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Assignment 2

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1 compute the lower bound of congestion when embedding a hypercube into a ring with same number of nodes

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1.1 solution:

congestion is the max number of edges mapped onto one edge.

supppose the lower bound is x;

for a q-cube, its degree is q. but the degree of every node in the ring is 2. a q-cube has 2^q nodes and $\frac{2^q \times q}{2}$,while a ring which has 2^q nodes has 2^q edges. if embedded 2^q nodes hypercube in a 2^q nodes ring, then the map is from $\frac{2^q \times q}{2}$ edges to 2^q edges. and every edge in the ring is mapped from the $\frac{q}{2}$ edges in the hypercube on the average. so in this situation, $x \geq \frac{q}{2}$.

define a map $y = f(x)$ from $x \in 2^q$ to $y \in 2^q$, an indicate function is defined:

$$\mathrm{h}(\mathrm{i}_1, i_2) = \begin{cases} 1, & \text{if } v_i \text{ and } v_j \text{ has an edge} \\ 0, & \text{otherwise} \end{cases}$$

then the problem becomes: how many nodes in a ring can make the congestion lower? then we define $y = f_k(x)$, which is a map from a 2^q node to a k node map. so $congestion_k$ is the low bound of congestion in the $y = f_k(x)$ map. then we want to prove that $congestion_k$ is monotonous decreasing. for every k,there exists a map which make the low bound of congestion in the map is $congestion_k$. add a node, if a node is mapped from multiple nodes, map a node in the cube to the node. then the $congestion_{k+1}$ is less than or equal to the $congestion_k$. so we can consider the map from 2^q nodes to 2^q nodes. a node in the cube has q edges, which means a node mapped to the ring must has q edges. so the congestion $x \geq q - 2 + 1 = q - 1$. and the problem is modelled as follows:

Definition 1 (definition 1). *Given Q={0,1,2,...2^{q-1}},find a function h(s,t), such as*

$$\begin{aligned} \min: \quad \max & \sum_{i \in Q, m <= i, n > i, (n-m) < 2^{q-1}} h(m,n) \\ & s.t. \sum_{t \neq s, t \in Q} h(s,t) = q. \end{aligned} \tag{1}$$

which can be solved by a definite q. so the low bound of congestion is q-1.

2 Prove that icube contains $2^k \times 2^j$mesh where i = j + k.

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2.1 solution:

可以具体说明怎样连接构造出新的mesh

we can use the mathematical induction to solve the problem.

for q=1, the conclusion is true.

if q=i, the conclusion is true.

then q=i+1, the hypercube is a recursive network. we can find 2 mirrorin $2^k \times 2^j$mesh where i = j + k. and it can be linked with each other. let k=k+1 or j=j+1. so the (i+1)cube contains $2^k \times 2^j$mesh where i = j + k.

3 Prove that the diameter of faulty i-cube is i + 1. Note that there are at most i1 faulty nodes in faulty icube.

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3.1 solution:

the degree of a i-cube is i, so for a node in the cube, if the neighbor of the node is fault, the graph is not a connected graph, and the diameter is infinite. so the faulty nodes$\leq i-1$.

and we can easily proof that if i-1 nodes are faulty, the graph is also a connected graph.

so we only need to proof that Prove that the diameter of faulty i-cube is i + 1 which the faulty number is i-1.

we can use the mathematical induction to solve the problem.

q=2,if 1 node is faulty, the diameter is ~~also 2~~.

q=3, if 2 nodes is faulty, there are 2 conditions, the 2 nodes are linked or the nodes are not linked. if the nodes are linked, the diameter is also 3, and if the nodes are not linked, the diameter is 4.

应指出坏点的分布

if q=i, the diameter of the faulty i-cube is i+1,

for q=i+1, the (i+1)-cube can be divided into 2 i-cubes, and the diameter of the i-cubes is lower than i+1. we define the two i-cubes A and B. and for a_i node in the cube A, there exists a corresponding node b_i, and there is an edge between node a_i and node b_i. assume the diameter of the (i+1)-cube is the length of a_k and b_l, if a_k or b_l has the corresponding b_k or a_l,the length between a_k and b_l is less than i+1+1=i+2. when both a_k and b_l don't have the corresponding b_k and a_l, and the diameter in the i-cube is the route between a_k and a_l, this situation is difficult. let's assume $a_k \rightarrow a_{s_1} \rightarrow a_{s_2} \rightarrow a_{s_3} \rightarrow ... \rightarrow a_{s_i} \rightarrow a_l$, we can find a $x \in 1, 2, 3, ..., i$ that both a_x and b_x is not faulty. and the length between a_k and b_l is less than i+1+1=i+2. so, the proposition is proved.

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4 Figure out a permutation that cannot be supported by 8x8 butterfly network.

4.1 solution:

the full permutation of 8x8 network is $7! = 5040$. and the full permutation of 8x8 butterfly network is $16^3 = 4096$.so there are many permutations that cannot supported by 8x8 butterfly network.

for example: $\begin{Bmatrix} 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7 \\ 0\ 1\ 5\ 3\ 4\ 2\ 6\ 7 \end{Bmatrix}$? 对应没有 block

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5 Calculate how many permutations n x n Omega network can support.

5.1 solution:

the number of permutations which $n \times n$ Omega network can support is: $2^{\frac{n}{2}\times\lceil\log_2 n\rceil}$

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