Additive and multiplicative decompositions

Time series decomposition

Contents

INTRODUCE IDEA OF ADDITIVE AND MULTIPLICATIVE DECOMPOSITIONS

DISCUSS WHEN TO USE AN ADDITIVE OR MULTIPLICATIVE DECOMPOSITIONS

Time series decomposition

- Aim: Decompose a time series into components
- Choice / assumption:

$$y(t) = \text{trend}(t) \times \text{seasonal}(t) \times \text{residual}(t)$$

$$y(t) = \text{trend}(t) + \text{seasonal}(t) + \text{residual}(t)$$

- How to choose?
  - Domain knowledge around data generating process
  - Looking at certain behaviours of the time series

Multiplicative decomposition - Example

- T-shirt sales of a growing online clothing brand
- Visitors to website increasing over time $\Rightarrow N(t) \propto \exp(at)$
- The product has seasonal demand. Each customer has higher probability, $p(t)$, of purchasing during summer then winter

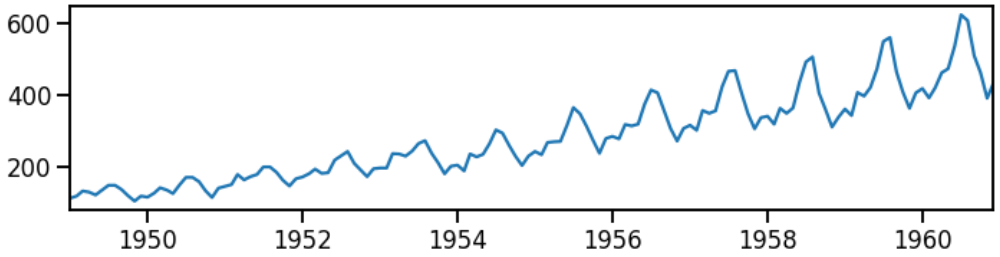
$$\text{sales}(t) \propto N(t) \times p(t)$$

Long term trend

yearly seasonality

Multiplicative decomposition

Note: The magnitude of the seasonal fluctuations is proportional to the level the time series



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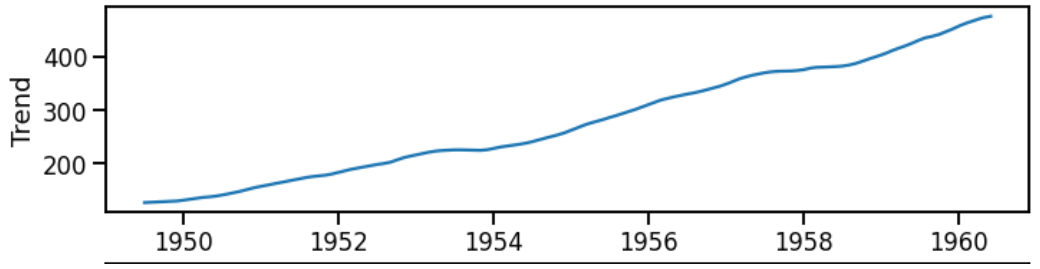

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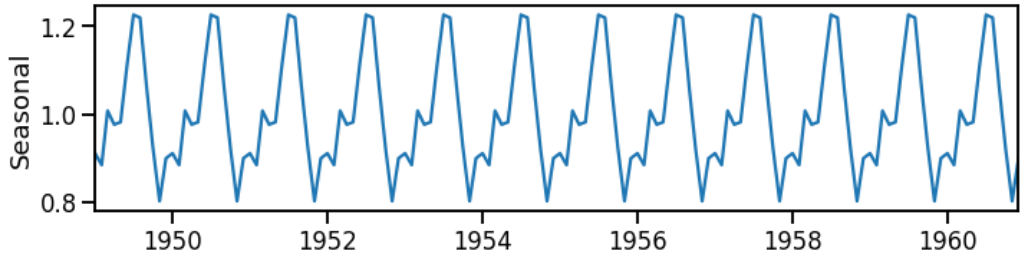

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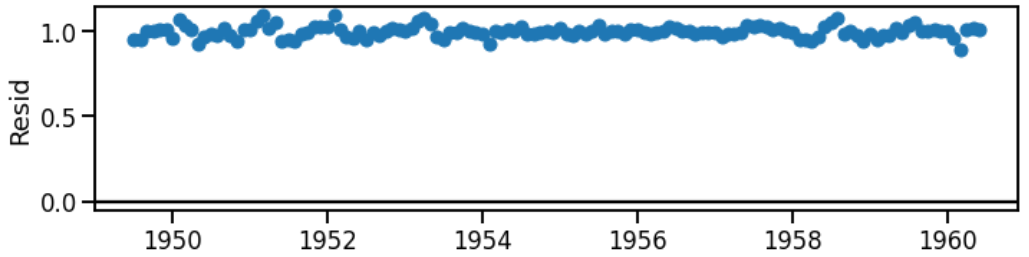


Additive decomposition - Example

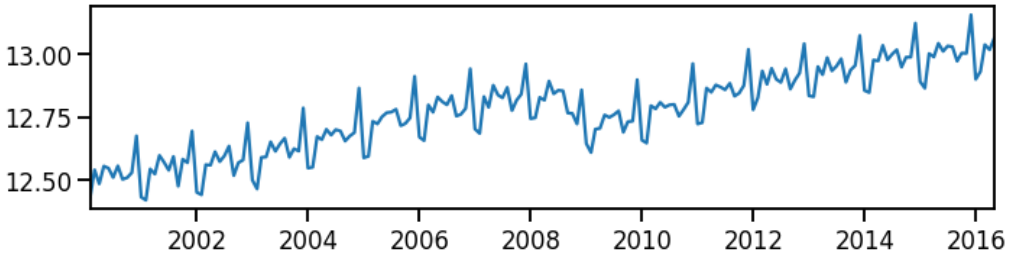
- Air pollution on a particular road, $\mathbf{pollution}(t)$
- There is some background level of air pollution, $B(t)$
- Traffic from vehicles can add air pollution, $T(t)$
- Traffic is busier during peak hours, hence, there is a daily seasonality
- So we expect: $\mathbf{pollution}(t) \propto B(t) + T(t)$

Long term trend

Daily seasonality

Additive decomposition

Note: The magnitude of the seasonal fluctuations **does not** change with the level of the time series



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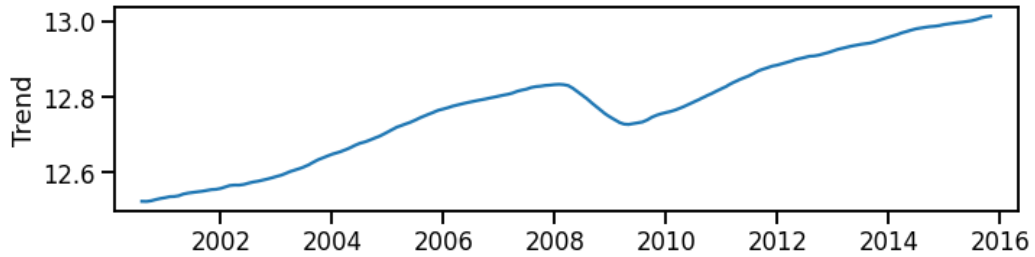

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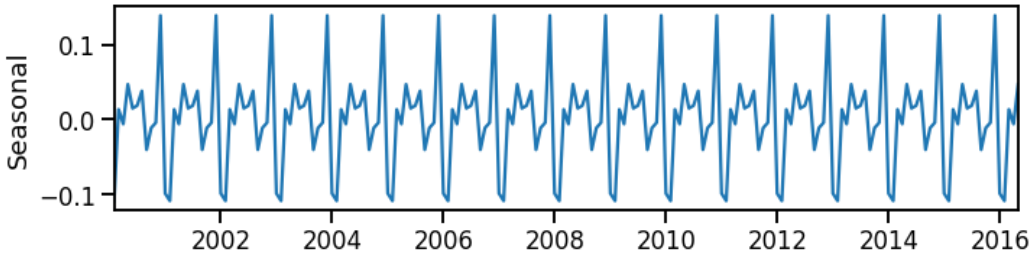

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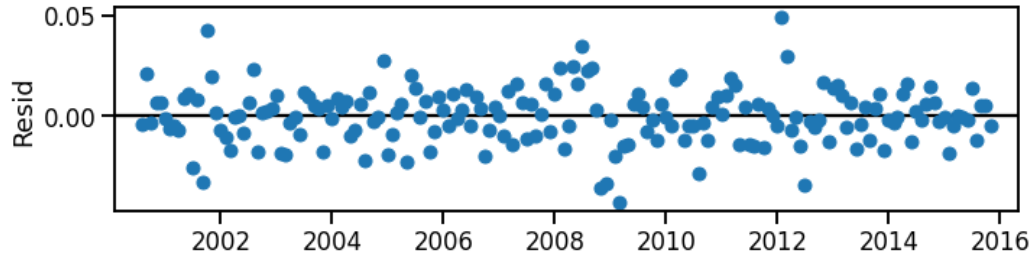

Summary

Time series can be decomposed into a multiplicative or additive decomposition/model

If the seasonal variation is proportional to the level of the time series it would indicate a multiplicative decomposition