# Periodic or Cyclical Features

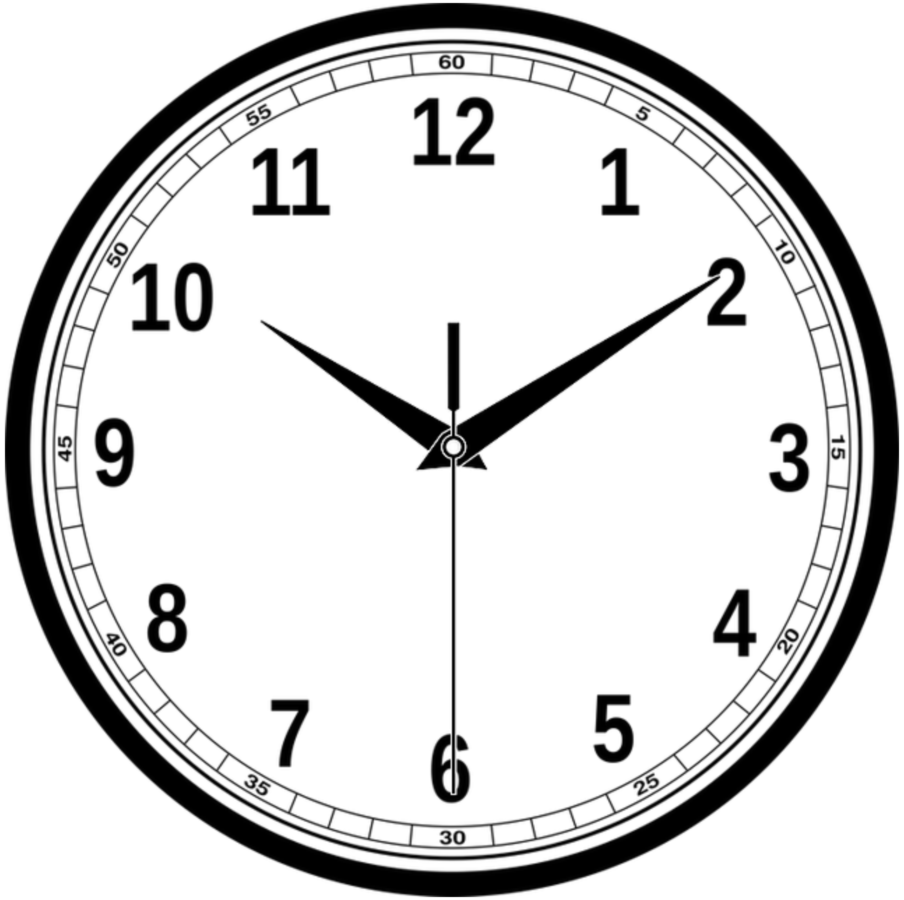
Capturing the periodicity

# Periodic Features

- Periodic features **repeat their values** at regular intervals.
- **Time examples**: hour, days of a week, week of the year, months, quarter, semester, seasons.
- **Other examples**: tides, moon cycle, position in an orbit.

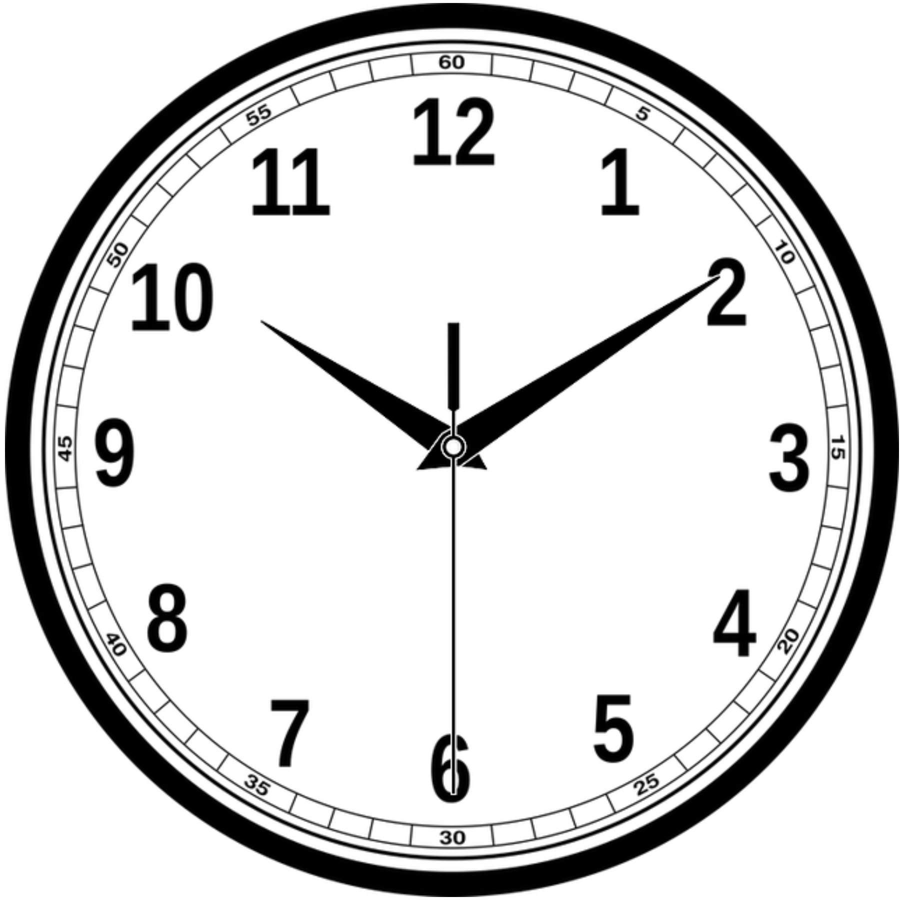
# Periodic Features

- Also known as **cyclical features**:
  - They reach a maximum value and start over again.



# Periodic Features

- Values that are very different in maginute are actually closer to each other.
  - Hour: 1 is closer to 23 than to 6.
  - Month: December is closer to January than July.



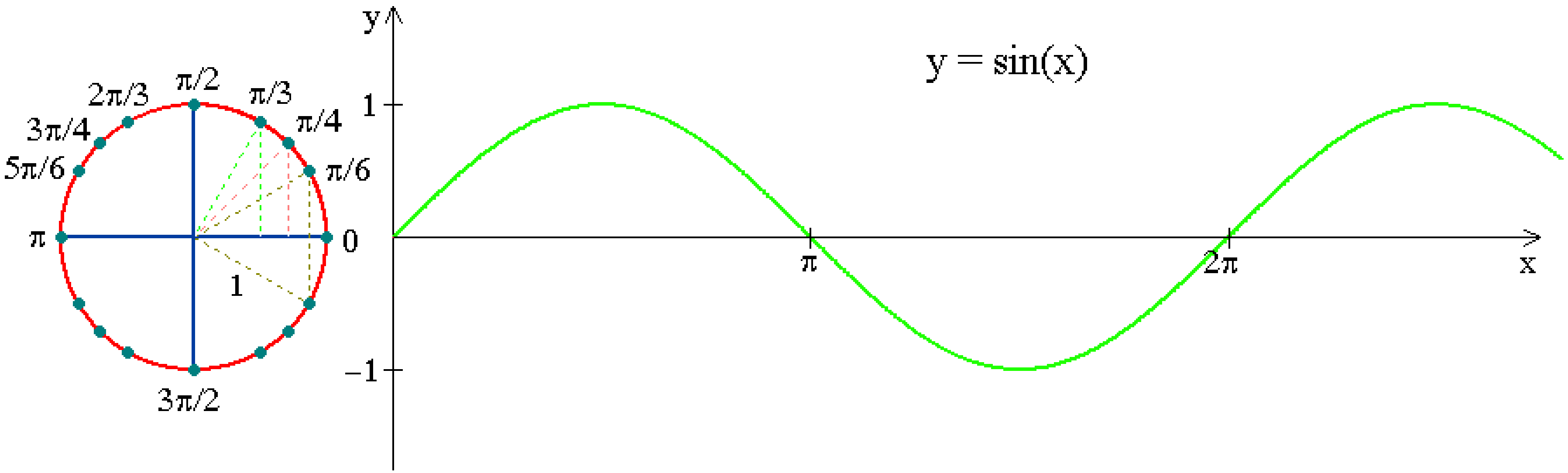
How can we let our machine learning models know that the features are cyclical?

# Trigonometric functions

- Trigonometric functions are periodic functions.

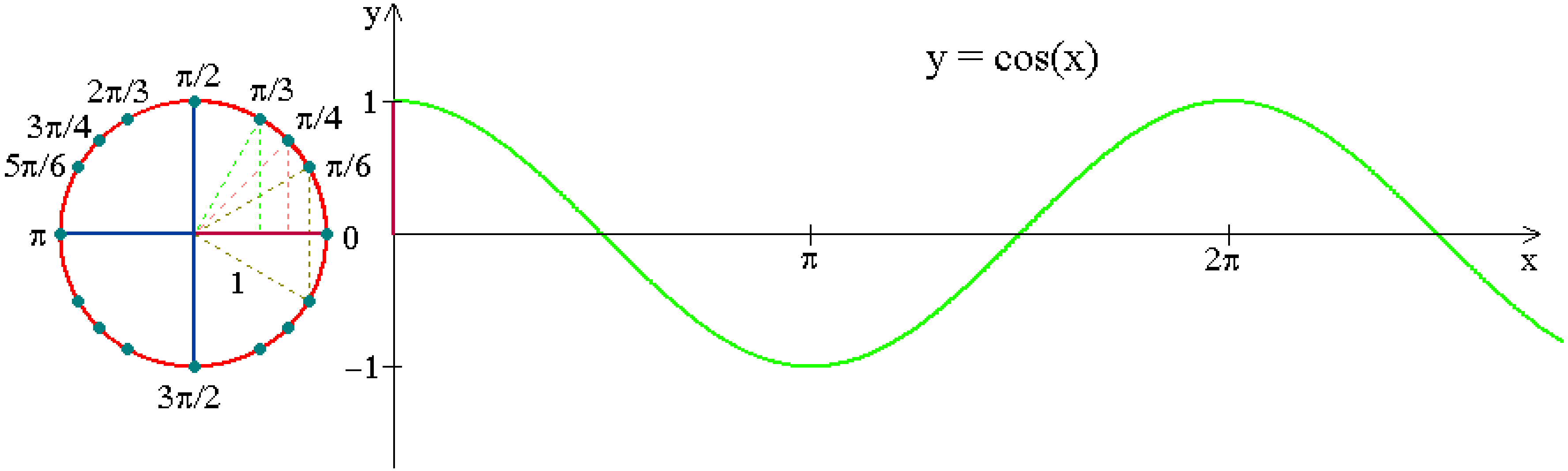
- The sine and cosine functions repeat their values at intervals of 2 π radians.

# Sine




https://www.math.hkust.edu.hk/~machiang/1013/Notes/tri_func.html

# Cosine




https://www.math.hkust.edu.hk/~machiang/1013/Notes/tri_func.html

# Periodic Transformation

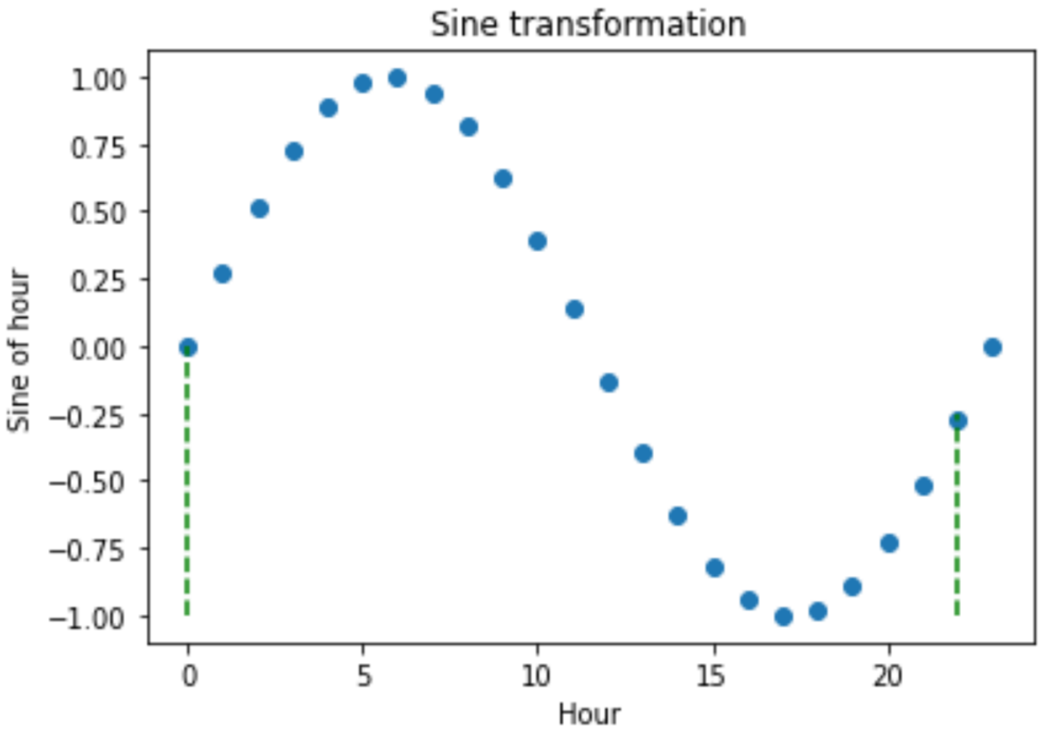
We know that the period of the sine and cosine is 2 π.

To transform the cyclical features:

$$\text{Sin(var)} = sin(variable \times \frac{2 \times \Pi}{\max value})$$

$$\text{Cos(var)} = cos(variable \times \frac{2 \times \Pi}{\max value})$$

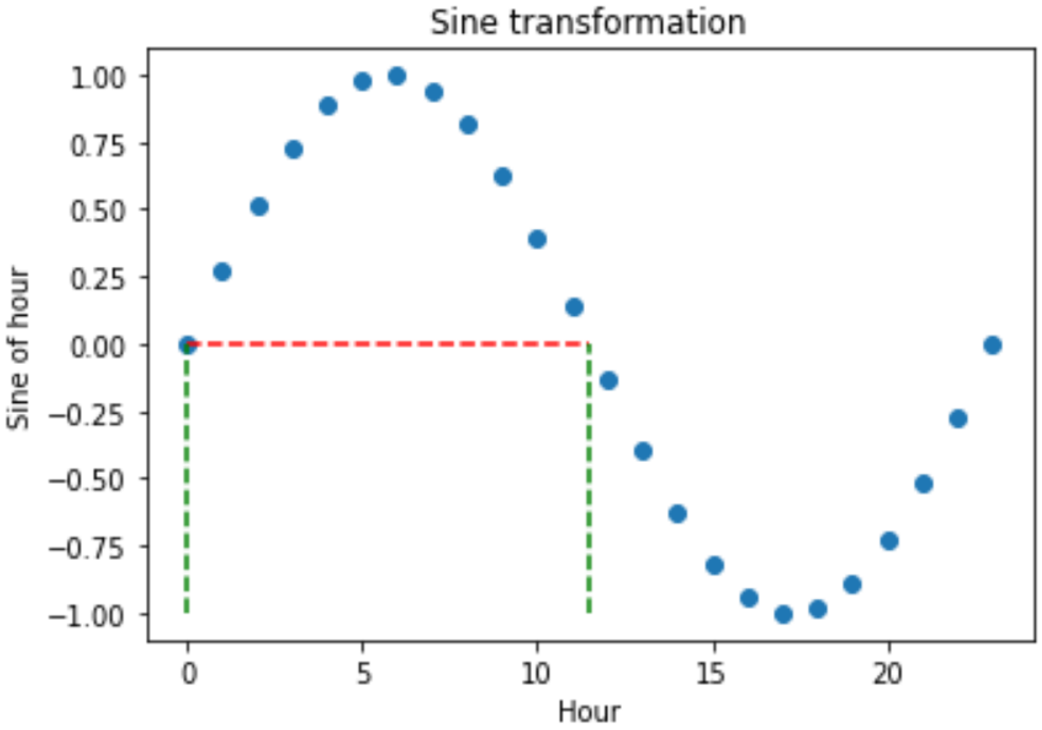
# Periodic transformation - advantage




When we transform the hour with the sine, we obtain the plot on the left.

Now, the hours 0 and 22 are much closer in value.

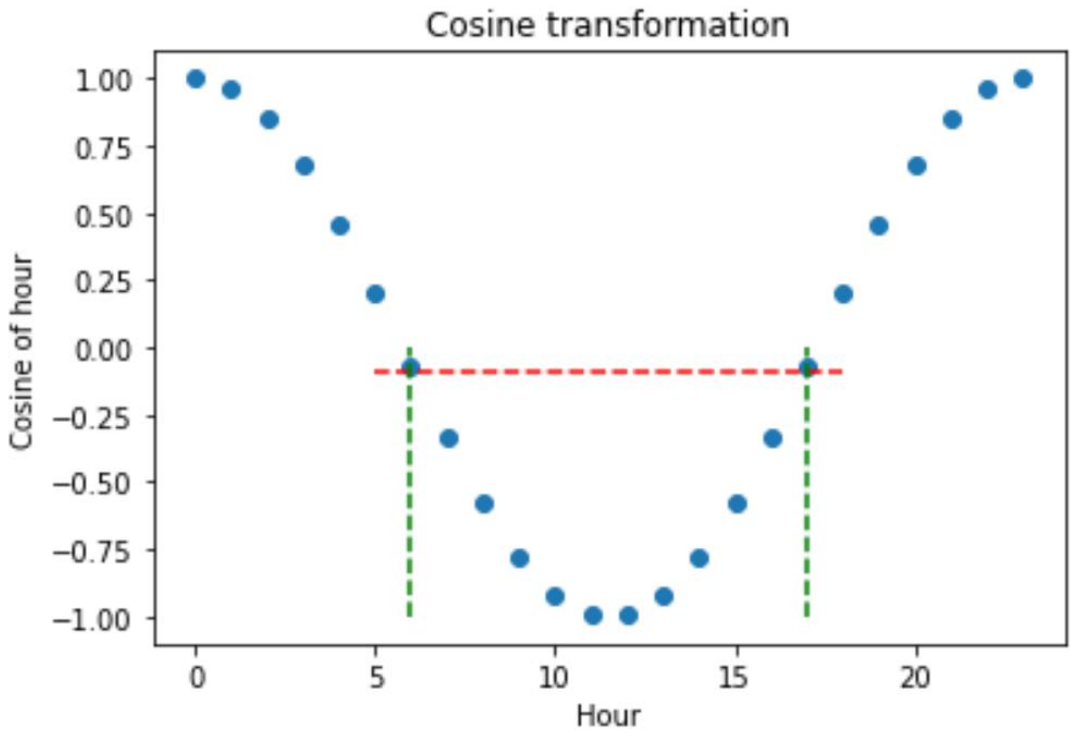
# Periodic transformation - limitation




However, different hours could take the same value after the transformation.

See for example the hours 0 and 11.5.

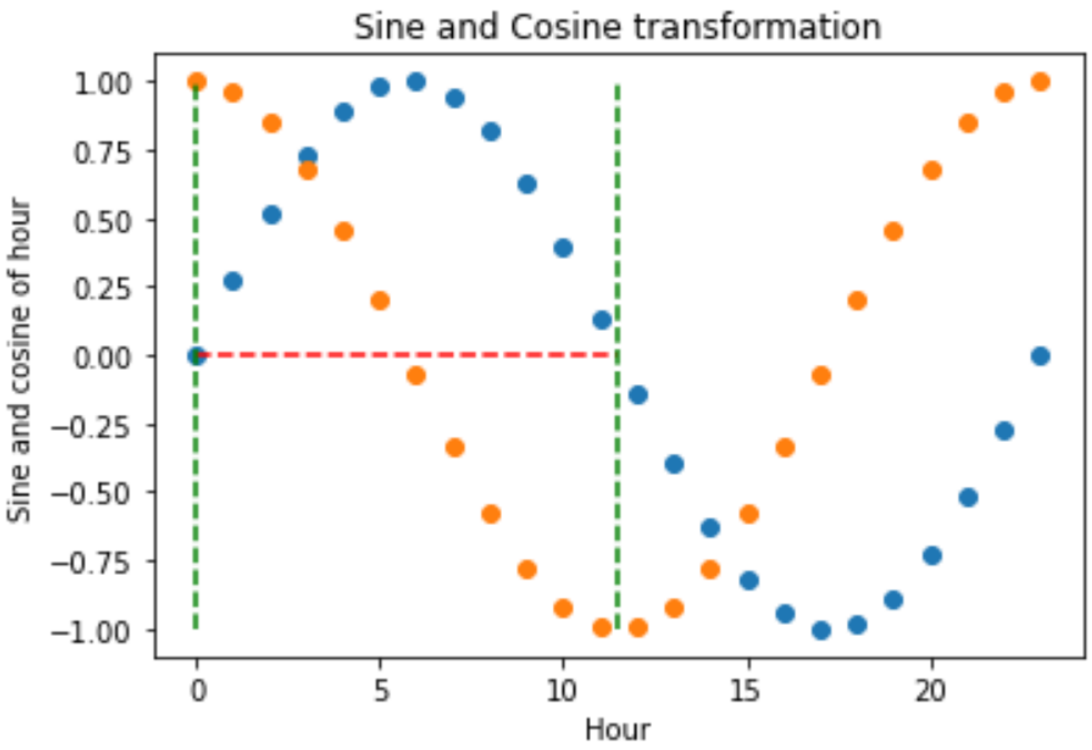
# Periodic transformation




The same is true for the cosine transformation.

Different hours could take the same cosine value.

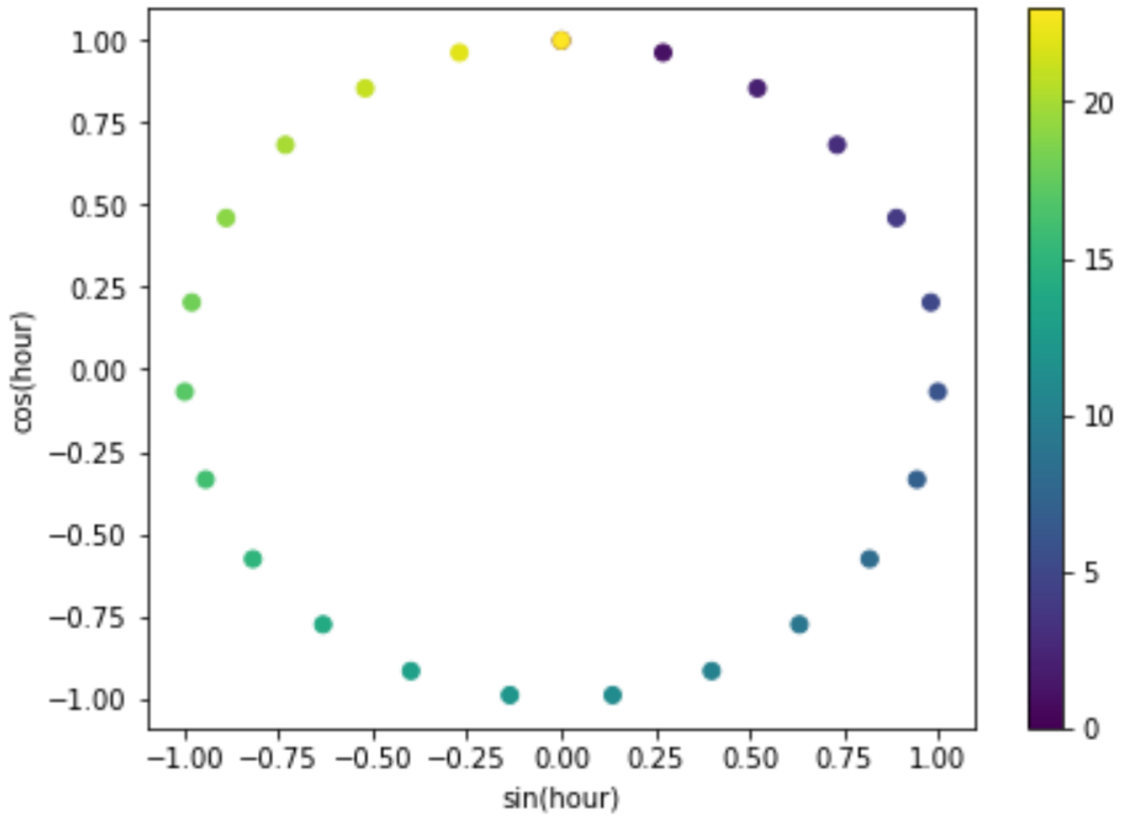
# Both features to fully encode the original




With the 2 functions together we can unequivocally identify each hour.

That is because sin and cosine are out of phase.

# Both features to fully encode the cycle




An intuitive way to show the new representation is to plot the sine vs the cosine transformation of the hour.

Now, the distance between two points corresponds to the difference in time as we would expect from a 24-hour cycle.