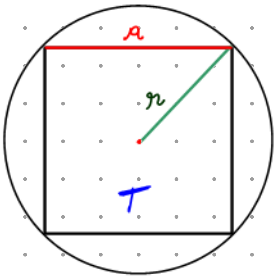
Devoir 02 : exercice 02 page 185

Shéma et Observation



$T = \frac{a^2}{2}\pi$

Observation

Le rayon :

$\sigma_r = 0{,}05\ mm$

$r_1 = 99{,}7\ mm$

$r_2 = 100{,}5\ mm$

$r_3 = 100{,}1\ mm$

$r_4 = 99{,}5\ mm$

Le Coté

$\sigma_a = 0{,}05\ mm$

$a_1 = 141{,}5\ mm$

$a_2 = 140{,}9\ mm$

$a_3 = 141{,}4\ mm$

$a_4 = 141{,}2\ mm$

La surface

$\sigma_p = 50\ mm^2$

$S_1 = 31311\ mm^2$

$S_2 = 31437\ mm^2$

$S_3 = 31360\ mm^2$

$S_4 = 31442\ mm^2$

Analyse de Problème

Le nombre des observations : $n = 12$

Le nombre de variable distinct : $n_0 = 1$

Le nombre de paramètre : $u = 1$

Le nombre de degrés de libertés : $\nu = 11$

Le nombre des equations : $r = n = 12$

Identification des Paramètres

Vecteur des observations :

$$\bar{L}_{(n,1)} = [\bar{r}_1\ \bar{r}_2\ \bar{r}_3\ \bar{r}_4\ \bar{a}_1\ \bar{a}_2\ \bar{a}_3\ \bar{a}_4\ \bar{S}_1\ \bar{S}_2\ \bar{S}_3\ \bar{S}_4]^T$$

Vecteur des valeurs approchées des Paramètres :

$$\bar{X}^0_{(u,1)} = [\,S_1\,] = [\,31311\,]^{mm^2}$$

Vecteur des résiduelles

$$\hat{V}_{(n,1)} = \hat{L} - \bar{L} = [\hat{v}_1\ \hat{v}_2\ \hat{v}_3\ \hat{v}_4\ \hat{v}_5\ \hat{v}_6\ \hat{v}_7\ \hat{v}_8\ \hat{v}_9\ \hat{v}_{10}\ \hat{v}_{11}\ \hat{v}_{12}]^T$$

Correction des Paramètres

$$\hat{x}_{(u,1)} = [\,\hat{s}_1\,] = \hat{X} - \bar{X}^0$$

Vecteur des estimés des Paramètres : $\hat{X}_{(u,1)}$

Vecteur des observations compensées : $\hat{L}_{(n,1)}$

Modèle mathématique

Forme générale

$$\hat{L} = \begin{bmatrix} \hat{r}_1 \\ \hat{r}_2 \\ \hat{r}_3 \\ \hat{r}_4 \\ \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \\ \hat{a}_4 \\ \hat{S}_1 \\ \hat{S}_2 \\ \hat{S}_3 \\ \hat{S}_4 \end{bmatrix} = \begin{bmatrix} (\hat{X}/\pi)^{\frac{1}{2}} \\ (\hat{X}/\pi)^{\frac{1}{2}} \\ (\hat{X}/\pi)^{\frac{1}{2}} \\ (\hat{X}/\pi)^{\frac{1}{2}} \\ (2\hat{X}/\pi)^{\frac{1}{2}} \\ (2\hat{X}/\pi)^{\frac{1}{2}} \\ (2\hat{X}/\pi)^{\frac{1}{2}} \\ (2\hat{X}/\pi)^{\frac{1}{2}} \\ \hat{X} \\ \hat{X} \\ \hat{X} \\ \hat{X} \end{bmatrix} = F(\hat{X})$$

Avec :
- $\rightarrow \pi \hat{r}^2 = \hat{X}$
- $\rightarrow 2a^2 = (2r)^2$
- $\rightarrow \dfrac{\pi \quad a^2}{2} = \hat{X}$

Forme linéarisée :

$$A\hat{X} + W = \hat{V}$$

Evaluer A

$$A = \left.\frac{\partial F(\hat{X})}{\partial \hat{X}}\right|_{\bar{X}^{\circ}} = \begin{bmatrix} 0,5\cdot(\hat{X}\pi)^{-\frac{1}{2}} \\ 0,5\cdot(\hat{X}\pi)^{-\frac{1}{2}} \\ 0,5\cdot(\hat{X}\pi)^{-\frac{1}{2}} \\ 0,5\cdot(\hat{X}\pi)^{-\frac{1}{2}} \\ \cdot(2\hat{X}\pi)^{-\frac{1}{2}} \\ \cdot(2\hat{X}\pi)^{-\frac{1}{2}} \\ \cdot(2\hat{X}\pi)^{-\frac{1}{2}} \\ \cdot(2\hat{X}\pi)^{-\frac{1}{2}} \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.0016 \\ 0.0016 \\ 0.0016 \\ 0.0016 \\ 0,00225 \\ 0,00225 \\ 0,00225 \\ 0,00225 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Evaluer W

$$W = F(\bar{X}^{\circ}) - \bar{L} = \begin{bmatrix} 0.1329 \text{ mm} \\ -0.6671 \text{ mm} \\ -0,2671 \text{ mm} \\ 0,3329 \text{ mm} \\ -0.315 \text{ mm} \\ 0.285 \text{ mm} \\ -0,215 \text{ mm} \\ -0,015 \text{ mm} \\ 0 \text{ mm}^2 \\ -126 \text{ mm}^2 \\ -49 \text{ mm}^2 \\ -131 \text{ mm}^2 \end{bmatrix}$$

Solution des Paramètres $\hat{X}$ et Système d'equation Normale

Calcul de $\hat{x}$

Le système d'equation normale est le suivant :

$$\boxed{N\hat{x} + U = 0}$$

$$\hat{x} = -[A^T P A]^{-1} A^{-1} P W$$

$$= -N^{-1} U$$

Calcul de P

On a $\Sigma_I = \begin{bmatrix} (0{,}05)^2 & & & & & & & & & & & \\ & (0{,}05)^2 & & & & & & & & & & \\ & & (0{,}05)^2 & & & & & & & & & \\ & & & (0{,}05)^2 & & & & & 0 & & & \\ & & & & (0{,}05)^2 & & & & & & & \\ & & & & & (0{,}05)^2 & & & & & & \\ & & & & & & (0{,}05)^2 & & & & & \\ & & & & & & & (0{,}05)^2 & & & & \\ & & & 0 & & & & & (50)^2 & & & \\ & & & & & & & & & (50)^2 & & \\ & & & & & & & & & & (50)^2 & \\ & & & & & & & & & & & (50)^2 \end{bmatrix}$

On prend $\sigma_0^2 = (50)^2$

$P\ \Sigma_I = \begin{bmatrix} 10^6 & & & & & & & & & & & \\ & 10^6 & & & & & & & & & & \\ & & 10^6 & & & & & & 0 & & & \\ & & & 10^6 & & & & & & & & \\ & & & & 10^6 & & & & & & & \\ & & & & & 10^6 & & & & & & \\ & & & & & & 10^6 & & & & & \\ & & & & & & & 10^6 & & & & \\ & & 0 & & & & & & 1 & & & \\ & & & & & & & & & 1 & & \\ & & & & & & & & & & 1 & \\ & & & & & & & & & & & 1 \end{bmatrix}$

Calcul de N^{-1}

$$N = A^T P A = 34{,}49$$

Calcul de U

$$U = A^T P . W = -1572.94$$

$$\boxed{\hat{x} = -N^{-1}.U = 45{,}61 \ mm^2}$$

Calcul de $\hat{X}$

$$\hat{X} = X^0 + \hat{x} = 31356.61 \ mm^2$$

Solution de coefficient de Lagrange $\hat{K}$

Calcul de $\hat{K}$

$$\hat{K} = -M^{-1}(A\hat{X} + W)$$

$$\hat{K} = \begin{bmatrix} -2{,}05 \cdot 10^{5} \\ 5.94 \cdot 10^{5} \\ -4{,}06 \cdot 10^{5} \\ 2{,}12 \cdot 10^{5} \\ -3{,}86 \cdot 10^{5} \\ 1{,}12 \cdot 10^{5} \\ -1{,}17 \cdot 10^{5} \\ -4{,}56 \cdot 10^{1} \\ 8{,}03 \cdot 10 \\ 3{,}39 \\ 8{,}53 \cdot 10 \end{bmatrix}$$

Calcul des résiduelle $\hat{V}$

Calcul de $\hat{V}$

$$\hat{V} = -P^{-1}\hat{K} = A\hat{X} + W$$

$$\hat{V} = \begin{bmatrix} 0{,}21 \\ -0{,}59 \\ -0{,}19 \\ 0{,}41 \\ -0{,}21 \\ 0{,}38 \\ -0{,}11 \\ 0{,}12 \\ 45{,}61 \\ -80{,}39 \\ -3{,}39 \\ -85{,}39 \end{bmatrix}$$

Calcul de $\hat{L}$

$$\hat{L} = L + \hat{V} = \begin{bmatrix} 99{,}91 \\ 99{,}91 \\ 99{,}91 \\ 99{,}91 \\ 141{,}29 \\ 141{,}29 \\ 141{,}29 \\ 141{,}29 \\ 31356{,}61 \\ 31356{,}61 \\ 31356{,}61 \\ 31356{,}61 \end{bmatrix}$$

estimé $\hat{\sigma}_0^2$ Choisie à priori

$$\hat{\sigma}_0^2 = \frac{V^T P V}{11} = 75950,24\ mm^2$$

La Matrice de Coefficient de Poid $Q_{\hat{x}}$

$\boxed{\text{Calcul } Q_x}$

$$Q_{\hat{x}} = N^{-1} = 0.03$$

Matrice de variance covariance $\Sigma\hat{x}$

$\boxed{\text{Calcul de } \Sigma\hat{x}}$

$$\Sigma_{\hat{x}} = \hat{\sigma}_0^2 \cdot Q_{\hat{x}} = 2202.09$$

$\boxed{\text{Calcul } Q_{\hat{l}}}$

$$Q_{\hat{l}} = P[I - A(A^T P A)^{-1} A^T P]$$

925775.59	-74224.41	-74224.41	-74224.41	-104378.08	-104378.08	-104378.08	-104378.08	-46.39	-46.39	-46.39	-46.39
-74224.41	925775.59	-74224.41	-74224.41	-104378.08	-104378.08	-104378.08	-104378.08	-46.39	-46.39	-46.39	-46.39
-74224.41	-74224.41	925775.59	-74224.41	-104378.08	-104378.08	-104378.08	-104378.08	-46.39	-46.39	-46.39	-46.39
-74224.41	-74224.41	-74224.41	925775.59	-104378.08	-104378.08	-104378.08	-104378.08	-46.39	-46.39	-46.39	-46.39
-104378.08	-104378.08	-104378.08	-104378.08	853218.32	-146781.68	-146781.68	-146781.68	-65.24	-65.24	-65.24	-65.24
-104378.08	-104378.08	-104378.08	-104378.08	-146781.68	853218.32	-146781.68	-146781.68	-65.24	-65.24	-65.24	-65.24
-104378.08	-104378.08	-104378.08	-104378.08	-146781.68	-146781.68	853218.32	-146781.68	-65.24	-65.24	-65.24	-65.24
-104378.08	-104378.08	-104378.08	-104378.08	-146781.68	-146781.68	-146781.68	853218.32	-65.24	-65.24	-65.24	-65.24
-46.39	-46.39	-46.39	-46.39	-65.24	-65.24	-65.24	-65.24	0.97	-0.03	-0.03	-0.03
-46.39	-46.39	-46.39	-46.39	-65.24	-65.24	-65.24	-65.24	-0.03	0.97	-0.03	-0.03
-46.39	-46.39	-46.39	-46.39	-65.24	-65.24	-65.24	-65.24	-0.03	-0.03	0.97	-0.03
-46.39	-46.39	-46.39	-46.39	-65.24	-65.24	-65.24	-65.24	-0.03	-0.03	-0.03	0.97

$\boxed{\text{Calcul de } \Sigma_{\hat{K}}}$ $\Sigma_{\hat{K}} = \hat{\sigma}_0^2 (Q_{\hat{K}})$

$$\begin{bmatrix}
0.006 & 0.006 & 0.006 & 0.006 & 0.008 & 0.008 & 0.008 & 0.008 & 3.523 & 3.523 & 3.523 & 3.523 \\
0.006 & 0.006 & 0.006 & 0.006 & 0.008 & 0.008 & 0.008 & 0.008 & 3.523 & 3.523 & 3.523 & 3.523 \\
0.006 & 0.006 & 0.006 & 0.006 & 0.008 & 0.008 & 0.008 & 0.008 & 3.523 & 3.523 & 3.523 & 3.523 \\
0.006 & 0.006 & 0.006 & 0.006 & 0.008 & 0.008 & 0.008 & 0.008 & 3.523 & 3.523 & 3.523 & 3.523 \\
0.008 & 0.008 & 0.008 & 0.008 & 0.011 & 0.011 & 0.011 & 0.011 & 4.955 & 4.955 & 4.955 & 4.955 \\
0.008 & 0.008 & 0.008 & 0.008 & 0.011 & 0.011 & 0.011 & 0.011 & 4.955 & 4.955 & 4.955 & 4.955 \\
0.008 & 0.008 & 0.008 & 0.008 & 0.011 & 0.011 & 0.011 & 0.011 & 4.955 & 4.955 & 4.955 & 4.955 \\
0.008 & 0.008 & 0.008 & 0.008 & 0.011 & 0.011 & 0.011 & 0.011 & 4.955 & 4.955 & 4.955 & 4.955 \\
3.523 & 3.523 & 3.523 & 3.523 & 4.955 & 4.955 & 4.955 & 4.955 & 2202 & 2202 & 2202 & 2202 \\
3.523 & 3.523 & 3.523 & 3.523 & 4.955 & 4.955 & 4.955 & 4.955 & 2202 & 2202 & 2202 & 2202 \\
3.523 & 3.523 & 3.523 & 3.523 & 4.955 & 4.955 & 4.955 & 4.955 & 2202 & 2202 & 2202 & 2202 \\
3.523 & 3.523 & 3.523 & 3.523 & 4.955 & 4.955 & 4.955 & 4.955 & 2202 & 2202 & 2202 & 2202
\end{bmatrix}$$

$$\boxed{\Sigma_{\hat{V}} = \left(P^{-1} - A N^{-1} A^T\right) \hat{\sigma}_0^2}$$

$$\begin{bmatrix}
0.07 & -0.006 & -0.006 & -0.006 & -0.008 & -0.008 & -0.008 & -0.008 & -3.523 & -3.523 & -3.523 & -3.523 \\
-0.006 & 0.07 & -0.006 & -0.006 & -0.008 & -0.008 & -0.008 & -0.008 & -3.523 & -3.523 & -3.523 & -3.523 \\
-0.006 & -0.006 & 0.07 & -0.006 & -0.008 & -0.008 & -0.008 & -0.008 & -3.523 & -3.523 & -3.523 & -3.523 \\
-0.006 & -0.006 & -0.006 & 0.07 & -0.008 & -0.008 & -0.008 & -0.008 & -3.523 & -3.523 & -3.523 & -3.523 \\
-0.008 & -0.008 & -0.008 & -0.008 & 0.065 & -0.011 & -0.011 & -0.011 & -4.955 & -4.955 & -4.955 & -4.955 \\
-0.008 & -0.008 & -0.008 & -0.008 & -0.011 & 0.065 & -0.011 & -0.011 & -4.955 & -4.955 & -4.955 & -4.955 \\
-0.008 & -0.008 & -0.008 & -0.008 & -0.011 & -0.011 & 0.065 & -0.011 & -4.955 & -4.955 & -4.955 & -4.955 \\
-0.008 & -0.008 & -0.008 & -0.008 & -0.011 & -0.011 & -0.011 & 0.065 & -4.955 & -4.955 & -4.955 & -4.955 \\
-3.523 & -3.523 & -3.523 & -3.523 & -4.955 & -4.955 & -4.955 & -4.955 & 73747 & -2202 & -2202 & -2202 \\
-3.523 & -3.523 & -3.523 & -3.523 & -4.955 & -4.955 & -4.955 & -4.955 & -2202 & 73747 & -2202 & -2202 \\
-3.523 & -3.523 & -3.523 & -3.523 & -4.955 & -4.955 & -4.955 & -4.955 & -2202 & -2202 & 73747 & -2202 \\
-3.523 & -3.523 & -3.523 & -3.523 & -4.955 & -4.955 & -4.955 & -4.955 & -2202 & -2202 & -2202 & 73747
\end{bmatrix}$$

Matrice de variences covarience des observations $\Sigma_{\hat{L}}$:

$$\Sigma_{\hat{L}} = \Sigma_{L} - \Sigma_{\hat{V}}$$