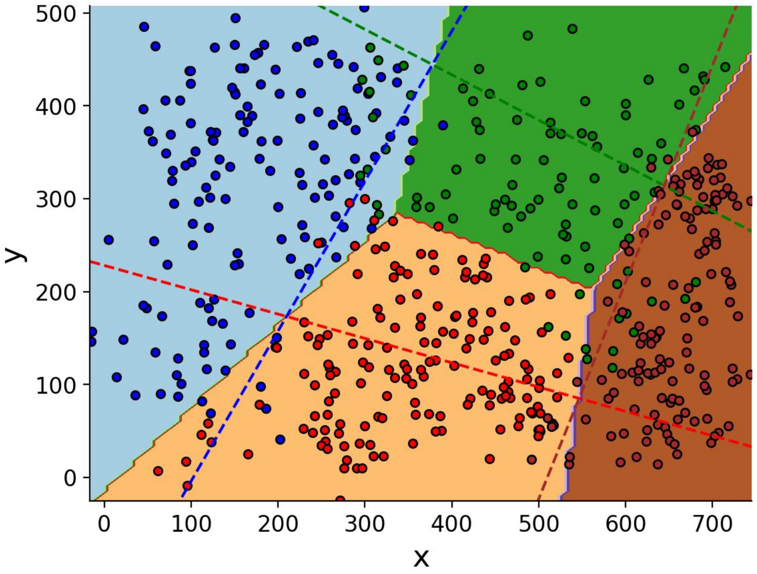
$$s \left(\overrightarrow{\mathbf{x_i}} \cdot \overrightarrow{\beta_{\mathbf{c'}}} \right) = \frac{e^{\overrightarrow{\mathbf{x_i}} \cdot \overrightarrow{\beta_{\mathbf{c'}}}}}{\sum_{c=1}^{C} e^{\overrightarrow{\mathbf{x_i}} \cdot \overrightarrow{\beta_{\mathbf{c}}}}}$$

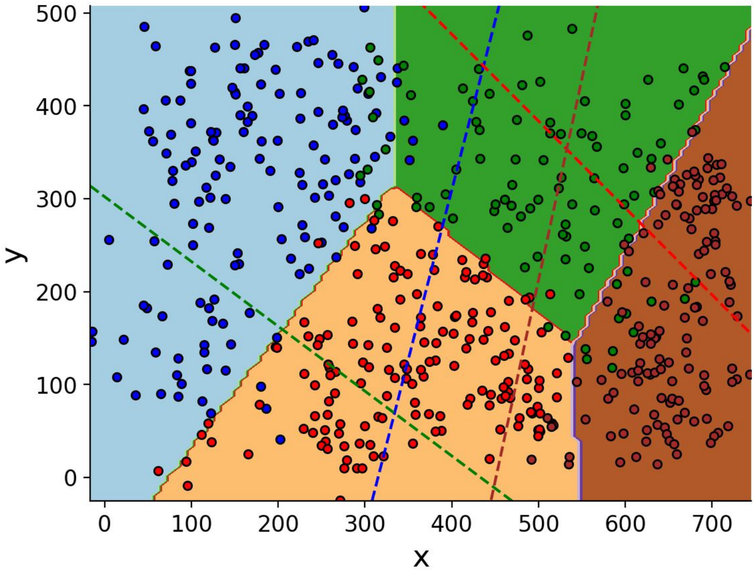
~ Probability sample belongs to class c'

Softmax function



Multiclass Classification - OVR vs Multinomial

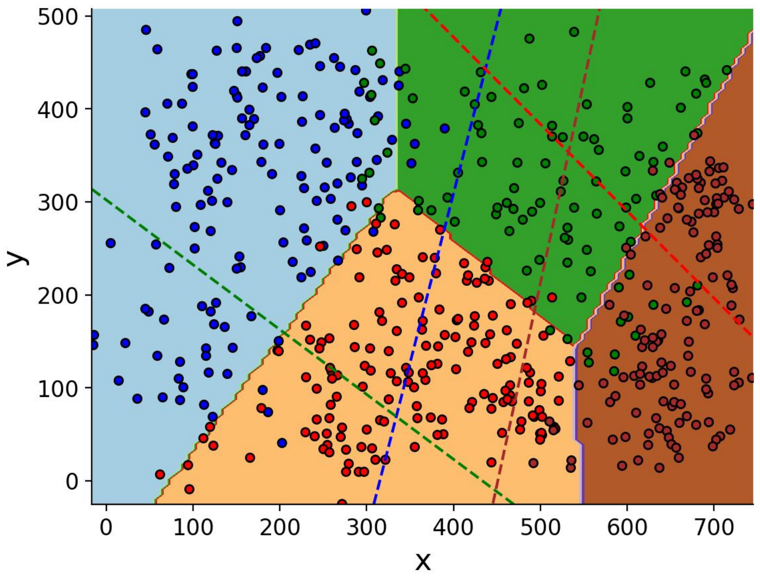




Multiclass Classification Evaluation

Confusion Matrix

Predicted Labels / True Labels	0	1	2	4
0	135	6	6	0
1	9	71	3	13
2	6	5	150	0
4	0	5	5	121



Multiclass Classification Evaluation

Accuracy is now harder!

Binary: For 50/50 labels, random classifier gets 50% accuracy

Multiclass: for equal numbers of C labels, random classifier gets 1 / C accuracy

For 4 classes, random classifier gets 25% accuracy.

Multiclass Classification Evaluation

- Precision, recall, and F1 all have multiclass equivalents.
- You can compute them for any individual class.

$$Precision_c = \frac{TP_c}{TP_c + FP_c}$$

- For computing across all classes:
  - Macro-average: Calculate metric for each class and then take the average across all classes.

$$\cancel{Precision = \frac{\sum_c TP_c}{\sum_c TP_c + FP_c}}$$

  - Micro-average: Compute numerators and denominators for all classes.

$$Precision = \frac{\sum_c TP_c}{\sum_c TP_c + FP_c}$$

Language Models are Multiclass Classifiers

- Starting Data: “Sounds like fun!”

Language Models are Multiclass Classifiers

- Starting Data: “Sounds like fun!”
- Break it up into data + answers where we try to predict the next word:

Data	Answer
Sounds	like
Sounds like	fun
Sounds like fun	!

Language Models are Multiclass Classifiers

- Starting Data: “Sounds like fun!”
- Break it up into data + answers where we try to predict the next word:

Data	Answer
Sounds	like
Sounds like	fun
Sounds like fun	!

- One-hot-encode words into numbers:

0	0	1	0	1	1	0	1	…
hi	you	like	dog	fun	sounds	me	!	

Language Models are Multiclass Classifiers

- Starting Data: “Sounds like fun!”
- Break it up into data + answers where we try to predict the next word:

Data	Answer
Sounds	like
Sounds like	fun
Sounds like fun	!

- One-hot-encode words into numbers:

0	0	1	0	1	1	0	1	…
hi	you	like	dog	fun	sounds	me	!	

- Feed into a model, treat all words as classes, and predict the answer word (class)