National University of Computer & Emerging Sciences, Karachi
Fall-2018 Department of Computer Science
Final Exam
27th December 2017, 9:00 am– 12:00 noon

Course Code: CS302	**Course Name:** Design and Analysis of Algorithm
Instructor Name / Names: Dr. Muhammad Atif Tahir, Subhash Sagar and Zeshan Khan	
Student Roll No:	**Section:**

Instructions:

- Return the question paper.
- Read each question completely before answering it. There are **9 questions.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.

Time: 180 minutes.   **Max Marks**: 50 points

Question 1:   **(0.5*10=5 Points)**

Answer the following. You should explain in only 3-4 otherwise answer will not be checked.

(a) What is meant by Design and Analysis of Algorithms?
(b) List two topics in Computer Science that are more important than studying computer program performance.
(c) Write down the formal definition of Big-Oh Notation i.e. in terms of f(n) and g(n)
(d) List one condition / scenario where Master Theorem cannot be applied.
(e) What are the average running time complexities of Insertion and counting sort?
(f) List two algorithms that belong to Divide and conquer domain.
(g) List two algorithms that belong to Greedy Algorithm domain.
(h) List two algorithms that belong to Dynamic Programming domain.
(i) What is the complexity of Polynomial time algorithms?
(j) What are the limitations of Greedy Algorithms?

Question 2:   **(4 points)**

Recurrence Equation of Quick Sort is $T(N) = T(i) + T(N - i - 1) + cN$. For worst case, pivot point is the smallest element while for best case, pivot is the middle element. Show by using either substitution or recurrence / iterative method that the worst-case complexity is $O(n^2)$ and best-case complexity is $O(nlogn)$.

Question 3:   **(3+2=5 points)**

a) What is meant by P and NP Problems? Explain P = NP [3 Points]
b) A problem that is solvable in time complexity of $T(n) = 3 * n^n$ and space complexity of $S(n) = n^2$ and it can be validated in $T(n) = 2^n$ time. Is it a NP-Complete or NP-Hard?

Question 4:   **(3 points)**

For TSP graph, Proof by Contradiction that Weight of $MST \leq OPT$ where OPT denotes the cost of the minimum weight tour in TSP graph.

Question 5: (6 points)

Consider following points in 2D
(6,2), (9,5), (-2,2), (-3,4), (-8,8), (-10,4), (-10,3), (-8,-6), (-4,-4), (6,4), (6,-6), (-6,-10), (8,0)
Find the smallest convex set containing all the points using Package Wrap (Jarvis March) and Graham Scan (Show all iterations). Pseudo code of both algorithms is given in Appendix 1.
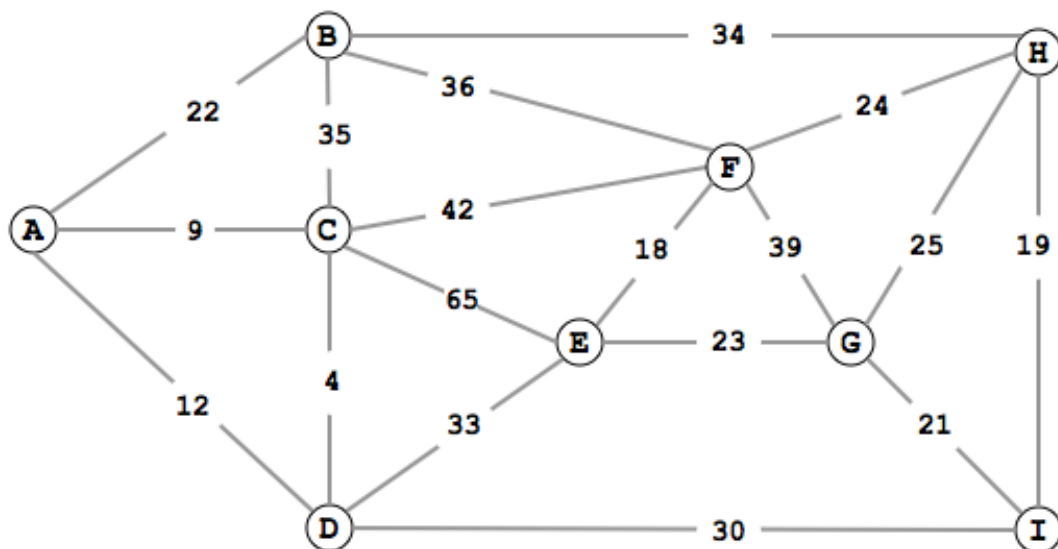
Question 5: (5 points)

Given a list of **n-1** integers and these integers are in the range of 1 to **n**. There are no duplicates in list. One of the integers is missing in the list. Design O(n) running time algorithm to find the missing integer.

Question 6: (2+3=5 points)

We are interested to send some of the students to visit different universities in the city and let them know about Procom (Fast University Event). There are "*n*" universities and *n* students are selected (1 for each university). The sample map of the universities is available below with the point "A" as the main campus of FAST where the students are waiting to start visit.
   A. Design a time optimal algorithm within $O(n^2)$ to compute the shortest path for all students from A to visit all universities.
   B. Apply your algorithm on the provided example graph in "**Figure: 1**" and write the shortest path for each of the student from A.

Figure: 1

Question 7: (6 points)

Suppose that we have a given set of five matrices A1, A2, A3, A4 and A5 with the following dimensions:

Matrix	A1	A2	A3	A4	A5
Dimension	5 X 2	2 X 3	3 X 1	1 X 4	4 X 3

Use the Matrix Chain Multiplication algorithm to discover the optimal parenthesization of the matrices with the cost of multiplication.

Question 8: (6 points)

Minimum Bottleneck Spanning Tree (MBST) in an undirected graph is a spanning tree in which the most weighted edge is as cheap as possible among all the possible combinations of spanning trees. A bottleneck edge is the highest weighted edge in a spanning tree. A spanning tree is minimum bottleneck spanning trees if the graph contains a spanning tree with a minimum bottleneck edge weight. More clearly, for a graph; list all the possible spanning trees. Among these pick the spanning tree whose maximum edge weight is minimum.

Prove or Disprove with example using graph? (Graph must be of at least 4 vertices).

1) Whether a MST is always a MBST?
2) A MBST is always a MST?

Question 9: (5 points)

For a graph given in **Figure: 2**, Solve the vertex cover problem using the following two methods.

1) 2 approximation greedy algorithm for vertex cover problem
2) Using the following algorithm
   "To solve the vertex-cover problem. Repeatedly select a vertex of highest degree, and remove all of its incident edges."

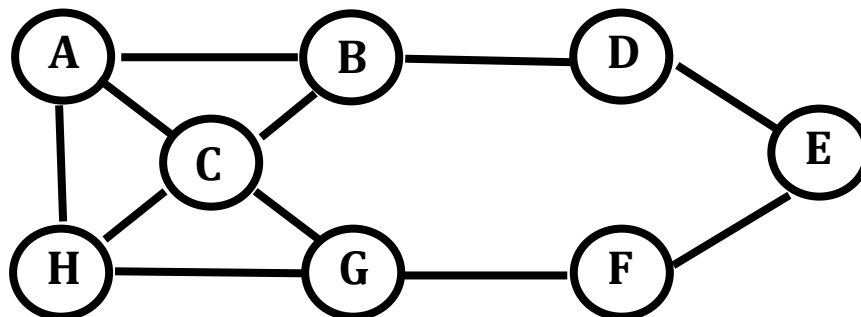For a given graph, do both the algorithms give the same result or not.

Figure: 2

BEST OF LUCK

Appendix 1:

Package wrap.
- Start with point with smallest y-coordinate.
- Rotate sweep line around current point in ccw direction.
- First point hit is on the hull.
- Repeat.

Graham scan.
- Choose point p with smallest y-coordinate.
- Sort points by polar angle with p to get simple polygon.
- Consider points in order, and discard those that would create a clockwise turn.