Lab05-Linear Programming

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2019.

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1. A company intends to invest 0.3 million dollars in 2018, with a proper combination of the following 3 projects:

   - **Project 1:** Invest at the beginning of a year, and can receive a 20% profit of the investment in this project at the end of this year. Both the capital and profit can be invested at the beginning of next year;
   - **Project 2:** Invest at the beginning of 2018, and can receive a 50% profit of the investment in this project at the end of 2019. The investment in this project cannot exceed 0.15 million dollars;
   - **Project 3:** Invest at the beginning of 2019, and can receive a 40% profit of the investment in this project at the end of 2019. The investment in this project cannot exceed 0.1 million dollars.

   Assume that the company will invest *all* its money at the beginning of a year. Please design a scheme of investment in 2018 and 2019 which maximizes the overall sum of capital and profit at the end of 2019.

   (a) Formulate a linear programming with necessary explanations.

   (b) Transform your LP into its standard form and slack form.

   (c) Transform your LP into its dual form.

   (d) Use the simplex method to solve your LP by step.

Solution. (a) The variables are defined as follows.

	P_1	P_2	P_3
2018	x_1	$0.3 - x_1$	–
2019	x_2	–	x_3

The question is transferred to the following linear programming form.

$$\begin{aligned}
\max \quad & 1.5 \times (0.3 - x_1) + 1.2x_2 + 1.4x_3 \\
s.t. \quad & x_2 + x_3 \leq 1.2x_1 \\
& 0 \leq 0.3 - x_1 \leq 0.15 \\
& x_3 \leq 0.1 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

(b) i. Standard form:

$$\begin{aligned}
\max \quad & 1.5 \times (0.3 - x_1) + 1.2x_2 + 1.4x_3 \\
s.t. \quad & -1.2x_1 + x_2 + x_3 \leq 0 \\
& x_1 \leq 0.3 \\
& -x_1 \leq -0.15 \\
& x_3 \leq 0.1 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

ii. Slack form:

$$\begin{aligned} \max \quad & 1.5 \times (0.3 - x_1) + 1.2x_2 + 1.4x_3 \\ s.t. \quad & -1.2x_1 + x_2 + x_3 + x_4 = 0 \\ & x_1 + x_5 = 0.3 \\ & -x_1 + x_6 = -0.15 \\ & x_3 + x_7 = 0.1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \end{aligned}$$

Here, x_4, x_5, x_6, x_7 are slack variables.

(c) Dual form:

$$\begin{aligned} \min \quad & 0.3y_2 - 0.15y_3 + 0.1y_4 + 0.45 \\ s.t. \quad & -1.2y_1 + y_2 - y_3 \geq -1.5 \\ & y_1 \geq 1.2 \\ & y_1 + y_4 \geq 1.4 \\ & y_1, y_2, y_3, y_4 \geq 0 \end{aligned}$$

(d) i. Obtaining basic solution: Set $x_1 = 0.3, x_2 = 0, x_3 = 0$.

ii. Selecting nonbasic variable:
We choose x_1. Currently the basic solution is $\overline{x} = \{0.3, 0, 0, 0.36, 0, 0.15, 0.1\}$ If $x_1 \downarrow, then x_5 \uparrow, x_6 \downarrow, x_4 \uparrow$. x_6 needs to be greater than 0. Thus the tight bound is $x_1 = x_6 + 0.15$.

iii. Pivoting:
Exchange x_1 and x_6. The equation becomes

$$\begin{aligned} 0.45 - 1.5x_1 + 1.2x_2 + 1.4x_3 &= 0.45 - 1.5(x_6 + 0.15) + 1.2x_2 + 1.4x_3 \\ &= 0.225 - 1.5x_6 + 1.2x_2 + 1.4x_3 \end{aligned}$$

The nonbasic variable becomes x_2, x_3, x_6. The first constraint becomes $-1.2(x_6 + 0.15) + x_2 + x_3 + x_4 = 0$.

iv. Selecting nonbasic variable:
We choose x_3. Currently the basic solution is $\overline{x} = \{0.15, 0, 0, 0.36, 0, 0, 0.1\}$. If $x_3 \uparrow$, then $x_7 \downarrow, x_4 \downarrow$.
$x_7 = 0.1 - x_3 \geq 0 \Rightarrow x_3 \leq 0.1$.
$x_4 = 1.2(x_6 + 0.15) - x_2 - x_3 \geq 0 \Rightarrow x_3 \leq 0.18$
Thus, the first one is tighter.

v. Pivoting:
Exchange x_3 and x_7. The equation becomes

$$\begin{aligned} 0.225 - 1.5x_6 + 1.2x_2 + 1.4x_3 &= 0.225 - 1.5x_6 + 1.2x_2 + 1.4(0.1 - x_7) \\ &= 0.365 - 1.5x_6 - 1.4x_7 + 1.2x_2 \end{aligned}$$

The nonbasic variable becomes x_2, x_7, x_6. The first constraint becomes $-1.2x_6 - 0.08 + x_2 - x_7 + x_4 = 0$.

vi. Selecting nonbasic variable: We choose x_2. Currently the basic solution is $\overline{x} = \{0.15, 0, 0.1, 0.36, 0, 0, 0\}$. If $x_2 \uparrow$, then $x_4 \downarrow$.
$x_4 = x_7 - x_2 + 1.2x_6 + 0.08 \geq 0 \Rightarrow x_2 \leq x_7 + 1.2x_6 + 0.08$.

vii. Pivoting:
Exchange x_2 and x_4. The equation becomes

$$\begin{aligned} 0.365 - 1.5x_6 - 1.4x_7 + 1.2x_2 &= 0.365 - 1.5x_6 - 1.4x_7 + 1.2(x_7 + 1.2x_6 + 0.08) \\ &= 0.461 - 0.06x_6 - 0.2x_7 \end{aligned}$$

Current solution is $\overline{x} = \{0.15, 0.08, 0.1, 0, 0, 0, 0\}$

viii. Conclusion:
Since $x_6 \geq 0, x_7 \geq 0$, the optimal solution is 0.461 million dollars.
In 2018, the company should invest 0.15 million dollars on **Project 1**, 0.15 million dollars on **Project 2**, and in 2019 invest 0.08 million dollars on **Project 1**, 0.1 million dollars on **project 3**.

□

2. An engineering factory makes seven products (PROD 1 to PROD 7) on the following machines: four grinders, two vertical drills, three horizontal drills, one borer and one planer. Each product yields a certain contribution to profit (in £/unit). These quantities (in £/unit) together with the unit production times (hours) required on each process are given below. A dash indicates that a product does not require a process.

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
Contribution to profit	10	6	8	4	11	9	3
Grinding	0.5	0.7	-	-	0.3	0.2	0.5
Vertical drilling	0.1	0.2	-	0.3	-	0.6	-
Horizontal drilling	0.2	-	0.8	-	-	-	0.6
Boring	0.05	0.03	-	0.07	0.1	-	0.08
Planing	-	-	0.01	-	0.05	-	0.05

There are marketing limitations on each product in each month, given in the following table:

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	300	600	0	0	500	400	100
April	200	300	400	500	200	0	100
May	0	100	500	100	1000	300	0
June	500	500	100	300	1100	500	60

It is possible to store up to 100 of each product at a time at a cost of £0.5 per unit per month (charged at the end of each month according to the amount held at that time). There are no stocks at present, but it is desired to have a stock of exactly 50 of each type of product at the end of June. The factory works six days a week with two shifts of 8h each day. It may be assumed that each month consists of only 24 working days. Each machine must be down for maintenance in one month of the six. No sequencing problems need to be considered.

When and what should the factory make in order to maximize the total net profit?

Symbol	Definition
g_{ij}	grinder i's status in month j. 1-maintenance, 0-work. $i = 1, 2, 3, 4, j = 1, 2, 3, 4, 5, 6$
v_{ij}	vertical drillings i's status in month j. $i = 1, 2, j = 1, 2, 3, 4, 5, 6$
h_{ij}	horizontal drillings i's status in month j. $i = 1, 2, 3, j = 1, 2, 3, 4, 5, 6$
b_i	boring's status in month i. $i = 1, 2, 3, 4, 5, 6$.
p_i	planing's status in month i. $i = 1, 2, 3, 4, 5, 6$.
x_{ij}	The total amount of producing of Product i in month j.
y_{ij}	The total amount of selling of Product i in month j.

(a) Use *CPLEX Optimization Studio* to solve this problem. Describe your model in *Optimization Programming Language* (OPL). Remember to use a separate data file (.dat) rather than embedding the data into the model file (.mod).

Detailed process is written in the model file.

(b) Solve your model and give the following results.

i. For each machine:

A. the month for maintenance.

Grinders: in February, April, April, April respectively.
Vertical drillings: in January,May respectively.
Horizontal drillings:in January, February, May respectively.
Borer:in April.
Planer: in April.

ii. For each product:

A. The amount to make in each month.

	Jan.	Feb.	Mar.	Apr.	May	Jun.
Product 1	500	600	400	0	0	550
Product 2	1000	500	700	0	100	550
Product 3	300	200	100	0	500	150
Product 4	300	0	100	0	100	350
Product 5	800	400	600	0	1000	1150
Product 6	200	300	400	0	300	550
Product 7	100	150	200	0	0	110

B. The amount to sell in each month.

	Jan.	Feb.	Mar.	Apr.	May	Jun.
Product 1	500	600	300	100	0	500
Product 2	1000	500	600	100	100	500
Product 3	300	200	0	100	500	100
Product 4	300	0	0	100	100	300
Product 5	800	400	500	100	1000	1100
Product 6	200	300	400	0	300	500
Product 7	100	150	100	100	0	60

iii. The amount to hold at the end of each month.

A. The total selling profit.

Jan.	Feb.	Mar.	Apr.	May	Jun.
25500	43650	59650	63850	82550	109330

B. The total holding cost.

Jan.	Feb.	Mar.	Apr.	May	Jun.
0	0	300	300	300	475

C. The total net profit (selling profit minus holding cost).

Jan.	Feb.	Mar.	Apr.	May	Jun.
25500	43650	59350	63550	82250	108855

Remark: You need to include your .mod, .dat, .pdf and .tex files in your uploaded .zip file.