

# Automata and machines

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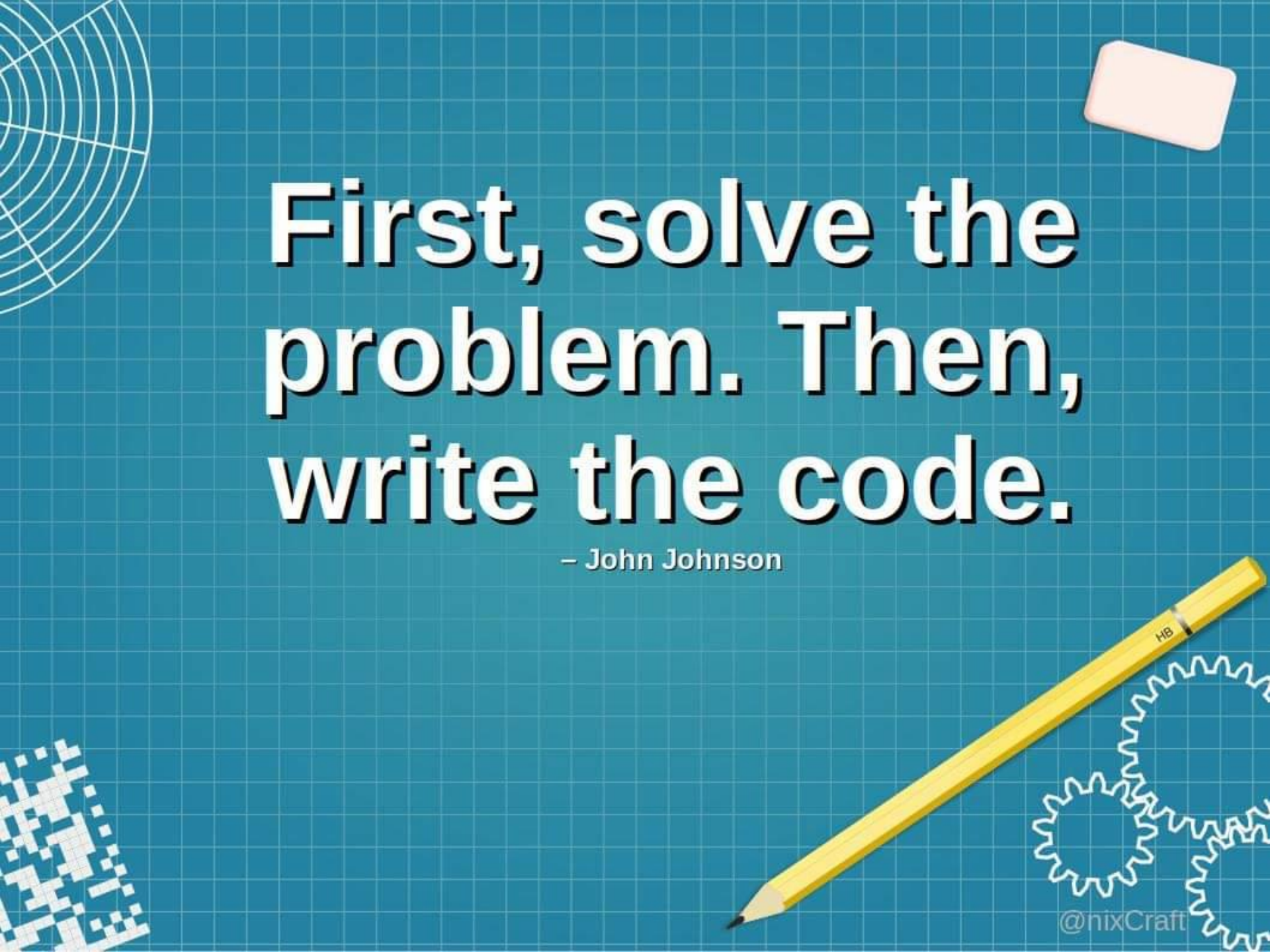
Paolo Burgio

paolo.burgio@unimore.it



**UNIMORE**  
UNIVERSITÀ DEGLI STUDI DI  
MODENA E REGGIO EMILIA

High Performance  
Real Time **Lab**



**First, solve the  
problem. Then,  
write the code.**

– John Johnson



# Industrial embedded systems

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What they do

- › Monitor physical properties of the system/plant (via *sensors*)
- › Might perform some control, or part of, control algos
- › Via *actuators*

Control can be

- › Continuous in time
  - › Discrete in time
- ➔ Control theory



# Industrial controls in a nutshell



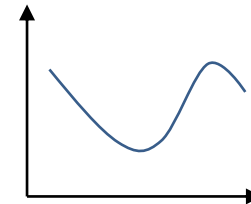
In their generic form,

$$F: \{S, I\} \rightarrow \{O\}$$

computed ...when?

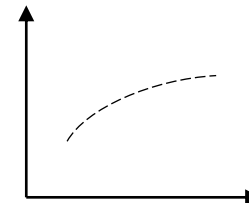
If continuous

- › Physical properties and actuators are continuous in time
- ›  $F(t)$  continuous
- › Combinatorial logic/analogic systems



If discrete

- › Computed at pre-determined instance in time
- › Event-driven (e.g., timeout, interrupt)
- › Sequential logics/digital systems





# Finite state automations for discrete controls

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E.g., an elevator, reacts to multiple events

- › Typically in idle state
- › If you are press the button, the door opens
- › You select the floor, doors close
- › Then, it reaches the floor (feat. velocity control)
- › Then, it opens the door, which subsequently closes after X seconds

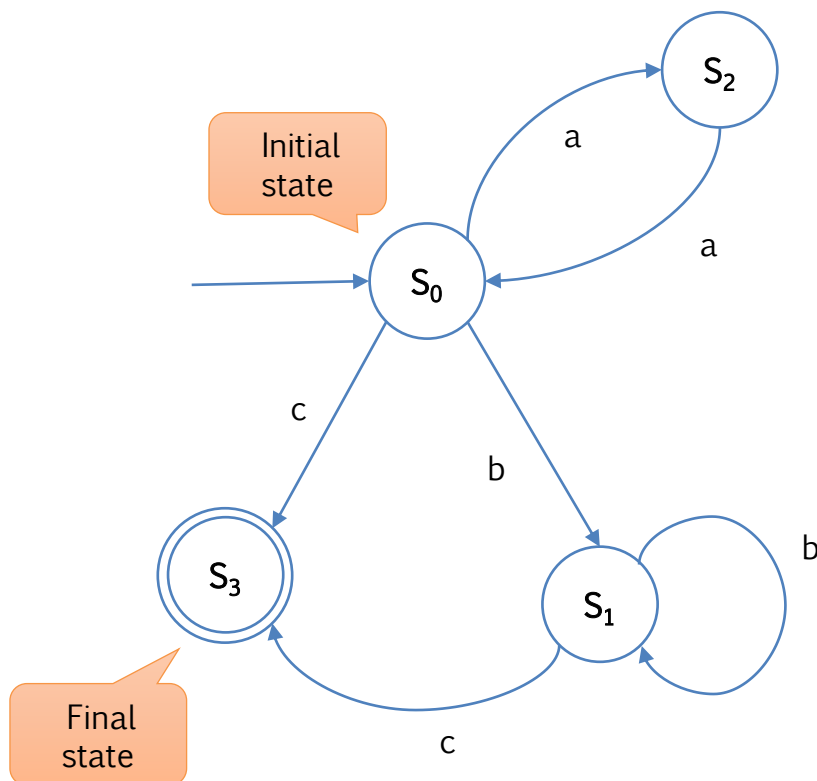
This behavior is controlled by a **finite state automations/machine**



# Finite State Automations/Machines

## Problem

- › Identify even sequences of  $a$  (even empty), followed by one, or more, or no,  $b$ , ended by  $c$



Given an alphabet  $V$ ,

*...that identifies a language (we'll see)..*

define FSA as

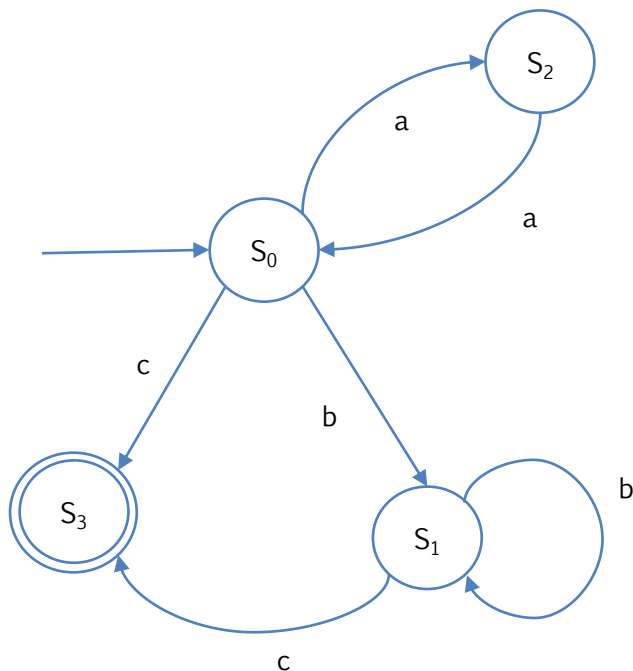
- ›  $S$  : a non-empty states set
- ›  $s_0 \in S$  : initial state
- ›  $S_f \subseteq S$  : final states set
- ›  $t: S \times V \rightarrow S$  : states transaction func



# FSMs and languages

Let  $V^* = \{ v, w, \dots \}$  contain all the combinations of words using  $V$  symbols

- › Including the empty word  $\epsilon$
- › For instance,  $ac$ ,  $aabbc$ ,  $abbabbbc$  belong to  $V^*$
- › (note that, we can associate words in  $V^*$  to inputs, or combination of them)



A **language**  $L$  is a subset of  $V^*$

(abbabbbc does **not** belong  
to  $L$ , as previously defined)

*“Identify even sequences of  $a$  (even empty),  
followed by one, or more, or no,  $b$ , ended by  $c$ ”*



# State transaction function

- ›  $t(s_0, b) = s_1 \quad | \quad s_0 \xrightarrow{b} s_1$
- ›  $s_y$  is reachable by  $s_x$  if there exists a path from  $s_x$  to  $s_y$ 
  - a combination of alphabet symbols  $l$  (letters in our case)

$$t: S \times V \xrightarrow{b} S$$

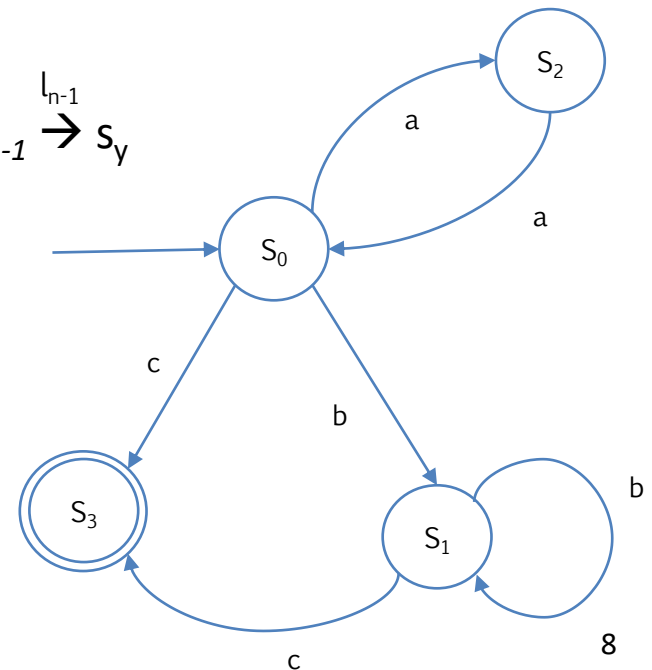
$$\rightarrow^* : S \times V^* \times S : s_x \xrightarrow{w^*} s_y$$

iff

$$w = l_1 l_2 \dots l_n \quad \exists s_1, s_2, \dots, s_n : s_x \xrightarrow{l_1} s_1 \xrightarrow{l_2} s_2 \dots s_{n-1} \xrightarrow{l_{n-1}} s_y$$

$$s_2 \xrightarrow{w^*} s_1$$

$$w = aaab \quad \exists s_1, s_2, \dots, s_n : s_2 \xrightarrow{a} s_0 \xrightarrow{a} s_2 \xrightarrow{a} s_0 \xrightarrow{b} s_1$$







# Exercise

Let's  
code!

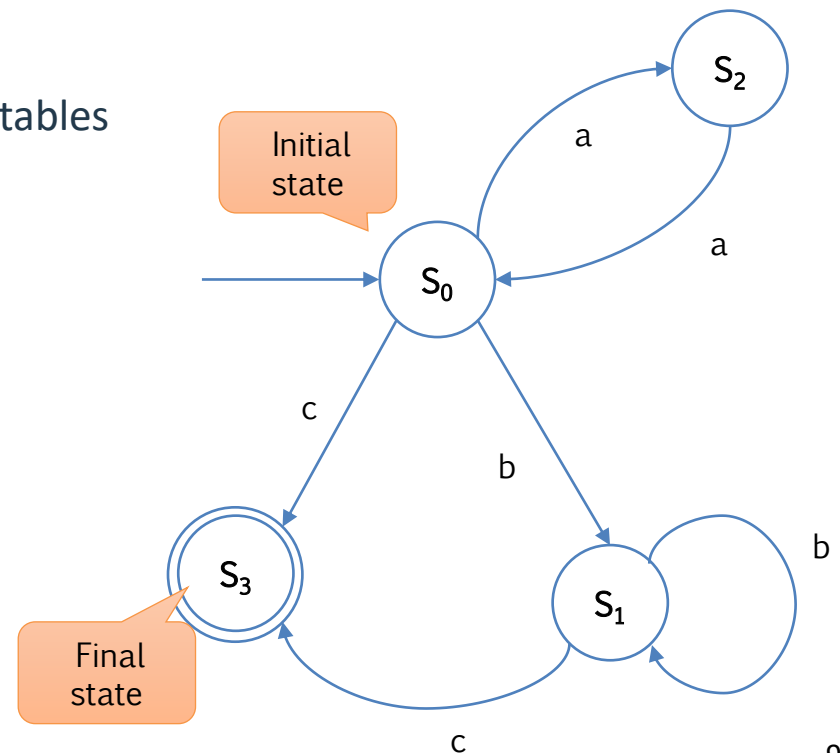
Implement the automata that understands whether a words is from L

*“Identify even sequences of a (even empty),  
followed by one, or more, or no, b, ended by c”*

Use the language that you want

- › You just need IFs, CASE-SWITCH, recursion, tables
- › Receive the target word from stdin
- › Hint: start simple...

What's missing?





# Exercise

Let's  
code!

Implement the automata that understands whether a words is from L

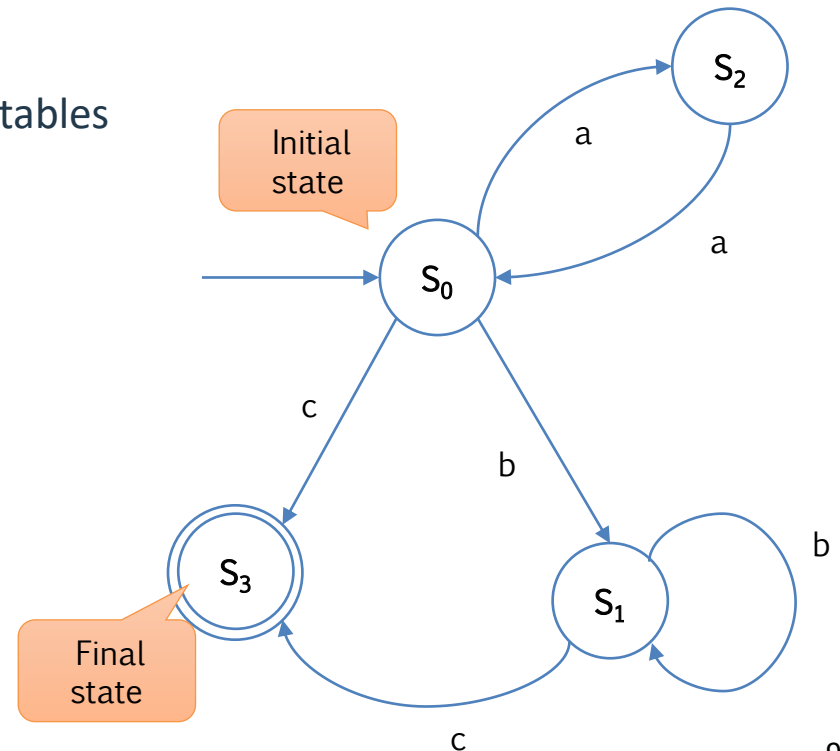
*“Identify even sequences of a (even empty),  
followed by one, or more, or no, b, ended by c”*

Use the language that you want

- › You just need IFs, CASE-SWITCH, recursion, tables
- › Receive the target word from stdin
- › Hint: start simple...

What's missing?

- › In case of error => default error state
- › Typically implicit in state diagrams





# Grammars

- › A standard way of representing languages (Noam Chomsky, 1950)

$$G = \langle VT, VN, P, S \rangle$$

*VT and VN disjoint*

$$VT \cap VN = \emptyset$$

- › VT : terminal symbols  $\subseteq V$
- › VN : non-terminal symbols  $\subseteq V$  (aka: syntax categories)
- › P : production rules  $P \subseteq VN \times (VN \cup VT)$
- ›  $S \in VN$ : initial symbol

*VT and VN are V*

$$VT \cup VN = V$$

A language  $L_G$  generated by grammar  $G$  is the set of  $V^*$  elements  
derived by start symbol  $S$  through productions in  $P$



# Backus-Naur Form

- › Productions rules have form

$$\alpha ::= \beta, \alpha \in VN \beta \in V$$

- ›  $x \in VN$  have the form  $\langle \text{name} \rangle$
- ›  $|$  specifies an option

```
VT = { il, gatto, topo, sasso, mangia, beve }
```

```
VN = { <frase>, <soggetto>, <verbo>, <compl-ogg>, <articolo>, <nome> }
```

```
S = <frase>
```

```
P = {  
  <frase> ::= <soggetto> <verbo> <compl-ogg>  
  <soggetto> ::= <articolo> <nome>  
  <articolo> ::= il  
  <nome> ::= gatto | topo | sasso  
  <verbo> ::= mangia | beve  
  <compl-ogg> ::= <articolo> <nome>  
}
```

Automata states





# Another example

## › Natural numbers

```
VT = { 0, 1, ..., 9 }
```

```
VN = { <num>, <cifra>, <cifra-non-nulla> }
```

```
S = <num>
```

```
P = {  
  <num> ::= <cifra>|<cifra-non-nulla>{<cifra>}  
  <cifra> ::= 0|<cifra-non-nulla>  
  <cifra-non-nulla> ::= 1|2|3|4|5|6|7|8|9  
}
```

“Recursion”  
Extended BNF

## › **Challenge:** extend it with sign (+, -)!



# Another example: solution

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## › Natural numbers

```
VT = { 0, 1, ..., 9, +, - }
```

```
VN = { <int>, <num>, <cifra>, <cifra-non-nulla> }
```

```
S = <int>
```

```
P = {
```

```
  <int> ::= [+|-] <num>
```

```
  <num> ::= <cifra> | <cifra-non-nulla> {<cifra>}
```

```
  <cifra> ::= 0 | <cifra-non-nulla>
```

```
  <cifra-non-nulla> ::= 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

```
}
```

## › Challenge: extend it with sign (+, -)!



# In reality...

- › We only give production rules: VN, VT, S are implicitly defined..

```
P = {  
  <frase> ::= <soggetto> <verbo> <compl-ogg>  
  <soggetto> ::= <articolo> <nome>  
  <articolo> ::= il  
  <nome> ::= gatto | topo | sasso  
  <verbo> ::= mangia | beve  
  <compl-ogg> ::= <articolo> <nome>  
}
```

Let's  
code!

Want to try?

- › Implement a machine that recognizes whether a sentence (aka: **a word of the Language L**) is legal for that language
- › (“our” words are symbols of L)



# Chomsky classification

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- › 4 types of grammars, with increasing constraints on production rules structures

## Type 0

- › No restriction on productions
- › Phrases can even become shorter!





# Type 1 grammars/languages

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- › *Context-sensitive*
- › Production must be in the form

$$x A y \rightarrow x \alpha y$$

where

$$x, y, \alpha \in (VT \cup VN)^*, A \in VN, \alpha \neq \varepsilon$$

- › A can be replaced with  $\alpha$  only if in the context of (surrounded by) x and y
- › Phrases never get shortened
- ›  $\alpha \rightarrow \beta$  con  $|\beta| \geq |\alpha|$



# Type 2 grammars/languages

---

- › *Context-free*
- › Production must be in the form

$$A \rightarrow \alpha$$

where

$$\alpha \in (VT \cup VN)^*, A \in VN$$

- ›  $\alpha$  can be  $\epsilon$
- ›  $A$  can always be replaced with  $\alpha$



# Type 3 grammars/languages

---

- › *Regular*
- › Production must be in the **linear** form

$$\begin{aligned} A &\rightarrow \alpha \\ A &\rightarrow B \alpha \end{aligned}$$

$$\begin{aligned} A &\rightarrow \alpha \\ A &\rightarrow \alpha B \end{aligned}$$

where

$$\alpha \in VT^*, A, B \in VN$$

- ›  $\alpha$  can be  $\epsilon$
- › Either left, or right linear: not both in the same grammar



# ...and..?

---

We can build specific machines to recognize/process specific grammar Types

- › Type 0 => Turing machine (if  $L(G)$  is recognizable)
- › Type 1 => **Turing machine** with constrained tape length
- › Type 2 => Finite state automations with stack (**Push down automations**)
- › Type 3 => **Finite state automations**



# Hierarchy of machine types

---

- › Base (combinatorial) machine
- › Finite state machines – FSM
- › FSM with stack (PDA)
- › Turing machine





# Base combinatorial machine

$\langle I, O, \text{mfn} \rangle$

$I$  : (finite) set of Input symbols

$O$  : (finite) set of output symbols

$\text{mfn}: I \rightarrow O$  machine function

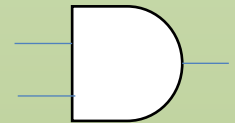
- › E.g., Logical ports, gates
- › Suitable for continuous control
- › Non suitable if you need state/memory
  - Need to model all possible cases!

Example: logical AND

$I = \{ \{0,1\} \times \{0,1\} \}$

$O = \{0,1\}$

**mfn** defined by a table



	0	1
0	0	0
1	0	1



# Finite state machine

$\langle I, O, S, mfn, sfn \rangle$

$I$  : (finite) set of Input symbols

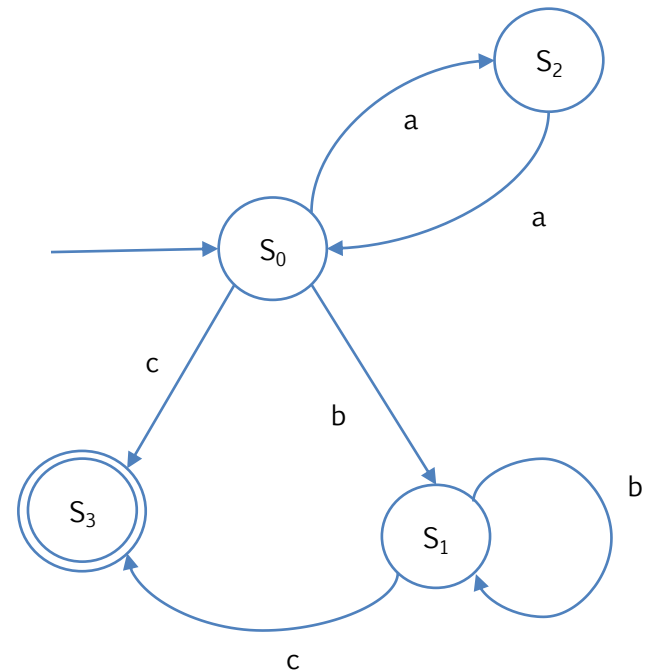
$O$  : (finite) set of output symbols

$S$  : (finite) set of states

$mfn: I \times S \rightarrow O$  machine function

$sfn: I \times S \rightarrow S$  state function

- › Partly already seen
- › Has memory
- › Memory is a limitation





# Finite state machine with stack

$\langle I, O, A, S, \text{mfn}, \text{sfn} \rangle$

$I$  : (finite) set of Input symbols

$A$  : (finite) set of stack alphabet symbols

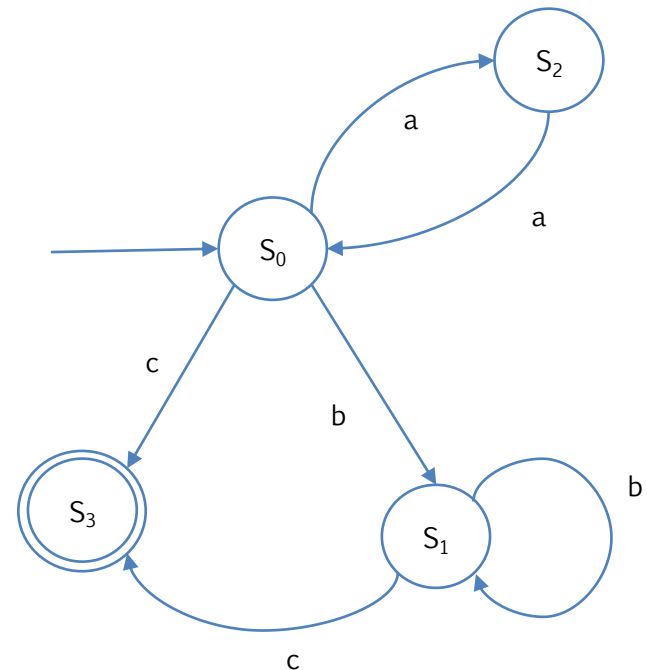
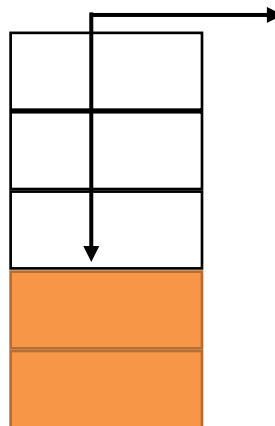
$O$  : (finite) set of output symbols

$S$  : (finite) set of states

$\text{mfn}: I \times S \times A \rightarrow O$  machine function

$\text{sfn}: I \times S \times A \rightarrow S$  state function

- › Also known as Push-Down Automata (PDA)
- › Uses a stack
- › We'll see them...







# Turing machine

**$\langle A, S, \text{mfn}, \text{sfn}, \text{dfn} \rangle$**

$A$  : (finite) set of in/out symbols

$S$  : (finite) set of states

$\text{mfn}: A \times S \rightarrow A$  machine function

$\text{sfn}: A \times S \rightarrow S$  state function (inc. HALT)

$\text{dfn}: A \times S \rightarrow \{ \text{left}, \text{right}, \text{none} \}$   
direction function

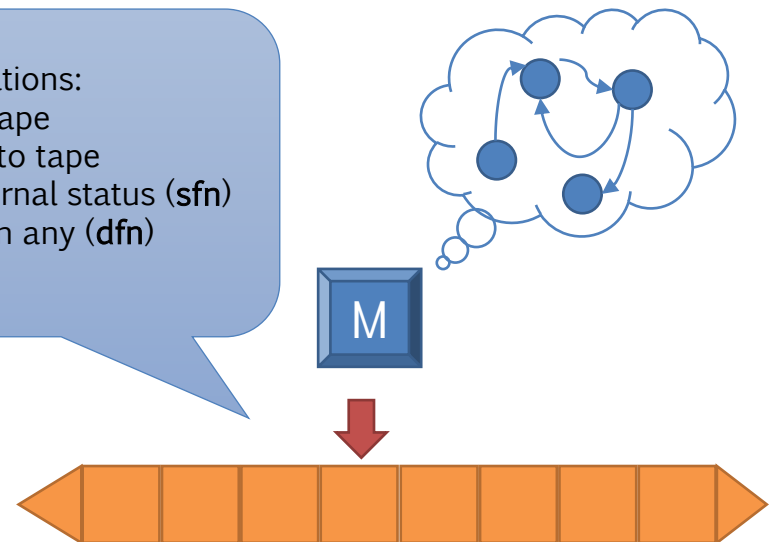
› Unlimited memory

Possible operations:

- Read from tape
- Write (**mfn**) to tape
- Change internal status (**sfn**)
- Move tape in any (**dfn**) direction

## Church-Turing thesis

*A function on the natural numbers can be calculated by an effective method if and only if it is computable by a Turing machine*





# A Universal Turing Machine

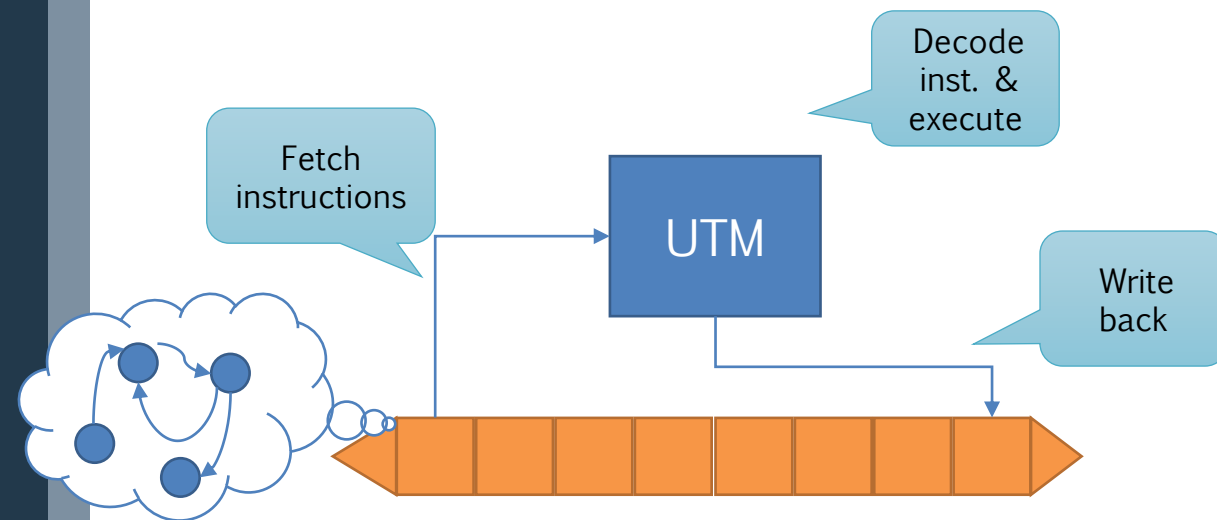
- › In TM, the algorithm is inside the machine M, we write results in the tape

What if instruction as well is in the tape?

- › We have a programmable machine, with a memory
- › .....does this remind something?

Which are the catch? What do we miss?

- › Ok, the infinite tape makes it infeasible
- › ..but what else?

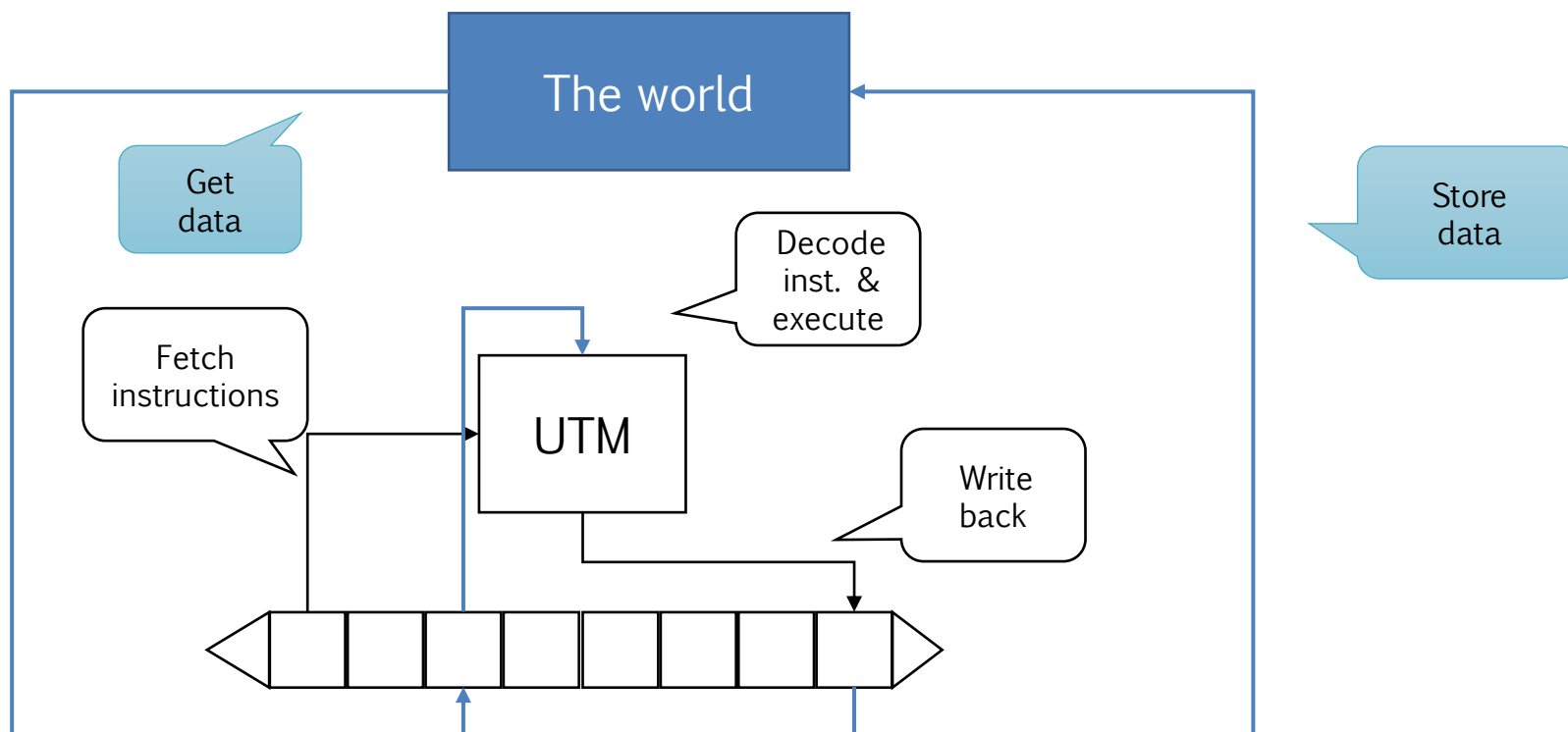




# The Van Neumann Machine

We also need to model the interaction with the environment!

- › Aka: I/O (HD/SSD is also I/O)
- › Where data comes from!
- › It is a real machine: we can **build** it



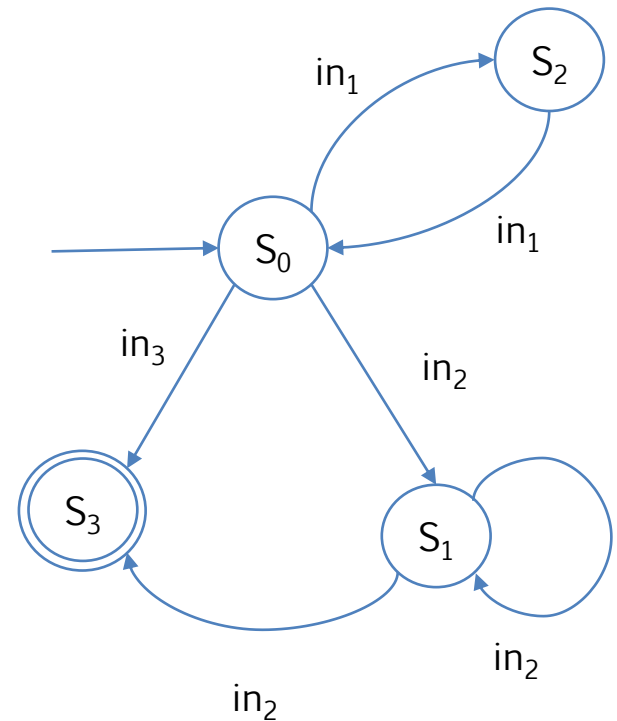


# How to implement a FSM



# A generic FSM

- › Till now, we only saw machines that can recognize a **word** from a language
  - I say “word”, you might want to understand “sentence”
- › Let’s now see how a machine can actually **produce** an output





# The Machine of Mealy

- › When crossing an edge, produce an output

**$\langle I, O, S, \text{mfn}, \text{sfn} \rangle$**

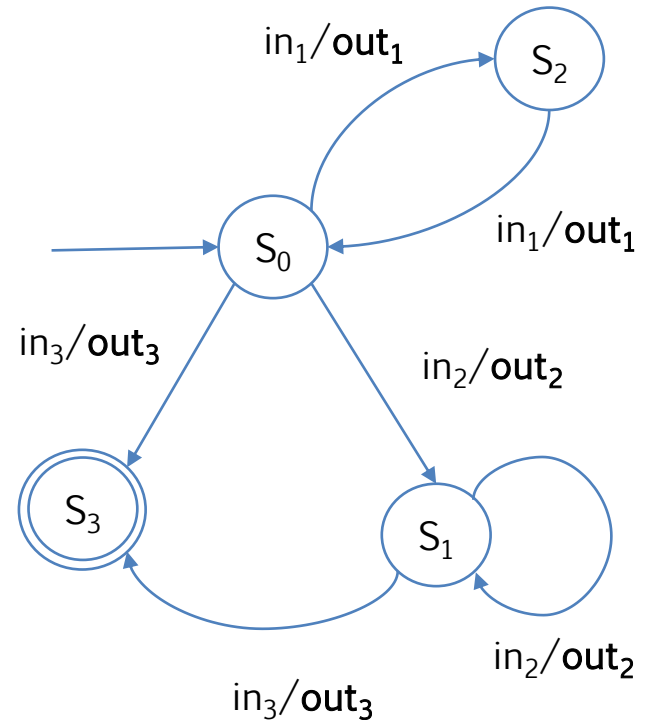
$I$  : (finite) set of Input symbols

$O$  : (finite) set of output symbols

$S$  : (finite) set of states ( $s_0$  initial state)

$\text{mfn}: I \times S \rightarrow O$  machine/output function

$\text{sfn}: I \times S \rightarrow S$  state transition function





# The Machine of Moore

- › When in a state an edge, produce an output

**$\langle I, O, S, \text{mfn}, \text{sfn} \rangle$**

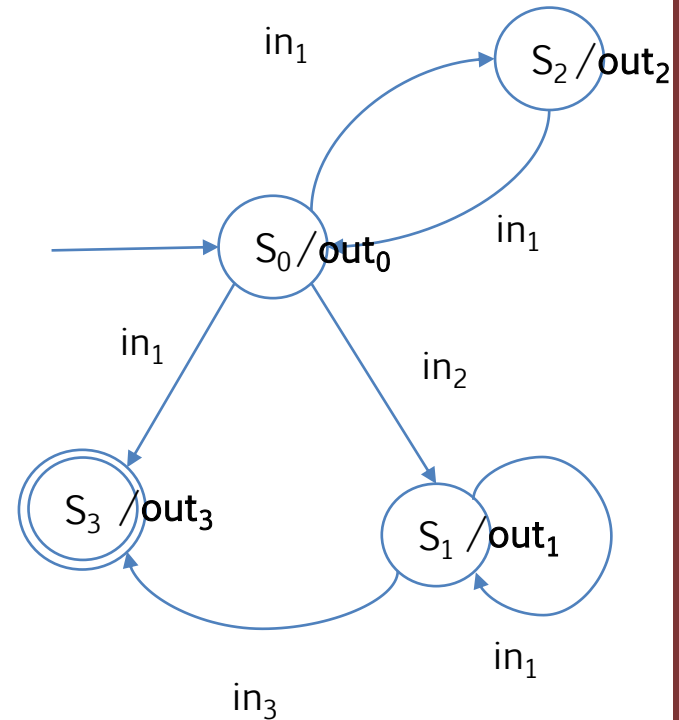
$I$  : (finite) set of Input symbols

$O$  : (finite) set of output symbols

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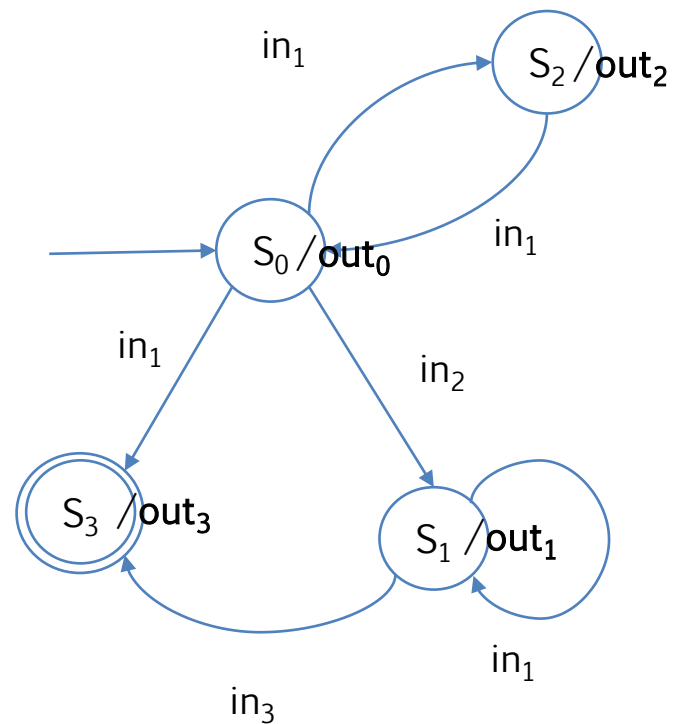
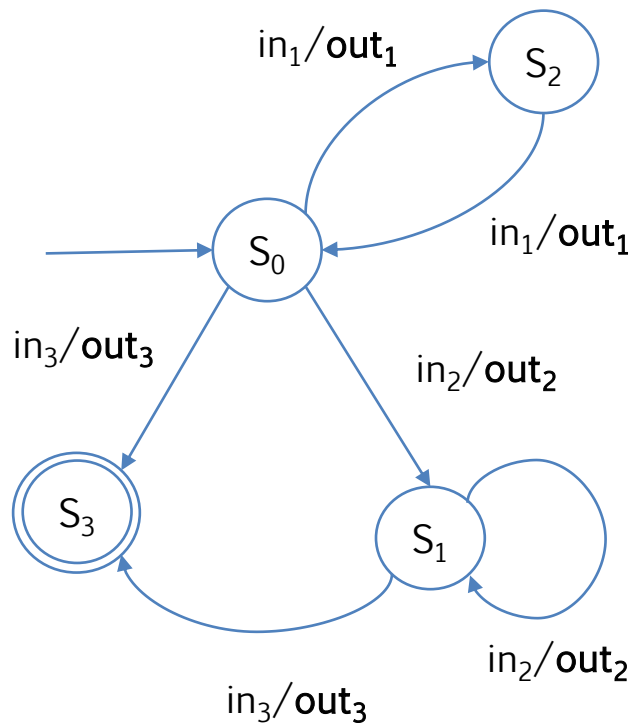
$\text{mfn}: S \rightarrow O$  machine/output function

$\text{sfn}: I \times S \rightarrow S$  state transition function





# What's the difference?







# What's the difference?

---

Mathematically equivalent

- › One can be transformed in another

..but..

- › Mealy can potentially have different outs, to different inputs/transitions
  - Less states, if output depends on inputs one can add an edge to the machine
- › Moore potentially keeps the output stable for all the state
  - Moore requires more states, in case out depends on input and not only on state



# Exercise

Let's  
code!

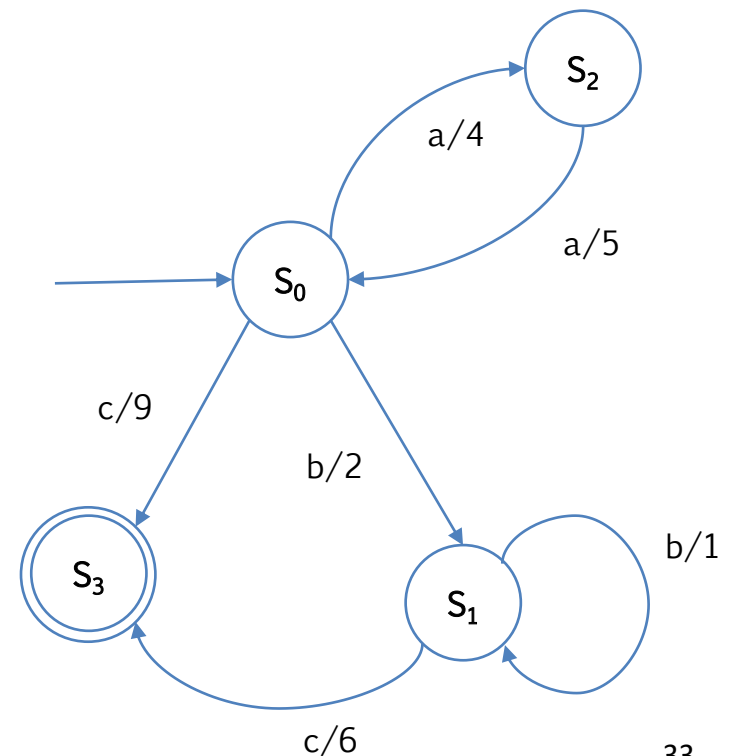
- › Implement the automata that understands whether a words is from L

*“Identify even sequences of a (even empty),  
followed by one, or more, or no, b, ended by c”*

- › ..and writes the corresponding number  
(I choose them randomly)
- › Mealy? Moore? You choose
  - Here, I show Mealy

Hint

- › If not already done, use tables  
for state/output transactions





# What else?

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Several tools to support the design

- › Matlab Stateflow, UML

Several grammar interpreters to rely the burden of writing FSM code

- › FSF's GNU Bison – Included in GCC
- › YACC – Yet Another Compiler-Compiler



# GNU Bison



Converts a context-free grammar into a deterministic LR parser  
(but not only) in C

- › Recognizes correct sentences from a grammar
- › <https://www.gnu.org/software/bison/>
- › **Not part of exam** 😊

Input format: Bison grammar files

```
%{  
    Prologue  
%}  
  
Bison declarations  
  
%%  
Grammar rules  
%%  
  
Epilogue
```



# Bison prologue



C-style code that will be appended at the beginning of the generated file

- › Useful for defining macros, includes, headers..
- › `ptypes.h` contains Bison internal data structures: trees, tokens...

```
%{
    #define _GNU_SOURCE
    #include <stdio.h>
    #include "ptypes.h"
}%

%union {
    long n;
    tree t; /* tree is defined in ptypes.h. */
}

%{
    static void print_token (yytoken_kind_t token, YYSTYPE val);
}%
```



# Grammar rules



- › Like-BNF syntax
- › Can also include (C) language-specific rules

```
// results => non-terminal;  
// components => any  
result: components...;
```

```
// C statement  
{C statements}
```

```
// Multiple rules  
result:  
    rule1-components...  
| rule2-components...  
...  
;
```

```
// recursive rule  
expseq1:  
    exp  
| expseq1 ',' exp  
;
```



# Example - Reverse-polish notation calculator

rpcalc.y

```
input:      /* empty */
            | input line
;

line:       '\n'
            | exp '\n' { printf ("\t%.10g\n", $1); }
;

exp:        NUM                { $$ = $1;          }
            | exp exp '+'      { $$ = $1 + $2;    }
            | exp exp '-'      { $$ = $1 - $2;    }
            | exp exp '*'      { $$ = $1 * $2;    }
            | exp exp '/'      { $$ = $1 / $2;    }
            /* Exponentiation */
            | exp exp '^'      { $$ = pow ($1, $2); }
            /* Unary minus */
            | exp 'n'          { $$ = -$1;        }
;

%%
```



# Example - Reverse-polish notation calculator

"A complete input is either an empty string, or a complete input followed by an input line"

rpcalc.y

```
input:      /* empty */
           | input line
;

line:       '\n'
           | exp '\n' { printf ("\t%.10g\n", $1); }
;

exp:        NUM { $$ = $1; }
           | exp exp '+' { $$ = $1 + $2; }
           | exp exp '-' { $$ = $1 - $2; }
           | exp exp '*' { $$ = $1 * $2; }
           | exp exp '/' { $$ = $1 / $2; }
           /* Exponentiation */
           | exp exp '^' { $$ = pow ($1, $2); }
           /* Unary minus */
           | exp 'n' { $$ = -$1; }
;
%%
```





# Example - Reverse-polish notation calculator

rpcalc.y

```
input:      /* empty */
           | input line
;

line:       '\n'
           | exp '\n' { printf ("\t%.10g\n", $1); }
;

exp:        NUM { $1 }
           | exp exp '+' { $1+$2 }
           | exp exp '-' { $1-$2 }
           | exp exp '*' { $1*$2 }
           | exp exp '/' { $1/$2 }
           /* Exponentiation */
           | exp exp '^' { $1^$2 }
           /* Unary minus */
           | exp 'n' { -$1 }
;

%%
```

“Can be either a newline, or an expression followed by a newline”

Also, specifies an **action** that prints this value (exp, indicated by \$1)

Note: we use language-specific features and libraries, such as printf (in prologue, I included stdio.h)



# Example - Reverse-polish notation calculator

rpcalc.y

Multi-rules expression (“pure” numbers + six arithm operators)

Actions specify how to translate it in C

- `$$ => result`
- `$1, $2 => operators`
- (remember to `#include math.h` ☺)

```
; }  
;  
  
exp:      NUM                { $$ = $1;          }  
      | exp exp '+'          { $$ = $1 + $2;    }  
      | exp exp '-'          { $$ = $1 - $2;    }  
      | exp exp '*'          { $$ = $1 * $2;    }  
      | exp exp '/'          { $$ = $1 / $2;    }  
      /* Exponentiation */  
      | exp exp '^'          { $$ = pow ($1, $2); }  
      /* Unary minus */  
      | exp 'n'              { $$ = -$1;        }  
  
;  
%%
```



# Exercise (optional)

Let's  
code!

Write a parser for the following grammar using Bison

```
P = {  
  <frase> ::= <soggetto> <verbo> <compl-ogg>  
  <soggetto> ::= <articolo><nome>  
  <articolo> ::= il  
  <nome> ::= gatto | topo | sasso  
  <verbo> ::= mangia | beve  
  <compl-ogg> ::= <articolo> <nome>  
}
```



# Non-deterministic automata





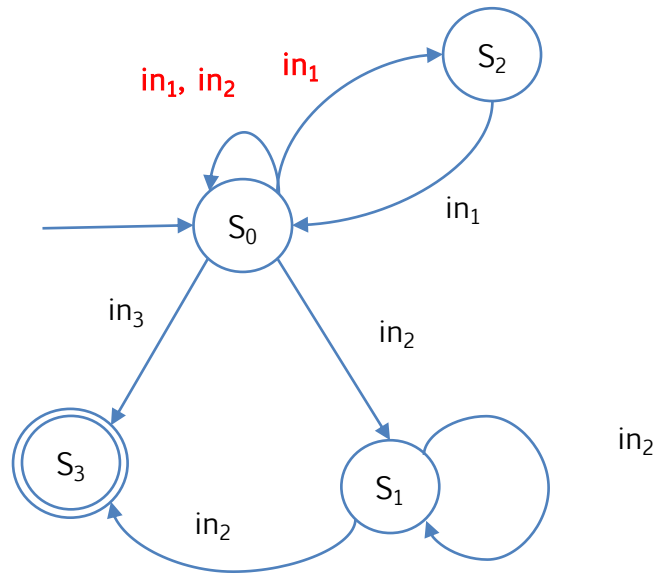
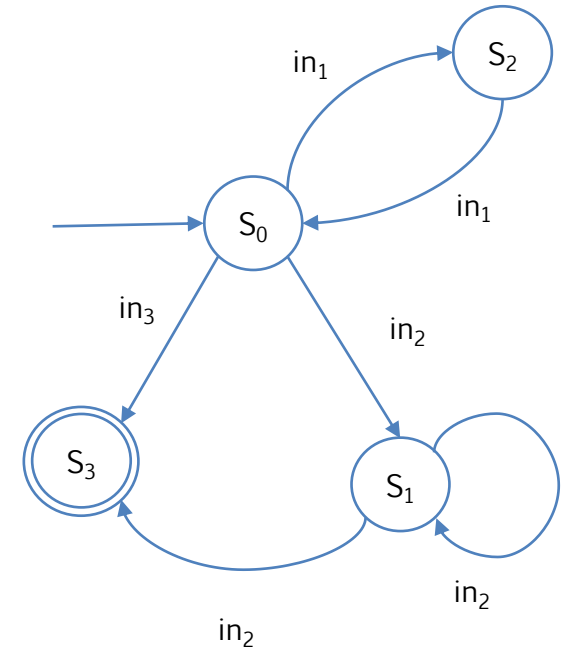
# Deterministic vs. non-deterministic

Till now, deterministic automations - **DFA**

- › uniquely identifies a transaction with the couple state-input
- › Only one edge/input connects nodes/states

Non-deterministic finite automations (**NFA**) can have multiple edges connecting nodes/states

- › Also, same inputs can lead to more than one nodes





# NFA-to-DFA conversion


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- › Non-deterministic automata can be translated into equivalent deterministic one
- › For each NFA, there is a DFA such that it recognizes the same formal language
- › If a DFA cannot recognize a formal language, neither a NFA can


## Rabin–Scott powerset construction

- › Catch: if a NFA has  $n$  states, the corresponding DFA can have up to  $2^n$  states!
- › We won't see this...

M. O. Rabin and D. Scott, "Finite Automata and Their Decision Problems," in IBM Journal of Research and Development, vol. 3, no. 2, pp. 114-125, April 1959, doi: 10.1147/rd.32.0114.



# Event driven Systems





# Event driven systems

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A system that reacts from external stimula

- › Instantly?
- › Aka: Cyber-Physical Systems (CPS)

Can be

- › Synchronous
- › Asynchronous





# Synchronous (Active polling)

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- › Infinite loop

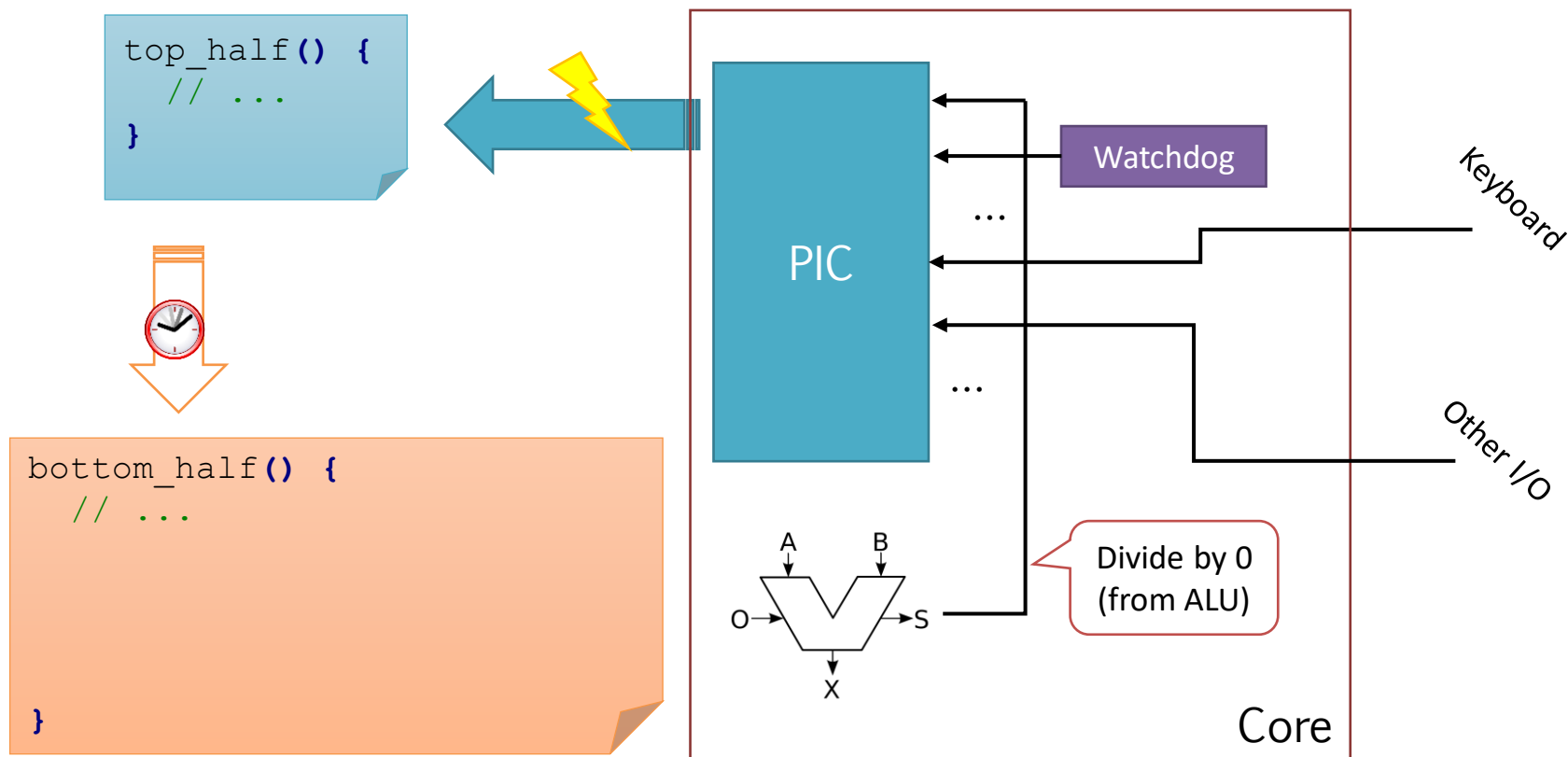
```
char c;  
while (c != SOME_VALUE)  
    c = readC();  
  
// We can go, now
```

- › **Pros:** extremely fast and reactive
- › **Cons:** waste of resources as one core is busy
  - Possible workaround: insert a sleep



# Asynchronous (Interrupt Service Routine)

- › Programmable interrupt controller (hierarchy)



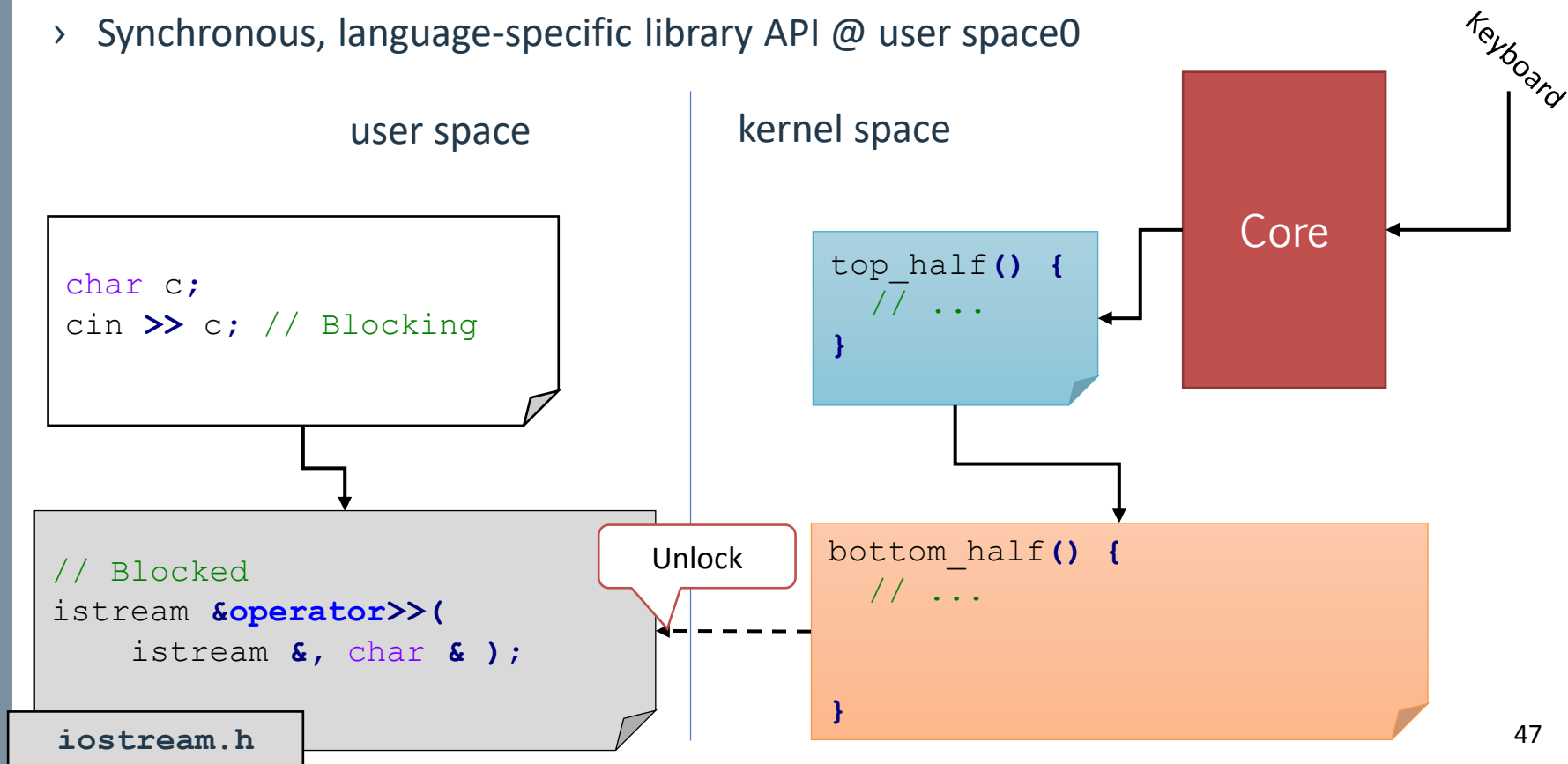
- › **Pros:** “pay-as-you-go”
- › **Cons:** takes more time to issue a ISP



# ...a mix of the two

Keyboard management in a General-Purpose system

- › GNU/Linux
- › ISP with bottom-half and top-half @ kernel space
- › Synchronous, language-specific library API @ user space0





# Petri nets

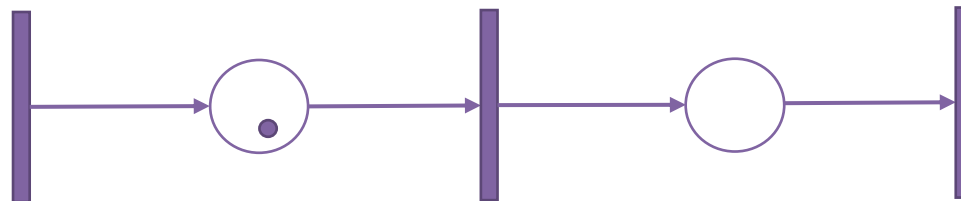


# Petri nets - definition

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A directed bipartite graph

- › **Transitions** triggered by **events** (*bars*)
- › **Places**, i.e., conditions (*circles*)
- › *Arcs* connect only places to transitions (or vice-versa), and specify which places are **pre- or post-conditions** for events
- › Every place collects **tokens** (*dots*) which might trigger an event (if multiple events are triggered in the same net, which fires first is non-deterministic)



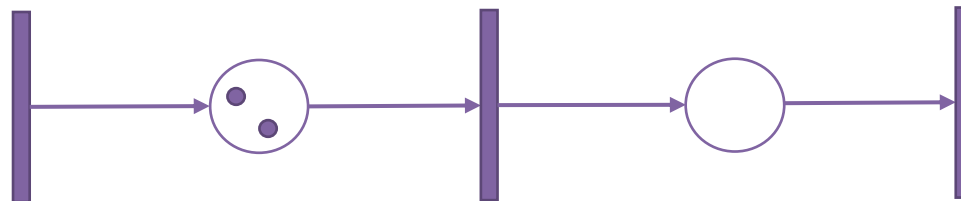
Model distributed systems, discrete events dynamic systems

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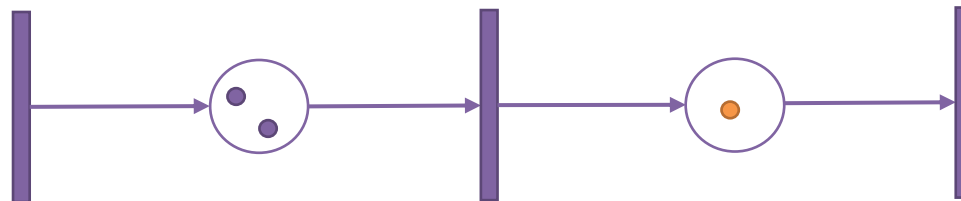
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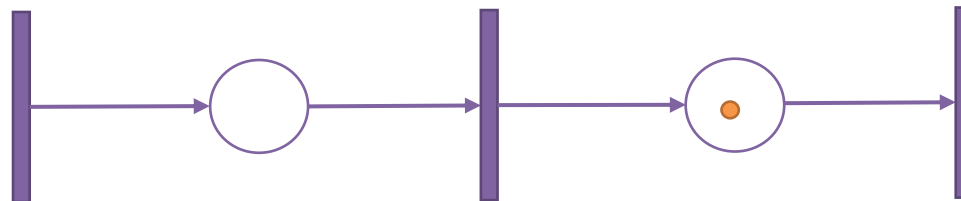
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Model distributed systems, discrete events dynamic systems





# (Marking) Petri net - formalism

A tuple  $(S, T, W, M_0)$

Subject to:

- ›  $S$  and  $T$  are disjoint
- › No arc can connect two states or two transitions among them

- ›  $S$  : finite set of states
- ›  $T$  : finite set of transitions
- ›  $W : (S \times T) \cup (T \times S) \rightarrow \mathbb{N}$  multiset of arcs
- ›  $M$  : (marking) a mapping  $S \rightarrow \mathbb{N}$  that assigns to each place a number of tokens
- ›  $M_0$  : initial marking

How they execute

- › firing a transition  $t$  in a marking  $M$  consumes  $W(s, t)$  tokens from each of its input places, and produces  $W(t, s)$  tokens in each of its output places
- › a transition is *enabled* (it may fire) in  $M$  if there are enough tokens in its input places for the consumptions to be possible, i.e. if and only if  $\forall s : M(s) \geq W(s, t)$



# How to run the examples

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Let's  
code!

- › Find them in Code/ folder from the course website

For C++: compile

- › `$ gcc code.cpp -o code -Wall -lstdc++`

Run (Unix/Linux)

`$ ./code`

Run (Win/Cygwin)

`$ ./code.exe`



# References

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## Course website

- › [http://hipert.unimore.it/people/paolob/pub/Industrial\\_Informatics/index.html](http://hipert.unimore.it/people/paolob/pub/Industrial_Informatics/index.html)

## My contacts

- › [paolo.burgio@unimore.it](mailto:paolo.burgio@unimore.it)
- › <http://hipert.mat.unimore.it/people/paolob/>

## Resources

- › Alessandro Fantechi, «Informatica Industriale», Città Studi Edizioni
- › For interrupts
  - Robert Love, «Linux kernel development», Pearson
- › For GNU Bison
  - [http://dinosaur.compilertools.net/bison/bison\\_5.html](http://dinosaur.compilertools.net/bison/bison_5.html)
- › A "small blog"
  - <http://www.google.com>