Coordinate systems and units in HiSPARC

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February 18, 2015

Abstract

The coordinate systems and units used in HiSPARC data and analysis are illustrated and described. We also have to deal with other coordinate systems such as the one used in CORSIKA and some used as intermediary in coordinate transformations. The conversions and relations between these systems are given.

1 Introduction

Since we have to work with many coordinate systems it can be hard to keep track of the definitions of each. This document is meant as a reference to easily find the relations between the different systems. First coordinate systems used by HiSPARC will be discussed. Including the units that are used and where it is used. Then other coordinate systems that we have to deal with are discussed, including ways to convert from those to our usual coordinate systems.

In Section 2 we discuss geographic coordinates to define the coordinates of the observer. In Section 3 we discuss different time keeping systems and how we can relate a time of observation to the rotation of the Earth w.r.t the celestial sphere. The systems to define a point on the celestial sphere are described in Section 4. In Section 5 the coordinate systems which are used in CORSIKA simulations are described. Finally in Section 6 an example shows the process to go from a HiSPARC detection to equatorial coordinates.

Make sure that the angles are converted to the correct units for the cosine and sine functions when converting between coordinate systems. The use of radians or degrees depends on the programming language.

2 Geographic

Geographic coordinate systems define a point on the Earth. For HiSPARC two systems are used: WGS84 and ENU. Figure 1 illustrates the relationships between these systems. Additionally a compass based system is used to determine detector locations relative to the GPS antenna. These systems are described in the following sections. The conversion formulae are taken from [1, sec. K].

2.1 World Geodetic System 1984 (WGS84)

The location of a HiSPARC station is determined by means of a GPS antenna (one antenna for each station). The GPS returns coordinates in the WGS84 coordinate system. This defines a position with a latitude (φ), longitude (λ) and altitude (h). The latitude and longitude are defined in decimal degrees, the altitude in meters. Latitude is the angular distance between a location and the equator, angles towards north are positive. The longitude is the

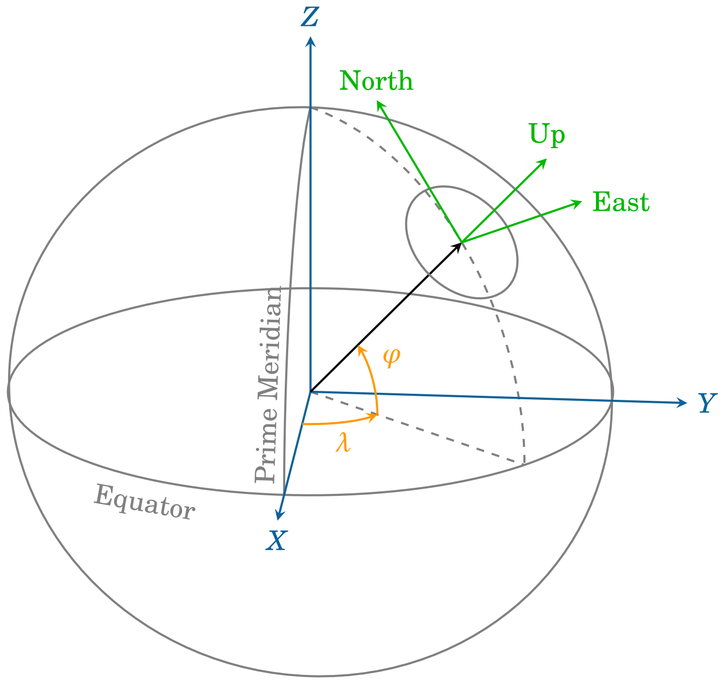

Figure 1 – Relationship between the WGS84 (orange), ECEF (blue) and ENU (green) coordinate systems. The Prime meridian is the IERS Reference Meridian.

angular distance of a location to the Reference Meridian (from now on referred to as the prime meridian) defined by the International Earth Rotation and Reference Systems Service (IERS), angles towards east are positive. The altitude is the height above an ellipsoid that approximates the shape of the Earth. The parameters which define this ellipsoid are given in Equation 2 [2].

2.2 Earth-Centered, Earth-Fixed (ECEF)

The ECEF coordinate system is used as intermediate between WGS84 and ENU. ECEF has its origin as the center of the Earth (Earth-Centered), and its axes are aligned with the WGS84 system. The z-axis points to North. The x-axis crosses the Earth surface where the equator (0° latitude) intersects the prime meridan (0° longitude). The y-axis intersects the equator at 90° longitude. The axes rotate with the Earth (Earth-Fixed). A position is defined by the X, Y, and Z coordinates, each given in meters.

The formulae to convert WGS84 coordinates to ECEF coordinates are:

$$\begin{aligned} X &= (N+h)\cos\varphi\cos\lambda\ , \\ Y &= (N+h)\cos\varphi\sin\lambda\ , \\ Z &= \left(\frac{b^2}{a^2}N+h\right)\sin\varphi\ . \end{aligned} \tag{1}$$

For the paramaters that define the ellipsoidal approximation of the Earth surface we have

$$\begin{aligned} a &= 6\,378\,137.0\,\mathrm{m}\ , \\ f &= \frac{1}{298.257223563}\ , \\ b &= a(1-f)\ , \\ e &= \sqrt{2f-f^2}\ , \\ N &= \frac{a}{\sqrt{1-e^2\sin^2\varphi}}\ , \end{aligned} \tag{2}$$

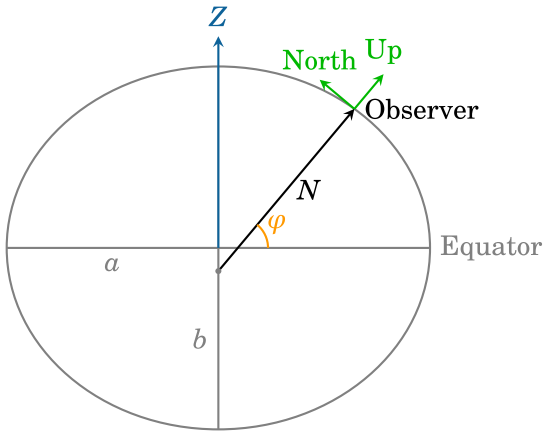

Figure 2 – Ellipsoidal approximation of Earth. Showing the definitions of the semi-major axis, the semi-minor axis, and the Normal.

here a is the semi-major axis, f the flattening, b the semi-minor axis and e the eccentricity of the ellipsoid. N is the Normal, which is the distance between a location on the ellipsoid and the intersection of its normal and the z-axis of the ellipsoid. For a position on the equator (i.e. $\varphi = 0°$) $N = a$.

2.3 East, North, Up (ENU)

East, North, Up is used to obtain relative locations and angles with respect to a reference position. It is a local coordinate system in a plane tangential to the WGS84 elipsoid. From the reference position the positive x-axis (East) is in the eastern direction, the positive y-axis (North) is towards north, and the positive z-axis (Up) is towards the Zenith. All distances are in meters.

The formulae to convert ECEF to ENU with a given reference position are:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sin(\lambda_r) & \cos(\lambda_r) & 0. \\ -\sin(\varphi_r)\cos(\lambda_r) & -\sin(\varphi_r)\sin(\lambda_r) & \cos(\varphi_r) \\ \cos(\varphi_r)\cos(\lambda_r) & \cos(\varphi_r)\sin(\lambda_r) & \sin(\varphi_r) \end{bmatrix} \begin{bmatrix} X - X_r \\ Y - Y_r \\ Z - Z_r \end{bmatrix} \tag{3}$$

where (X, Y, Z) and (x, y, z) are the location in ECEF and ENU respectively. (φ_r, λ_r) and (X_r, Y_r, Z_r) are the reference position in WGS84 and ECEF respectively.

2.4 'Compass'

For reconstruction of events the precise location of the individual detectors in each station is required. The location of detectors are taken relative to the GPS antenna, because that position is known. We use a system that requires a distance and angle measurement, see Figure 3. The distance (r) between the GPS and a detector (center of the scintillator) is measured (in meters), the angle (α) relative to the geographic north pole (not magnetic north!) is determined using a compass and the magnetic declination. This angle is in degrees and goes from North to East (NESW). The rotation angle of the detector itself (β) is defined as the angle of the long side of the detector relative to the North Pole (positive towards East). The height difference (z) between the GPS and detectors can also be measured, however, this is only important if the detectors are not all at the same altitude or are at a very different altitude from the GPS.

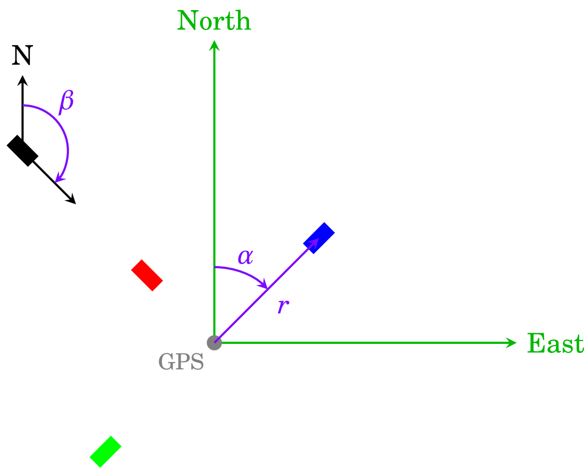

Figure 3 – Relationship between the ENU (green) and Compass (purple) coordinate system. The detectors of station 503 are shown. Its GPS is used as the origin for the ENU coordinates. The α and r coordinate of detector 4 (blue) and the β coordinate of detector 1 (black) are illustrated. The values are given in Section 6.

When a compass is used the angle needs to be compensated for the difference between magnetic and geographic north, the difference is called the magnetic declination. The magnetic declination is date and location dependent [3]. The magnetic declination can be up to 3° for some HiSPARC station locations.

The formulae to convert the location of a station in Compass coordinates to ENU are:

$$\begin{aligned} x &= r * \sin\alpha \ , \\ y &= r * \cos\alpha \ , \\ z &= z \ . \end{aligned} \tag{4}$$

3 Time

For HiSPARC events it is important to know precisely when they occur, a GPS is used to get synchronized and precise timing. It gives us a unique timestamp, this needs to be converted to Local Sidereal Time for the conversion of celestial coordinates in Section 4. The full transformation chain is as follows: GPS → UTC → JD → GMST → LST.

3.1 GPS time

GPS time started counting on 6 January 1980 00:00 UTC (Coordinated Universal Time), i.e. 315 964 800 UNIX seconds after the UNIX epoch of 1 January 1970 00:00 UTC. GPS time is a continuously increasing number. This is different from UTC time in which certain seconds are repeated when 'leap seconds' are added [4]. Currently (6 February 2015) the difference between UTC and GPS time is 16 seconds. From the HiSPARC electronics we get a GPS timestamp, adjusted to the UNIX epoch, i.e. GPS + UNIX(GPS = 0). The timestamp is given in nanoseconds. The GPS timestamp is therefore a large number which requires 64-bits to be represented digitally. For instance the GPS timestamp for 1 December 2014 at 00:00:29.886222166 (GPS time) is

$$1\,101\,427\,229\,886\,222\,166\,\mathrm{ns} + 315\,964\,800\,000\,000\,000\,\mathrm{ns} = 1\,417\,392\,029\,886\,222\,166\,\mathrm{ns}\ . \tag{5}$$

Date	Leap seconds
31 December, 1998	13
31 December, 2005	14
31 December, 2008	15
30 June, 2012	16
30 June, 2015	17

Table 1 – Leap seconds in effect or introduced after 2002. This is the amount of seconds that GPS time is ahead of UTC time.

3.2 UTC time

Similar to GPS but corrected with leap seconds to keep it more in sync with solar time. A leap second means that a second is counted twice. The IERS decides when to add leap seconds. HiSPARC does not use UTC time because of the duplicate timestamps, which makes it possible for a station to detect two events at the 'same time'. Moreover, it causes issues when looking for coincidences between stations.

3.2.1 Leap seconds

Leap seconds can be introduced at the end of either 30 June or 31 December, the leap second is represented in UTC time as 23:59:60. Table 1 gives a list of the recent leap seconds, which are important for HiSPARC. Leap seconds are announced about 6 months before they take effect.

3.3 Julian Date (JD)

JD is a decimal number counting the number of days since 1 January 4713 B.C. 12:00:00. Calculating this is not trivial since one has to account for the fact that the 5th up to and including the 14th of October in 1582 do not exist. Those ten days were skipped because the Julian calendar, which was used before that gap, introduced too many leap years, every year divisible by 4 was a leap year. The Gregorian calendar used afterwards compensated by having less leap years. Leap years still occur when years are divisible by 4, except when they are also divisible by 100 (e.g. 1900), except when they are also divisible by 400 (e.g. 2000) [5]. Consider these examples: 1999 is not divisible by 4 so not a leap year, 2004 is divisible by 4 and not by 100 and is a leap year, 1900 is divisible by 4 and 100 but not 400 so is not a leap year, and 2000 is divisible by 4, 100 and 400 and is therefore a leap year.

The following conversion is valid for dates after 15 October 1582. The brackets $\lfloor\ \rfloor$ indicate that the decimal part of the value should be discarded. The *month* starts at 1 for January and *hour* is the decimal number of hours.

$$\begin{aligned}
a &= \left\lfloor \frac{14 - month}{12} \right\rfloor \ , \\
A &= \left\lfloor \frac{year - a}{100} \right\rfloor \ , \\
B &= 2 - A + \left\lfloor \frac{A}{4} \right\rfloor \ , \\
C &= \lfloor 365.25(year - a) \rfloor \ , \\
D &= \lfloor 30.6001(month + 12a + 1) \rfloor \ , \\
JD &= B + C + D + day + \frac{hour}{24} + 1720994.5 \ .
\end{aligned} \tag{6}$$

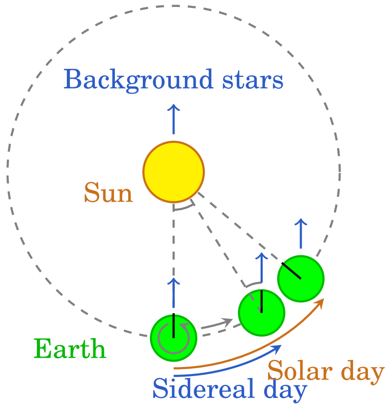

Figure 4 – This shows the orbit of Earth (green) around the Sun (yellow). The black line on the Earth represents the prime meridian. A full rotation of the Earth causes the prime meridian to point to the same background stars, this is a sidereal day. It takes longer, due to the orbit of the Earth, for the prime meridian to point to the Sun again, which is a solar day. Used sizes and angles are illustrative, not realistic.

3.3.1 Modified Julian Date (MJD)

Modified Julian Date to keep number shorter uses epoch 17 November 1858 00:00:00. The difference between a Modified Julian Date and a Julian Date is 2400000.5 days. The .5 days moves the reference time from noon to midnight.

$$MJD = JD - 2400000.5\ . \tag{7}$$

3.4 Sidereal Time

In sidereal time the rotation of Earth relative to the 'fixed' background stars is taken. A sidereal day is a full rotation of Earth around its axis, relative to background stars. In a solar day the Earth rotates more than 360° around its axis because it orbits around the Sun, causing the position of the Sun relative to the Earth to change. The Earth orbits the Sun every 365.25 days, so in one day the orbit proceeds by $\frac{360°}{365.25\,\mathrm{d}} \approx 1\,°\,\mathrm{d}^{-1}$. So an extra rotation of approximately 1° around its axis is required to make the same part of the Earth face the Sun (a solar day). A sidereal day is therefore around 4 min shorter than a solar day. In Figure 4 the difference between a solar and sidereal day is illustrated.

3.4.1 Greenwich Mean Sidereal Time (GMST)

GMST is the hour angle of the vernal equinox (Section 4) with respect to the prime meridian at Greenwich, expressed in decimal hours [6].

$$GMST = 6.697374558 + 0.06570982441908\,D0 + 1.00273790935\,H + 0.000026\,T^2\ . \tag{8}$$

Here $D0$ is the Julian Date of the previous midnight using the J2000 epoch (1 January, 2000 12:00 UTC or Julian date 2451545.0), H is the number of hours since the last midnight, and T is the number of centuries since J2000.

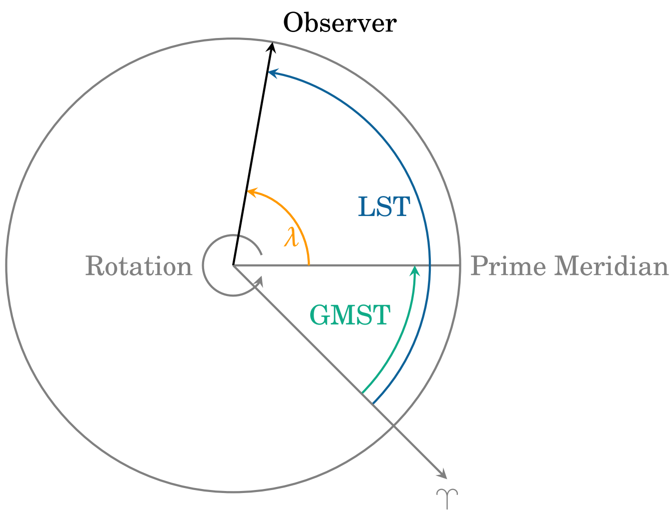

Figure 5 – Top-down view of Earth showing the relation between GMST (teal) and LST (blue) for an observer at a longitude λ (orange). The observer and prime meridian move with the rotation of the Earth, the vernal equinox does not.

3.4.2 Local Sidereal Time (LST)

Similar to GMST but takes observers longitude into account. To get the Local Sidereal Time the observers longitude has to be added to the GMST. To convert the longitude from degrees to hours it has to be divided by 15, $\frac{360°}{24\,\mathrm{h}} = 15\,°\,\mathrm{h}^{-1}$.

$$LST = GMST + \tfrac{\lambda}{15}\ . \tag{9}$$

The relation between LST and GMST is illustrated in Figure 5.

4 Celestial

Here we describe the different coordinate systems that are used to define the direction (origin) of an air shower. The zenith and azimuth coordinates rotate with the rotation of the Earth, because they are anchored to the location of the observer. The Equatorial system is linked to the celestial sphere and is independent from the rotation of Earth, the time of observation, and the location of the observer. To transform from zenith and azimuth to Equatorial the position of the observer and time of observation are required. The Horizontal coordinate system is used as intermediate, it also defines an azimuth coordinate, this is different from the azimuth coordinate in the zenith and azimuth system.

4.1 Zenith and azimuth coordinates

When a station detects a shower we try to reconstruct the direction of its origin, i.e. the position on the sky the shower came from. This direction is then given by a zenith (θ) and azimuth (ϕ) coordinate. These coordinates define a point on the semi-sphere that is the sky above the detection station. The Zenith is the point directly above the observer. The zenith angle is the angle between the direction and the Zenith point, so straight up is 0 rad and the horizon $\pi/2$ rad. The azimuth is the direction in the horizontal plane, it starts at East (0 rad) then goes to North (ENWS). This is illustrated in pink in Figure 6.

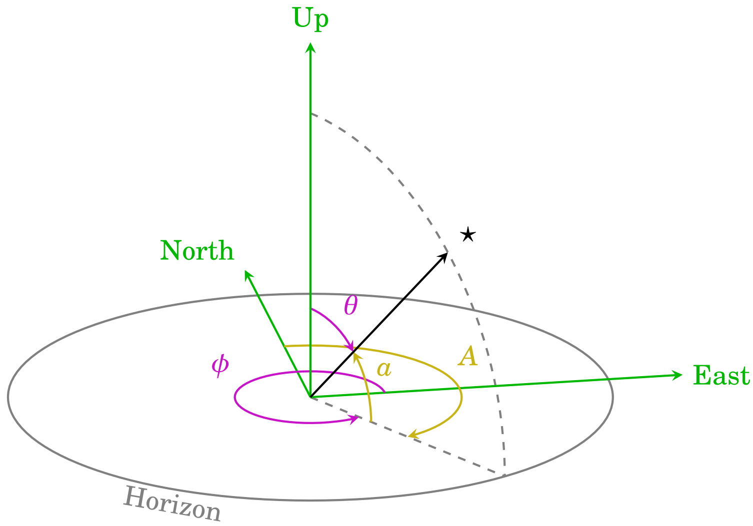

Figure 6 – Relationship between the ENU (green), Zenith Azimuth (pink) and horizontal (gold) coordinate systems.

We neither expect nor consider air showers from below the horizon, so the zenith angles, defined in radians, are an angle in the range $[0, \frac{\pi}{2})$. The azimuth is restricted to the range $[-\pi, \pi)$.

4.2 Horizontal coordinate system

This is a system used as intermediary for some coordinate conversions. It azimuth (A) and altitude (a) to define a direction. The altitude is the complement of the zenith, so 0 rad is horizontal and $\pi/2$ rad is the zenith. The azimuth definition also differs, in Horizontal coordinates it moves from North to East (NESW). This is illustrated in gold in Figure 6.

The formulae to convert from zenith (θ) and azimuth (ϕ) to altitude (a) and azimuth (A) are:

$$\begin{aligned} a &= \tfrac{\pi}{2} - \theta \, , \\ A &= \tfrac{\pi}{2} - \phi \, . \end{aligned} \tag{10}$$

Conversions can cause values to go beyond the allowed range of values for angles or times. They may have to be brought back into the range after the conversion.

4.3 Equatorial (J2000)

Figure 7 shows the relation between the celestial sphere and Equatorial coordinates. The right ascension (α) is the angle between the projection of the position on the celestial sphere ($\star$) on the plane of the Celestial equator and the vernal equinox (♈). The declination (δ) is the angle between the plane of the Celestial equator and the sky position. Due to precession (change of direction of Earth's rotational axis) the celestial positions slowly change over time. To compensate for precession coordinates are calculated as they would have been on a specific date, the J2000 epoch is commonly used. The equatorial coordinates are fixed to the celestial sphere and not influenced by the rotation of the Earth.

Given an observers position in WGS84, a time of observation in LST (derived from the GPS timestamp and observer longitude) and horizontal coordinates (converted from the

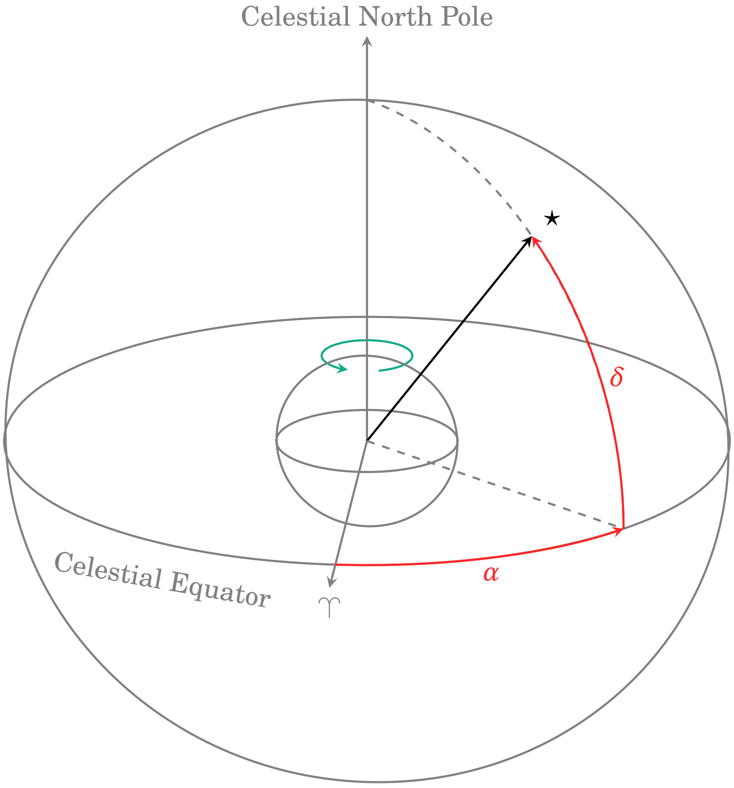

Figure 7 – This figures shows the relation between a position on the sky (black), Equatorial coordinates (red), and the Celestial sphere.

zenith and azimuth coordinates) pointing to the source the following formulae can be used to get the corresponding equatorial coordinates [7, p. 37].

$$\begin{aligned}
&\delta = \arcsin\left((\sin a \sin\varphi) + (\cos a \cos\varphi \cos A)\right) , \\
&HA = \arccos\left(\frac{\sin a - (\sin\varphi \sin\delta)}{\cos\varphi \cos\delta}\right) , \\
&\alpha = LST - HA \ .
\end{aligned} \tag{11}$$

The output range of arccos is $[0, \pi)$, while that of HA is $[0, 2\pi)$. If the azimuth is positive use: $HA = 2\pi - HA$.

Where φ the geographical latitude, a the altitude, A the azimuth, LST the Local sidereal time, δ is the declination, α the right ascension and the intermediate variable HA is the hour angle. This is illustrated in Figure 8.

5 CORSIKA

CORSIKA is an extensive air shower simulation program which we use. It uses coordinate systems which are slightly different from the ones we employ [8]. The SAPPHiRE framework provides a script to convert CORSIKA output to HDF5 format. During this conversion the coordinates are transformed to the HiSPARC coordinate system. The following sections describe these transformations.

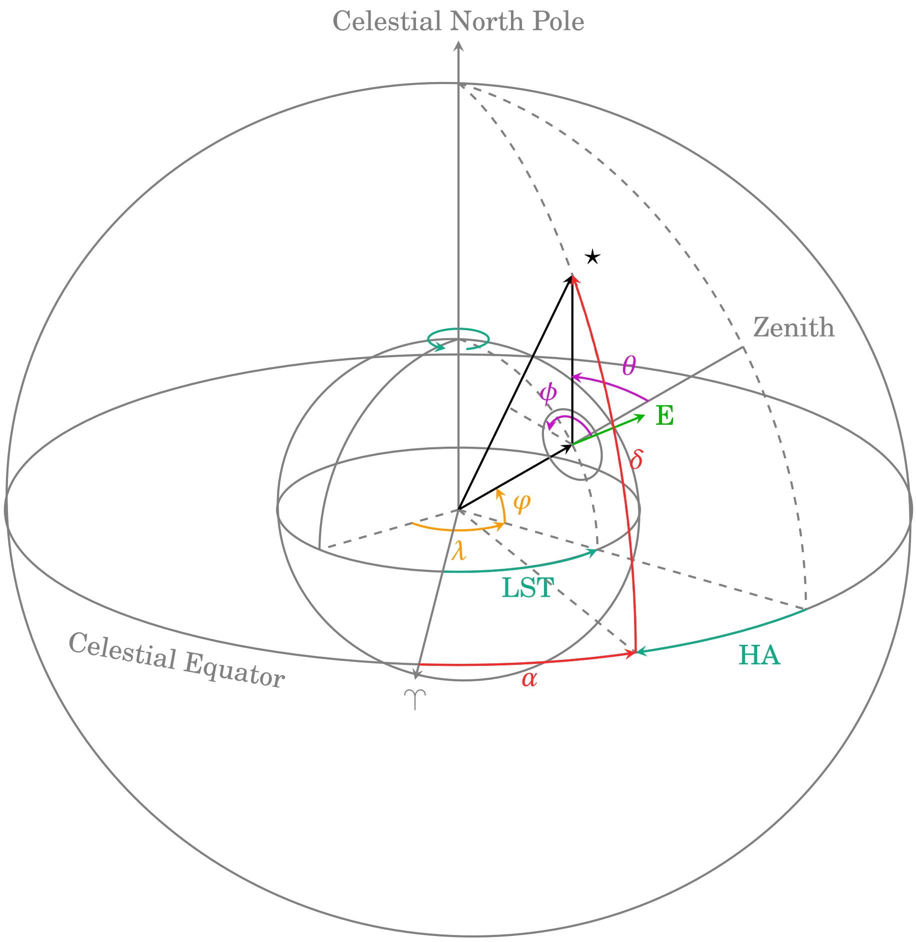

Figure 8 – Relationship between the WGS84 (orange), ENU (green), Zenith azimuth (pink), and clock (teal) and Equatorial (red) coordinate systems. ♈ points to the vernal equinox, the point where the ecliptic and celestial equator cross in the Aries zodiac. WGS84 defines the position of the observer, LST defines the time of observation, ENU is the local coordinate system (only East is shown because it is used for the azimuth), and zenith and azimuth give the direction of shower origin. Combined these give the same position on the sky as the equatorial coordinates.

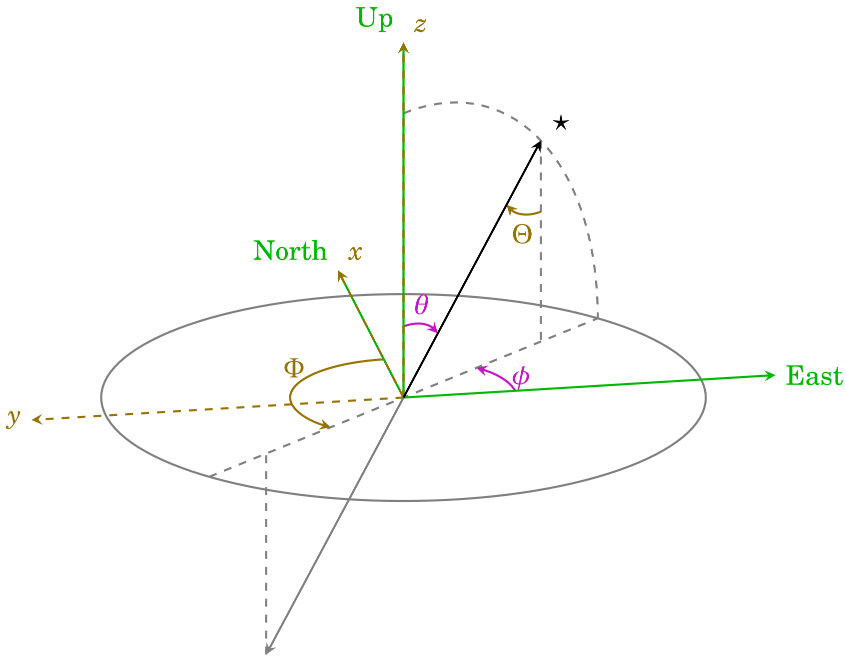

Figure 9 – Relationship between the ENU (green), Zenith Azimuth (pink), and CORSIKA (brown) coordinates.

5.1 Geographic

CORSIKA defines positions on the ground (or observation level) relative to the point where the shower axis intersects the observation level. Positive x axis points to magnetic North, positive y axis to the West, and the z axis upwards. This is shown by the dashed brown lines in Figure 9.

The formulae to convert CORSIKA to ENU are:

$$\begin{aligned} x_{\mathrm{ENU}} &= -y_{\mathrm{CORSIKA}} \,, \\ y_{\mathrm{ENU}} &= x_{\mathrm{CORSIKA}} \,, \\ z_{\mathrm{ENU}} &= z_{\mathrm{CORSIKA}} \,. \end{aligned} \tag{12}$$

5.2 Celestial

CORSIKA looks from the point of view of the shower, so not the direction it came form, but the direction it goes to. The CORSIKA zenith angle (Θ) is identical to the HiSPARC zenith (θ), 0 rad is a shower from the zenith and $\pi/2$ rad is a horizontal shower. The CORSIKA azimuth angle (Φ) differs from the HiSPARC definition of azimuth (ϕ). The CORSIKA azimuth is the angle the shower is heading to with respect to the North, while the HiSPARC azimuth is the angle the shower is coming from with respect to the East. $\Phi = 0$ rad is a shower heading towards North, so coming from South, which we would define as $\phi = -\pi/2$ rad. The (positive) rotation of the angle is in the same direction, from North to West. This is shown by the brown arced lines in Figure 9.

The formulae to convert the CORSIKA direction to HiSPARC zenith and azimuth are:

$$\begin{aligned} \theta &= \Theta \,, \\ \phi &= \Phi - \tfrac{\pi}{2} \,. \end{aligned} \tag{13}$$

6 Example

An example will now be given for a real HiSPARC event detected by station 503.

6.1 Measured

From the GPS of station 503 we know its geographical location in WGS84,

$$(\varphi, \lambda, h) = (52.356\,260\,0°, 4.952\,944\,0°, 51.4\,\text{m}) \; . \tag{14}$$

The location of its four detectors have been measured in compass coordinates (see Figure 3)

Detector	α [°]	r [m]	β [°]	z [m]
1	315	8.97	135	0
2	315	3.15	135	0
3	225	5.09	225	0
4	45	4.89	225	0

An event was detected on GPS timestamp 1 333 018 296 870 008 589 ns. The relative arrival times in each detector are 0 ns, 2.5 ns, 5 ns and 12.5 ns.

6.2 Conversions

To reconstruct the direction of the shower we use SAPPHiRE. First it converts the detector compass coordinates to ENU (relative to the GPS)

Detector	x [m]	y [m]	z [m]
1	-6.34	6.34	0
2	-2.23	2.23	0
3	-3.60	-3.60	0
4	3.46	3.46	0

The local zenith and azimuth direction is then reconstructed using the relative arrival times and the ENU positions. In this case

$$\begin{aligned} \theta &= 0.3818\,\text{rad} \\ \phi &= 3.0030\,\text{rad} \; , \end{aligned} \tag{15}$$

which is equal to these horizontal coordinates

$$\begin{aligned} a &= 1.1890\,\text{rad} \; , \\ A &= -1.4322\,\text{rad} \; . \end{aligned} \tag{16}$$

The GPS timestamp represents the date 29 March 2012 at 10:51:21. On that date 15 leap seconds were in effect. The corresponding UTC timestamp, Julian Date, and GMST are

$$\begin{aligned} UTC &= 1\,333\,018\,281\,870\,008\,589\,\text{ns} \; , \\ JD &= 2\,456\,015.952\,336\,\text{d} \; , \\ GMST &= 23.3389\,\text{h} = 23^{\text{h}}20^{\text{m}}20^{\text{s}} \; . \end{aligned} \tag{17}$$

Using the longitude of the station the Local Sidereal Time can be calculated from the GMST

$$LST = 23.6691\,\mathrm{h} = 23^{\mathrm{h}}40^{\mathrm{m}}9^{\mathrm{s}}\ . \tag{18}$$

Now the equatorial coordinates can be calculated

$$\begin{aligned} \alpha &= 5.5848\,\mathrm{rad} = 21^{\mathrm{h}}19^{\mathrm{m}}57^{\mathrm{s}}\ , \\ \delta &= 0.8730\,\mathrm{rad} = 50°1'\ . \end{aligned} \tag{19}$$

7 Acknowledgements

We want to thank Dr. J.J.M. Steijger for his careful checking of our methods.

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