# Webs on trees Understanding the interplay between ecological network motifs and phylogenetic trees

Giulio Valentino Dalla Riva

Supervisors:

Prof. Mike Steel (Mathematics and Statistics)
Dr. Daniel Stouffer (Biology)



Research Proposal for the Degree of Doctor of Philosophy in the Department of Mathematics and Statistics University of Canterbury

· 2013 ·

## Contents

1	Introduction			3
<b>2</b>	Stat	te of tl	he art	4
	2.1	Model	ling	5
	2.2	Extino	etion, hybrids and diversity	5
	2.3		s stability	6
3	Res	earch	objectives and approach	8
	3.1	Coupl	ing ecological roles and phylogenetic structure	9
		3.1.1	Foodweb and tree motifs	9
		3.1.2	Does predation separate clans?	12
	3.2	Toward a speciation/predation model		
		3.2.1		
		3.2.2	Non trivial rewiring	
		3.2.3	Ecological and Phylogenetic informations	18
	3.3	Extino	etion under complex ecological constraints	
		3.3.1	Clade size weighted extinction	19
		3.3.2	Survival through hybridization	
		3.3.3	Network diversity	

## Chapter 1

### Introduction

The anthropogenic pressure on the landscape and the living species communities is every day more evident. The biodiversity of the world is recognized as a precious, yet fragile, treasure and more and more efforts are done in order to preserve it, both in the local and in the global scale. A sensible conservation policy requires science to understand the current status of the ecological world and to develop tools with which to forecast its future status.

Biodiversity is, at the same time, the product of fast time processes, such as the one ruling the dynamics of the web of life, and long time processes, such as the ones involved in the growth of the tree of life.

Hence, it is of great importance to understand how this different time scale processes interact.

We approach the problem of studying the interplay between the building blocks of structures of the web and tree of life: motifs. In foodwebs, motifs are given by small subgraphs, which are seen to be over-abundant in the empirical observed webs. In phylogenies we can focus our attention on the mutual evolutionary distance between pair, triplets, quadruplets, ... of species.

In this scenario many elementary question, regarding trophic linking, speciation or extinction events, are yet to be answered.

We will explore the interplay between webs and trees following different approaches, including statistical description, model development and analytical investigation.

## Chapter 2

### State of the art

Foodwebs are currently analysed in ever growing detail: a number of network characteristics are considered of particular ecological interest - e.g., connectivity, stability, keyroles, degree distribution, motifs distribution. Analogously, phylogenetic trees also topical and their reconstruction is becoming sounder and sounder thanks to the developing sequencing techniques. The relationship between trophic networks and phylogenies is studied within different approaches including statistical descriptions, modelling efforts and analytical studies.

The evolutionary history of a species ensemble has non trivial influence on the species' role in the food webs, as shown by Stouffer et al. [46], Naisbit et al. [30]. Moreover, Eklöf et al. [15] showed that the taxonomy of a species community is a valid predictor of its trophic interactions: the authors find, indeed, a phylogenetic signal in the formation of food webs; similar results are found in Krasnov et al. [23], Ives et al. [22], Bersier et al. [5].

Interpretations of motifs in terms of ecological properties has been reviewed by Bascompte and Stouffer [2], while various field investigations have been and are being attempted (see for example Rip et al. [38]). It has been argued that Phylogenetic Diversity arising from the evolutionary history of a species community may be a predictor of ecosystems functions: see Srivastava et al. [44], Gravel et al. [19], Mouquet et al. [29].

#### 2.1 Modelling

A survey of classical models, up to the models developed in 2002, can be found in Drossel and McKane [13]. The author focused on stability issues, covering both static and dynamical models. More recently, a number of models can be found in Pascual and Dunne [32].

Drossel and McKane proposed a model taking into account predator-prey dynamics (see [12,27] and references within) implementing Arditi-Ginzburg Predator-Prey ratio-dependent efforts [1] and traits evolution (the proportion of a prey p in a predator P's is given as the score of a traits matching matrix). In [36] they implemented other predator-prey dynamics, e.g., Lotka-Volterra, and observed that the model food web outcome didn't fit the empirical observed data. The model has been developed in order to study immigration-speciation processes [34], and complexity-stability relation [33].

Caldarelli and Garlaschelli proposed a general framework for the ecological network evolution, the Webworld model [8]. Cannon et al. introduced a Lotka-Volterra niche model starting from a void universe with a single steady source of biomass [10] and gave an interpretation of two peculiar motifs in terms of network expansion within total biomass constraint. A food web model implementing niche model and the hypothesis that speciation and extinction rates decrease with increasing body mass was proposed in 2005 by Rossberg et al. [41] and then further developed [42] implementing the signal of an evolutionary history where new species avoid to compete with their ancestors.

It would be useful to reconstruct foodwebs structure along their historical evolution. Although detailed direct observations are yet to come, some attempts have been done, see Roopnarine et al. on paleocommunities [39, 40] or Dunne et al. for the Cambrian period [14]. Another possibility has been introduced by Doi et al. [11]: the author examined lakes spanning six order of magnitude different ages, and hence showing the result of shorter or longer evolution times.

#### 2.2 Extinction, hybrids and diversity

Extinction events prune the trees of life. This results in a loss of phylogenetic diversity, which depends clearly on which species goes extinct and which are retained.

A basic model which is widely used to predict on the future biodiversity of a species community under extinction threat is the Field of Bullets model, introduced by Raup in 1984 [37]. Since its introduction the model has been variously generalised and applied with good results.

An immediately related problem asks, under limited resources, to find a conservation policy maximizing the expected biodiversity. A first answer comes from just considering the number of saved species. Prioritizing the length of the saved branches means taking into account Phylogenetic Diversity (Faith [16]): this approach was introduced by Witting and Loeschcke [49].

Weitzmann [47] considered probabilities of survival and budget limitation and introduced the so-called "Noah's Ark Problem". Hartmann and Steel [20] showed that, under certain limitation, we can optimally solve the problem through a greedy algorithm.

Bordewich and Semple [7] considered an important generalization of the Noah's Ark Problem where conservational policy can be applied to groups of species, or nature reserves, hence introducing relationship between the survival probability of different species.

Recently, Billionnet [6] showed that fast mixed-integer linear programming techniques can give a near-optimal solution to both the original Noah's Ark Problem and its nature reserves generalization. This techniques also yield an upper bound to the optimal solution.

With a different approach, a relationship between the survival probability of species taking into account their phylogenetic diversity (in terms of shared common evolutionary history) was introduced as a trait-based extinction model, introduced by Steel and Faller [18,45]: the model allowed trait evolution along the tree branches following a markov chain model.

An explicit introduction of ecological constraints into a field of bullets extinction model has been proposed by Moulton et al. [28], where the authors used the concept of "viability", that is taking into account that every predator species has to predate at least on a species to survive. The model complexity has been studied by Faller et al. [17].

#### 2.3 Species stability

Ecological networks can be seen as dynamical system, where the dynamic is ruled by the population dynamics process, such as predation processes, competition or cooperation. The quest for a proper definition of ecological network stability and for the identification of drivers of stability, date at least at May's works [25,26] on complexity and stability, where the author showed that the two don't always grow together.

Stability may be approached as global property, hence focusing on the overall structure of the foodweb, or as a local property, hence focusing on the contribute of certain species to the overall stability.

Foodwebs have been described in terms of ecosystem stability under a perturbation regime (which means, altering the population dynamic parameters, see for example Solé and Bascompte [43] for a recent overview). An usual approach is to consider linear or approximately linear population dynamics – i.e., systems were the applied perturbation can be considered to produce a linear effect on the stable solution. A review of the approaches for measuring

the response of ecological systems to perturbation can be found in Neubert and Caswell [31].

More recently Berg [3] (see also [4]) developed sensitivity based measures to identify keystone species in foodwebs.

Working on protein transcription networks and neuron networks, Prill et al. [35] gave a numerical characterization of 3-motifs and 4-motifs in terms of stability through a Structural Stability Score, defined as the probability that the dynamical system corresponding to a given motif relaxes monotonically to the stable equilibrium after a small linear perturbation. Significantly the authors found a positive correlation between the SSS score of a motif and its over or under representation in an empirical network.

In a recent work [46] Stouffer et al. suggest that "species dynamic importance, the degree to which a species acts as a keystone species, may indeed be an intrinsic and inherent species attribute that arises as a consequence of species' evolutionary histories", mediated by the species motifs distributions, which indeed exhibit a phylogenetic signal.

## Chapter 3

## Research objectives and approach

We aim to discover the relationship between phylogenetic evolution and foodweb evolution.

A main focus of the thesis will be on the correlation between basic trophic network structural elements, such as elementary predator-preys linking, foodweb motifs, node or edge roles (as depicted in figure 3.1), and phylogenetic tree properties, such as phylogenetic diversity, pairwise distance between leaves, clades and clans. In this scenario, some foundational question are yet to be answered.

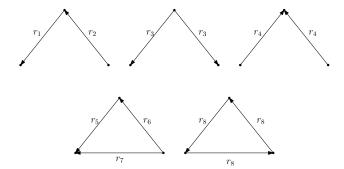


Figure 3.1: The different edge roles in the five most common predation motifs.

One problem, that will need to be solved, arises from the merging of data coming from two different empirical sources: genetic sequencing studies and foodwebs observation. The usual empirical data are given in the form of an ecological community which phylogeny is reconstructed and the species are mapped in a foodweb. This gives a straightforward mapping of the phylogenetic tree leaves into the trophic relationship graph. The food can be variably detailed in terms of guilds census (how many extant species and trophic relationship are present in the given foodweb) and taxonomic resolution (how

coarse is the identification of the guilds taxonomy). Analog uncertainty can be present in the phylogeny reconstruction, which may vary from a topological tree given by the taxonomic classification to a tree reconstructed from the actual genome sequencing of the observed species.

## 3.1 Coupling ecological roles and phylogenetic structure

#### 3.1.1 Foodweb and tree motifs

Let  $\tau$  be the phylogenetic tree of the species in a certain ecological community and  $\omega$  its trophic relationship graph, the foodweb. Call  $l_1, l_2, \ldots, l_n$  the leaves of  $\tau$ , we will use the same name to identify the species in the foodweb.

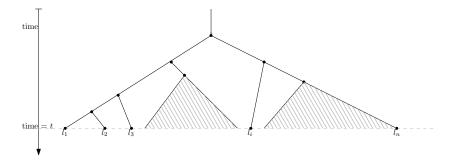


Figure 3.2: A Phylogenetic Tree, the shadowed zones hides other branches and leaves.

We will express  $\omega$  in terms of its community adjacency matrix  $M_{\omega}$  which elements  $m_{\omega}^{ij}$  are equal to 1 if and only if there's a link between node i and j, and hence if and only if the species  $l_i$  predates on the species  $l_j$ . Through the tree  $\tau$  we can define a phylogenetic distance matrix  $M_{PD}$  which elements  $m_{PD}^{ij}$  are given by the phylogenetic distance  $Pd_{ij}$  between the species  $l_i$  and  $l_j$ . As  $Pd_{ij} = Pd_{ji}$ ,  $M_{PD}$  is a symmetric matrix.

The phenomenon we are looking at is apt to two complementary analysis approaches: 1) looking from the phylogeny to the foodweb structure, 2) looking from the foodweb structure to the phylogeny.

**Couples** We will consider basic components of both the predation network and the graph, starting with couples and adding one element at time.

1. What is the probability of  $m_{omega}^{ij} = 1$  given that the species  $l_i$  and  $l_j$  have phylogenetic distance  $Pd_{ij} = k$ ? Or, similarly, give a good

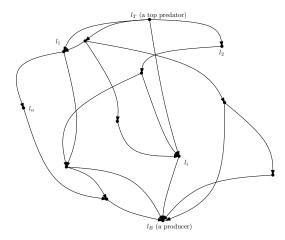


Figure 3.3: In this foodweb, and in all the proposal, we draw a link from a node i to a node j whenever species i predates on j.

approximating function

$$f(k) \approx \mathbb{P}(m_{omega}^{ij} = 1 | Pd_{ij} = k).$$

Notice that, in a model where evolution and predation are independent, we would have

$$\mathbb{P}(m_{omega}^{ij} = 1 | Pd_{ij} = k) = \mathbb{P}(m_{omega}^{ij} = 1).$$

2. On the other hand, given  $m_{omega}^{ij} = 1$ , what is  $\mathbb{E}[Pd_{ij}]$ ? In a null model we have  $\mathbb{E}[Pd_{ij}|m_{omega}^{ij} = 1] = \mathbb{E}[Pd_{ij}]$ .

**Triplets** We will, then, study the identity mapping of foodweb elements triplets, called 3-motifs, into phylogenetic tree elements triplets, called triangles. We will call  $\omega(i, j, ...)$  the subgraph induced by the node i, j, ...

We will call triangle of distances a triplet  $T_{ijk} = \{Pd_{ij}, Pd_{jk}, Pd_{ki}\}$  of mutual phylogenetic distances between three elements of the ecological community. Figure 3.4 represents a triangle between the species  $l_i, l_k, l_j$ .

There are 13 possible 3-nodes: eight of these have at least a double link, that is two nodes linking each other, while five have only single links. In empirical foodwebs double links are rare (and so are cannibalistic links), see for example Appendix A in Camacho et al. [9]. Indeed, the five motifs with only single links constitute the large majority of the overall motifs<sup>1</sup>. Hence, we will focus initially just on the single links motifs: however, the analysis

<sup>&</sup>lt;sup>1</sup>CITATION NEEDED

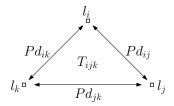


Figure 3.4: A triangle of phylogenetic distances.

can be extended to the eight double links motifs and the greater dimension motifs including more than three nodes.

In the basic five 3-motifs, the ones without mutual predation relationship, we have 8 topologically different relationship  $r_i$ , as depicted in figure 3.1. If a connected subgraph  $\omega(i, j, k)$  is isomorphic to a certain motif with its edges in positions  $r_z$ ,  $r_{z'}$ ,  $r_{z''}$  we will write

$$\omega(i, j, k) \rightarrow m(z, z', z'')$$
.

The question 2) has now a straightforward extension: we want to give

$$\mathbb{E}\left[T_{ijk}|\omega(i,j,k)\to m(z,z',z'')\right].$$

The generalization of question 1) requires more attention.

We should notice that each predation relationship, the link (i, j), can be part of various motifs including the species  $l_i$ ,  $l_j$  and a third species completing the motif, as shown in figure 3.5.

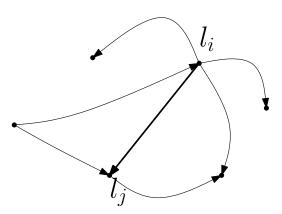


Figure 3.5: The neighborhood of (i, j). Here  $\underline{m}(i, j) = \{0, 1, 2, 1, 1, 1, 0, 0\}$ .

We will call the *motif distribution* of a link (i, j) the vector  $\underline{m}(i, j) = \{c_1, c_2, \ldots, c_8\}$  where  $c_i$  is given by the number of times the links (i, j) occurs in position  $r_i$ . Each motif  $\omega(i, j, k)$  is associated to a triangle of distances

 $T_{ijk}$  and to each motif distribution  $\underline{m}(i,j)$  is associated a vector of triangles  $T_{m(i,j)}$ .

When considering the linking between two species in this scenario we should take into account also their 3-motifs neighborhood. Hence, the probability of having a predation relationship (i, k) may be influenced by the phylogenetic distance between  $l_i$  and  $l_k$ , the motif distribution such a relationship would have and all the triangles of distances induced by the motifs containing  $l_i$  and  $l_i$  if that relationship would exist.

Question 1) in this scenario asks: what's the probability of  $m_{omega}^{ij} = 1$  given phylogenetic distance  $Pd_{ij} = k$ , motif distribution  $\underline{m}(i,j) = \vec{v}$  and vector of triangles  $T_{m(i,j)}$ . Or, similarly, give a good approximating function

$$f(k, \vec{v}, T_{\vec{v}}) \approx \mathbb{P}(m_{omega}^{ij} = 1 | Pd_{ij} = k, \underline{m}(i, j) = \vec{v}, T_{\underline{m}(i, j)} = T_{\vec{v}}).$$

Again, notice that, in a null model, the motif distribution should have no role.

#### 3.1.2 Does predation separate class?

Let's observe again a foodweb  $\omega$  and the phylogenetic tree  $\tau$  whose leaves set  $L = \{l_1, l_2, \ldots\}$  is a subset of the elements of  $\omega$ . Explicitly denote the mapping of the leaves in  $\omega$  as  $p(l_i)$ .

For each species  $l_i$  we can define one couple of subsets in the foodweb and one couple in the phylogenetic tree. Indeed, we can split the neighbourhood of  $p(l_i)$  in  $\omega$ , which is denoted  $\star(p(l_i))$ , in the set of species predating  $p(l_i)$ , denoted  $T(p(l_i))$ , and the set of species  $p(l_i)$  predates,  $B(p(l_i))$ . Hence  $\star(p(l_i)) = T(p(l_i)) \cup B(p(l_i))$ . In a similar fashion, consider  $l_i$  and the subset of L given by the mapping of the species in  $\star(p(l_i))$  into the phylogenetic tree, that is  $p^{-1}(\star(p(l_i)))$ , which we will denote as  $\star(l_i)$ . We define a rooted subtree by pruning all the species but the ones in  $star(l_i)$  and placing the subtree root in  $l_i$ , which we denote  $\tau_{|\star(l_i)}$ . The subtree can be split in two principal clades,  $C_r(l_i), C_l(l_i)$ , such that  $\tau_{|\star(l_i)} = C_r(l_i) \cup l_i \cup C_l(l_i)$ .

We can, at this point, ask what is the distribution of the predators and preys of  $p(l_i)$  on the two clades  $C_r(l_i)$ ,  $C_l(l_i)$ .

As a working null hypothesis, let's assume the predation relationship is a random process independent from the evolutionary history. Consider a species  $l_i$ , in the foodweb  $\omega$ . Suppose  $l_i$  has in-degree  $\delta^i(p(l_i))$  and out-degree  $\delta^o(p(l_i))$ . In- and out-degree of the species gives respectively the size of the  $l_i$  predators set  $T(p(l_i))$  and of the  $l_i$  preys set  $B(p(l_i))$ . Predators and preys of  $l_i$  are distributed uniformly at random on the leaves of  $\tau$ . This means that, taking a leaf from  $\tau$ , the probability of getting a  $l_i$  predator, a  $l_i$  prey or a

species that as not direct trophic relationship with  $l_i$  depends only the relative size of L,  $B(p(l_i))$  and  $T(p(l_i))$ : drawing the lead uniformly at random or with a bias based on the tree structure doesn't change.

Calling  $C_r(l_i)$  the larger of the two clans, the number of predators in it, which is the size of the set of leaves  $C_r(l_i) \cap B(l_i)$ , will be distributed as a hypergeometric random variable. A similar reasoning can be proposed for the prey.

A more refined null model can be retrieved by considering the probabilistic niche model [48] or the evolutionary niche model proposed by Ingram [21]: indeed, we can study the distribution of predators and preys on the two principle clades, assuming that, instead of being independent from the evolutionary history, the predation is shaped by a probabilistic niche model, where the niche position is a quantitative trait depending on the evolutionary history of the species.

Moreover, we can compare the model data with the data retrieved from the empirical observed network.

#### 3.2 Toward a speciation/predation model

An insight into the mechanisms relating foodwebs evolution and speciation/extinction events can be acquired through the development of a suitable model which will be studied both from an analytical point of view and software implemented.

We will develop the model starting from a the most basic implementation possible, adding gradually complexity at each step. The key element will be defining what happens to the population of predators and preys of a species  $l_i$  when  $l_i$  speciate.

For sake of simplicity, we will allow only just bifurcation events, where one species originates two daughters. Moreover, we will assume that one of the daughter will be identical to the mother while the other, the active daughter, may change its place in the foodweb. Another strong simplification for the base model will be to consider only topological phylogenetic trees, hence omitting any information on the length of the branches or the phylogenetic distance between two species. Such a simplistic model is described in 3.2.1.

Generalisations and developments of the model will come considering more complex rewiring algorithms for the active daughter, as summarized in 3.2.2, considering both the daughter active, and taking explicitly into account more complex ecological and phylogenetic information, as sketched in 3.2.3, possibly including the statistical insight retrieved studying the interplay between motifs and triplets (see section 3.1). Beside the analytical definition of the models, software simulation of them will be done, possibly in collaboration with Dr. Jordan Ferenc and the use of the stochastic simulation language L developed by COSBI lab.

During the research it would be necessary to develop a set of benchmarks with which to judge the quality of the proposed model: possible benchmarks include the distribution of top and base species, the connectivity, the overall foodweb nodes degree distribution, the overall foodweb motifs distribution, the nodes motifs distribution, ....

#### 3.2.1 A most elementary model

Consider a certain ecological community given by a set of species  $C_t$  at time t, let  $\tau_t$  be its phylogenetic tree at time t and  $\omega_t$  its trophic relationship graph (the foodweb) at time t. Call  $l_1, l_2, \ldots, l_n$  the leaves of  $\tau_t$ .

We will use the same naming also for the species in  $\omega_t$  without fear of confusion<sup>2</sup>.

**Evolution event** Each species  $l_i$  can be chosen for a speciation event at the same rate  $\sigma$ . When the species  $l_i$  is chosen at time t for speciation it gives birth to two species  $l'_i$  and  $l''_i$ , see fig. 3.6. Hence, we will observe the building of a classical Yule random tree.

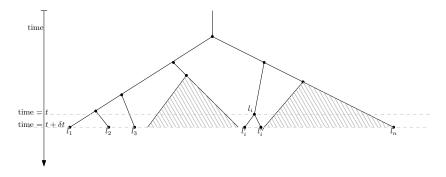


Figure 3.6: The phylogenetic tree  $\tau$  at time  $t + \delta t$  has two new species,  $l'_i$  and  $l''_i$ .

**Imperfect duplication** To keep the model to the most simple case possible, we will assume that the place of one of the two siblings,  $l_i''$ , in  $\omega_{t+\delta t}$  will duplicate exactly the place of  $l_i$  in  $\omega_t$ . The other daughter will get rid of one

<sup>&</sup>lt;sup>2</sup>In a more formal way we should introduce a bijective mapping i of the leaves  $l_i$  of  $\tau_t$  to the species  $x_i$  in  $\omega_t$ , so that  $x_i = i(l_i)$ .

of her ancestor predators with probability p and, independently from this, will add one new prey to her diet with probability q. We impose initially p = q.

In this model the predator will be drawn uniformly at random in the set  $P(l_i)$  of the predators of  $l_i$  and the prey will be chosen uniformly at random from  $pp_t(l_i)$ , which is the set of species  $l_p$  in  $C_t$  such that there are no path from  $l_p$  to  $l_i$ . Hence, we do not allow for the formation of predation chain relationship. The mechanism is depicted in figure 3.7

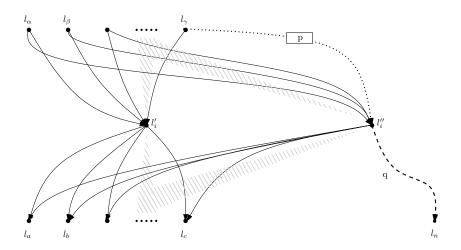


Figure 3.7: The rewiring of a species' foodweb neighbourhood during a speciation event in the imperfect duplication model.

What happens to the in- (and out-) degree of the graph during a speciation event?

If both siblings just duplicated their mother,

$$\delta_{l'}^{in}(t+\delta t) = \delta_{l''}^{in}(t+\delta t) = \delta_{l_i}^{in}(t) .$$

At the same time all predators of  $l_i$  will acquire one more prey and all prey of  $l_i$  will have one more predator. Hence

$$\delta_{tot}^{in}(t+\delta t) = \delta_{tot}^{in}(t) + \delta_{l_i}(t).$$

If  $l'_i$  get rid of a predator,  $l'_i$  in-degree decreases by one and the out-degree of the lost predator decreases by one. On the other hand, if  $l'_i$  acquire a new prey,  $l'_i$  out-degree increases by one and the in-degree of the new prey increases by one. Hence, if  $l_i$  speciates at time t and we don't have other speciation before  $t + \delta t$ , we have that, assuming p = q,

$$\mathbb{E}\left[\delta_{tot}^{in}(t+\delta t)\right] = \delta_{tot}^{in}(t) + \delta_{l_i}(t) . \tag{3.1}$$

Moreover, naming  $\widehat{\delta^{in}}(t)$  the mean degree at time t, during one speciation event occurring in the interval  $[t,t+\delta t]$ , assuming no other speciation event happen in that time, and remembering that  $\widehat{\delta}(t)=2\widehat{\delta^{in}}(t)$  for each time t, we'll have that, for p=q,

$$\mathbb{E}\left[\widehat{\delta^{in}}(t+\delta t)\right] = \widehat{\delta^{in}}(t) + \frac{\widehat{\delta^{in}}(t)}{|C_t|+1} . \tag{3.2}$$

#### 3.2.2 Non trivial rewiring

More natural rewiring mechanism can be given following various possible generalisation.

We can consider different p, q values, so to distinguish between predators lost and preys adding. Moreover, we may consider also predators adding and preys lost, for example due to invasion or geographic drift, hence considering four different probability values  $p_l$ ,  $p_a$ ,  $q_l$ ,  $q_a$ . In this scenario the set of potential predators of  $l_i$ , which is  $Pp_t(l_i)$ , will be given by those species  $l_j$  such that a predation link between  $l_j$  and  $l_i$  would not result in a predation chain.

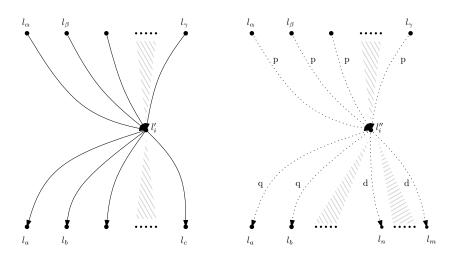


Figure 3.8: The rewiring of a species' foodweb neighbourhood during a speciation event in the niche partition model.

**Niche partition model** Instead of considering just the adding or erasing of one link, multiple link can be concurrently rewired, producing what we may call a niche partition.

As before one of the two siblings keep the same place of her ancestor in the foodweb. The active daughter,  $l_i$ , preserves each of her ancestor predator with a probability p and preserves each prey with probability q. Moreover, she may acquire new preys from  $pp_t(l_i)$  or acquire new predators from  $Pp_tl_i$ . The active daughter add each new prey or predator to her diet with probability respectively

$$d = \frac{k}{|pp_t(l_i)|}$$

and

$$D = \frac{K}{|Pp_t(l_i)|}$$

so that the overall probability of adding at least a new prey is, respectively,

$$\mathbb{P}(\text{add at least one prey}) = 1 - (1 - d)^{|pp_t(l_i)|} ,$$

and

$$\mathbb{P}(\text{add at least one predator}) = 1 - (1 - D)^{|Pp_t(l_i)|}$$
.

The mechanism is depicted in figure 3.8.

#### 3.2.3 Ecological and Phylogenetic informations

The model can be further developed in order to include more detailed ecological and phylogenetic network.

In the previous basic models the rewiring were independent from the phylogenetic distance between the active node, the original target of the link and the new target of the link. The results of section 3.1 can be used to define rewiring mechanism where the rewiring targets are chosen with different probabilities depending on their position on the phylogenetic tree.

More complex ecological constraints can be taken into account: species may tend to preserve their ancestor degree distribution (which is in some amount granted by the basic models), the overall foodweb motifs distribution (a global rule), their ancestor motifs distribution (a local rule) or their neighbourhood motifs distribution (which may be due to a cost of altering their neighbours motifs distribution).

Moreover, the model may include extinction events: in the previous sections we explored how speciation events may rewire a foodweb. Similarly, we may ask how extinction events rewire the foodweb. Different extinction dependent rewiring models can be used in connections with ecological aware generalisation of the Field of Bullets model to include network properties in the Noah's ark problem, as sketched in section 3.3.3.

## 3.3 Extinction under complex ecological constraints

The most common simple mathematical model of extinction events are given by the *field of bullets* models and its generalization. A problem closely related is finding the, under a set of properly defined constraints, i.e. a cost/budget bound, the conservation policy maximizing the expected phylogenetic diversity.

A natural generalization of this approach may be given taking into account hybridization events and ecological constraint. Natural question regards the expected Phylogenetic Diversity, its variance and its comparison with the classical models.

Insights can be gathered from both a pure analytical approach, working with artificial data, and from the application of the models to empirical observed species community. Preliminary results shows that the behaviour of models where the extinction depends on ecological constraints and hybridization events are allowed is sensibly different from the classical models.

We just show briefly two possible initial model, which will be developed during the thesis research.

#### 3.3.1 Clade size weighted extinction

Very poor branches of the tree of life, such as just saved from extinction species or small isolated population may have lower probability to survive from a changing habitat.

We propose a model of random extinction which takes into account this issue.

Let  $\tau$  be the phylogeny we are considering and  $x_i$  its leaves We will call  $C_r^0$  and  $C_l^0$  the two principal clades at time 0.

At each extinction event either a species or a clade goes extinct. To select which one to prune we think of extinction as an event running from the root to the leaves and stopping randomly. At each bifurcation the events steps on, it choose to stop with probability

$$\mathbb{P}(\text{stops at } C^y) = \left(\frac{1}{|C^y|}\right)^{\alpha}$$

or decides to proceed on the right or left clade with probability

$$\mathbb{P}(C_r^y) = \left(\frac{|C_r^y|}{|C_l^y|}\right)^{\beta} \mathbb{P}(C_l^y)$$

where  $C_r^y$  ( $C_l^y$ ) is one of the two principal clades descending from  $C^y$ . A more precise notation can be given registering the r and l path followed in order to come to  $C^y$ .

When the extinction walk ends, all the species descending from the selected node go extinct.

In general, varying the values of  $\beta$  and  $\alpha$  (as long as they're both positive), a continuous of scenario where isolated branches goes extinct can be modeled. Notice that if  $\alpha \to \infty$  and  $\beta = 0$  then the model is simply the classic Field of Bullets model.

#### 3.3.2 Survival through hybridization

**Motivation** Let's consider a very simple case where we observe two groups of closely related taxa (A, B) and (B, C). Each of them have almost null probability of survival if they are not chosen for a conservation policy and probability p of survival otherwise. Moreover there's a probability  $H(A, \neg B)$  that the clade (A, B) generate an hybrid  $h_a b$  when A is protected but B is not; there's a probability H(A, B) that the clade (A, B) generate an hybrid  $h_a b$  when both A and B are protected; there's a probability  $H(\neg A, \neg B)$  that the clade (A, B) generate an hybrid  $h_a b$  when both A and B are not protected. We'll use a similar notation for the clade (C, D) or for hybrid coming from a couple of non-closely related species (as A and D or B and C, which are not considered in the Becker et al. paper).

Notice that, without hybridization, protecting the species A and C (scenario 1) brings to a greater expected biodiversity than saving species A and B (scenario 2). On the other hand, if we allow for hybridization events, the previous inequality can be shown to be no more valid.

We will now analyze the expected Phylogenetic Diversity scenario, in the case with hybridization, in the spirit of Becker et al. A careful choice should be made when defining the weight of a hybrid species. We will introduce our choice with a suitable example.

Shared history proportion Let  $h_{bc}$  be a hybrid generated by the species B and C, as in figure 3.9. The hybrid will inherit a certain proportion of the genetic sequence of B, while the remaining part will be inherited from C. Let

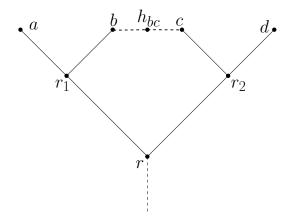


Figure 3.9: A phylogenetic tree where the species B and C generated an hybrid  $h_{bc}$ .

 $\delta$  be the proportion of the genetic sequence of  $h_{bc}$  coming from B, we will write  $h_{bc} = \delta C + (1 - \delta)B$ , with  $0 < \delta < 1$ .

When computing the expected PD, we will consider that a part of the evolutionary history of C is conserved in  $h_{bc}$ , precisely a proportion equal to  $\delta$ , and, at the same time a proportion  $(1 - \delta)$  of the evolutionary history of B is conserved in  $h_{bc}$ . These can be expressed imposing that the branch connecting  $h_{bc}$  to C is attached at a  $\delta$  of its length, as in figure 3.10.

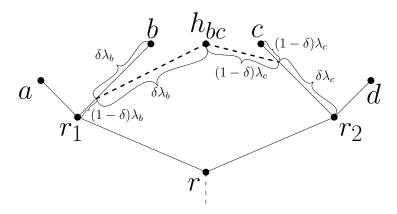


Figure 3.10: The hybrid  $h_{bc}$  conserve  $\delta$  ((1 –  $\delta$ )) of the evolutionary history of species C (B), nearby the branches we show their length.

Given the length of the various branches in the original phylogeny (the one without the hybrid) and the survival probability for each of the extant species (and the hybrids), it is straightforward to compute the expected Phylogenetic Diversity of the tree.

We will model hybridization through a random process occurring with a probability depending on the evolutionary history of the species

$$\mathbb{P}(H(i,j)) = \frac{1}{k \ d(i,j)}$$

where k is a scale factor  $(k \ge 1)$ , d(i, j) is the phylogenetic distance between species i and j normalized so that the shortest branch in the phylogeny has length 1, so to have  $\min_{i,j}(d(i,j)) \ge 2$ . Moreover, if a species is extinct or doomed, that is, has zero survival probability, its probability to generate a hybrid is also zero.

In the case where a hybrid may be originated, its PD contribution will be weighted by both the hybridization probability, H(i, j), and the survival probability,  $p_h$ , of the hybrid.

#### 3.3.3 Network diversity

The Noah's Ark Problem requires to maximize the Phylogenetic Diversity under certain constraints. The problem may be generalised taking into account other desired property of the surviving ecological network of species: we know, for example, that different foodweb structures can be more or less resilient to small perturbation. Moreover, as species show different motifs distribution, which means they have a different weight in the overall stability, their extinction produce greater or smaller impact in the ecosystem.

On one hand, hence, we can take into account local network properties, i.e. the motifs distribution of each species, possibly expressed as a stability index. On the other hand, we can decide to observe global network properties, i.e. one of the various foodweb overall stability definitions (see the literature cited in section 2.3).

In the local prospective, the extinction of a species produces a certain loss of Phylogenetic Diversity and the ecological network loses that species contribute to the overall stability. Hence, under a Field of Bullets model, we can ask how much Phylogenetic Diversity and overall stability index are expected to be lost. A solution to a so generalised Noah's Ark Problem is the conservation policy which maximize, under certain constrains, the expected PD and overall stability sum (or a more proper functional of them).

In a global prospective, studying the changes of ecological network properties such as network stability, overall motifs distribution, connectedness, due to an extinction event requires to understand how the extinction event rewires the foodweb structure (which will be explored in the task 3.2.3). Once adopted a rewiring model, we can state again a generalised Fields of Bullets extinction model and a related Noah's Ark Problem.

A third approach may be obtained considering a network property, developed in the context of control theory, named controllability. As recently proposed by Liu et al. [24] the problem of controllability may be posed in terms of identifying the minimum number of driver nodes (nodes on which we can act directly) are required to have a controllable system, that is a system that can be forced, from any initial state, to reach any required final state in finite time, through a proper set of inputs. Again, we can pose the problem within a local or a global point of view: in the latter prospective we can compute how many extinctions may occur in an ecosystems without losing controllability, in the former prospective we can express the desire to retain leaves of the tree which are driver nodes necessary for the controllability of the system.

## Bibliography

- [1] R. Arditi and L. R. Ginzburg, Coupling in predator-prey dynamics: Ratio-dependence, Journal of Theoretical Biology, 139 (1989), pp. 311–326.
- [2] J. BASCOMPTE AND D. B. STOUFFER, *The assembly and disassembly of ecological networks*, Philosophical Transactions of the Royal Society B: Biological Sciences, 364 (2009), pp. 1781–1787. PMID: 19451127 PMCID: PMC2685423.
- [3] S. Berg, Community Robustness Analysis: Theoretical Approaches to Identifying Keystone Structures in Ecological Communities, PhD thesis, Linköping, 2013.
- [4] S. Berg, M. Christianou, T. Jonsson, and B. Ebenman, Using sensitivity analysis to identify keystone species and keystone links in size-based food webs, Oikos, 120 (2011), pp. 510–519.
- [5] L.-F. Bersier and P. Kehrli, *The signature of phylogenetic constraints on food-web structure*, Ecological Complexity, 5 (2008), pp. 132–139.
- [6] A. BILLIONNET, Solution of the generalized noah's ark problem, Systematic Biology, 62 (2013), pp. 147–156.
- [7] M. BORDEWICH AND C. SEMPLE, Nature reserve selection problem: a tight approximation algorithm, Computational Biology and Bioinformatics, IEEE/ACM Transactions on, 5 (2008), pp. 275–280.
- [8] G. CALDARELLI, P. G. HIGGS, AND A. J. MCKANE, Modelling coevolution in multispecies communities, Journal of Theoretical Biology, 193 (1998), pp. 345– 358.
- [9] J. CAMACHO, D. STOUFFER, AND L. AMARAL, Quantitative analysis of the local structure of food webs, Journal of theoretical biology, 246 (2007), pp. 260–268. PMID: 17292921 PMCID: PMC2128744.
- [10] J. CANNON, G. FAY, A. HEIN, AND V. WEINBERGER, Diversification in simulated food webs: The role of closed motifs, (2010).

- [11] H. Doi, M. J. Vander Zanden, and H. Hillebrand, Shorter food chain length in ancient lakes: Evidence from a global synthesis, PLoS ONE, 7 (2012), p. e37856.
- [12] B. Drossel, P. G. Higgs, and A. J. McKane, The influence of predatorprey population dynamics on the long-term evolution of food web structure, arXiv:nlin/0002032, (2000).
- [13] B. Drossel and A. J. McKane, *Modelling food webs*, arXiv:nlin/0202034, (2002).
- [14] J. A. Dunne, R. J. Williams, N. D. Martinez, R. A. Wood, and D. H. Erwin, *Compilation and network analyses of cambrian food webs*, PLoS biology, 6 (2008), p. e102.
- [15] A. EKLÖF, M. R. HELMUS, M. MOORE, AND S. ALLESINA, *Relevance of evolutionary history for food web structure*, Proceedings of the Royal Society B: Biological Sciences, 279 (2012), p. 1588–1596.
- [16] D. P. Faith, Conservation evaluation and phylogenetic diversity, Biological Conservation, 61 (1992), pp. 1–10.
- [17] B. Faller, C. Semple, and D. Welsh, Optimizing phylogenetic diversity with ecological constraints, Annals of Combinatorics, 15 (2011), pp. 255–266.
- [18] B. FALLER AND M. STEEL, Trait-dependent extinction leads to greater expected biodiversity loss, SIAM Journal on Discrete Mathematics, 26 (2012), pp. 472– 481.
- [19] D. GRAVEL, T. BELL, C. BARBERA, T. BOUVIER, T. POMMIER, P. VENAIL, AND N. MOUQUET, Experimental niche evolution alters the strength of the diversity-productivity relationship, Nature, 469 (2010), pp. 89–92.
- [20] K. Hartmann and M. Steel, Maximizing phylogenetic diversity in biodiversity conservation: greedy solutions to the noah's ark problem, Systematic Biology, 55 (2006), pp. 644–651.
- [21] T. Ingram, L. Harmon, and J. Shurin, When should we expect early bursts of trait evolution in comparative data? predictions from an evolutionary food web model, Journal of Evolutionary Biology, 25 (2012), pp. 1902–1910.
- [22] A. R. IVES AND H. C. J. GODFRAY, *Phylogenetic analysis of trophic associations.*, The American Naturalist, 168 (2006), pp. E1–E14. ArticleType: research-article / Full publication date: July 2006 / Copyright © 2006 The University of Chicago.

- [23] B. R. Krasnov, M. A. Fortuna, D. Mouillot, I. S. Khokhlova, G. I. Shenbrot, and R. Poulin, *Phylogenetic signal in module composition and species connectivity in compartmentalized host-parasite networks.*, American Naturalist, 179 (2012), p. 501.
- [24] Y.-Y. LIU, J.-J. SLOTINE, AND A.-L. BARABÁSI, Controllability of complex networks, Nature, 473 (2011), pp. 167–173.
- [25] R. M. MAY, Stability in multispecies community models, Mathematical Biosciences, 12 (1971), pp. 59–79.
- [26] —, Will a large complex system be stable?, Nature, 238 (1972), pp. 413–414.
- [27] A. J. MCKANE AND B. DROSSEL, *Models of food web evolution*, Ecological networks: linking structure to dynamics in food webs, (2005), p. 223.
- [28] V. MOULTON, C. SEMPLE, AND M. STEEL, Optimizing phylogenetic diversity under constraints, Journal of theoretical biology, 246 (2007), pp. 186–194.
- [29] N. MOUQUET, V. DEVICTOR, C. N. MEYNARD, F. MUNOZ, L.-F. BERSIER, J. CHAVE, P. COUTERON, A. DALECKY, C. FONTAINE, D. GRAVEL, ET AL., Ecophylogenetics: advances and perspectives, Biological reviews, 87 (2012), pp. 769–785.
- [30] R. E. NAISBIT, R. P. ROHR, A. G. ROSSBERG, P. KEHRLI, AND L.-F. BERSIER, Phylogeny versus body size as determinants of food web structure, Proceedings of the Royal Society B: Biological Sciences, 279 (2012), pp. 3291–3297.
- [31] M. G. NEUBERT AND H. CASWELL, Alternatives to resilience for measuring the responses of ecological systems to perturbations, Ecology, 78 (1997), pp. 653– 665.
- [32] M. PASCUAL AND J. A. DUNNE, Ecological Networks: Linking Structure to Dynamics in Food Webs, Oxford University Press, Nov. 2005.
- [33] S. J. PLITZKO, B. DROSSEL, AND C. GUILL, Complexity-stability relations in generalized food-web models with realistic parameters, Journal of Theoretical Biology, 306 (2012), pp. 7–14.
- [34] C. R. POWELL AND A. J. MCKANE, Comparison of food webs constructed by evolution and by immigration, Ecological Complexity, 6 (2009), pp. 316–327.
- [35] R. J. Prill, P. A. Iglesias, and A. Levchenko, Dynamic properties of network motifs contribute to biological network organization, PLoS biology, 3 (2005), p. e343.

- [36] C. Quince, P. G. Higgs, and A. J. McKane, Topological structure and interaction strengths in model food webs, Ecological Modelling, 187 (2005), pp. 389–412.
- [37] D. RAUP, Evolutionary radiations and extinctions, in Patterns of Change in Earth Evolution, Springer, 1984, pp. 5–14.
- [38] J. M. K. RIP, K. S. MCCANN, D. H. LYNN, AND S. FAWCETT, An experimental test of a fundamental food web motif, Proceedings of the Royal Society B: Biological Sciences, 277 (2010), pp. 1743–1749.
- [39] P. ROOPNARINE, Red queen for a day: models of symmetry and selection in paleoecology, Evolutionary Ecology, 26 (2012), pp. 1–10.
- [40] P. D. ROOPNARINE, Networks, extinction, and palecommunity food webs, Quantitative methods in paleobiology, (2010), p. 143–161.
- [41] A. ROSSBERG, H. MATSUDA, T. AMEMIYA, AND K. ITOH, An explanatory model for food-web structure and evolution, Ecological Complexity, 2 (2005), pp. 312–321.
- [42] A. G. ROSSBERG, H. MATSUDA, T. AMEMIYA, AND K. ITOH, Food webs: experts consuming families of experts, Journal of theoretical biology, 241 (2006), pp. 552–563. PMID: 16466654.
- [43] R. V. Solé and J. Bascompte, Self-Organization in Complex Ecosystems. (MPB-42), vol. 42, Princeton University Press, 2006.
- [44] D. S. SRIVASTAVA, M. W. CADOTTE, A. A. M. MACDONALD, R. G. MARUSHIA, AND N. MIROTCHNICK, *Phylogenetic diversity and the functioning of ecosystems*, Ecology Letters, 15 (2012), p. 637–648.
- [45] M. Steel and B. Faller, Markovian log-supermodularity, and its applications in phylogenetics, Applied Mathematics Letters, 22 (2009), pp. 1141–1144.
- [46] D. B. Stouffer, M. Sales-Pardo, M. I. Sirer, and J. Bascompte, Evolutionary conservation of species' roles in food webs, Science, 335 (2012), pp. 1489–1492.
- [47] M. L. Weitzman, *The noah's ark problem*, Econometrica, (1998), pp. 1279–1298.
- [48] R. J. WILLIAMS, A. ANANDANADESAN, AND D. PURVES, The probabilistic niche model reveals the niche structure and role of body size in a complex food web, PloS one, 5 (2010), p. e12092.
- [49] L. WITTING AND V. LOESCHCKE, The optimization of biodiversity conservation, Biological Conservation, 71 (1995), pp. 205–207.

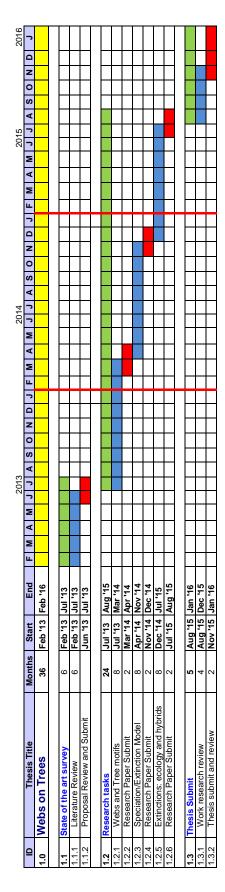


Figure 3.11: Research work plan