Mobile Radio Propagation: Small-Scale Fading and Multi-path

Small-scale Fading

- Small-scale fading, or simply *fading* describes the rapid fluctuation of the amplitude of a radio signal over a short period of time or travel distance
- It is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at different times
  - This interference can vary widely in amplitude and phase over time

Small-scale Fading Effects

- The three most important fading effects are
  1. Rapid changes in signal strength over a small travel distance or time interval
  2. Random frequency modulation due to varying Doppler shifts (described later) on different multi-path signals
  3. Time dispersions (echos) caused by multi-path propagation delays

Factors Influencing Small-scale Fading

- The following physical factors in the radio propagation channel influence small-scale fading
  - multi-path propagation
  - speed of the mobile
  - speed of the surrounding objects
  - the transmission bandwidth of the signal

Multi-path Propagation

- The presence of reflecting objects and scatterers in the channel creates a constantly changing environment
  - this results in multiple versions of the transmitted signal that arrive at the receiving antenna, displaced with respect to one another in time and spatial orientation

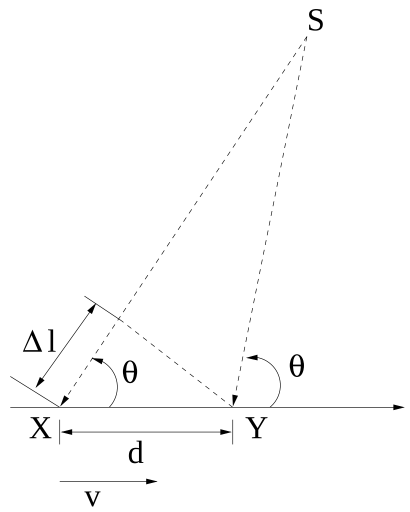
Multipath Propagation (continued)

- The random phase and amplitudes of the different multipath components cause fluctuations in signal strength, thereby inducing *small-scale fading*, signal distortion, or both
- Multipath propagation often lengthens the time required for the baseband portion of the signal to reach the receiver which can cause signal smearing due to intersymbol interference

Doppler Shift

- **Definition:** *The shift in received signal frequency due to motion is called the Doppler shift*
  - It is directly proportional to
    * the velocity of the mobile
    * the direction of motion of the mobile with respect to the direction of arrival of the received wave

Doppler Shift (continued)



- Consider a mobile moving at a constant velocity v, along a path segment having length d between points X and Y
- The mobile receives signals from a remote source S

Doppler Shift (continued)

- **Assumptions:**
  - d is small and S is very remote
- When the distance of $S \gg d \longrightarrow SX$ is almost parallel to SY

Doppler Shift (continued)

- The difference in path lengths traveled by the wave from source S to the mobile at points X and Y is

$$\Delta l = d \cos\theta = v \Delta t \cos\theta$$

where

  * Δt = time required for the mobile to travel from X to Y
  * θ = angle of arrival of the wave, which is the same at points X and Y due to the assumptions in the previous slide

Doppler Shift (continued)

- The transmitted signal can be expressed as

$$s(t) = A\{\exp[j2\pi f_c t]\}$$

where

* A = amplitude of the signal
* f_c = carrier frequency

- The received signal at point X is given by

$$r_x(t) = A\{\exp[j2\pi f_c\,(t - \tau_x)]\}$$

* τ_x = propagation delay

Doppler Shift (continued)

- The received signal at point Y is given by

$$
\begin{aligned}
r_y(t) &= A\exp[j2\pi f_c\,(t-\tau_y)] \\
&= A\exp[j2\pi f_c\{t-(\tau_x-\Delta t)\}] \\
&= A\exp\left[j2\pi f_c\left\{(t-\tau_x)+\frac{\Delta l}{c}\right\}\right] \\
&= A\exp\left[j2\pi\left\{f_c\,(t-\tau_x)+\frac{\Delta l}{\lambda}\right\}\right] \\
&= A\exp\left[j2\pi\left\{f_c\,(t-\tau_x)+\frac{v\cos\theta}{\lambda}\Delta t\right\}\right]
\end{aligned}
$$

Doppler Shift (continued)

- From the previous slide, let

$$\Phi_y = 2\pi f_c t - 2\pi f_c \tau_x + 2\pi \frac{v\cos\theta}{\lambda}\Delta t$$

- Received frequency at point Y is

$$\begin{aligned} f_y &= \frac{1}{2\pi}\frac{d\Phi_y}{dt} \\ &= f_c + \frac{v\cos\theta}{\lambda} \\ &= f_c + f_d \end{aligned}$$

  where f_d is the Doppler shift due to the motion of the mobile

- **Note:** f_d is positive when the mobile is moving towards the source S

Doppler Shift (continued)

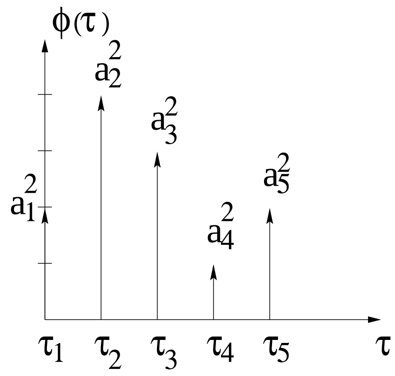
- If the mobile is moving away from the base station then

$$r_x(t) = A\exp\left[j2\pi\left\{f_c\left(t-\tau_y\right)-\frac{v\cos\theta}{\lambda}\Delta t\right\}\right]$$

- Thus the received frequency at X is

$$f_x = f_c - f_d = f_c - \frac{v\cos\theta}{\lambda}$$

Power Delay Profile



Multipath Power Delay Profile

- Power delay profiles are
  - used to derive many multipath channel parameters
  - generally represented as plots of relative received power (a_k^2) as a function of excess delay (τ) with respect to a fixed time delay reference

Power Delay Profile (continued)

- Power delay profiles are found by averaging instantaneous power delay profile measurements over a local area in order to determine an average small-scale power delay profile

Time Dispersion Parameters

- The time dispersion parameters that can be determined from a power delay profile are
  - *mean excess delay*
  - *rms delay spread*
  - *excess delay spread*
- The time dispersive properties of wide band multipath channels are most commonly quantified by their mean excess delay ($\overline{\tau}$) and rms delay spread ($\overline{\sigma}_\tau$)

Mean Excess Delay

- The mean excess delay is the first moment of the power delay profile and is defined as

$$\begin{aligned}\overline{\tau} &= \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} \\ &= \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}\end{aligned}$$

where

$$P(\tau_k) = \frac{a_k^2}{\sum_i a_i^2}$$

RMS Delay Spread

- The rms delay spread is the square root of the second central moment of the power delay profile and is defined to be

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\overline{\tau})^2}$$

where

$$\begin{aligned}\overline{\tau^2} &= \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} \\ &= \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}\end{aligned}$$

Notes

- The mean excess delay and rms delay spread are measured relative to the first detectable signal arriving at the receiver at $\tau_0 = 0$
- $\overline{\tau}$ and $\overline{\tau^2}$ do not rely on the absolute power level, but only the relative amplitudes of the multipath components
- Typical values of rms delay spread are on the order of
  - microseconds in outdoor mobile radio channel
  - nanoseconds in indoor radio channels

Notes (continued)

- The rms delay spread and mean excess delay are defined from a single power delay profile which is the temporal or spatial average of consecutive impulse response measurements collected and averaged over a local area
- Typically many measurements are made at many local areas in order to determine a statistical range of multipath channel parameters for a mobile communication system over a large-scale area

Maximum Excess Delay

- The maximum excess delay (X dB) of the power delay profile is defined to be the time delay during which multipath energy falls to X dB below the maximum
- If τ_0 is the first arriving signal and τ_X is the maximum delay at which a multipath component is with X dB of the strongest multipath signal (which does not necessarily arrive at τ_0), then the maximum excess delay is defined as

$$\tau_{\max}(X\mathrm{dB}) = \tau_X - \tau_0$$

Relation between B_c and σ_τ

- The rms delay spread and coherence bandwidth are inversely proportional to one another, although their exact relationship is a function of the exact multipath structure
- If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9, then the coherence bandwidth is approximately

$$B_c \approx \frac{1}{50\sigma_\tau}$$

where σ_τ is the rms delay spread

Relation between B_c and σ_τ (continued)

- If the definition is relaxed so that the frequency correlation function is above 0.5, then the coherence bandwidth is approximately

$$B_c \approx \frac{1}{5\sigma_\tau}$$

Coherence Time

- Coherence time T_c is the time domain dual of Doppler spread and is used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain
- What is **Doppler spread,** B_d?
- $B_d \propto 1/T_c$
- **Remark:** A slowly changing channel has a large coherence time or, equivalently, a small Doppler spread

Coherence Time (continued)

- If the coherence time is defined as the time over which the time correlation function is above 0.5, then it is approximated as

$$T_c \cong \frac{9}{16\pi f_m} = \frac{0.179}{f_m}$$

where

∗ f_m = maximum Doppler shift and is given by

$$\begin{aligned} f_m &= f_{d,\max} \\ &= \frac{v}{\lambda} \end{aligned}$$

Coherence Time (continued)

- The approximation of the coherence time in the previous slide is too restrictive and a popular rule of thumb defines the coherence time as

$$T_c \cong \sqrt{\frac{9}{16\pi f_m^2}} = \frac{0.423}{f_m}$$

- **Note:** The definition of coherence time implies that two signals arriving with a time separation greater than T_c are affected differently by the channel

Types of Small-Scale Fading

- The types of small-scale fading experienced by a signal propagating through a mobile radio channel depends on the relation between the
  1. Signal parameters such as
     - bandwidth
     - symbol period
  2. Channel parameters such as
     - rms delay spread
     - Doppler spread

Types of Small-Scale Fading (continued)

- Based on multipath time delay spread, two types of small-scale fading are
  1. Flat fading or frequency nonselective fading
  2. Frequency selective fading
- Based on Doppler spread, two types of small-scale fading are
  1. Fast fading
  2. Slow fading

Frequency Nonselective (flat) Fading

- **Definition:** *If the mobile radio channel has a constant gain and linear phase response over the bandwidth B_c which is greater than the bandwidth of the transmitted signal B_s, then the received signal will undergo flat fading*
- In flat fading, the multipath structure of the channel is such that
  - the spectral characteristics of the transmitted signal are preserved at the receiver
  - the strength of the received signal changes with time, due to fluctuations in the gain of the channel caused by multipath

Flat Fading (continued)

- In a flat fading channel, all of the frequency components in $S_l(f)$ undergo the same attenuation and phase shift in transmission through the channel, which implies
  - within the bandwidth occupied by $S_l(f)$, the time variant transfer function $H_l\,(f, t)$ is a complex-valued constant in the frequency variable

Flat Fading (continued)

- Thus the equivalent lowpass received signal can be expressed as

$$r_l(t) = \alpha(t) e^{-j\phi(t)} s_l(t)$$

where

  * $\alpha(t)$ = envelope of the equivalent lowpass channel
  * $\phi(t)$ = phase of the equivalent lowpass channel

Transfer Function (continued)

- When $\alpha(t)e^{-j\phi(t)}$ is modeled as a zero-mean complex valued Gaussian random process
  - the envelope $\alpha(t)$ is Rayleigh distributed for any fixed value of t
  - the phase $\phi(t)$ is uniformly distributed over the interval $(-\pi, \pi)$

Remarks

- In a flat fading channel, the reciprocal bandwidth of the transmitted signal is much larger than the multipath time delay spread of the channel and thus
  - $h_b(t, \tau)$ can be approximated as having no excess delay $\longrightarrow$ a single delta function with $\tau = 0$

Remarks (continued

- Flat fading channels are known as *amplitude varying channel* and are sometimes referred to as *narrow-band channels*, since the bandwidth of the applied signal is *narrow* compared to the channel flat fading bandwidth
- Typical flat fading channels may cause deep fades, and thus may require 20 or 30 dB more transmitter power to achieve low bit error rates during times of deep fades as compared to systems operating over non-fading channel

Summary of Flat Fading

- A signal undergoes flat fading if

$$B_s \ll B_c$$

and

$$T_s \gg \sigma_\tau$$

where $B_s, B_c, T_s, \sigma_\tau$ are as defined previously

Frequency Selective Fading

- **Definition:** *If the channel possesses a constant-gain and linear phase response over a bandwidth (coherence bandwidth) that is smaller than the bandwidth of transmitted signal, then the channel creates* **frequency selective fading** *on the received signal*
  - in this case, the received signal includes multiple versions of the transmitted waveform which are attenuated (faded) and delayed in time, and hence the received signal is **strongly** distorted by the channel

Remarks

- Frequency selective fading is caused by multipath delays which approach or exceed the symbol period of the transmitted symbol
- Frequency selective fading channels are also known as *wideband channels* since the bandwidth of the signal is wider than the bandwidth of the channel impulse response
- As time varies, the channel varies in gain and phase across the spectrum of $s_l(t)$, resulting in time varying distortion in the received signal $r_l(t)$

Summary of Frequency Selective Fading

- A signal undergoes frequency selective fading if

$$B_s > B_c$$

and

$$T_s < \sigma_\tau$$

where

* B_s = bandwidth of the transmitted signal
* T_s = reciprocal bandwidth (e.g., symbol period) of the transmitted signal
* σ_τ = rms delay spread of the channel
* B_c = coherence bandwidth of the channel

Remark

- A common rule of thumb is that a channel is frequency selective if

$$T_s \leq 10\sigma_\tau$$

although this is dependent on the specific type of modulation used

Fast Fading

- In a fast fading channel, the channel impulse response changes rapidly within the symbol duration
- A signal undergoes fast fading if

$$T_s > T_c$$

and

$$B_s < B_d$$

where

  * T_c = coherence time of the channel
  * B_d = Doppler spread

Fast Fading (continued)

- Since in a fast fading channel the coherence time of the channel is smaller than the symbol period of the transmitted signal
  - this causes frequency dispersion (also called *time selective fading*) due to Doppler spreading $\longrightarrow$ leads to signal distortion
- Viewed in the frequency domain, signal distortion due to fast fading increases with increasing Doppler spread relative to the bandwidth of the transmitted signal

Flat and Fast Fading

- In the case of a flat fading channel, we can approximate the impulse response to be simply a delta function (no time delay)
  - *a flat and fast fading channel is a channel in which the amplitude of the delta function varies faster than the rate of change of the transmitted baseband signal*

Frequency Selective and Fast Fading

- *In the case of a frequency selective and fast fading channel, the amplitude, phases, and time delays of any one of the multipath components vary faster than the rate of change of the transmitted signal*
- **Remark:** In practice, fast fading only occurs for very low data rates

Slow Fading

- In a slow fading channel, the channel impulse response changes at a rate much slower than the transmitted baseband signal
- A signal undergoes slow fading if

$$T_s \ll T_c$$

and

$$B_s \gg B_d$$

Slow Fading (continued)

- Since in a slow fading channel, signal duration is smaller than the coherence time of the channel, the channel attenuation and phase shift are fixed for the duration of at least one signaling interval
  - in the frequency domain this implies that the Doppler spread of the channel is much less than the bandwidth of the baseband signal
- **Note:** Fast and slow fading deal with the relationship between the time rate of change in the channel and the transmitted signal, and not with propagation path loss model

Flat and Slow Fading

- When $B_s \approx \frac{1}{T_s}$, the conditions that the channel be frequency non-selective and slowly fading imply that the product of σ_τ and B_d must satisfy the condition

$$\sigma_\tau B_d < 1$$

- The product $\sigma_\tau B_d$ is called the *spread factor* of the channel
  - if $\sigma_\tau B_d < 1$, the channel is said to be *under-spread*
  - if $\sigma_\tau B_d > 1$, the channel is said to be *over-spread*

Rayleigh Fading Distribution

- In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component
- **Remark:** The envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution

Rayleigh Distribution

- The probability density function (pdf) of the Rayleigh distribution is given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) & (0 \le r \le \infty) \\ 0 & (r < 0) \end{cases}$$

where

- σ = rms value of the received voltage signal before *envelop detection*
- σ^2 = time-average power of the received signal before *envelop detection*

Rayleigh Distribution (continued)

- The probability that the envelope of the received signal does not exceed a specified value R is given by the corresponding cumulative distribution function (CDF)

$$\begin{aligned} P(R) &= Prob\,(r \leq R) \\ &= \int_0^R p(r)dr \\ &= 1 - \exp\left(-\frac{R^2}{2\sigma^2}\right) \end{aligned}$$

Ricean Fading Distribution

- When there is a dominant stationary (non-fading) signal component present, such as a line-of-sight propagation path, the small-scale fading envelope distribution is Ricean
- As the dominant signal becomes weaker, the Ricean distribution degenerates to a Rayleigh distribution

Ricean Distribution

- The pdf of the Ricean distribution is given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{\left(r^2+A^2\right)}{2\sigma^2}} I_0\left(-\frac{Ar}{\sigma^2}\right) & (A \geq 0, r \geq 0) \\ 0 & (r < 0) \end{cases}$$

  where

  * $A =$ peak amplitude of the dominant signal
  * $I_0(\cdot) =$ modified Bessel function of the first kind and zero-order
  * $k = \frac{A^2}{2\sigma^2} =$ Ricean factor

Performance of Digital Modulation

- **Goal:** To evaluate the probability of error of a any digital modulation scheme in a slow, flat fading channel
- Recall that the flat fading channels cause a multiplicative (gain) variation in the transmitted signal $s(t)$

Performance of Digital Modulation (continued)

- Since slow fading channels change much slower than the applied modulation
  - it can be assumed that the attenuation and phase shift of the signal is constant over at least one symbol interval
  - the received signal $r(t)$ may be expressed as

$$\begin{aligned} r(t) &= \alpha(t)e^{-j\theta(t)}s(t) + n(t) \qquad 0 \le t \le T \\ &\cong \alpha(0)e^{-j\theta(0)}s(t) + n(t) \end{aligned}$$

where

* $\alpha(t)$ = gain of the channel
* $\theta(t)$ = phase shift of the channel
* $n(t)$ = additive Gaussian noise

Performance of Digital Modulation (continued)

- If $\theta(t)$ is varying slowly compared to the speed of the receiver processing, then
  - we can estimate $\theta(t)$ and implement coherent receivers
  - otherwise, we have to use non-coherent receivers

Probability of Error

- The probability of error in slow, flat fading channels can be obtained by averaging the error in additive white Gaussian noise (AWGN) channels over the fading probability density function
- **Remark:** The probability of error in AWGN channels is viewed as a conditional error probability, where the condition is that α is fixed

Probability of Error (continued)

- For BPSK signals, the probability of error in AWGN channels is expressed as

$$
\begin{aligned}
P_{e,\text{BPSK}} &= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \\
&= \frac{1}{2} erf\left(\sqrt{\Gamma_b}\right)
\end{aligned}
$$

where

* E_b = energy per bit
* $\Gamma_b = \frac{E_b}{N_0}$ = SNR

Probability of Error (continued)

- The probability of error in a slow, flat fading may be evaluated as

$$P_e = \int_0^\infty P_e(X)p(X)dX$$

where

* $P_e(X)$ = probability of error for an arbitrary modulation at a specific value of signal-to-noise ratio X
* $X = \alpha^2 \frac{E_b}{N_0}$
* $p(X)$ = probability density function of X due to the fading channel
* α = amplitude values of the fading channel with respect to E_b/N_0

Probability of Error (continued)

- For Rayleigh fading channels, α has a Rayleigh distribution $\longrightarrow \alpha^2$ and consequently X have a chi-square distribution with two degrees of freedom (which is the exponential distribution)
- Thus, the pdf of X due to fading channel is expressed as

$$p(X) = \frac{1}{\Gamma} \exp\left(-\frac{X}{\Gamma}\right) \qquad X \geq 0$$

where

$$\begin{aligned} \Gamma &= \text{average value of the signal-to-noise ratio} \\ &= \frac{E_b}{N_0} \overline{\alpha}^2 \end{aligned}$$

Probability of Error (continued)

- Average error probability of coherent binary PSK and coherent binary FSK in a slow, flat Rayleigh fading channel are given by

$$\begin{cases} P_{e,\text{PSK}} = \frac{1}{2}\left[1 - \sqrt{\frac{\Gamma}{1+\Gamma}}\right] & \text{(coherent binary PSK)} \\ P_{e,\text{FSK}} = \frac{1}{2}\left[1 - \sqrt{\frac{\Gamma}{2+\Gamma}}\right] & \text{(coherent binary FSK)} \end{cases}$$

Probability of Error (continued)

- Average error probability of differential PSK and orthogonal non-coherent FSK in a slow, flat Rayleigh fading channel are given by

$$\begin{cases} P_{e,\text{DPSK}} = \frac{1}{2(1+\Gamma)} & \text{(differential binary PSK)} \\ P_{e,\text{NCFSK}} = \frac{1}{2+\Gamma} & \text{(non-coherent orthogonal binary FSK)} \end{cases}$$

Probability of Error (continued)

- For large values of E_b/N_0 (i.e., large values of X) the error probability equations may be simplified as

$$\begin{cases} P_{e,\text{PSK}} = \frac{1}{4\Gamma} & \text{(coherent binary PSK)} \\ P_{e,\text{FSK}} = \frac{1}{2\Gamma} & \text{(coherent FSK)} \\ P_{e,\text{DPSK}} = \frac{1}{2\Gamma} & \text{(differential PSK)} \\ P_{e,\text{NCFSK}} = \frac{1}{\Gamma} & \text{(non-coherent orthogonal binary FSK)} \end{cases}$$

- **Note:** At higher values of E_b/N_0, P_e is a linear function of $\frac{1}{\Gamma}$

Level Crossing Rate and Fade Duration

- **Level Crossing Rate:**
  - it describes how often the envelope crosses a specified level
- **Average Fade Duration:**
  - it describes how long the envelope remains below a specified level

Envelope Level Crossing Rate

- **Definition:** *The envelope level crossing rate N_R at a specified level R is defined as the rate at which the envelope crosses the level R in the positive (or negative) going direction*
- Let
  - $r(t) =$ received signal
  - $z(t) =$ envelope $= |r(t)|$
  - $p(R, \dot{z}) =$ joint pdf of the signal envelope z and the time derivative of z, $\dot{z}$ at the point where $z = R$

Envelope Level Crossing Rate (continued)

- The expected number of times R occurs for a given slope $\dot{z}$ and time duration dt

$$N_{R,\dot{z}} = p\left(R, \dot{z}\right) dz d\dot{z}$$

- Since in small time dt, the number of times R can occur with slope $\dot{z}$ is either 1 or 0 and $dz = \dot{z}dt$

$$N_{R,\dot{z}} = \dot{z}p\left(R, \dot{z}\right) d\dot{z}dt$$

- The expected number of crossings $N_{R,\dot{z}}(T)$ of the envelope level R with slope $\dot{z}$ over time interval $[0, T]$ is given by

$$N_{R,\dot{z}}(T) = \int_0^T \dot{z}p\left(R, \dot{z}\right) d\dot{z}dt$$

Envelope Level Crossing Rate (continued)

- Now, since the derivative $\dot{z}$ in the positive direction will range from zero to infinity, we obtain the expected number of crossings $N_R(T)$ of the envelope level R over time interval $[0, T]$ in the positive direction (for any derivative) by integrating $N_{R,\dot{z}}(T)$ over all possible derivatives

$$
\begin{aligned}
N_R(T) &= \int_0^T \left[\int_0^\infty \dot{z} p\,(R, \dot{z})\, d\dot{z}\right] dt \\
&= T \int_0^\infty \dot{z} p\,(R, \dot{z})\, d\dot{z}
\end{aligned}
$$

Envelope Level Crossing Rate (continued)

- Thus, the expected number of crossings N_R of the envelope level R per unit time is obtained by dividing $N_R(T)$ by the time interval T

$$\begin{aligned} N_R &= \frac{N_R(T)}{T} \\ &= \int_0^\infty \dot{z} p\left(R, \dot{z}\right) d\dot{z} \end{aligned}$$

Envelope Level Crossing Rate (continued)

- For Ricean fading, the expected number of crossings N_R of the envelope level R per unit time is expressed as

$$N_R = \sqrt{2\pi (K+1)} f_m \rho e^{-K-(K+1)\rho^2} I_0 \left(2\rho\sqrt{K(K+1)} \right)$$

where

* f_m = maximum Doppler frequency
* $K = \frac{A^2}{2\sigma^2}$ = Ricean factor
* $\rho = \frac{R}{R_{\text{rms}}}$ = value of the specified level R, normalized to the local rms amplitude of the fading envelope (i.e., $\sqrt{2\sigma^2}$)

Envelope Level Crossing Rate (continued)

- When the received envelope is Rayleigh distributed, $K = 0$
- Thus, the expected number of crossings N_R of the envelope level R per unit time for Rayleigh fading envelope is given by

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

- **Note:** The level crossing rate is a function of the mobile speed as is apparent from the presence of f_m in the above equation

Maximum Level Crossing Rate

- The maximum level crossing rate occurs when the derivative of N_R with respect to ρ is zero, i.e.,

$$\frac{dN_R}{d\rho} = e^{-\rho^2}\left(1 + 2\rho^2\right) = 0$$

$$\Rightarrow \rho = \frac{1}{\sqrt{2}}$$

- There are few crossings at both high and low levels, with the maximum rate occurring at $\rho = 1/\sqrt{2}$
- **Remark:** The signal envelope experiences very deep fades only occasionally, but shallow fades are frequent

Average Fade Duration

- **Definition:** *The average fade duration is defined as the average period of time for which the received signal is below a specified level R*
- For a Rayleigh fading signal, the average fade duration is given by

$$\overline{\tau} = \frac{1}{N_R} Prob[r \leq R]$$

Average Fade Duration (continued)

- In the previous slide,

$$
\begin{aligned}
Prob[r \leq R] &= \frac{1}{T}\sum_i \tau_i \\
&= \text{average time } z(t) \text{ stays below } R \text{ in one second}
\end{aligned}
$$

where

* τ_i = duration of the fade
* T = observation interval of the fading signal

Average Fade Duration (continued)

- The probability that the received signal r is less than the threshold R is found from the Rayleigh distribution as

$$\begin{aligned} Prob[r \leq R] &= \int_0^R p(r)dr \\ &= \int_0^R \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \\ &= 1 - \exp\left(-\frac{R^2}{2\sigma^2}\right) \\ &= 1 - \exp\left(-\rho^2\right) \end{aligned}$$

Average Fade Duration (continued)

- The average fade duration as a function of ρ and f_m can be expressed as

$$\begin{aligned}\overline{\tau} &= \frac{1}{N_R} Prob[r \leq R] \\ &= \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho}\end{aligned}$$

Remarks

- The average fade duration of a signal fade helps determine the most likely numbers of signaling bits that may be lost during a fade
- Average fade duration primarily depends upon the speed of the mobile, and decreases as the maximum Doppler frequency f_m becomes large assuming that ρ is fixed

$$v \uparrow \rightarrow f_m \uparrow \rightarrow N_R \uparrow \rightarrow \overline{\tau} \downarrow$$

- When the maximum Doppler f_m frequency becomes small, the results will be the other way round