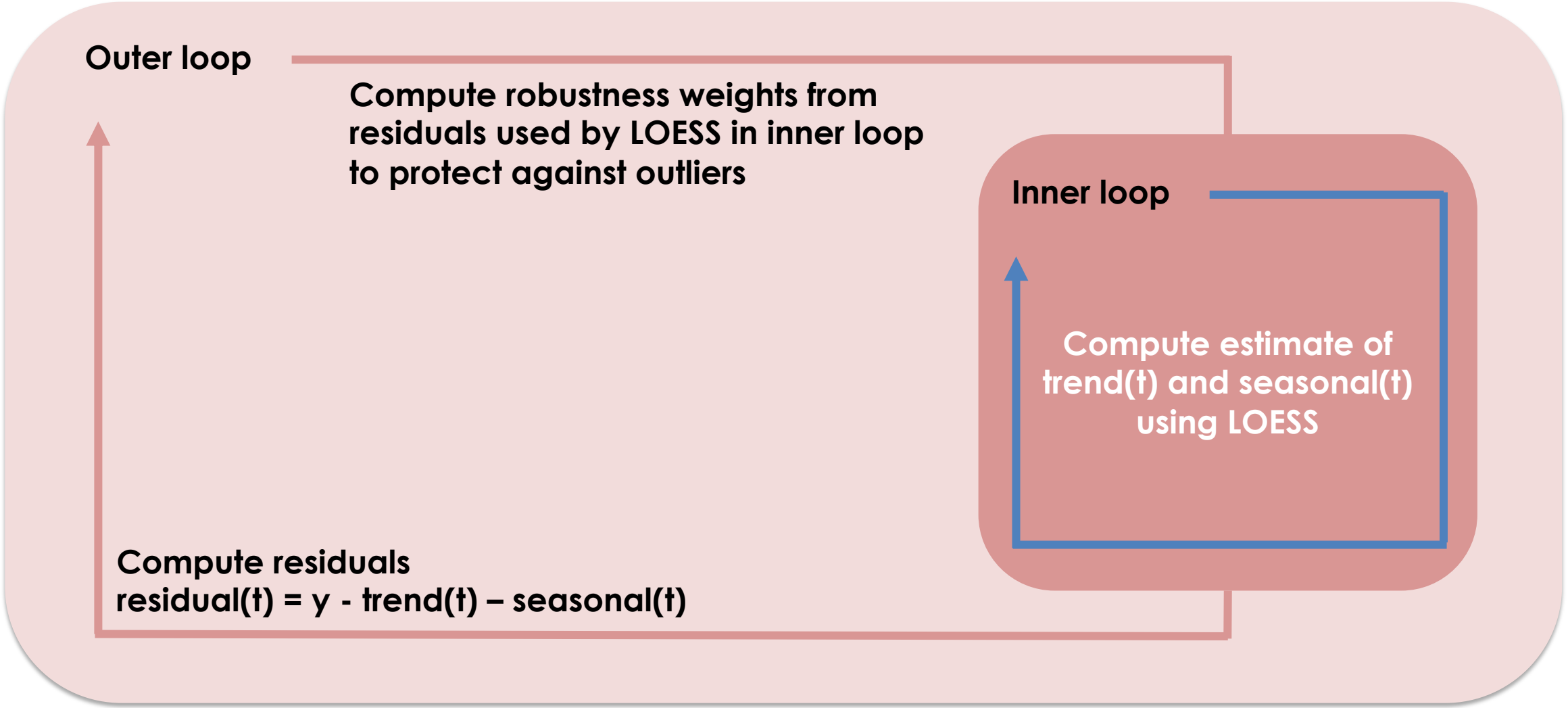
STL (Theory) – Outer Loop

Time series decomposition

The main idea



Outer loop summary

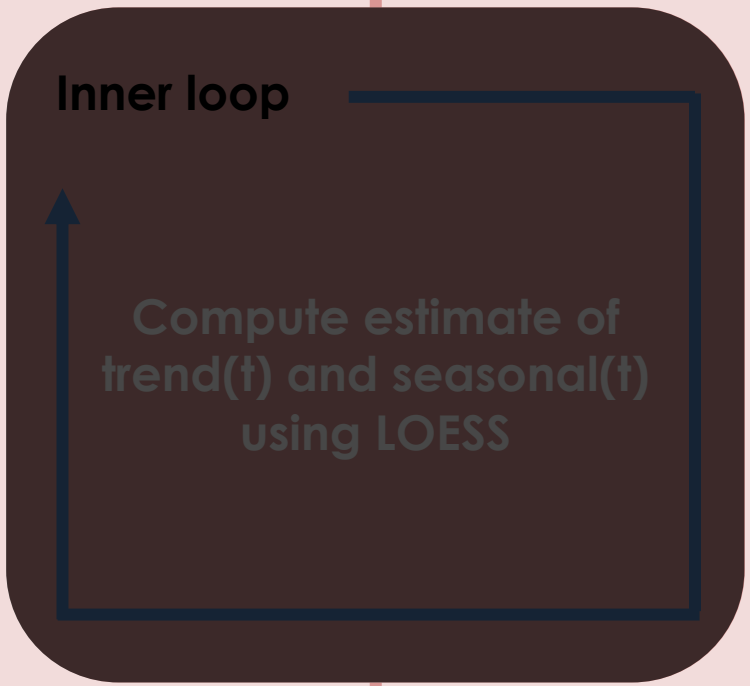
Outer loop

ITERATE

1. **Extract seasonal(t) and trend(t)** from inner loop

2. **Compute residuals:**
   y(t) – trend(t) – seasonal(t)

3. **Compute weights from residuals** ρ_t to pass to LOESS in the inner loop . This is to down-weight outliers.



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Inner loop

Compute estimate of trend(t) and seasonal(t) using LOESS

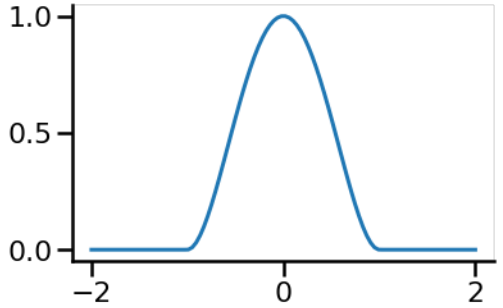
Compute weights from residuals

1. Compute residuals: $R_t = y_t - T_t - S_t$

2. Compute robustness weights: $\rho_t = B(\frac{|R_t|}{h})$ where $h = 6 \times median(|R_t|)$

3. Use ρ_t as robustness weights in LOESS in the inner loop for 1) cycle-subseries smoothing and 2) trend smoothing. For the first iteration set $\rho_t = 1$

$$B(x) = \begin{matrix} (1 - x^2)^2, & |x| < 1 \\ 0, & |x| \geq 1 \end{matrix}$$



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ρ_t

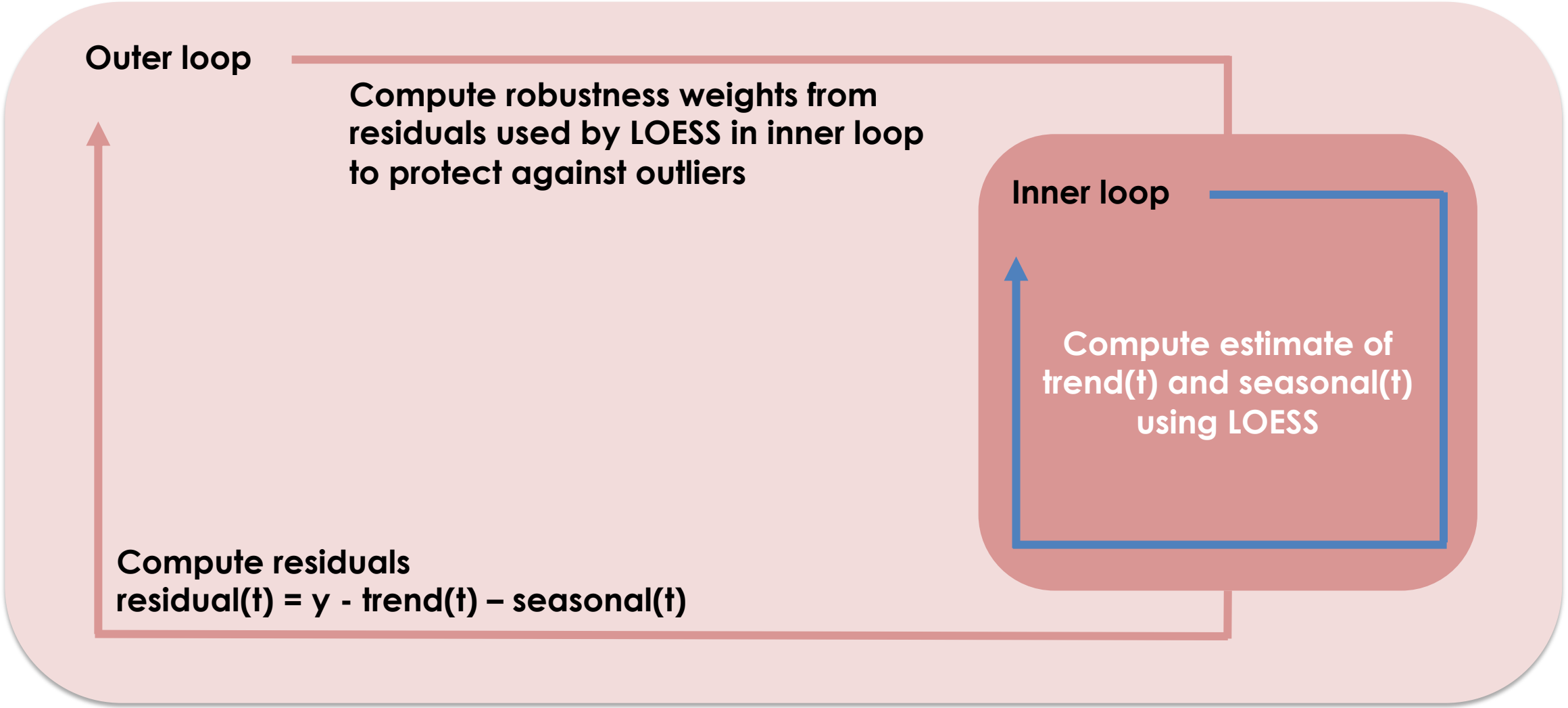
Inner loop

Compute estimate of trend(t) and seasonal(t) using LOESS

Parameters from outer loop

Symbol	Statsmodels	Description	Typical value
n_o	outer_iter	Number of iterations in the outer loop	1 or 2
N/A	robust	A flag to indicate whether to use robustness weights	Set true if suspect outliers exist

The main idea



Summary

- STL extracts the seasonality and trend iteratively using LOESS
- STL is robust to outliers
- There are two main parameters to set in practice, the remaining default parameters are normally sufficient
- The seasonal component can vary in time and is not necessarily periodic