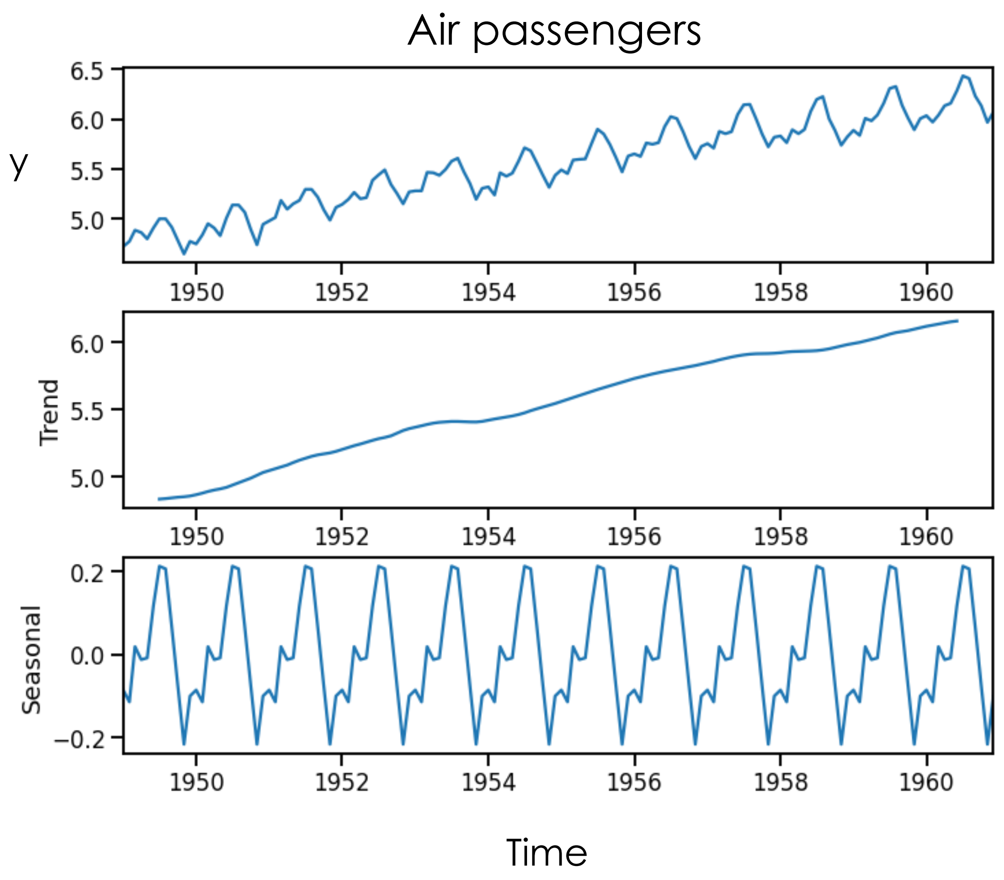
White noise

Time series decomposition

Contents

Motivation

- We will want to show how tools we introduce later (e.g., lag plots, correlation functions) behave for time series with various properties.
- Two properties already covered in the course so far are trend and seasonality.
- In this lecture we discuss a new property that can characterise the behaviour of a time series: white noise.



White noise

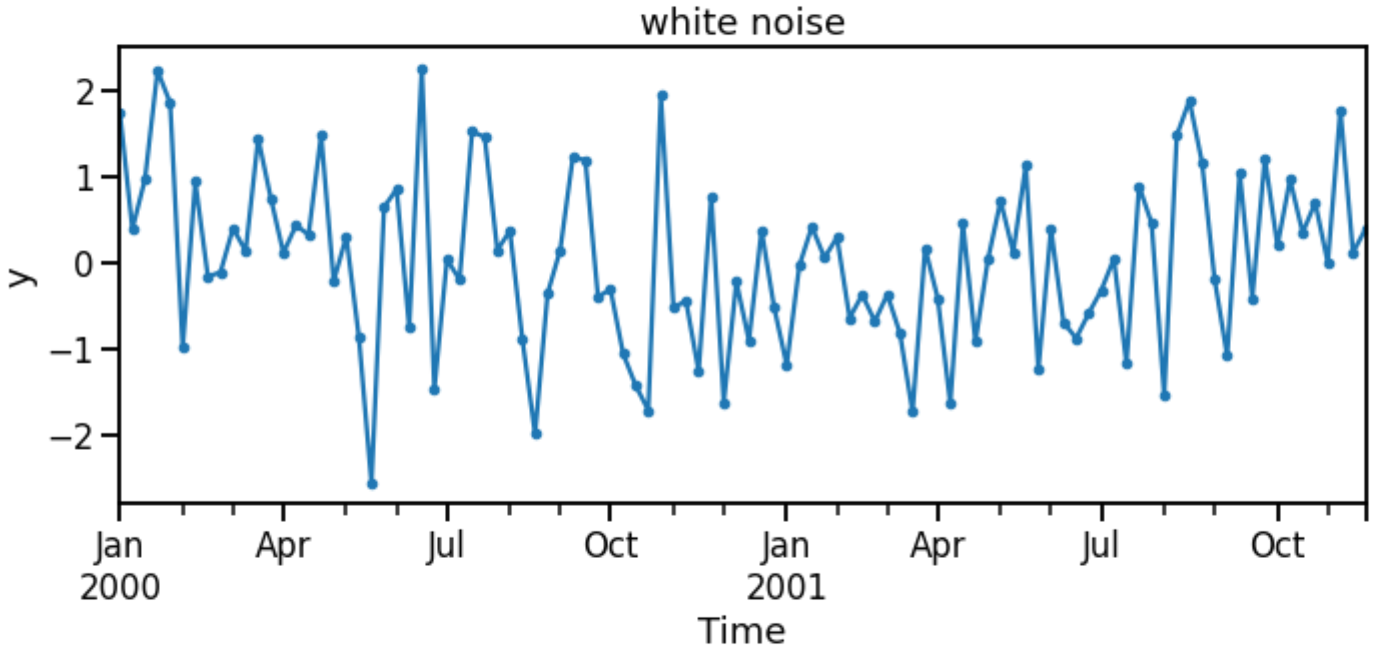
- Let's generate a time series y_t by taking a random sample from any distribution of our choice (e.g., Normal distribution with zero mean and unit variance aka $N(0,1)$) at each time step t .

```
num_timesteps = 100 # Length of time series we want
np.random.seed(0) # Ensures we generate the same random numbers every time
y = np.random.normal(loc=0, scale=1, size=num_timesteps)
ts = pd.date_range(start='2000-01-01', periods=num_timesteps, freq='W')
df = pd.DataFrame(data={'y': y}, index=ts)
```

White noise

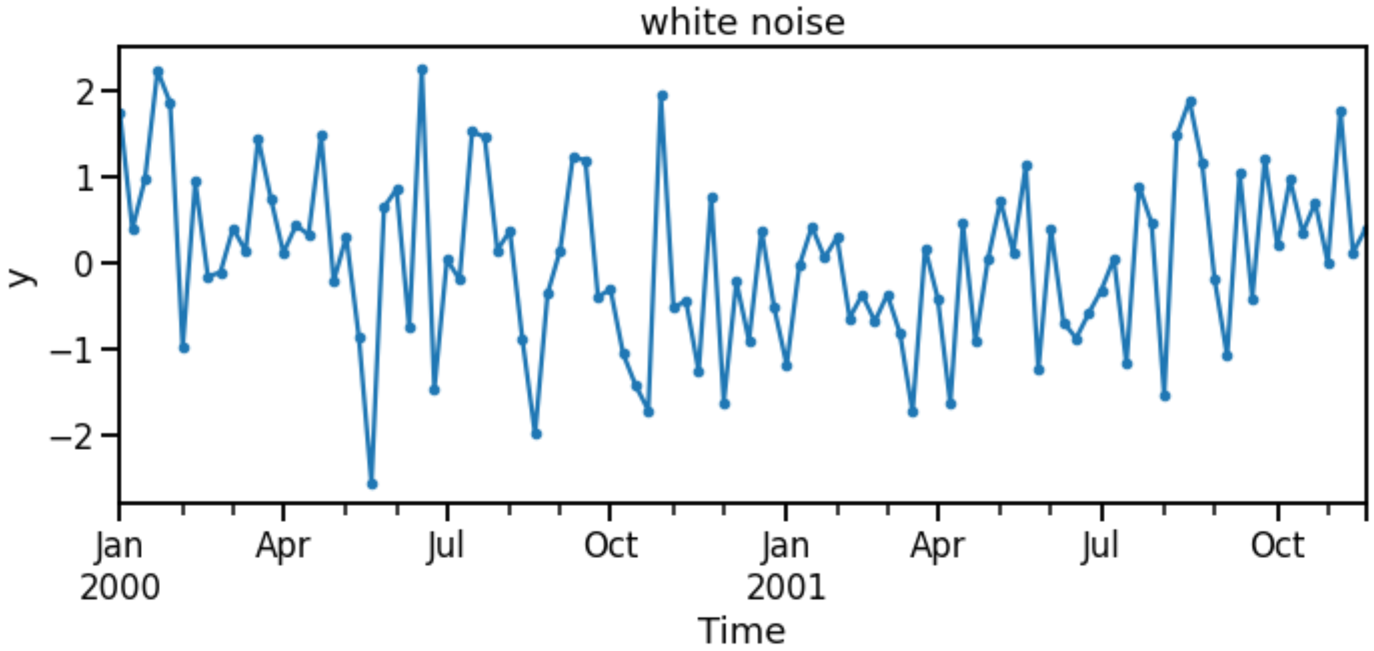
- Let’s generate a time series y_t by taking a random sample from any distribution of our choice (e.g., Normal distribution with zero mean and unit variance aka $N(0,1)$) at each time step t .

- Each value of y_t is **identically** and **independently** distributed. There is no correlation between any two points in time. This is called white noise.



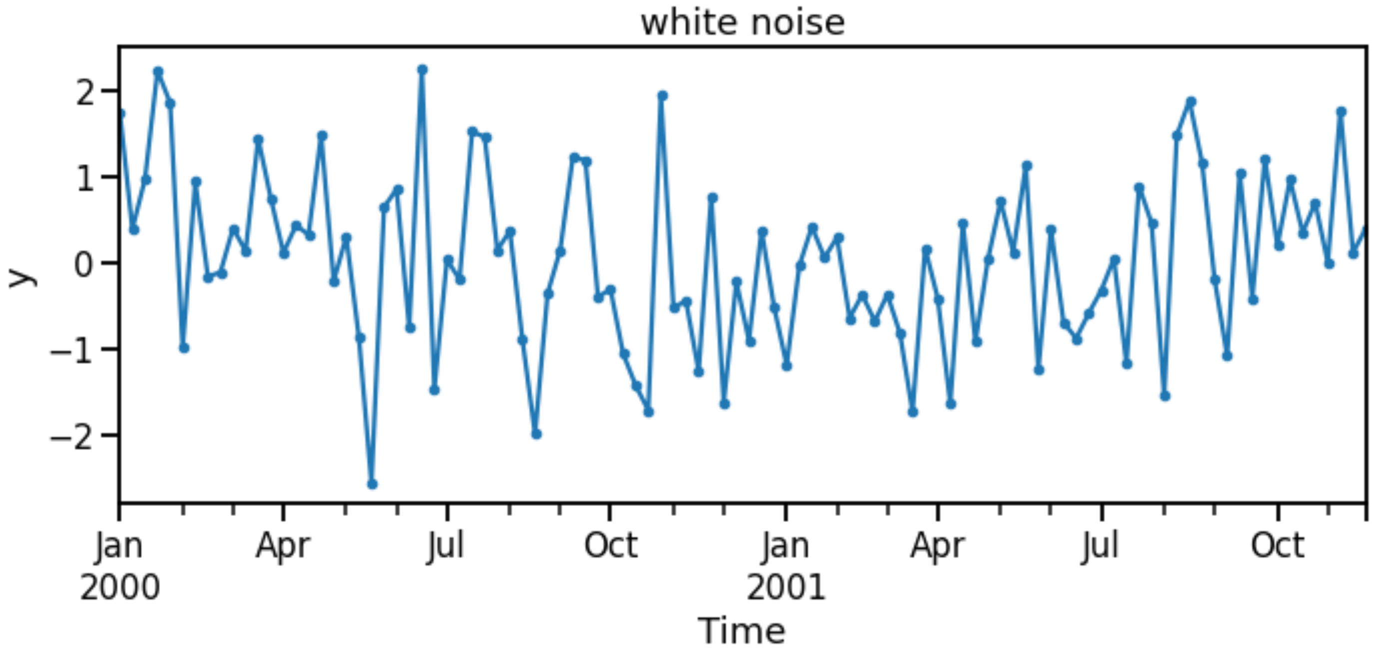
White noise

- If we sample from $N(0,1)$ then it is known as Gaussian white noise.
- It is common to see this written as: $y_t = \epsilon_t$ where $\epsilon_t \sim N(0,1)$



White noise

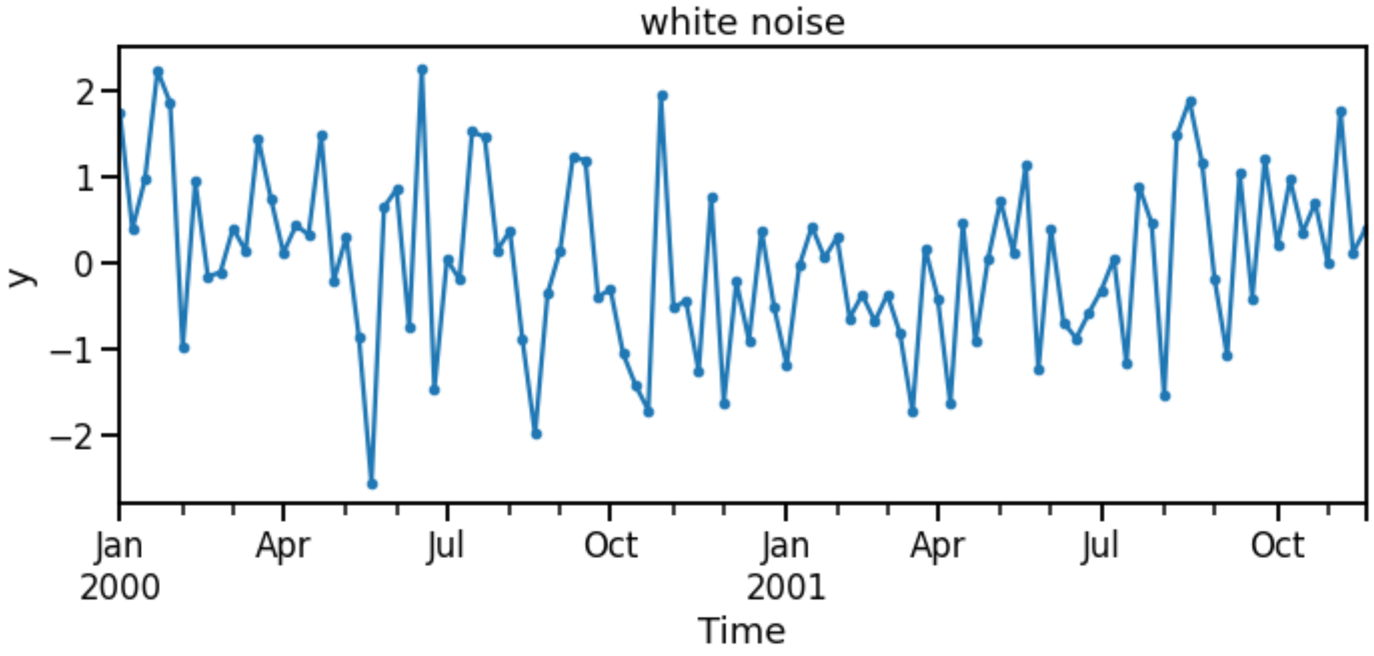
- White noise has no predictive information in past values as there is no correlation at any two points in time.
- It is useful to see how the tools we introduce later in the course behave when a time series is just white noise or contains white noise.



White noise

- An important application of white noise is in the analysis of the residuals of a forecast or time series decomposition.

- The residuals are the difference between the actuals, $\mathrm{y_t}$, and estimated values, $\hat{y}_t$, obtained through a forecast or time series decomposition.



White noise applied to residuals

- It’s common to think of time series as having a non-random component $x(t)$ (e.g., trend, seasonality, autoregressive) & a noise component $\epsilon(t)$

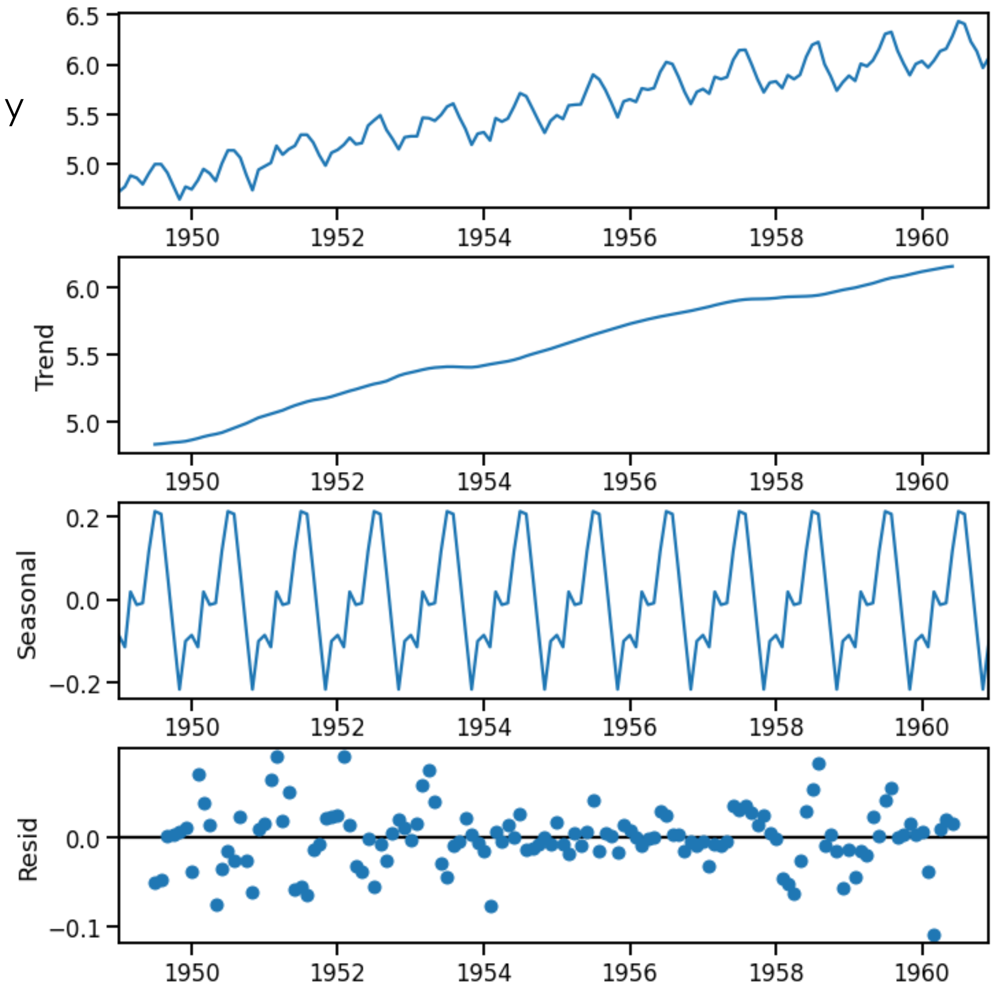
$$y(t) = x(t) + \epsilon(t)$$

where $\epsilon(t)$ is modelled as white noise to represent random effects for example from:

  - Imperfect sensors (e.g., air pollution, temperature, humidity etc.)

  - Random fluctuations in the underlying process (e.g., purchasing behaviours and sales data)

Air passengers



Time

White noise applied to residuals

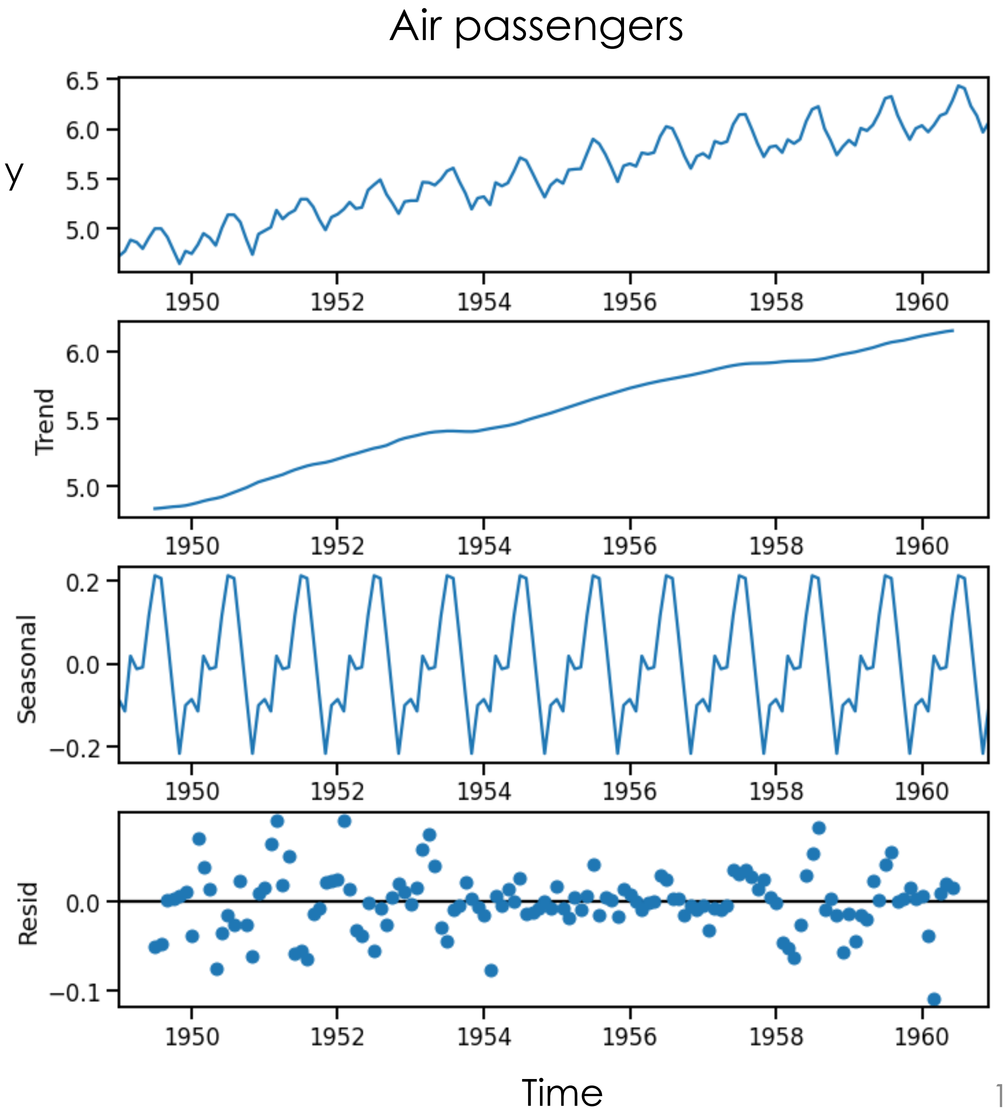
- A forecast or time series decomposition, $\hat{x}(t)$, can be thought of as estimating the non-random component $x(t)$.

- Assume we estimate perfectly, $\hat{x}(t) = x(t)$, then the residual, would be equal to white noise:

$$y(t) = x(t) + \epsilon(t)$$

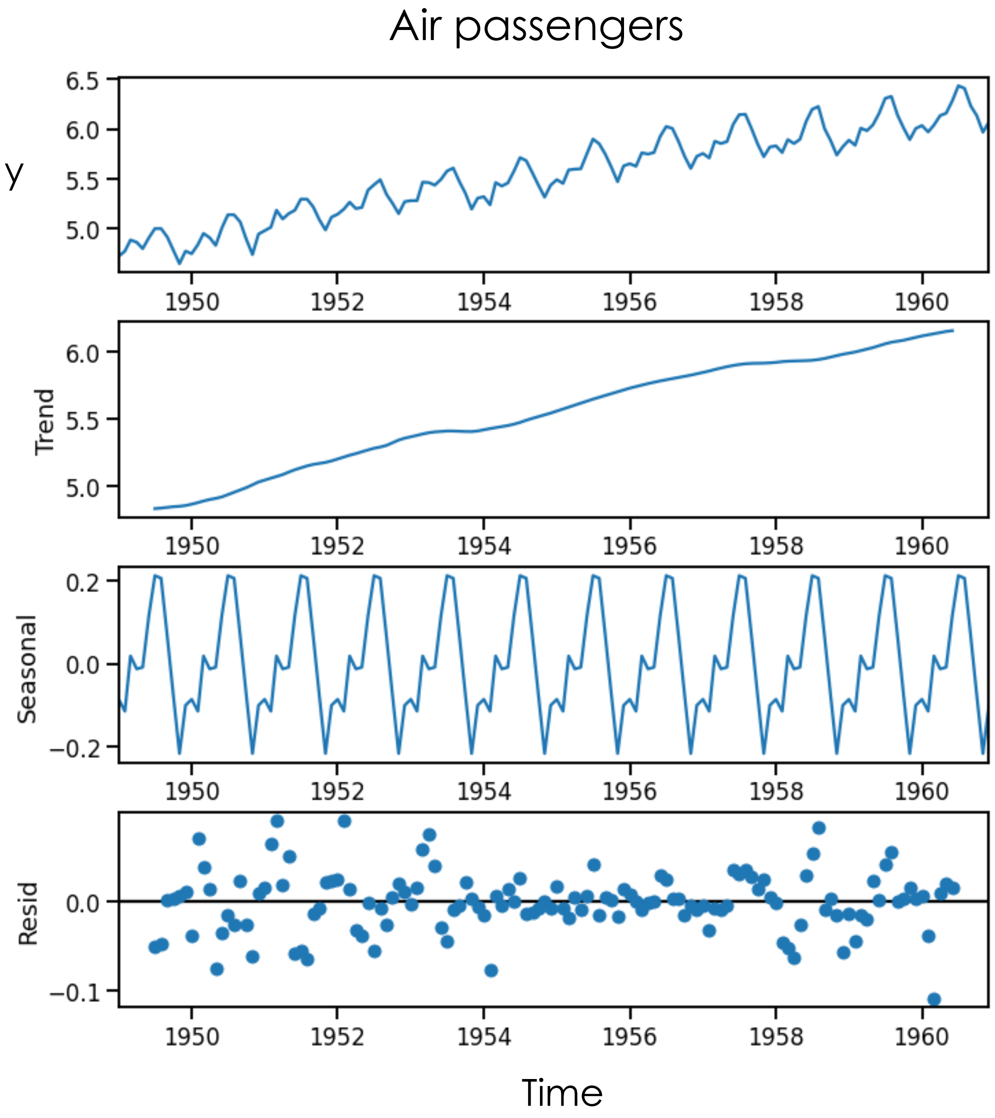
$$r(t) = \mathrm{y(t)} - \hat{x}(t) = y(t) - x(t) = \epsilon(t)$$

- Hence, if the residuals of our forecast or decomposition look like white noise it means there is no more predictive information to be extracted from the data.



White noise applied to residuals

- Some simple checks to see if your residuals are like white noise:
  - Mean is zero.
  - Mean and variance don't change over time (implies no trend or seasonality in residuals).
  - No autocorrelation. This means that there is no correlation between a time point and any other in the past (we discuss this in a lot more detail in later sections).



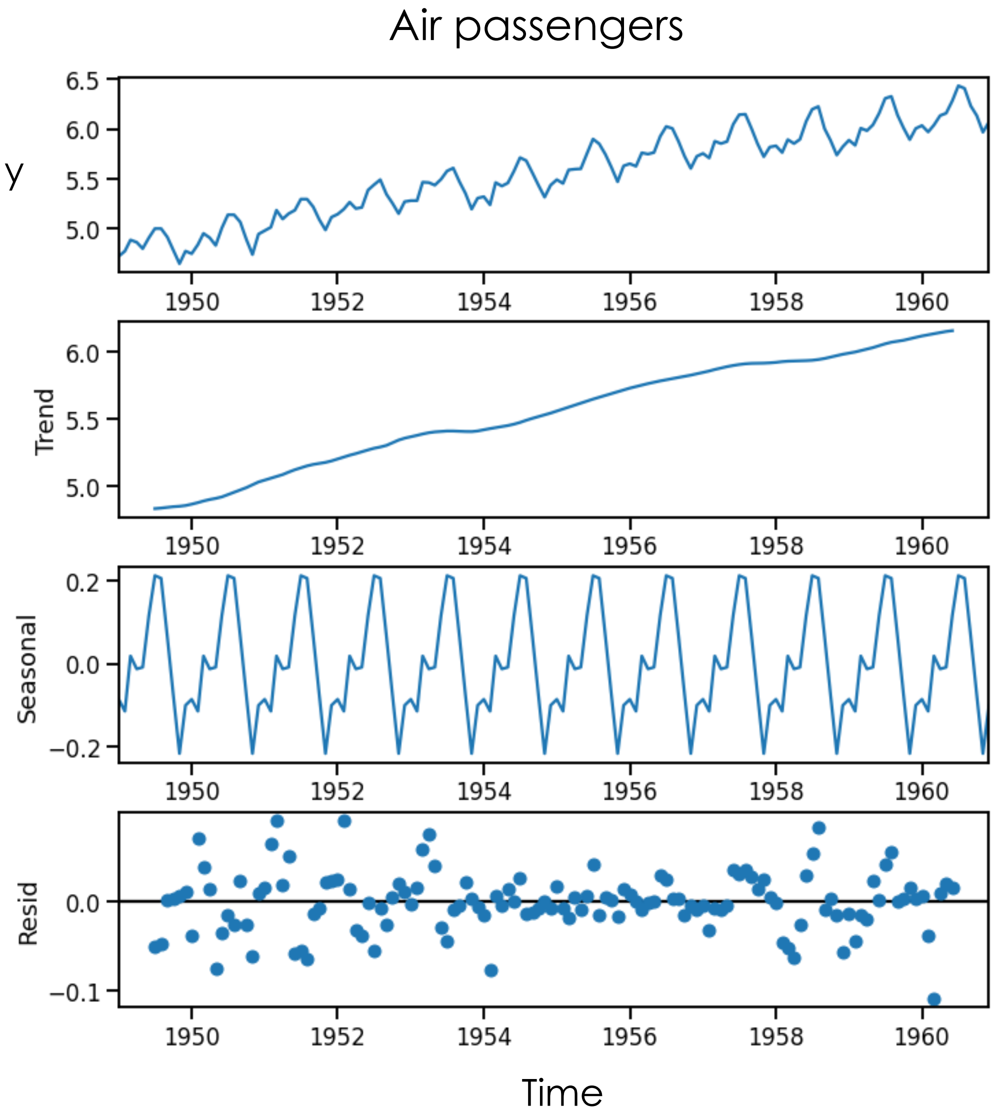
White noise applied to residuals

- This decomposition produces residuals which:
  - have zero mean.
  - the mean doesn't change in time.
  - the variance **does** change in time.

- This means that there is still some non white noise component left in the time series which could be caused by:

  - imperfect extraction of trend and seasonality.

  - an additional component exists which is not caught by trend or seasonality (e.g., an autoregressive component).



Summary

A white noise time series is one generated by repeatedly sampling from any distribution where each sample is independent.

White noise has no correlation between any two time points. Therefore, past values cannot be used to predict the future.

If the residuals of a decomposition or forecast look like white noise it means there is no more information left to extract and we did a good job!