作业 19　三角恒等变换综合 参考答案

1．已知$\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{3}-\theta\right)=1$，则$\cos\left(2\theta-\frac{\pi}{3}\right)=$（　）

A．$\frac{1}{3}$　　B．$-\frac{1}{3}$　　C．$\frac{\sqrt{3}}{3}$　　D．$-\frac{\sqrt{3}}{3}$

【答案】B

【详解】由$\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{3}-\theta\right)=1$得$\cos\theta+\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta=1$，进而可得

$\frac{3}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta=1$，

结合辅助角公式得$\sqrt{3}\cos\left(\theta-\frac{\pi}{6}\right)=1$，

则$\cos\left(\theta-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$，$\therefore\cos\left(2\theta-\frac{\pi}{3}\right)=2\cos^2\left(\theta-\frac{\pi}{6}\right)-1=-\frac{1}{3}$，

2．式子$\frac{2\sin18^\circ\left(3\cos^2 9^\circ-\sin^2 9^\circ-1\right)}{\cos6^\circ+\sqrt{3}\sin6^\circ}$化简的结果为（　）

A．$\frac{1}{2}$　　B．1　　C．$2\sin9^\circ$　　D．2

【答案】B

【详解】原式$=\frac{2\sin18^\circ\left(3\cos^2 9^\circ-\sin^2 9^\circ-\cos^2 9^\circ-\sin^2 9^\circ\right)}{2\sin\left(6^\circ+30^\circ\right)}$

$=\frac{2\sin18^\circ\left(2\cos^2 9^\circ-2\sin^2 9^\circ\right)}{2\sin36^\circ}=\frac{2\sin18^\circ\cos18^\circ}{\sin36^\circ}=\frac{\sin36^\circ}{\sin36^\circ}=1$．

3．已知$\sin\left(\alpha+\frac{\pi}{3}\right)=\frac{\sqrt{3}}{6}$，则$\sin\left(2\alpha+\frac{\pi}{6}\right)=$（　）

A．$\frac{5}{6}$　　B．$-\frac{5}{6}$　　C．$\frac{11}{12}$　　D．$-\frac{11}{12}$

【答案】B

【详解】由$2\alpha+\frac{\pi}{6}=2\left(\alpha+\frac{\pi}{3}\right)-\frac{\pi}{2}$，

得$\sin\left(2\alpha+\frac{\pi}{6}\right)=\sin\left[2\left(\alpha+\frac{\pi}{3}\right)-\frac{\pi}{2}\right]=-\cos\left[2\left(\alpha+\frac{\pi}{6}\right)\right]$

$=-\left[1-2\sin^2\left(\alpha+\frac{\pi}{6}\right)\right]=-\left(1-\frac{1}{6}\right)=-\frac{5}{6}$．

4．已知$\sin\alpha=\frac{\sqrt{5}}{5},\sin\beta=\frac{\sqrt{10}}{10}$，且$\alpha$和$\beta$均为钝角，则$\alpha+\beta$的值为（　）

A．$\frac{\pi}{4}$　　　　B．$\frac{5\pi}{4}$　　　　C．$\frac{5\pi}{4}$或$\frac{7\pi}{4}$　　　　D．$\frac{7\pi}{4}$

【答案】D

【详解】∵α和β均为钝角，

∴$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\frac{2\sqrt{5}}{5}$，$\cos\beta=-\sqrt{1-\sin^2\beta}=-\frac{3\sqrt{10}}{10}$.

∴$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=-\frac{2\sqrt{5}}{5}\times\left(-\frac{3\sqrt{10}}{10}\right)-\frac{\sqrt{5}}{5}\times\frac{\sqrt{10}}{10}=\frac{\sqrt{2}}{2}$.

由α和β均为钝角，得$\pi<\alpha+\beta<2\pi$，∴$\alpha+\beta=\frac{7\pi}{4}$.

5．(多选)设函数$f(x)=\sqrt{3}\cos^2x+\frac{1}{2}\sin2x-\frac{\sqrt{3}}{2}$，则下列结论正确的是（　）

A．$f(x)$的最小正周期为π

B．$f(x)$的图象关于直线$x=\frac{7\pi}{12}$对称

C．$f(x)$的一个零点为$\frac{\pi}{6}$

D．$f(x)$的最大值为1

【答案】ABD

【详解】函数$f(x)=\sqrt{3}\times\frac{1+\cos2x}{2}+\frac{1}{2}\sin2x-\frac{\sqrt{3}}{2}=\frac{1}{2}\sin2x+\frac{\sqrt{3}}{2}\cos2x=\sin\left(2x+\frac{\pi}{3}\right)$.

对于A，$f(x)$的最小正周期为$\frac{2\pi}{2}=\pi$，故A正确；

对于B，$f\left(\frac{7\pi}{12}\right)=\sin\left(2\times\frac{7\pi}{12}+\frac{\pi}{3}\right)=\sin\frac{3\pi}{2}=-1$，所以$f(x)$的图象关于直线$x=\frac{7\pi}{12}$对称，故B正确；

对于C，$f\left(\frac{\pi}{6}\right)=\sin\left(2\times\frac{\pi}{6}+\frac{\pi}{3}\right)=\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$，所以$x=\frac{\pi}{6}$不是$f(x)$的一个零点，故C错误；

对于D，函数$f(x)=\sin\left(2x+\frac{\pi}{3}\right)$，则$f(x)$的最大值为1，故D正确.

6．(多选)下列等式成立的有（　）

A．$\sin^2\frac{\pi}{12}-\frac{1}{2}=\frac{\sqrt{3}}{4}$

B．$\tan80^\circ-\tan35^\circ-\tan80^\circ\tan35^\circ=1$

C．$\cos20^\circ\cos40^\circ\cos60^\circ\cos80^\circ=\frac{1}{8}$

D．$\frac{2\cos10^\circ-\sin20^\circ}{\cos20^\circ}=\sqrt{3}$

【答案】BD

【详解】对于A选项，$\sin^2\frac{\pi}{12}-\frac{1}{2}=-\frac{1}{2}\left(1-2\sin^2\frac{\pi}{12}\right)=-\frac{1}{2}\cos\frac{\pi}{6}=-\frac{\sqrt{3}}{4}$，A错；

对于B选项，因为$\tan45^\circ=\tan\left(80^\circ-35^\circ\right)=\frac{\tan80^\circ-\tan35^\circ}{1+\tan80^\circ\tan35^\circ}=1$，

所以，$\tan80^\circ-\tan35^\circ-\tan80^\circ\tan35^\circ=\tan80^\circ\tan35^\circ+1-\tan80^\circ\tan35^\circ=1$，B对；

对于 C 选项，$\cos 20^{\circ}\cos 40^{\circ}\cos 60^{\circ}\cos 80^{\circ}=\dfrac{\frac{1}{2}\sin 20^{\circ}\cos 20^{\circ}\cos 40^{\circ}\cos 80^{\circ}}{\sin 20^{\circ}}$

$=\dfrac{\frac{1}{4}\sin 40^{\circ}\cos 40^{\circ}\cos 80^{\circ}}{\sin 20^{\circ}}=\dfrac{\frac{1}{8}\sin 80^{\circ}\cos 80^{\circ}}{\sin\left(180^{\circ}-20^{\circ}\right)}=\dfrac{\frac{1}{16}\sin 160^{\circ}}{\sin 160^{\circ}}=\dfrac{1}{16}$，C 错；

对于 D 选项，$\dfrac{2\cos 10^{\circ}-\sin 20^{\circ}}{\cos 20^{\circ}}=\dfrac{2\cos\left(30^{\circ}-20^{\circ}\right)-\sin 20^{\circ}}{\cos 20^{\circ}}$

$=\dfrac{2\left(\cos 30^{\circ}\cos 20^{\circ}+\sin 30^{\circ}\sin 20^{\circ}\right)-\sin 20^{\circ}}{\cos 20^{\circ}}=\dfrac{\sqrt{3}\cos 20^{\circ}+\sin 20^{\circ}-\sin 20^{\circ}}{\cos 20^{\circ}}=\sqrt{3}$，D 对.

7．(多选)已知$\alpha,\beta\in(0,\pi)$，$\sin\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{5}{13}$，$\cos\left(\beta-\dfrac{\pi}{3}\right)=\dfrac{4}{5}$，则$\sin(\alpha-\beta)=$（　　）

A．$-\dfrac{33}{65}$　　B．$-\dfrac{63}{65}$　　C．$\dfrac{33}{65}$　　D．$\dfrac{63}{65}$

【答案】CD

【详解】$\alpha,\beta\in(0,\pi)$，$\alpha+\dfrac{\pi}{6}\in\left(\dfrac{\pi}{6},\dfrac{7\pi}{6}\right)$，

$\sin\left(\alpha+\dfrac{\pi}{6}\right)=\dfrac{5}{13}<\dfrac{1}{2},\alpha+\dfrac{\pi}{6}\in(\dfrac{\pi}{2},\pi),\therefore\cos\left(\alpha+\dfrac{\pi}{6}\right)=-\dfrac{12}{13}$；

$\beta-\dfrac{\pi}{3}\in\left(-\dfrac{\pi}{3},\dfrac{2\pi}{3}\right)$，$\cos\left(\beta-\dfrac{\pi}{3}\right)=\dfrac{4}{5}\therefore\sin\left(\beta-\dfrac{\pi}{3}\right)=\pm\dfrac{3}{5}$；

$\sin(\alpha-\beta)=\sin\left[\left(\alpha+\dfrac{\pi}{6}\right)-\left(\beta-\dfrac{\pi}{3}\right)-\dfrac{\pi}{2}\right]=-\cos\left[\left(\alpha+\dfrac{\pi}{6}\right)-\left(\beta-\dfrac{\pi}{3}\right)\right]$

$=-[\cos\left(\alpha+\dfrac{\pi}{6}\right)\cos\left(\beta-\dfrac{\pi}{3}\right)+\sin\left(\alpha+\dfrac{\pi}{6}\right)\sin\left(\beta-\dfrac{\pi}{3}\right)]$

当$\sin\left(\beta-\dfrac{\pi}{3}\right)=\dfrac{3}{5}$，所以$\sin(\alpha-\beta)=-(-\dfrac{12}{13}\times\dfrac{4}{5}+\dfrac{5}{13}\times\dfrac{3}{5})=\dfrac{33}{65}$，

当$\sin\left(\beta-\dfrac{\pi}{3}\right)=-\dfrac{3}{5}$，所以$\sin(\alpha-\beta)=-[-\dfrac{12}{13}\times\dfrac{4}{5}+\dfrac{5}{13}\times(-\dfrac{3}{5})]=\dfrac{63}{65}$，

8．$\left(\tan 30^{\circ}+\tan 70^{\circ}\right)\sin 10^{\circ}=$___________.

【答案】$\dfrac{\sqrt{3}}{3}$

【详解】$\left(\tan 30^{\circ}+\tan 70^{\circ}\right)\sin 10^{\circ}=(\dfrac{\sin 30^{\circ}}{\cos 30^{\circ}}+\dfrac{\sin 70^{\circ}}{\cos 70^{\circ}})\sin 10^{\circ}$

$=\dfrac{(\sin 30^{\circ}\cos 70^{\circ}+\cos 30\sin 70^{\circ})\sin 10^{\circ}}{\cos 30^{\circ}\cos 70^{\circ}}$

$=\dfrac{\sin 100^{\circ}\sin 10^{\circ}}{\frac{\sqrt{3}}{2}\sin 20^{\circ}}=\dfrac{2\sin 10^{\circ}\cos 10^{\circ}}{\sqrt{3}\sin 20^{\circ}}=\dfrac{\sqrt{3}}{3}$.

9．若$\dfrac{1+2\sqrt{3}\sin\theta\cos\theta+\cos 2\theta}{\sin\left(\theta+\dfrac{3\pi}{2}\right)}=\dfrac{1}{2}$，则$\sin\left(2\theta-\dfrac{\pi}{6}\right)=$______.

【答案】$-\frac{31}{32}$

【详解】因为$\frac{1+2\sqrt{3}\sin\theta\cos\theta+\cos2\theta}{\sin\left(\theta+\frac{3\pi}{2}\right)}=\frac{2\sqrt{3}\sin\theta\cos\theta+2\cos^2\theta}{-\cos\theta}$

$=-2\sqrt{3}\sin\theta-2\cos\theta=-4\sin\left(\theta+\frac{\pi}{6}\right)=\frac{1}{2}$,

所以$\sin\left(\theta+\frac{\pi}{6}\right)=-\frac{1}{8}$,

所以$\sin\left(2\theta-\frac{\pi}{6}\right)=\sin\left[2\left(\theta+\frac{\pi}{6}\right)-\frac{\pi}{2}\right]=-\cos2\left(\theta+\frac{\pi}{6}\right)=-1+2\sin^2\left(\theta+\frac{\pi}{6}\right)=-\frac{31}{32}$.

10．已知函数$f\left(x\right)=\sin^2x+\sqrt{3}\sin x\cos x-\frac{1}{2}\left(x\in\mathrm{R}\right)$.

(1)若函数$f\left(x+\theta\right)$的图象过点$P\left(\frac{\pi}{3},0\right)$，且$\theta\in\left(0,\frac{\pi}{2}\right)$，求$\theta$的值；

(2)若$f\left(\alpha\right)=\frac{2\sqrt{2}}{3}$，且$\alpha\in\left(0,\frac{\pi}{3}\right)$，求$\sin\left(\alpha+\frac{5\pi}{12}\right)$的值.

【答案】(1)$\frac{\pi}{4}$　　　　(2)$\frac{\sqrt{6}}{3}$

【详解】(1）因为$f\left(x\right)=\frac{1-\cos2x}{2}+\frac{\sqrt{3}}{2}\sin2x-\frac{1}{2}=\sin\left(2x-\frac{\pi}{6}\right)$.

所以$f\left(x+\theta\right)=\sin\left(2x+2\theta-\frac{\pi}{6}\right)$.

因为函数$f\left(x+\theta\right)$的图象过点$P\left(\frac{\pi}{3},0\right)$,

所以$\sin\left(\frac{2\pi}{3}+2\theta-\frac{\pi}{6}\right)=\sin\left(2\theta+\frac{\pi}{2}\right)=\cos2\theta=0$.

因为$\theta\in\left(0,\frac{\pi}{2}\right)$，所以$2\theta\in\left(0,\pi\right)$，所以$2\theta=\frac{\pi}{2}$，解得$\theta=\frac{\pi}{4}$.

（2）因为$\alpha\in\left(0,\frac{\pi}{3}\right)$，所以$2\alpha-\frac{\pi}{6}\in\left(-\frac{\pi}{6},\frac{\pi}{2}\right)$.

因为$f\left(\alpha\right)=\sin\left(2\alpha-\frac{\pi}{6}\right)=\frac{2\sqrt{2}}{3}$，所以$\cos\left(2\alpha-\frac{\pi}{6}\right)=\sqrt{1-\sin^2\left(2\alpha-\frac{\pi}{6}\right)}=\frac{1}{3}$.

所以$\cos\left(2\alpha+\frac{5\pi}{6}\right)=\cos\left(2\alpha-\frac{\pi}{6}+\pi\right)=-\cos\left(2\alpha-\frac{\pi}{6}\right)=-\frac{1}{3}$,

又$\cos\left(2\alpha+\frac{5\pi}{6}\right)=1-2\sin^2\left(\alpha+\frac{5\pi}{12}\right)$，所以$\sin^2\left(\alpha+\frac{5\pi}{12}\right)=\frac{2}{3}$.

因为$\alpha\in\left(0,\frac{\pi}{3}\right)$，所以$\alpha+\frac{5\pi}{12}\in\left(\frac{5\pi}{12},\frac{3\pi}{4}\right)$，所以$\sin\left(\alpha+\frac{5\pi}{12}\right)=\frac{\sqrt{6}}{3}$.

11．已知α,β为锐角，$\cos\alpha=\frac{\sqrt{10}}{10},\sin(\alpha+\beta)=\frac{\sqrt{5}}{5}$,

(1)求$\cos\beta$；

(2)求 $2\alpha+\beta$.

【答案】(1) $\dfrac{\sqrt{2}}{10}$ (2) $2\alpha+\beta=\dfrac{5\pi}{4}$

【详解】(1) $\because \sin(\alpha+\beta)=\dfrac{\sqrt{5}}{5}, \therefore \cos(\alpha+\beta)=\pm\dfrac{2\sqrt{5}}{5}$.

$\because \alpha,\beta$ 为锐角，$\therefore \pi>\alpha+\beta>\alpha>0, \therefore \sin\alpha=\dfrac{3\sqrt{10}}{10}$，

又 $y=\cos x$ 在 $[0,\pi]$ 上单调递减，$\therefore \cos(\alpha+\beta)<\cos\alpha=\dfrac{\sqrt{10}}{10}$，

$\therefore \cos(\alpha+\beta)=-\dfrac{2\sqrt{5}}{5}$，

$\therefore \cos\beta=\cos[(\alpha+\beta)-\alpha]=\cos(\alpha+\beta)\cos\alpha+\sin(\alpha+\beta)\sin\alpha$

$=\dfrac{\sqrt{10}}{10}\times(-\dfrac{2\sqrt{5}}{5})+\dfrac{3\sqrt{10}}{10}\times\dfrac{\sqrt{5}}{5}=\dfrac{\sqrt{2}}{10}$.

（2） $\sin(2\alpha+\beta)=\sin[\alpha+(\alpha+\beta)]$

$=\sin\alpha\cos(\alpha+\beta)+\cos\alpha\sin(\alpha+\beta)$

$=\dfrac{\sqrt{10}}{10}\times(-\dfrac{2\sqrt{5}}{5})-\dfrac{3\sqrt{10}}{10}\times\dfrac{\sqrt{5}}{5}=-\dfrac{\sqrt{2}}{2}$，

$\because \alpha,\beta$ 为锐角，$\therefore 2\alpha+\beta\in(0,\dfrac{3\pi}{2})$，$\therefore 2\alpha+\beta=\dfrac{5\pi}{4}$.

【拓展探究※选做】

12．(多选)下列结论正确的有：(　　)

A．$\cos4\alpha+4\cos2\alpha+3=8\cos^4\alpha$

B．$\dfrac{1+\sin2\alpha}{2\cos^2\alpha+\sin2\alpha}=\dfrac{1}{2}\tan\alpha+\dfrac{1}{2}$

C．$\dfrac{\sin(2\alpha+\beta)}{\sin\alpha}-2\cos(\alpha+\beta)=\dfrac{\sin\beta}{\sin\alpha}$

D．$\dfrac{3-4\cos2A+\cos4A}{3+4\cos2A+\cos4A}=\tan^4A$

【答案】ABCD

【详解】对于 A 选项：左边 $=2\cos^2 2\alpha-1+4\cos2\alpha+3$

$=2\left(\cos^2 2\alpha+2\cos2\alpha+1\right)$

$=2(\cos2\alpha+1)^2$

$=2\left(2\cos^2\alpha\right)^2$

$=8\cos^4\alpha$

$=$右边，

故该选项正确；

对于 B 选项：左边 $=\dfrac{\sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha}{2\cos^2\alpha+2\sin\alpha\cos\alpha}$

$=\dfrac{(\sin\alpha+\cos\alpha)^2}{2\cos\alpha(\cos\alpha+\sin\alpha)}$

$=\dfrac{\sin\alpha+\cos\alpha}{2\cos\alpha}$

$=\dfrac{1}{2}\tan\alpha+\dfrac{1}{2}$

= 右边，

故该选项正确；

对于 C 选项：左边$=\dfrac{\sin(2\alpha+\beta)-2\cos(\alpha+\beta)\sin\alpha}{\sin\alpha}$

$=\dfrac{\sin[(\alpha+\beta)+\alpha]-2\cos(\alpha+\beta)\sin\alpha}{\sin\alpha}$

$=\dfrac{\sin(\alpha+\beta)\cos\alpha-\cos(\alpha+\beta)\sin\alpha}{\sin\alpha}$

$=\dfrac{\sin\left[(\alpha+\beta)-\alpha\right]}{\sin\alpha}$

$=\dfrac{\sin\beta}{\sin\alpha}$

= 右边，

故该选项正确；

对于 D 选项：左边$=\dfrac{3-4\cos 2A+2\cos^2 2A-1}{3+4\cos 2A+2\cos^2 2A-1}$

$=\dfrac{2\left(\cos^2 2A-2\cos 2A+1\right)}{2\left(\cos^2 2A+2\cos 2A+1\right)}$

$=\dfrac{(1-\cos 2A)^2}{(1+\cos 2A)^2}$

$=\dfrac{\left(2\sin^2 A\right)^2}{\left(2\cos^2 A\right)^2}$

$=\tan^4 A$

= 右边，

故该选项正确；

故选：ABCD.

13．已知$\cos\left(2\alpha-\dfrac{\pi}{3}\right)=\dfrac{p}{2}$，$\tan\alpha\tan\left(\alpha-\dfrac{\pi}{3}\right)=p$，则正常数 p 的值为________.

【答案】$\sqrt{2}-1$

【详解】设 $A=\sin\alpha\sin\left(\alpha-\dfrac{\pi}{3}\right)$，$B=\cos\alpha\cos\left(\alpha-\dfrac{\pi}{3}\right)$.

故$\cos\left(2\alpha-\dfrac{\pi}{3}\right)=\cos\left(\alpha+\alpha-\dfrac{\pi}{3}\right)=B-A=\dfrac{p}{2}$，

$\cos\left(-\dfrac{\pi}{3}\right)=\cos\left(\alpha-\dfrac{\pi}{3}-\alpha\right)=B+A=\dfrac{1}{2}$，故 $A=\dfrac{1-p}{4}$，$B=\dfrac{1+p}{4}$.

$\tan\alpha\tan\left(\alpha-\frac{\pi}{3}\right)=\frac{\sin\alpha\sin\left(\alpha-\frac{\pi}{3}\right)}{\cos\alpha\cos\left(\alpha-\frac{\pi}{3}\right)}=\frac{A}{B}=\frac{1-p}{1+p}=p$，且 $p>0$，解得 $p=\sqrt{2}-1$.

故答案为：$\sqrt{2}-1$.

14．已知函数 $f(x)=\sin\left(\frac{5\pi}{6}-2x\right)-2\sin\left(x-\frac{\pi}{4}\right)\cos\left(x+\frac{3\pi}{4}\right)$.

(1)解不等式 $f(x)\geq-\frac{1}{2}$；

(2)若 $x\in\left[\frac{\pi}{12},\frac{\pi}{3}\right]$，且 $F(x)=-4\lambda f(x)-\cos\left(4x-\frac{\pi}{3}\right)$的最小值是$-\frac{3}{2}$，求实数$\lambda$的值.

【答案】(1)$\left[k\pi,k\pi+\frac{2\pi}{3}\right]$，$k\in Z$；(2)$\lambda=\frac{1}{2}$.

【详解】(1)∵ $f(x)=\sin\left(\frac{5\pi}{6}-2x\right)-2\sin\left(x-\frac{\pi}{4}\right)\cos\left(x+\frac{3\pi}{4}\right)$

$=\frac{1}{2}\cos 2x+\frac{\sqrt{3}}{2}\sin 2x+(\sin x-\cos x)(\sin x+\cos x)$

$=\frac{1}{2}\cos 2x+\frac{\sqrt{3}}{2}\sin 2x+\sin^2 x-\cos^2 x$

$=\frac{1}{2}\cos 2x+\frac{\sqrt{3}}{2}\sin 2x-\cos 2x$

$=\sin\left(2x-\frac{\pi}{6}\right)$

由 $2k\pi-\frac{\pi}{6}\leq 2x-\frac{\pi}{6}\leq 2k\pi+\frac{7\pi}{6}$，得 $k\pi\leq x\leq k\pi+\frac{2\pi}{3}$，

解集为$\left[k\pi,k\pi+\frac{2\pi}{3}\right]$，$k\in Z$

(2) $F(x)=-4\lambda f(x)-\cos\left(4x-\frac{\pi}{3}\right)=-4\lambda\sin\left(2x-\frac{\pi}{6}\right)-\left[1-2\sin^2\left(2x-\frac{\pi}{6}\right)\right]$

$=2\sin^2\left(2x-\frac{\pi}{6}\right)-4\lambda\sin\left(2x-\frac{\pi}{6}\right)-1=2\left[\sin\left(2x-\frac{\pi}{6}\right)-\lambda\right]^2-1-2\lambda^2$

∵ $x\in\left[\frac{\pi}{12},\frac{\pi}{3}\right]$，∴ $0\leq 2x-\frac{\pi}{6}\leq\frac{\pi}{2}$，$0\leq\sin\left(2x-\frac{\pi}{6}\right)\leq 1$，

①当$\lambda<0$时，当且仅当$\sin\left(2x-\frac{\pi}{6}\right)=0$ 时，$f(x)$取得最小值-1，这与已知不相符；

②当$0\leqslant\lambda\leqslant 1$时，当且仅当$\sin\left(2x-\frac{\pi}{6}\right)=\lambda$时，$f(x)$取最小值$-1-2\lambda^2$，由已知得

$-1-2\lambda^2=-\frac{3}{2}$，解得$\lambda=\frac{1}{2}$；

③当$\lambda>1$时，当且仅当$\sin\left(2x-\frac{\pi}{6}\right)=1$时，$f(x)$取得最小值$1-4\lambda$，由已知得$1-4\lambda=-\frac{3}{2}$，

解得$\lambda=\frac{5}{8}$，这与$\lambda>1$相矛盾.综上所述，$\lambda=\frac{1}{2}$.