STAT4710 Data Science and Analytics using Python

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Question 1

a

$$B = \begin{bmatrix} 2 & 2 & 2 \\ 5 & 8 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 10 \end{bmatrix}$$

b

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

c

$$AB\vec{v_2} = \vec{x}$$

$$\begin{bmatrix} 9 & 12 & 4 \\ 7 & 12 & 15 \\ 0 & 0 & 100 \end{bmatrix} \cdot \vec{v_2} = \begin{bmatrix} 80 \\ 80 \\ 100 \end{bmatrix}$$

Solving for $\vec{v_2}$ using matrix row reduction

$$\left.\begin{array}{ccc|c} 9 & 12 & 4 & 80 \\ 7 & 12 & 15 & 80 \\ 0 & 0 & 100 & 100 \end{array}\right. \sim \left.\begin{array}{ccc|c} 6 & 9 & 1 & 40 \\ 7 & 12 & 15 & 80 \\ 0 & 0 & 1 & 1 \end{array}\right. \sim \left.\begin{array}{ccc|c} 6 & 9 & 1 & 40 \\ 7 & 12 & 15 & 80 \\ 0 & 0 & 1 & 1 \end{array}\right. \sim$$

$$\left.\begin{array}{ccc|c} 9 & 0 & 0 & 99/2 \\ 7 & 12 & 15 & 53 \\ 0 & 0 & 1 & 1 \end{array}\right. \sim \left.\begin{array}{ccc|c} 1 & 0 & 0 & 11/2 \\ 0 & 1 & 0 & 53/24 \\ 0 & 0 & 1 & 1 \end{array}\right.$$

$$\vec{v_2} = \begin{bmatrix} 11/2 \\ 53/24 \\ 1 \end{bmatrix}$$

Problem 2

a

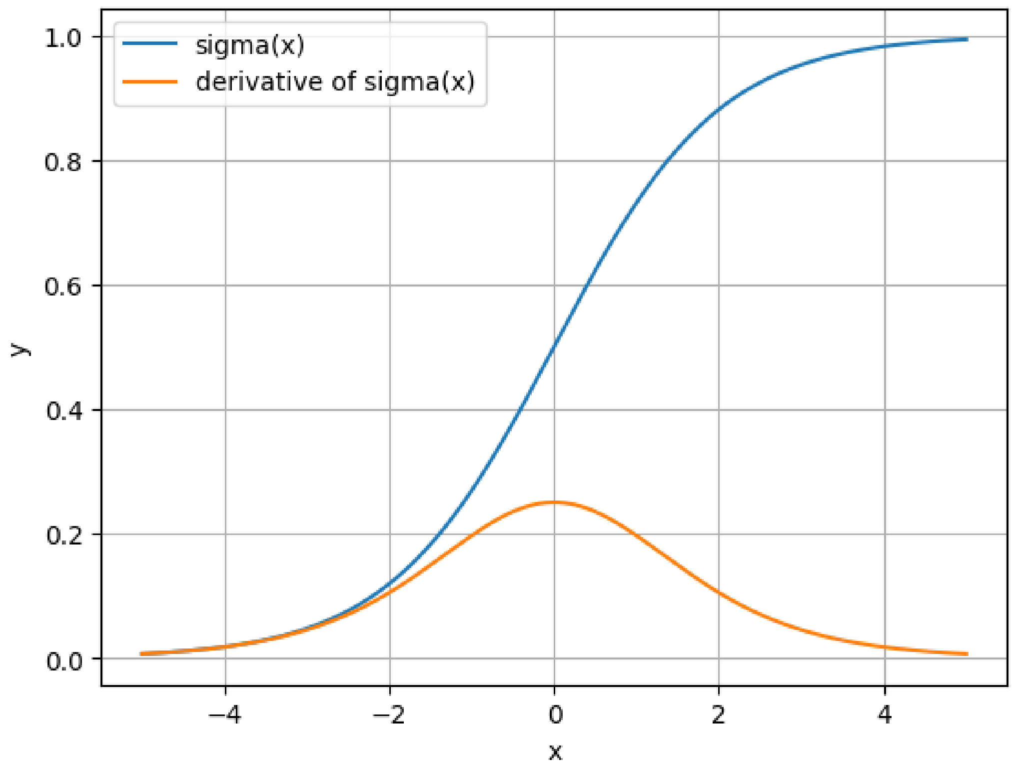
$$\begin{aligned}
\sigma(-x) &= \frac{1}{1+e^x} \\
&= \frac{1}{e^{x-x}+e^x} \\
&= \frac{1}{e^x(1+e^{-x})} \\
&= \frac{e^x}{1+e^{-x}} \\
&= \frac{e^x-1+1}{1+e^{-x}} \\
&= 1-\sigma(x)
\end{aligned}$$

b

$$\begin{aligned}
\frac{d}{dx}\sigma(x) &= (1+e^{-x})^{-1} \\
&= (1+e^{-x})^{-2}(e^{-x})(-1) \\
&= \frac{e^{-x}}{(1+e^{-x})^2} \\
&= (1+e^{-x})^{-1}\frac{e^{-x}}{(1+e^{-x})} \\
&= \sigma(x)(1-\sigma(x))
\end{aligned}$$

c

where the red line represents the $\sigma(x)$, and green line $\frac{d}{dx}\sigma(x)$



Problem 3

$$\begin{aligned} f(c) &= \frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2 \\ &= \frac{1}{n}\sum_{i=1}^{n}(x_i^2 - 2x_i c + c^2) \\ &= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{2c}{n}\sum_{i=1}^{n} x_i + c^2 \end{aligned}$$

To find c which minimises f(c),

$$\begin{aligned} \frac{d}{dc}f(c) &= -\frac{2}{n}\sum_{i=1}^{n} x_i + 2c \\ c &= \frac{1}{n}\sum_{i=1}^{n}(x_i) \end{aligned}$$

To prove that it is the mininum, we check
the 2nd derivative

$$\frac{d2}{dc2}f(c) = 2$$

Since the 2nd derivative is positive, then the turning point computed must be a minimum point

Question 4

Given the following,

$$\begin{aligned} P(C) &= 0.01 \\ P(Pos|C) &= 0.8 \\ P(Pos|'C) &= 0.096 \end{aligned}$$

Solve for $P(C|Pos)$

$$\begin{aligned} P(C|Pos) &= \frac{P(Pos|C)P(C)}{P(Pos)} \\ &= \frac{P(Pos|C)P(C)}{P(Pos|C)P(C) + P(Pos|`C)P(`C)} \\ &= \frac{0.8 \times 0.01}{0.8 \times 0.01 + 0.096 \times 0.99} \\ &= 0.0776 \end{aligned}$$

Problem 5

Since the distribution is approximately normal with $n \geq 30$, we can use the fact that the area under a normal distribution within 1SD from mean is 68% and 2SD is 95%. Then, based on the graph, 2SD is likely to be 12+. Working backwards, 1SD can be approximated to be 6.1.

Answer: b

Problem 6

a

From the lecture slides, we can obtain the following information,
Population = Entire population of America (45million)
Sample = 10million

b

Null hypothesis is 61% of the population voted for Roosevelt.
Alternate hypothesis from sample suggest that the population should be smaller

$H_0 : p = 0.61$

$H_A : \hat{p} < 0.61$

c

Note that for this question we can model the problem as a binomial proportion since in this problem, the person can either vote or not vote for Roosevelt. Since in this problem n is reduced to 100, we can still assume that the distribution is approximately normal since $n > 30$.

$$Z = \frac{\bar{p} - p_0}{\frac{p_0 q_0}{\sqrt{n}}}$$

For a significance level of 1%, the critical value is -2.326 for a left tail test.

$$Z = \frac{0.43 - 0.61}{\sqrt{\frac{0.61 * 0.39}{100}}} = -3.69$$

We assume that the H_0 is true, that is $p_0 = 0.61$ and we compute the Z score. Since the Z score falls in the rejection region, we should reject H_0 in favour of H_A.