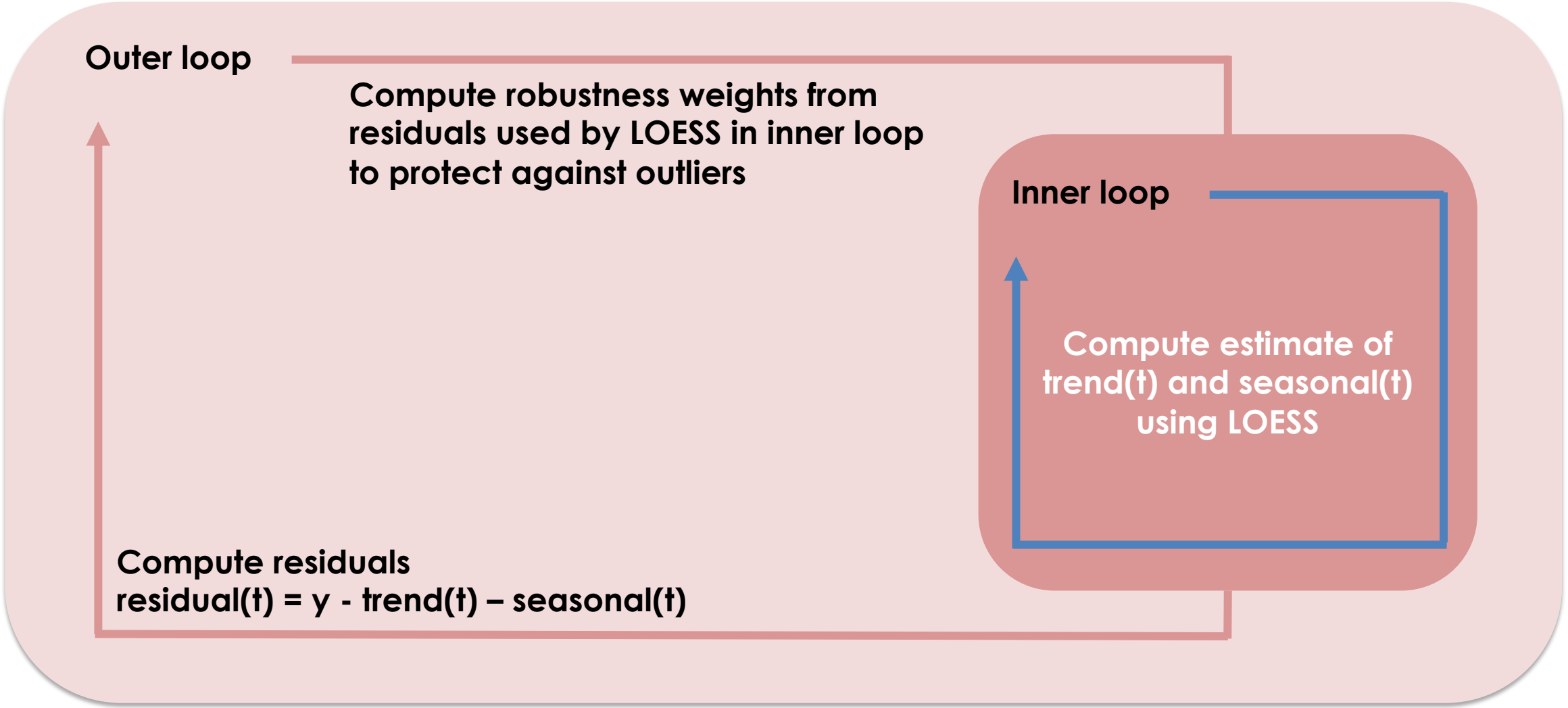
STL (Theory) – Inner Loop

Time series decomposition

The main idea



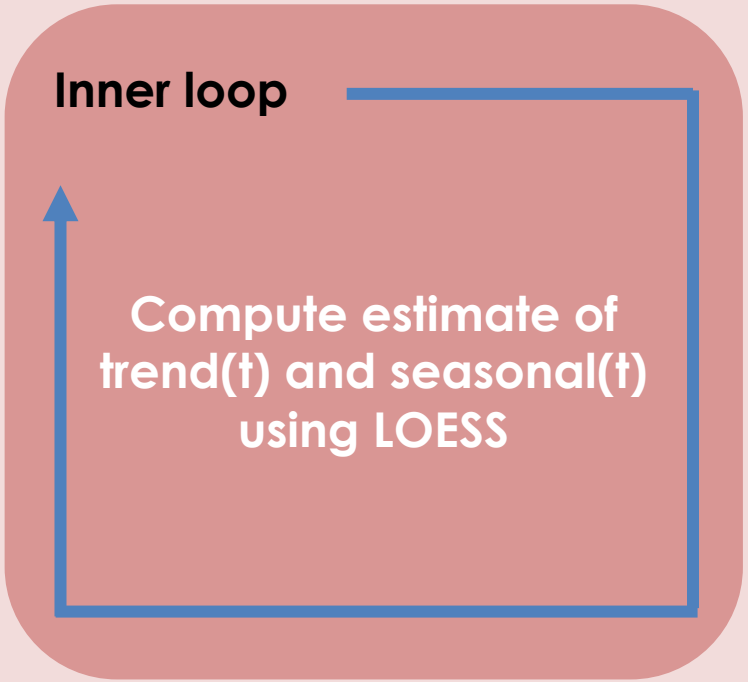
Inner loop summary

ITERATE

1. **De-trend the data**: y(t) – trend(t). For the first iteration assume trend(t) = 0

2. **Extract seasonal(t)** using LOESS on the cycle-subseries of y_detrend(t)

3. **Extract trend(t)** by subtracting seasonal(t) from y(t) and smoothing using LOESS



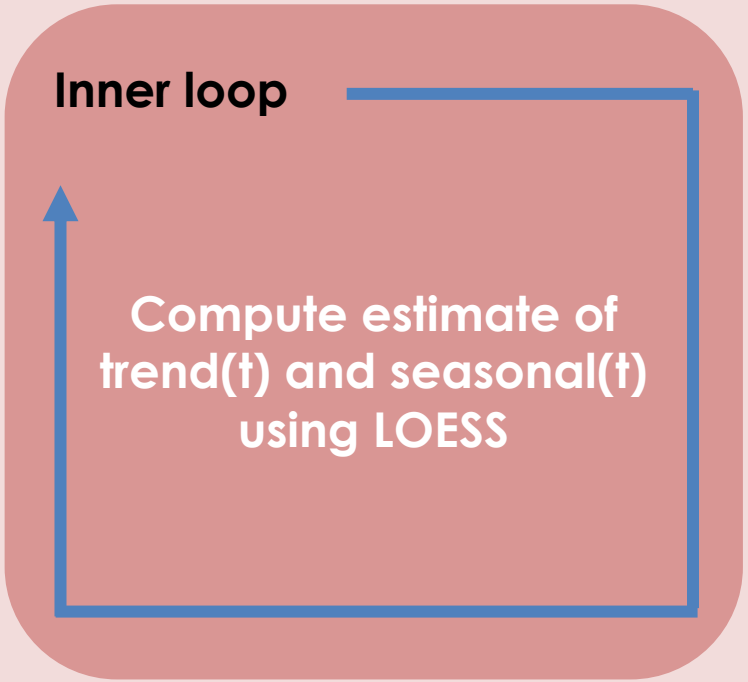
Inner loop summary

ITERATE

1. **De-trend the data**: y(t) – trend(t). For the first iteration assume trend(t) = 0

2. **Extract seasonal(t)** using LOESS on the cycle-subseries of y_detrend(t)

3. **Extract trend(t)** by subtracting seasonal(t) from y(t) and smoothing using LOESS



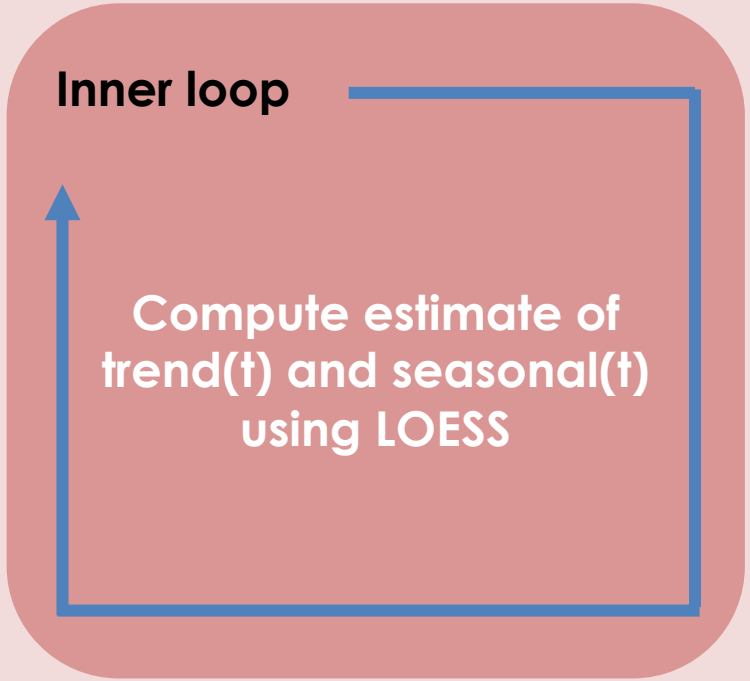
Inner loop summary

ITERATE

1. **De-trend the data**: y(t) – trend(t). For the first iteration assume trend(t) = 0

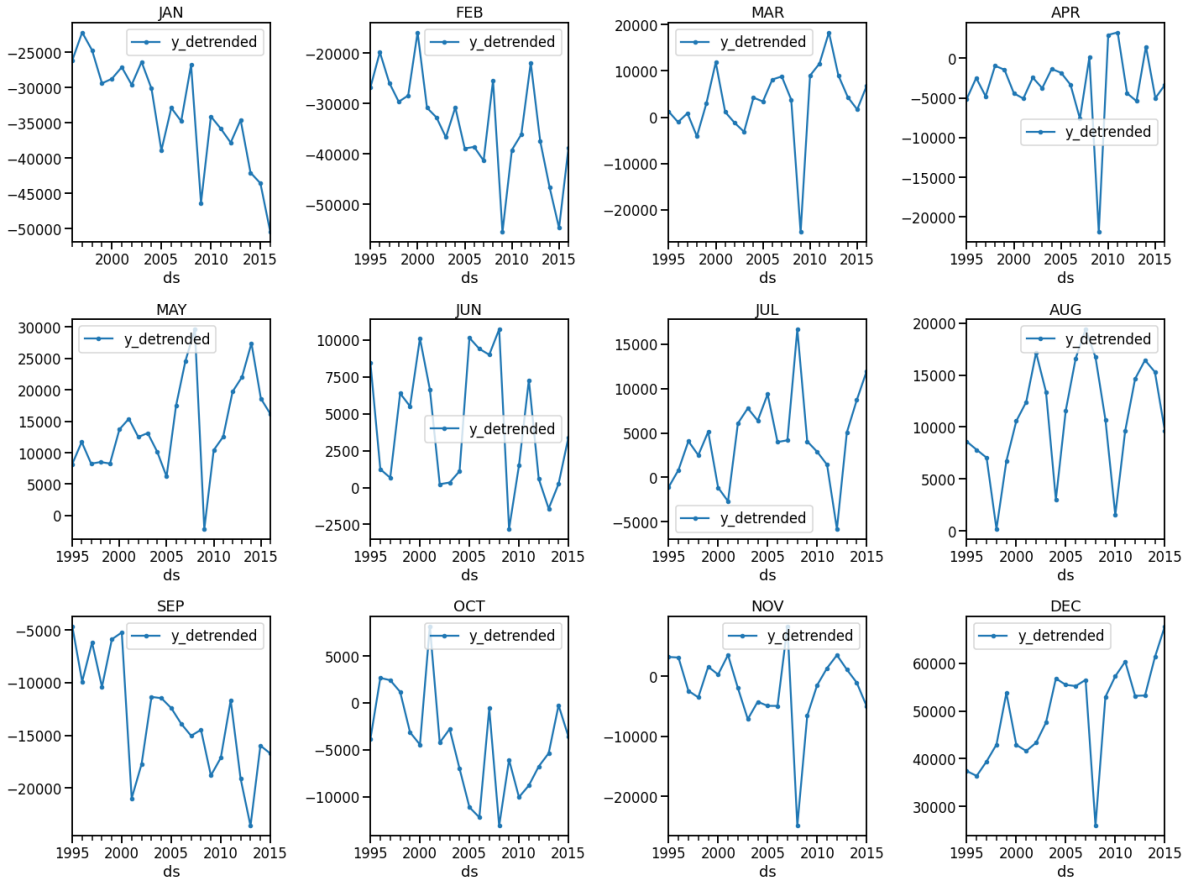
2. **Extract seasonal(t)** using LOESS on the cycle-subseries of y_detrend(t)

3. **Extract trend(t)** by subtracting seasonal(t) from y(t) and smoothing using LOESS



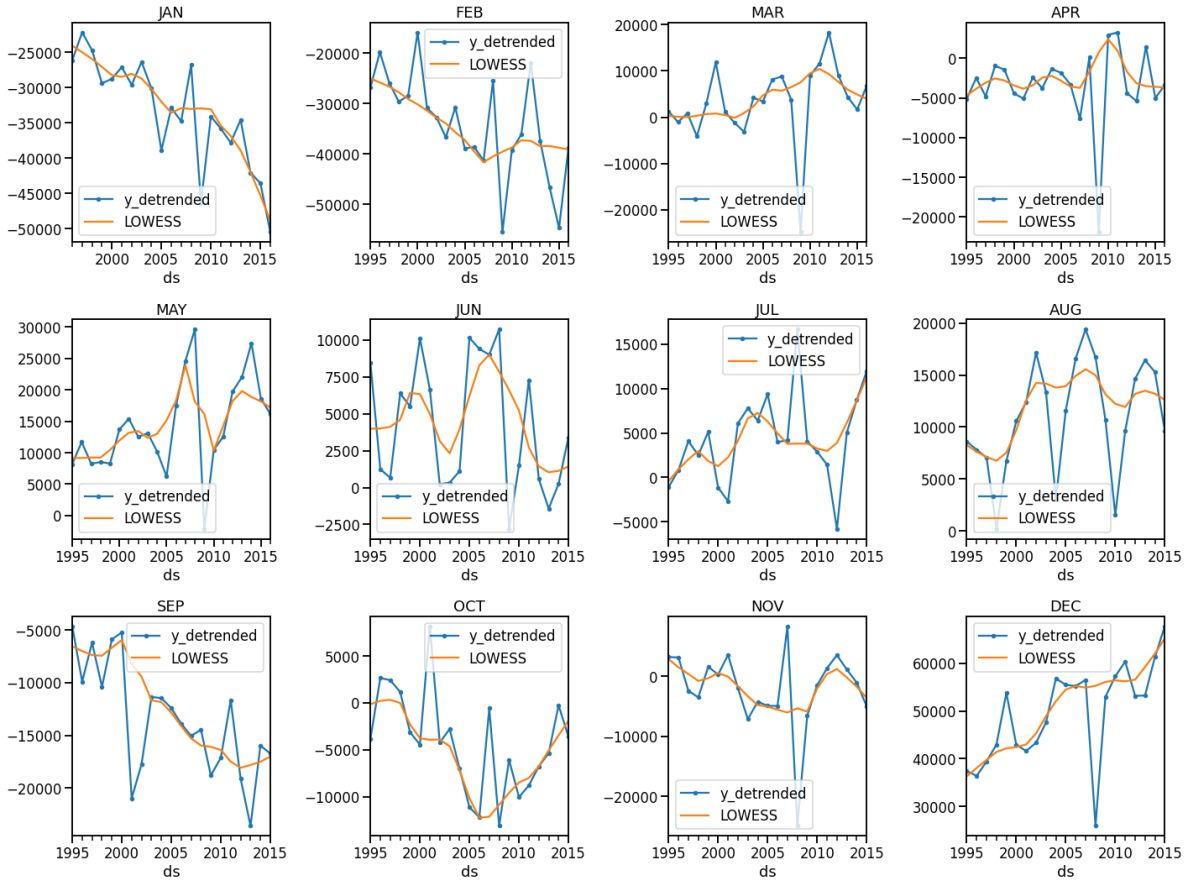
Extract seasonal component: Step 1 – Apply LOESS to Cycle-Subseries

- User specifies n_p (aka period in statsmodels): number of periods in a seasonal cycle
- Compute all the cycle-subseries for y_detrend(t)
- Apply LOESS to each cycle-subseries with smoothing parameter n_s. n_s is set by the user.



Extract seasonal component: Step 1 – Apply LOESS to Cycle-Subseries

- User specifies n_p (aka period in statsmodels): number of periods in a seasonal cycle
- Compute all the cycle-subseries for y_detrend(t)
- Apply LOESS to each cycle-subseries with smoothing parameter n_s. n_s is set by the user.
- Denote output as C_t



$n_s = 7$

Extract seasonal component: Step 2 – De-trend the cycle-subseries

- Apply a series of moving averages to each smooth cycle-subseries:

$$[3\times MA][n_p\times MA][n_p\times MA]$$

- Apply LOESS, with smoothing parameter n_l, to each smooth cycle-subseries:

$$L_t = [LOESS]\ [3\times MA][n_p\times MA][n_p\times MA](C_t)$$

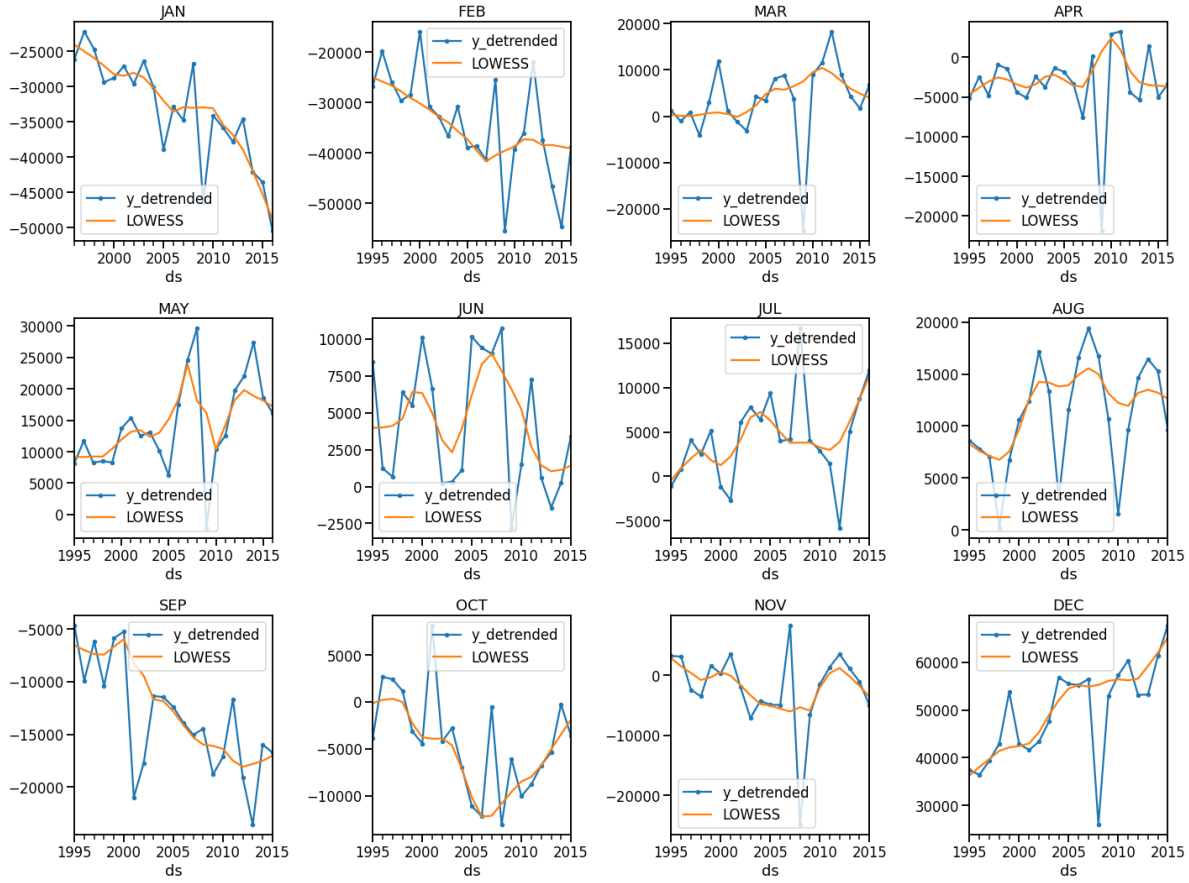
- Acts as a "low-pass filter" meaning that L_t represents any long term trend in the cycle-subseries

- De-trend the smoothed cycle-subseries. The gives the seasonality

$$S_t = C_t - L_t$$

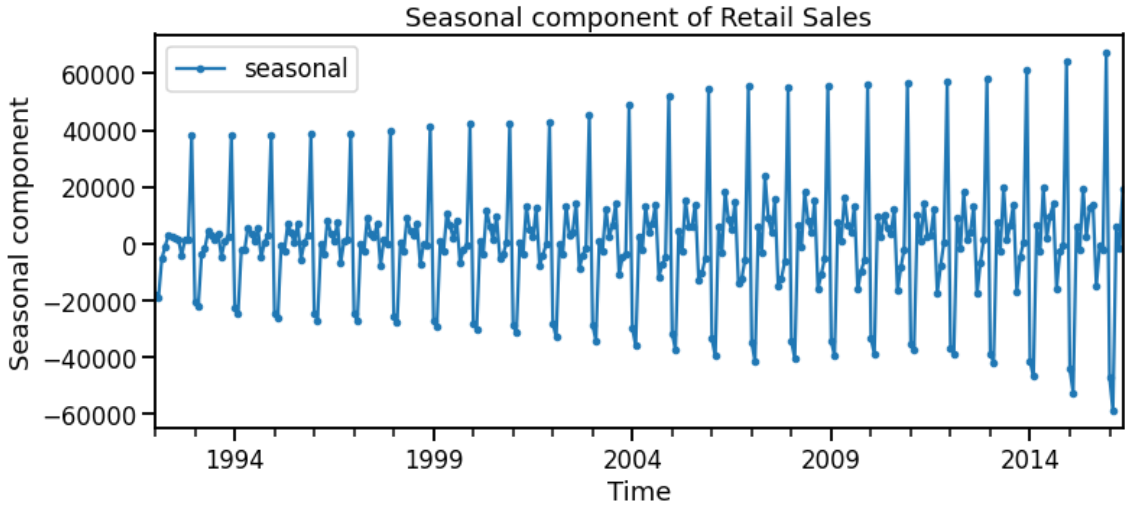
Extract seasonal component: Step 2 – De-trend the cycle-subseries

- As the seasonal component is derived from cycle-subseries it can vary in time!
- So seasonal component isn't exactly periodic over each cycle



Extract seasonal component: Step 2 – De-trend the cycle-subseries

- As the seasonal component is derived from cycle-subseries it can vary in time!
- So seasonal component isn't exactly periodic over each cycle



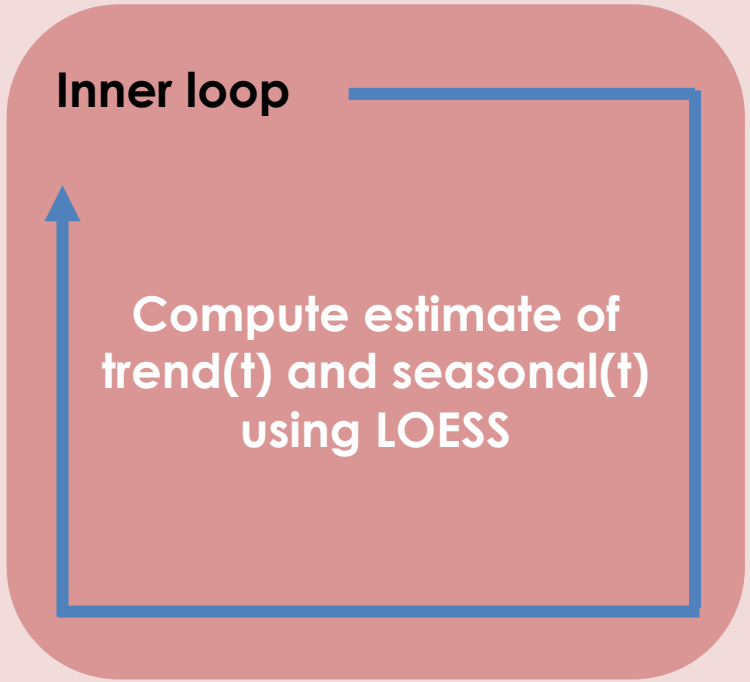
Inner loop summary

ITERATE

1. **De-trend the data**: y(t) – trend(t). For the first iteration assume trend(t) = 0

2. **Extract seasonal(t)** using LOESS on a set of time series derived from y_detrend(t) called cycle-subseries. Will require us to specify the seasonal period.

3. **Extract trend(t)** by subtracting seasonal(t) from y(t) and smoothing using LOESS



Extract trend component

- De−seasonalize original series:

$$T_t^* = \mathrm{y}_t - S_t$$

- Smooth T_t^* using LOESS, with smoothing parameter n_T, to give trend estimate:

$$T_t = LOESS(T_t^*)$$

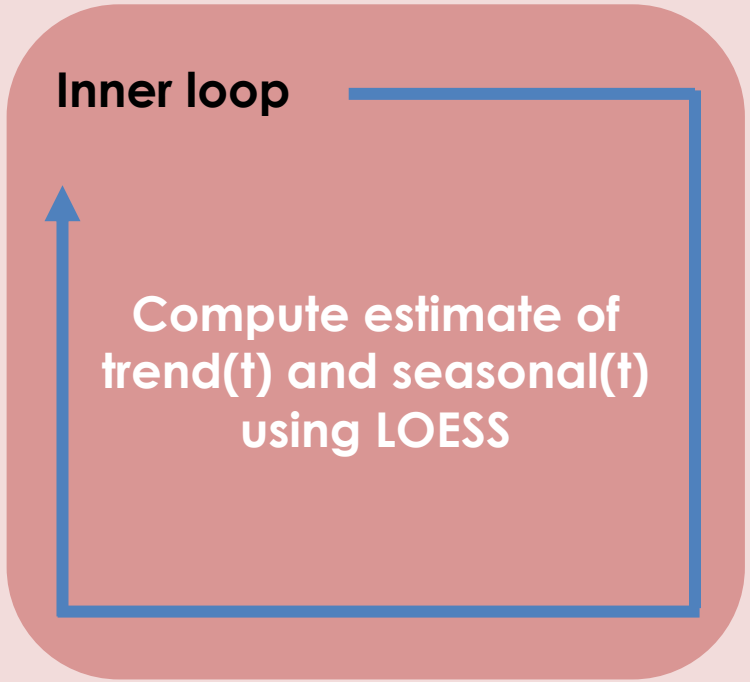
Inner loop summary

ITERATE

1. **De-trend the data**: y(t) – trend(t). For the first iteration assume trend(t) = 0

2. **Extract seasonal(t)** using LOESS on a set of time series derived from y_detrend(t) called cycle-subseries. Will require us to specify the seasonal period.

3. **Extract trend(t)** by subtracting seasonal(t) from y(t) and smoothing using LOESS



Inner loop summary

ITERATE

1. **De-trend the data**: y(t) – trend(t). For the first iteration assume trend(t) = 0
2. **Extract seasonal(t)** using LOESS on a set of time series derived from y_detrend(t) called cycle-subseries. Will require us to specify the seasonal period.
3. **Extract trend(t)** by subtracting seasonal(t) from y(t) and smoothing using LOESS

How many times to iterate, n_i?

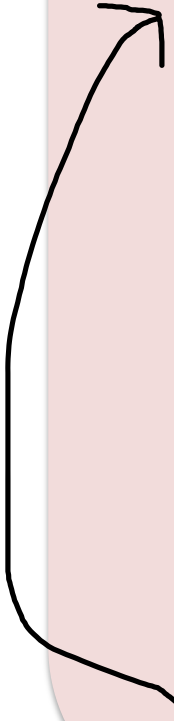
Until the outputted seasonal(t) and trend(t) are stable

Typically $n_i = 2$ or 3

Inner loop summary

ITERATE

1. **De-trend the data**: y(t) – trend(t). For the first iteration assume trend(t) = 0
2. **Extract seasonal(t)** using LOESS on a set of time series derived from y_detrend(t) called cycle-subseries. Will require us to specify the seasonal period.
3. **Extract trend(t)** by subtracting seasonal(t) from y(t) and smoothing using LOESS



Robustness to outliers

Remember in LOESS that residuals are used to re-weight data during regression

Outer loop provides these weights for all uses of LOESS in inner loop

Parameters from inner loop

Symbol	Statsmodels	Description	Typical value
n_i	inner_iter	Number of inner loops	2 or 3
n_p	period	Number of periods in a seasonal cycle	Depends on data
n_s	seasonal	Window size for LOESS for cycle-subseries (aka seasonal component)	Depends on data
d_s	seasonal_deg	Degree for LOESS for cycle-subseries (aka seasonal component)	1 or in rare cases 0 (see notebook)
n_l	low_pass	Window size for LOESS for low pass filter of cycle-subseries	n_p or the next largest odd integer
d_l	low_pass_deg	Degree for LOESS for low pass filter of cycle-subseries	1
n_T	trend	Window size for LOESS for Trend	1.5 to 2 x n_p
d_T	trend_deg	Degree for LOESS for Trend	1

The main idea