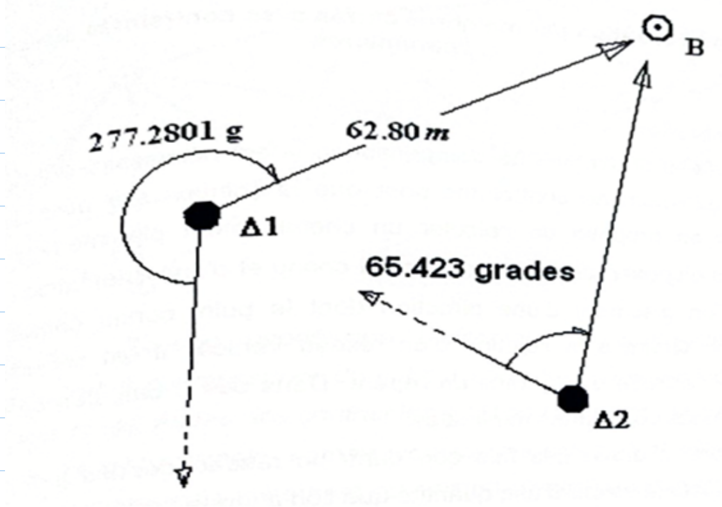
Shéma



Analyse de problème

→ Le nombre des observations : $n = 5$

→ Le nombre de variable distinct : $n_0 = 4$

→ Le nombre des paramètre : u $u = 4$. $(X_B , Y_B , G_{B\Delta_1} , G_{B\Delta_2})$

→ Le nombre de degres de liberté : v

→ Le nombre des equations : $n = 5$

Identification des variables

$L = [\, d_{17} \;\; d_{1B} \;\; d_{21} \;\; d_{2B} \;\; \Delta_1 D \,]^T$

	pt visée	D_{ij}	distance	Sigma
station Δ1	Δ7	0,0000		20 cc
	B	277,2801	62,80	2 cm (20 cc
Station Δ2	Δ1	0,0000		20 cc
	B	65,4230		20 cc

$\underset{(n,1)}{L} = [\, 0{,}0000 ,\; 277{,}2801 \;\; 0{,}0000 \;\; 65.4230 \;\; 62.80 \,]^T$ Vecteur d'observateur

$\underset{(u,1)}{\hat{X}} = [\, dG_1 \;\; dG_2 \;\; \hat{X}_B \;\; \hat{Y}_B \,]^T$ Vecteur destimeés des paramètre.

$\underset{(u,1)}{\overline{X}^{\circ}} = [\, dG_1^{\circ} ,\; dG_2^{\circ} ,\; X_b^{\circ} ,\; Y_b^{\circ} \,]^T$ Vecteur des valeur approché des paramètres.

$\underset{(u,1)}{\hat{x}} = [\, d\hat{G}_1 \;\; d\hat{G}_2 \;\; \hat{X}_b \;\; \hat{Y}_b \,] = \hat{X} - \overline{X}^{\circ}$ Correction des Paramètres

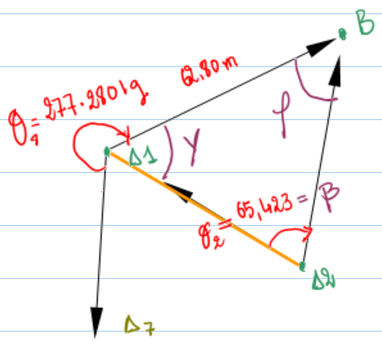
$\underset{(n,1)}{\hat{V}} = [\, \hat{v}_1 \;\; \hat{v}_2 \;\; \hat{v}_3 \;\; \hat{v}_4 \;\; \hat{v}_5 \,]^T = \underset{(n,1)}{\hat{L}} - \underset{(n,1)}{L}$ Valeur Résiduelle par Moindre Carrés

Calcul de $\overline{X}^{\circ}$

On va calculer les valeurs Approchés de $(\overline{X}_b^{\circ}$ et $\overline{Y}_b^{\circ})$ par intersection angulaire

Travaont Y

d'abord calculons : $\alpha_{17} \quad \alpha_{1B} \quad \alpha_{12} \quad \alpha_{2B}$



$\cdot\ \alpha_{17}$? On a $\begin{cases} \Delta X_{17} = -45.53 \\ \Delta Y_{17} = -28{,}85 \end{cases}$

$\alpha_{T17} = \text{Arctan}\left(\frac{\Delta X}{\Delta Y}\right) + 200$

$\alpha_{T17} = 264.0441$ grades

- α_{12} ?

$\begin{cases} \Delta X_{12} = 48{,}35 \\ \Delta Y_{12} = -45{,}69 \end{cases}$

$\alpha_{12} = \text{Arctan}\left(\left|\frac{\Delta Y}{\Delta X}\right|\right) + 100$

$\alpha_{12} = 148{,}1997$ grades

$\alpha_{21} = 348{,}1997$ grades

α_{1B} ?

$\alpha_{1B} = \alpha_{17} + \theta = 141.3242$ grades

α_{2B} ?

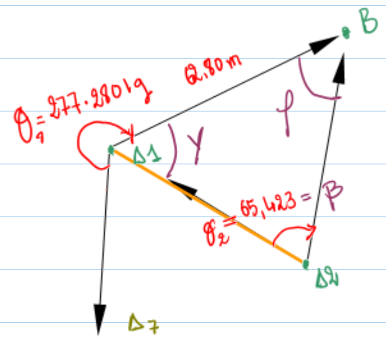
$\alpha_{2B} = \alpha_{21} + \beta = 13{,}6227$ grades

Calcule de γ ?

$\gamma = 400 - (\theta_1 + (\alpha_{17} - \alpha_{12}))$

$\gamma = 6{,}8756$ grades

$(\overline{X}_b^{\circ}, \overline{Y}_b^{\circ})$ D'après la règle de sinus



$\frac{\Delta_1\Delta_2}{\sin(\varphi)} = \frac{\overline{\Delta_1 B}^{\circ}}{\sin(\beta)}$

$\Leftrightarrow \overline{\Delta_1 B}^{\circ} = \sin(\beta) \cdot \frac{\Delta_1\Delta_2}{\sin(\varphi)}$

Calculons $\Delta_1\Delta_2$, φ

- $\Delta_1\Delta_2 = \sqrt{(X_2-X_1)^2 + (Y_2-Y_1)^2} = 66.529$ m

- $\varphi = 200 - \beta - \gamma = 127.7014$ grades

Donc $\Delta_1 B = 62.8021$ m

$\overset{\circ}{X}_b, \overset{\circ}{Y}_b$

$$\overset{\circ}{X}_b = X_1 + \Delta_1 B \cdot \cos(\alpha_{1b})$$

$$\overset{\circ}{Y}_b = Y_1 + \Delta_1 B \cdot \sin(\alpha_{1b})$$

$$\begin{cases} \overset{\circ}{X}_b = 361\,685.169 \text{ m} \\ \overset{\circ}{Y}_b = 371\,722.8971 \text{ m} \end{cases}$$

Tableau de gisement et distance approchée

	direction ij	gisement approché $\overset{\circ}{\alpha}_{ij}$(grade)	distance app (m)	Sigma
station Δ_1	Δ7	264.0441		20 cc
	B	141.3242	62.802	2 cm (20 cc)
Station Δ_2	Δ_1	348.1997		20 cc
	B	13.6227		20 cc

Calcul de la Constante d'orientation approchée des station Δ_1 Δ_2

	direction ij	gisement approché $\overset{\circ}{\alpha}_{ij}$(grade)	Lecture Horizontale D_{ij}	G_i°	G_m°
station Δ_1	Δ7	264.0441	0,0000	264.0441	264.0441
	B	141.3242	277.2801	264.0441	
Station Δ_2	Δ_1	348.1997	0,0000	348.1997	349.1996
	B	13.6227	65.423	348.1995	

On pose les equations d'observation:

$$\hat{v}''_{17} = -d\hat{G}''_1 + \omega''_{17}$$

$$\hat{v}''_{1B} = -d\hat{G}''_1 + \rho'' \frac{\cos \overset{\circ}{\alpha}_{1B}}{(ij)_0} \hat{x}_b - \rho'' \frac{\sin \overset{\circ}{\alpha}_{1B}}{(ij)_0} \hat{y}_b + \omega''_{1B}$$

$$\hat{v}_{21} = -d\hat{G}''_2 + \omega''_{21}$$

$$\hat{v}_{2B} = -d\hat{G}''_2 + \rho'' \frac{\cos \overset{\circ}{\alpha}_{2B}}{(ij)_0} \hat{x}_b - \rho'' \frac{\sin \overset{\circ}{\alpha}_{2B}}{(ij)_0} \hat{y}_b + \omega''_{2B}$$

$$\hat{v}''_{l_{1B}} = \underset{\text{Appr}}{\rho'} \sin(\overset{\circ}{\alpha}_{1B}) \hat{x}_b + \underset{\text{Appr}}{\rho'} \cos(\overset{\circ}{\alpha}_{1B}) \hat{y}_B + \omega_{1B}$$

Le système d'equations d'observation du type précedent s'écrit sous la forme matricielle suivante:

$$\hat{V} = A\hat{X} + W$$

Où;

$$\hat{V}_{n,1} = \left[\hat{v}_1 \quad \hat{v}_2 \quad \hat{v}_3 \quad \hat{v}_4 \quad \hat{v}_5\right]^T$$

$$\hat{X}_{u,1} = \left[d\hat{G}_1 \quad d\hat{G}_2 \quad \hat{X}_b \quad \hat{Y}_b\right]^T$$

$$W_{n,1} = \left[\omega''_{17} \quad \omega''_{1b} \quad \omega''_{21} \quad \omega''_{2b} \quad \omega_{1b}\right]^T$$

Calcul de W:

On Sait que: $\omega''_{ij} = (\overline{\alpha}^{\circ}_{ij} - \overline{D}_{ij}) - \overline{G}^{\circ}_{mi}$ | $\omega_{ij} = (ij)_0 - \overline{ij}$

$$\omega''_{17} = \overline{\alpha}^{\circ}_{17} - \overline{D}_{17} - \overline{G}^{\circ}_{m1} \qquad \omega''_{21} = \overline{\alpha}^{\circ}_{21} - \overline{D}_{21} - \overline{G}^{\circ}_{m2}$$

$$\omega''_{1b} = \overline{\alpha}^{\circ}_{1b} - \overline{D}_{1b} - \overline{G}^{\circ}_{m1} \qquad \omega''_{2b} = \overline{\alpha}^{\circ}_{2b} - \overline{D}_{2b} - \overline{G}^{\circ}_{m2}$$

$$\omega_{1b} = \overline{\Delta_1 B}^{\circ} - \overline{\Delta_1 B}$$

$$\omega''_{17} = 0 \qquad \omega''_{21} = 1,48^{cc}$$

$$\omega''_{1b} = 0 \qquad \omega''_{2b} = 1,48^{cc}$$

$$\omega_{1b} = 2.09 \text{ mm}$$

Calculer A

$$A = \begin{array}{cccc} \frac{\partial V}{\partial dG_1} & \frac{\partial V}{\partial G_2} & \frac{\partial V}{\partial X_b} & \frac{\partial V}{\partial Y_b} \end{array}$$
$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & 0 & -6,13 \cdot 10^3 & -8,08\ 10^3 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 7,87 \cdot 10^4 & -1,71 \cdot 10^4 \\ 0 & 0 & 5.07 \cdot 10^5 & -3,85\ 10^5 \end{pmatrix}$$

$$W = \begin{bmatrix} 0^{cc} & 0^{cc} & 1,48^{cc} & 1,48^{cc} & 2,09^{mm} \end{bmatrix}$$

$$P = \sigma_0^2 \begin{pmatrix} 400^{cc} & 0 & 0 & 0 & 0 \\ 0 & 400^{cc} & 0 & 0 & 0 \\ 0 & 0 & 400^{cc} & 0 & 0 \\ 0 & 0 & 0 & 400^{cc} & 0 \\ 0 & 0 & 0 & 0 & 400^{mm} \end{pmatrix}^{-1}$$

$$P = I_5$$

On a $\hat{X} = -(A^T M^{-1} A)^{-1} (A^T M^{-1} W)$

$M = B P^{-1} B^T = P^{-1} \doteq I_5$

$N = A^T M^{-1} A$

$$= \begin{pmatrix} 2 & 0 & 6.10^{+3} & 8{,}07.10^{3} \\ 0 & 2 & -7.86\,10^{4} & 9{,}71\ 10^{4} \\ 6{,}13.10^{3} & -7{,}86.10^{4} & 6{,}22.10^{9} & -1.29.10^{9} \\ 8{,}08.10^{3} & 9{,}71.10^{4} & -1{,}3.10^{9} & 3{,}57\,10^{8} \end{pmatrix}$$

$$U = A^{-1} M^{-1} W = \begin{pmatrix} -5.68 . 10^{-10} \\ -2{,}96 \\ 1{,}16 . 10^{5} \\ -2{,}52 . 10^{4} \end{pmatrix}$$

$$\boxed{\hat{X} = \begin{pmatrix} 0{,}31^{cc} \\ 1{,}38^{cc} \\ 1{,}23^{mm} .10^{-6} \\ 6{,}78^{mm} .10^{-6} \end{pmatrix}}$$

σ_0

$\hat{x} = (A^T M^{-1} A)^{-1} (A^T M^{-1} W)$

$= \left(\frac{cc}{m}\right) \cdot \left(\frac{cc}{mm}\right)\left(\ \ /cc\right)\left(\frac{cc}{m} \cdot \frac{cc}{mm}\ \frac{cc}{mm}\right)$

$$\boxed{\hat{V} = \begin{pmatrix} 0{,}3^{cc} \\ -0{,}3^{cc} \\ 0{,}09^{cc} \\ 0{,}09^{cc} \\ 0{,}1\ mm \end{pmatrix}}$$

$\hat{L} \simeq \bar{L}$

$(cc\ cc\ cc\ cc\ mm)$

$= (1cc\ 1cc\ 1cc\ 1cc\ 1mm)$

$\hat{\sigma}_0 = 0{,}22 \qquad = \frac{D.P.D}{1}$

test $= A^T.P.V = \begin{pmatrix} 10^{-5} \\ 10^{-15} \\ 10^{-9} \\ 10^{-10} \end{pmatrix}$

$\frac{D.\hat{\sigma}_0^2}{\chi^2_{D,\alpha}} = \frac{1.\ 0{,}22}{\chi^2_{1;0{,}05}} = \frac{0{,}22}{3{,}84} = 0{,}05$

don $\sigma_0^2 > 0{,}05$ donc σ_0^2 est acceptable