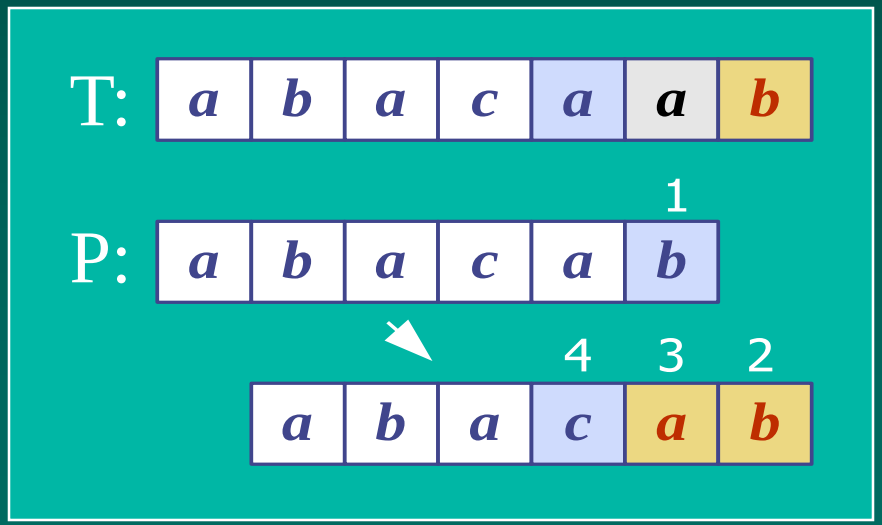
String / Pattern Matching Algorithms



Overview

1. What is Pattern Matching?
2. The Brute Force Algorithm
3. The Boyer-Moore Algorithm
4. The Knuth-Morris-Pratt Algorithm
5. More Information

1. What is Pattern Matching?

- Definition:
  - given a text string T and a pattern string P, find the pattern inside the text
    - T: "the rain in spain stays mainly on the plain"
    - P: "n th"
- Applications:
  - text editors, Web search engines (e.g. Google), image analysis

String Concepts

- Assume S is a string of size m.
- A *substring* S[i .. j] of S is the string fragment between indexes i and j.
- A *prefix* of S is a substring S[0 .. i]
- A *suffix* of S is a substring S[i .. m-1]
  - i is any index between 0 and m-1

Examples

S

- Substring S[1..3] == "ndr"
- All possible prefixes of S:
  - "andrew", "andre", "andr", "and", "an", "a"
- All possible suffixes of S:
  - "andrew", "ndrew", "drew", "rew", "ew", "w"

2. The Brute Force Algorithm

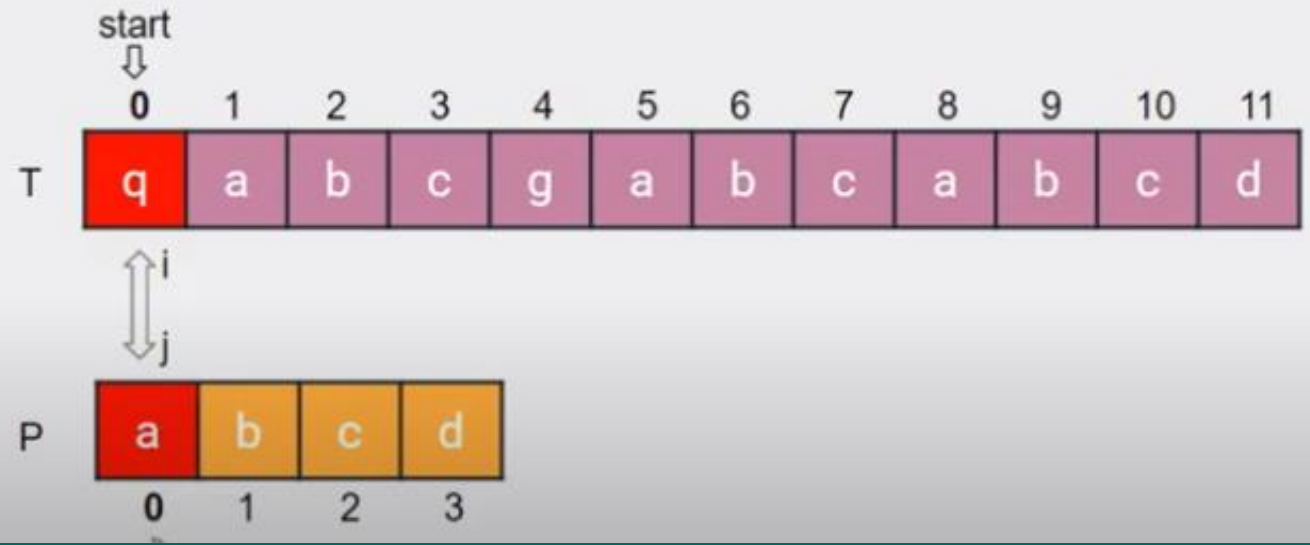
- Check each position in the text T to see if the pattern P starts in that position

P moves 1 char at a time through T

. . . .

Brute Force Approach (Naive)

Check every possible fit.



Brute Force Approach (Naive)

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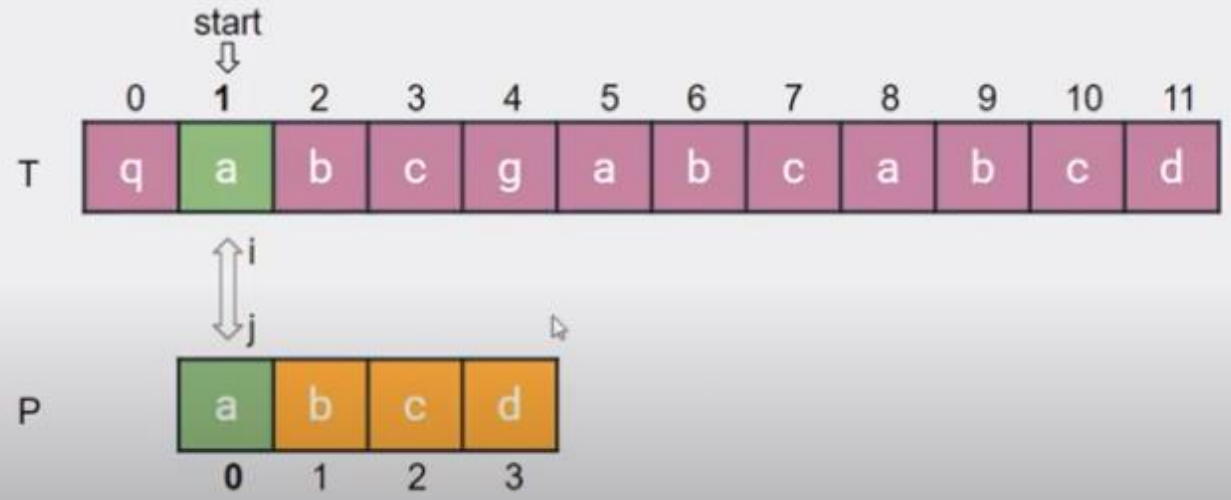

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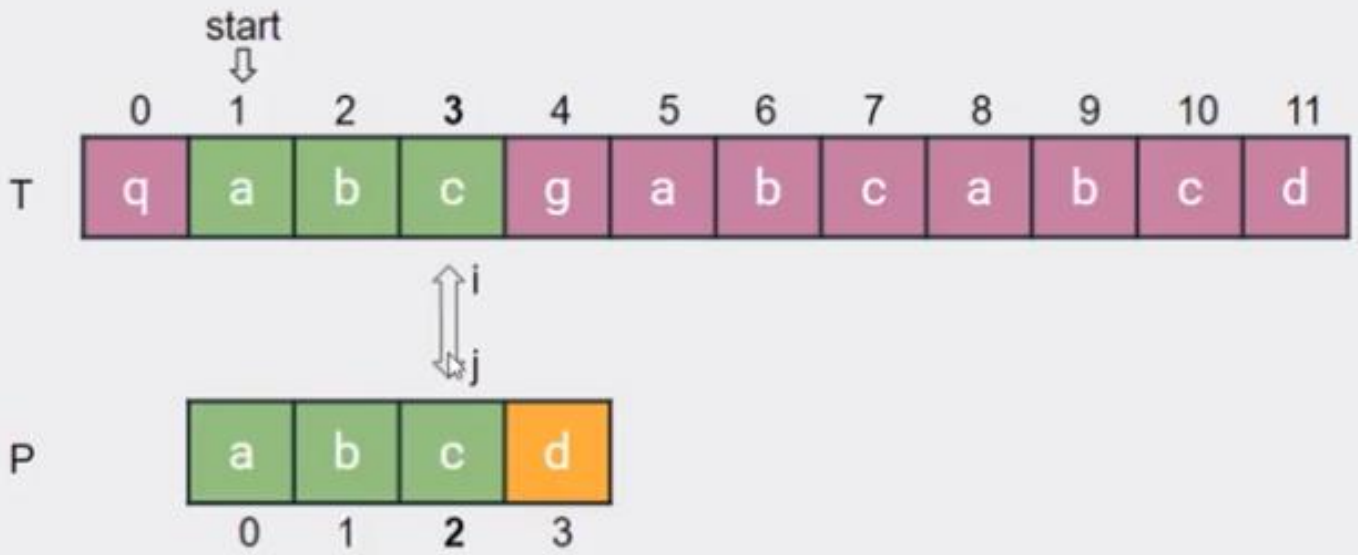

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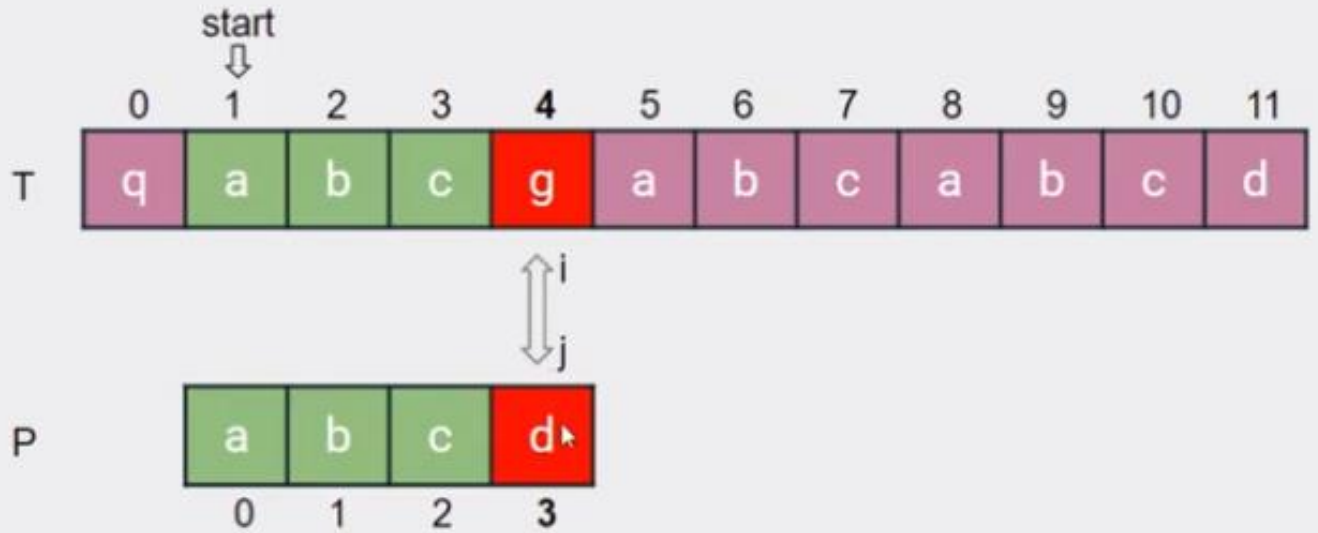

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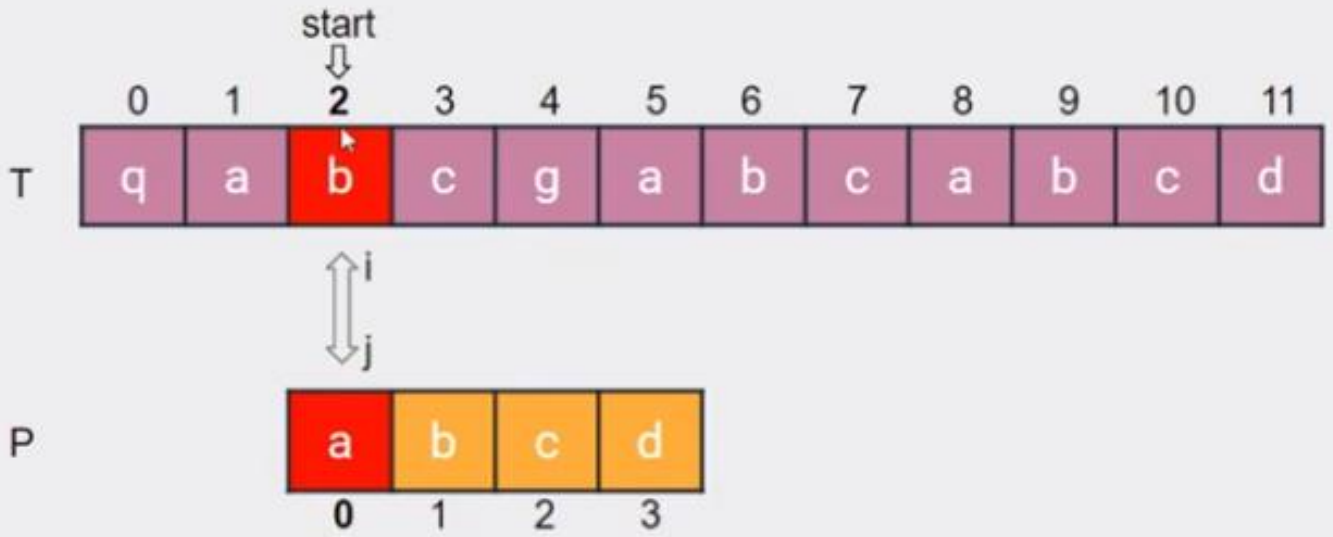
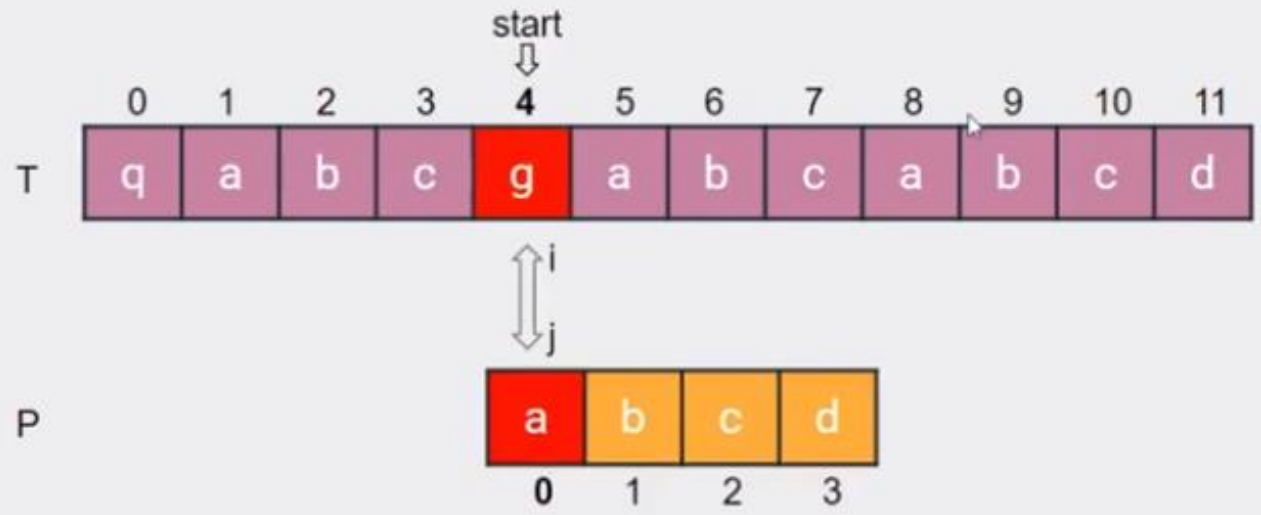


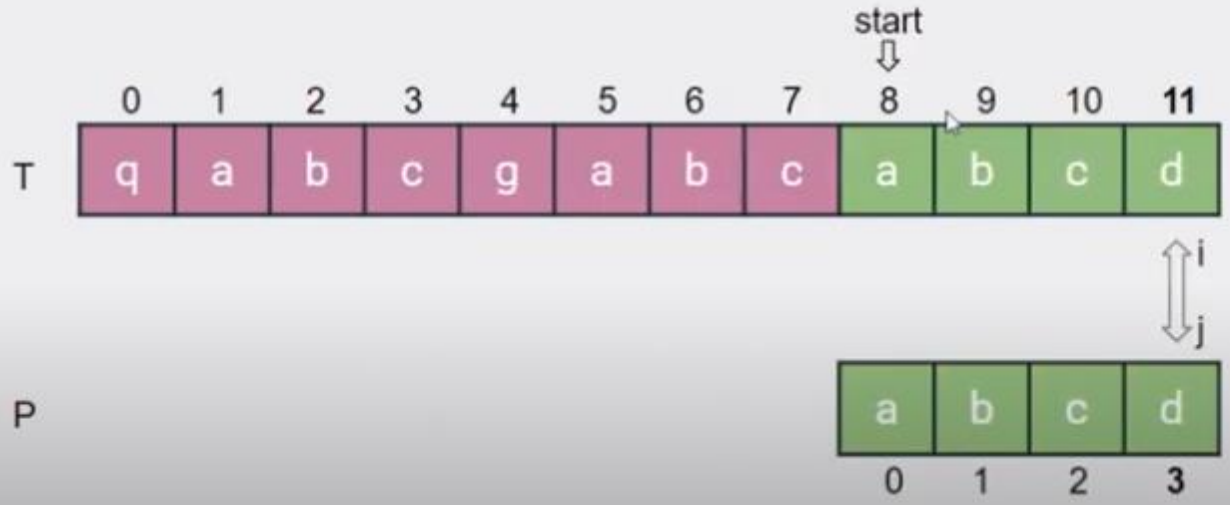
Brute Force Approach (Naive)

Check every possible fit.



Brute Force Approach (Naive)

Check every possible fit.



Return index where pattern starts, or -1

The Naive string-matching algorithm

The naive algorithm finds all valid shifts using a loop that checks the condition $P[1 . .m] = T[s + 1 . . s + m]$ *for each of the* $n - m + 1$ *possible values of* s.

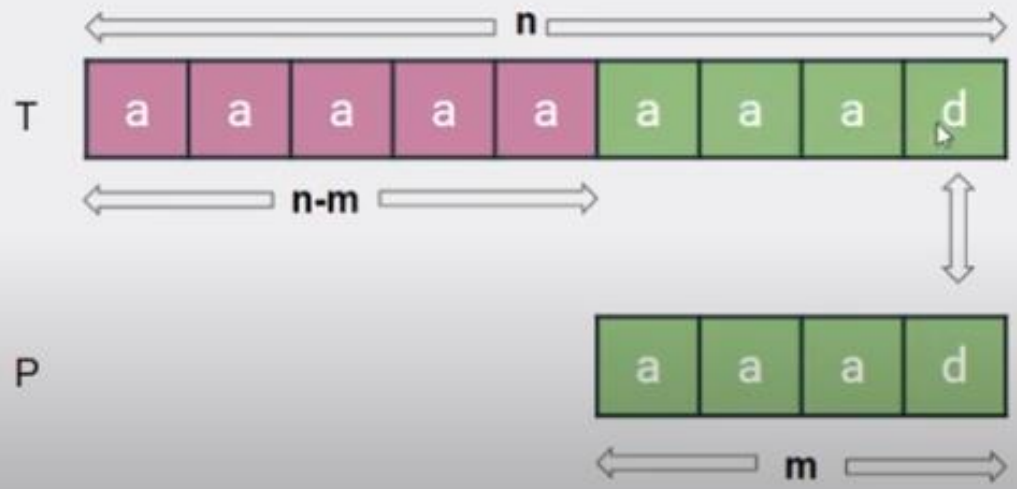
```
NAIVE-STRING-MATCHER(T, P)
1  n ← length[T]
2  m ← length[P]
3  for s ← 0 to n − m
4       do if P[1..m] = T[s + 1..s + m]
5            then print "Pattern occurs with shift" s
```

Procedure NAIVE-STRING-MATCHER takes time $O((n - m + 1)m)$, *and this* bound is tight in the worst case.

Time Complexity

Brute Force Approach (Naive)

Worst Case



Time Complexity

O([n-m+1]*m)

~ O(nm)

when n >>> m

- The brute force algorithm is fast when the alphabet of the text is large
  - e.g. A..Z, a..z, 1..9, etc.
- It is slower when the alphabet is small
  - e.g. 0, 1 (as in binary files, image files, etc.)

continued

- Example of a worst case:
  - T: "aaaaaaaaaaaaaaaaaaaaaaaaaaab"
  - P: "aaab"
- Example of a more average case:
  - T: "a string searching example is standard"
  - P: "store"

4. The KMP Algorithm

- The Knuth-Morris-Pratt (KMP) algorithm looks for the pattern in the text in a *left-to-right* order (like the brute force algorithm).

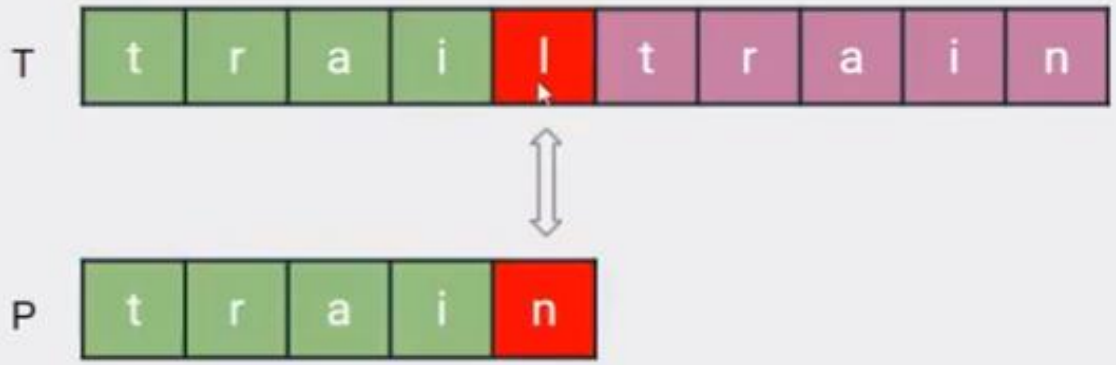
- But it shifts the pattern more intelligently than the brute force algorithm.

continued

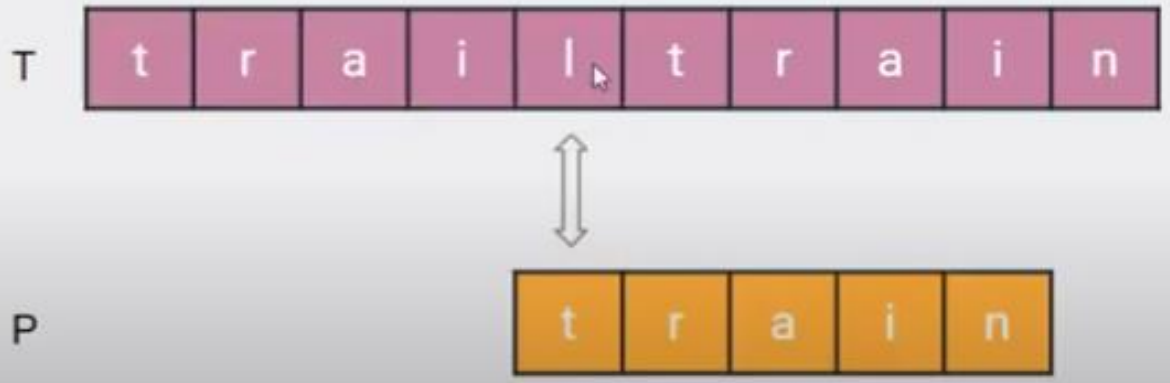
- If a mismatch occurs between the text and pattern P at P[j], what is the *most* we can shift the pattern to avoid wasteful comparisons?

- *Answer*: the largest prefix of P[0 .. j-1] that is a suffix of P[1 .. j-1]

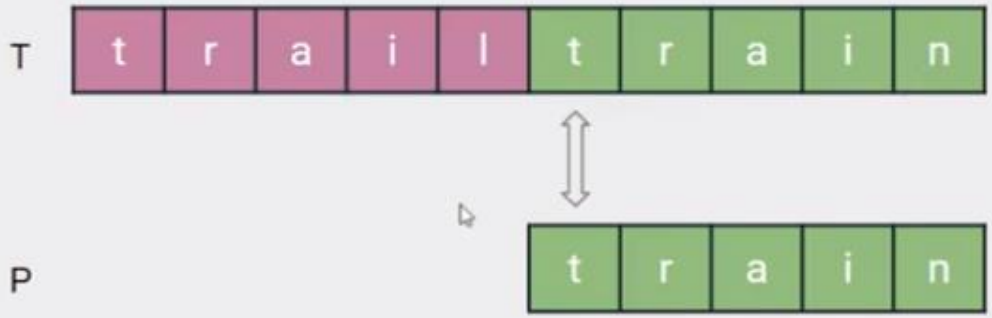
KMP algorithm



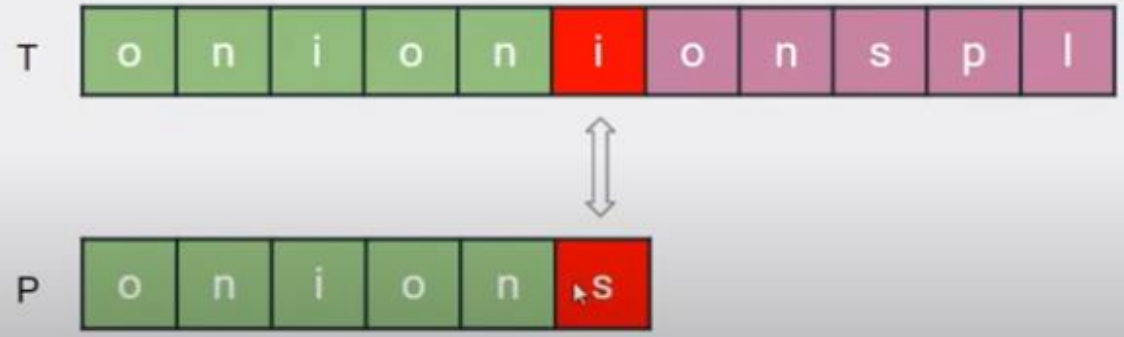
KMP algorithm



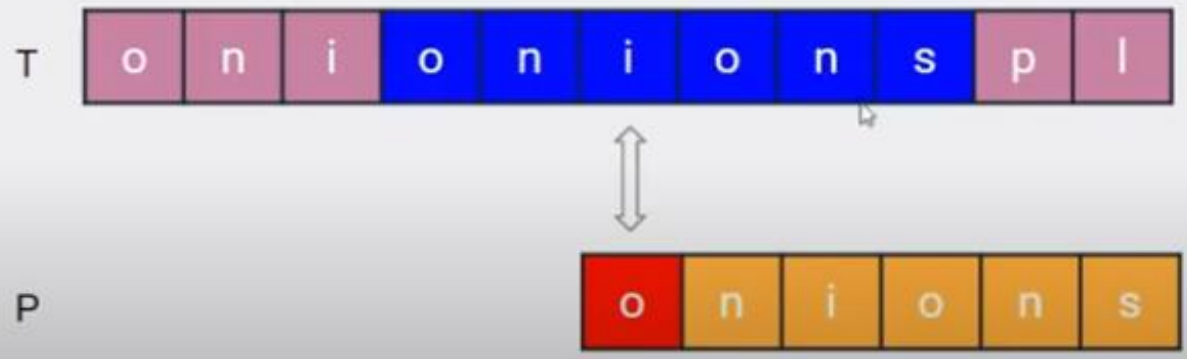
KMP algorithm



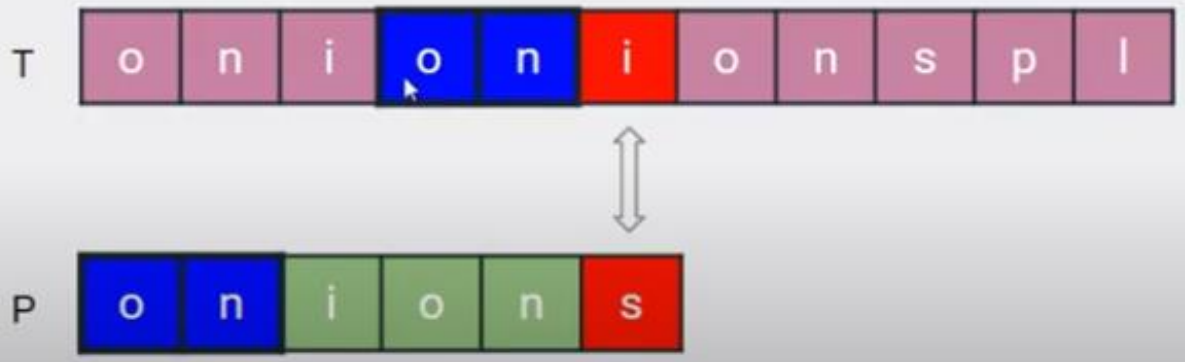
KMP algorithm



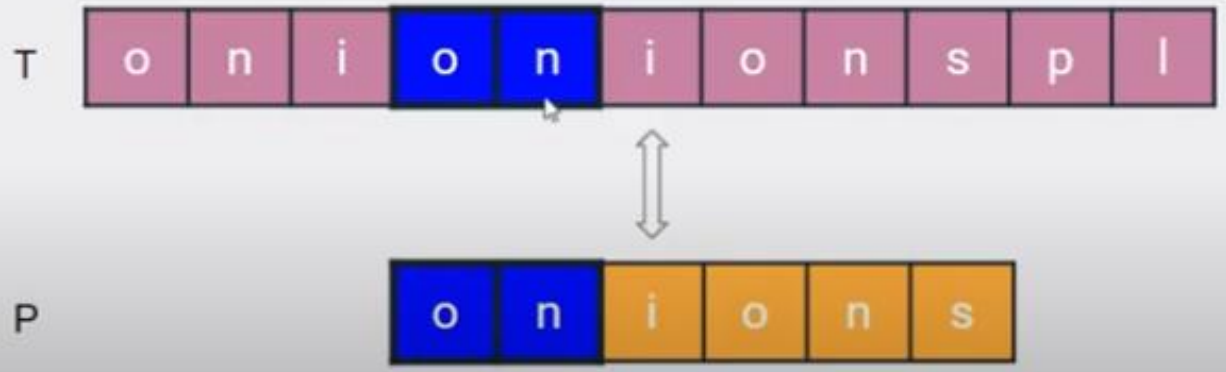
KMP algorithm



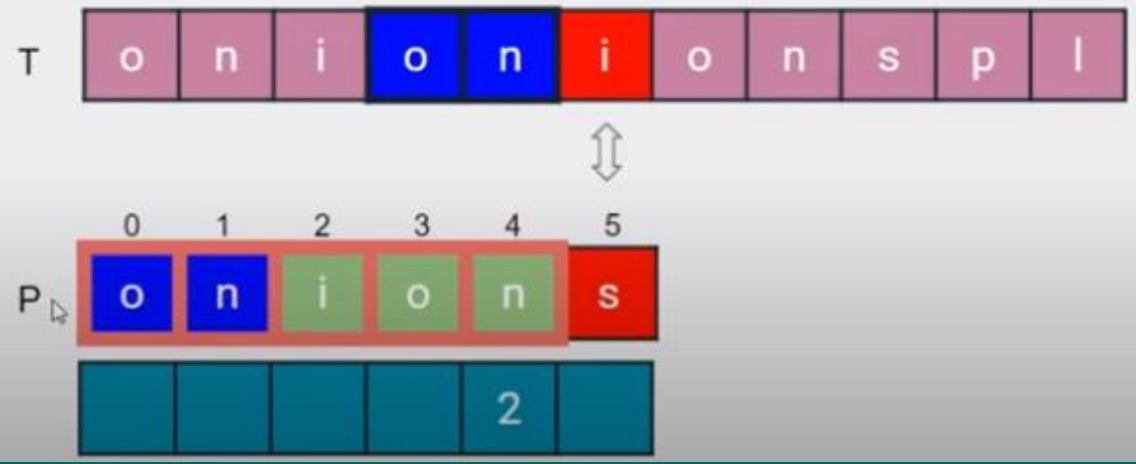
KMP algorithm



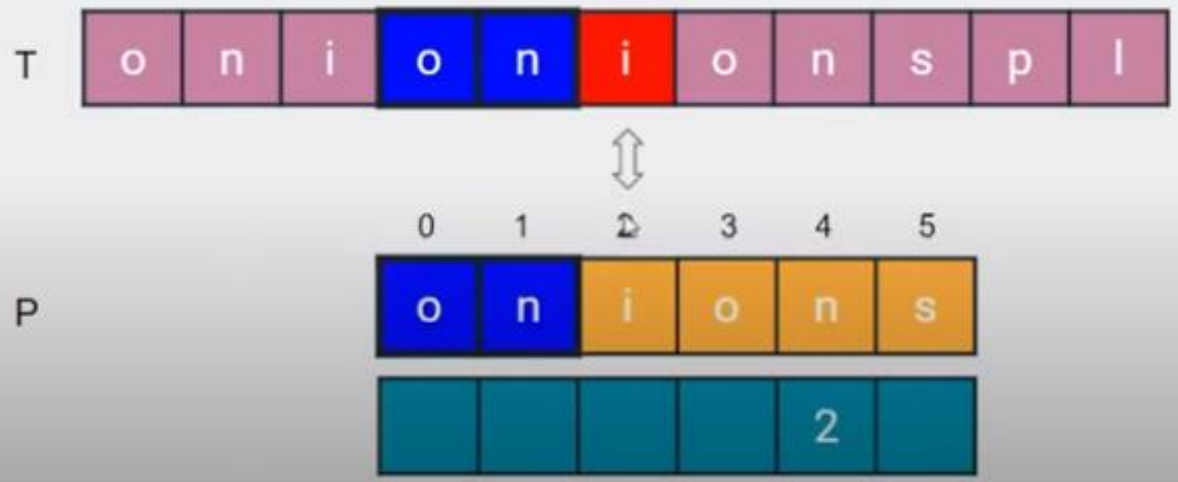
KMP algorithm



KMP algorithm

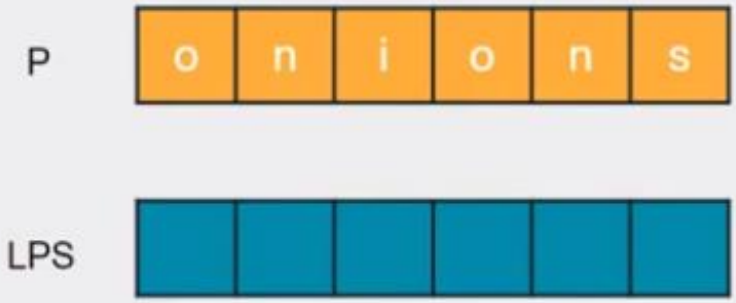


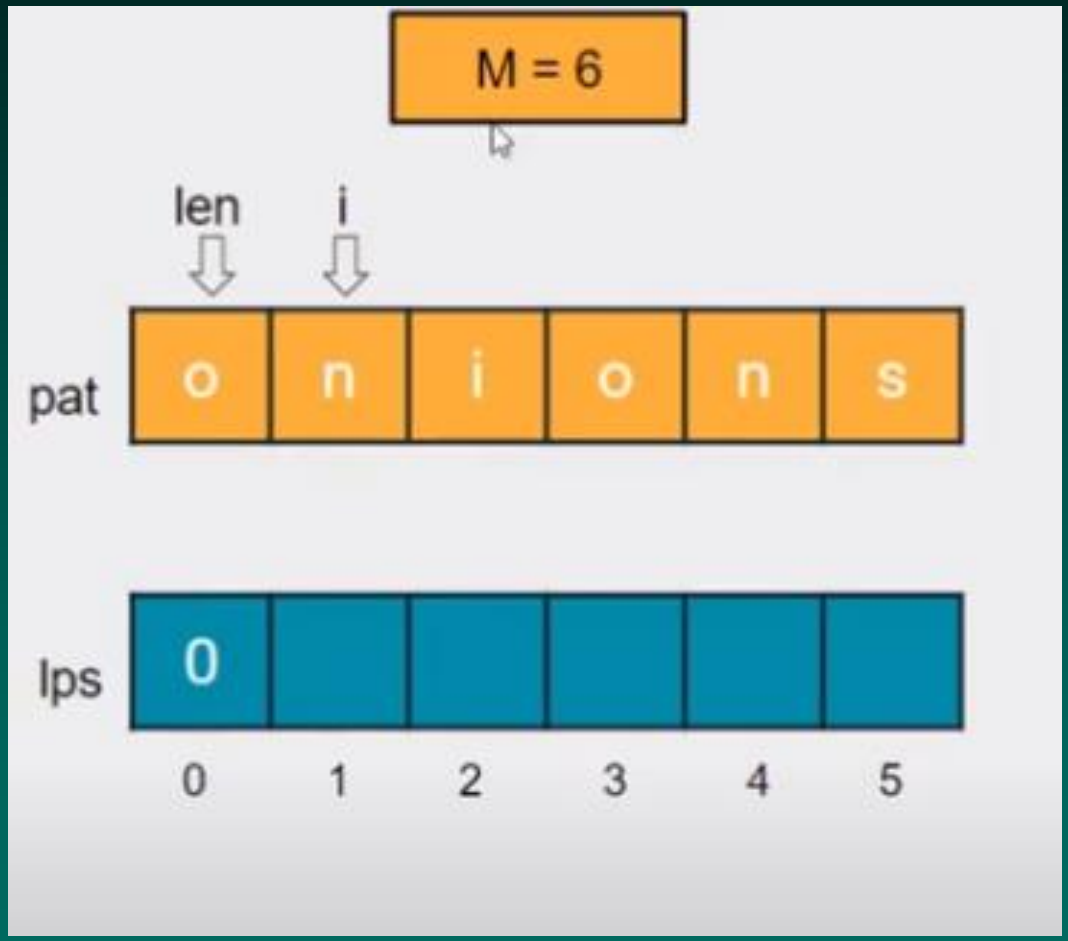
KMP algorithm



LPS array

Longest prefix that is also a suffix

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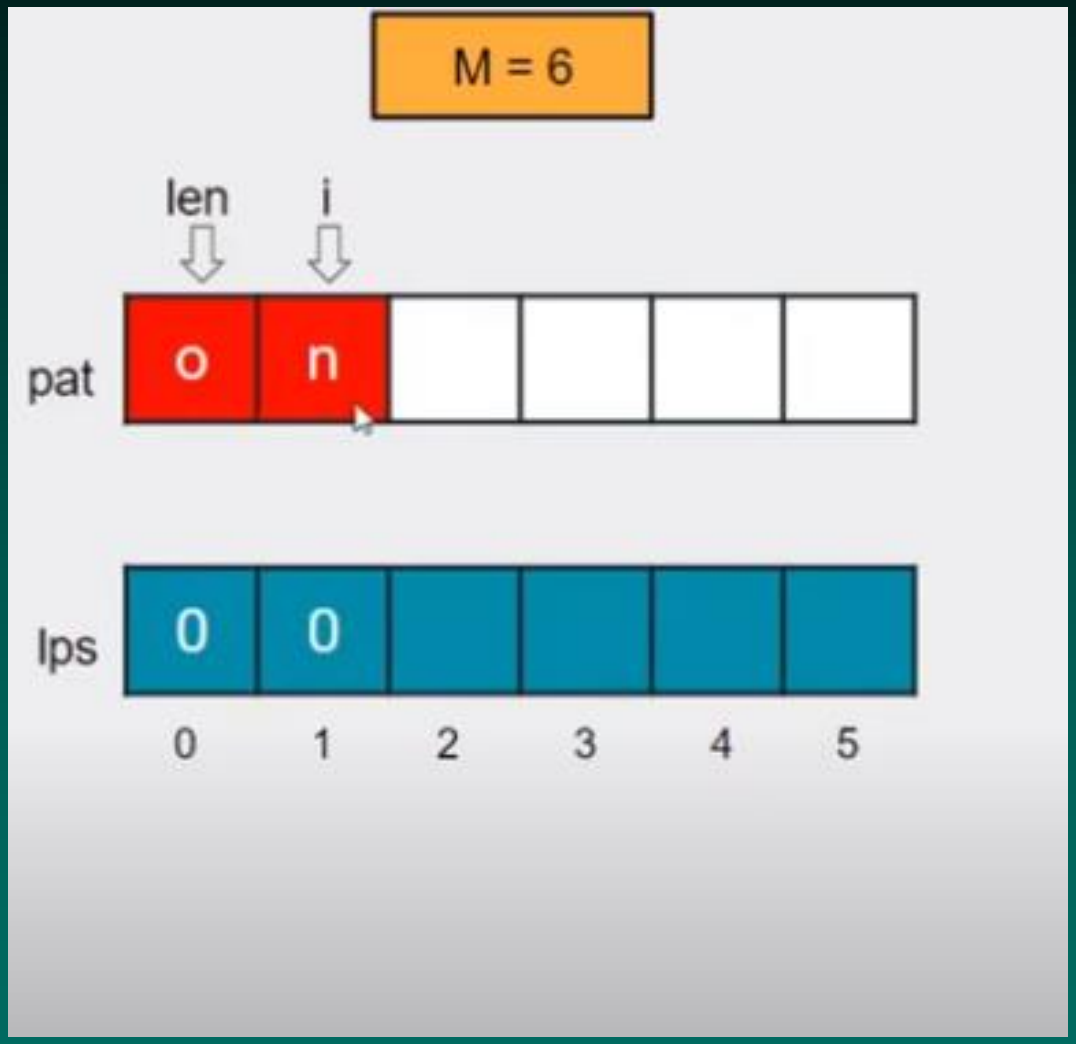
i
pat
o
n
i
o
n
s
lps
0
0
1
2
3
4
5


M = 6
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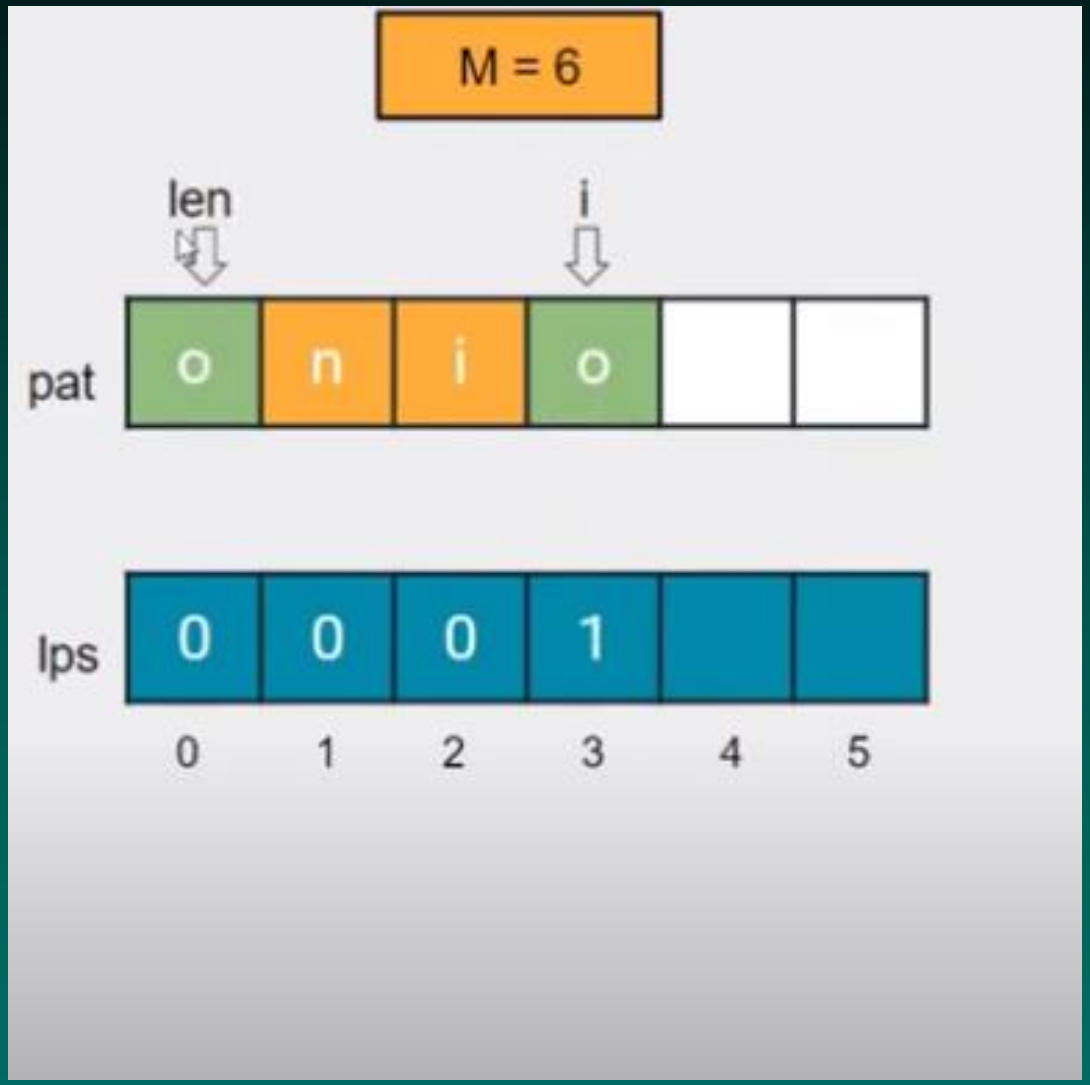
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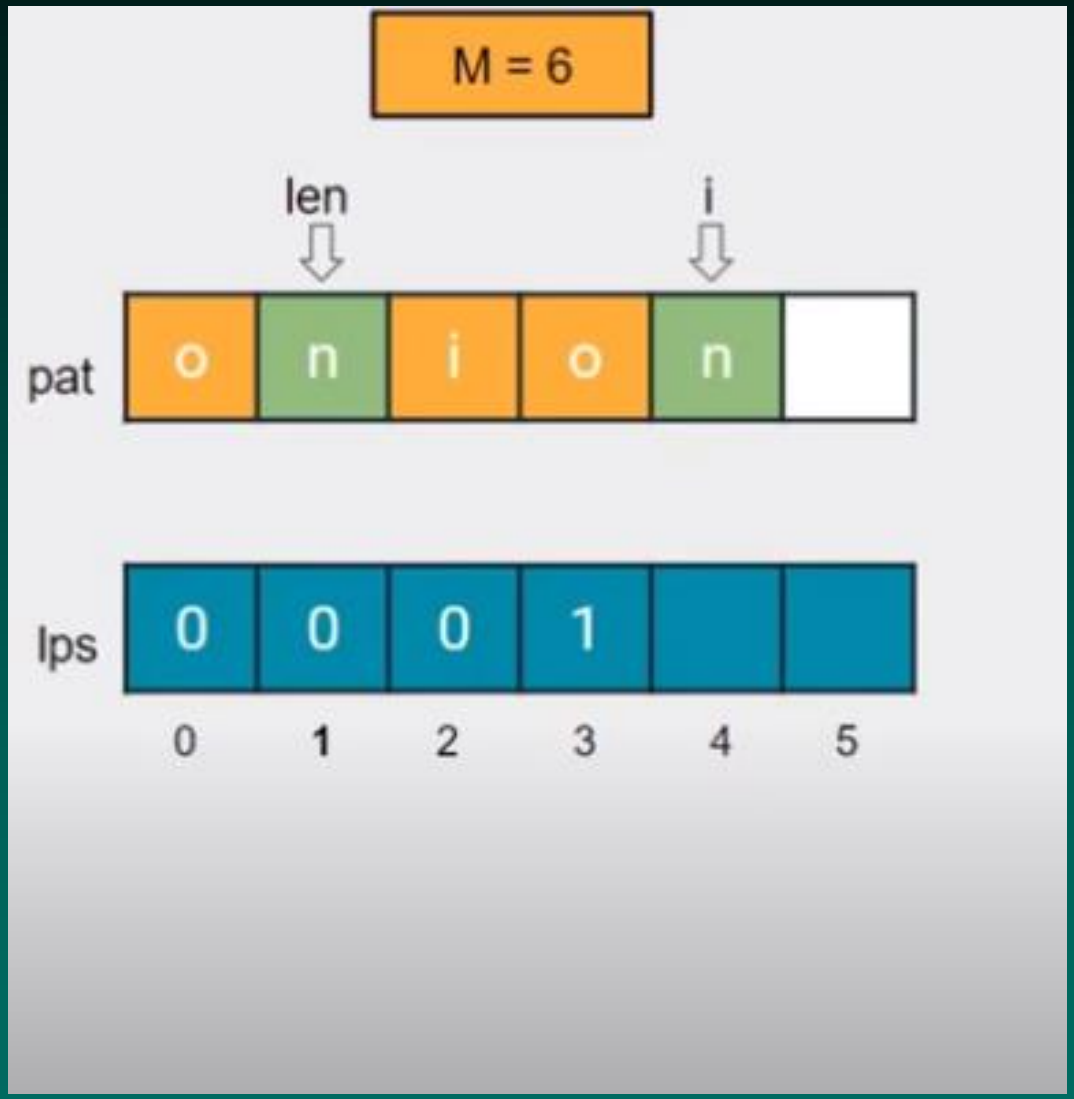
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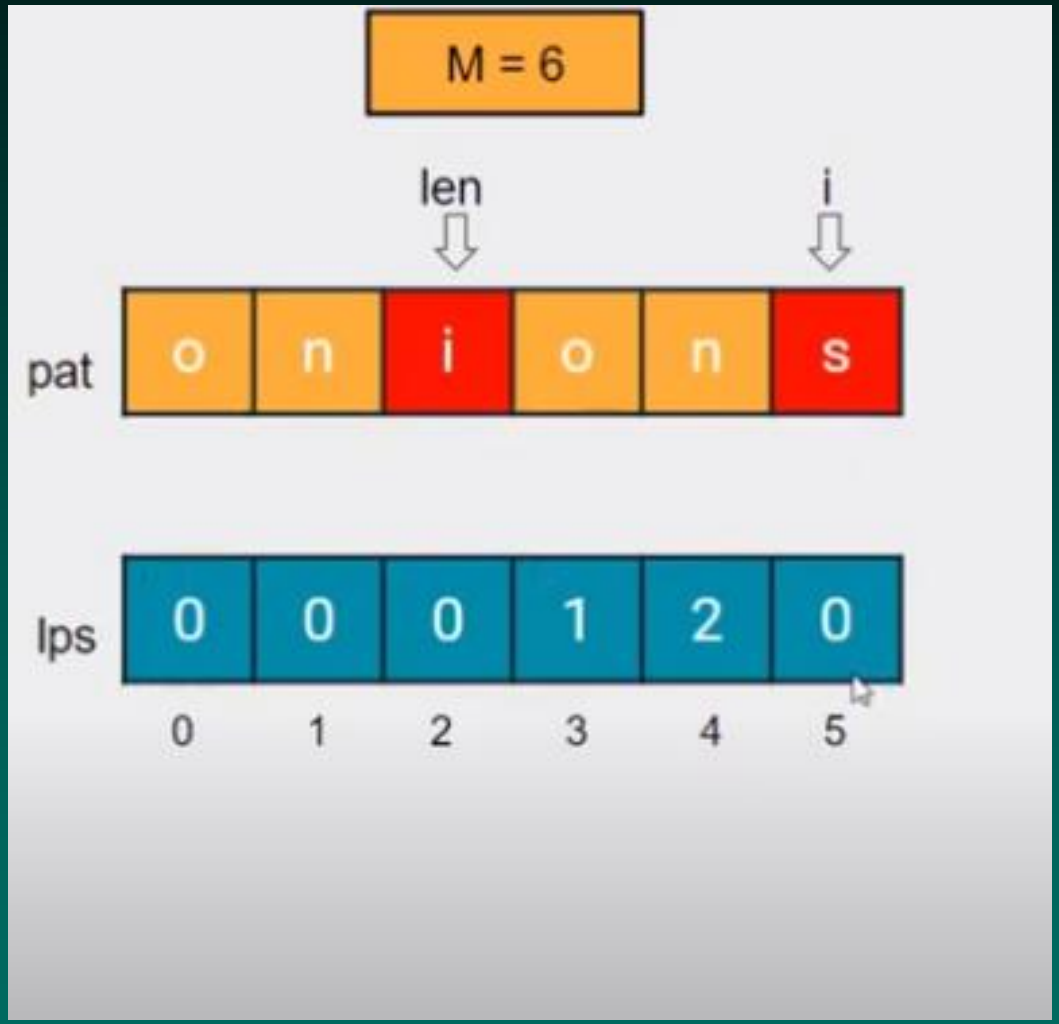
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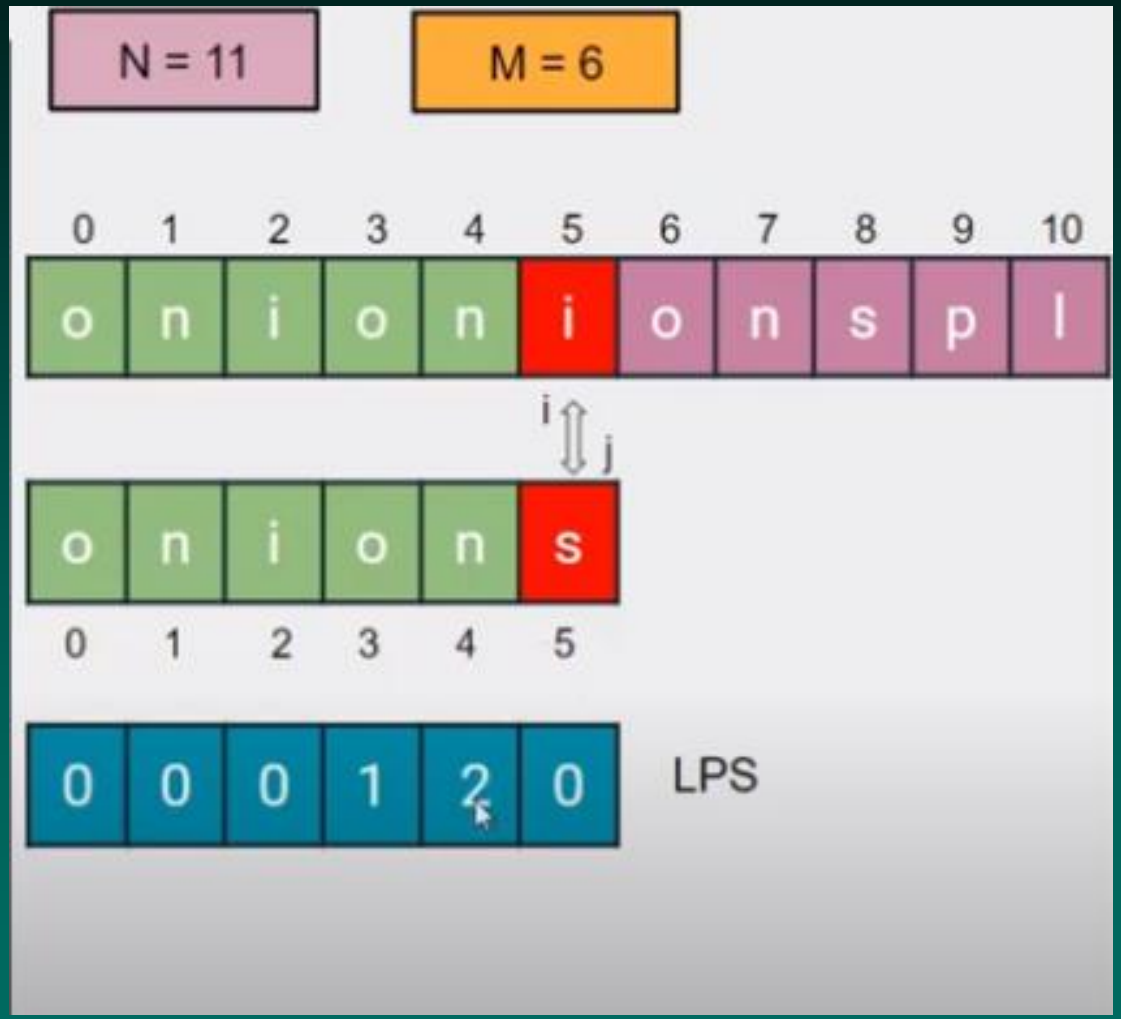
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M = 6
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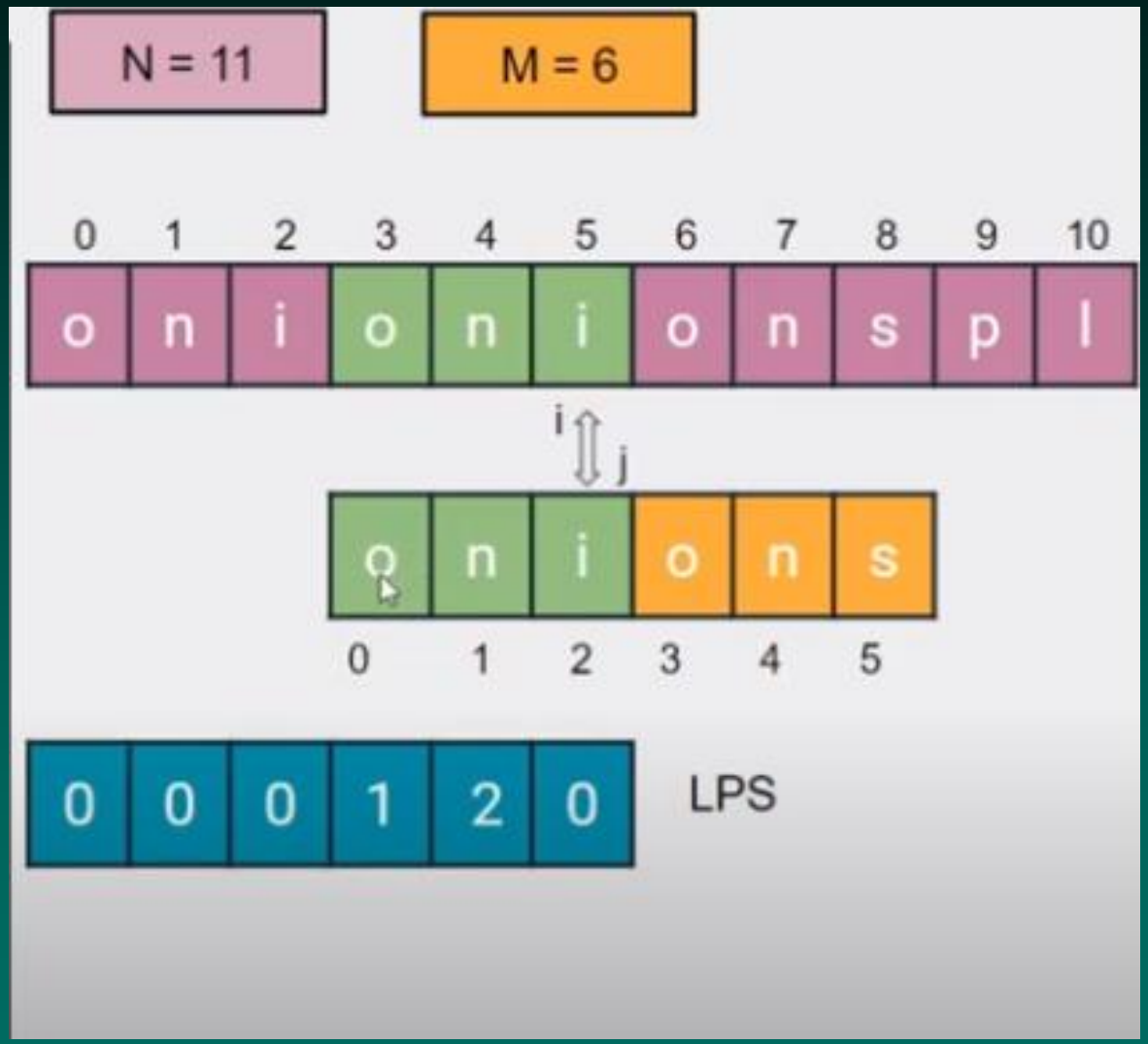
o n i o n i o n s p l
i
j
o n i o n s
0 1 2 3 4 5
0 0 0 1 2 0
LPS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M = 6
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o n i o n s
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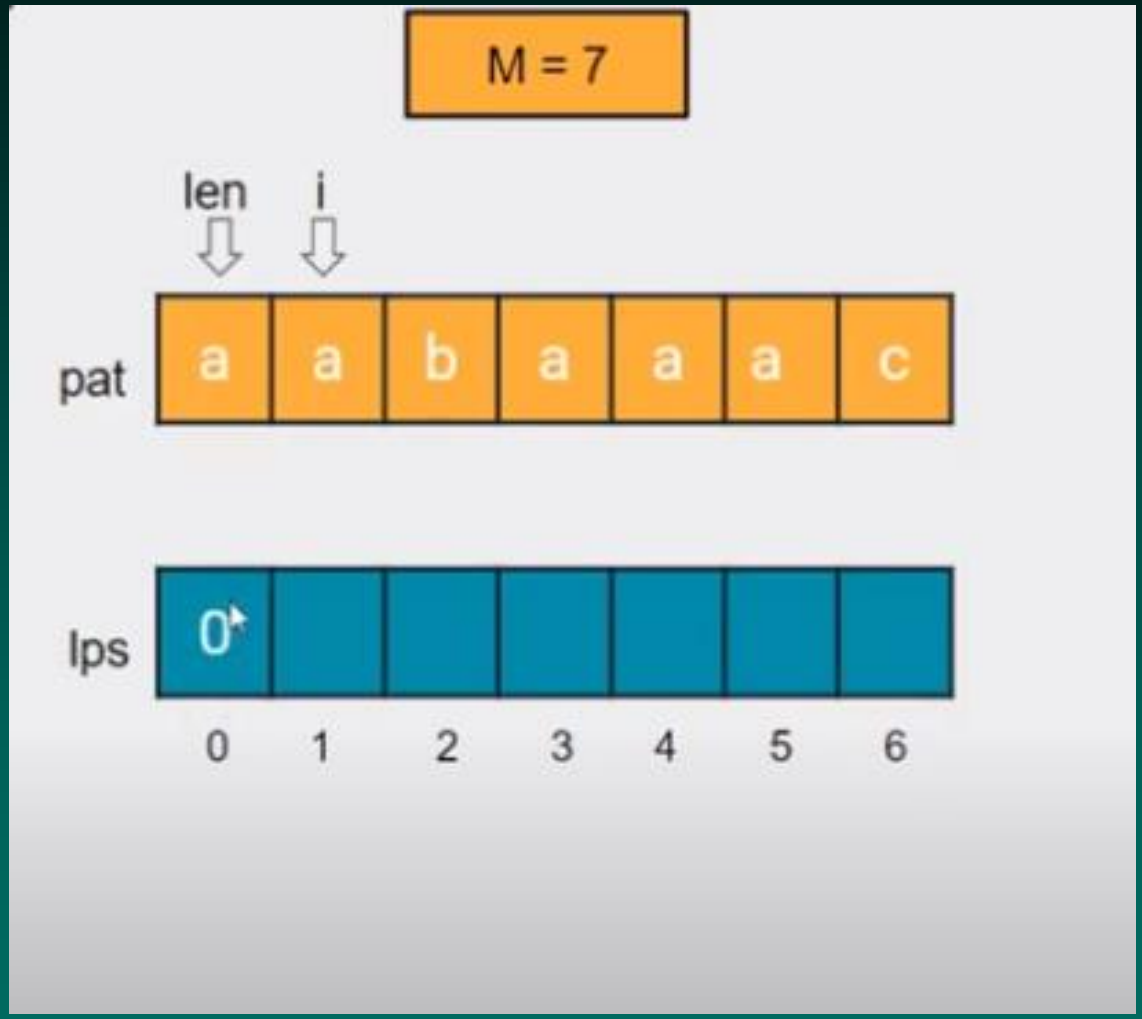
LPS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pat
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lps
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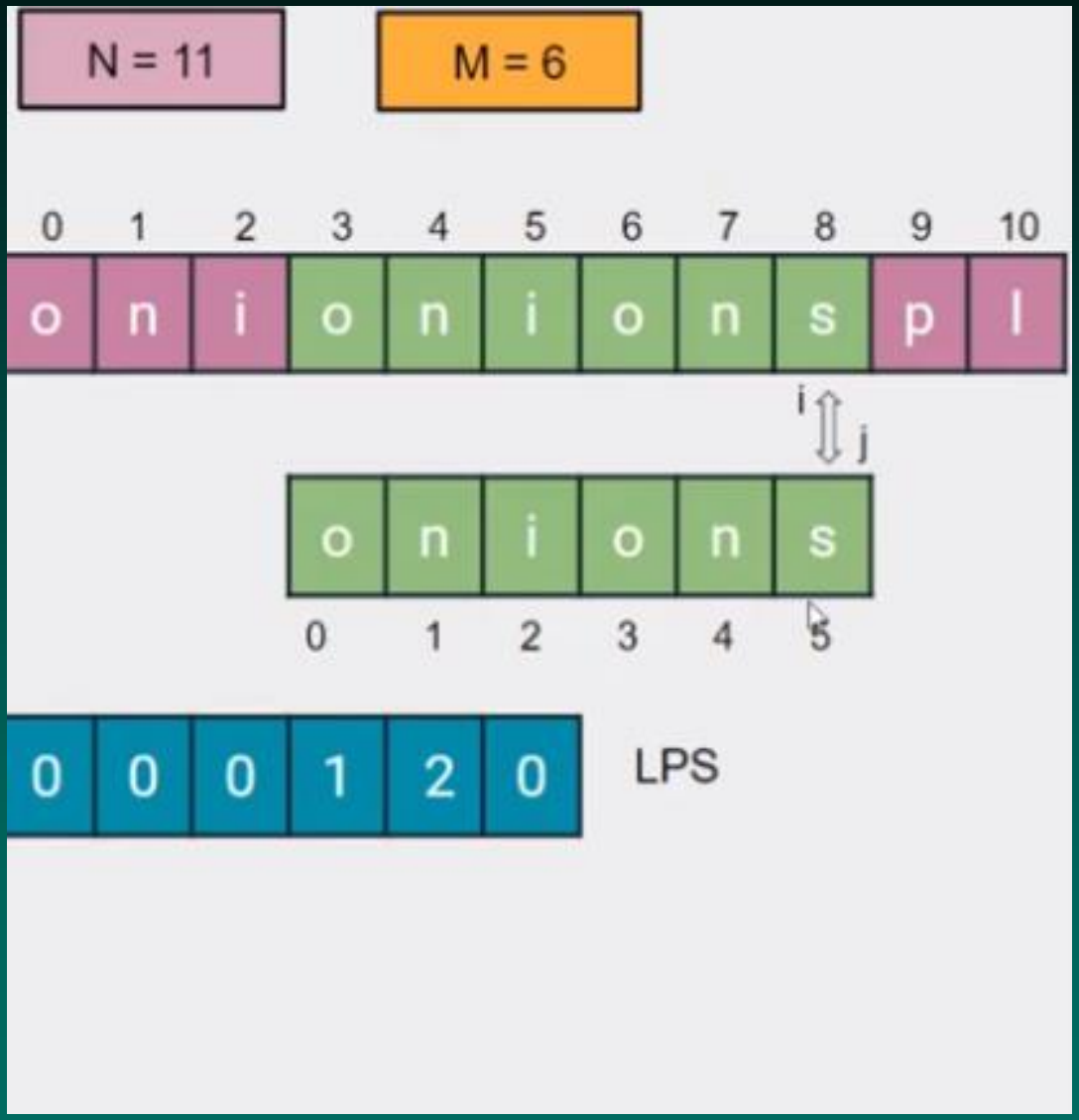
0 1 2 3 4 5 6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M = 6
0 1 2 3 4 5 6 7 8 9 10
o n i o n i o n s p l
i
j
o n i o n s
0 1 2 3 4 5
0 0 0 1 2 0
LPS

Time complexity

Overall -> O(n+m)

=> O(n) for n>>m

Space complexity

O(m) extra space for LPS array