Basic Mathematical Tool

Tuesday, 4 August 2020 10:58 PM

- $\boxed{\text{ALGEBRA}}$

1. Some Basic Identities

(i) $(a+b)^2 = a^2+b^2+2ab$

(ii) $(a-b)^2 = a^2+b^2-2ab$

(iii) $(a+b)(a-b) = a^2-b^2$

(iv) $(a+b)^3 = a^3+b^3+3ab(a+b)$

(v) $(a-b)^3 = a^3+b^3-3ab(a-b)$

(vi) $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

(vii) $a^3+b^3 = (a+b)(a^2+b^2-ab)$

(viii) $a^3-b^3 = (a-b)(a^2+b^2+ab)$

2. Linear Equation

(i) For one variable equation :- $\boxed{ax+b=0}$

(ii) For two variable equation :- $\boxed{ax+by+c=0}$

3. Quadratic Equation

$ax^2+bx+c=0$

has two roots $\alpha = \dfrac{-b+\sqrt{(b^2-4ac)}}{2a}$

$\beta = \dfrac{-b-\sqrt{(b^2-4ac)}}{2a}$

* $\alpha+\beta = -b/a$

* $\alpha \cdot \beta = c/a$

4. Binomial Theorem

* For +ve integer power

→ $$\boxed{(x+a)^n = x^n + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + {}^nC_3 x^{n-3} a^3 + \dots + {}^nC_n a^n}$$

here • $${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

• ${}^nC_0 = 1$; ${}^nC_1 = n$; ${}^nC_n = 1$

FOR any number.

→ $$\boxed{(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + x^n}$$

* If x is very less than 1 then

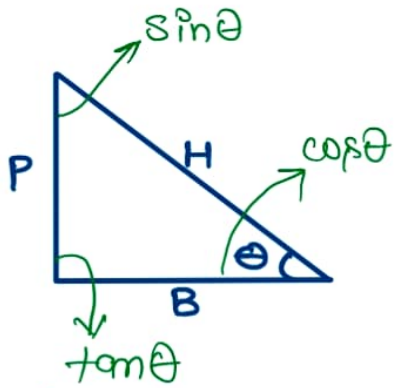
- $(1+x)^n = 1+nx$
- $(1+x)^{-n} = 1-nx$
- $(1-x)^{+n} = 1-nx$
- $(1-x)^{-n} = 1+nx$

• 5. ANGLE

* $\text{Angle} = \dfrac{\text{Arc}}{\text{Radius}}$
* SI unit = Radian
* π Radian = 180°
* 2π Radian = 360°
* $1^\circ = 60'$ (minutes)
* $1' = 60''$ (seconds)

6. TRIGONOMETRY

- $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$
- $\cos\theta = \dfrac{1}{\sec\theta}$
- $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{\sin\theta}{\cos\theta}$



- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

★ 1→ $\sin 2A = 2\sin A\cos A$

2→ $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

3→ $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

{Put $2A \Rightarrow A$

4→ $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$

5→ $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = 2\cos^2\frac{A}{2} - 1$
$= 1 - 2\sin^2\frac{A}{2}$

6→ $\tan A = \dfrac{2\tan A/2}{1-\tan^2 A/2}$

7→ $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

8→ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

{Put A=B get 1,2}

9→ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

10→ $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

11→ $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

12→ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

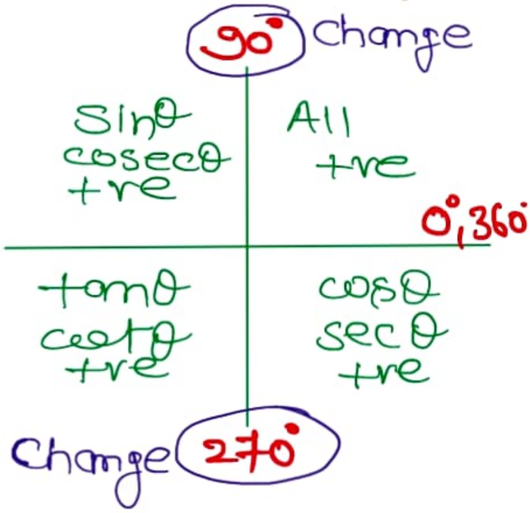
13→ $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

14→ $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

15→ $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

16→ $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

* find different values of Angle



* Change when you 180° 90 and 270

sin $\longleftrightarrow$ cos

tan $\longleftrightarrow$ cot

sec $\longleftrightarrow$ cosec

for eg. $\sin(90+\theta) = +\cos\theta$

$\cos(180-\theta) = -\cos\theta$

$\tan(270-\theta) = +\cot\theta$

$\tan(90-\theta) = +\cot\theta$

Small angle approximation

$\sin\theta \approx \theta$; $\cos\theta \approx 1$; $\tan\theta \approx \theta$

Range of trigometric function

$-1 \leq \sin\theta \leq +1$

$-1 \leq \cos\theta \leq +1$

$-\infty \leq \tan\theta \leq +\infty$

7.ARITHMETIC PROGRESSION

$a, a+d, a+2d, \ldots$

(i) $a_n = a + (n-1)d$

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

or

$= \frac{n}{2}[a + l]$

a = first term

d = common diff

a_n = nth term

S_n = sum of n term

l = last term

8.GEOMETRIC PROGRESSION

$a, ar, ar^2, ar^3, \ldots$

(i) $a_n = ar^{n-1}$

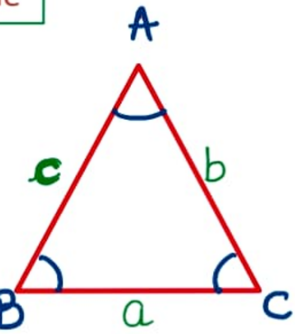
(ii) $S_n = \frac{a(1-r^n)}{1-r}$ If $r < 1$

(iii) $S_n = \frac{a(r^n-1)}{r-1}$ If $r > 1$

(iv) $S_\infty = \frac{a}{1-r}$

9. The sine and cosine formula for a triangle

In ΔABC of side a, b, c and angle $A, B,$ and C then



(i) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

(ii) $a^2 = b^2 + c^2 - 2bc\cos A$

(iii) $b^2 = c^2 + a^2 - 2ca\cos B$

(iv) $c^2 = a^2 + b^2 - 2ab\cos C$

(v) $ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$

10. LOGARITHM

* Logarithm of a number with respect to a given base is the power to which the base must be raised to represent that number.

eg. $1000 = 10^3$ $\therefore \log_{10} 1000 = 3$

In general if $N = a^x$

then $\boxed{\log_a N = x}$

$a^0 = 1$

$\log_a 1 = 0$

$a^1 = a$

$\log_a a = 1$

* $\log_a mn = \log_a m + \log_a n$

* $\log_a \frac{m}{n} = \log_a m - \log_a n$

* $\log_a m^n = n \log_a m$

* $\log_e m = 2.303 \times \log_{10} m$

* Two system of logarithm in common use are (i) Nepearian log or Natural log.

base is e

where $e = 2.718$ approx.

(ii) Common log. base is 10.

11. IMPORTANT GRAPHS AND THEIR EQUATION

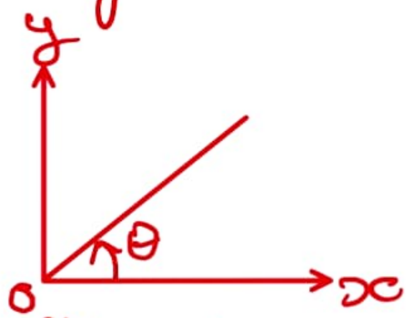
* $\text{Slope} = \dfrac{y\ \text{axis}}{x\ \text{axis}} = m = \tan\theta = \dfrac{\Delta y}{y}$

$= \dfrac{y_2 - y_1}{x_2 - x_1}$

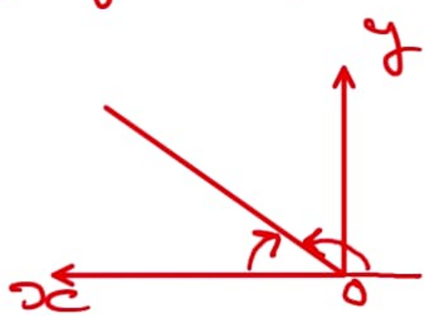
* Area under the curve = y axis $\times$ x axis

(1) Straight line →

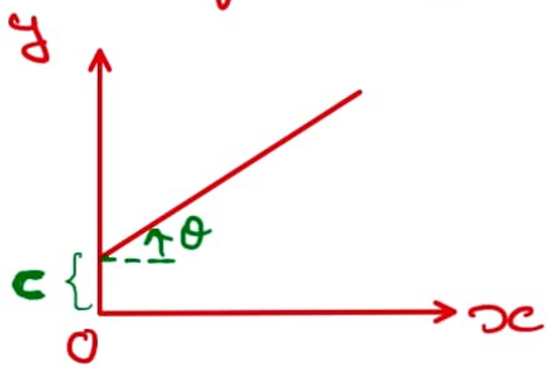
(i) $y = mx$



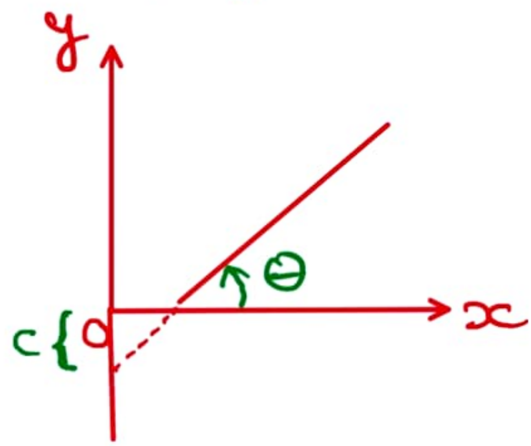
(ii) $y = -mx$



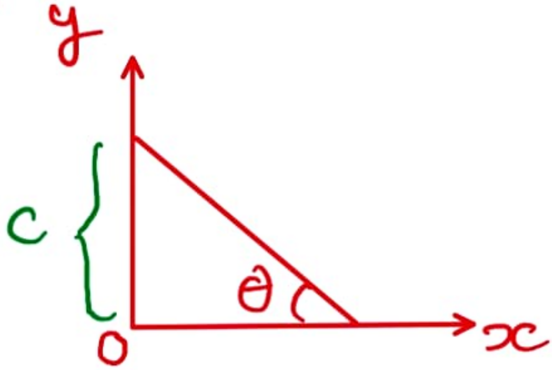
(iii) $y = mx + c$



(iv) $y = mx - c$

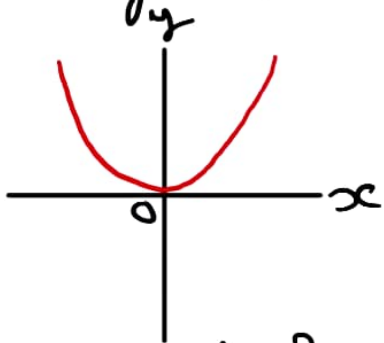


(v) $y = -mx + c$

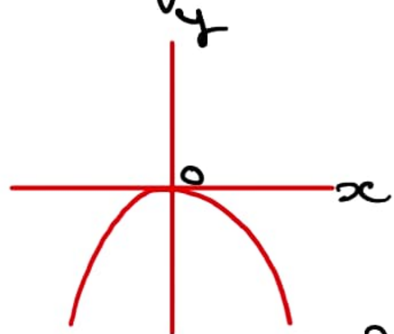


(2) Parabola

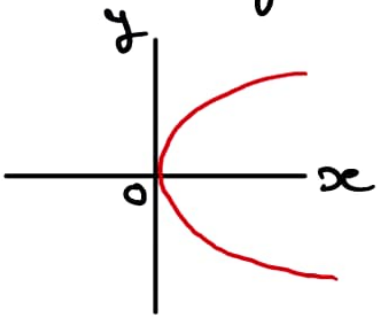
(i) $y = kx^2$



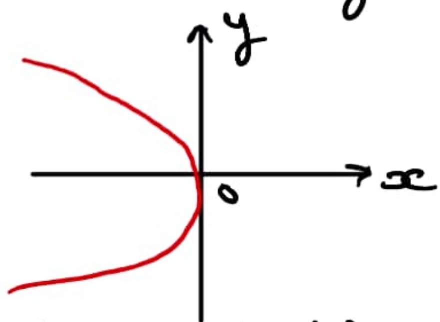
(ii) $y = -kx^2$



(iii) $x = ky^2$

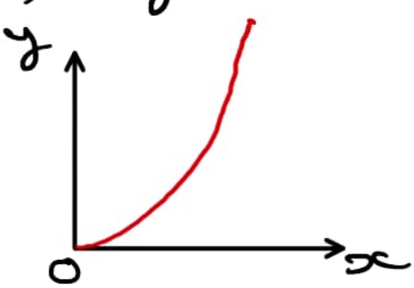


(iv) $x = -ky^2$

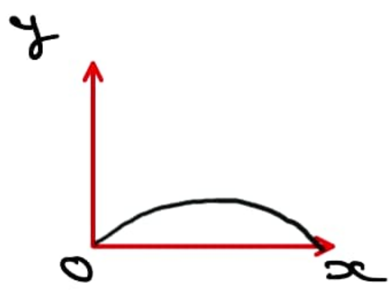


These are symmetric parabolic curve

(v) $y = ax + bx^2$



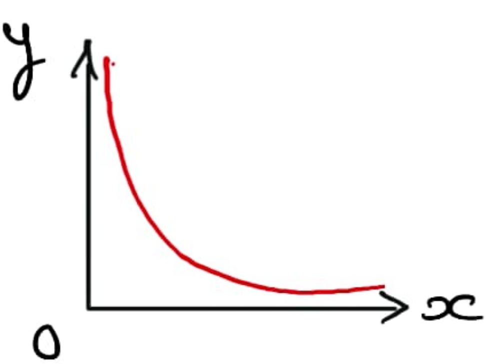
a & b → +ve



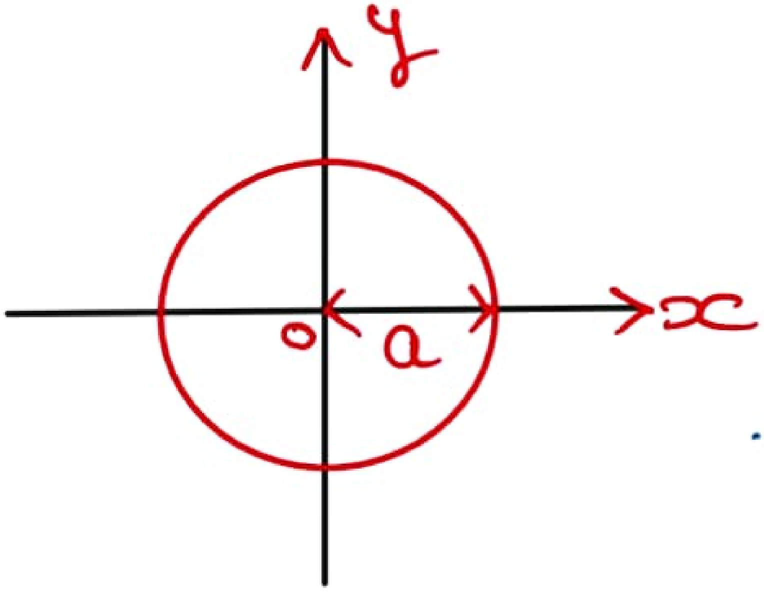
a → +ve b → -ve

Asymmetric parabola

(vi) $xy = c$ or $x \propto \frac{1}{y}$

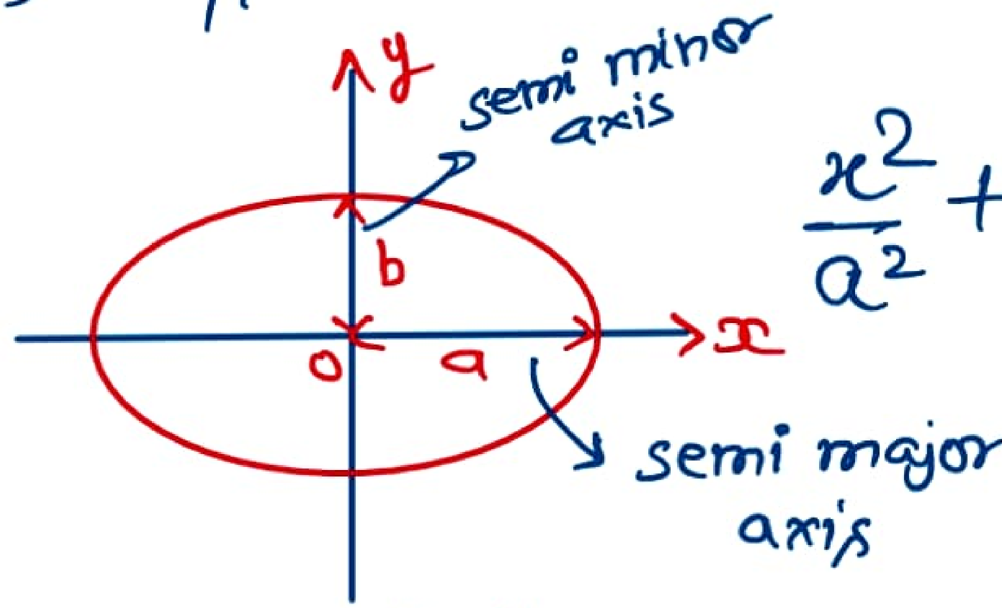


(3) circle



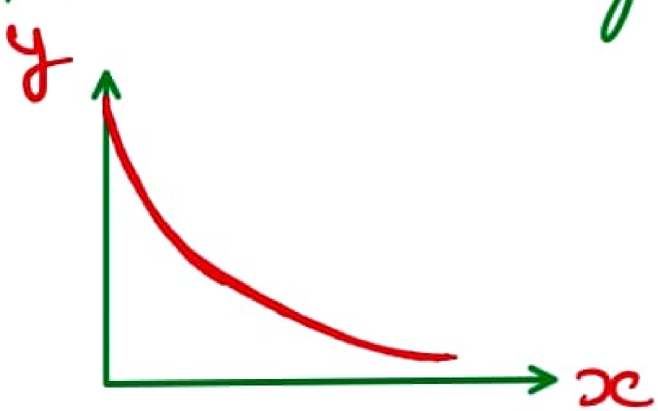
$$x^2+y^2=a^2$$

(4) Ellipse



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

(5) Exponential decay curve



$$y=e^{-kx}$$

12.DIFFERENTIATION

The derivative of y with respect to variable x is defined as the instantaneous rate of change of y w.r.t. x it is denoted by $\frac{dy}{dx}$.

Geographically the differential coefficient of y=f(x),with respect to x at any point is equal to the slope of the tangent to the curve.

Fundamental formulae

(1) $\frac{d(K)}{dx} = 0$ where K=constant

(2) $\frac{d(Ku)}{dx} = K\frac{du}{dx}$ where u=f(x)

(3) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

where $u = f(x)$; $v = g(x)$

(4) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

where $u = f(x)$; $v = g(x)$

(5) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$

(6) $\frac{d}{dx}(x^n) = nx^{n-1}$

(7) $\frac{d}{dx}x = 1$ (8) $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

(9) $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

(10) $\frac{d}{dx}e^x = e^x$

(11) $\frac{d}{dx}\log_e x = \frac{1}{x}$

(12) $\frac{d}{dx}(\sin x) = \cos x$ (13) $\frac{d}{dx}(\cos x) = -\sin x$

(14) $\frac{d}{dx}(\tan x) = \sec^2 x$ (15) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(16) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(17) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(18) $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

(19) $\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$

(20) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

(21) $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$

(22) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

(23) $\frac{d}{dx}(\mathrm{cosec}^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$

no need to learn now.

Condition for maxima or minima

If $y = f(x)$ then for maximum or minimum value of y for value of x

(i) $\boxed{\frac{dy}{dx} = 0}$ find value of x from here (ii)

(iii) Now find $\frac{d^2y}{dx^2}$

(iv) and put value of x in this

(v) If $\frac{d^2y}{dx^2} < 0$ then y is maximum

(vi) If $\frac{d^2y}{dx^2} > 0$ then y is minimum

13.INTEGRATION

The process of integration is just the reverse of differentiation.

symbol $\int$

$$\boxed{y = \int f(x)dx = F(x)}$$

(i) Definite integrals
(ii) Indefinite integrals

Fundamental formulae

(1) $\int x^n dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$

(2) $\int dx = x$

(3) $\int (u+v)dx = \int u dx + \int v dx$

(4) $\int k u dx = k \int u dx$

(5) $\int \frac{1}{x} dx = \log_e x + c$

(6) $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$

(7) $\int e^x dx = e^x + c$

(8) $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$

(9) $\int \cos x dx = \sin x + c$

(10) $\int \sin x dx = -\cos x + c$

(11) $\int \sec^2 x dx = \tan x + c$

(12) $\int \operatorname{cosec}^2 x dx = -\cot x + c$

(13) $\int \sec x \tan x dx = \sec x + c$

OVER