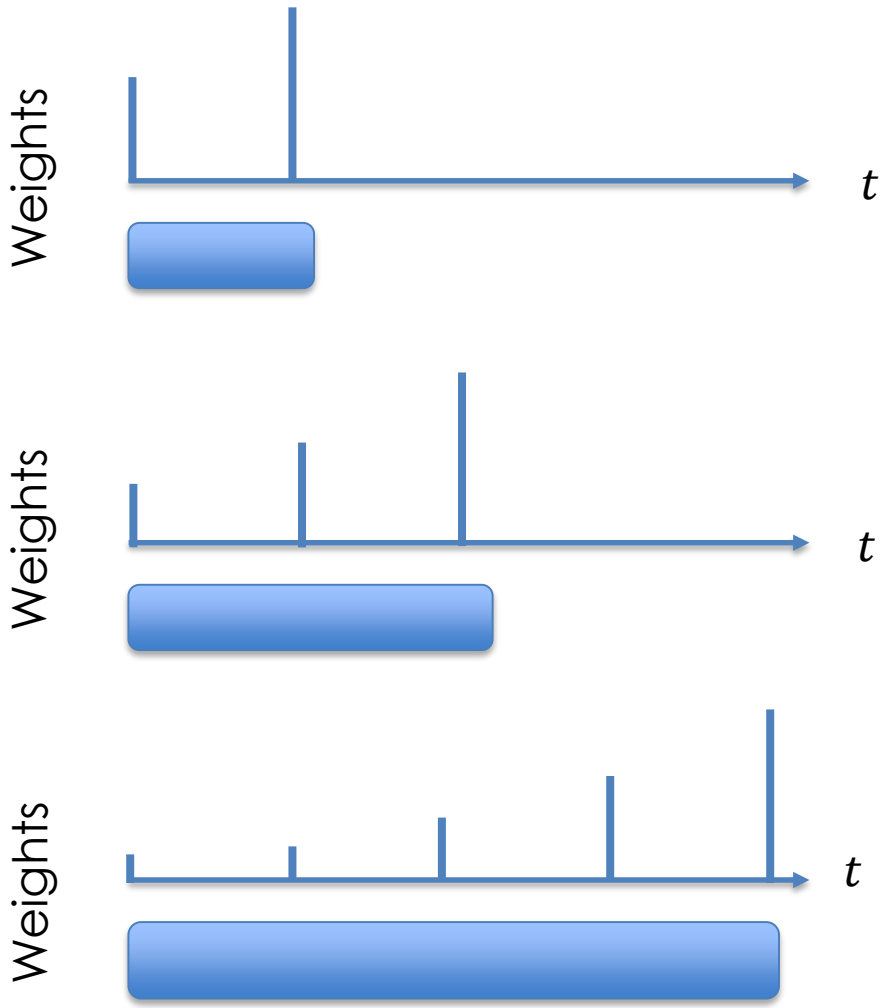
Exponential weights: part 2

Window features

Exponential weights and expanding windows

- Exponential weights are commonly applied to expanding windows.
- Allows use of the whole history but weighs recent values more than the distant past.
- The weights change as the window expands.
- Exponentially weighted moving statistics.

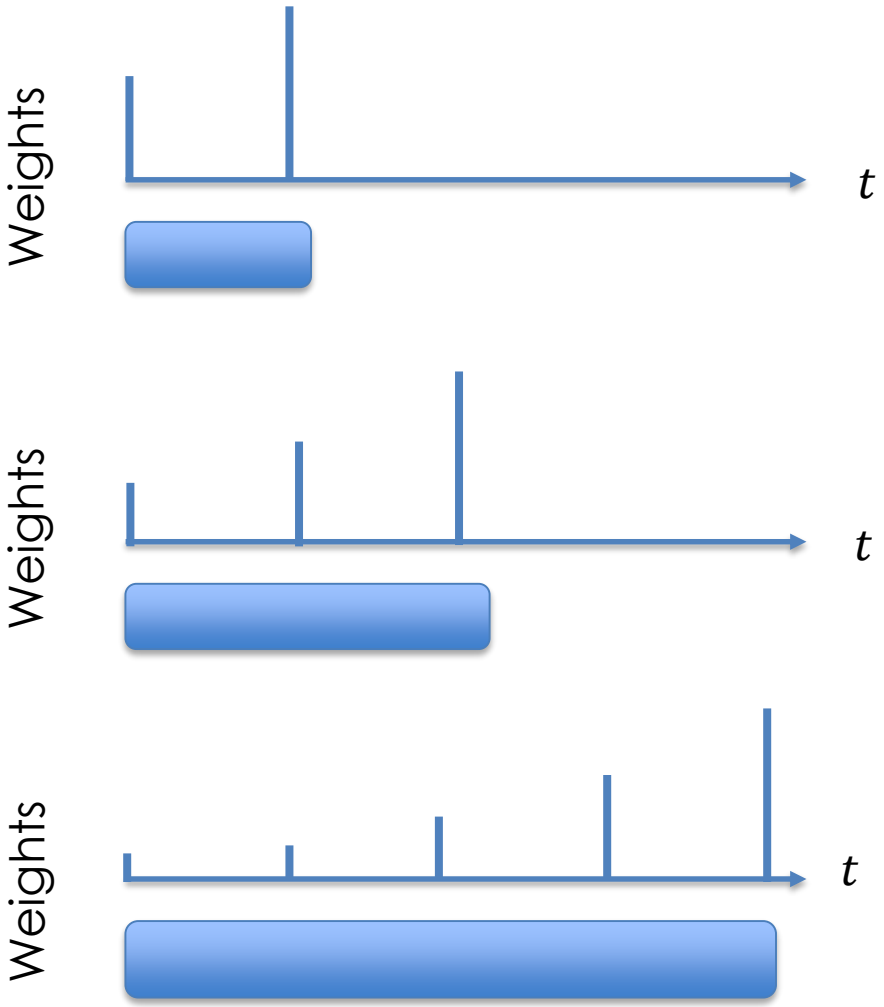


Exponential weights and expanding windows

- When computing the mean of the target this becomes Simple Exponential Smoothing (SES):

$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \cdots$$

- Simple exponential smoothing is used commonly as a baseline forecasting model.
- α is learned by minimizing the forecasting error, for example, MSE = $\sum_t (\hat{y}_t - y_t)^2$.



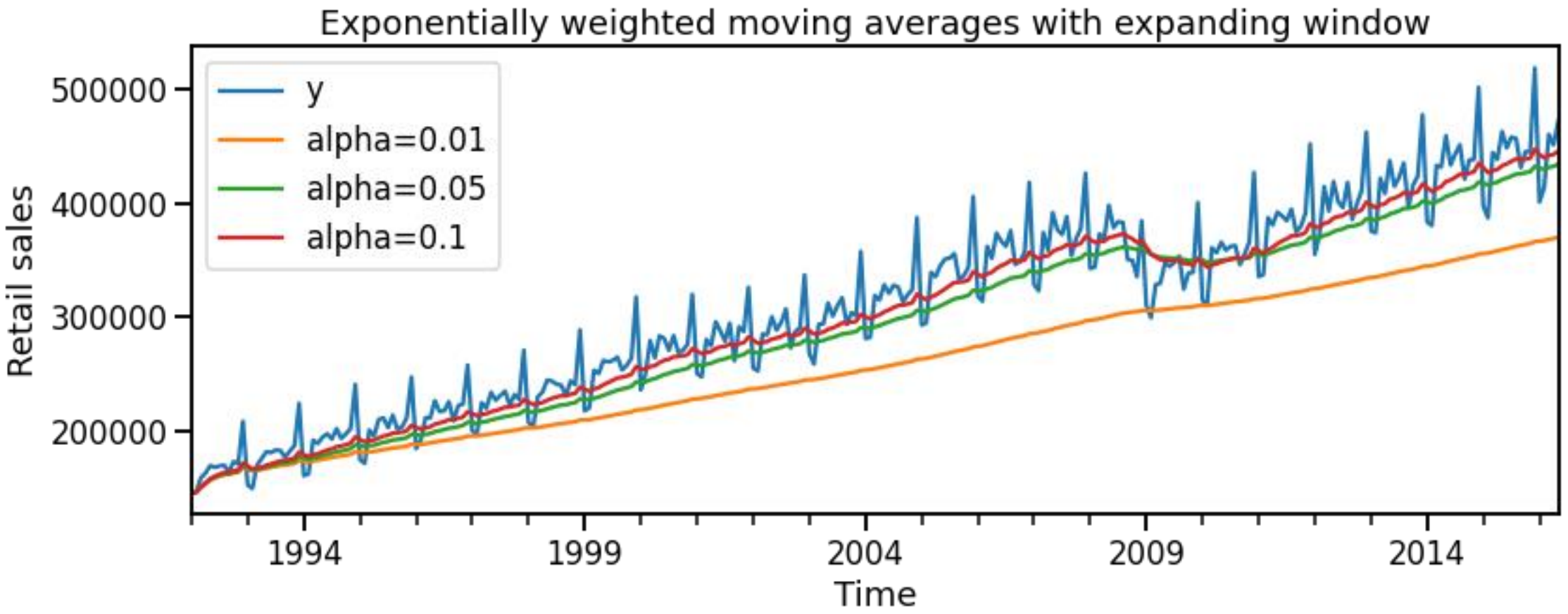
How to pick α?

- If using target variable and the mean:
  - the α from a SES model is a good start.
  - other α could still be helpful in presence of other features though…
- If using other features and/or other metrics:
  - then trial and error is required (e.g., try many values of α and use a feature selection method like LASSO).

```python
from statsmodels.tsa.api import SimpleExpSmoothing
```

```python
result = SimpleExpSmoothing(df["y"]).fit(optimized=True)
alpha = result.params["smoothing_level"]
```

Example: Retail sales



Implementation in Pandas

```
# Expanding window, exponential weights
df[['y_ewm_mean', 'y_ewm_std']] = (
                          df['y'].ewm(alpha=0.5)
                                 .agg(['mean', 'std']) # Multiple statistics
                                 .shift(periods=1) # Lag to avoid data leakage
                           )
```

Implementation in Pandas

ds	y	y_ewm_mean	y_ewm_std
1992-01-01	146376	NaN	NaN
1992-02-01	147079	146376.00	NaN
1992-03-01	159336	146844.67	497.10
1992-04-01	163669	153982.57	8182.53
1992-05-01	170068	159148.67	8137.89
...	...	...	...
2016-01-01	400928	481216.66	45733.06
2016-02-01	413554	441072.33	58848.17
2016-03-01	460093	427313.17	44894.61
2016-04-01	450935	443703.08	37559.38
2016-05-01	471421	447319.04	26925.20

Summary

Exponential weights requires specifying another parameter α.

α determines how quickly the weights decay going back in time.

Exponential weights can be used with expanding window functions to give less weight to the distant past.