Trajectory optimization

Obstacles
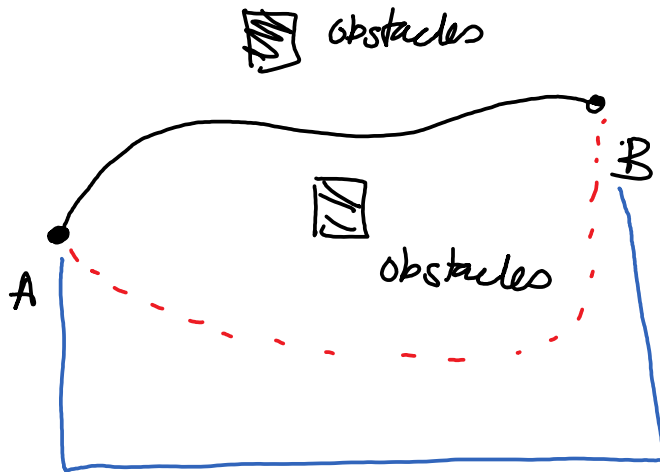
Optimization

Unconstrained optimization

$$\min_{X_1, X_2} \quad F(x) = 100 \left(X_2 - X_1^2 \right)^2 + \left(1 - X_1 \right)^2$$

① Graphing

surface
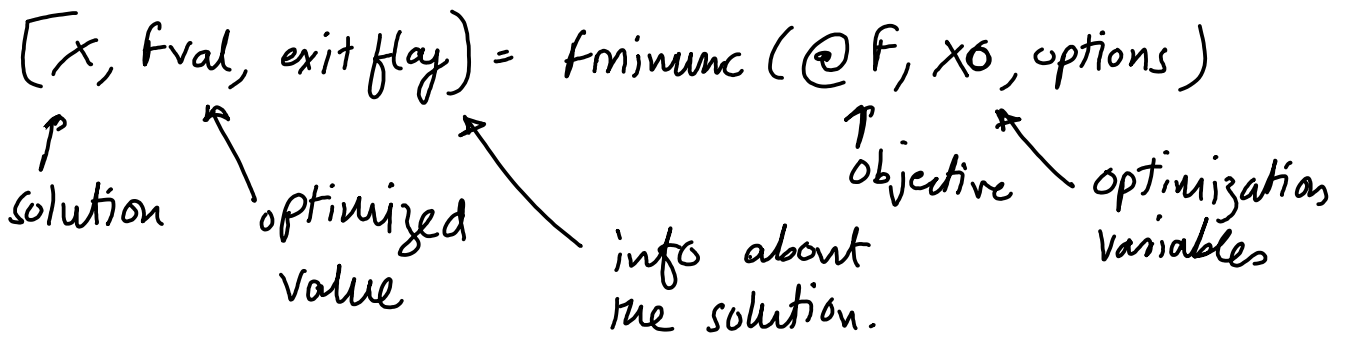
$f(x)$

X_2

minimum

X_1
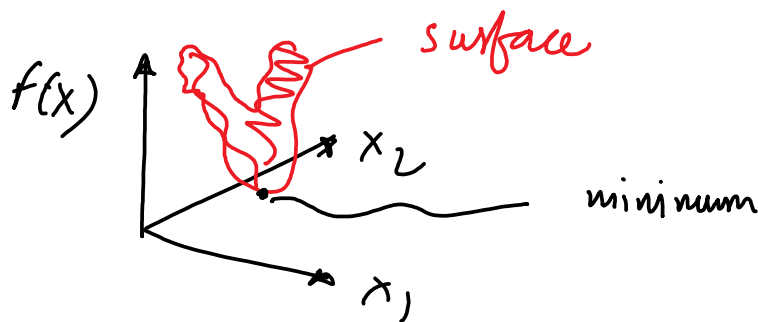
② Guess: $\quad 1 - X_1 = 0 \quad X_1 = 1$

$$X_2 - X_1^2 = 0 \quad \Rightarrow \quad X_2 - (1)^2 = 0 \quad \Rightarrow X_2 = 1$$

$$(X_1, X_2) = (1, 1) \text{ minimize } f(x) \Rightarrow \underline{f(1,1) = 0}$$

③ Numerically solving for the optimum
    ⌐ – graphing only works for 1, 2, 3 →D
    ⌐ – guessing works only for special
    └→D
        – always works, fminunc

$$[x, fval, exit\ flag] = fminunc\ (@\ F, x0,\ options\ )$$

↑   ↖   ↖   ↑   ↖
solution   optimized   objective   optimization
                value   variables
                            info about
                            the solution.

Constrained optimization

$$\min_{x} \quad f(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$$

$$\{x_1, x_2, x_3, x_4, x_5\}$$

subject to:

$$x_1 + x_2 + x_3 = 5$$

$$x_3^2 + x_4 = 2$$

$$x_1 \geq 0.3$$

$$x_3 \leq 5$$

$$x_4^2 + x_5^2 <= 5$$

$\implies$ fmincon $\longrightarrow$ constrained optimization

$$\min_{x} \quad f(x)$$

subject : $lb \leq x \leq ub$ Linear Equality constraints

$A_{eq} x = b_{eq}$

$A x < b$ Linear inequality constraints

$C_{eq} = 0$ Non linear equality constraints

$C < 0$ Non linear inequality constraints

$$\min_{X} \ f(x) = X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2$$

⚝ $X_1 + X_2 + X_3 = 5$ ☞

$X_3^2 + X_4 = 2$

→☞ $X_1 \geq 0.3$ X

→ $X_3 \leq 5$ X

∼ $X_4^2 + X_5^2 <= 5$

$$0.3 \leq X_1 \leq \infty$$
$$-\infty \leq X_3 \leq 5$$
$$-\infty \leq X_2, X_4, X_5 \leq \infty$$

$$lb < X < ub$$

✦ $\underline{A_{eq} X = b_{eq}}$

→☞ $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = 5$

$C_{eq} = 0 \implies X_3^2 + X_4 = 2$ ➤

∼ $C < 0 \implies X_4^2 + X_5 <= 5$ ➤

$$A X < b \quad (\text{not there})$$

$$A = [\quad]$$
$$b = [\quad]$$