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1. MSE (LSB - k)

$$= \frac{1}{2^k \times 2^k} \sum_{i=0}^{2^k-1} \sum_{j=0}^{2^k-1} (i-j)^2$$

$$= \frac{1}{2^k \times 2^k} \left[2^k \sum_{i=0}^{2^k-1} i^2 - 2 \left(\frac{(0+2^k-1) \cdot 2^k}{2} \right)^2 + 2^k \sum_{j=0}^{2^k-1} j^2 \right]$$

$$= \frac{1}{2^k \times 2^k} \left[2^k \cdot \frac{(2^k-1) 2^k (2^{k+1}-1)}{6} \times 2 - 2 \frac{(2^k-1)^2 \cdot 2^{2k}}{4} \right]$$

$$= \frac{(2^{k+1}-2)(2^{k+1}-1)}{6} - \frac{(2^k-1)^2}{2}$$

$$= \frac{2^{2k+2} - 3 \cdot 2^{k+1} + 2 - 3(2^{2k} - 2 \cdot 2^k + 1)}{6}$$

$$= \frac{2^{2k+2} - 3 \cdot 2^{k+1} + 2 - 3 \cdot 2^{2k} + 6 \cdot 2^k - 3}{6}$$

$$= \frac{2^{2k} - 1}{6}$$

2. MSE(OPAP-k)

$$= \frac{1}{2^k}\left[2\cdot\left(1^2+2^2+\cdots+(2^{k-1})^2\right)-(2^{k-1})^2\right]$$

$$= \frac{1}{2^k}\left[2\cdot\frac{(2^{k-1})(2^{k-1}+1)(2^k+1)}{6}-(2^{k-1})^2\right]$$

$$= \left[\frac{(2^{k-1}+1)(2^k+1)-3\cdot(2^{k-1})}{6}\right]$$

$$= \frac{2^{2k-1}+2^{k-1}+2^k+1-3\cdot 2^{k-1}}{6}$$

$$= \frac{2^{2k-1}+1}{6}.$$

3.

LSB：

k	embedding rate	MSE	PSNR	embedding efficiency
1	1	0.5	51.1411	2
2	2	2.5	44.1514	0.8
3	3	10.5	37.9189	0.2857
4	4	42.5	31.8469	0.0941
5	5	170.5	25.8136	0.0293
6	6	682.5	19.7898	0.0088
7	7	2730.5	13.7684	0.0026

OPAP：

k	embedding rate	MSE	PSNR	embedding efficiency
1	1	0.5	51.1411	2
2	2	1.5	46.3699	1.3333
3	3	5.5	40.7272	0.5455
4	4	21.5	34.8064	0.186
5	5	85.5	28.8111	0.0585
6	6	341.5	22.7969	0.0176
7	7	1365.5	16.7779	0.0051