LOWESS vs LOESS

Time series decomposition

LOWESS vs LOESS

LOWESS

- **Lo**cally **w**eighted **s**catterplot **s**moothing
- W. Cleveland 1979

Robust Locally Weighted Regression and Smoothing Scatterplots

WILLIAM S. CLEVELAND*

The visual information on a scatterplot can be greatly enhanced, with little additional cost, by computing and plotting smoothed points. Robust locally weighted regression is a method for smoothing a scatterplot, (x_i, y_i), $i = 1, \ldots, n$, in which the fitted value at x_k is the value of a polynomial fit to the data using weighted least squares, where the weight for (x_i, y_i) is large if x_i is close to x_k and small if it is not. A robust fitting procedure is used that guards against deviant points distorting the smoothed points. Visual, computational, and statistical issues of robust locally weighted regression are discussed. Several examples, including data on lead intoxication, are used to illustrate the methodology.

KEY WORDS: Graphics; Scatterplots; Nonparametric regression; Smoothing; Robust estimation.

An early example of smoothing scatterplots is given by Ezekiel (1941, p. 51). The points are grouped according to x_i, and for each group the mean of the y_i is plotted against the mean of the x_i. More recently, Stone (1977) proves the consistency of a wide class of nonparametric regression estimates under very general conditions and presents a discussion and bibliography of methods that have appeared in the literature. Another method, which appeared after Stone's review, is that of Clark (1977), who proposes a technique for smoothing scatterplots in which the plot is interpolated by joining successive

Cleveland, William S. "Robust locally weighted regression and smoothing scatterplots." *Journal of the American statistical association* 74.368 (1979): 829-836.

LOESS

- **Lo**cally **e**stimated **s**catterplot **s**moothing
- W. Cleveland & S. Devlin 1988

Locally Weighted Regression: An Approach to Regression Analysis by Local Fitting

WILLIAM S. CLEVELAND and SUSAN J. DEVLIN*

Locally weighted regression, or *loess*, is a way of estimating a regression surface through a multivariate smoothing procedure, fitting a function of the independent variables locally and in a moving fashion analogous to how a moving average is computed for a time series. With local fitting we can estimate a much wider class of regression surfaces than with the usual classes of parametric functions, such as polynomials. The goal of this article is to show, through applications, how loess can be used for three purposes: data exploration, diagnostic checking of parametric models, and providing a nonparametric regression surface. Along the way, the following methodology is introduced: (a) a multivariate smoothing procedure that is an extension of univariate locally weighted regression; (b) statistical procedures that are analogous to those used in the least-squares fitting of parametric functions; (c) several graphical methods that are useful tools for understanding loess estimates and checking the assumptions on which the estimation procedure is based; and (d) the *M* plot, an adaptation of Mallows's C_p procedure, which provides a graphical portrayal of the trade-off between variance and bias, and which can be used to choose the amount of smoothing.

Cleveland, William S., and Susan J. Devlin. "Locally weighted regression: an approach to regression analysis by local fitting." *Journal of the American statistical association* 83.403 (1988): 596-610.

See https://stats.stackexchange.com/questions/161069/difference-between-loess-and-lowess for a detailed discussion.

LOWESS vs LOESS

LOWESS

- **Lo**cally **w**eighted **s**catterplot **s**moothing
- W. Cleveland 1979

LOESS

- **Lo**cally **e**stimated **s**catterplot **s**moothing
- W. Cleveland & S. Devlin 1988

See https://stats.stackexchange.com/questions/161069/difference-between-loess-and-lowess for a detailed discussion.

LOWESS vs LOESS

LOWESS

- **Lo**cally **w**eighted **s**catterplot **s**moothing
- W. Cleveland 1979

Similarity

- Fits a locally weighted curve using regression.
- Uses tri-cubic weight function.
- Uses bi-square weights for robustness.

LOESS

- **Lo**cally **e**stimated **s**catterplot **s**moothing
- W. Cleveland & S. Devlin 1988

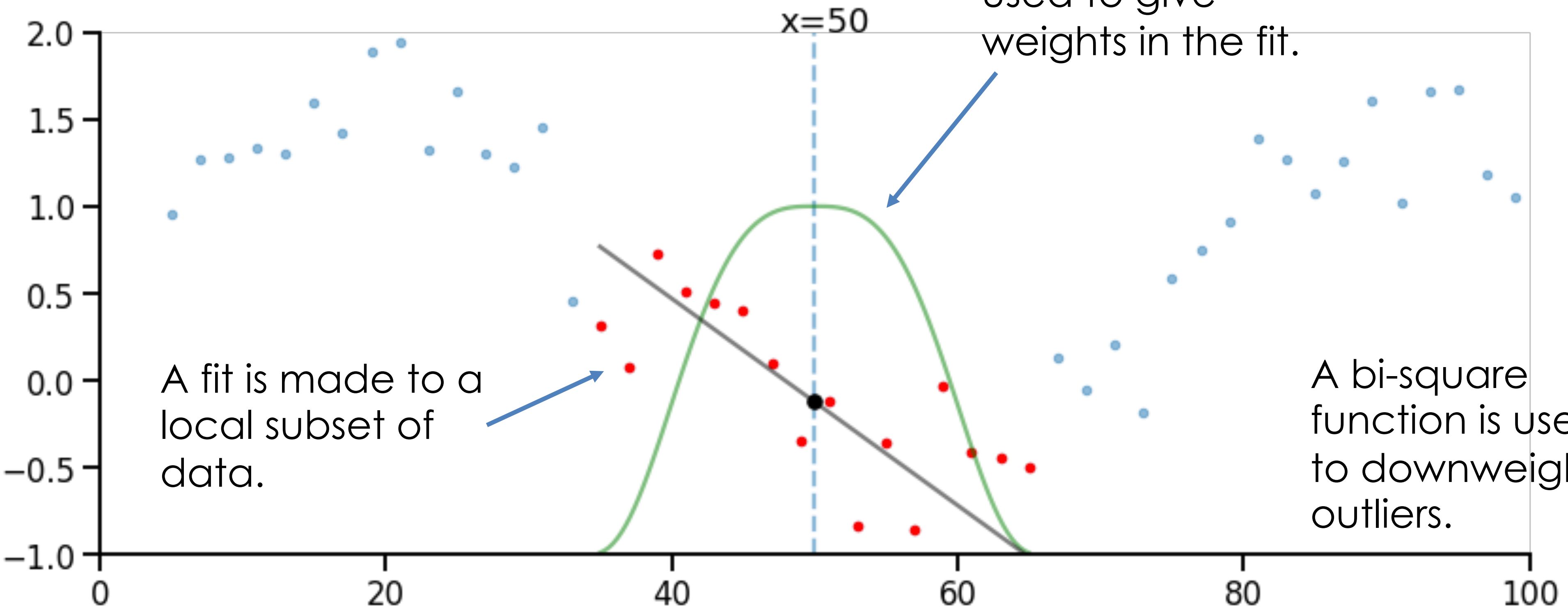
Similarity

- Fits a locally weighted curve using regression.
- Uses tri-cubic weight function.
- Uses bi-square weights for robustness (depends on implementation).

See https://stats.stackexchange.com/questions/161069/difference-between-loess-and-lowess for a detailed discussion.

LOWESS and LOESS fit local weighted curves

Tri-cubic function used to give weights in the fit.



A fit is made to a local subset of data.

A bi-square function is used to downweight outliers.

LOWESS vs LOESS

LOWESS

- **Lo**cally **w**eighted **s**catterplot **s**moothing
- W. Cleveland 1979

Similarity

- Fits a locally weighted curve using regression.
- Uses tri-cubic weight function.
- Uses bisquare weights for robustness.

Difference

- Uses linear regression.
- Fits a curve to univariate data (y is 1-D).

LOESS

- **Lo**cally **e**stimated **s**catterplot **s**moothing
- W. Cleveland & S. Devlin 1988

Similarity

- Fits a locally weighted curve using regression.
- Uses tri-cubic weight function.
- Uses bi-square weights for robustness (depends on implementation).

Difference

- Uses polynomial regression.
- Fits a surface to multivariate data (y is N-D).

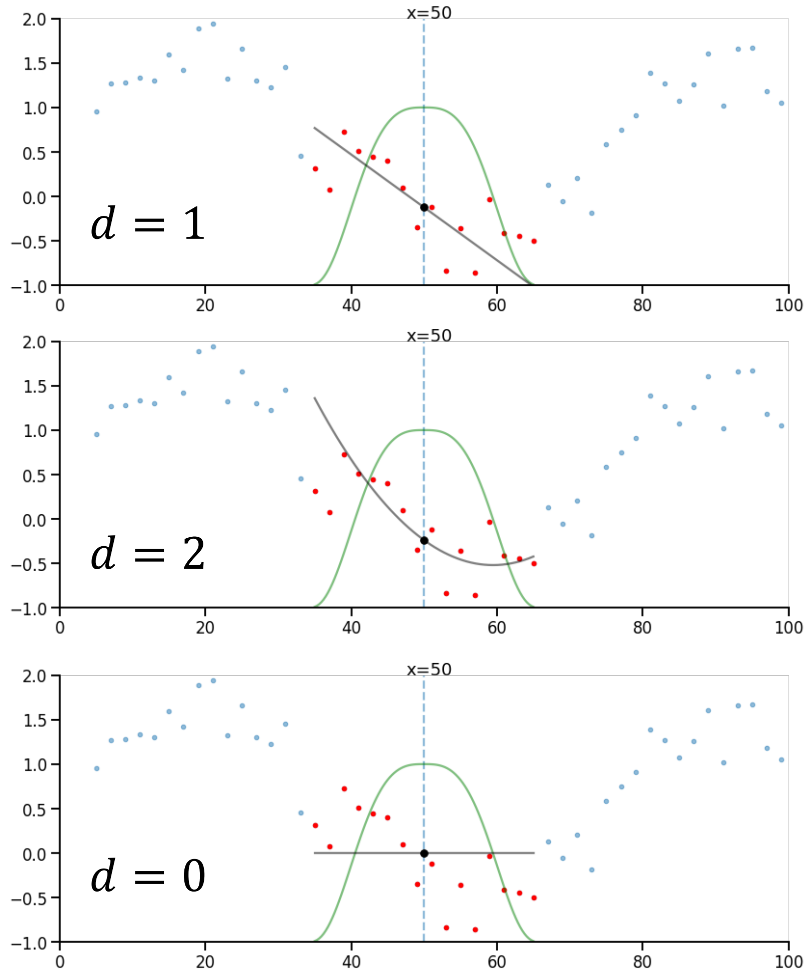
See https://stats.stackexchange.com/questions/161069/difference-between-loess-and-lowess for a detailed discussion.

LOESS fits a local polynomial regression

- Previously showed that LOWESS uses a local, robust, and weighted linear regression.
- LOESS uses a polynomial regression

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_d x^d$$

- d: Degree of polynomial to use in LOESS.
- Most applications use $d = 1$ and in some rarer cases $d = 2$ or $d = 0$.



LOWESS vs LOESS

LOWESS

- **Lo**cally **w**eighted **s**catterplot **s**moothing
- W. Cleveland 1979

Similarity

- Fits a locally weighted curve using regression
- Uses tri-cubic weight function
- Uses bisquare weights for robustness

Difference

- Uses linear regression
- Fits a curve to univariate data (y is 1-D)

LOESS

- **Lo**cally **e**stimated **s**catterplot **s**moothing
- W. Cleveland & S. Devlin 1988

Similarity

- Fits a locally weighted curve using regression
- Uses tri-cubic weight function
- Uses bi-square weights for robustness (depends on implementation)

Difference

- Uses polynomial regression
- Fits a surface to multivariate data (y is N-D)

See https://stats.stackexchange.com/questions/161069/difference-between-loess-and-lowess for a detailed discussion.

Summary

LOWESS and LOESS fit a curve to local partitions of the data

LOWESS fits a line and LOESS fits a polynomial

LOWESS only works with 1-D data and LOESS can work with N-D data