Tutorial-2 Low Noise Amplifier (LNA) Design

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Objective:

Low noise amplifiers are one of the basic building blocks of any communication system. The purpose of the LNA is to amplify the received signal to acceptable levels with minimum self generated additional noise. Gain, NF, non-linearity and impedance matching are four most important parameters in LNA design.

The objective of this tutorial is to outline the basic tradeoffs between different amplifying topologies w.r.t gain, NF and impedance matching. After this comparison it is concluded that inductor degenerated common source topology gives the best performance to meet the gain, NF, and impedance matching goals with minimum power consumption in case of narrow band designs.

Goals:

After this tutorial, students should be able to

- Calculate the gain, input impedance and NF of common gate, common source, and shunt feedback amplifiers.
- Understand the basic equations and tradeoff between different LNA topologies.
- Perform the calculation for inductor degenerated common source topology and understand the tradeoff between the gain, NF, and impedance matching.

A supplement tutorial LNA lab is also part of this course which takes the circuit from Problem-2.8 and guides through different analysis to design and practical LNA.

Problem-2.1(Tutorial)

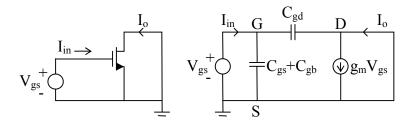
 ω_T is single figure of merit for high frequency transistors. This is defined as frequency at which current gain is extrapolated to fall to unity.

Although the dc gate current of an MOS transistor is essentially zero, the high-frequency behavior of the transistor is controlled by the capacitive elements in the small- signal model, which cause the gate current to increase as frequency increases.

- a) Derive the expression for $\omega_{\rm T}$.
- b) For RF design we always use minimum length transistors. Why?

Solution:

a).



$$V_{sb} = V_{ds} = 0$$

So g_{mb}, r_o, C_{sb}, C_{db} have no effect on calculations. (This is drawback of ω t definition)

$$i_i = j\omega(C_{gs} + C_{gb} + C_{gd})V_{gs}$$

$$i_i \approx g_m V_{gs}$$

$$\frac{i_o}{i_{in}} = \frac{g_m}{j\omega(C_{gs} + C_{gb} + C_{gd})}$$

According to definition $\frac{i_o}{i_{in}} = 1$ at ω_T

$$\omega = \omega_T = \frac{g_m}{C_{gs} + C_{gb} + C_{gd}}$$

 C_{gb} and C_{gd} are small compared to C_{gs}

So,
$$\omega_T \approx \frac{g_m}{C_{gs} + C_{gd}} \approx \frac{g_m}{C_{gs}}$$
 ----(1)

b).
$$I_D = \frac{\mu_o C_{ox}}{2} \frac{W}{L} (V_{gs} - V_T)^2$$

$$g_m = \frac{\partial I_D}{\partial V_{os}} = \mu_o C_{ox} \frac{W}{L} (V_{gs} - V_T) \qquad (2)$$

And
$$C_{gs} = C_o \times WL$$
 -----(3)

Put (2) & (3) in (1)

$$\omega_{T} = \frac{g_{m}}{C_{gs}} = \frac{\mu_{o}C_{ox}W(V_{gs} - V_{t})}{L.C_{ox}.W.L} = \frac{\mu_{o}(V_{gs} - V_{t})}{L^{2}}$$

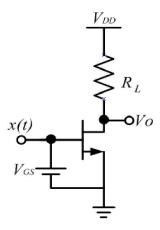
$$\omega_T = \frac{\mu_o \left(V_{gs} - V_t \right)}{L^2}$$

This means that $\omega_T \alpha \frac{1}{L^2}$ so that's why minimum L is preferred. But this approximation holds

for long channel devices for short channel $\omega_{\scriptscriptstyle T} \alpha \frac{1}{L}$ instead of $\frac{1}{L^2}$.

Problem-2.2(Tutorial)

NMOS transistor is racing horse in LNA design arena due to its higher mobility compared to PMOS transistors. Calculate the IP3 of NMOS CS amplifier shown below. Assume that NMOS transistor is in saturation.



a) Consider simplified square law model. (HW)

$$I_{D} = \frac{K_{n}}{2} (V_{GS} - V_{T})^{2}$$

b) Consider the short channel effects as (Tutorial)

$$I_{D} = \frac{K_{n}}{2} \left[\frac{(V_{GS} - V_{T})^{2}}{1 + \theta(V_{GS} - V_{T})} \right]$$

 $\theta = Velocity\ Saturation, Mobility\ Degradation$

$$V_{GS} - V_T = 0.2V$$
 and $\theta = 0.1V^{-1}$

c) What conclusion can be drawn from part b) about the bias current and transconductance of transistor for higher IP3?

Solution:

a). Home work Ans: $IP3 = \alpha$

b). From Razavi

$$y(x) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$
 -----(1)

$$x(t) = A\cos\omega_1 t + A\cos\omega_2 t \Rightarrow IP3 = \sqrt{\frac{4}{3} \left| \frac{\alpha_1}{\alpha_3} \right|}$$

$$I_{D} = \frac{K_{n}}{2} \frac{(V_{GS} - V_{T})^{2}}{1 + \theta(V_{GS} - V_{T})}$$

Here we assume that small signal x(t) over-rides $(V_{GS} - V_T)$.

So,

$$V_{DD}$$

$$R_{L}$$

$$X(t) \bigcirc V_{GS}$$

$$V_{DS}$$

$$NMOS$$

$$DC-bias$$

$$I_{D} = \frac{K_{n}}{2} \frac{\left[\left(V_{GS} - V_{T} \right) + x(t) \right]^{2}}{1 + \theta(V_{GS} - V_{T} + x(t))}$$

&
$$V_{GS} - V_T = \Delta V$$
 ------ Large signal

X(t) ----- Small signal

$$I_D = \frac{K_n}{2} \frac{\left[x(t) + \Delta V\right]^2}{\theta(x(t) + \Delta V) + 1}$$

$$V_o = I_D R_L \Rightarrow V_o = \frac{K_n R_L}{2} \frac{(x(t) + \Delta V)^2}{1 + \theta(x(t) + \Delta V)}$$
 put $\frac{K_n R_L}{2} = K$

$$\theta << 1$$
 So $(x(x) + \Delta V)$ is also small $\Rightarrow \frac{1}{1+x} = 1 - \frac{x}{2}$

$$\frac{1}{1 + \theta(x(t) + \Delta V)} \approx 1 - \frac{\theta(x(t) + \Delta V)}{2}$$

$$V_o = K(x(t) + \Delta V)^2 \left(1 - \frac{\theta(x(t) + \Delta V)}{2}\right)$$

$$V_o = K(x(t) + \Delta V)^2 - (x(t) + \Delta V)^3 \frac{K\theta}{2}$$

$$V_o = K\Delta V^2 - \frac{K\theta}{2}\Delta V^3 + \left(2K\Delta V - \frac{3K\theta}{2}\Delta V^2\right)x(t) + \left(K - \frac{3K\theta}{2}\Delta V\right)x^2(t) - \frac{K\theta}{2}x^3(t)$$

Small signal components

$$V_o = \left(2K\Delta V - \frac{3K\theta}{2}\Delta V^2\right)x(t) + \left(K - \frac{3K\theta}{2}\Delta V\right)x^2(t) - \frac{K\theta}{2}x^3(t) \qquad -----(3)$$

Comparing (1) & (3)

$$\begin{split} \alpha_1 &= 2K\Delta V - \frac{3K\theta}{2}\Delta V^2, \quad \alpha_2 = K - \frac{3K\theta}{2}\Delta V \quad, \quad \alpha_3 = -\frac{K\theta}{2} \\ IP3 &= \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} = \sqrt{\frac{4}{3}}\times\frac{2K\Delta v - \frac{3}{2}K\theta\Delta V^2}{\frac{K\theta}{2}} = \sqrt{\frac{8}{3}\left(\frac{2\Delta V}{\theta} - 3\Delta V^2\right)} \\ IP3 &= \sqrt{\frac{8}{3}\frac{2\Delta V}{\theta}} = \sqrt{\frac{16}{3}\frac{\Delta V}{\theta}} \\ IP3 &= \sqrt{\frac{16}{3}\frac{\Delta V}{\theta}} = \sqrt{\frac{16}{3}\frac{(V_{GS} - V_T)}{\theta}} \\ IP3 &= \sqrt{\frac{16}{3}\frac{\Delta V}{\theta}} = \sqrt{\frac{16}{3}\frac{(V_{GS} - V_T)}{\theta}} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = 3.27Volts \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = 3.27Volts \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = 3.27Volts \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{V_{GS} - V_T}} = \frac{2}{\Delta V} \\ IP3 &= \sqrt{\frac{16}{3}\frac{0.2}{0.1}} = \frac{2}{\sqrt{\frac{16}{3}\frac{0.2}{0.1}}} =$$

$$\frac{g_m}{I_D} = \frac{2}{V_{GS} - V_T} = \frac{2}{\Delta V}$$

$$\Rightarrow IP3 \cong \sqrt{\frac{32}{3\theta}} \sqrt{\frac{I_D}{g_m}} \qquad \therefore g_m = \sqrt{\mu_n C_{ox} \frac{W}{L} I_D}$$

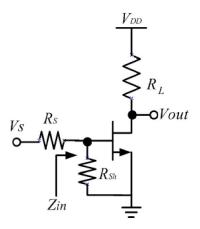
c).

- To increase IIP3 I_D \uparrow (high power) or g_m \downarrow (high noise)
- g_m also depends upon I_{D,} so when ID \uparrow g_m \uparrow but at that rate $\alpha \sqrt{I_D}$
- When W increases gm \uparrow for same I_D (Power consumption) so this decreases IP3
- The above observations are for long-channel. But for short channel $(4) \Rightarrow (V_{GS} V_T) \uparrow$ then ID \uparrow any how.

Problem-2.3 (Tutorial)

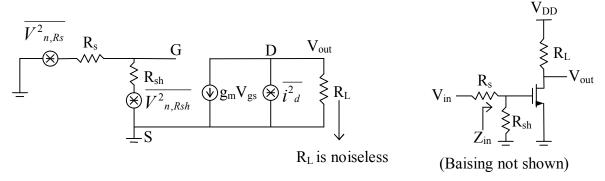
It is preferred in current RF designs that the input of LNA be matched to 50 Ω (Razavi, Pg168). The easiest way is to shunt the gate with a resistor of 50 Ω .

- a) Calculate the gain, input impedance and NF in absence of gate noise. Assume that $R_{sh}=R_L$ for NF derivation.
- b) What are the disadvantage of shunt resistor with reference to gain and NF?



Solution:

a). (Please read assumption in problem statement carefully)



$$F = \frac{Total\ output\ noise\ power}{Output\ noise\ due\ to\ input\ source}$$

$$\overline{V^{2}_{m,Rs}} = 4KTR_{s}\Delta f$$

$$Gain|_{Gate} = -g_{m}R_{L}$$

$$\overline{V^{2}_{m,Rsh}} = 4KTR_{sh}\Delta f$$

$$A = g_{m}R_{L}\left(\frac{R_{sh}}{R_{s} + R_{sh}}\right) \text{ for } R_{sh} = R_{s}$$

$$\overline{i^{2}_{d}} = 4KT\gamma g_{m}\Delta f$$

$$A = -g_{m}\frac{R_{L}}{2}$$

Using superposition, considering one at a time and shorting / opening other sources.

$$\overline{V^{2}_{on,Rs}} = \overline{V^{2}_{n,Rs}} \times g^{2}_{m} R_{L}^{2} \times \left(\frac{R_{sh}}{R_{s} + R_{sh}}\right)^{2}$$

$$\overline{V^{2}_{on,Rsh}} = \overline{V^{2}_{n,Rsh}} \times g^{2}_{m} R_{L}^{2} \times \left(\frac{R_{s}}{R_{s} + R_{sh}}\right)^{2}$$

$$\overline{V^{2}_{no,d}} = \overline{i^{2}_{d}} \times R^{2}_{L}$$

$$F = \frac{\overline{V^{2}_{on,Rs}} + \overline{V^{2}_{on,Rsh}} + \overline{V^{2}_{no,d}}}{\overline{V^{2}_{on,Rs}}} = 1 + \frac{\overline{V^{2}_{on,Rsh}} + \overline{V^{2}_{o,d}}}{\overline{V^{2}_{on,Rs}}}$$

$$F = 1 + \frac{4KTR_{sh}\Delta f \times \frac{g^{2}_{m}R^{2}_{L} \times R^{2}_{sh}}{(R_{s} + R_{sh})^{2}}}{4KTR_{s}\Delta f \times \frac{g^{2}_{m}R^{2}_{L} \times R^{2}_{s}}{(R_{s} + R_{sh})^{2}}} + \frac{4KT\gamma g_{m}\Delta f \times R^{2}_{L}}{4KTR_{s}\Delta f \times \frac{g^{2}_{m}R^{2}_{L} \times R^{2}_{s}}{(R_{s} + R_{sh})^{2}}}$$

In case of impedance match $R_s = R_{sh}$

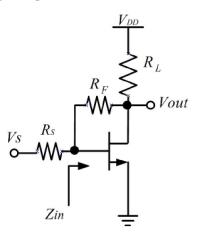
$$F = 1 + 1 + \frac{R^{2} L \gamma g_{m}}{R_{s} \times \frac{g^{2} m R^{2} s \times R^{2} L}{4R^{2} s}} = 2 + \frac{R^{2} L \gamma g_{m}}{R_{s} \times \frac{g^{2} m \times R^{2} L}{4}} = 2 + \frac{4\gamma}{g_{m} R_{s}}$$

b).

- Poor Noise Figure
- Input signal attenuated by voltage divider
- R_{sh} adds extra noise.
- At high frequency, shunt L is needed to tune out $C_{\rm gs}$
- Reduced gain

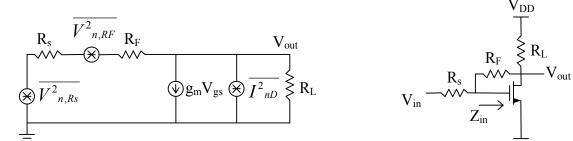
Problem-2.4 (Tutorial)

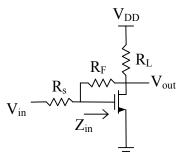
Another approach to get 50 Ω input impedance match is shunt feedback amplifier shown below.



- a) Calculate the gain, input impedance and NF in absence on gate noise. Neglect gate drain and gate to bulk and gate to source capacitance.
- b) What are the disadvantage of shunt feedback amplifier with reference to gain and NF?

Solution:





(Equivalent noise model ignoring gate noise), RL is noiseless

(Baising not shown)

$$\overline{I_{nD}^{2}} = 4KT\gamma g_{m} \Delta f, \overline{V_{RS}^{2}} = 4KTR_{S} \Delta f$$

$$NF = \frac{\overline{V^2}_{n,out}}{\overline{A^2}_{v,tot}\overline{V^2}_{RS}} = \frac{Total\ input\ noise\ power}{Output\ noise\ power\ due\ to\ input\ source}$$

Here $A_{v,tot}$ = Gain from V_{in} to V_{out}

Again using superposition theorem

$$NF = \frac{\overline{V^{2}}_{n,out}}{A^{2}_{v,tot}\overline{V^{2}_{RS}}} = \frac{\overline{V_{n}^{2}}_{RS,out} + \overline{V_{n}^{2}}_{RF,out} + \overline{V_{n}^{2}}_{D,out}}{A^{2}_{v,tot}\overline{V^{2}}_{RS}}$$

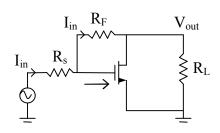
Gain Calculation

$$V_{in} = i_{in}(R_S + R_F) + V_{out}$$

$$V_{out} = (i_{in} - g_m V_{gs})R_L$$

$$V_{gs} = i_{in}R_F + V_o$$

$$A_{v,tot} = \frac{V_{out}}{V_{in}} = \frac{R_L(1 - g_m R_L)}{R_S + R_F + R_L + g_m R_S R_L}$$



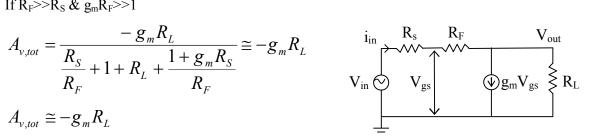
If $R_F >> R_S \& g_m R_F >> 1$

$$A_{v,tot} = \frac{-g_m R_L}{\frac{R_S}{R_F} + 1 + R_L + \frac{1 + g_m R_S}{R_F}} \cong -g_m R_L$$

$$A_{v,tot} \cong -g_m R_L$$

Also
$$Z_{in} = \frac{R_F + R_L}{1 + g_m R_L}$$

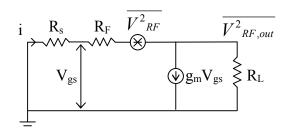
By ignoring $C_{gs,}$ we have considered real part only.



For source resistance

$$\overline{V^2}_{nRS,out} = A^2_{v,tot} \overline{V^2}_{nRS} \qquad -----(1)$$

For feedback resistance



$$\begin{split} V_{gs} &= -iR_{S} = iR_{F} - V_{RF} + V_{RF,out} \\ V_{RF,out} &= R_{L} \Big(i - g_{m} V_{gs} \Big) \\ V_{RF,out} &= V_{RF} \frac{1}{1 + \frac{R_{S} + R_{F}}{R_{L} \Big(1 + g_{m} R_{S} \Big)}} = V_{RF} \frac{R_{L}}{R_{F}} \Big(1 + g_{m} R_{S} \Big) \end{split}$$

$$\overline{V_{n,RF,out}^2} = \overline{V_{n,RF}^2} \left[\frac{R_L}{R_F} \left(1 + g_m R_S \right) \right]^2 \quad ----(2)$$

Similarly

$$i \quad R_{s} \quad R_{F} \qquad \overline{V^{2}_{nD,out}}$$

$$V_{gs} \quad g_{m}V_{gs} \quad \bigotimes \overline{I^{2}_{n,D}} \quad R_{L}$$

$$\frac{V_{nD,out}}{R_L} + \overline{I_{nD}} + g_m V_{gs} + \frac{V_{nD,out}}{R_S + R_F} = 0$$

$$V_{gs} = R_S \frac{V_{nD,out}}{R_S + R_E}$$

$$V_{nD,out} = \frac{I_{n,D}}{\frac{1}{R_{I}} + \frac{1}{R_{S} + R_{E}} + \frac{g_{m}R_{S}}{R_{S} + R_{E}}} \approx I_{nD}R_{L}$$

So.

$$\overline{V^2}_{nD,out} = \overline{I^2}_{nD} R_L^2 \qquad -----(3)$$

Combaining (1)(2) & (3)

$$NF = 1 + \frac{\overline{V^{2}}_{n,RF} \left[\frac{R_{L}}{R_{F}} (1 + g_{m} R_{S}) \right]^{2}}{A^{2}_{v,tot} \overline{V^{2}}_{n,RS}} + \frac{\overline{I^{2}}_{nD} R^{2}_{L}}{A^{2}_{v,tot} \overline{V^{2}}_{n,RS}}$$

$$A_{v,tot} = -g_{m} R_{L}, \ \overline{V^{2}}_{n,RS} = 4KTR_{S} \Delta f, \ \overline{V^{2}}_{M,RF} = 4KTR_{F} \& \ \overline{I^{2}}_{nD} = 4KT \gamma g_{m}$$

$$NF = 1 + \frac{R_{S}}{R_{F}} \left(1 + \frac{1}{g_{m} R_{S}} \right)^{2} + \frac{\gamma}{g_{m} R_{S}}$$

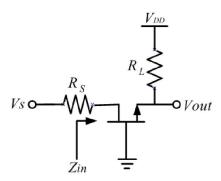
b).

NF
$$\downarrow$$
 g_mR_S \uparrow & R_F \uparrow usually $R_S = 50\Omega$

- Better performance than CS amplifier
- R_F induces noise
- At f \uparrow need shunt inductor to tune out C_{gs}
- Broadband Amp @ Lower frequency
- To make NF \downarrow R_F > R_S g_mR_S >> 1

Problem-2.5 (HW)

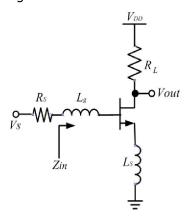
Common gate amplifier also offers 50 Ω input impedance match and solves the input matching problem.



- c) Calculate the gain, input impedance and NF in absence on gate noise. Neglect gate drain and gate to bulk and gate to source capacitance.
- a) What are the disadvantage of common gate amplifier with reference to gain and NF?

Problem-2.6 (Tutorial)

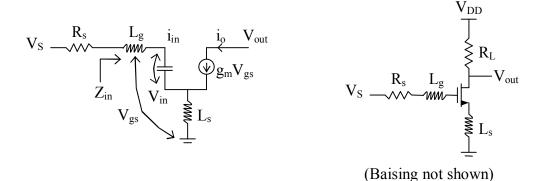
The disadvantages of three types of amplifiers in Problem-3, 4 & 5 can be circumvented by using source degenerated LNA shown below.



- a) Calculate the input impedance. This inductor source degenerated amplifier presents a noiseless resistance for 50Ω for input power match. How we can cancel the imaginary part of complex input impedance so that the LNA presents 50Ω real input resistance at input port.
- b) Calculate the NF in absence on gate noise. Neglect gate drain and gate to bulk and gate to source capacitance.
- c) C_{gd} bridges the input and output ports. The reverse isolation of this LNA is very poor. Why reverse isolation is important? Suggest the modification to improve reverse isolation.

Solution:

a).



From model above we can write

$$V_{in} = i_{in} \left(j\omega L_g + j\omega L_s \right) + i_{in} \left(\frac{1}{j\omega c} \right) + i_o j\omega L_s \quad -----(1)$$

$$i_o = g_m V_{gs} = g_m i_{in} \times \frac{1}{j\omega C_{gs}} \quad -----(2)$$

Substituting (2) in (1)

$$V_{in} = i_{in} \left[j\omega \left(L_g + L_s \right) + \frac{1}{j\omega C_{gs}} + \frac{g_m L_s}{C_{gs}} \right]$$

$$Z_{in} = \frac{V_{in}}{i_{in}} = j\omega \left(L_g + L_s\right) + \frac{1}{j\omega C_{gs}} + \frac{g_m L_s}{C_{gs}}$$

$$Z_{in} = j\omega (L_g + L_s) + \frac{1}{j\omega C_{gs}} + \frac{g_m L_s}{C_{gs}}$$

For matching $L_g + L_s$ are canceled out by C_{gs} . So at frequency of interest

$$\omega_o(L_g + L_s) = \frac{1}{\omega_o C_{gs}} \Rightarrow \omega_o^2 = \frac{1}{(L_g + L_s)C_{gs}}$$

And
$$R_S = 50\Omega = \frac{g_m}{C_{os}} L_s$$

Notes:

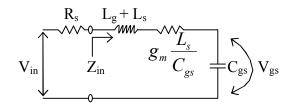
- a). L_s is typically small and may be realized by bond wire.
- b). L_g can be implemented by spiral/external inductor.

b).

From part a)

$$Z_{in} = j\omega (L_g + L_s) + \frac{1}{j\omega C_{gs}} + \frac{g_m L_s}{C_{gs}}$$

We can draw this circuit as



Here

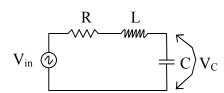
$$Q_{in} = \frac{\omega_o \left(L_g + L_s \right)}{R_S + \frac{g_m L_S}{C_{cos}}} = \frac{\omega_o \left(L_g + L_s \right)}{R_S + W_T L_S}$$

$$: \omega_T \cong \frac{g_m}{C_{gs}}$$

$$Q_{in} = \frac{1}{\omega_o \left(R_S + \frac{g_m L_S}{C_{gs}} \right) C_{gs}}$$
 For match load $R_S = \frac{g_m L_S}{C_{gs}}$

Reference:

For series RLC Circuit



$$Q_s = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$$

And
$$V_C = Q_S V_{in}$$

For problem (1)

$$\omega_T \cong \frac{g_m}{C_{gs}}$$
 Unity gain frequency

for current

$$Q_{in} = \frac{1}{2\omega_o R_s C_{gs}}$$

Gain

$$\overline{V_{gs}} = Q_{in}V_{in}$$

$$g_m = \frac{V_{out}}{V_{gs}}$$

$$G_m = \frac{V_{out}}{V_{in}} = \frac{V_{gs}g_m}{V_{in}} = Q_{in}g_m$$

$$G_m = Qg_m$$
So, $Gain = -G_m R_L$ or $-Q_{in}g_m R_L$ & $G_m = Q_{in}g_m$

Noise Figure:

$$NF = \frac{Total\ noise\ power\ at\ output}{noise\ power\ at\ output\ due\ to\ input\ source}$$

For this calculation we ignore channel noise.

$$F = \frac{\overline{V^{2}_{nRS,OUT}} + \overline{V^{2}_{nD,OUT}}}{\overline{V^{2}_{nRS,OUT}}} = 1 + \frac{\overline{V^{2}_{nD,OUT}}}{\overline{V^{2}_{nRS,OUT}}}$$

$$\overline{V^{2}_{nD,OUT}} = \overline{i^{2}_{n,D}R^{2}_{L}}$$

$$\overline{V^{2}_{nRS,OUT}} = \overline{V^{2}_{n,RS}G^{2}_{m}R^{2}_{L}}$$

$$F = 1 + \frac{\overline{i^{2}_{n,D}R^{2}_{L}}}{\overline{V^{2}_{n,RS}Q^{2}_{in}g^{2}_{m}R^{2}_{L}}}$$

$$\overline{i^{2}_{n,D}} = 4KT\gamma g_{m} \wedge f$$

$$\overline{i^{2}_{n,D}} = 4KT\gamma g_{m} \wedge \overline{V^{2}_{n,RS}} = 4KTR_{S}$$

$$\overline{i^{2}_{n,D}} = 4KT\gamma g_{m} \wedge \overline{V^{2}_{n,RS}} = 4KTR_{S}$$

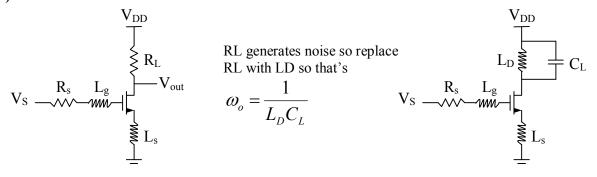
$$F = 1 + \frac{\gamma}{g_{m}R_{S}Q^{2}_{in}}$$

Notes:

- Very good NF value
- Narrow band matching
- NF \downarrow with Q^2
- The Q is dependent upon $L_{\text{g}} + L_{\text{s}},\, L_{\text{s}}$ is small so Q depend upon L_{g}

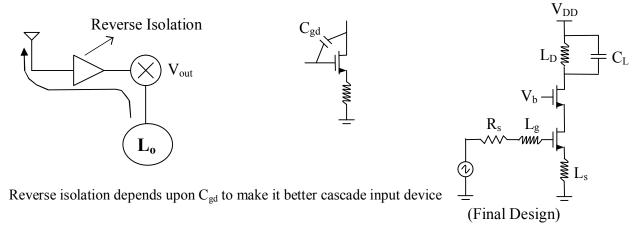
C). Draw Backs

i).



The C_L can be the input capacitance of mixer or filter.

ii).



Problem-2.7 (HW)

Fill-in the Table below, use the data from Problem-2.4, 2.5, 2.6 and 2.7

Type of LNA	Z_{in}	Noise Factor	Gain	NF (dB)
Shunt Resistor	R_{sh}	$2 + \frac{4\gamma}{g_m R_S}$	$\frac{-g_{_m}R_{_L}}{2}$	
Common Gate				
Shunt Feedback				
Source Degenerated				

- a) Calculate the NF for all above amplifiers. Assume γ =2, g_m = 20mS, Rs = 50 Ω , R_F = 500 Ω , and Q_{in} = 2.
- b) Which is best topology for Narrow Band LNA design at high frequency?

Problem-2.8 (Tutorial)

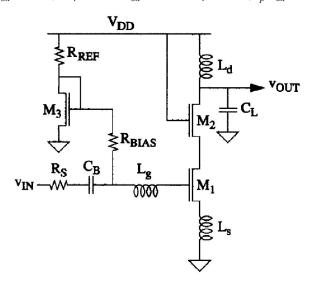
Real Design: We will design the inductor source degenerated LNA shown in Fig below to meet the specification outlined for IEEE802.11 (b) standard. The first cut approximate values are calculated as a starting point for simulation. In **LAB3: Design of LNA** you will take the same design and modify these component values to meet the specification.

LNA Specification:

NF < 2.5 db, Gain > 15dB, IP3 > -5dBm, Centre Frequency = 2.4 GHz S11 < -20dB, S22 < -10dB, Load Capacitance = 1pF

Technology Parameters for 0.35um CMOS:

$$L_{eff} = 0.35 \,\mu m$$
, $\mu_n C_{ox} = 170 \,\mu A/V^2$, $C_{ox} = 4.6 \,m F/m^2$, $\mu_n C_{ox} = 58 \,\mu A/V^2$, $\gamma = 2$



Solution:

$$\text{Technology: } \left\{ \begin{aligned} \mu_o C_{ox} &= 170 \, \mu A/V^2 \,, \mu_p C_{ox} = 58 \, \mu A/V^2 \,, 0.35 \, \mu m \\ CMOS, C_{ox} &= 4.6 \, mF/m^2 \,, \gamma = 2, f_o = 2.4 \, GHz \\ \left(IEE802.11(b) \, S \tan dard \, \right), \, \mu_o C_{ox} = 170 \, \mu A/V^2 \end{aligned} \right\}$$

Design Parameters

$$R_S = 50\Omega$$
, $V_{DD} = 3.3V$, $C_L = 10PF$, $NF < 2.5dB$

$$\begin{array}{c} V_{DD} \\ \hline \\ V_{ID} \\ \hline \\ V_{In} \\ \hline \end{array}$$

$$\begin{array}{c} R_{S} \\ C_{B} \\ \hline \end{array}$$

$$\begin{array}{c} C_{L} = 10pF \\ \hline \end{array}$$

Component Description

L_s – Matches input impedance

 L_g – Set the Resonant Frequency f_O = 2.4 GHz

M₃ – Biasing transistor which forms current mirror with M₁

L_d – Tuned output increases the gain and also work as band pass filter with C_L

 M_2 – Isolate tuned input and tuned output increases reverse isolation, also reduces the effect of Miller capacitance $C_{\rm gd}$

 $C_B - BC$ Blocking capacitor chosen to have negligible reactance at $f_O = 2.4$ GHz

R_{BIAS} – Large enough so that its equivalent current noise is small enough to be ignored. (Don't consider it as voltage noise source. Why??)

Design Procedure

Size of M1:

We will not go for global minimum noise figure as given by two-point noise theory (See lecture on LNA Slide # 10)

$$G_{opt} = \alpha \omega C_{gs} \sqrt{\frac{\delta}{5\gamma} \left(1 - \left|C\right|^2\right)} = \frac{1}{50\Omega} \qquad -----(1)$$

$$C_{os} \cong 4 pF \Rightarrow W_{M1} \approx 4mm!!$$
 (not possible)

Solution:

A & B are from Thomos. H. Lee book (LNA Chapter)

LNA NF will be optimized for given Power

* It will not be best NF globally.

$$W_{opt} = \frac{1}{3\omega_o L_{eff} C_{ox} R_S}$$

$$F_{\min,p} = 1 + 2.4 \frac{\gamma}{\alpha} \frac{\omega}{\omega_T} \Rightarrow F_{\min,p} = 1 + 5.6 \frac{\omega}{\omega_T}$$
 -----(A)

From (1) we can derive

$$F_{\min,p} = 1 + \frac{2}{\sqrt{5}} \frac{\omega}{\omega_T} \sqrt{\gamma C \left(1 - \left|C\right|^2\right)} = 1 + 2.3 \frac{\omega}{\omega_T}$$

$$F_{\min,p} = 1 + 2.3 \frac{\omega}{\omega_T} \qquad -----(B)$$

- (a) is minimum NF for a given power consumption.
- (b) is global minimum noise figure.

The difference is usually 0.5dB to 1dB (no big deal for Lower Power)

Step - 1:

$$I_1 = I_2 = 5mA$$
 (Low Power consumption)

Step - 2:

$$W_{M1} = \frac{1}{3WL_{eff}C_{ox}R_S}$$

$$W_{M1} = \frac{1}{3 \times 0.35 \mu \times 4.6 m \times 50 \times \omega_o} \quad \begin{cases} R_S = 50 \Omega, C_{ox} = 4.6 mF/m^2, \\ \mu_n C_{ox} = 170 \, \mu A/V, L_{eff} = 0.35 \, \mu m, \\ \omega_o = 2 \pi f_o, f_o = 2.4 GHz \end{cases}$$

$$W_{M1} = 3.9 \times 10^{-4}$$

$$W_{M1} = 3.9 \times 10^{-4} = 390 \mu m$$

Step - 3:

$$C_{gs1} = \frac{2}{3} W_{M1} L_{eff} C_{ox}$$

$$C_{gs1} = \frac{2}{3} \times 390 \,\mu \times 0.35 \,\mu \times 4.6 m = 0.41 \,pF$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{M1} I_{DM1}}$$

$$g_{m1} = \sqrt{2 \times 170 \mu \times \left(\frac{390}{0.35}\right) \times 5m} = 43 \, mA/V$$

$$\omega_T \approx \frac{g_{m1}}{C_{osl}} = \frac{43 \, mA/V}{0.41 \, pF} = 104 \, Grad/Sec$$

Assuming $\gamma = 2$

Now
$$F_{\min} = 1 + 5.6 \frac{\omega_o}{\omega_T}$$

$$F_{\min} = 1 + 5.6 \frac{2\pi 2.4G}{104G} \approx 2.55 dB$$

$$NF \approx 2.55 dB$$

It's very close to what we derive, if the value is higher we can increase I_D to increase ω_T and hence low NF on expense of power.

Step - 4:

Source and gate inductance such that they cancel $C_{\rm gs}$ and set 50Ω input impedance

$$\omega_o = 2\pi f_o = 2\pi 2.4 = 15 Grad/Sec$$

From previous problem

$$R_{S} = R_{Transformed} = g \frac{L_{S}}{C_{gs}} = L_{S} \omega_{T}$$

$$L_S = \frac{R_S}{\omega_T} = \frac{50}{100G} \cong 0.5nH$$

 $L_S = 0.5nH$ can be implemented using Band wire.

Now
$$L_g + L_s = \frac{1}{\left(\omega_o^2 C_{gs1}\right)}$$

$$L_g + L_s = \frac{1}{(15G)^2 \times 0.41pF} = 10.81nH$$

$$L_g \approx 10nH$$

Step - 5:

$$L_d = \frac{1}{\omega_o^2 C_L} \qquad \because C_L = 1pF$$

$$L_d = \frac{1}{(15G)^2 \times 1pF} \cong 4.4nH$$

$$L_d = 4.4nH$$

Step - 6:

Size of M3 is chosen to minimize power consumption

$$W_{M3} = 70 \,\mu\text{m}, R_{RFF} = 2K\Omega \Longrightarrow I_3 = 0.6 \,\text{mA}$$

 $R_{\rm BIAS}=2K\Omega$ (Large enough so that it's equivalent current noise can be neglected)

$$C_B = 10 pF$$
 ($X_C \approx 6.6 \Omega$ so good value @ 2.4G $X_B = \frac{1}{2\pi f_a C_B} = 6.6 \Omega$)

Step - 7:

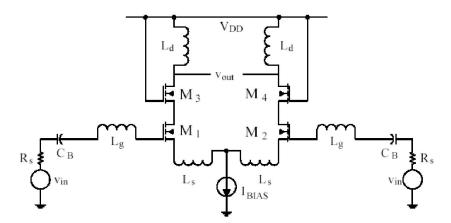
Size M2 = M3

So that they can have shared Drain Area..

(Note: We will simulate same design in LAB # 2)

Problem-2.9 (Point to Ponder):

Connecting two Inductor source degenerated LNA as shown in Figure make the differential LNA. Differential LNA has many advantages: higher common mode rejection ratio, less sensitivity to the ground inductance variation *Ls* compared to single ended counterpart..



- a) Compare intuitively the NF of single ended and differential if both have same power consumption.
- b) If low power is not parameter on interest, which LNA has lower NF?

Instructions:

For hand calculation of NF you can ignore the gate noise of the device and noise generated by the load resistance R_L .

Acknowledgement: The major part of this tutorial was developed, while author was employed by Linkoping University, Sweden.