COMP232 - Mathematics for Computer Science

Tutorial 4

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Exercise 4

What rule of inference is used in each of these arguments

- **a)** Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
  Answer: Simplification
- **b)** It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
  Answer: Disjunctive syllogism
- **c)** Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard
  Answer: Modus ponens

Exercise 4 cont...

What rule of inference is used in each of these arguments

- **d)** Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.
  Answer: Addition
- **e)** If I work all night on this homework, then I can answer all of the exercises. If I answer all of the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.
  Answer: Hypothetical syllogism

Exercise 10

For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

- **a)** If I play hockey, then I am sore the next day. I use the whirlpool if I am sore. I did not use the whirlpool.

  Predicates:

  $H(x)$: I play hokey on day x

  $S(x)$: I am sore on day x

  $W(x)$: I use whirlpool on day x

  Answer: Premises:

  $H(x-1) \to S(x)$

  $S(x) \to W(x)$

  $\neg W(x)$

  Conclusions:

  $S(x) \to W(x)$ and $\neg W(x)$ implies $\neg S(x)$

  $H(x-1) \to S(x)$ and $\neg S(x)$ implies $\neg H(x-1)$

  $H(x-1) \to S(x)$ and $S(x) \to W(x)$ implies $H(x-1) \to W(x)$

Exercise 10 cont...

For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

- **b)** If I work, it is either sunny or partly sunny. I worked last Monday or I worked last Friday. It was not sunny on Tuesday. It was not partly sunny on Friday

  Predicates: $W(x)$: I work on day x, $S(x)$: day x is sunny, $P(x)$: day x is partly sunny.

  Answer: Premises:

  $W(x) \to S(x) \vee P(x)$, $W(Monday) \vee W(Friday)$
  $\neg S(Tuesday)$, $\neg P(Friday)$

  Conclusions:

  $W(x) \to S(x) \vee P(x)$ and $W(Monday) \vee W(Friday)$ implies
  $S(Monday) \vee P(Monday) \vee S(Friday) \vee P(Friday)$
  $S(Monday) \vee P(Monday) \vee S(Friday) \vee P(Friday)$ and
  $\neg S(Tuesday)$ and $\neg P(Friday)$ implies
  $S(Monday) \vee P(Monday) \vee S(Friday)$

Exercise 10 cont...

For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the used rules of inference

- **c)** All insects have six legs. Dragonflies are insects. Spiders do not have six legs. Spiders eat dragonflies.

Predicates

$I(x)$: x is insect $S(x)$: x has six legs $SP(x)$: x is spider
$D(x)$: x is dragonfly $E(x, y)$: x eats y

Answer: Premises:

$\forall x\,(I(x) \to S(x))$, $\forall x\,(D(x) \to I(x))$, $\forall x\,(SP(x) \to \neg S(x))$
$\forall x, y\,(SP(x) \wedge D(y) \to E(x, y))$

Conclusions:

$\forall x\,(D(x) \to I(x))$ implies $D(a) \to I(a)$ for any a
$\forall x\,(I(x) \to S(x))$ implies $I(a) \to S(a)$ for any a
$D(a) \to I(a)$ and $I(a) \to S(a)$ implies $D(a) \to S(a)$ for any a
$D(a) \to S(a)$ for any a implies $\forall x\,(D(x) \to S(x))$

Exercise 11

Show that the argument form with premises $p_1, p_2, \ldots, p_n$ and conclusion $q \to r$ is valid if the argument form with premises $p_1, p_2, \ldots, p_n, q$ and conclusion r is valid.

Answer: It is sufficient to see that if $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \to (q \to r)$ is false then $(p_1 \wedge p_2 \wedge \cdots \wedge p_n \wedge q) \to r$ is false.

Let $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \to (q \to r)$ be false, then $p_1 \wedge p_2 \wedge \cdots \wedge p_n$ is true and $q \to r$ is false.

Moreover, for $q \to r$ to be false, q has to be true and r false.

So, $p_1, p_2, \ldots, p_n$ and q are true, and r is false.

Consequently, $(p_1 \wedge p_2 \wedge \cdots \wedge p_n \wedge q) \to r$ is false.

Exercise 12

Show that the argument form with premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\neg s$ and conclusion $q \rightarrow r$ is valid by first using exercise 11 and then using rules of inferences from table 1.

Answer:

using exercise 11 we have: the argument form with premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\neg s$ and q and conclusion r.

from q and $q \rightarrow (u \wedge t)$ we get $u \wedge t$

from $u \wedge t$ we get u and t

from u and $u \rightarrow p$ we get p

from t and p we get $p \wedge t$

from $p \wedge t$ and $(p \wedge t) \rightarrow (r \vee s)$ we get $r \vee s$

from $r \vee s$ and $\neg s$ we get r, which is what we expected.

Exercise 16

For each of these arguments determine whether the argument is correct or incorrect and explain why.

a) Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.
Answer: Valid

b) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.
Answer: Invalid

c) Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie
Answer: Invalid

d) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.
Answer: Valid

Exercise 20

Determine whether these are valid arguments.

a) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.
   Answer: Invalid

b) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$; then $a \neq 0$.
   Answer: Valid

Exercise 24

Identify the error or errors in this argument that supposedly shows that if $\forall x\,(P(x) \vee Q(x))$ is true then $\forall x\, P(x) \vee \forall x\, Q(x)$ is true.

1) $\forall x\,(P(x) \vee Q(x))$ Premise
   Validity: Valid
2) $(P(c) \vee Q(c))$ Universal instantiation from 1
   Validity: Valid
3) $P(c)$ Simplification from 2
   Validity: Invalid
4) $\forall x\, P(x)$ Universal generalization from 3
   Validity: Valid
5) $Q(c)$ Simplification from 2
   Validity: Invalid
6) $\forall x\, Q(x)$ Universal generalization from 5
   Validity: Valid
7) $\forall x\, P(x) \vee \forall x\, Q(x)$ Conjunction from of 4 and 6
   Validity: Invalid (Error)

Exercise 25

Justify the rule of **universal modus tollens** by showing that the premises $\forall x(P(x) \to Q(x))$ and $\neg Q(a)$ for a particular element a in the domain, imply $\neg P(a)$.

Remark

- $\forall x(P(x) \to Q(x))$
- $\neg Q(a)$

Answer:

$$\begin{array}{ll} \quad \forall x(P(x) \to Q(x)) & \text{Universal Instantiation} \\ \hline \therefore P(a) \to Q(a) & \\ \quad \neg Q(a) & \\ \hline \therefore \neg P(a) & \text{Modus tollens} \end{array}$$

Exercise 26

Justify the rule of **universal transitivity** which states that if $\forall x\ (P(x) \to Q(x))$ and $\forall x\ (Q(x) \to R(x))$ are true, the $\forall x\ (P(x) \to R(x))$ is true, where the domains of all quantifiers are the same.

Answer:

$$\frac{\forall P(x) \to Q(x)}{\therefore P(a) \to Q(a)}$$

Universal Instantiation

$$\frac{\forall Q(x) \to R(x)}{\therefore Q(a) \to R(a)}$$

Universal Instantiation

Exercise 26 (cont...)

Justify the rule of **universal transitivity** which states that if $\forall x\ (P(x) \to Q(x))$ and $\forall x\ (Q(x) \to R(x))$ are true, the $\forall x\ (P(x) \to R(x))$ is true, where the domains of all quantifiers are the same.

Answer (cont...):

1. $P(a) \to Q(a)$
2. $Q(a) \to R(a)$

$$\begin{array}{ll}
P(a) \to Q(a) & \\
\underline{Q(a) \to R(a)} & \\
\therefore \underline{P(a) \to R(a)} & \text{Hypothetical syllogism} \\
\therefore \forall x(P(x) \to R(x)) & \text{Universal generalization}
\end{array}$$

Exercise 29

Use rules of inference to show that if $\forall x\ (P(x) \vee Q(x))$, $\forall x\ (\neg Q(x) \vee S(x))$, $\forall x\ (R(x) \to \neg S(x))$, and $\exists x\ \neg P(x)$ are true, then $\exists x\ \neg R(x)$ is true.

Answer:

1) $\exists x\ \neg P(x)$ Premise
2) $\neg P(c)$ Existential instantiation using (1)
3) $\forall x\ (P(x) \vee Q(x))$ Premise
4) $P(c) \vee Q(c)$ Universal instantiation using (3)
5) $Q(c)$ Disjunctive syllogism using (4) and (2)
6) $\forall x\ (\neg Q(x) \vee S(x))$ Premise
7) $\neg Q(c) \vee S(c)$ Universal instantiation using (6)
8) $S(c)$ Disjunctive syllogism using (5) and (7)
9) $\forall x\ (R(x) \to \neg S(x))$ Premise
10) $R(c) \to \neg S(c)$ Universal instantiation using (9)
11) $\neg R(c)$ Modus tollens using (8) and (10)
12) $\exists x\ \neg R(x)$ Existential generalization using (11)

Exercise 32

Show that the equivalence $p \wedge \neg p \equiv F$ can be derived using resolution together with the fact that a conditional statement with a false hypothesis is true. [Hint: Let $q = r = F$ in resolution.]

Answer:

Resolution: $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

Let $q = r = F$, then:

$$((p \vee F) \wedge (\neg p \vee F)) \rightarrow (F \vee F)$$

Hence,

$$(p \wedge \neg p) \rightarrow F$$

To make above conditional statement *true*, hypothesis should be *false*,

$$p \wedge \neg p \equiv F$$

Exercise 33

Use resolution to show that the compound proposition $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$ is not satisfiable
Answer:
Resolution: $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
Let $r = q$, hence $((p \vee q) \wedge (\neg p \vee q)) \rightarrow (q \vee q)$, and

$$((p \vee q) \wedge (\neg p \vee q)) \rightarrow q$$

If hypothesis is TRUE, then $q =$ TRUE.Therefor, either $(p \vee \neg q)$ or $(\neg p \vee \neg q)$ is FALSE.
On the other hand if hypothesis is FALSE, then the truth value of above mentioned compound is FALSE.