COMP232 - Mathematics for Computer Science

Tutorial 3

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Table of Contents

Exercise 4

Express quantifications in English

Remark

$P(x, y)$: Student x has taken class y
Domain of x: all students
Domain of y: all computer science courses at your school

- **a)** $\exists x \exists y\ P(x, y)$
  Answer: Some students have taken some computer science courses
- **b)** $\exists x \forall y\ P(x, y)$
  Answer: Some students have taken all computer science courses
- **c)** $\forall x \exists y\ P(x, y)$
  Answer: All students have taken some computer science courses

Exercise 4 (cont...)

Express quantifications in English

Remark

$P(x, y)$: Student x has taken class y
Domain of x: all students
Domain of y: all computer science courses at your school

- **d)** $\exists y \forall x \ P(x, y)$
  Answer: Some computer science courses have been taken by all students
- **e)** $\forall y \exists x \ P(x, y)$
  Answer: All computer science courses have been taken by some students
- **f)** $\forall x \forall y \ P(x, y)$
  Answer: All students have taken all computer science courses

Exercise 7

Express sentences by simple English sentence

Remark

$T(x, y)$: Student x likes cuisine y
Domain of x: all students at your school
Domain of y: all cuisines

- **a)** $\neg T(AbdallahHussein, Japanese)$
  Answer: Abdallah Hussein does not like Japanese cuisine
- **b)** $\exists x \; T(x, Korean) \land \forall x T(x, Mexican)$
  Answer: Some students at our school like Korean cuisine and all students at our school like Mexican cuisine
- **c)** $\exists y \; (T(MoniqueArsenault, y) \lor T(JayJohnson, y))$
  Answer: There exists some cuisines which Monique or Jay like them

Exercise 7 (cont...)

Express sentences by simple English sentence

Remark

$T(x, y)$: Student x likes cuisine y
Domain of x: all students at your school
Domain of y: all cuisines

- **d)** $\forall x \forall y \exists z\ ((x \neq z) \rightarrow \neg(T(x, y) \wedge T(z, y))$
  Answer: There is no restaurant that you can find a pair of students which both of them like it.
- **e)** $\exists x \exists z \forall y\ (T(x, y) \leftrightarrow T(z, y))$
  Answer: There are some pair of students, which for all cuisines, both of them either like it or not
- **f)** $\forall x \forall z \exists y\ (T(x, y) \leftrightarrow T(z, y))$
  Answer: For every couple of students, there are some cuisines which both of them either like it or not

Exercise 9

Use quantifiers to express each of these statements

Remark

$L(x, y)$: x loves y
Domain of x and y: all people in the world

a) Everybody loves Jerry
Answer: $\forall x\ L(x, Jerry)$

b) Everybody loves somebody
Answer: $\forall x \exists y\ L(x, y)$

c) There is somebody whom everybody loves
Answer: $\exists y \forall x\ L(x, y)$

d) Nobody loves everybody
Answer: $\neg \exists x \forall y\ L(x, y) \equiv \forall x \exists y\ \neg L(x, y)$ *De Morgan's law*

e) There is somebody whom Lydia does not love
Answer: $\exists y\ \neg L(Lydia, y)$

Exercise 9 (cont...)

Use quantifiers to express each of these statements

Remark

$L(x, y)$: x loves y
Domain of x and y: all people in the world

f) There is somebody whom no one loves
Answer: $\exists y \neg \exists x\ L(x, y) \equiv \exists y \forall x\ \neg L(x, y)$ *De Morgan's law*

g) There is exactly one person whom everybody loves
Answer: $\exists y (\forall x\ L(x, y) \wedge (\forall z \forall w\ (L(w, z) \rightarrow z = y)))$

h) There are exactly two people whom Lynn loves
Answer: $\exists x \exists y\ (x \neq y \wedge L(Lynn, x) \wedge L(Lynn, y) \wedge \forall z\ (L(Lynn, z) \rightarrow (z = x \vee z = y)))$

i) Everyone loves himself or herself
Answer: $\forall x\ L(x, x)$

j) There is someone who loves no one besides himself or herself.
Answer: $\exists x \forall y\ (L(x, y) \leftrightarrow x = y)$

Exercise 11

Use quantifiers to express each of these statements

Remark

$S(x)$: x is a student
$F(x)$: x is a faculty member
$A(x, y)$: x has asked y a question
Domain of x and y: all people associated with your school

a) Lois has asked Professor Michaels a question
Answer: $A(Lois, ProfessorMichaels)$

b) Every student has asked Professor Gross a question
Answer: $\forall x\ (S(x) \rightarrow A(x, ProfessorGross))$

c) Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller
Answer: $\forall x\ (F(x) \rightarrow (A(x, ProfessorMiller) \oplus A(ProfessorMiller, x)))$

d) Some student has not asked any faculty member a question
Answer: $\exists x \forall y\ (S(x) \land (F(y) \rightarrow \neg A(x, y)))$

Exercise 11(cont...)

Use quantifiers to express each of these statements

Remark

$S(x)$: x is a student
$F(x)$: x is a faculty member
$A(x, y)$: x has asked y a question
Domain of x and y: all people associated with your school

e) There is a faculty member who has never been asked a question by student
Answer: $\exists x \forall y (F(x) \land (S(y) \to \neg A(y, x)))$

f) Some student has asked every faculty member a question
Answer: $\exists x \forall y \; (S(x) \land (F(y) \to A(x, y)))$

g) There is a faculty member who has asked every other faculty member a question Answer: $\exists x \forall y \; (F(x) \land ((F(y) \land x \neq y) \to A(x, y)))$

h) Some student has never been asked a question by a faculty member
Answer: $\exists x \forall y \; (S(x) \land (F(y) \to \neg A(y, x)))$

Exercise 20

Express each of the statements using predicates, quantifiers, logical connectives, and mathematical operators

Remark

Domain consists of all integers

a) The product of two negative integers is positive
Answer: $\forall x \forall y ((x < 0 \land y < 0) \rightarrow xy > 0)$

b) The average of two positive integers is positive
Answer: $\forall x \forall y ((x > 0 \land y > 0) \rightarrow \frac{x+y}{2} > 0)$

c) The difference of two negative integers is not necessarily negative
Answer: $\exists x \exists y ((x < 0 \land y < 0) \rightarrow \neg(x - y < 0))$

d) The absolute value of the sum of two integers does not exceed the sum of the absolute values of the integers
Answer: $\forall x \forall y (|x + y| \leq |x| + |y|)$

Exercise 32

Express the negations of each of these statements so that all negation symbols immediately precede predicates

a) $\exists z \forall y \forall x T(x, y, z)$

Answer:

$\neg \exists z \forall y \forall x T(x, y, z) \equiv \forall z \neg \forall y \forall x T(x, y, z) \equiv \forall z \exists y \neg \forall x T(x, y, z) \equiv \forall z \exists y \exists x \neg T(x, y, z)$

b) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

Answer:

$\neg(\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)) \equiv \neg \exists x \exists y P(x, y) \vee \neg \forall x \forall y Q(x, y) \equiv \forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)$

Exercise 32 (cont...)

Express the negations of each of these statements so that all negation symbols immediately precede predicates

c) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$

Answer:

$\neg \exists x \exists y (Q(x, y) \leftrightarrow Q(y, x)) \equiv \forall x \forall y \neg (Q(x, y) \leftrightarrow Q(y, x)) \equiv$
$\forall x \forall y (Q(x, y) \oplus Q(y, x))$

d) $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$

Answer:

$\neg \forall y \exists x \exists z (T(x, y, z) \vee Q(x, y)) \equiv \exists y \forall x \forall z \neg (T(x, y, z) \vee Q(x, y)) \equiv$
$\exists y \forall x \forall z (\neg T(x, y, z) \wedge \neg Q(x, y)) \equiv$

Exercise 34

Find a common domain for the variable x, y, z for which below statement is true and another domain for which it is false.

Statement

$$\forall x \forall y ((x \neq y) \rightarrow \forall z ((z = x) \vee (z = y)))$$

Answer:

- **TRUE**: On the domain $D = \{a, b\}$ the statement is true. If $x = a$ and $y = b$ (or $x = b, y = a$), any choice of z must be either equal to x or y.
- **FALSE**: With other domains more than two elements, the statement is false.
  Let $D = \{a, b, c\}$, and $x = a, y = b$. Then $z = c$, makes above statement truth value, FALSE.