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FAST- National University of Computer & Emerging Sciences, Karachi.
Department of Computer Science
Quiz- II, fall 2018

Course Code: CS 211	Course Name: Discrete Structures
Instructors: Mr. Shoaib Raza	
Student Roll No:	Section:

Time Allowed: 50 minutes. Maximum Points: 25 points

Question # 1: (03 points)

Determine a relation on {a, b, c} that is reflexive and transitive, but not antisymmetric.

Solution: {(a, a), (b, b), (c, c), (a, b), (b, a)}.

Question # 2: (03 points)

Let g: Z x Z → Z be defined by g (m, n) = 6m + 3n. Is the function g an injection? Is the function g a surjection? Prove it.

Solution.
Let g: Z x Z → Z be defined by *g* (m, n) = 6m+3n. The function is not an injection since (0, 2) and (1, 0) both map to 6. The function is also not surjective. Note that 6m + 3n = 3(2m + n) and so all of the outputs will be multiples of 3. Thus 2 is not a possible output of g.

Question #3: (03 points)

Consider the following relation on the set of positive integers. R = {(x, y) | y divides x}
Prove or disprove that the above relation is partial order relation.
Solution:

- R is reflexive since x | x for every positive integer x, so (x, x) ∈ R for all x.
- R is antisymmetric since y | x and x | y imply that x = y if x and y are positive integers.
- R is transitive since y | x and z | y imply that z | x.

Hence the above relation is partial order relation.

Question # 4: (03 points)

Solve the linear congruence 54x ≡ 12 (mod 73).

Solution x= 57

Question # 5: (03 points)

Decrypt the message "AHFXVHFBGZ" that was encrypted using the shift cipher f(x) = (x + 19) mod 26.

Solution: HOMECOMING

Question # 6: (03 points)

Suppose that a computer has only the memory locations 0, 1, 2. . . 24. Use the hashing function h where h(x) = (x + 3) mod 25 to determine the memory locations in which 76, 132, and 26 are stored.

Solution: 76 on 4,132 on 10 and 26 on 5.

Question # 7: (03 points)

Express gcd (84, 18) as a linear combination of 18 and 84.

Solution: gcd(84,18): 6 = 18 · (−9) + 84 · 2.

Question # 8: (04 points)

An old man goes to market and a camel steps on her basket and crushes the oranges. The camel rider offers to pay for the damages and asks him how many oranges he had brought. He does not remember the exact number, but when he had taken them out five at a time, there were 3 oranges left. When he took them six at a time, there were also three oranges left, when he had taken them out seven at a time, there was only one orange was left and when he had taken them out eleven at a time, there was no orange left. What is the number of oranges he could have had?

Solution:

We will follow the notation used in the proof of the Chinese remainder theorem.

We have $m=m_1 * m_2 * m_3 * m_4 = 2310$.

Also, by simple inspection we see that:

$y_1 = 3$ is an inverse for $M_1 = 462$ modulo5,

$y_2 = 1$ is an inverse for $M_2 = 385$ modulo 6,

$y_3 = 1$ is an inverse for $M_3 = 330$ modulo 7 and

$Y_4 = 1$ is an inverse for $M_3 = 210$ modulo 11.

The solutions to the system are then all numbers x such that

$x = a_1M_1y_1 + a_2M_2y_2 + a_3M_3y_3 = (3 * 462 * 3) + (3* 385* 1) + (1* 330* 1) + (0*210*1) = 5643 \pmod{2310} = 1023$.

He could have 1023 oranges.

BEST OF LUCK!