Trajectory generation

$$q(t) = a_0 + a_1 t \qquad (1^{st} \text{ order})$$

$$a_0, a_1 \quad \text{constants}$$
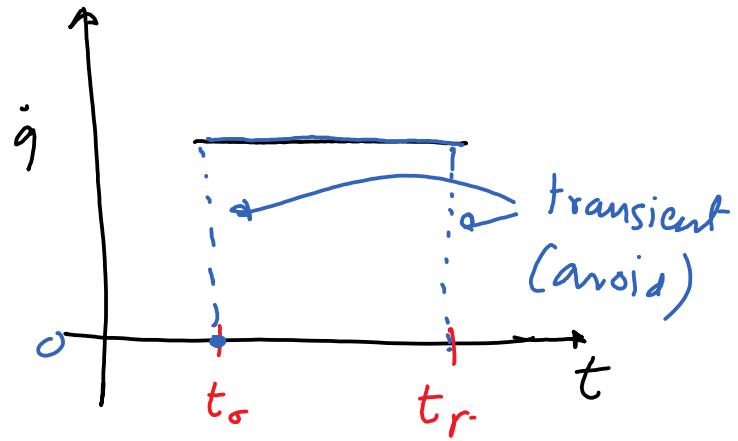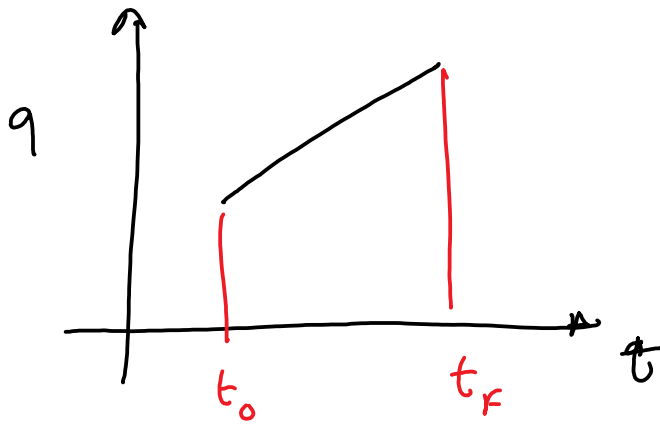
$$q_0 = a_0 + a_1 t_0 \qquad \Big\} \quad \text{—①}$$
$$q_f = a_0 + a_1 t_f \qquad \quad \text{—②}$$

$$2 \text{ unknowns} \quad a_0, a_1$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{1}{(t_f - t_0)} \begin{bmatrix} q_0 t_f - q_f t_0 \\ q_f - q_0 \end{bmatrix}_{2 \times 1}$$

$$q(t) = \frac{q_0 t_f - q_f t_0}{(t_f - t_0)} + \left(\frac{q_f - q_0}{t_f - t_0} \right) t$$
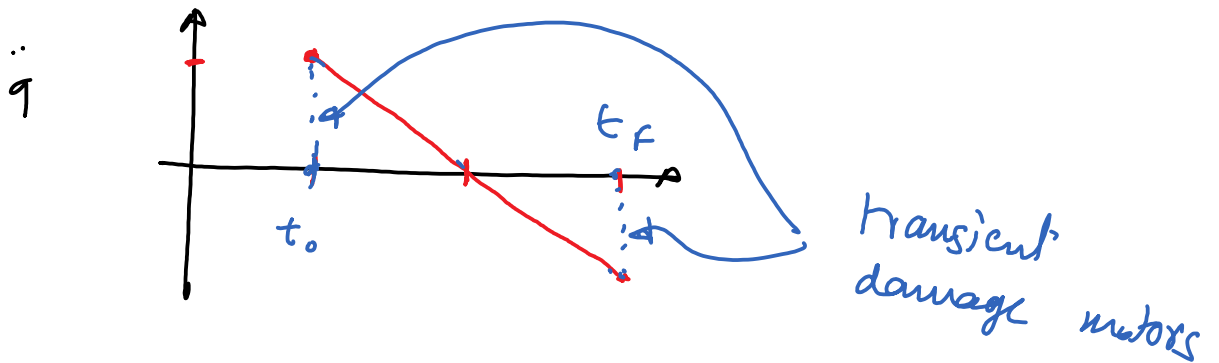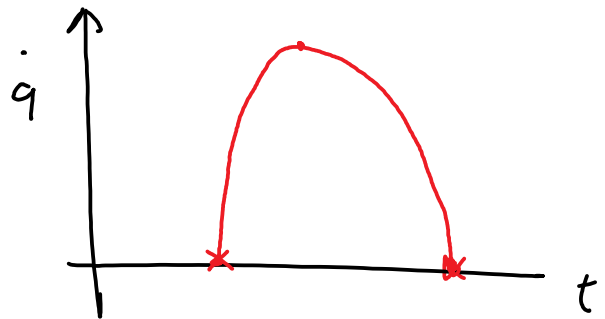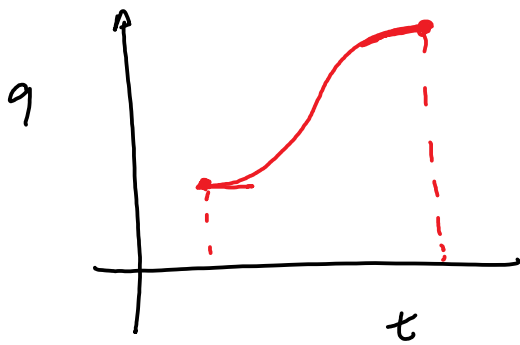
$$\dot{q}(t) = \frac{q_f - q_0}{t_f - t_0} = \text{constant}$$

transient
(avoid)

Cubic polynomial

$$
\begin{array}{ll}
t = t_o & q = q_o \\
t = t_f & q = q_f \\
t = t_o & \dot{q} = 0 \\
t = t_f & \dot{q} = 0
\end{array}
\left.\rule{0pt}{60pt}\right\}
\begin{array}{l}
4 \text{ conditions} \\
\\
\text{we need cubic} \\
\text{profile}
\end{array}
$$

$$
q(t) = a_o + a_1 t + a_2 t^2 + a_3 t^3
$$

4 conditions, 4 equations

transient
damage motors

Extend this idea to via points

start   goal
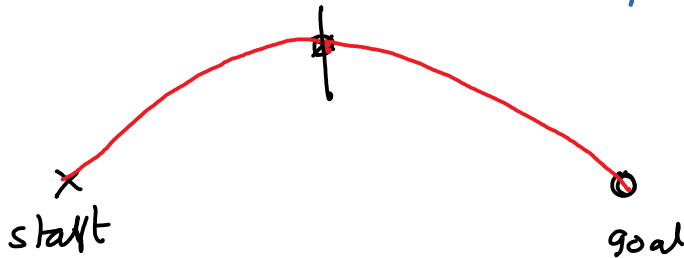
Example 1

Problem: Find $q(t)$ such that

$q(t=0) = \underline{\underline{0}}$, $q(t=1) = 0.5$, $q(t=2) = \underline{\underline{1}}$

↑ via point

$\dot{q}(t=0) = 0$, $\dot{q}(t=2) = 0$

at via point lets assume $\dot{q}(t) = \underline{\underline{0.2}}$

$q_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3 \qquad 0 \leq t \leq 1$

$q_2(t) = a_{20} + a_{21}t + a_{32}t^2 + a_{23}t^3 \qquad 1 \leq t \leq 2$

8 unknowns

$q_1(t=0) = 0$ \qquad $q_2(t=1) = 0.5$

$q_1(t=1) = 0.5$ \qquad $q_2(t=2) = 1$

$\dot{q}_1(t=0) = 0$ \qquad $\dot{q}_2(t=1) = 0.2$

$\dot{q}_1(t=1) = 0.2$ \qquad $\dot{q}_2(t=2) = 0$

8 equations