STL (Theory) – LOESS and Cycle-Subseries

Time series decomposition

Contents

LOESS PARAMETERS IN STL

CYCLE-SUBSERIES

LOESS Parameters in STL

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- f: Fraction of data for window size
  - Determines smoothness of data

→

- n: Number of data points in window size
  - Must be odd so can have centred window

- i: Number of iterations for robust regression
  - Ensures robustness to outliers

→

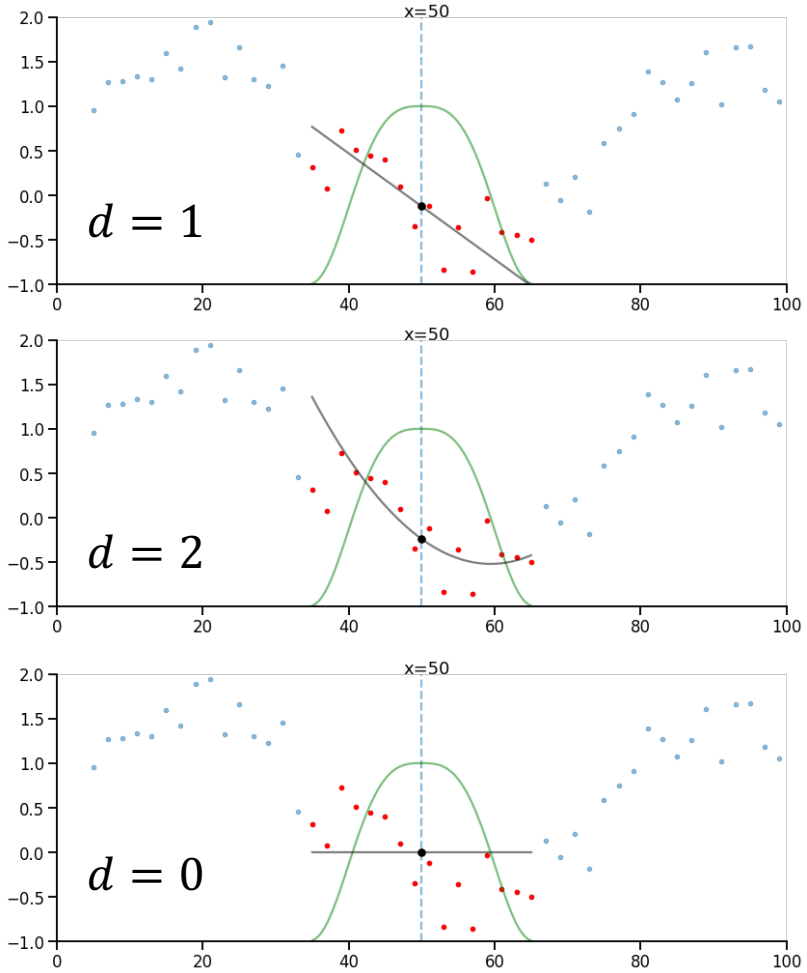
- n_o: Number of outer loop iterations
  - In a single outer loop we compute robustness weights to input into LOESS in the inner loop
  - Ensures robustness to outliers

LOESS Parameters in STL

- Previously showed that LOWESS uses a local, robust, and weighted linear regression
- LOESS uses a polynomial regression

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_d x^d$$

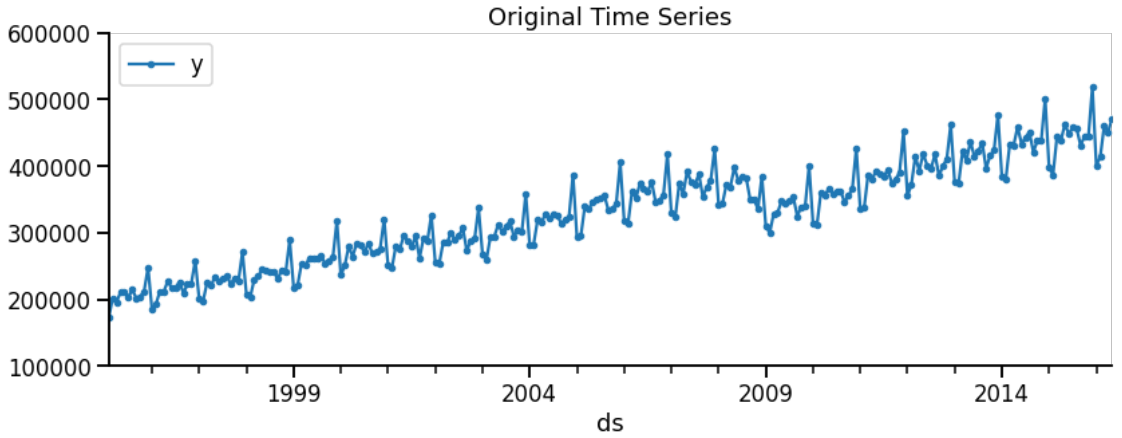
- d: Degree of polynomial to use in LOESS
- Most applications use $d = 1$ and in some rarer cases $d = 2$ or $d = 0$

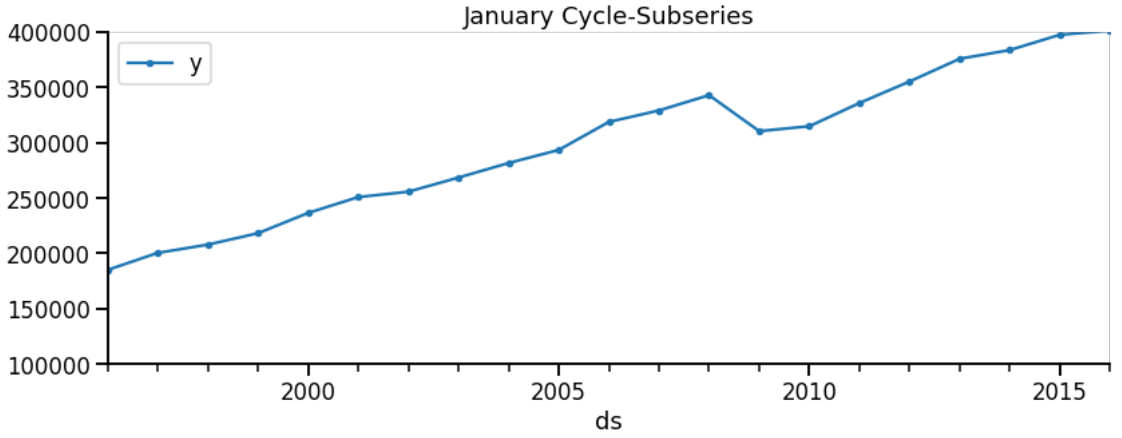


Cycle-subseries

Cycle-subseries

- n_p: Number of data points in one seasonal cycle
- Example: Monthly data with yearly seasonality, $n_p = 12$
- Cycle-subseries is the time series formed from looking at the value of each period within a seasonal cycle over time
- Example: The time series of all January values over multiple years





Cycle-subseries

Year-Month	y
2011-Jan	112
2011-Feb	146
2011-Mar	80
2011-Apr	90
…	…

Month/Year	2011	2012	2013	…
Jan	112	134	156	…
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Note relation to classical decomposition for seasonality:

1) De-trend original time series
2) Average over seasonal index

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Month/Year	2011	2012	2013	…	Mean
Jan	112	134	156	…	130
Feb	146	145	151	…	148
Mar	80	85	86	…	82
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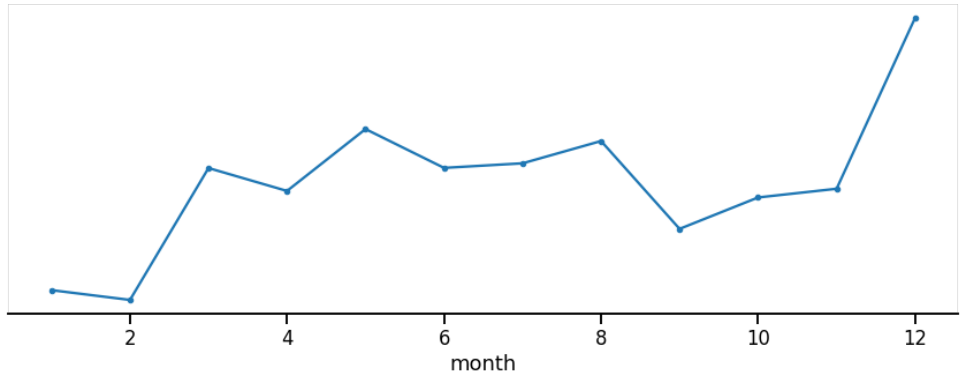

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This view of the data motivates a seasonality that can change with time (i.e., each year)!

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Useful to enable STL to extract a time varying seasonal component

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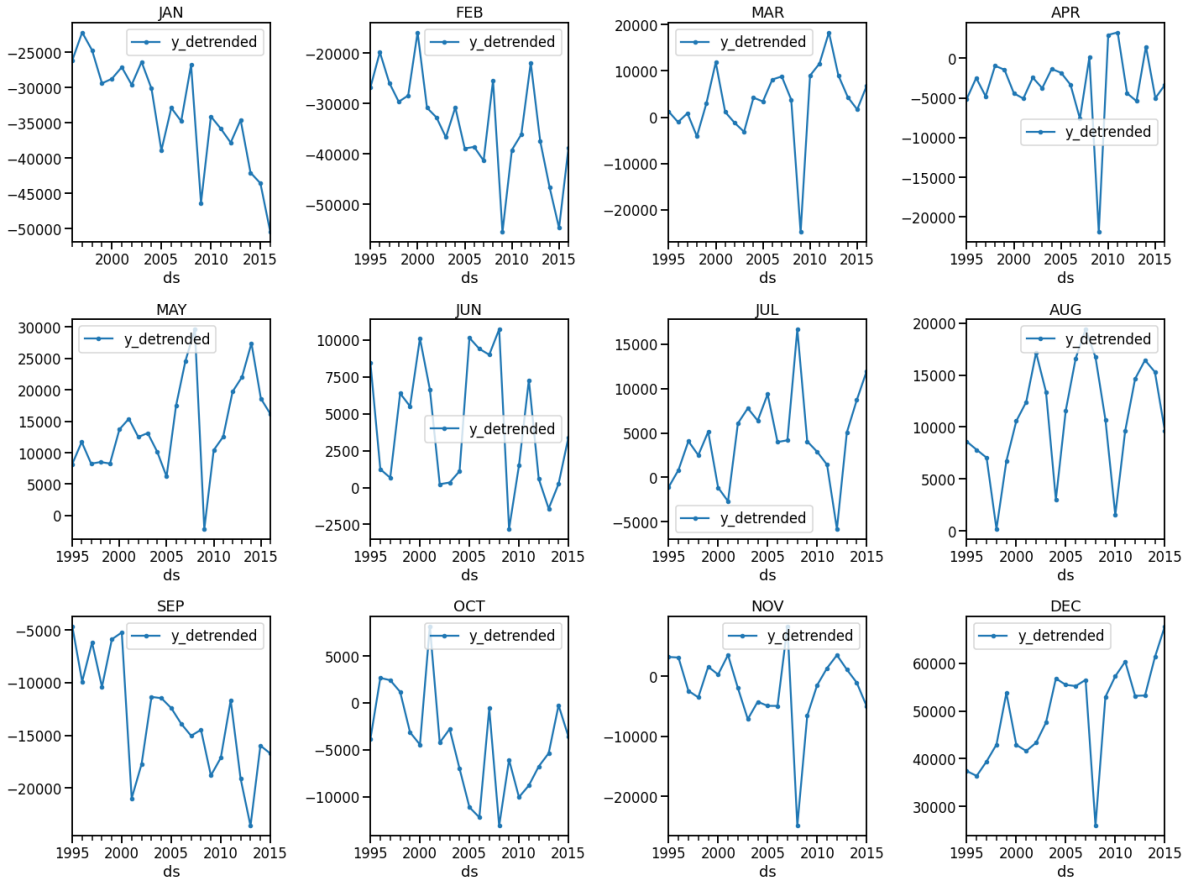
Cycle-subseries

- If seasonal cycle has n_p periods then there are n_p cycle-subseries
- In this example, $n_p = 12$ hence 12 one cycle-subseries for each month
- This view allows us to think about how seasonality changes with time and how we model it
- Example: Simple average to get constant seasonality over time like in classical decomposition



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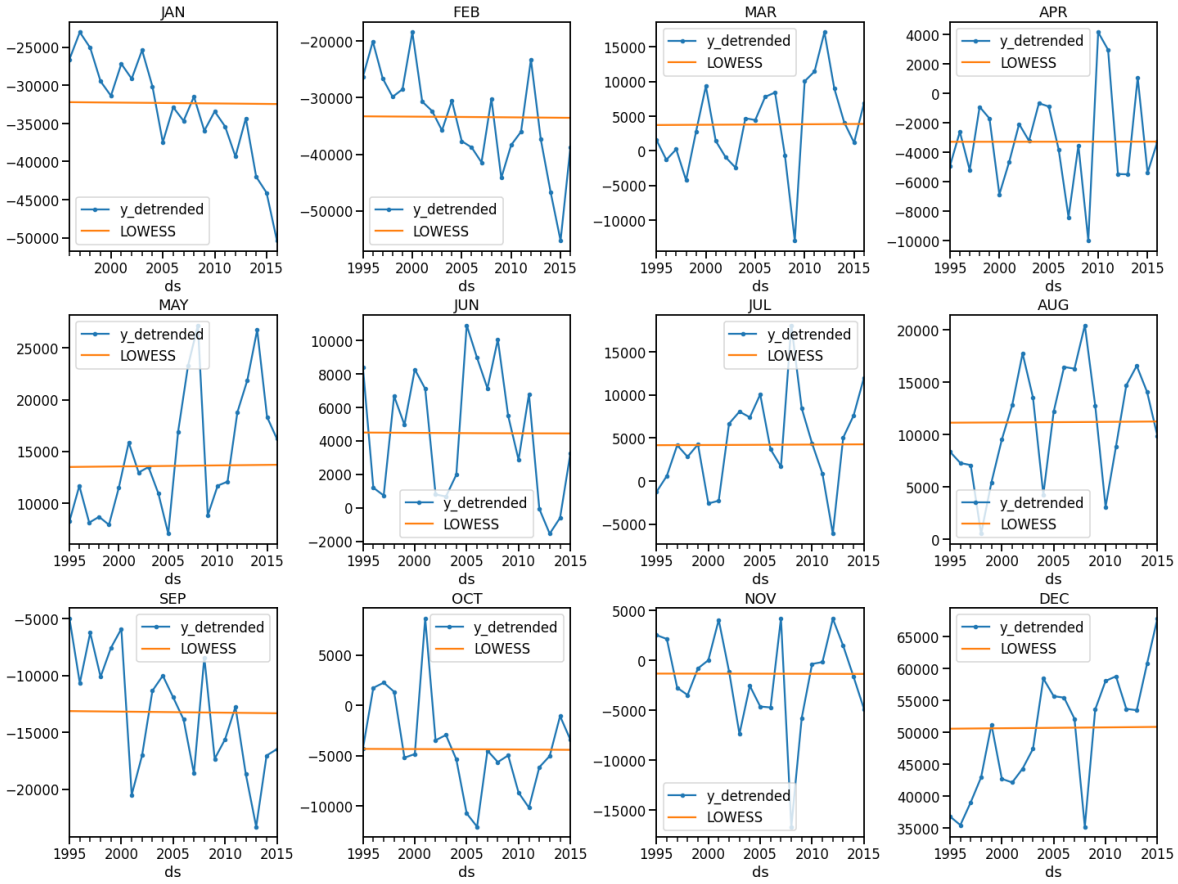

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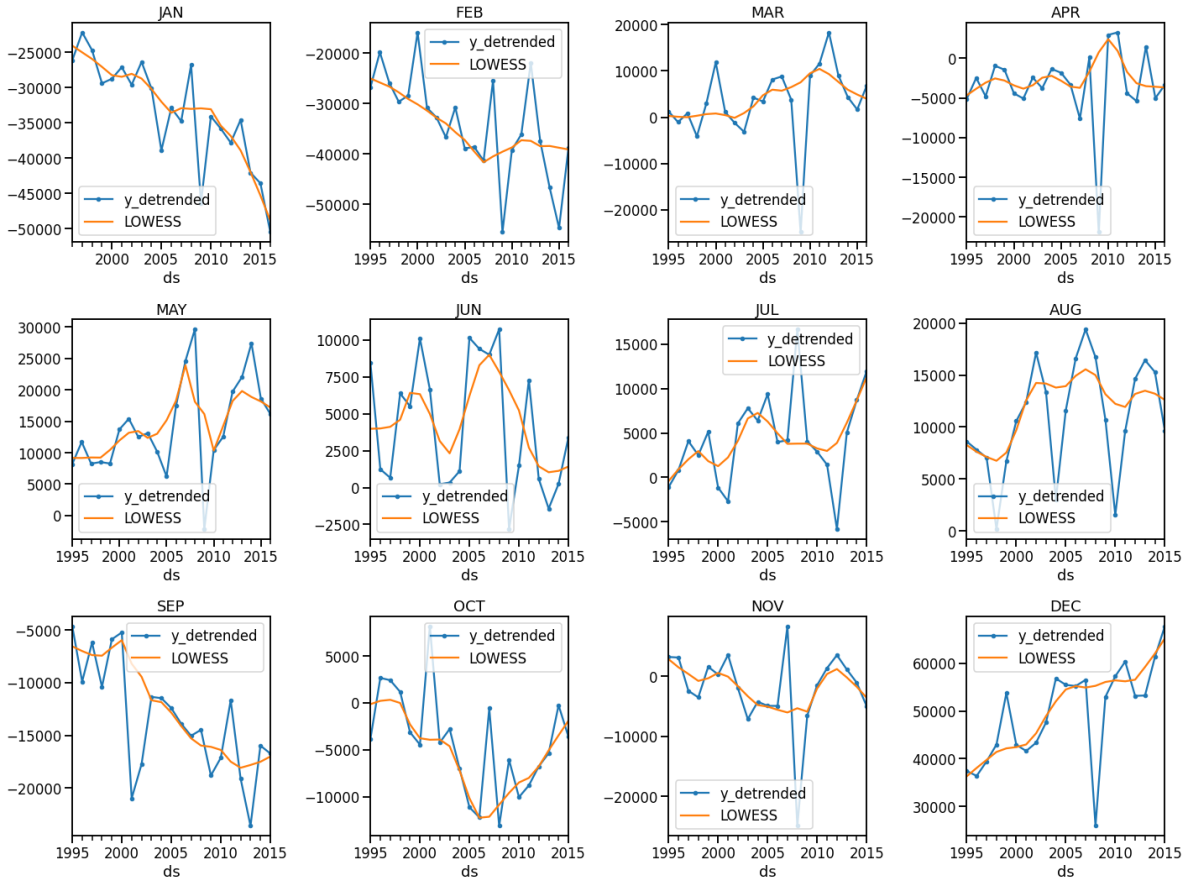

Summary

LOESS parameters used in STL are different from LOWESS

Cycle-subseries is a time series of one of the seasonal indexes

Cycle-subseries is a useful way to look at the data to understand and model seasonality