Lab05-Linear Programming

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2019.

∗ If there is any problem, please contact TA Jiahao Fan.

∗ Name: Guohang Liu Student ID: 517030910039 Email: 1092327841@qq.com

1. A company intends to invest 0.3 million dollars in 2018, with a proper combination of the following 3 projects:

   - **Project 1:** Invest at the beginning of a year, and can receive a 20% profit of the investment in this project at the end of this year. Both the capital and profit can be invested at the beginning of next year;
   - **Project 2:** Invest at the beginning of 2018, and can receive a 50% profit of the investment in this project at the end of 2019. The investment in this project cannot exceed 0.15 million dollars;
   - **Project 3:** Invest at the beginning of 2019, and can receive a 40% profit of the investment in this project at the end of 2019. The investment in this project cannot exceed 0.1 million dollars.

   Assume that the company will invest *all* its money at the beginning of a year. Please design a scheme of investment in 2018 and 2019 which maximizes the overall sum of capital and profit at the end of 2019.

   (a) Formulate a linear programming with necessary explanations.

   Solution. The investments in each project are defined in the following table:

Year	**Project 1**	**Project 2**	**Project 3**
2018	$0.3 - x_1$	x_1	-
2019	$1.2(0.3 - x_1) - x_2$	-	x_2

   Then we have

$$\begin{cases} x_2 \leq 1.2(0.3 - x_1) \\ x_1 \leq 0.15 \\ x_2 \leq 0.1 \\ x_1, x_2 \geq 0 \end{cases}$$

   Then we can get the overall sum of capital and profit at the end of 2019:

$$\begin{aligned} W &= 1.5x_1 + 1.2(1.2(0.3 - x_1) - x_2) + 1.4x_2 \\ &= 0.432 + 0.06x_1 + 0.2x_2 \end{aligned}$$

   What we should do is to gain the maximum of W. Ignoring the constant term in W, we can define the objective function of the LP as

$$f(x_1, x_2) = 0.06x_1 + 0.2x_2$$

   □

   (b) Transform your LP into its standard form and slack form.

Solution.
Standard Form.

$$\max \quad 0.06x_1 + 0.2x_2$$

$$\begin{aligned} s.t. \quad 1.2x_1 + x_2 &\leq 0.36 \\ x_1 &\leq 0.15 \\ x_2 &\leq 0.1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Slack Form.

$$\max \quad 0.06x_1 + 0.2x_2$$

$$\begin{aligned} s.t. \quad x_3 &= 0.36 - 1.2x_1 - x_2 \\ x_4 &= 0.15 - x_1 \\ x_5 &= 0.1 - x_2 \\ x_i &\geq 0 \quad , i = 1,2,3,4,5 \end{aligned}$$

□

(c) Transform your LP into its dual form.

Solution.
Dual Form.

$$\min \quad 0.36y_1 + 0.15y_2 + 0.1y_3$$

$$\begin{aligned} s.t. \quad 1.2y_1 + y_2 &\geq 0.06 \\ y_1 + y_3 &\geq 0.2 \\ y_1, y_2, y_3 &\geq 0 \end{aligned}$$

□

(d) Use the simplex method to solve your LP by step.

Solution.
Converting LP into Slack Form
Using the slack form shown in (b).

Obtaining Basic Solution
Nonbasic variable: x_1, x_2
Basic variable: x_3, x_4, x_5
The basic solution: $\mathbf{x} = (0, 0, 0.36, 0.15, 0.1)$

Selecting Nonbasic Variable
Choose the nonbasic variable x_1. When $x_1 \uparrow$, $x_3 \downarrow$ and $x_4 \downarrow$
$x_1 = 0.15 - x_4$ is the tightest constraint for x_1. So we exchange x_1 and x_4.

Pivoting

The new slack form is

$$
\begin{aligned}
\max \quad & 0.09 - 0.06x_4 + 0.2x_2 \\
\\
s.t. \quad & x_3 = 0.18 - x_2 + 1.2x_4 \\
& x_1 = 0.15 - x_4 \\
& x_5 = 0.1 - x_2 \\
& x_i \geq 0 \quad , i = 1, 2, 3, 4, 5
\end{aligned}
$$

The new basic solution: $\bar{\mathbf{x}} = (0.15, 0, 0.18, 0, 0.1)$

Repeating Selecting Nonbasic Variable and Pivoting

Choose the nonbasic variable x_2. When $x_2 \uparrow$, $x_3 \downarrow$ and $x_5 \downarrow$
$x_2 = 0.1 - x_5$ is the tightest constraint for x_2. So we exchange x_2 and x_5.
The new slack form is

$$
\begin{aligned}
\max \quad & 0.029 - 0.06x_4 - 0.2x_5 \\
\\
s.t. \quad & x_3 = 0.28 + 1.2x_4 - x_5 \\
& x_1 = 0.15 - x_4 \\
& x_2 = 0.1 - x_5 \\
& x_i \geq 0 \quad , i = 1, 2, 3, 4, 5
\end{aligned}
$$

The new basic solution: $\bar{\mathbf{x}} = (0.15, 0.1, 0.28, 0, 0)$

Getting Answer

All coefficients in the final objective function are negative. Hence the basic solution $\bar{\mathbf{x}} = (0.15, 0.1, 0.28, 0, 0)$ is the optimal.
Therefore, the best scheme of investment is shown in the following table: (million dollar)

Year	**Project 1**	**Project 2**	**Project 3**
2018	0.15	0.15	-
2019	0.08	-	0.1

The maximum of the overall sum of capital and profit at the end of 2019 is (million dollar)

$$W_m = 0.432 + f(\bar{x_1}, \bar{x_2}) = 0.432 + 0.06\bar{x_1} + 0.2\bar{x_2} = 0.461$$

□

2. An engineering factory makes seven products (PROD 1 to PROD 7) on the following machines: four grinders, two vertical drills, three horizontal drills, one borer and one planer. Each product yields a certain contribution to profit (in £/unit). These quantities (in £/unit) together with the unit production times (hours) required on each process are given below. A dash indicates that a product does not require a process.

   There are marketing limitations on each product in each month, given in the following table:

   It is possible to store up to 100 of each product at a time at a cost of £0.5 per unit per month (charged at the end of each month according to the amount held at that time). There are no

	PROD 1	PROD 2	PROD 3	PROD 4	PROD 5	PROD 6	PROD 7
Contribution to profit	10	6	8	4	11	9	3
Grinding	0.5	0.7	-	-	0.3	0.2	0.5
Vertical drilling	0.1	0.2	-	0.3	-	0.6	-
Horizontal drilling	0.2	-	0.8	-	-	-	0.6
Boring	0.05	0.03	-	0.07	0.1	-	0.08
Planing	-	-	0.01	-	0.05	-	0.05

	PROD 1	PROD 2	PROD 3	PROD 4	PROD 5	PROD 6	PROD 7
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	300	600	0	0	500	400	100
April	200	300	400	500	200	0	100
May	0	100	500	100	1000	300	0
June	500	500	100	300	1100	500	60

stocks at present, but it is desired to have a stock of exactly 50 of each type of product at the end of June. The factory works six days a week with two shifts of 8h each day. It may be assumed that each month consists of only 24 working days. Each machine must be down for maintenance in one month of the six. No sequencing problems need to be considered.

When and what should the factory make in order to maximize the total net profit?

(a) Use *CPLEX Optimization Studio* to solve this problem. Describe your model in *Optimization Programming Language* (OPL). Remember to use a separate data file (.dat) rather than embedding the data into the model file (.mod).

(b) Solve your model and give the following results.

   i. For each machine:

      A. the month for maintenance.

   ii. For each product:

      A. The amount to make in each month.

      B. The amount to sell in each month.

      C. The amount to hold at the end of each month.

   iii. The total selling profit.

   iv. The total holding cost.

   v. The total net profit (selling profit minus holding cost).

Solution.

The month for maintenance.

grinders: 1 in February, 1 in March, 2 in April.

vertical drill: 1 in February, 1 in April.

horizontal drill: 1 in January, 2 in February.

borer: 1 in April.

planer: 1 in April.

The total selling profit.

$$TSP = \pounds 109330$$

The total holding cost.

$$THC = \pounds 475$$

The total net profit.

$$TNP = \pounds 108855$$

The amount to make in each month.

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	400	700	100	100	600	400	200
April	0	0	0	0	0	0	0
May	0	100	500	100	1000	300	0
June	550	550	150	350	1150	550	110

The amount to sell in each month.

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	300	600	0	0	500	400	100
April	100	100	100	100	100	0	100
May	0	100	500	100	1000	300	0
June	500	500	100	300	1100	500	60

The amount to hold in each month.

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
January	0	0	0	0	0	0	0
February	0	0	0	0	0	0	0
March	100	100	100	100	100	0	100
April	0	0	0	0	0	0	0
May	0	0	0	0	0	0	0
June	50	50	50	50	50	50	50

□

Remark: You need to include your .mod, .dat, .pdf and .tex files in your uploaded .zip file.