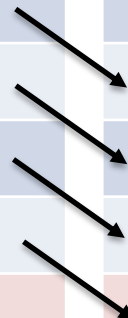


Recursive forecasting with trend features

Trend features

Recursive forecasting: features built with target

Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	NaN	NaN
2020-02-13	30	23	NaN
2020-02-14	35	30	NaN
2020-02-15	30	35	29.3
2020-02-16		30	31.7
2020-02-17			
2020-02-18			



Recursive forecasting: features built with target

Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	NaN	NaN
2020-02-13	30	23	NaN
2020-02-14	35	30	NaN
2020-02-15	30	35	29.3
2020-02-16		30	31.7
2020-02-17			
2020-02-18			

Impute or **drop** the missing values at the start of the time series.

Recursive forecasting: features built with target

Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16		30	31.7
2020-02-17			
2020-02-18			

Impute or **drop** the missing values at the start of the time series.

Recursive forecasting: features built with target

Target y		Features X	
Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16		30	31.7
2020-02-17			
2020-02-18			

`model.fit(X_train, y_train)`

`model.predict(X_pred)`

Recursive forecasting: features built with target

Target y		Features X	
Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16		30	31.7
2020-02-17			
2020-02-18			

`model.fit(X_train, y_train)`

`model.predict([30, 31.7])`



\hat{y}_{T+1}

Recursive forecasting: features built with target

Target y		Features X	
Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16	\hat{y}_{T+1}	30	31.7
2020-02-17			
2020-02-18			

`model.fit(X_train, y_train)`

`model.predict([30, 31.7])`



Recursive forecasting: features built with target

Target y		Features X	
Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16	\hat{y}_{T+1}	30	31.7
2020-02-17		\hat{y}_{T+1}	
2020-02-18			

`model.fit(X_train, y_train)`

Recursive forecasting: features built with target

Time	Target y	Features X	
	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16	\hat{y}_{T+1}	30	31.7
2020-02-17		\hat{y}_{T+1}	$\frac{\hat{y}_{T+1} + 30 + 35}{3}$
2020-02-18			

`model.fit(X_train, y_train)`

Recursive forecasting: features built with target

Target y		Features X	
Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16	\hat{y}_{T+1}	30	31.7
2020-02-17		\hat{y}_{T+1}	$\frac{\hat{y}_{T+1} + 30 + 35}{3}$
2020-02-18			

`model.fit(X_train, y_train)`

`model.predict([\hat{y}_{T+1} , ...])`



\hat{y}_{T+2}

Recursive forecasting: features built with target

Target y		Features X	
Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16	\hat{y}_{T+1}	30	31.7
2020-02-17	\hat{y}_{T+2}	\hat{y}_{T+1}	$\frac{\hat{y}_{T+1} + 30 + 35}{3}$
2020-02-18			

`model.fit(X_train, y_train)`

Recursive forecasting: features built with target

	Target y	Features X	
Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16	\hat{y}_{T+1}	30	31.7
2020-02-17	\hat{y}_{T+2}	\hat{y}_{T+1}	$\frac{\hat{y}_{T+1} + 30 + 35}{3}$
2020-02-18		\hat{y}_{T+2}	

`model.fit(X_train, y_train)`

Recursive forecasting: features built with target

Target y		Features X	
Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16	\hat{y}_{T+1}	30	31.7
2020-02-17	\hat{y}_{T+2}	\hat{y}_{T+1}	$\frac{\hat{y}_{T+1} + 30 + 35}{3}$
2020-02-18		\hat{y}_{T+2}	$\frac{\hat{y}_{T+1} + \hat{y}_{T+2} + 35}{3}$

`model.fit(X_train, y_train)`

Recursive forecasting: features built with target

Target y		Features X	
Time	y	Lag 1 y	Rolling mean y
2020-02-12	23	23	29.3
2020-02-13	30	23	29.3
2020-02-14	35	30	29.3
2020-02-15	30	35	29.3
2020-02-16	\hat{y}_{T+1}	30	31.7
2020-02-17	\hat{y}_{T+2}	\hat{y}_{T+1}	$\frac{\hat{y}_{T+1} + 30 + 35}{3}$
2020-02-18		\hat{y}_{T+2}	$\frac{\hat{y}_{T+1} + \hat{y}_{T+2} + 35}{3}$

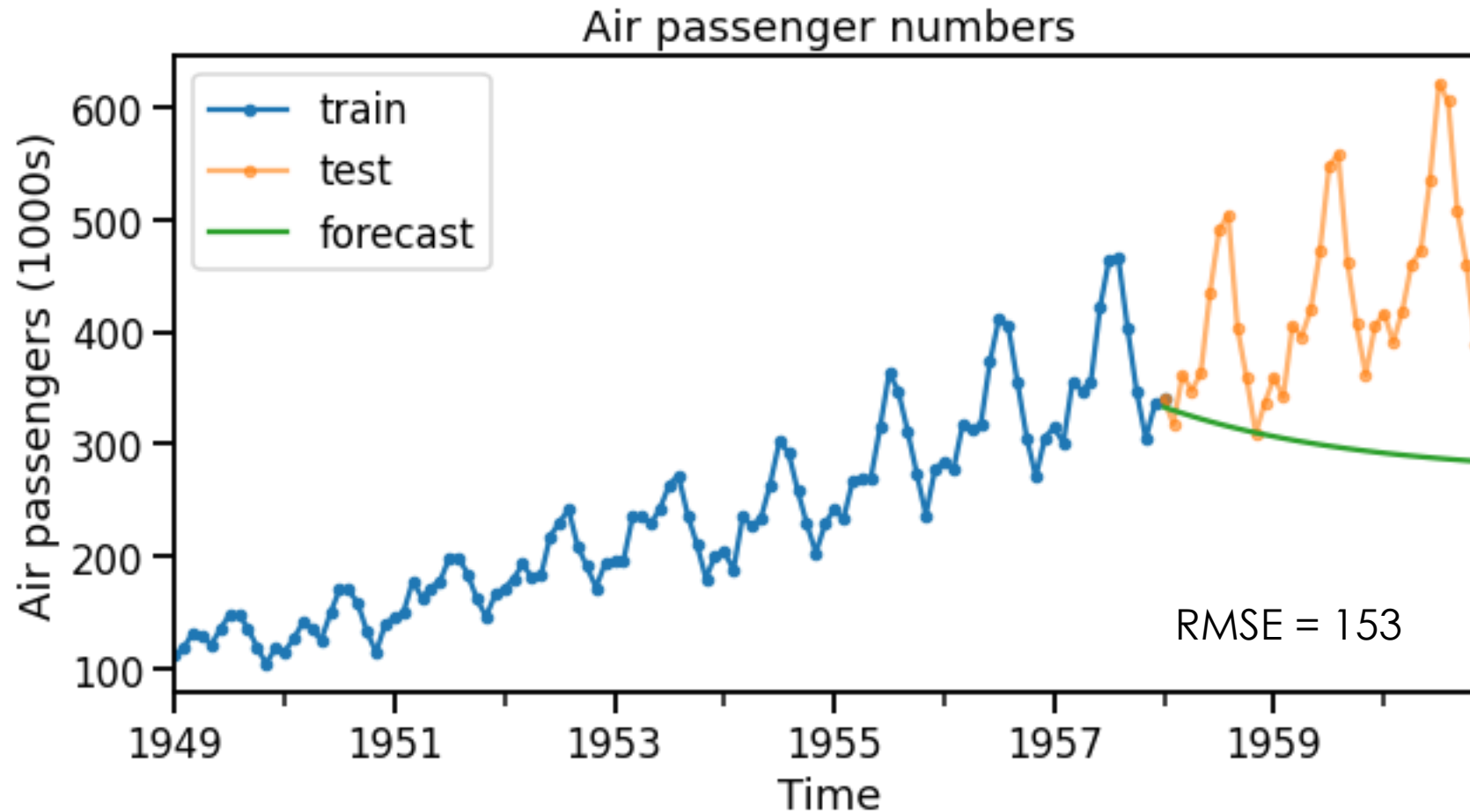
`model.fit(X_train, y_train)`

`model.predict($[\hat{y}_{T+1}, \dots]$)`



\hat{y}_{T+3}

Example: Linear regression



Features:

- Lag y of 1

Why does it decay?

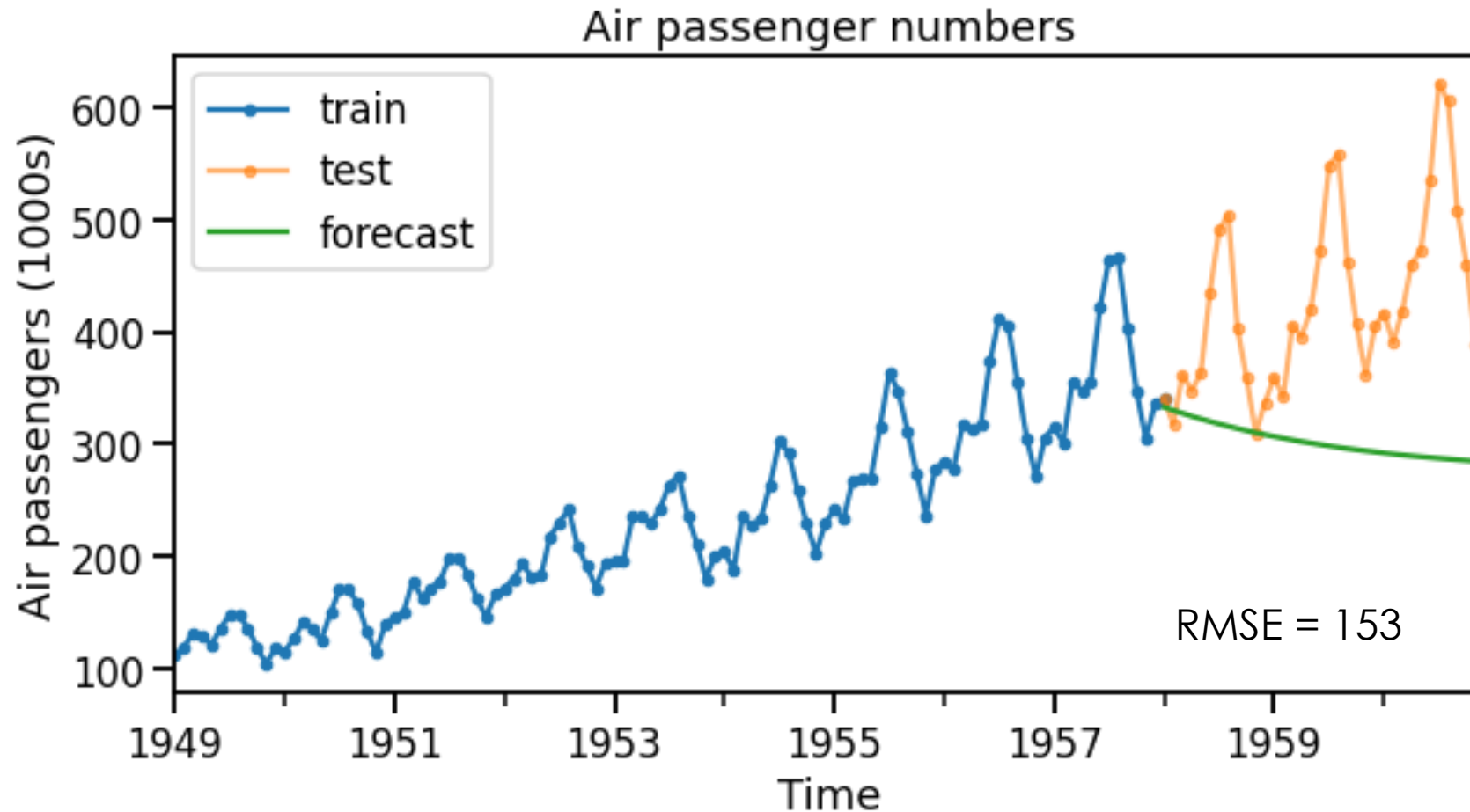
$$y_{t+1} = \beta_0 + \beta_1 y_t$$

Consider a simple case

$$y_{t+1} = \beta_1 y_t$$

If $\beta_1 < 1$ then y_{t+1} will decay exponentially as we recursively iterate forward in time.

Example: Linear regression



Features:

- Lag y of 1

Why does it decay?

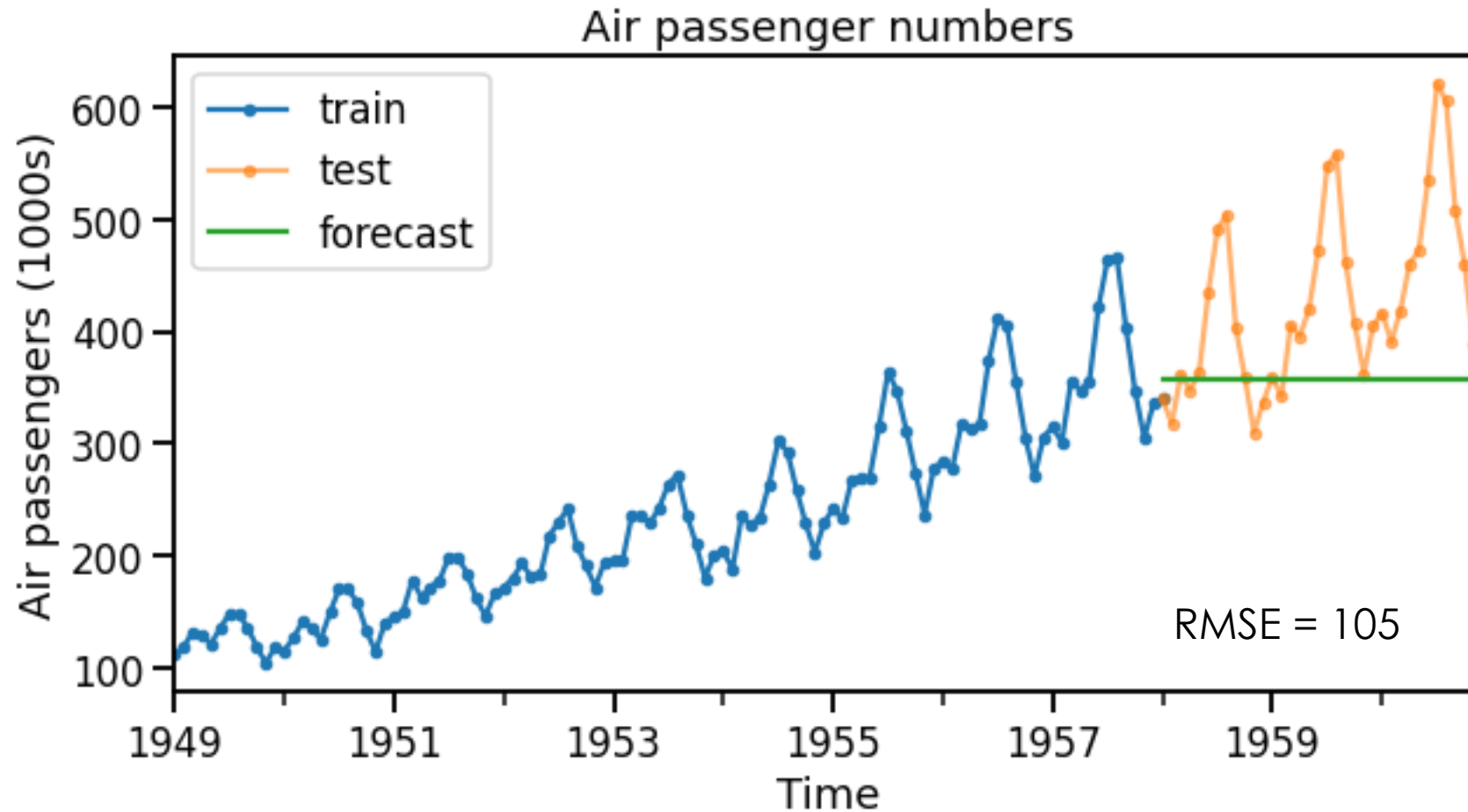
$$y_{t+1} = \beta_0 + \beta_1 y_t$$

Consider a simple case

$$y_{t+1} = \beta_1 y_t$$

If $\beta_1 > 1$ then y_{t+1} will grow exponentially as we recursively iterate forward in time.

Example: Gradient Boosted Trees



Features:

- Lag y of 1

Recursive forecasting: lags & window & time


Time	y	Lag 1 y	Rolling mean y	t (days)
2020-02-12	23	23	29.3	0
2020-02-13	30	23	29.3	1
2020-02-14	35	30	29.3	2
2020-02-15	30	35	29.3	3
2020-02-16		30	31.7	4
2020-02-17				5
2020-02-18				6

Recursive forecasting: lags & window & time

Target y		Features X			
Time	y	Lag 1 y	Rolling mean y	t (days)	
2020-02-12	23	23	29.3	0	model.fit(X_train, y_train)
2020-02-13	30	23	29.3	1	
2020-02-14	35	30	29.3	2	
2020-02-15	30	35	29.3	3	
2020-02-16		30	31.7	4	model.predict(X_pred)
2020-02-17				5	
2020-02-18				6	

Recursive forecasting: lags & window & time

Target y		Features X			
Time	y	Lag 1 y	Rolling mean y	t (days)	
2020-02-12	23	23	29.3	0	model.fit(X_train, y_train)
2020-02-13	30	23	29.3	1	
2020-02-14	35	30	29.3	2	
2020-02-15	30	35	29.3	3	
2020-02-16		30	31.7	4	model.predict(X_pred)
2020-02-17				5	
2020-02-18				6	



 \hat{y}_{T+1}

Recursive forecasting: lags & window & time

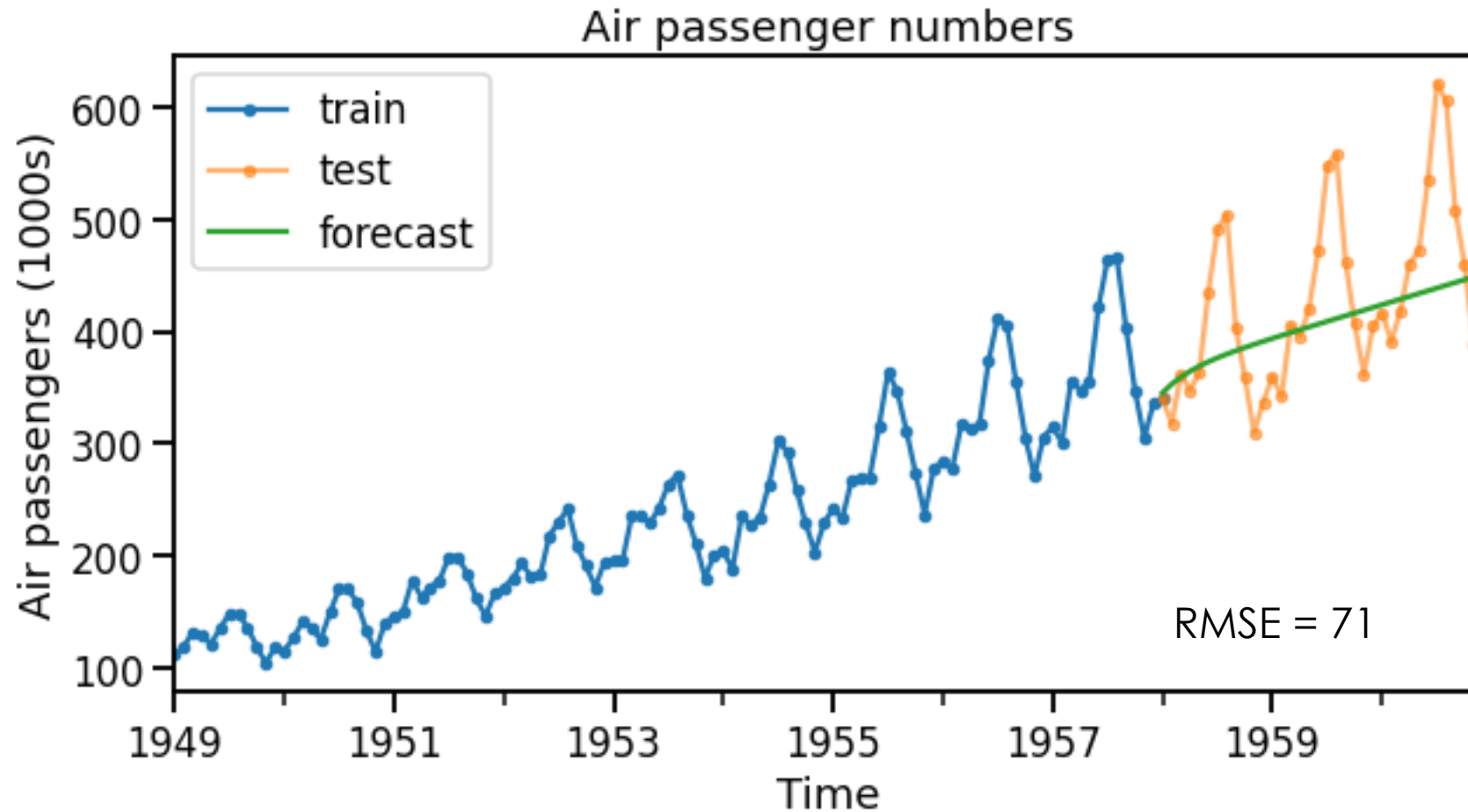
Target y		Features X			
Time	y	Lag 1 y	Rolling mean y	t (days)	
2020-02-12	23	23	29.3	0	model.fit(X_train, y_train)
2020-02-13	30	23	29.3	1	
2020-02-14	35	30	29.3	2	
2020-02-15	30	35	29.3	3	
2020-02-16	\hat{y}_{T+1}	30	31.7	4	model.predict(X_pred)
2020-02-17		\hat{y}_{T+1}	$\frac{\hat{y}_{T+1} + 30 + 35}{3}$	5	
2020-02-18				6	

Recursive forecasting: lags & window & time

Target y		Features X			
Time	y	Lag 1 y	Rolling mean y	t (days)	
2020-02-12	23	23	29.3	0	model.fit(X_train, y_train)
2020-02-13	30	23	29.3	1	
2020-02-14	35	30	29.3	2	
2020-02-15	30	35	29.3	3	
2020-02-16	\hat{y}_{T+1}	30	31.7	4	model.predict(X_pred)
2020-02-17		\hat{y}_{T+1}	$\frac{\hat{y}_{T+1} + 30 + 35}{3}$	5	
2020-02-18				6	


 \hat{y}_{T+2}

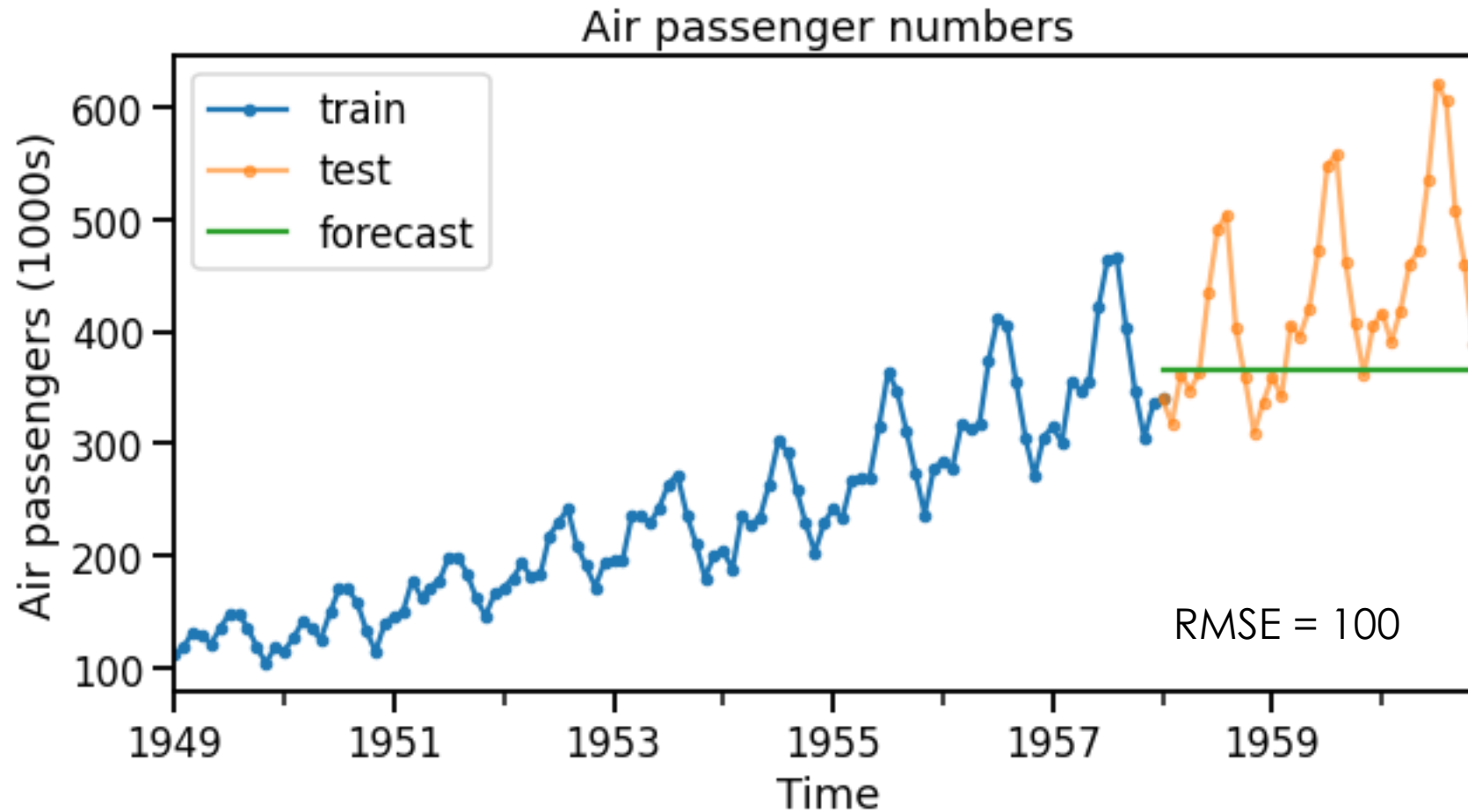
Example: Linear regression



Features:

- Lag y of 1
- Time since start (t)

Example: Gradient Boosted Trees



Features:

- Lag y of 1
- Time since start (t)