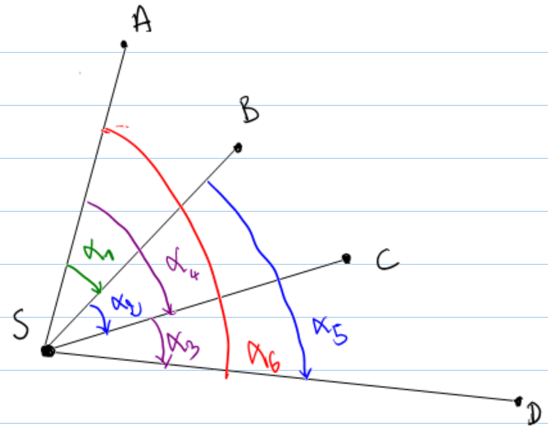
Shème



Analyse de problème :

$n = 6$
$n_0 = 3$
$\nu = 3$
$u = 3$
$r = 6$

Identifier les variables :

$$L_{(n,1)} = \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 & \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 & \bar{\alpha}_6 \end{bmatrix}^T$$

$$\hat{V}_{(n,1)} = \begin{bmatrix} \hat{v}_1 & \hat{v}_2 & \hat{v}_3 & \hat{v}_4 & \hat{v}_5 & \hat{v}_6 \end{bmatrix}^T$$

$$\hat{X}_{(u,1)} = \begin{bmatrix} \hat{\alpha}_1 & \hat{\alpha}_2 & \hat{\alpha}_3 \end{bmatrix}^T$$

$$\hat{X}_{(u,1)} = \begin{bmatrix} \hat{X}_1 & \hat{X}_2 & \hat{X}_3 \end{bmatrix}^T$$

Model Mathematique

Forme Générale

Méthode de Variation de Paramètre

$$\begin{cases} \hat{X}_1 = \hat{\alpha}_1 \\ \hat{X}_2 = \hat{\alpha}_2 \\ \hat{X}_3 = \hat{\alpha}_3 \\ \hat{X}_1 + \hat{X}_2 = \hat{\alpha}_4 \\ \hat{X}_2 + \hat{X}_3 = \hat{\alpha}_5 \\ \hat{X}_1 + \hat{X}_2 + \hat{X}_3 = \hat{\alpha}_6 \end{cases}$$

$$F(\hat{X}) = \hat{L}$$

forme linéarisée

$$A\hat{X} + W = \hat{V}$$

A et W ; X^0 :

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ & 1 & 1 \end{pmatrix}$$

$\vec{X}^{\circ} = [\vec{\alpha}_1 \quad \vec{\alpha}_2 \quad \alpha_3]^T = [39.1852, 18.7510, 32.4142]$

$W = [0, 0, 0, -4.10^{4}, 12.10^{-3}, -1{,}4.10^{-3}]^T$

$\bar{X}$?

On a : $\boxed{\bar{X} = -(A^T M^{-1} A)^{-1} (A^T M^{-1} W)} = \begin{pmatrix} 0{,}00075 \\ -0{,}0002 \\ -0{,}00005 \end{pmatrix}$ grades

Avec :

- $M = B P^{-1} B^T = P^{-1} = I_6$
- $N = A^T M^{-1} A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{pmatrix}$
- $N^{-1} = (A^T M^{-1} A)^{-1} = \begin{pmatrix} 0{,}5 & -0{,}25 & 0 \\ -0{,}25 & 0{,}5 & -0{,}25 \\ 0 & -0{,}25 & 0{,}5 \end{pmatrix}$
- $U = A^T M^{-1} W = \begin{pmatrix} -0{,}0018 \\ -0{,}006 \\ -0{,}002 \end{pmatrix}$

$$\boxed{\hat{X} = \overset{\circ}{X} + \hat{x}} = [39,18595 ; 17,7508 ; 32,41415]^T$$

Coefficient du Poids

$$Q_{\hat{x}} = (A^T M^{-1} A)^{-1} = \begin{pmatrix} 0,5 & -0,25 & 0 \\ -0,25 & 0,5 & -0,25 \\ 0 & -0,25 & 0,5 \end{pmatrix}$$

La Solution de Lagrange $\hat{K}$

$$\boxed{\hat{K} = -M^{-1}(A\hat{X} + W)}$$

Vecteur des Résiduelle $\hat{V}$

$$\boxed{\hat{V} = P^{-1} B \hat{K}} = -\hat{K} = \begin{pmatrix} 0,00075 \\ -0,0002 \\ -0,00005 \\ 0,00015 \\ 0,00095 \\ -0,0009 \end{pmatrix}$$

$$Q_{\hat{K}} = Q_{\hat{V}} = P^{-1}$$

$$\hat{L} = \begin{pmatrix} 39,18595 \\ 17,7508 \\ 32,4145 \\ 57,93675 \\ 51,16495 \\ 90,3509 \end{pmatrix}$$

Règle de Résolution Avec Contrainte

équation de Contraint Additionnel:

$$\left\{\begin{array}{l}\hat{X}_2 = 18.7500 \\ \hat{X}_3 = 32.4150\end{array}\right.$$

La forme linearisée:

$$C\hat{X} + W_c = 0$$

$$\boxed{C = \left.\frac{\partial F(\hat{X})}{\partial \hat{X}}\right|_{\bar{X}^{\circ}}} = \begin{array}{ccc} \hat{X}_1 & \hat{X}_2 & \hat{X}_3 \\ \end{array} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$W_c = \left[1.10^{-3},\ -8.10^{-4}\right]$$

La Solution est donnée par:

$$\hat{X} = (A^T M A)^{-1}\left[-A^T M^{-1} W + C^T\left(C(A^T M^{-1} A)^{-1} C^T\right)^{-1} \cdot \left[-W_c + C(A^T M^{-1} A)^{-1} A^T M^{-1} W\right]\right]$$

Dans notre Cas: $P = M^{-1}$; $B = -I$; $N = A^T P A$; $U = A^T M^{-1} W$

$$\hat{X}_c = \hat{X} + N^{-1} C^T \left(C\left[C N^{-1} C^T\right]^{-1}\left[-W_c + C N^{-1} U\right]\right)$$

$$\hat{X}_c = \begin{pmatrix} 0{,}001 \\ -0{,}001 \\ 0{,}0008 \end{pmatrix} = \left[39.1862 \quad 17{,}75 \quad 32.415\right]^T$$

$$\hat{V}_c = -\hat{K}_c = \left[0{,}001,\ -0{,}001 \quad 0{,}0008 \quad -0{,}0004 \quad 0{,}001 \quad -0{,}0006\right]^T$$

$$\boxed{\sigma_0^2 = \frac{V^T P V}{3}} = 7{,}8 \times 10^{-7}$$

$$\hat{L}_c = L + V_c = \left[39{,}1862 \quad 17{,}75 \quad 32.415 \quad 57.9362 \quad 51.165 \quad 90.3512\right]^T$$

Variation de Paramètre

Analyse de problème :

$n = 6$

$n_0 = 3$

$\nu = 3$

$u = 0$

$r = 3$

$$\bar{L}_{(n,1)} = \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 & \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 & \bar{\alpha}_6 \end{bmatrix}^T$$

$$\hat{V}_{(n,1)} = \begin{bmatrix} \hat{v}_1 & \hat{v}_2 & \hat{v}_3 & \hat{v}_4 & \hat{v}_5 & \hat{v}_6 \end{bmatrix}^T$$

Identifier les variables.

Modèle Mathématique :

modèle explicite :

$$\begin{cases} \hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_3 - \hat{\alpha}_6 = \\ \hat{\alpha}_1 + \hat{\alpha}_2 - \hat{\alpha}_4 = \\ \hat{\alpha}_2 + \hat{\alpha}_3 - \hat{\alpha}_5 = \end{cases}$$

Forme de méthode de Condition :

$$F(\hat{L}) = C$$

Forme linearisée $\quad B\hat{V} + W = 0$

Evo

Les B et W :

M

$$B = \left.\frac{\partial F(\hat{L})}{\partial \hat{L}}\right|_{\bar{L}} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & -1 \\ 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \end{pmatrix}$$

$$W = F(\bar{L}) - C = \left[-4 \cdot 10^{-4},\ 12 \cdot 10^{-3},\ -1{,}4 \cdot 10^{-3}\right]^T$$

La valeur de K

$$\boxed{K = -M^{-1} W} = \left[0{,}00015 \quad -0{,}0007 \quad 0{,}0006\right]^T$$

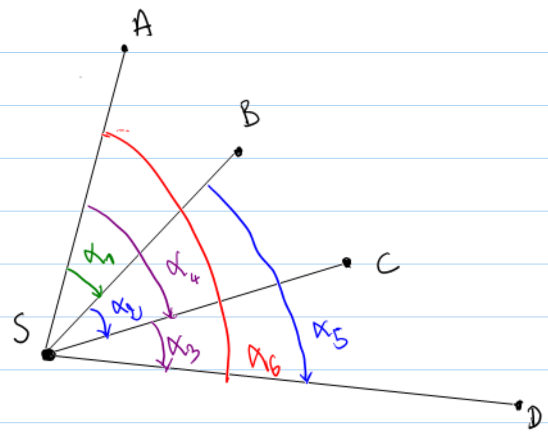
La Valeur de V

$$\boxed{V = P^{-1} B^T K} = \left[-0{,}55 \quad 0{,}05 \quad 0{,}75 \quad 0{,}7 \quad -0{,}6 \quad -0{,}15\right]^T$$

Observation estimé

$$\boxed{\hat{L} = \bar{L} + \hat{V}} = \left[39{,}1847 \quad 17.7511 \quad 32{,}4150 \quad 57.9373 \quad 51.1634 \quad 90{,}3517\right]^T$$

Shème



Analyse de problème :

$n = 6$

$n_0 = 3$

$\partial = 3$

$u = 1$

$r = 4$

Identifier les variables :

$$L_{(n,1)} = \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 & \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 & \bar{\alpha}_6 \end{bmatrix}^T$$

$$\hat{V}_{(n,1)} = \begin{bmatrix} \hat{v}_1 & \hat{v}_2 & \hat{v}_3 & \hat{v}_4 & \hat{v}_5 & \hat{v}_6 \end{bmatrix}^T$$

$$\hat{X}_{(u,1)} = \begin{bmatrix} \hat{\alpha}_1 \end{bmatrix}^T$$

$$\hat{\hat{X}}_{(u,1)} = \begin{bmatrix} \hat{X}_1 \end{bmatrix}^T$$

Model Mathematique

Forme Générale

$$\begin{cases} \hat{X}_1 - \hat{\alpha}_1 = 0 \\ \hat{X}_1 + \hat{\alpha}_2 - \hat{\alpha}_4 = 0 \\ \hat{X}_1 - \hat{\alpha}_6 + \hat{\alpha}_5 = 0 \\ \hat{X}_1 + \hat{\alpha}_6 + \hat{\alpha}_2 + \hat{\alpha}_3 = 0 \end{cases} \iff F(\hat{X}, \hat{L}) = 0$$

Forme linéarisée :

$$A\hat{X} + B\hat{V} + W = 0$$

- évaluer A, B et W :

$$\boxed{A = \frac{\partial F(\hat{\hat{X}}, \hat{L})}{\partial \hat{\hat{X}}}\bigg|_{L, X^0}} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\boxed{B = \left.\frac{\partial F(\hat{X}, \hat{L})}{\partial \hat{L}}\right|_{L, \bar{X}^{\circ}}} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 & -1 \end{pmatrix}$$

$$\boxed{W = F(L, \bar{X}^{\circ})} = [0 \;,\; -4\times10^{-4} \;,\; -2{,}6\times10^{-3} \;,\; -1{,}4 \cdot 10^{-3}]^T$$

Le Poids : $P = I_6$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 1 & -1 & 3 \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0{,}625 & -0{,}125 & -0{,}25 \\ 0 & -0{,}125 & 0{,}625 & 0{,}25 \\ 0 & -0{,}25 & 0{,}25 & 0{,}5 \end{pmatrix}$$

$$N = (A^T M^{-1} A)^{-1} = 2{,}5 \qquad N^{-1} = 0{,}4 \qquad U = -0{,}00275$$

$$\boxed{\hat{X} = -(A^T M^{-1} A)^{-1} (A^T M^{-1} W)} = 0{,}0011$$

$$\hat{X} = \bar{X}^{\circ} + \hat{X} = 39{,}1863$$

$$\hat{K} = [-0{,}0011 \quad -0{,}0017 \quad 0{,}0011 \quad 0{,}0007]^T$$

$$\hat{V} = [1{,}1 \quad 0 \quad 0{,}7 \quad 0{,}7 \quad -1{,}1 \quad 0{,}4]^T$$

$$\hat{L} = L + \hat{V} = [39.1863 \quad 17{,}751 \quad 32.4149 \quad 57.9373 \quad 51.1629 \quad 90{,}3522]^T$$