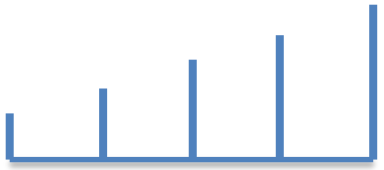
Exponential weights: part 1

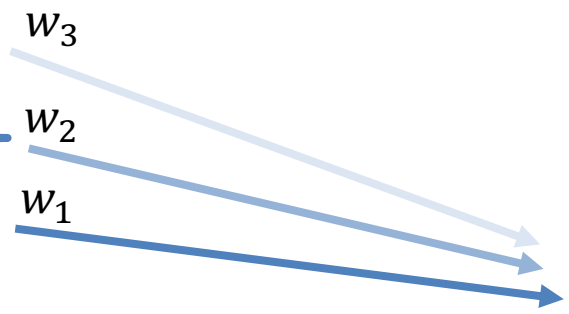
Window features

How to pick the weights?

	Constant	Linear	Exponential
			
Example:	[20%, 20%, 20%, 20%, 20%]	[6.5%, 13%, 19.5%, 26%, 32.5%]	[3.2%, 6.5%, 13%, 26%, 52%]
	Even weighting	Weight decays linearly	Weight decays exponentially Parameter: Rate of decay, α

Exponential weights

Date	Sales
…	…
2020-02-12	23
2020-02-13	30
2020-02-14	35
2020-02-15	?



Exponential weighted window (mean)
…
x

$$= \frac{w_1 35 + w_2 30 + w_3 23}{\sum_i w_i}$$

Exponential weights

User specifies the parameter α.

Date	Sales
…	…
2020-02-12	23
2020-02-13	30
2020-02-14	35
2020-02-15	?

$w_1 = 1$

Exponential weighted window (mean)
…
x

$$= \frac{1*35}{1}$$

Exponential weights

User specifies the parameter α.

Date	Sales
…	…
2020-02-12	23
2020-02-13	30
2020-02-14	35
2020-02-15	?

$w_2 = 1 * (1 - \alpha)$

Exponential weighted window (mean)
…
x

$$= \frac{1*35+(1-\alpha)*30}{1+(1-\alpha)}$$

Exponential weights

User specifies the parameter α.

Date	Sales
…	…
2020-02-12	23
2020-02-13	30
2020-02-14	35
2020-02-15	?

$w_3 = 1 * (1-\alpha)^2$

Exponential weighted window (mean)
…
X

$$= \frac{1*35+(1-\alpha)*30+(1-\alpha)^2*23}{1+(1-\alpha)+(1-\alpha)^2}$$

Exponential weights

User specifies the parameter $\alpha = 0.5$.

Date	Sales
…	…
2020-02-12	23
2020-02-13	30
2020-02-14	35
2020-02-15	?

$w_3 = 0.25$

$w_2 = 0.5$

$w_1 = 1$

Exponential weighted window (mean)
…
31.9

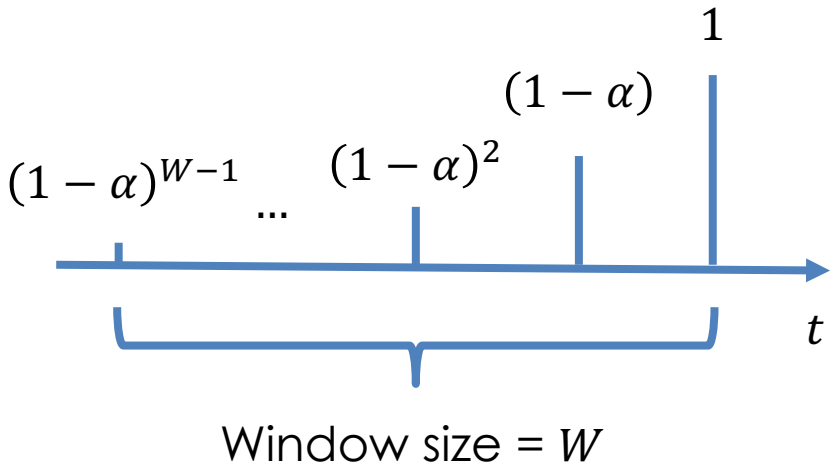
$$= \frac{1*35 + \frac{1}{2}*30 + \frac{1}{4}*23}{1+\frac{1}{2}+\frac{1}{4}}$$

Exponential weights

Date	Sales
…	…
2020-02-12	23
2020-02-13	30
2020-02-14	35

Window size = W

…	…
$t-2$	$(1-\alpha)^2$
$t-1$	$(1-\alpha)$
t	1



Exponential weighted moving average (EWMA) at time t:

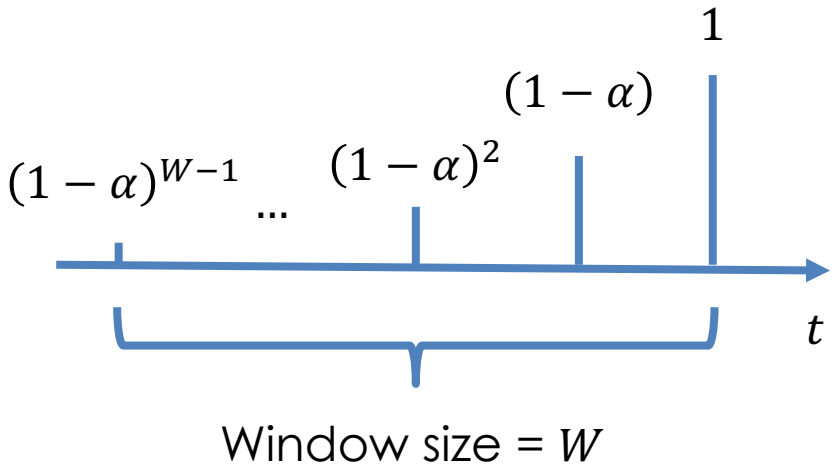
$$\frac{1 * y_t + (1-\alpha) * y_{t-1} + (1-\alpha)^2 * y_{t-2} + \ \ldots + (1-\alpha)^{W-1} y_{t-W-1}}{1 + (1-\alpha) + (1-\alpha)^2 + \ \ldots + (1-\alpha)^{W-1}}$$

Exponential weights

Date	Sales
…	…
2020-02-12	23
2020-02-13	30
2020-02-14	35

Window size = W

…	…
$t-2$	$(1-\alpha)^2$
$t-1$	$(1-\alpha)$
t	1



Exponential weighted moving average (EWMA) at time t, if the window is large:

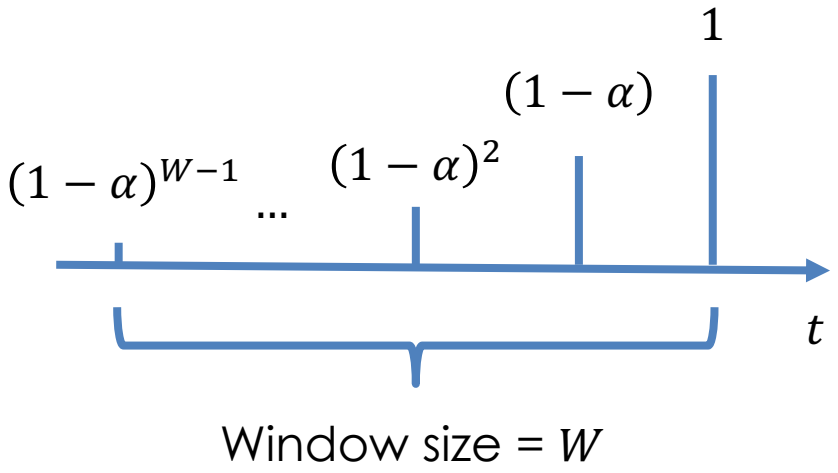
$$\frac{1 * y_t + (1-\alpha) * y_{t-1} + (1-\alpha)^2 * y_{t-2} + \ \ldots + (1-\alpha)^{W-1} y_{t-W-1}}{\frac{1}{\alpha}}$$

Exponential weights

Date	Sales
…	…
2020-02-12	23
2020-02-13	30
2020-02-14	35

Window size = W

…	…
$t-2$	$(1-\alpha)^2$
$t-1$	$(1-\alpha)$
t	1



Exponential weighted moving average (EWMA) at time t, if the window is large:

$$\alpha y_t + \alpha(1-\alpha) * y_{t-1} + \alpha(1-\alpha)^2 * y_{t-2} + \ \ldots + \alpha(1-\alpha)^{W-1} y_{t-W-1}$$

Exponential weights