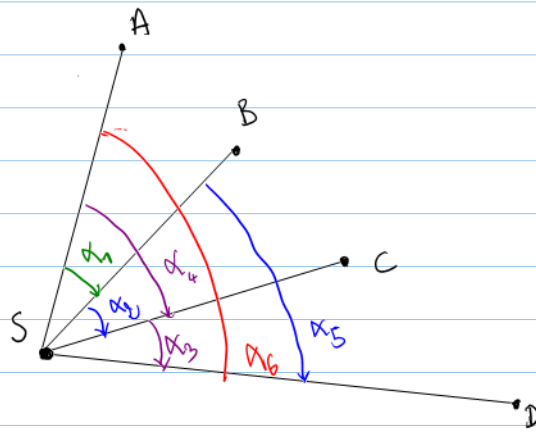


# Méthode de Variation de Paramètre

Schéma

Kiba Doi



Analyse de problème :

$$\begin{aligned} n &= 6 \\ n_0 &= 3 \\ \mathcal{O} &= 3 \\ \mu &= 3 \\ n &= 6 \end{aligned}$$

$$L_{(n,1)} = \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 & \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 & \bar{\alpha}_6 \end{bmatrix}^T$$

Identifie les variables :

$$\hat{V}_{(n,1)} = \begin{bmatrix} \hat{v}_1 & \hat{v}_2 & \hat{v}_3 & \hat{v}_4 & \hat{v}_5 & \hat{v}_6 \end{bmatrix}^T$$

$$\hat{X}_{(u,1)} = \begin{bmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 \end{bmatrix}^T$$

$$\hat{\tilde{X}}_{(u,1)} = \begin{bmatrix} \hat{\tilde{x}}_1 & \hat{\tilde{x}}_2 & \hat{\tilde{x}}_3 \end{bmatrix}^T$$

Model Mathématique

Forme Générale

Méthode de Variation de Paramètre

$$\begin{cases} \hat{x}_1 &= \hat{x}_1 \\ \hat{x}_2 &= \hat{x}_2 \\ \hat{x}_3 &= \hat{x}_3 \\ \hat{x}_1 + \hat{x}_2 &= \hat{x}_4 \\ \hat{x}_2 + \hat{x}_3 &= \hat{x}_5 \\ \hat{x}_1 + \hat{x}_2 + \hat{x}_3 &= \hat{x}_6 \end{cases}$$

$$F(\hat{X}) = \hat{L}$$

$$A \hat{X} + W = \hat{V}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

forme linéarisée

A et W; X°:

$$\bar{X}^0 = [\bar{x}_1 \quad \bar{x}_2 \quad \bar{x}_3]^T = [39.1852, 18.7510, 32.4142]^T$$

$$W = [0, 0, 0, -4 \cdot 10^4, 12 \cdot 10^3, -114 \cdot 10^3]^T$$

$\bar{X}?$

$$O_{na}: \quad \bar{X} = (A^T M^{-1} A)^{-1} (A^T M^{-1} W) = \begin{pmatrix} 0,00075 \\ -0,0002 \\ -0,00005 \end{pmatrix}^{\text{grades}}$$

Avec:

$$\star M = B P^a B^T = P^a = I_6$$

$$\star N = A^T M^{-1} A = \begin{pmatrix} 3 & 0 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\star N^{-1} = (A^T M^{-1} A)^{-1} = \begin{pmatrix} 0,5 & -0,25 & 0 \\ -0,25 & 0,5 & -0,25 \\ 0 & -0,25 & 0,5 \end{pmatrix}$$

$$\star U = A^T M^{-1} W = \begin{pmatrix} -0,0019 \\ -0,006 \\ -0,002 \end{pmatrix}$$

$$\hat{\bar{X}} = \bar{X}^0 + \hat{X} = [39,18595; 17,7508; 32,41415]^T$$

Coefficient du Poids

$$Q_{\hat{X}} = (A^T M^{-1} A)^{-1} = \begin{pmatrix} 0,5 & -0,25 & 0 \\ -0,25 & 0,5 & -0,25 \\ 0 & -0,25 & 0,5 \end{pmatrix}$$

La Solution de Lagrange  $\hat{K}$

$$\hat{K} = -M^{-1} (A \hat{X} + W)$$

$$Q_{\hat{K}} = Q_{\hat{X}} = P^{-1}$$

$$\hat{V} = P^{-1} \beta \hat{K} = -\hat{K} = \begin{pmatrix} 0,00075 \\ -0,0002 \\ -0,00005 \\ 0,00015 \\ 0,00095 \\ -0,0009 \end{pmatrix}$$

$$\hat{L} = \begin{pmatrix} 39,18595 \\ 17,7508 \\ 32,4145 \\ 57,93675 \\ 51,16495 \\ 90,3509 \end{pmatrix}$$

Vecteur des Résiduelle  $\hat{V}$

Règle de Résolution  
Avec  
Contrainte

Equation de Contraint Additionnel:

La forme linéarisée:

$$\begin{cases} \hat{X}_2 = 18.7500 \\ \hat{X}_3 = 32.4150 \end{cases}$$

$$C\hat{X} + W_c = 0$$

$$C = \left. \frac{\partial F(\hat{X})}{\partial \hat{X}} \right|_{\hat{X}^0} = \begin{pmatrix} \hat{X}_1 & \hat{X}_2 & \hat{X}_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$W_c = [1 \cdot 10^3, -8 \cdot 10^{-4}]$$

La Solution est donnée par:

$$\hat{X} = (A^T M A)^{-1} \begin{bmatrix} -A^T M^{-1} W + C^T (C (A^T M^{-1} A)^{-1} C^T)^{-1} \\ [-W_c + C (A^T M^{-1} A)^{-1} A^T M^{-1} W] \end{bmatrix}$$

Dans notre cas:  $P = M^{-1}$ ;  $B = -I$ ;  $N = A^T P A$ ;  $V = A^T M^{-1} W$

$$\hat{X}_c = \hat{X} + N^{-1} C^T (C [C N^{-1} C^T]^{-1} [-W_c + C N^{-1} V])$$

$$\hat{X}_c = \begin{pmatrix} 0,001 \\ -0,001 \\ 0,0008 \end{pmatrix} = [39.1862 \quad 17,75 \quad 32.415]^T$$

$$\hat{V}_c = -\hat{K}_c = [0,001, -0,001 \quad 0,0008 \quad -0,0004 \quad 0,001 \quad -0,0006]^T$$

$$\sigma_0^2 = \frac{V^T P V}{3} = 7,8 \times 10^{-7}$$

$$\hat{E}_c = \hat{E} + \hat{V}_c = [39,1862 \quad 17,75 \quad 32,415 \quad 57,9362 \quad 51,115 \quad 90,3512]^T$$

# M Condition

Analyse de  
problème :

$$\begin{aligned}n &= 6 \\ n_0 &= 3 \\ \mathcal{O} &= 3 \\ \mu &= 0 \\ n &= 3\end{aligned}$$

$$\begin{aligned}\bar{L}_{(n,1)} &= \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 & \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 & \bar{\alpha}_6 \end{bmatrix}^T \\ \hat{V}_{(n,1)} &= \begin{bmatrix} \hat{v}_1 & \hat{v}_2 & \hat{v}_3 & \hat{v}_4 & \hat{v}_5 & \hat{v}_6 \end{bmatrix}^T\end{aligned}$$

Identifier  
les variables .

Modèle Mathématique :

modèle Explicite :

$$\begin{cases} \hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_3 - \hat{\alpha}_6 = \\ \hat{\alpha}_1 + \hat{\alpha}_3 - \hat{\alpha}_4 = \\ \hat{\alpha}_2 + \hat{\alpha}_3 - \hat{\alpha}_5 = \end{cases}$$

Forme de méthode de Condition :

$$F(\hat{L}) = C$$

Forme linéarisée

$$B\hat{V} + W = 0$$

Evo

M

$$*B = \frac{\partial F(\hat{L})}{\partial \hat{L}} \bigg|_{\bar{L}} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & -1 \\ 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \end{pmatrix}$$

don B et W :

$$*W = F(\bar{L}) - C = \begin{bmatrix} -4 \cdot 10^4 & 12 \cdot 10^{-3} & -1,4 \cdot 10^{-3} \end{bmatrix}^T$$

La valeur de K

$$K = -M^{-1}W = \begin{bmatrix} 0,00015 & -0,0007 & 0,0001 \end{bmatrix}^T$$

La valeur de V

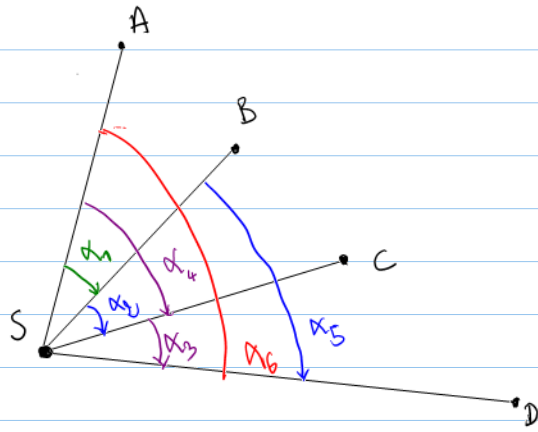
$$V = P^{-1}B^TK = \begin{bmatrix} -0,55 & 0,05 & 0,7 & -0,6 & -0,15 \end{bmatrix}^T$$

Observation estimée

$$\hat{L} = \bar{L} + \hat{V} = \begin{bmatrix} 39,1847 & 17,7511 & 32,4150 \\ 57,9373 & 51,1634 & 90,3517 \end{bmatrix}^T$$

# Méthode Générale

## Schéma



Analyse de problème :

$$\begin{aligned} n &= 6 \\ n_0 &= 3 \\ D &= 3 \\ u &= 1 \\ n &= 4 \end{aligned}$$

Identifier les variables :

$$L_{(n,1)} = \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 & \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 & \bar{\alpha}_6 \end{bmatrix}^T$$

$$\hat{V}_{(n,1)} = \begin{bmatrix} \hat{v}_1 & \hat{v}_2 & \hat{v}_3 & \hat{v}_4 & \hat{v}_5 & \hat{v}_6 \end{bmatrix}^T$$

$$\hat{X}_{(u,1)} = \begin{bmatrix} \hat{x}_1 \end{bmatrix}^T$$

$$\hat{\bar{X}}_{(u,1)} = \begin{bmatrix} \hat{\bar{x}}_1 \end{bmatrix}^T$$

Model Mathématique

Forme Générale

$$\begin{cases} \hat{x}_1 - \hat{x}_1 & = 0 \\ \hat{x}_1 + \hat{\alpha}_2 - \hat{x}_4 & = 0 \\ \hat{x}_1 - \hat{\alpha}_6 + \hat{\alpha}_5 & = 0 \\ \hat{x}_1 + \hat{\alpha}_6 + \hat{\alpha}_2 + \hat{\alpha}_3 & = 0 \end{cases} \Rightarrow F(\hat{X}, \hat{L}) = 0$$

Forme linéarisée :

$$A \hat{X} + B \hat{V} + W = 0$$

• evaluer A, B et W :

$$A = \frac{\partial F(\hat{X}, \hat{L})}{\partial \hat{X}} \bigg|_{\hat{L}, \hat{X}^0}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$B = \left. \frac{\partial F(\hat{X}, \hat{L})}{\partial \hat{L}} \right|_{\bar{L}, \bar{X}^0}$$

$$= \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 & -1 \end{pmatrix}$$

$$\omega = F(\bar{L}, \bar{X}^0) = [0, -4 \times 10^{-4}, -2,6 \times 10^{-3}, -1,4 \times 10^{-3}]^T$$

LePoids:  $P = I_6$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 1 & -1 & 3 \end{pmatrix} \quad M^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0,625 & -0,125 & -0,25 \\ 0 & -0,125 & 0,625 & 0,25 \\ 0 & -0,25 & 0,25 & 0,5 \end{pmatrix}$$

$$N = (A^T M^{-1} A)^{-1} = 2,5 \quad N^A = 0,4 \quad U = -0,00275$$

$$\hat{X} = -(A^T M^{-1} A)^{-1} (A^T M^{-1} \omega) = 0,0011$$

$$\hat{X} = \bar{X}^0 + \hat{X} = 39,1863$$

$$\hat{L} = [-0,0011 \quad -0,0017 \quad 0,0011 \quad 0,0007]^T$$

$$\hat{V} = [1,1 \quad 0 \quad 0,7 \quad 0,7 \quad -1,1 \quad 0,6]^T$$

$$\hat{L} = \bar{L} + \hat{V} = [39,1863 \quad 17,751 \quad 32,4149 \quad 57,9373 \quad 51,1629 \quad 90,3522]^T$$