Chapter 4
Radio Propagation – Large-Scale Path Loss

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Outline

- Introduction to Radio Wave Propagation
- Three Basic Propagation Mechanisms
- Free Space Propagation Model
- Practical Link Budget Design using Path Loss Models
- Outdoor Propagation Models
- Indoor Propagation Models
- Signal Penetration into Buildings

Introduction to Radio Wave Propagation

- The mobile radio channel places fundamental limitations on the performance of wireless communication systems.
- Radio channels are extremely random and do not offer easy analysis.
- Modeling radio channel is important for:
  - Determining the coverage area of a transmitter
    - Determine the transmitter power requirement
    - Determine the battery lifetime
  - Finding modulation and coding schemes to improve the channel quality
    - Determine the maximum channel capacity

Introduction to Radio Wave Propagation

- The mechanisms behind electromagnetic wave propagation are diverse, but can generally be attributed to reflection, diffraction, and scattering.
- Propagation models have traditionally focused on predicting the average received signal strength at a given distance from the transmitter, as well as the variability of the signal strength in close spatial proximity to a particular location.

Introduction to Radio Wave Propagation

- Propagation models that predict the mean signal strength for an arbitrary transmitter-receiver (T-R) separation distance are useful in estimating the radio coverage area of a transmitter and are called large-scale propagation models.
- On the other hand, propagation models that characterize the rapid fluctuations of the received signal strength over very short travel distances (a few wavelengths) or short time durations (on the order of seconds) are called small-scale or fading models.

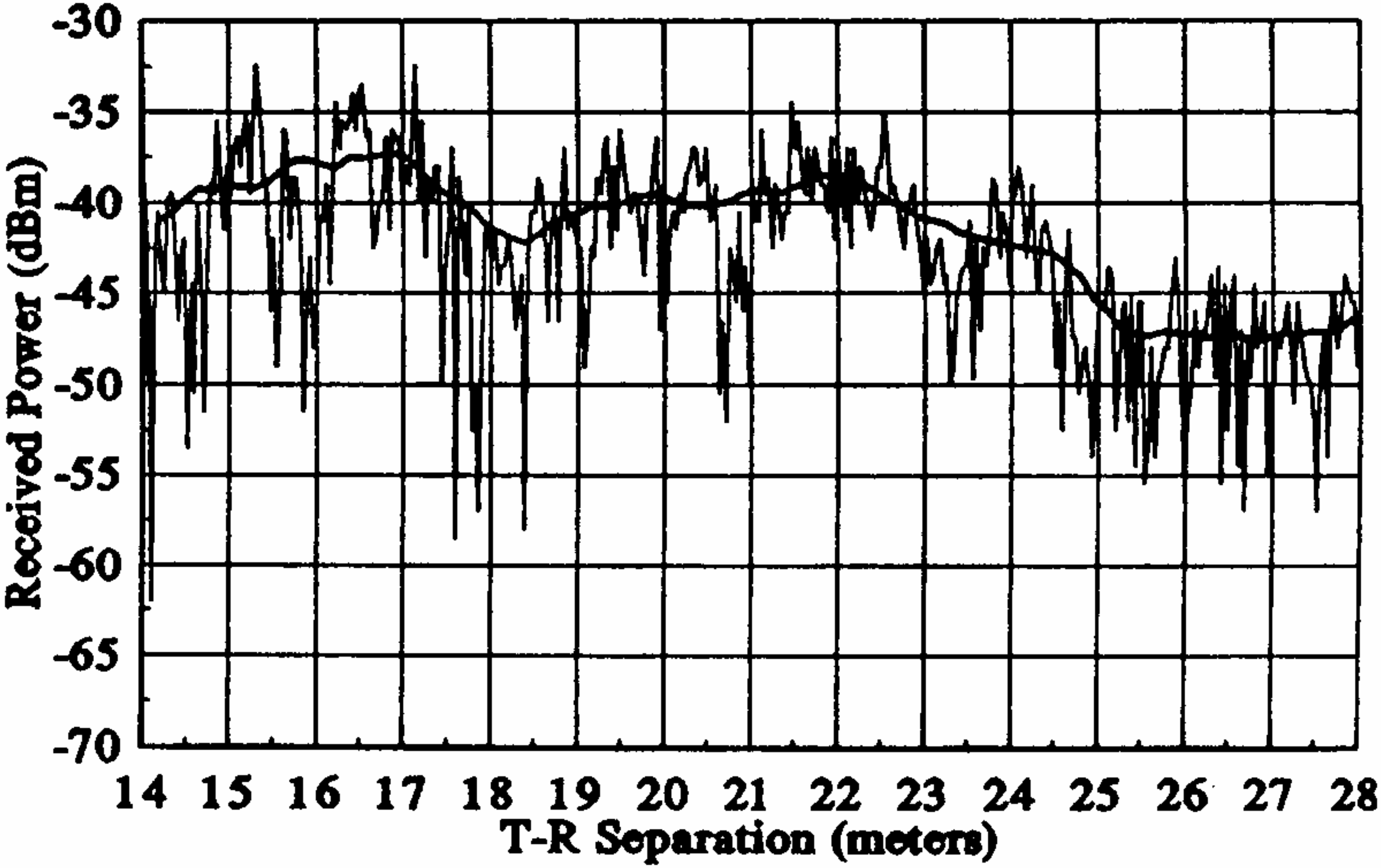
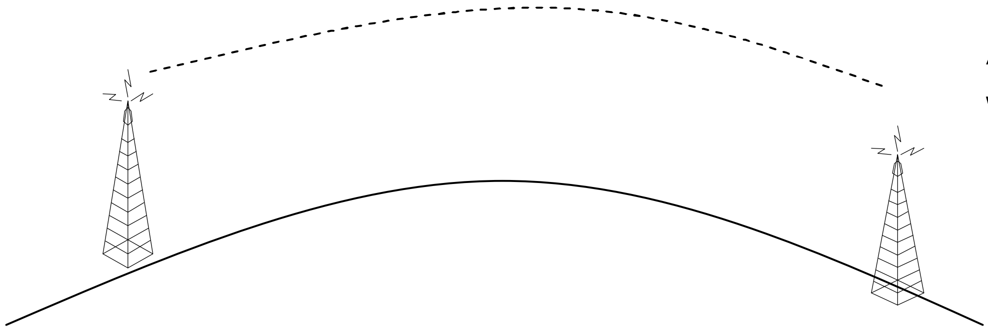
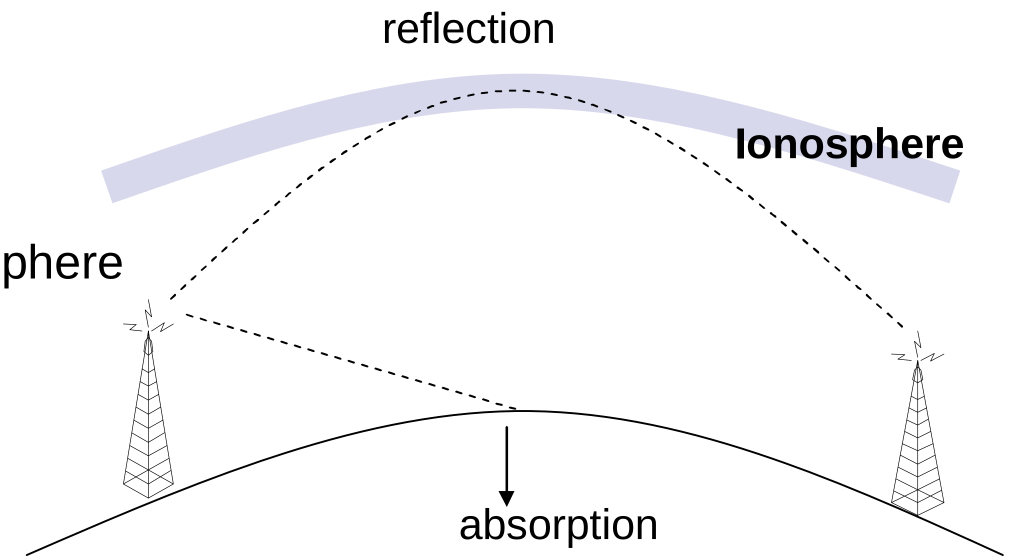


Figure 4.1 Small-scale and large-scale fading.

Basics - Propagation



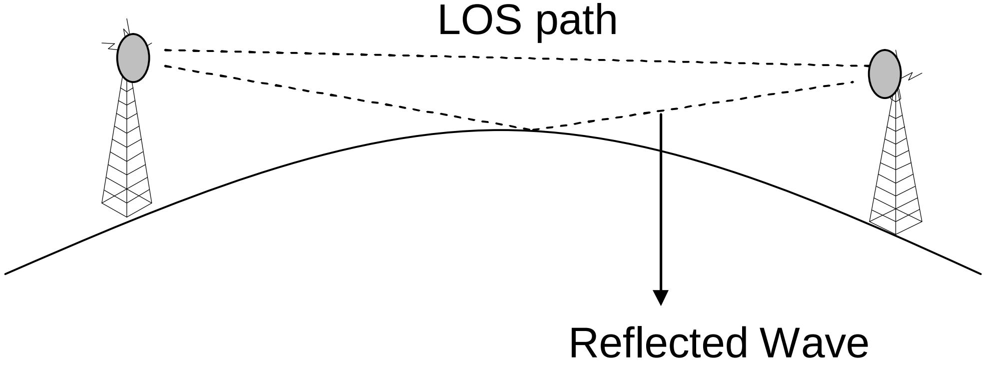
At **VLF, LF, and MF** bands, radio waves follow the ground. AM radio broadcasting uses MF band



At **HF** bands, the ground waves tend to be absorbed by the earth. The waves that reach ionosphere (100-500km above earth surface), are refracted and sent back to earth.

Basics - Propagation

VHF Transmission



-**Directional antennas are used**
-**Waves follow more direct paths**
- LOS: Line-of-Sight Communication
- Reflected wave interfere with the original signal

Basics - Propagation

- Waves behave more like light at higher frequencies
  - Difficulty in passing obstacles
  - More direct paths
- They behave more like radio at lower frequencies
  - Can pass obstacles

Radio Propagation Models

- Transmission path between sender and receiver could be
  - Line-of-Sight (LOS)
  - Obstructed by buildings, mountains and foliage
- Even speed of motion effects the fading characteristics of the channel

Three Radio Propagation Mechanisms

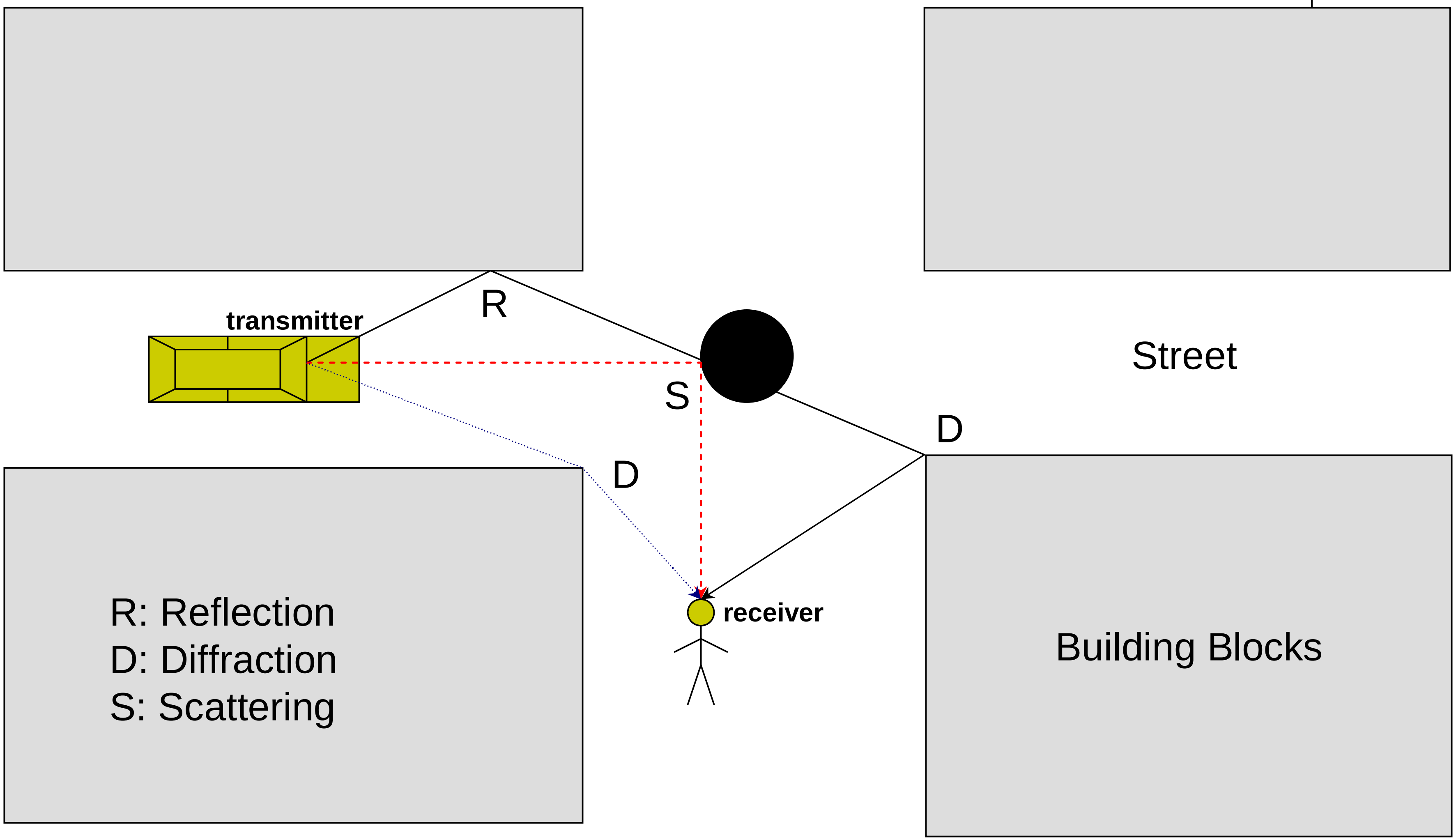
- The physical mechanisms that govern radio propagation are complex and diverse, but generally attributed to the following three factors
  1. Reflection
  2. Diffraction
  3. Scattering
  - Reflection
    - Occurs when waves impinges upon an obstruction that is much larger in size compared to the wavelength of the signal
    - Example: reflections from earth and buildings
    - These reflections may interfere with the original signal constructively or destructively

Three Radio Propagation Mechanisms

- Diffraction
  - Occurs when the radio path between sender and receiver is obstructed by an impenetrable body and by a surface with sharp irregularities (edges)
  - Explains how radio signals can travel urban and rural environments without a line-of-sight path

- Scattering
  - Occurs when the radio channel contains objects whose sizes are on the order of the wavelength or less of the propagating wave and also when the number of obstacles are quite large.
  - They are produced by small objects, rough surfaces and other irregularities on the channel
  - Follows same principles with diffraction
  - Causes the transmitter energy to be radiated in many directions
  - Lamp posts and street signs may cause scattering

Three Radio Propagation Mechanisms



Three Radio Propagation Mechanisms

- As a mobile moves through a coverage area, these 3 mechanisms have an impact on the instantaneous received signal strength.
  - If a mobile does have a clear line of sight path to the base-station, than diffraction and scattering will not dominate the propagation.
  - If a mobile is at a street level without LOS, then diffraction and scattering will probably dominate the propagation.

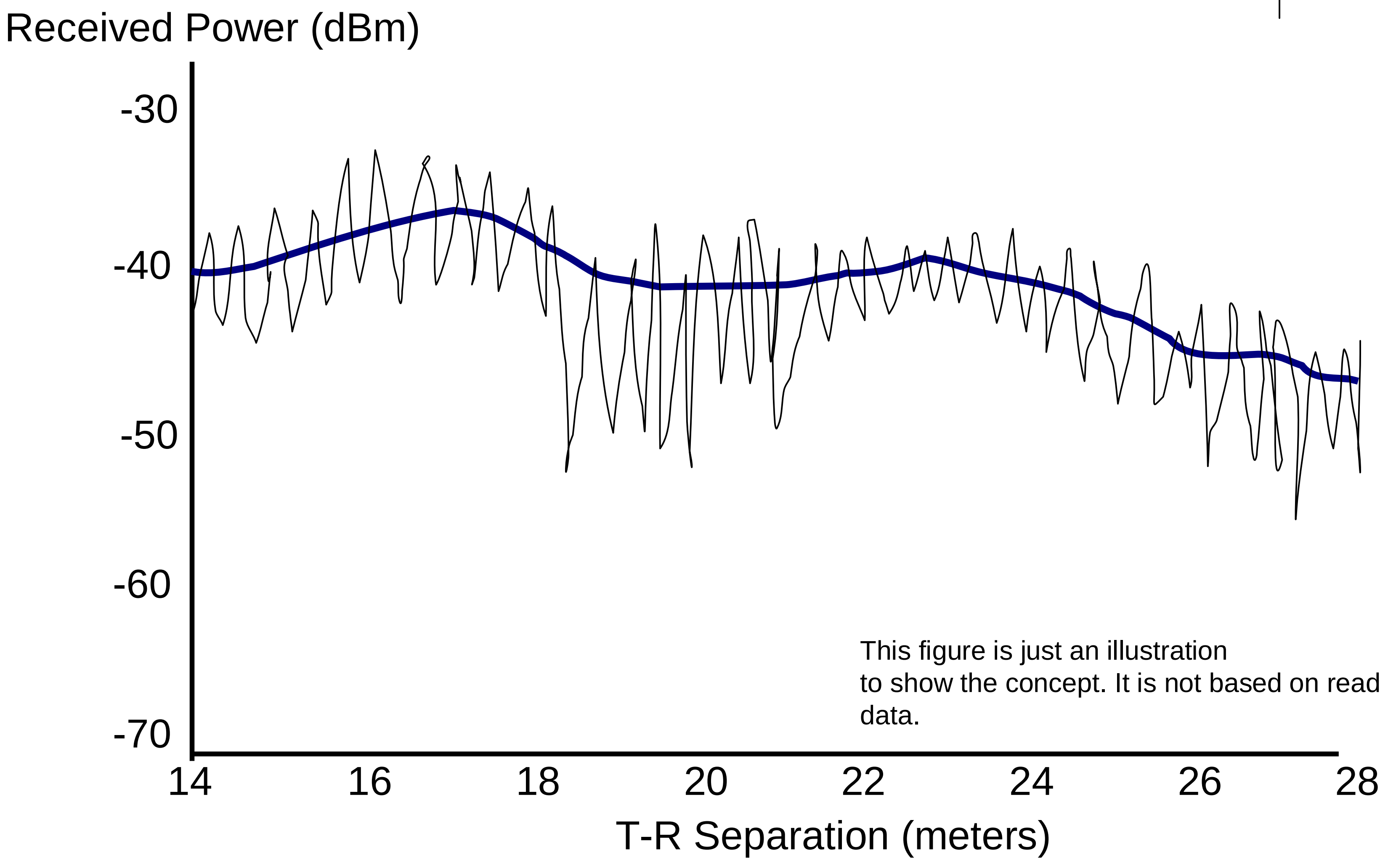
Radio Propagation Models

- As the mobile moves over small distances, the instantaneous received signal will fluctuate rapidly giving rise to small-scale fading
  - The reason is that the signal is the sum of many contributors coming from different directions and since the phases of these signals are random, the sum behave like a noise (Rayleigh fading).
  - In small scale fading, the received signal power may change as much as 3 or 4 orders of magnitude (30dB or 40dB), when the receiver is only moved a fraction of the wavelength.

Radio Propagation Models

- As the mobile moves away from the transmitter over larger distances, the local average received signal will gradually decrease. This is called large-scale path loss.
  - Typically the local average received power is computed by averaging signal measurements over a measurement track of 5λ to 40λ. (For PCS, this means 1m-10m track)
- The models that predict the mean signal strength for an arbitrary-receiver transmitter (T-R) separation distance are called large-scale propagation models
  - Useful for estimating the coverage area of transmitters

Small-Scale and Large-Scale Fading



Free-Space Propagation Model

- Used to predict the received signal strength when transmitter and receiver have clear, unobstructed LOS path between them.
- The received power decays as a function of T-R separation distance raised to some power.
- Path Loss: Signal attenuation as a positive quantity measured in dB and defined as the difference (in dB) between the effective transmitter power and received power.

Free-Space Propagation Model

- Free space power received by a receiver antenna separated from a radiating transmitter antenna by a distance d is given by Friis free space equation:

$$P_r(d) = (P_t G_t G_r \lambda^2) / ((4\pi)^2 d^2 L) \qquad \text{[Equation 1]}$$

- P_t is transmited power
- $P_r(d)$ is the received power
- G_t is the trasmitter antenna gain (dimensionless quantity)
- G_r is the receiver antenna gain (dimensionless quantity)
- d is T-R separation distance in meters
- L is system loss factor not related to propagation (L >= 1)
  - L = 1 indicates no loss in system hardware (for our purposes we will take L = 1, so we will igonore it in our calculations).
- λ is wavelength in meters.

Free-Space Propagation Model

- The gain of an antenna G is related to its affective aperture A_e by:
  - $G = 4\pi A_e / \lambda^2$ [Equation 2]
    - The effective aperture of A_e is related to the physical size of the antenna,
  - λ is related to the carrier frequency by:
    - $\lambda = c/f = 2\pi c / \omega_c$ [Equation 3]
      - f is carrier frequency in Hertz
      - ω_c is carrier frequency in radians per second.
      - c is speed of light in meters/sec

Free-Space Propagation Model

- An *isotropic* radiator is an ideal antenna that radiates power with unit gain uniformly in all directions. It is as the reference antenna in wireless systems.
- The *effective isotropic radiated power* (EIRP) is defined as:
  - $EIRP = P_t G_t$ [Equation 4]
- Antenna gains are given in units of dBi (dB gain with respect to an isotropic antenna) or units of dBd (dB gain with respect to a half-wave dipole antenna).
  - Unity gain means:
    - G is 1 or 0dBi

Free-Space Propagation Model

- Path loss, which represents signal attenuation as positive quantity measured in dB, is defined as the difference (in dB) between the effective transmitted power and the received power.

$$PL(dB) = 10 \log (P_t/P_r) = -10\log[(G_t G_r \lambda^2)/(4\pi)^2 d^2] \quad \text{[Equation 5]}$$

(You can drive this from equation 1)

- If antennas have unity gains (exclude them)

$$PL(dB) = 10 \log (P_t/P_r) = -10\log[\lambda^2/(4\pi)^2 d^2] \quad \text{[Equation 6]}$$

Free-Space Propagation Model

- For Friis equation to hold, distance *d* should be in the far-field of the transmitting antenna.
- The far-field, or Fraunhofer region, of a transmitting antenna is defined as the region beyond the far-field distance d_f given by:
  - $d_f = 2D^2/\lambda$ [Equation 7]
    - D is the largest physical dimension of the antenna.
  - Additionally, $d_f \gg D$ and $d_f \gg \lambda$

Free-Space Propagation Model – Reference Distance d_0

- It is clear the Equation 1 does not hold for $d = 0$.
- For this reason, models use a close-in distance d_0 as the receiver power reference point.
  - d_0 should be >= d_f
  - d_0 should be smaller than any practical distance a mobile system uses
- Received power $P_r(d)$, at a distance $d > d_0$ from a transmitter, is related to P_r at d_0, which is expressed as $P_r(d_0)$.
- The power received in free space at a distance greater than d_0 is given by:

$$P_r(d) = P_r(d_0)(d_0/d)^2 \quad d \geq d_0 \geq d_f \qquad \text{[Equation 8]}$$

Free-Space Propagation Model

- Expressing the received power in dBm and dBW
  - $P_r(d)$ (dBm) $= 10 \log [P_r(d_0)/0.001W] + 20\log(d_0/d)$
    where $d >= d_0 >= d_f$ and $P_r(d_0)$ is in units of watts.
    [Equation 9]
  - $P_r(d)$ (dBW) $= 10 \log [P_r(d_0)/1W] + 20\log(d_0/d)$
    where $d >= d_0 >= d_f$ and $P_r(d_0)$ is in units of watts.
    [Equation 10]

- Reference distance d_0 for practical systems:
  - For frequncies in the range 1-2 GHz
    - 1 m in indoor environments
    - 100m-1km in outdoor environments

Example Question

- A transmitter produces 50W of power.
  - A) Express the transmit power in dBm
  - B) Express the transmit power in dBW
  - C) If d_0 is 100m and the received power at that distance is 0.0035mW, then find the received power level at a distance of 10km.
    - Assume that the transmit and receive antennas have unity gains.

Solution

- A)
  - Pt(W) is 50W.
  - Pt(dBm) = 10log[Pt(mW)/1mW)]
    Pt(dBm) = 10log(50x1000)
    Pt(dBm) = 47 dBm
- B)
  - Pt(dBW) = 10log[Pt(W)/1W)]
    Pt(dBW) = 10log(50)
    Pt(dBW) = 17 dBW

Solution

- $P_r(d) = P_r(d_0)(d_0/d)^2$
- Substitute the values into the equation:
  - $P_r(10km) = P_r(100m)(100m/10km)^2$
    $P_r(10km) = 0.0035mW(10^{-4})$
    $P_r(10km) = 3.5x10^{-10}W$
  - $Pr(10km)\ [dBm] = 10\log(3.5x10^{-10}W/1mW)$
    $= 10\log(3.5x10^{-7})$
    $= \underline{-64.5dBm}$

Two main channel design issues

- Communication engineers are generally concerned with two main radio channel issues:
  - Link Budged Design
    - Link budget design determines fundamental quantities such as transmit power requirements, coverage areas, and battery life
    - It is determined by the amount of received power that may be expected at a particular distance or location from a transmitter
  - Time dispersion
    - It arises because of multi-path propagation where replicas of the transmitted signal reach the receiver with different propagation delays due to the propagation mechanisms that are described earlier.
    - Time dispersion nature of the channel determines the maximum data rate that may be transmitted without using equalization.

Link Budged Design Using Path Loss Models

- Radio propagation models can be derived
  - By use of empirical methods: collect measurement, fit curves.
  - By use of analytical methods
    - Model the propagation mechanisms mathematically and derive equations for path loss
- Long distance path loss model
  - Empirical and analytical models show that received signal power decreases logarithmically with distance for both indoor and outdoor channels

Long distance path loss model

- The average large-scale path loss for an arbitrary T-R separation is expressed as a function of distance by using a path loss exponent *n*:
  - The value of *n* depends on the propagation environment: for free space it is 2; when obstructions are present it has a larger value.

$$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n$$

$$\overline{PL}(dB) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right)$$

Equation 11

$\overline{PL}(d)$ denotes the average large-scale path loss at a distance d (denoted in dB)

Path Loss Exponent for Different Environments

Environment	Path Loss Exponent, *n*
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

Selection of free space reference distance

- In large coverage cellular systems
  - 1km reference distances are commonly used
- In microcellular systems
  - Much smaller distances are used: such as 100m or 1m.
- The reference distance should always be in the far-field of the antenna so that near-field effects do not alter the reference path loss.

Log-normal Shadowing

- Equation 11 does not consider the fact the surrounding environment may be vastly different at two locations having the same T-R separation
- This leads to measurements that are different than the predicted values obtained using the above equation.
- Measurements show that for any value d, the path loss PL(d) in dBm at a particular location is random and distributed normally.

Log-normal Shadowing- Path Loss

Then adding this random factor:

$$PL(d)[dB] = \overline{PL}(d) + X_s$$

$$PL(d)[dB] = \overline{PL}(d_0) + 10n\log(\frac{d}{d_0}) + X_s$$

Equation 12

- $\overline{PL}(d)$ denotes the average large-scale path loss (in dB) at a distance d.
- X_S is a zero-mean Gaussian (normal) distributed random variable (in dB) with standard deviation σ (also in dB).
- $\overline{PL}(d_0)$ is usually computed assuming free space propagation model between transmitter and d_0 (or by measurement).

Equation 12 takes into account the shadowing affects due to cluttering on the propagation path. It is used as the propagation model for log-normal shadowing environments.

Log-normal Shadowing- Received Power

- The received power in log-normal shadowing environment is given by the following formula (derivable from Equation 12)

Equation 12

$$P_r(d)[dBm] = P_t[dBm] - PL(d)[dB]$$

$$P_r(d)[dBm] = P_t[dBm] - \left[\overline{PL}(d_0)[dB] + 10n\log\left(\frac{d}{d_0}\right) + X_s\,[dB] \right]$$

  - The antenna gains are included in PL(*d*).

Log-normal Shadowing, *n* and s

- The log-normal shadowing model indicates the received power at a distance d is normally distributed with a distance dependent mean and with a standard deviation of σ
- In practice the values of n and σ are computed from measured data using linear regression so that the difference between the measured data and estimated path losses are minimized in a mean square error sense.

Example of determining n and s

- Assume $P_r(d0)$ = 0dBm and d_0 is 100m
- Assume the receiver power P_r is measured at distances 100m, 500m, 1000m, and 3000m,
- The table gives the measured values of received power

Distance from Transmitter	**Received Power**
100m	0dBm
500m	-5dBm
1000m	-11dBm
3000m	-16dBm

Example of determining n and s

- We know the measured values.
- Lets compute the estimates for received power at different distances using long-distance path loss model. (Equation 11)
- $P_r(d_0)$ is given as 0dBm and measured value is also the same.
  - mean_P_r(d) = $P_r(d_0)$ – mean_PL(from_d_0_to_d)
  - Then mean_P_r(d) = 0 – 10log*n*(d/d0)
  - Use this equation to computer power levels at 500m, 1000m, and 3000m.

Example of determining n and s

- $\text{Average\_P}_r(500m) = 0 - 10\log n(500/100) = -6.99n$
- $\text{Average\_P}_r(1000m) = 0 - 10\log n(1000/100) = -10n$
- $\text{Average\_P}_r(3000m) = 0 - 10\log n(3000/100) = -14.77n$

- Now we know the estimates and also measured actual values of the received power at different distances
- In order approximate n, we have to choose a value for n such that the mean square error over the collected statistics is minimized.

Example of determining n and s: MSE(Mean Square Error)

The mean square error (MSE) is given with the following formula:

$$MSE = \sqrt{\sum_{i=1}^{k} (p_i - \hat{p}_i)^2} \qquad \text{[Equation 14]}$$

p_i is the actual measured value of power at some distance

$\hat{p}_i$ is the estimate of power at that distance

k is the number of measurement samples

Since power estimate at some distance depends on *n*, MSE(*n*) is a function of n.

We would like to find a value of n that will minimize this MSE(n) value. We
We will call it MMSE: minimum mean square error.

This can be achieved by writing MSE as a function of n. Then finding the value of n which minimizes this function. This can be done by derivating MSE(n) with respect to n and solving for n which makes the derivative equal to zero.

Example of determining n:

Distance	Measured Value of Pr (dBm)	Estimated Value of Pr (dBm)
100m	0	0
500m	-5	-6.99n
1000m	-11	-10n
3000m	-16	-14.77n

MSE = $(0-0)^2 + (-5-(-6.99n))^2 + (-11-(-10n)^2 + (-16-(-14.77n)^2$
MSE = $0 + (6.99n - 5)^2 + (10n - 11)^2 + (14.77n - 16)^2$

If we open this, we get MSE as a function of n which as second order polynomial.

We can easily take its derivate and find the value of n which minimizes MSE. (I will not show these steps, since they are trivial).

Example of determining σ:

We are interested in finding the standard deviation about the mean value
For this, we will use the following formula

$$\sigma^2 = \frac{\sum_{i=1}^{k}(p_i - \hat{p}_i)^2}{k}$$

Equation 14.1

p_i is the actual measured value of power at some distance d

$\hat{p}_i$ is the estimate of power at that distance d

k is the number of measurement samples

From the above definition of σ, we can derive that:

$$\sigma^2 = MSE(N)/k$$
$$\sigma^2 = MMSE/k$$
$$\sigma = \sqrt{MMSE/k}$$

Equation 14.2

where N is the value that minimizes $MSE(n)$

$MMSE$ is minimum mean square error.

$MSE(n)$ formula is given in the previous slides.

Some Statistics Knowledge: Computation of mean (m), variance (s^2) and standard deviation (s)

Assume we have k samples (k values) $X_1, X_2, \ldots, X_k$:

The mean is denoted by m.

The variance is denotes by s.

The standard deviation is denotes by s^2.

The formulas to computer m, s, and s^2 is given below:

$$m = \frac{\sum_{i=1}^{k} X_i}{k}$$ [Equation 15]

$$s^2 = \frac{\sum_{i=1}^{k} (X_i - m)^2}{k}$$ [Equation 16]

$$s = \sqrt{\frac{\sum_{i=1}^{k} (X_i - m)^2}{k}}$$ [Equation 17]

Path loss and Received Power

- In log normal shadowing environment:
  - PL(d) (path loss) and Pr(d) (received power at a distance d) are random variables with a normal distribution in dB about a distance dependent mean.
- Sometime we are interested in answering following kind of questions:
  - What is mean received $P_r(d)$ power (mean_$P_r(d)$)at a distance d from a transmitter
  - What is the probability that the receiver power $P_r(d)$ (expressed in dB power units) at distance d is above (or below) some fixed value g (again expressed in dB power units such as dBm or dBW).

Received Power and Normal Distribution

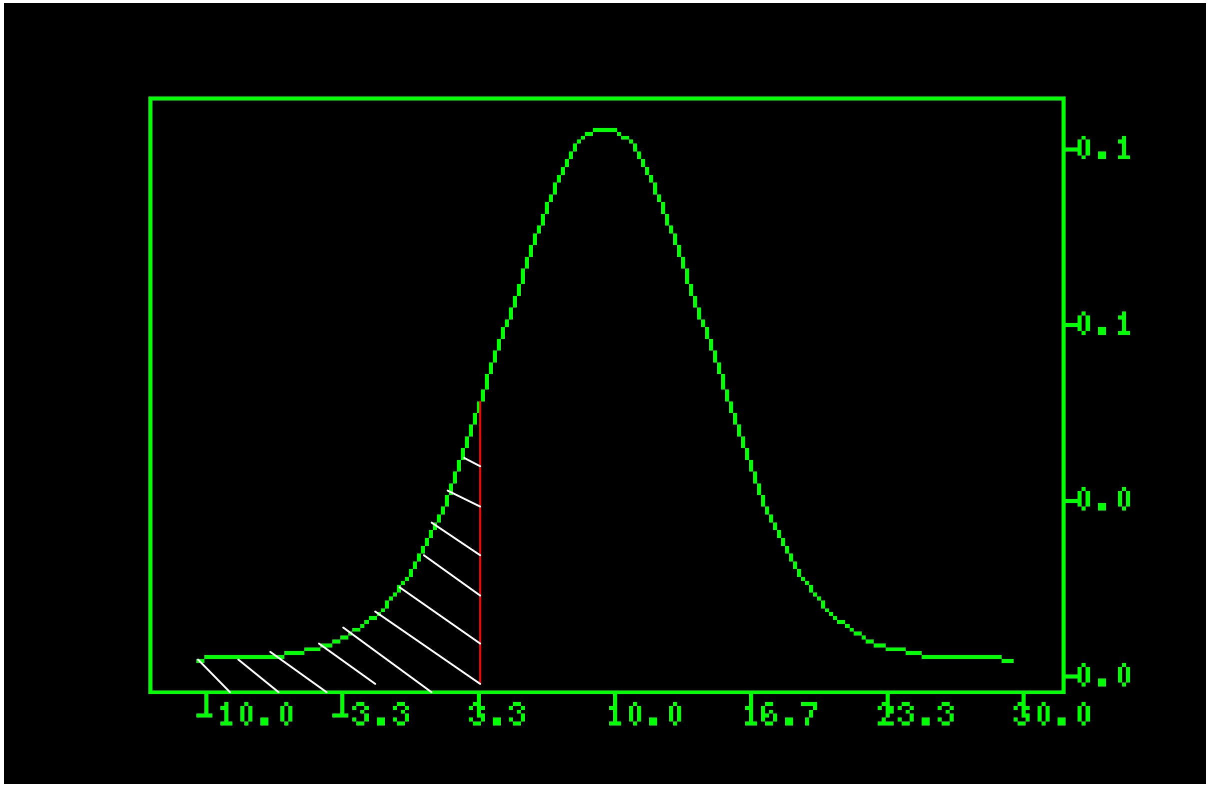
- In answering these kind of question, we have to use the properties of normal (gaussian distribution).
- $P_r(d)$ is normally distributed that is characterized by:
  - a mean (μ)
  - a standard deviation (σ)
- We are interested in Probability that

$$P_r(d) >= \gamma \text{ or } Pr(d) <= \gamma$$

Received Power and Normal Distribution PDF

Figure shows the PDF of a normal distribution for the received power P_r at some fixed distance d ($\mu = 10$, $\sigma = 5$)

(x-axis is received power, y-axis probability)

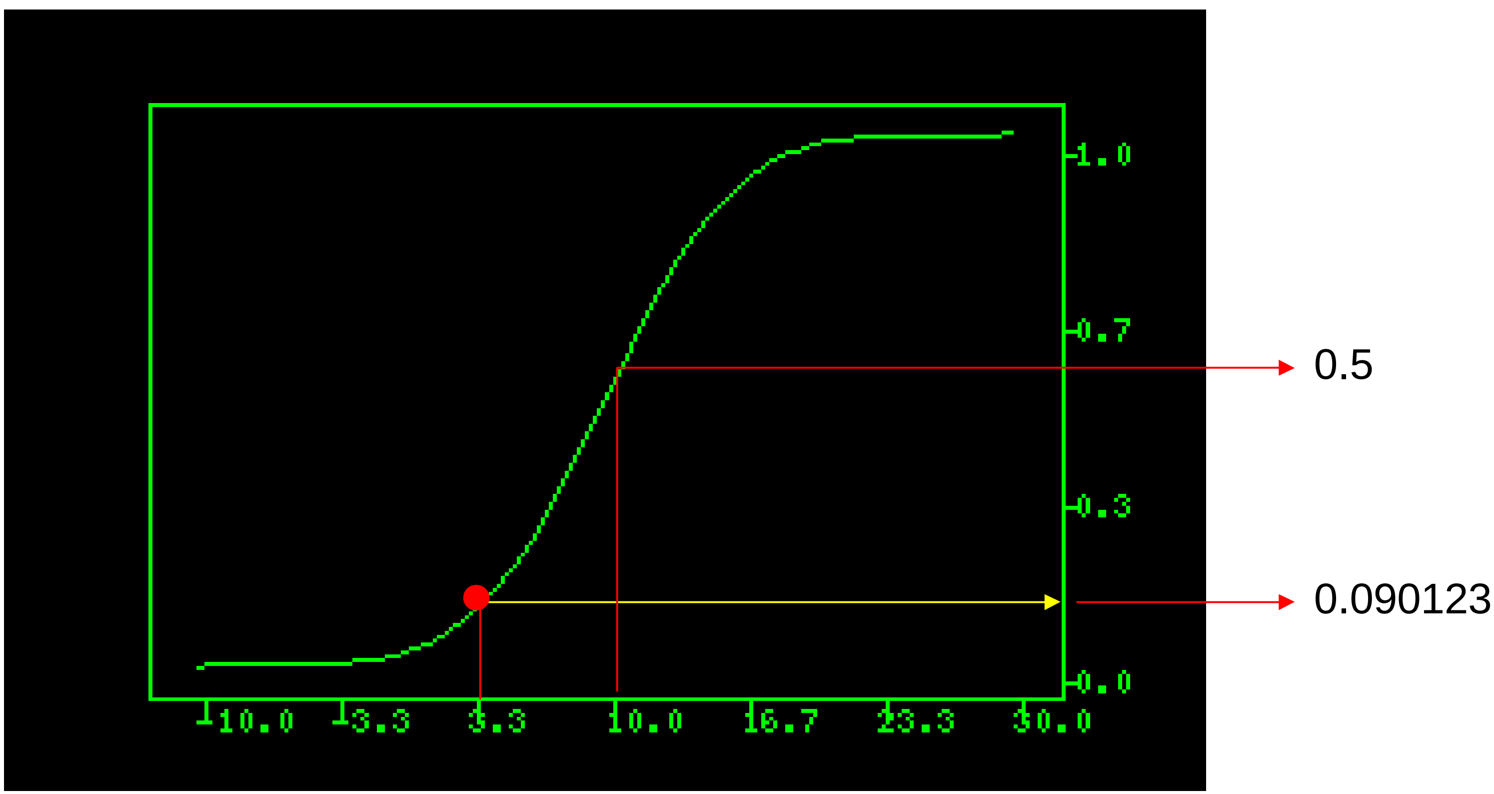


EXAMPLE:

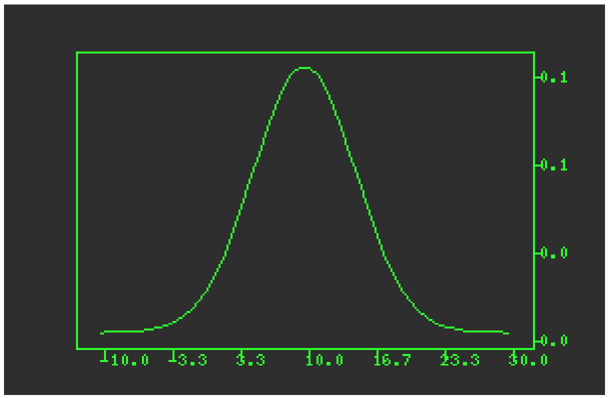
Probability that P_r is smaller than 3.3 (Prob(Pr <= 3.3)) is given with value of the stripped area under the curve.

Normal CDF

The figure shows the CDF plot of the normal distribution described previously. Prob(Pr <= 3.3) can be found by finding first the point where vertical line from 3.3 intersects the curve and then by finding the corresponding point on the y-axis. This corresponds to a value of 0.09. Hence Prob(Pr <= 3.3) = 0.09



Use of Normal Distribution



$$p(x) = \frac{1}{s\sqrt{2p}} e^{\frac{-(x-m)^2}{2s^2}}$$

[Equation 18]

PDF (probability density function of a normal distribution is characterized by two parameters, μ (mean)and σ (standard deviation), and given with the formula above.

Use of Normal Distribution

To find out the probability that a Gaussian (normal) random variable *X* is above a value *x0,* we have to integrate pdf.

$$\Pr(X > x_0) = \int_{x0}^{\infty} \frac{1}{s\sqrt{2p}} e^{\frac{-(x-m)^2}{2s^2}} dx$$

Equation 19

This integration does not have any closed form.

Any Gaussian PDF can be rewritten through substitution of y = x–µ / σ to yield

$$\Pr\left(y > \frac{x_0 - m}{s}\right) = \int_{\left(\frac{x0-m}{s}\right)}^{\infty} \frac{1}{s\sqrt{2p}} e^{\frac{-y^2}{2}} dy$$

Equation 20

Use of Normal Distribution

In the above formula, the kernel of the integral is normalized Gaussian PDF function with $\mu = 0$ and $\sigma = 1$.
Evaluation of this function is designed as Q-function and defined as

$$Q(z) = \int_{z}^{\infty} \frac{1}{s\sqrt{2p}} e^{\frac{-y^2}{2}} dy$$

Equation 21

Hence Equation 19 or 20 can be evaluated as:

$$\Pr(y > \frac{x_0 - m}{s}) = Q(\frac{x_0 - m}{s}) = Q(z)$$

Equation 22

Q-Function

Q-Function is bounded by two analytical expressions as follows:

$$(1-\frac{1}{z^2})\frac{1}{z\sqrt{2p}}e^{-z^2/2} \leq Q(z) \leq \frac{1}{z\sqrt{2p}}e^{-z^2/2} \qquad \text{Equation 23}$$

For values greater than 3.0, both of these bounds closely approximate *Q(z).*

Two important properties of *Q(z)* are:

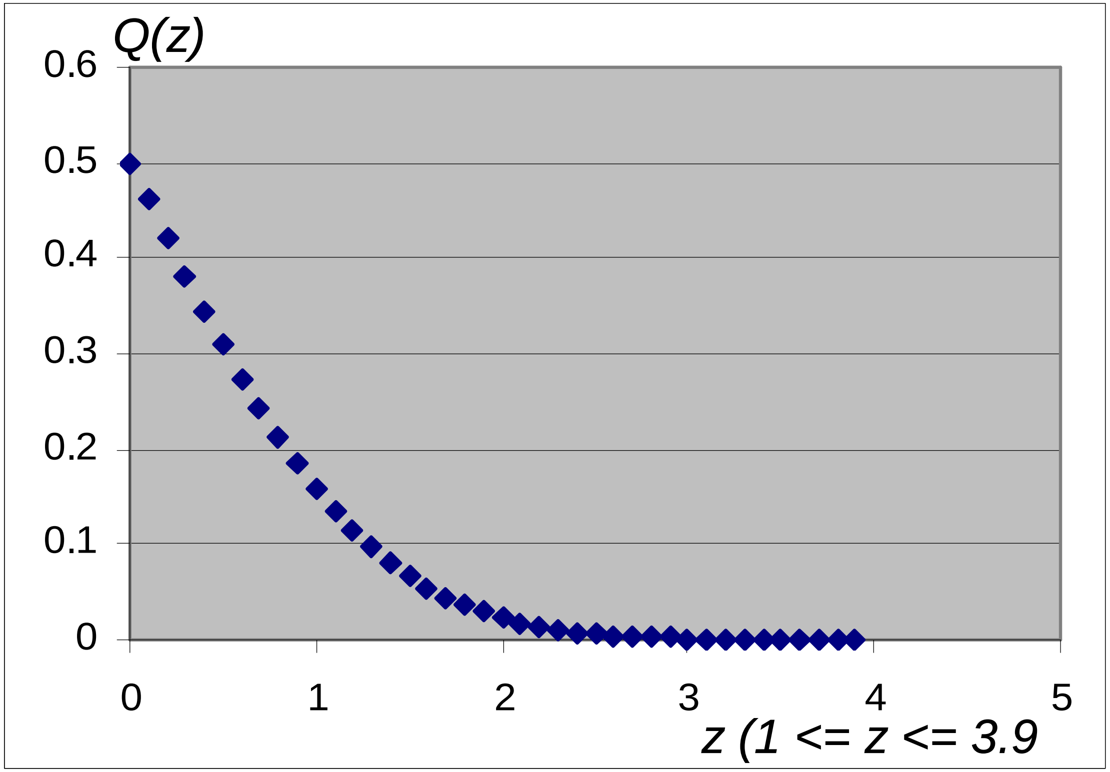
- *Q(-z)* = 1 – *Q(z)* Equation 24
- *Q(0)* = 1/2 Equation 25

Tabulation of Q-function ($0<=z<=3.9$)

z	Q(z)	z	Q(z)	z	Q(z)	z	Q(z)
0.0	0.5	1.0	0.15866	2.0	0.02275	3.0	0.00135
0.1	0.46017	1.1	0.13567	2.1	0.01786	3.1	0.00097
0.2	0.42074	1.2	0.11507	2.2	0.01390	3.2	0.00069
0.3	0.38209	1.3	0.09680	2.3	0.01072	3.3	0.00048
0.4	0.34458	1.4	0.08076	2.4	0.00820	3.4	0.00034
0.5	0.30854	1.5	0.06681	2.5	0.00621	3.5	0.00023
0.6	0.27425	1.6	0.05480	2.6	0.00466	3.6	0.00016
0.7	0.24196	1.7	0.04457	2.7	0.00347	3.7	0.00011
0.8	0.21118	1.8	0.03593	2.8	0.00256	3.8	0.00007
0.9	0.18406	1.9	0.02872	2.9	0.00187	3.9	0.00005

For values of z higher than 3.9, you should use the equations on the previous slide to compute $Q(z)$.

Q-Function Graph: z versus Q(z)



Erf and Erfc functions

The error function (erf) is defined as:

[Equation 26]

$$erfc(z) = \frac{2}{\sqrt{p}} \int_{z}^{\infty} e^{-x^2} dx$$

And the complementary error function (erfc) is defined as:

[Equation 27]

$$erf(z) = \frac{2}{\sqrt{p}} \int_{0}^{z} e^{-x^2} dx$$

The *erfc* function is related to *erf* function by:

[Equation 28]

$$erfc(z) = 1 - erf(z)$$

Erf and Erfc functions

The Q-function is related to erf and erfc functions by:

$$Q(z) = \frac{1}{2}[1 - erf(\frac{z}{\sqrt{2}})] = \frac{1}{2} erfc(\frac{z}{\sqrt{2}})$$ [Equation 29]

$$erfc(z) = 2Q(\sqrt{2}z)$$ [Equation 30]

$$erf(z) = 1 - 2Q(\sqrt{2}z)$$ [Equation 31]

Computation of probability that the received power is below/above a threshold

- We said that $P_r(d)$ is a random variable that is Gaussian distributed with mean μ and std deviation σ. Then:
  - Probability that $P_r(d)$ is above γ is given by:

$$\Pr(P_r(d) > g) = Q\left(\frac{g - \overline{P_r(d)}}{s}\right) \qquad \text{Equation 32}$$

  - Probability that $P_r(d)$ is below γ is given by:

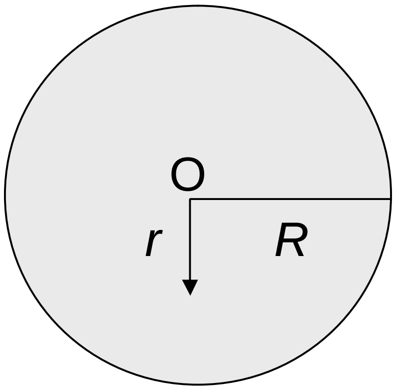
$$\Pr(P_r(d) < g) = Q\left(\frac{\overline{P_r(d)} - g}{s}\right) \qquad \text{Equation 33}$$

    - <u>P_r(d) bar</u> denotes the average (mean) received power at d.

Percentage of Coverage Area

- We are interested in the following problem
  - Given a circular coverage area with radius *R* from a base station
  - Given a desired threshold power level .
- Find out
  - *U*(γ), the percentage of useful service area
    - i.e the percentage of area with a received signal that is equal or greater than γ, given a known likelihood of coverage at the cell boundary

Percentage of Coverage Area



O is the origin of the cell

r: radial distance *d* from transmitter
0 <= r <= R

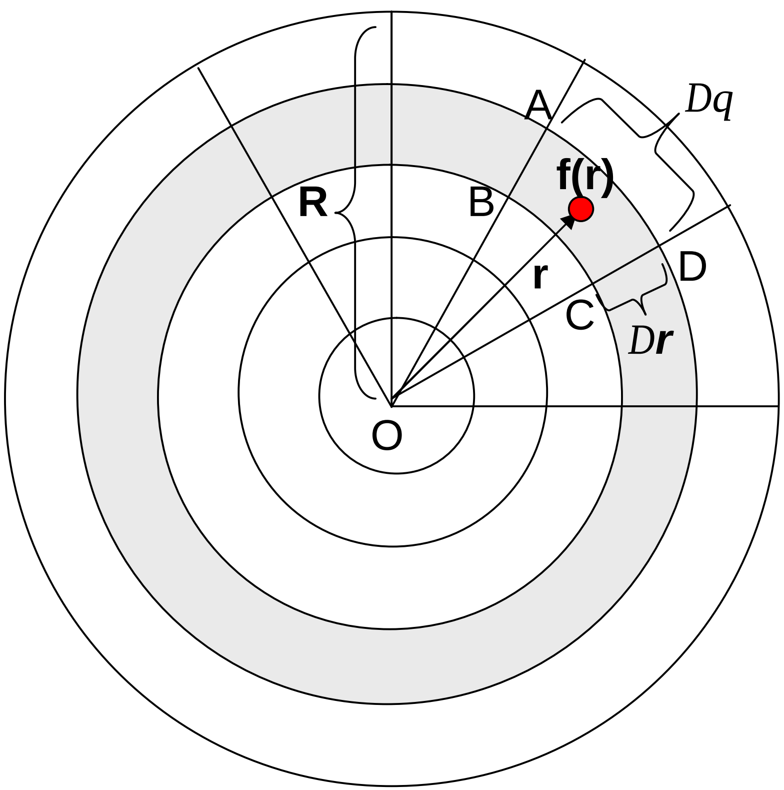
Definition: $P(P_r(r) > \gamma)$ denotes
probability that the random received power at a distance $d = r$ is greater than threshold γ within an incrementally small area dA

Then $U(\gamma)$ can be found by the following integration over the area of the cell:

$$U(g) = \frac{1}{pR^2}\int P[P_r(r) > g)dA = \frac{1}{pR^2}\int_0^{2p}\int_0^{R} P[P_r(r) > g)rdrdq$$

Equation 34

Integrating f(r) over Circle Area



Lets express the incremental area ΔA between points A, B, C, D.

The area could be approximated as the difference of areas of two sectors:

ΔA = Area(ABCD) ~= Area(OAD) ? Area(OBC)

$$AREA(OAD) = p\left(r + \frac{\Delta r}{2}\right)^2 \frac{\Delta q}{2p} = \frac{1}{2}\left(r + \frac{\Delta r}{2}\right)^2 \Delta q$$

q is expressed in radians : $0 \le q \le 2p$

$$AREA(OBC) = p\left(r - \frac{\Delta r}{2}\right)^2 \frac{\Delta q}{2p} = \frac{1}{2}\left(r - \frac{\Delta r}{2}\right)^2 \Delta q$$

$$\Delta A = \frac{1}{2}\left(r + \frac{\Delta r}{2}\right)^2 \Delta q - \frac{1}{2}\left(r - \frac{\Delta r}{2}\right)^2 \Delta q$$

$$\Delta A = \frac{\Delta q}{2}\left[\left(r + \frac{\Delta r}{2}\right)^2 - \left(r - \frac{\Delta r}{2}\right)^2\right]$$

$$\Delta A = \frac{\Delta q}{2}[2r\Delta r] = r\Delta r\Delta q$$

Then we can integrate a function f(r) over the surface area of the circle as follows:

$$F = \int f(r)dA$$

$$F = \int_0^{2p}\int_0^{R} f(r)rdrdq$$

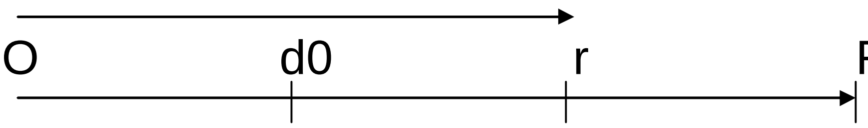
Percentage of Coverage Area

Using equation 32:

$$P(P_r(d) > g) = Q\left(\frac{g - \overline{P_r(d)}}{s}\right) = Q(g - [P_t - \overline{PL}(d_0) + 10n\log(r/d_0)])$$

Equation 33

The path loss at distance r can be expressed as:



PL(from O to r) = PL(from O to d0) + PL(from d0 to R) - PL(from r to R)
PL(from O to r) = PL(from O to d0) + PL(from d0 to R) + PL(from R to r)
(O is the point where base station is located)

Which can be formally expressed as:

$$\overline{PL}(r) = 10n\log(R/d_0) + 10n\log(r/R) + \overline{PL}(d_0)$$

Equation 34

Percentage of Coverage Area

Equation 33 can be expressed as follows using error function:

$$P(P_r(r) < g) = \frac{1}{2} - \frac{1}{2} erf\left(\frac{g - [P_t - (\overline{PL}(d) + 10n\log(r/d_0))]}{s\sqrt{2}}\right)$$

Equation 35

$$P(P_r(d) > g) = Q\left(\frac{g - \overline{P_r(d)}}{s}\right) = \frac{1}{2} - \frac{1}{2} erf\left(\frac{g - \overline{P_r(d)}}{s\sqrt{2}}\right)$$

By combining with Equation 34

$$P(P_r(r) > g) = \frac{1}{2} - \frac{1}{2} erf\left(\frac{g - [P_t - (\overline{PL}(d_0) + 10n\log(r/d_0) + 10n\log(r/R))]}{s\sqrt{2}}\right)$$

Equation 36

Percentage of Coverage Area

Let the following substitutions happen:

$$a = (g - P_t + \overline{PL}(d0) + 10n\log(R/d0))/s\sqrt{2}$$

$$b = (10n\log e)/s\sqrt{2}$$

Then

$$U(g) = \frac{1}{2} - \frac{1}{R^2}\int_0^R erf\left(a + b\ln\frac{r}{R}\right) r\,dr \qquad \text{Equation 37}$$

Substitute $t = a + b\log(r/R)$

$$U(g) = \frac{1}{2} - \left(1 - erf(a) + e^{\left(\frac{1-2ab}{b^2}\right)}\left[1 - erf\left(\frac{1-ab}{b}\right)\right]\right) \qquad \text{Equation 38}$$

Percentage of Coverage Area

- By choosing a signal level such that $\overline{P_r}(R) = g$ (i.e. a =), we obtain:

$$U(g) = \frac{1}{2} - \left(1 + e^{\left(\frac{1}{b^2}\right)}\left[1 - erf\left(\frac{1}{b}\right)\right]\right)$$

where $b = (10n \log e) / s\ \sqrt{2}$

Equation 39

The simplified formula above gives the percentage coverage assuming the mean received power at the cell boundary (r=R) is γ.
In other words, we are assuming: Prob($P_r(R)$ >= γ) = 0.5

Outdoor Propagation

- We will look to the propagation from a transmitter in an outdoor environment
  - The coverage area around a tranmitter is called a cell.
    - Coverage area is defined as the area in which the path loss is at or below a given value.
  - The shape of the cell is modeled as hexagon, but in real life it has much more irregular shapes.
  - By playing with the antenna (tilting and changing the height), the size of the cell can be controlled.
- We will look to the propagation characteristics of the three outdoor environments
  - Propagation in macrocells
  - Propagation in microcells
  - Propagation in street microcells

Macrocells

- Base stations at high-points
- Coverage of several kilometers
- The average path loss in dB has normal distribution
  - Avg path loss is result of many forward scattering over a great many of obstacles
    - Each contributing a random multiplicative factor
    - Converted to dB, this gives a sum of random variable
  - Sum is normally distributed because of central limit theorem

Macrocells

- In early days, the models were based on emprical studies
- Okumura did comprehesive measurements in 1968 and came up with a model.
  - Discovered that a good model for path loss was a simple power law where the exponent n is a function of the frequency, antenna heights, etc.
  - Valid for frequencies in: 100MHz – 1920 MHz
    for distances: 1km – 100km

Okumura Model

Equation 40

- $L_{50}(d)(dB) = L_F(d) + A_{mu}(f,d) - G(h_{te}) - G(h_{re}) - G_{AREA}$
  - L_{50}: 50th percentile (i.e., median) of path loss
  - $L_F(d)$: free space propagation pathloss.
  - $A_{mu}(f,d)$: median attenuation relative to free space
    - Can be obtained from Okumura's emprical plots shown in the book (Rappaport), page 151.
  - $G(h_{te})$: base station antenna heigh gain factor
  - $G(h_{re})$: mobile antenna height gain factor
  - G_{AREA}: gain due to type of environment

    - $G(h_{te}) = 20\log(h_{te}/200)$ $1000m > h_{te} > 30m$
    - $G(h_{re}) = 10\log(h_{re}/3)$ $h_{re} <= 3m$
    - $G(h_{re}) = 20\log(h_{re}/3)$ $10m > h_{re} > 3m$
      - h_{te}: transmitter antenna height
      - h_{re}: receiver antenna height

Hata Model

- Valid from 150MHz to 1500MHz
- A standard formula
- For urban areas the formula is:
  - $L_{50}(\text{urban},d)(\text{dB}) = 69.55 + 26.16\log f_c - 13.82\log h_{te} – a(h_{re}) + (44.9 – 6.55\log h_{te})\log d$ Equation 41

    where

    f_c is the ferquency in MHz

    h_{te} is effective transmitter antenna height in meters (30-200m)

    h_{re} is effective receiver antenna height in meters (1-10m)

    d is T-R separation in km

    $a(h_{re})$ is the correction factor for effective mobile antenna height which is a function of coverage area

    $a(h_{re}) = (1.1\log f_c – 0.7)h_{re} – (1.56\log f_c – 0.8)$ dB

    for a small to medium sized city

Microcells

- Propagation differs significantly
  - Milder propagation characteristics
  - Small multipath delay spread and shallow fading imply the feasibility of higher data-rate transmission
  - Mostly used in crowded urban areas
  - If transmitter antenna is lower than the surrounding building than the signals propagate along the streets: Street Microcells

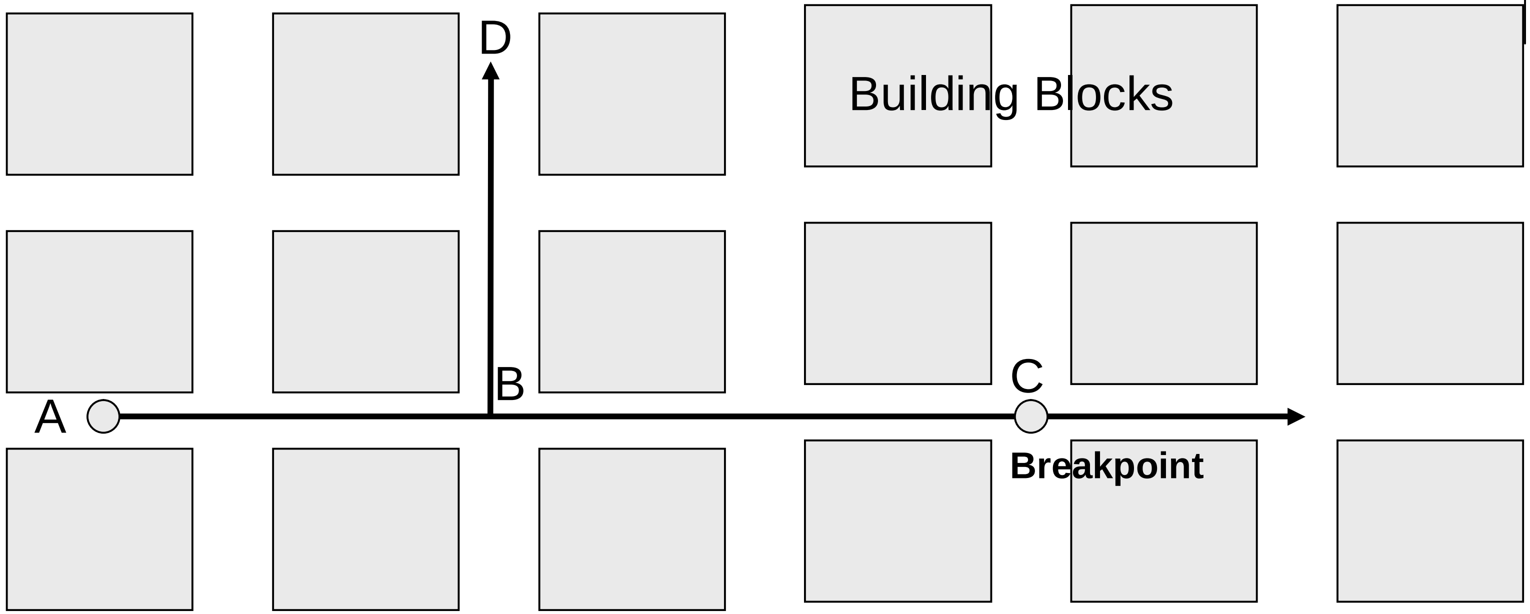
Macrocells versus Microcells

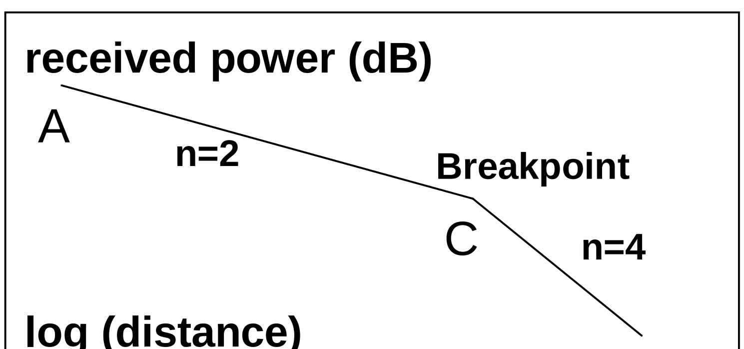
Item	Macrocell	Microcell
Cell Radius	1 to 20km	0.1 to 1km
Tx Power	1 to 10W	0.1 to 1W
Fading	Rayleigh	Nakgami-Rice
RMS Delay Spread	0.1 to 10μs	10 to 100ns
Max. Bit Rate	0.3 Mbps	1 Mbps

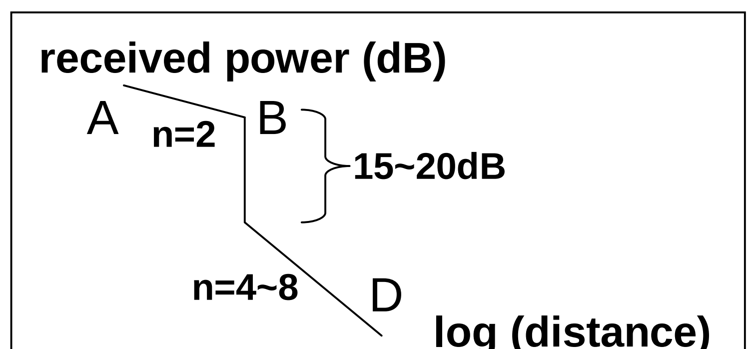
Street Microcells

- Most of the signal power propagates along the street.
- The sigals may reach with LOS paths if the receiver is along the same street with the transmitter
- The signals may reach via indirect propagation mechanisms if the receiver turns to another street.

Street Microcells







Indoor Propagation

- Indoor channels are different from traditional mobile radio channels in two different ways:
  - The distances covered are much smaller
  - The variablity of the environment is much greater for a much smaller range of T-R separation distances.
- The propagation inside a building is influenced by:
  - Layout of the building
  - Construction materials
  - Building type: sports arena, residential home, factory,...

Indoor Propagation

- Indoor propagation is domited by the same mechanisms as outdoor: reflection, scattering, diffraction.
  - However, conditions are much more variable
    - Doors/windows open or not
    - The mounting place of antenna: desk, ceiling, etc.
    - The level of floors
- Indoor channels are classified as
  - Line-of-sight (LOS)
  - Obstructed (OBS) with varying degrees of clutter.

Indoor Propagation

- Buiding types
  - Residential homes in suburban areas
  - Residential homes in urban areas
  - Traditional office buildings with fixed walls (hard partitions)
  - Open plan buildings with movable wall panels (soft partitions)
  - Factory buildings
  - Grocery stores
  - Retail stores
  - Sport arenas

Indoor propagation events and parameters

- Temporal fading for fixed and moving terminals
  - Motion of people inside building causes Ricean Fading for the stationary receivers
  - Portable receivers experience in general:
    - Rayleigh fading for OBS propagation paths
    - Ricean fading for LOS paths.
- Multipath Delay Spread
  - Buildings with fewer metals and hard-partitions typically have small rms delay spreads: 30-60ns.
    - Can support data rates excess of several Mbps without equalization
  - Larger buildings with great amount of metal and open aisles may have rms delay spreads as large as 300ns.
    - Can not support data rates more than a few hundred Kbps without equalization.
- Path Loss
  - The following formula that we have seen earlier also describes the indoor path loss:
    - $PL(d)[dBm] = PL(d_0) + 10n\log(d/d_0) + X\sigma$
      - n and σ depend on the type of the building
      - Smaller value for σ indicates the accuracy of the path loss model.

Path Loss Exponent and Standard Deviation Measured for Different Buildings

Building	Frequency (MHz)	n	s (dB)
Retail Stores	914	2.2	8.7
Grocery Store	914	1.8	5.2
Office, hard partition	1500	3.0	7.0
Office, soft partition	900	2.4	9.6
Office, soft partition	1900	2.6	14.1
Factory LOS			
Textile/Chemical	1300	2.0	3.0
Textile/Chemical	4000	2.1	7.0
Paper/Cereals	1300	1.8	6.0
Metalworking	1300	1.6	5.8
Suburban Home			
Indoor Street	900	3.0	7.0
Factory OBS			
Textile/Chemical	4000	2.1	9.7
Metalworking	1300	3.3	6.8

In building path loss factors

- Partition losses (same floor)
- Partition losses between floors
- Signal Penetration into Buildings

Partition Losses

- There are two kind of partition at the same floor:
  - Hard partions: the walls of the rooms
  - Soft partitions: moveable partitions that does not span to the ceiling

  - The path loss depends on the type of the partitions

Partition Losses

Average signal loss measurements reported by various researches for radio paths obscructed by some common building material.

Material Type	Loss (dB)	Frequency (MHz)
All metal	26	815
Aluminim Siding	20.4	815
Concerete Block Wall	3.9	1300
Loss from one Floor	20-30	1300
Turning an Angle in a Corridor	10-15	1300
Concrete Floor	10	1300
Dry Plywood (3/4in) – 1 sheet	1	9600
Wet Plywood (3/4in) – 1 sheet	19	9600
Aluminum (1/8in) – 1 sheet	47	9600

Partition Losses between Floors

- The losses between floors of a building are determined by
  - External dimensions and materials of the building
  - Type of construction used to create floors
  - External surroundings
  - Number of windows
  - Presence of tinting on windows

Partition Losses between Floors

Average Floor Attenuation Factor in dB for One, Two, Three and Four Floors in Two Office Buildings

Building	**FAF (dB)**	s **(dB)**
Office Building 1		
Through 1 Floor	12.9	7.0
Through 2 Floors	18.7	2.8
Through 3 Floors	24.4	1.7
Through 4 Floors	27.0	1.5
Office Building 2		
Through 1 Floor	16.2	2.9
Through 2 Floors	27.5	5.4
Through 3 Floors	31.6	7.2

Signal Penetration Into Buildings

- RF signals can penetrate from outside transmitter to the inside of buildings
  - However the siganls are attenuated
- The path loss during penetration has been found to be a function of:
  - Frequency of the signal
  - The height of the building

Signal Penetration Into Buildings

- Effect of Frequency
  - Penetration loss decreases with increasing frequency

Frequency (MHz)	Loss (dB)
441	16.4
896.5	11.6
1400	7.6

- Effect of Height
  - Penetration loss decreases with the height of the building up-to some certain height
    - At lower heights, the urban clutter induces greater attenuation
  - and then it increases
    - Shadowing affects of adjascent buildings

Conclusion

- More work needs to be done to understand the characteristics of wireless channels
- 3D numerical modeling approaches exist
- To achieve PCS, new and novel ways of classifying wireless environments will be needed that are both widely encompassing and reasonably compact.