National University of Computer & Emerging Sciences, Karachi

Fall-2018 CS-Department

CS211-Discrete Structures

Practice Assignment-II

Note:

1- This is hand written assignment.

2- Just write the question number instead of writing the whole question.

Submission date: Tuesday, 13th November, 2018 by 01 pm

1. Let R be the following relation defined on the set {a, b, c, d}:

   R = {(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)}.

   Determine whether R is:

   (a) Reflexive (b) Symmetric (c) Antisymmetric (d) Transitive

2. Let R be the following relation on the set of real numbers:

   $$aRb \leftrightarrow \lfloor a \rfloor = \lfloor b \rfloor, \text{ where } \lfloor x \rfloor \text{ is the floor of } x.$$

   Determine whether R is:

   (a) Reflexive (b) Symmetric (c) Antisymmetric (d) Transitive

3. Let $f(x) = \lfloor x^2/3 \rfloor$. Find $f(S)$ if

   a) $S = \{-2, -1, 0, 1, 2, 3\}$.

   b) $S = \{0, 1, 2, 3, 4, 5\}$.

   c) $S = \{1, 5, 7, 11\}$.

   d) $S = \{2, 6, 10, 14\}$.

4. Why is f not a function from $\mathbf{R}$ to $\mathbf{R}$ if

   a) $f(x) = 1/x$?

   b) $f(x) = \sqrt{x}$?

   c) $f(x) = \pm\sqrt{(x^2 + 1)}$?

5. Determine whether f is a function from $\mathbf{Z}$ to $\mathbf{R}$ if

   a) $f(n) = \pm n$.

   b) $f(n) = \sqrt{n^2 + 1}$.

   c) $f(n) = 1/(n^2 - 4)$.

6. Find these values.

   a) $\lceil \frac{3}{4} \rceil$

   b) $\lfloor \frac{7}{8} \rfloor$

   c) $\lceil -\frac{3}{4} \rceil$

   d) $\lfloor -\frac{7}{8} \rfloor$

   e) $\lceil 3 \rceil$

   f) $\lfloor -1 \rfloor$

   g) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$

   h) $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor$

7.

Determine whether each of these functions from $\{a, b, c, d\}$ to itself is one-to-one.

a) $f(a) = b,\ f(b) = a,\ f(c) = c,\ f(d) = d$
b) $f(a) = b,\ f(b) = b,\ f(c) = d,\ f(d) = c$
c) $f(a) = d,\ f(b) = b,\ f(c) = c,\ f(d) = d$

8.

Determine whether each of these functions is a bijection from $\mathbf{R}$ to $\mathbf{R}$.

a) $f(x) = 2x + 1$
b) $f(x) = x^2 + 1$
c) $f(x) = x^3$
d) $f(x) = (x^2 + 1)/(x^2 + 2)$

9.

Let $f: \mathbf{R} \to \mathbf{R}$ and let $f(x) > 0$ for all $x \in \mathbf{R}$. Show that $f(x)$ is strictly decreasing if and only if the function $g(x) = 1/f(x)$ is strictly increasing.

10.

Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from $\mathbf{R}$ to $\mathbf{R}$.

11.

Prove that if x is a real number, then $\lfloor -x \rfloor = -\lceil x \rceil$ and $\lceil -x \rceil = -\lfloor x \rfloor$.

12. For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
c) {(2, 4), (4, 2)}
d) {(1, 2), (2, 3), (3, 4)}
e) {(1, 1), (2, 2), (3, 3), (4, 4)}
f) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

13. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where (a, b) ∈ R if and only if

a) a is taller than b.
b) a and b were born on the same day.
c) a has the same first name as b.
d) a and b have a common grandparent.

14. Give an example of a relation on a set that is

a) both symmetric and antisymmetric.
b) neither symmetric nor antisymmetric.

15. Consider these relations on the set of real numbers:

R1 = {(a, b) ∈ R^2 | a > b}, the "greater than" relation,
R2 = {(a, b) ∈ R^2 | a ≥ b}, the "greater than or equal to "relation,
R3 = {(a, b) ∈ R^2 | a < b}, the "less than" relation,
R4 = {(a, b) ∈ R^2 | a ≤ b}, the "less than or equal to "relation,
R5 = {(a, b) ∈ R^2 | a = b}, the "equal to" relation,
R6 = {(a, b) ∈ R^2 | a ≠ b}, the "unequal to" relation.

i) Find:

a) $R2 \cup R4$. b) $R3 \cup R6$. c) $R3 \cap R6$. d) $R4 \cap R6$.
e) $R3 - R6$. f) $R6 - R3$. g) $R2 \oplus R6$. h) $R3 \oplus R5$.

ii) Find:

Find

a) $R2 \circ R1$. b) $R2 \circ R2$. c) $R3 \circ R5$. d) $R4 \circ R1$.
e) $R5 \circ R3$. f) $R3 \circ R6$. g) $R4 \circ R6$. h) $R6 \circ R6$.

16. What are the quotient and remainder when
a) 19 is divided by 7? b) −111 is divided by 11?
c) 789 is divided by 23? d) 1001 is divided by 13?
e) 0 is divided by 19? f) 3 is divided by 5?
g) −1 is divided by 3? h) 4 is divided by 1?

17. Let m be a positive integer. Show that $a \equiv b \pmod m$ if a mod m = b mod m.

18. Find a div m and a mod m when
a) a = −111, m = 99. b) a = −9999, m = 101.
c) a = 10299, m = 999. d) a = 123456, m = 1001.

19. Decide whether each of these integers is congruent to 5 modulo 17.
a) 80 b) 103 c) −29 d) −122

20. Determine whether the integers in each of these sets are pairwise relatively prime.
a) 11, 15, 19 b) 14, 15, 21 c) 12, 17, 31, 37 d) 7, 8, 9, 11

21. Find the prime factorization of each of these integers.
a) 88 b) 126 c) 729 d) 1001 e) 1111 f) 909,090

22. What are the GCD & LCM of these pairs of integers?
a) $37 \cdot 53 \cdot 73$, $211 \cdot 35 \cdot 59$ b) $11 \cdot 13 \cdot 17$, $29 \cdot 37 \cdot 55 \cdot 73$
c) 2331, 2317 d) $41 \cdot 43 \cdot 53$, $41 \cdot 43 \cdot 53$
e) $313 \cdot 517$, $212 \cdot 721$ f) 1111, 0

23. Use the extended Euclidean algorithm to express gcd (144, 89) and gcd (1001, 100001) as a linear combination.

24. Solve each of these congruences using the modular inverses.
a) $55x \equiv 34 \pmod{89}$ b) $89x \equiv 2 \pmod{232}$

25. Use the construction in the proof of the Chinese remainder theorem to find all solutions to the system of congruences.
a) $x \equiv 5 \pmod 6$, $x \equiv 3 \pmod{10}$, and $x \equiv 8 \pmod{15}$.
b) $x \equiv 7 \pmod 9$, $x \equiv 4 \pmod{12}$, and $x \equiv 16 \pmod{21}$.
c) $x \equiv 1 \pmod 2$, $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, and $x \equiv 4 \pmod{11}$.

26. Find an inverse of a modulo m for each of these pairs of relatively prime integers.
a) a = 2, m = 17 b) a = 34, m = 89
c) a = 144, m = 233 d) a = 200, m = 1001

27. Use Fermat's little theorem to compute 5^{2003} mod 7, 5^{2003} mod 11, and 5^{2003} mod 13.

28. Encrypt the message STOP POLLUTION by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters.
    a) f (p) = (p + 4) mod 26 b) f (p) = (p + 21) mod 26

29. Decrypt these messages encrypted using the shift cipher
    f (p) = (p + 10) mod 26.
    a) CEBBOXNOB XYG b) LO WI PBSOXN c) DSWO PYB PEX

30. What is the original message encrypted using the RSA system with n = 53 · 61 and e = 17 if the encrypted message is 3185 2038 2460 2550? (To decrypt, first find the decryption exponent d, which is the inverse of e = 17 modulo 52 · 60.)
31. Prove that for all integers a, b and c, if a|b and b|c then a|c.
32. Prove that for all integers a, b and c if a|b and a|c then a|(b+c)
33. Prove that the sum of any three consecutive integers is divisible by 3.
34. Prove the statement: There is an integer n > 5 such that 2n - 1 is prime.
35. Prove the statement: There are real numbers a and b such that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.
36. Prove or disprove that the product of any two irrational numbers is an irrational number.
37. Find a counter example to the proposition: For every prime number n, n + 2 is prime.
38. Prove by contradiction method, the statement: If n and m are odd integers, then n + m is an even integer.
39. Prove that the sum of any rational number and any irrational number is irrational.
40. Prove by contradiction that $6 - 7\sqrt{2}$ is irrational.
41. Prove by contradiction that $\sqrt{2} + \sqrt{3}$ is irrational.
42. Prove that for any integer a and any prime number p, if p | a, then $P \nmid (a + 1)$.
43. Show that the set of prime numbers is infinite.
44. Prove that if |x| > 1 then x > 1 or x < -1 for all x € R.
45. Prove the statement by contraposition: For all integers m and n, if m + n is even then m and n are both even or m and n are both odd.