Value Function Approximation

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Outline

Table of Contents

Large-Scale Reinforcement Learning

Reinforcement learning can be used to solve large problems, e.g.

- Backgammon: 10^{20} states
- Computer Go: 10^{170} states
- Helicopter: continuous state space

How can we scale up the model-free methods for prediction and control from the last two lectures?

Value Function Approximation

- So far we have represented value function by a *lookup* table
  - Every state s has an entry $V(s)$
  - Or every state-action pair s, a has an entry $Q(s, a)$
- Problem with large MDPs:
  - There are too many states and/or actions to store in memory
  - It is too slow to learn the value of each state individually
- Solution for large MDPs:
  - Estimate value function with *function approximation*

$$
\begin{aligned}
\hat{v}(s, \mathbf{w}) &\approx v_\pi(s) \\
\text{or } \hat{q}(s, a, \mathbf{w}) &\approx q_\pi(s, a)
\end{aligned}
$$

  - *Generalize* from seen states to unseen states
  - *Update* parameter $\mathbf{w}$ using MC or TD learning

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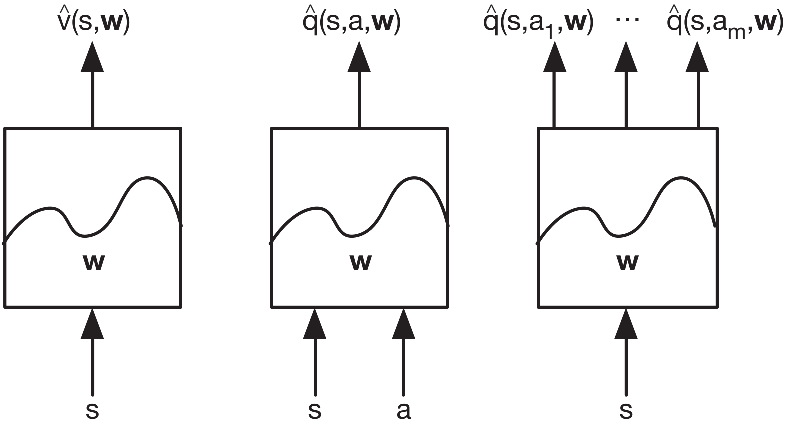
Types of Value Function Approximation



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Which Function Approximator?

There are many function approximators, e.g.

- Linear combinations of features
- Neural network
- Decision tree
- Nearest neighbor
- Fourier/wavelet bases
- $\cdots$

Which Function Approximator?

We consider differentiable function approximators, e.g.

- Linear combinations of features
- Neural network
- Decision tree
- Nearest neighbor
- Fourier / wavelet bases
- $\cdots$

Furthermore, we require a training method that is suitable for non-stationary, non-iid data

Table of Contents

Gradient Descent

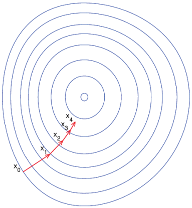
- Let $J(\mathbf{w})$ be a differentiable function of parameter vector $\mathbf{w}$
- Define the *gradient* of $J(\mathbf{w})$ to be

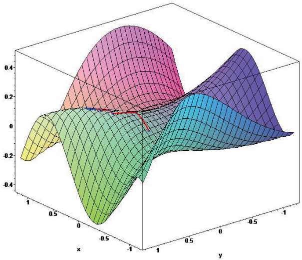
$$\nabla_{\mathbf{w}} J(\mathbf{w}) = \begin{pmatrix} \frac{\partial J(\mathbf{w})}{\partial \mathbf{w}_1} \\ \vdots \\ \frac{\partial J(\mathbf{w})}{\partial \mathbf{w}_n} \end{pmatrix}$$

- To find a local minimum of $J(\mathbf{w})$
- Adjust $\mathbf{w}$ in direction of -ve gradient

$$\Delta \mathbf{w} = -\frac{1}{2}\alpha \nabla_{\mathbf{w}} J(\mathbf{w})$$

where α is a step-size parameter





Value Function Approx. By Stochastic Gradient Descent

- Goal: find parameter vector $\mathbf{w}$ minimizing mean-squared error between approximate value function $\hat{v}(s, \mathbf{w})$ and true value function $v_\pi(s)$

$$J(\mathbf{w}) = \mathbb{E}_\pi[(v_\pi(S) - \hat{v}(S, \mathbf{w}))^2]$$

- Gradient descent finds a local minimum

$$\begin{aligned} \Delta \mathbf{w} &= -\frac{1}{2}\alpha \nabla_{\mathbf{w}} J(\mathbf{w}) \\ &= \alpha \mathbb{E}_\pi[(v_\pi(S) - \hat{v}(S, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})] \end{aligned}$$

- Stochastic gradient descent *sample* the gradient

$$\Delta \mathbf{w} = \alpha(v_\pi(S) - \hat{v}(S, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$$

- Expected update is equal to full gradient update

Feature Vectors

- Represent state by a *feature vector*

$$\mathbf{x}(S) = \begin{pmatrix} \mathbf{x}_1(S) \\ \vdots \\ \mathbf{x}_n(S) \end{pmatrix}$$

- For example:
  - Distance of robot from landmarks
  - Trends in the stock market
  - Piece and pawn configurations in chess

Linear Value Function Approximation

- Represent value function by a linear combination of features

$$\hat{v}(S, \mathbf{w}) = \mathbf{x}(S)^T \mathbf{w} = \sum_{j=1}^{n} \mathbf{x}_j(S) \mathbf{w}_j$$

- Objective function is quadratic in parameters $\mathbf{w}$

$$J(\mathbf{w}) = \mathbb{E}_\pi[(v_\pi(S) - \mathbf{x}(S)^T \mathbf{w})^2]$$

- Stochastic gradient descent converges on *global* optimum
- Update rule is particularly simple

$$\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) = \mathbf{x}(S)$$
$$\Delta \mathbf{w} = \alpha(v_\pi(S) - \hat{v}(S, \mathbf{w}))\mathbf{x}(S)$$

Update = *step-size* × *prediction error* × *feature value*

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Table Lookup Features

- Table lookup is a special case of linear value function approximation
- Using *table lookup features*

$$\mathbf{x}^{table}(S) = \begin{pmatrix} \mathbf{1}(S = s_1) \\ \vdots \\ \mathbf{1}(S = s_n) \end{pmatrix}$$

- Parameter vector $\mathbf{w}$ gives value of each individual state

$$\hat{v}(S, \mathbf{w}) = \begin{pmatrix} \mathbf{1}(S = s_1) \\ \vdots \\ \mathbf{1}(S = s_n) \end{pmatrix} \cdot \begin{pmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_n \end{pmatrix}$$

Incremental Prediction Algorithms

$$\Delta \mathbf{w} = \alpha(v_\pi(S) - \hat{v}(S, \mathbf{w}))\nabla_\mathbf{w} \hat{v}(S, \mathbf{w})$$

- Have assumed true value function $v_\pi(s)$ given by supervisor
- But in RL there is no supervisor, only rewards
- In practice, we substitute a target for $v_\pi(s)$
  - For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha(G_t - \hat{v}(S_t, \mathbf{w}))\nabla_\mathbf{w} \hat{v}(S, \mathbf{w})$$

  - For TD(0), the target is the TD target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha(R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}))\nabla_\mathbf{w} \hat{v}(S, \mathbf{w})$$

  - For TD(λ), the target is the λ-return G_t^λ

$$\Delta \mathbf{w} = \alpha(G_t^\lambda - \hat{v}(S_t, \mathbf{w}))\nabla_\mathbf{w} \hat{v}(S, \mathbf{w})$$

Monte-Carlo with Value Function Approximation

- Return G_t is an unbiased, noisy sample of true value $v_\pi(S_t)$
- Can therefore apply supervised learning to "training data":

$$\langle S_1, G_1 \rangle, \langle S_2, G_2 \rangle, \cdots, \langle S_T, G_T \rangle$$

- For example, using *linear Monte-Carlo policy evaluation*

$$\begin{aligned} \Delta \mathbf{w} &= \alpha({\color{red}G_t} - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \\ &= \alpha(G_t - \hat{v}(S_t, \mathbf{w}))\mathbf{x}(S_t) \end{aligned}$$

- Monte-Carlo evaluation converges to a local optimum
- Even when using non-linear value function approximation

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Monte-Carlo Prediction with Value Function Approximation

Gradient Monte Carlo Algorithm for Estimating $\hat{v} \approx v_\pi$

Input: the policy π to be evaluated
Input: a differentiable function $\hat{v} : \mathcal{S} \times \mathbb{R}^d \to \mathbb{R}$
Algorithm parameter: step size $\alpha > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, S_1, A_1, \ldots, R_T, S_T$ using π
  Loop for each step of episode, $t = 0, 1, \ldots, T-1$:
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[G_t - \hat{v}(S_t,\mathbf{w})\big]\nabla\hat{v}(S_t,\mathbf{w})$

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TD Learning with Value Function Approximation

- The TD-target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$ is a *biased* sample of true value $v_\pi(S_t)$
- Can still apply supervised learning to "training data":

$$\langle S_1, R_2 + \gamma \hat{v}(S_2, \mathbf{w}) \rangle, \langle S_2, R_3 + \gamma \hat{v}(S_3, \mathbf{w}) \rangle, \cdots, \langle S_{T-1}, R_T \rangle$$

- For example, using *linear* TD(0)

$$\begin{aligned} \Delta \mathbf{w} &= \alpha ({\color{red} R + \gamma \hat{v}(S', \mathbf{w})} - \hat{v}(S_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \\ &= \alpha \delta \mathbf{x}(S) \end{aligned}$$

- Linear TD(0) converges (close) to global optimum

TD(0) Prediction with Value Function Approximation

Semi-gradient TD(0) for estimating $\hat{v} \approx v_\pi$

Input: the policy π to be evaluated
Input: a differentiable function $\hat{v} : \mathcal{S}^+ \times \mathbb{R}^d \rightarrow \mathbb{R}$ such that $\hat{v}(\text{terminal},\cdot) = 0$
Algorithm parameter: step size $\alpha > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose $A \sim \pi(\cdot|S)$
    Take action A, observe R, S'
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R + \gamma\hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})\big]\nabla\hat{v}(S,\mathbf{w})$
    $S \leftarrow S'$
  until S is terminal

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TD(λ) with Value Function Approximation

- The λ-return G_t^λ is also a biased sample of true value $v_\pi(s)$
- Can again apply supervised learning to "training data":

$$\langle S_1, G_1^\lambda \rangle, \langle S_2, G_2^\lambda \rangle, \cdots, \langle S_{T-1}, G_{T-1}^\lambda \rangle$$

- Forward view linear TD(λ)

$$\begin{aligned} \Delta \mathbf{w} &= \alpha({\color{red}G_t^\lambda} - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \\ &= \alpha(G_t^\lambda - \hat{v}(S_t, \mathbf{w}))\mathbf{x}(S_t) \end{aligned}$$

- Backward view linear TD(λ)

$$\begin{aligned} \delta_t &= R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}) \\ E_t &= \gamma\lambda E_{t-1} + \mathbf{x}(S_t) \\ \Delta \mathbf{w} &= \alpha \delta_t E_t \end{aligned}$$

TD(λ) with Value Function Approximation

- The λ-return G_t^λ is also a biased sample of true value $v_\pi(s)$
- Can again apply supervised learning to "training data":

$$\langle S_1, G_1^\lambda \rangle, \langle S_2, G_2^\lambda \rangle, \cdots, \langle S_{T-1}, G_{T-1}^\lambda \rangle$$

- Forward view linear TD(λ)

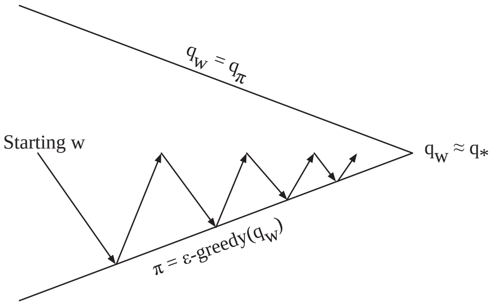
$$\begin{aligned} \Delta \mathbf{w} &= \alpha(G_t^\lambda - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{v}(S, \mathbf{w}) \\ &= \alpha(G_t^\lambda - \hat{v}(S_t, \mathbf{w}))\mathbf{x}(S_t) \end{aligned}$$

- Backward view linear TD(λ)

$$\begin{aligned} \delta_t &= R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}) \\ E_t &= \gamma\lambda E_{t-1} + \mathbf{x}(S_t) \\ \Delta \mathbf{w} &= \alpha \delta_t E_t \end{aligned}$$

Forward view and backward view linear TD(λ) are equivalent

Control with Value Function Approximation



Policy evaluation Approximation policy evaluation, $\hat{q}(\cdot, \cdot, \mathbf{w}) \approx q_\pi$
Policy improvement ϵ-greedy policy improvement

Action-Value Function Approximation

- Approximate the action-value function

$$\hat{q}(S, A, \mathbf{w}) \approx q_\pi(S, A)$$

- Minimize mean-squared error between approximate action-value function $\hat{q}(S, A, \mathbf{w})$ and true action-value function $q_\pi(S, A)$

$$J(\mathbf{w}) = \mathbb{E}_\pi[(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2]$$

- Use stochastic gradient descent to find a local minimum

$$-\frac{1}{2}\alpha\nabla_{\mathbf{w}} J(\mathbf{w}) = (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S, A, \mathbf{w})$$
$$\Delta\mathbf{w} = \alpha(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S, A, \mathbf{w})$$

Linear Action-Value Function Approximation

- Represent state and action by a feature vector

$$\mathbf{x}(S, A) = \begin{pmatrix} \mathbf{x}_1(S, A) \\ \vdots \\ \mathbf{x}_n(S, A) \end{pmatrix}$$

- Represent action-value function by linear combination of features

$$\hat{q}(S, A, \mathbf{w}) = \mathbf{x}(S, A)^T \mathbf{w} = \sum_{j=1}^{n} \mathbf{x}_j(S, A) \mathbf{w}_j$$

- Stochastic gradient descent update

$$\nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w}) = \mathbf{x}(S, A)$$
$$\Delta \mathbf{w} = \alpha(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\mathbf{x}(S, A)$$

Incremental Control Algorithms

- Like prediction, we must substitute a *target* for $q_\pi(S, A)$
  - For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha(G_t - \hat{q}(S, A, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S, A, \mathbf{w})$$

  - For TD(0), the target is the TD target $R_{t+1} + \gamma \hat{Q}(S_{t+1}, A_{t+1})$

$$\Delta \mathbf{w} = \alpha(R_{t+1} + \gamma \hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

  - For forward-view TD(λ), the target is the λ-return q_t^λ

$$\Delta \mathbf{w} = \alpha(q_t^\lambda - \hat{q}(S_t, A_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

  - For backward-view TD(λ), equivalent update is

$$\begin{aligned}
\delta_t &= R_{t+1} + \gamma \hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w}) \\
E_t &= \gamma \lambda E_{t-1} + \nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w}) \\
\Delta \mathbf{w} &= \alpha \delta_t E_t
\end{aligned}$$

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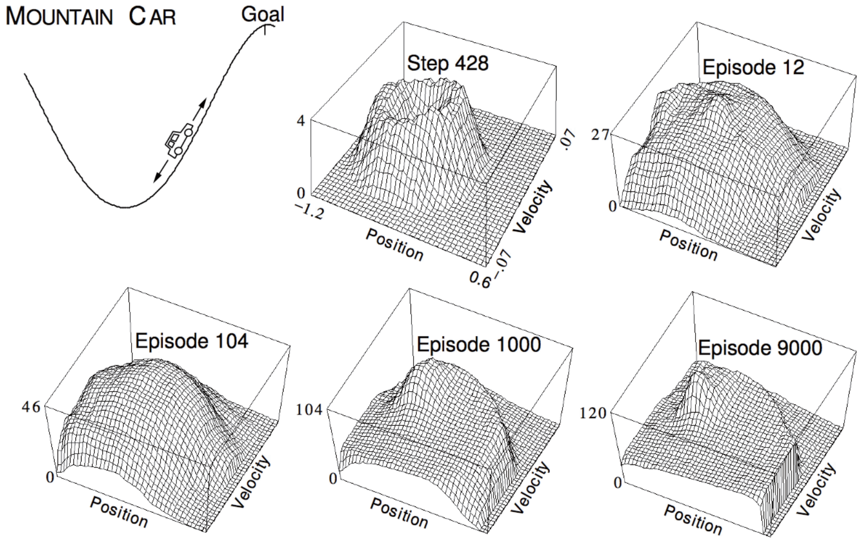
On-Policy Control with Value Function Approximation: Sarsa

Episodic Semi-gradient Sarsa for Estimating $\hat{q} \approx q_*$

Input: a differentiable action-value function parameterization $\hat{q} : \mathcal{S} \times \mathcal{A} \times \mathbb{R}^d \to \mathbb{R}$
Algorithm parameters: step size $\alpha > 0$, small $\varepsilon > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop for each episode:
  $S, A \leftarrow$ initial state and action of episode (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    If S' is terminal:
      $\mathbf{w} \leftarrow \mathbf{w} + \alpha \big[R - \hat{q}(S, A, \mathbf{w})\big] \nabla \hat{q}(S, A, \mathbf{w})$
      Go to next episode
    Choose A' as a function of $\hat{q}(S', \cdot, \mathbf{w})$ (e.g., ε-greedy)
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha \big[R + \gamma \hat{q}(S', A', \mathbf{w}) - \hat{q}(S, A, \mathbf{w})\big] \nabla \hat{q}(S, A, \mathbf{w})$
    $S \leftarrow S'$
    $A \leftarrow A'$

Linear Sarsa with Coarse Coding in Mountain Car



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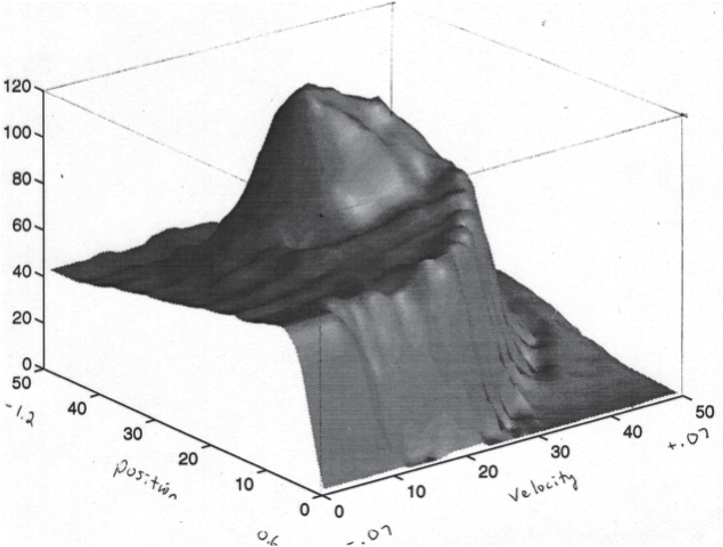
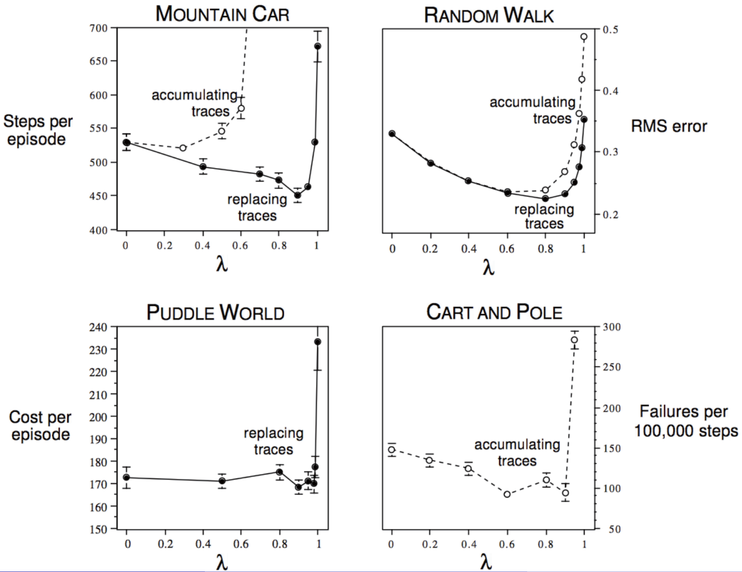
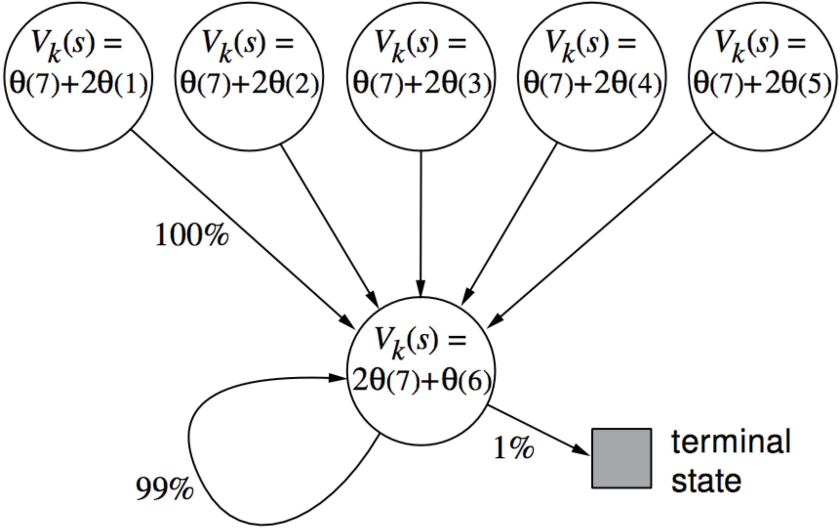
Linear Sarsa with Radial Basis Functions in Mountain Car



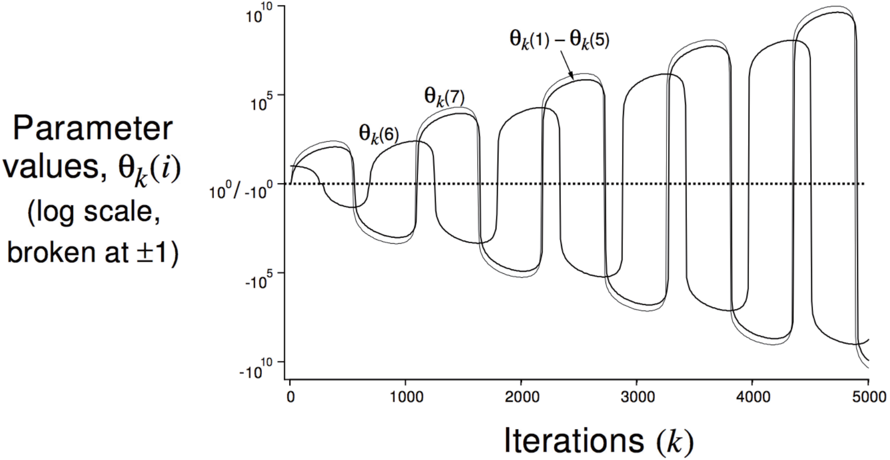
Study of λ: Should We Bootstrap?



Baird's Counterexample



Parameter Divergence in Baird's Counterexample



Convergence of Prediction Algorithms

On/Off-Policy	Algorithm	Table Lookup	Linear	Non-Linear
On-Policy	MC	✓	✓	✓
	TD(0)	✓	✓	✗
	TD(λ)	✓	✓	✗
Off-Policy	MC	✓	✓	✓
	TD(0)	✓	✗	✗
	TD(λ)	✓	✗	✗

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Gradient Temporal-Difference Learning

- TD does not follow the gradient of any objective function
- This is why TD can diverge when off-policy or using non-linear function approximation
- Gradient TD follows true gradient of projected Bellman error

On/Off-Policy	Algorithm	Table Lookup	Linear	Non-Linear
On-Policy	MC	✓	✓	✓
	TD	✓	✓	✗
	Gradient TD	✓	✓	✓
Off-Policy	MC	✓	✓	✓
	TD	✓	✗	✗
	Gradient TD	✓	✓	✓

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Convergence of Control Algorithms

Algorithm	Table Lookup	Linear	Non-Linear
Monte-Carlo Control	✓	(✓)	✗
Sarsa	✓	(✓)	✗
Q-learning	✓	✗	✗
Gradient Q-learning	✓	✓	✗

(✓) = chatters around near-optimal value function

Table of Contents

Batch Reinforcement Learning

- Gradient descent is simple and appealing
- But it is not sample efficient
- Batch methods seek to find the best fitting value function
- Given the agent's experience ("training data")

Least Squares Prediction

- Given value function approximation $\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$
- And *experience* $\mathcal{D}$ consisting of $\langle$ state, value $\rangle$ pairs

$$\mathcal{D} = \{\langle s_1, v_1^\pi \rangle, \langle s_2, v_2^\pi \rangle, \cdots, \langle s_T, v_T^\pi \rangle\}$$

- Which parameters $\mathbf{w}$ give the *best fitting* value function $\hat{v}(s, \mathbf{w})$?
- Least squares algorithms find parameter vector $\mathbf{w}$ minimizing sum-squared error between $\hat{v}(s, \mathbf{w})$ and target values v_t^π

$$\begin{aligned} LS(\mathbf{w}) &= \sum_{t=1}^{T} (v_t^\pi - \hat{v}(s_t, \mathbf{w}))^2 \\ &= \mathbb{E}_{\mathcal{D}}\left[(v_t^\pi - \hat{v}(s_t, \mathbf{w}))^2\right] \end{aligned}$$

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Deep Q-Networks (DQN): Q-learning Framework

DQN based on Q-learning framework, uses two tips: fixed target network and experience replay

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
    $S \leftarrow S'$;
  until S is terminal

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Deep Q-Networks (DQN): Fixed Target Network

Target Network

$$\cdots s_t, a_t, r_t, s_{t+1} \cdots$$

$$\boxed{\begin{aligned} &\mathrm{Q}^{\pi}(s_t, a_t) \\ &\quad = r_t + \mathrm{Q}^{\pi}\big(s_{t+1}, \pi(s_{t+1})\big) \end{aligned}}$$

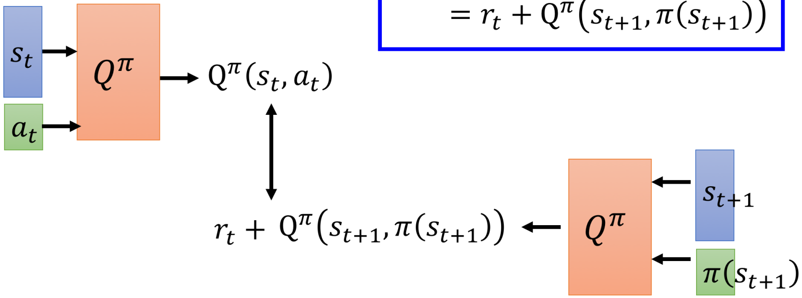
twork

Deep Q-Networks (DQN): Fixed Target Network

Target Network

$$\cdots s_t, a_t, r_t, s_{t+1} \cdots$$

$$Q^\pi(s_t, a_t) = r_t + Q^\pi\big(s_{t+1}, \pi(s_{t+1})\big)$$



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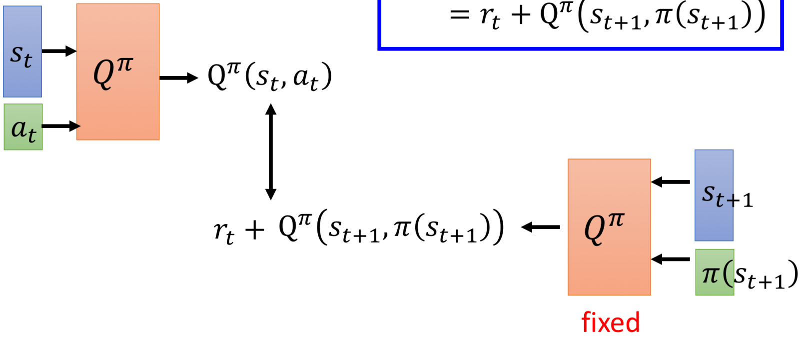
Deep Q-Networks (DQN): Fixed Target Network

Target Network

$$\cdots s_t, a_t, r_t, s_{t+1} \cdots$$

$$Q^\pi(s_t, a_t) = r_t + Q^\pi\big(s_{t+1}, \pi(s_{t+1})\big)$$



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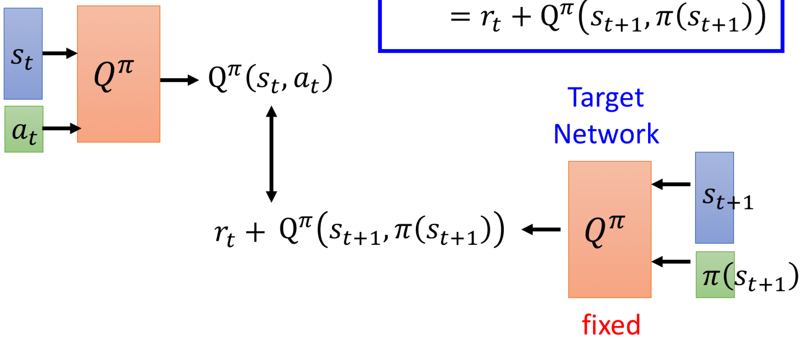
Deep Q-Networks (DQN): Fixed Target Network

Target Network

$$\cdots s_t, a_t, r_t, s_{t+1} \cdots$$

$$Q^\pi(s_t, a_t) = r_t + Q^\pi(s_{t+1}, \pi(s_{t+1}))$$



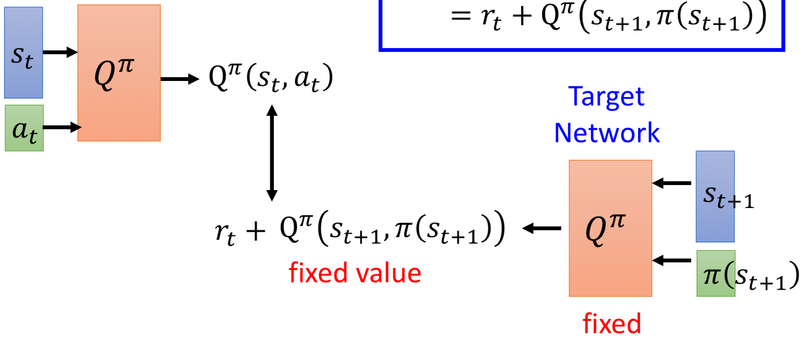
twork

Deep Q-Networks (DQN): Fixed Target Network

Target Network

$$\cdots s_t, a_t, r_t, s_{t+1} \cdots$$

$$\begin{aligned} Q^{\pi}(s_t, a_t) \\ &= r_t + Q^{\pi}\big(s_{t+1}, \pi(s_{t+1})\big) \end{aligned}$$



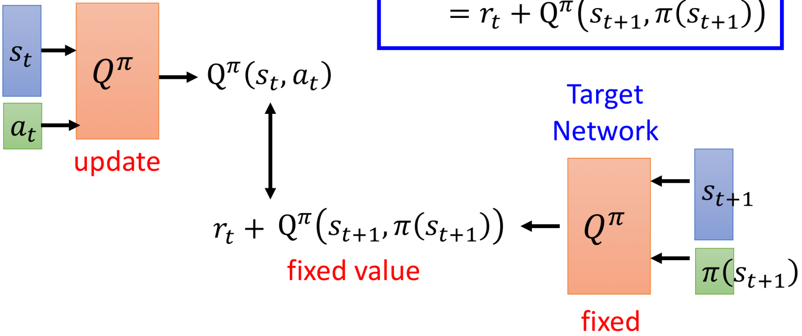
twork

Deep Q-Networks (DQN): Fixed Target Network

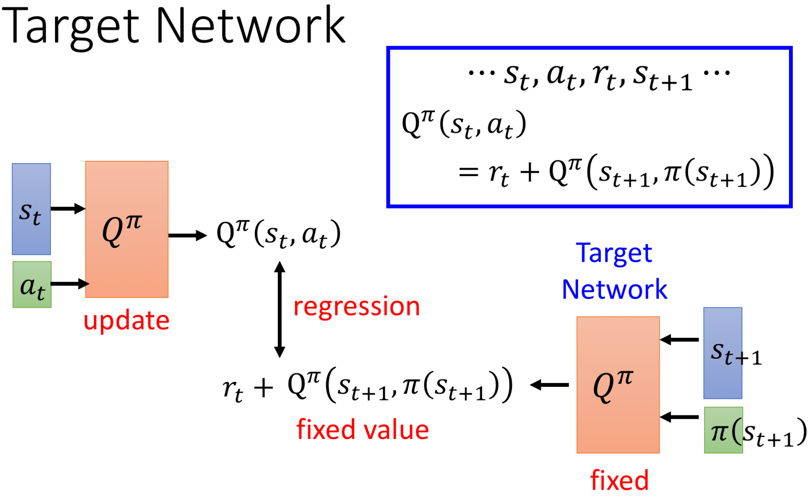
Target Network

$$\cdots s_t, a_t, r_t, s_{t+1} \cdots$$

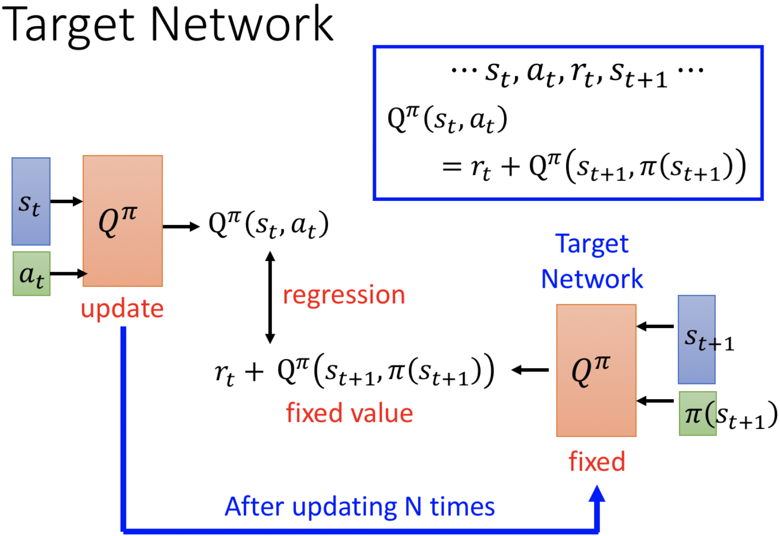
$$Q^{\pi}(s_t, a_t) \\ = r_t + Q^{\pi}\big(s_{t+1}, \pi(s_{t+1})\big)$$



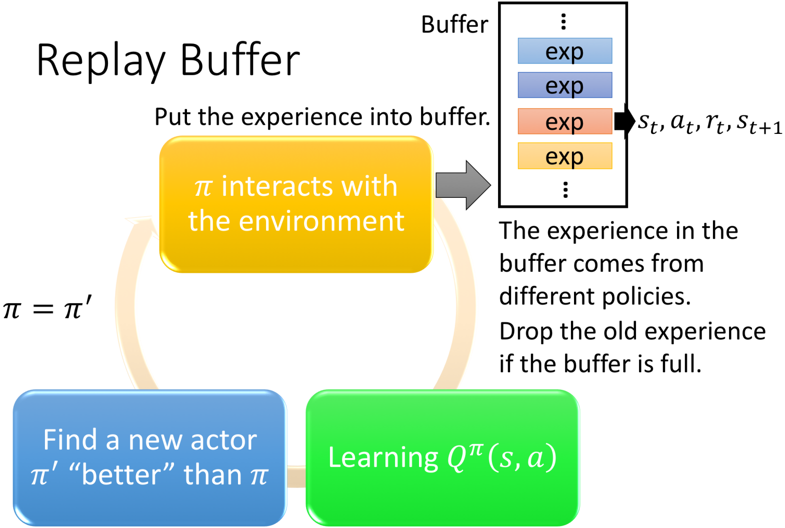
Deep Q-Networks (DQN): Fixed Target Network



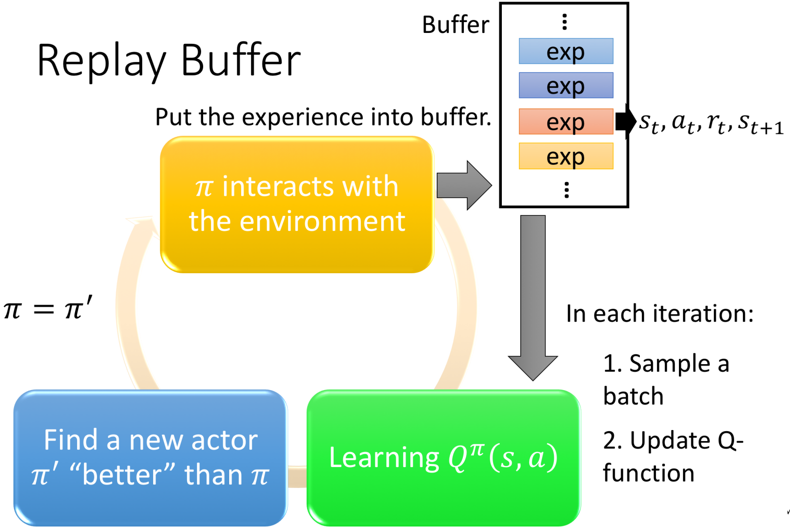
Deep Q-Networks (DQN): Fixed Target Network



Deep Q-Networks (DQN): Experience Replay



Deep Q-Networks (DQN): Experience Replay



DQN Algorithm

Typical Q-Learning Algorithm

- Initialize Q-function Q, target Q-function $\hat{Q} = Q$
- In each episode
  - For each time step t
    - Given state s_t, take action a_t based on Q (epsilon greedy)
    - Obtain reward r_t, and reach new state s_{t+1}
    - Store (s_t, a_t, r_t, s_{t+1}) into buffer
    - Sample (s_i, a_i, r_i, s_{i+1}) from buffer (usually a batch)
    - Target $y = r_i + \max_a \hat{Q}(s_{i+1}, a)$
    - Update the parameters of Q to make $Q(s_i, a_i)$ close to y (regression)
    - Every C steps reset $\hat{Q} = Q$

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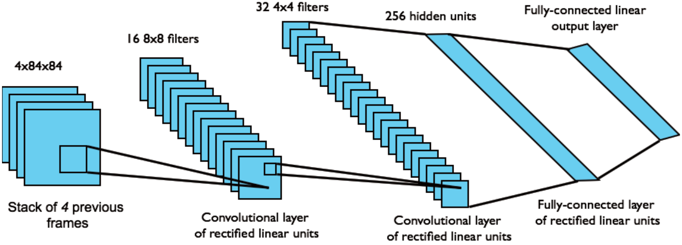
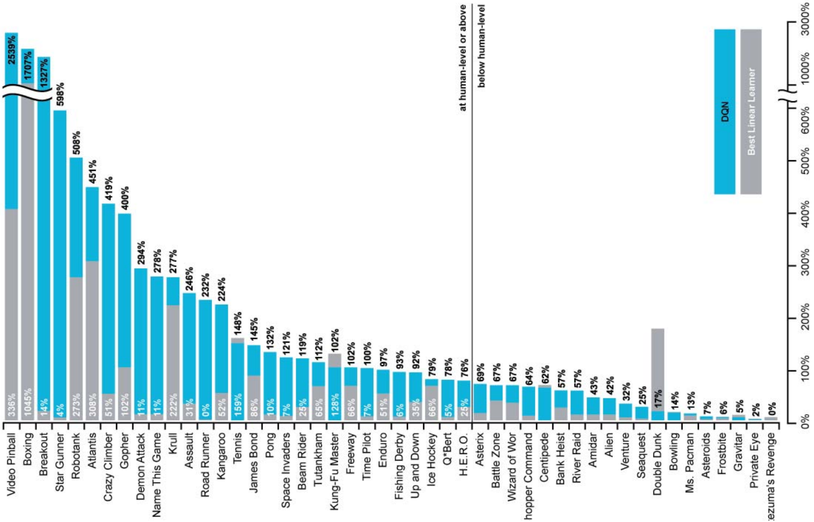
DQN in Atari

- End-to-end learning of values $Q(s, a)$ from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is $Q(s, a)$ for 18 joystick / button positions
- Reward is change in score for that step



Network architecture and hyperparameters fixed across all games

DQN Results in Atari



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How much does DQN help?

	Replay Fixed-Q	Replay Q-learning	No replay Fixed-Q	No replay Q-learning
Breakout	316.81	240.73	10.16	3.17
Enduro	1006.3	831.25	141.89	29.1
River Raid	7446.62	4102.81	2867.66	1453.02
Seaquest	2894.4	822.55	1003	275.81
Space Invaders	1088.94	826.33	373.22	301.99