Doping in sports

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- Part A (Begin to Explore): Complete all the attached tasks that will help you to explore some of the probabilities involved in medical tests (You can write on the sheet)
- Part B (Extend your Exploration): Reflect on your findings from Part A and decide how to extend your exploration to answer the inquiry question; then complete your exploration (attach as many extra sheets as you need - you might also want to use a spreadsheet)
- Part C (Report on your Exploration): Write a digital report on your exploration. You must communicate all the steps in your exploration process and answer the inquiry question. The report must be submitted in PDF format and must include an introduction, main body and conclusion (see extra guidelines for what to include in the introduction and conclusion)

Part A
Begin to Explore

1 A scientist designs a medical test for a banned substance in swimming. The name of the banned substance is Z. The following information is known:

- The probability that a professional swimming competitor is doping is 0.1
- If a swimmer is Z-doping the probability that the medical test will show Z-doping is 0.9
- If a swimmer is not Z-doping the probability that the medical test will show no Z-doping is 0.8

Use the information above to answer the following questions:

a) A professional swimming competition has 50 competitors. How many of the competitors do you expect to be Z-doping?

Number of people $\times$ probability of someone doping
$50 \cdot 0.1 = 5$

b) A different world-wide professional swimming competition includes 20 competitors who are actually Z-doping. All of the competitors take a medical test for Z-doping. How many of the 20 competitors who are actually Z-doping do you expect to have a medical test result that correctly shows they are Z-doping?

Number of competitors doping $\times$ probability of positive test
$20 \cdot 0.9 = 18$

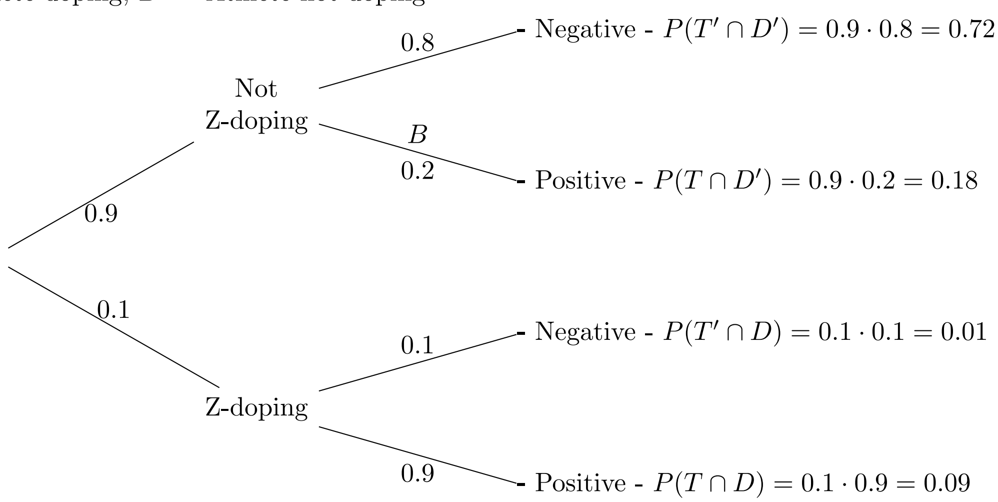
c) The world-wide professional swimming competition also includes 180 competitors who are not Z-doping. How many of the 180 competitors who are not Z-doping do you expect have a medical test result that incorrectly shows that they are Z-doping?

Number of competitors not Z-doping $\times$ probability of false positive
$180 \cdot 0.2 = 36$

2 Use the probability information about z-doping to draw a tree diagram

- The probability that a professional swimming competitor is Z-doping is 0.1
- If a swimmer is Z-doping the probability that the medical test will show Z-doping is 0.9
- If a swimmer is not z-doping the probability that the medical test will show no Z-doping is 0.8

T = Positive test result, T' = Negative test result
D = Athlete doping, D' = Athlete not doping



3 Use your tree diagram from A.2 to calculate the following probabilities:

a) The probability that a professional swimming competitor is actually Z-doping and the medical test correctly shows the competitor is Z-doping

$P(D \cap T) = 0.1 \cdot 0.9 = 0.09$

b) The probability that a professional swimming competitor is actually Z-doping and the medical test incorrectly shows the competitor is not Z-doping

$P(D \cap T') = 0.1 \cdot 0.1 = 0.01$

c) The probability that a professional swimming competitor is actually not Z-doping and the medical test incorrectly shows the competitor is Z-doping

$P(D' \cap T) = 0.9 \cdot 0.2 = 0.18$

d) The probability that a professional swimming competitor is actually not Z-doping and the medical test correctly shows the competitor is not Z-doping

$P(D' \cap T') = 0.9 \cdot 0.8 = 0.72$

e) The probability that the medical test gives a correct result for a competitor

$P((D' \cap T') \cup (D \cap T)) = (0.9 \cdot 0.8) + (0.1 \cdot 0.9) = 0.81$

f) The probability that the medical test gives an incorrect result for a competitor

$P((D \cap T') \cup (D' \cap T)) = (0.9 \cdot 0.2) + (0.1 \cdot 0.1) = 0.19$

4 Use the probabilities you calculated in A.3 to explore the data for the world-wide professional swimming competition with 200 competitors. Complete the two-way table to show how many of the 200 competitors we expect to see in each category

World-Wide Competition	Medical Test shows Z-doping	Medical Test shows not Z-doping	Total
Competitor actually Z-doping	18	2	20
Competitor actually not Z-doping	36	144	180
Total	54	146	200

Working:

World-Wide Competition	Medical Test shows Z-doping	Medical Test shows not Z-doping	Total
Competitor actually Z-doping	$20 \cdot P(T\|D) = 20 \cdot 0.9 = 18$	$20 \cdot P(T'\|D) = 20 \cdot 0.1 = 2$	20
Competitor actually not Z-doping	$180 \cdot P(T\|D') = 180 \cdot 0.2 = 36$	$180 \cdot P(T'\|D') = 180 \cdot 0.8 = 144$	180
Total	$18 + 36 = 54$	$2 + 144 = 146$	200

5 Use the two-way table from A.4 to estimate the probability that a competitor in the world-wide swimming competition with a medical test that shows Z-doping is actually Z-doping.

$P(D|T) = \frac{n(D \cap T)}{n(T)} = \frac{18}{54} = \frac{1}{3}$

6 Reflect on your answer to A.5, is there anything that surprises you about the result? Do you think that this medical test for Z-doping provides strong enough evidence to disqualify a competitor whose medical test showed Z-doping? If not, why not?

I think it's pretty surprising that if someone gets a positive Z-doping result, they're twice as likely to not actually be Z-doping that Z-doping. This means that taking Z-doping tests to disqualify athletes would be disqualifying more athletes who aren't Z-doping than those who are. I think that this means that Z-doping medical tests aren't accurate enough to be used to test every athlete as more athletes that aren't Z-doping will be disqualified than those who are. These tests are unreliable, and shouldn't be relied on for competition disqualifications. An accuracy of just $\frac{1}{3}$ is too low too make this a reliable test as most of the time you'd be accusing an athlete that hasn't been doping of doping. The chance of an athlete who is actually doping getting a negative test is small, with just a probability of $\frac{1}{73}$.

Part B

Extend your Exploration

Reflect on your findings from Part A and decide how to extend your exploration to answer the inquiry question; then complete your exploration. Remember that the inquiry question you need to answer in your report is

> *How do changes in the proportion of athletes doping affect the probability that an athlete whose medical test shows doping is actually doping?*

7 What important ideas came out of your exploration in Part A? What did you discover?

From my exploration in Part A, I have realized that given $\frac{1}{10}$ of athletes are doping, and that the medical test has a $\frac{1}{5}$ chance of giving a false positive and a $\frac{1}{10}$ of giving a false negative, that the probability of someone actually doping given a positive test result is half of the probability of them not doping, meaning that 2 out of every 3 people who get a positive medical test result aren't actually Z-doping. This means that it isn't a good idea to test everyone for Z-doping as most of the people who get a positive result aren't actually doping.

8 Explain how you will extend your exploration to get enough evidence to fully answer the inquiry question

The inquiry question is asking how changes in the proportion of athletes doping affect the probability that an athlete whose medical test shows doping is actually doping. This means that we need to change the proportion of athletes doping (the independent variable) and see how it impacts the probability that an athlete whose medical test shows doping is actually doping (the dependent variable). To do this, I will change the proportion of athletes doping from 0% to 100% and see how it impacts the dependent variable. I will see how the reliability and accuracy of the test changes and see whether or not it would be reliable enough to be relied upon to make career changing decisions for some athletes.

Part C

Report on your Exploration

How do changes in the proportion of athletes doping affect the probability that an athlete whose medical test shows doping is actually doping?

Athletes "doping" in large sporting competitions have been a problem for many years, giving them an unfair advantage over those who aren't doping. Doping is the act of using a banned athletic performance-enhancing drugs by athletic competitors, a form of cheating. The use of athletic performance-enhancing drugs was banned due the health risks, the equality of opportunity for athletes, and the exemplary effect of drug-free sport for the public. This is why numerous different medical tests to test for signs of athletes doping have been created. However, most tests only have a 90% chance of accurately giving a positive test result to someone who is actually doping and only an 80% chance of correctly giving a negative result to an athlete who wasn't doping, meaning 20% of athletes who weren't doping will get positive doping results, which could lead to serious consequences in that athletes career. This is problematic for athletic councils as they don't athletes doping in competitions but they also don't want to be incorrectly mislabeling 20% of athletes who aren't doping as dopers. Since classifying these athletes as doping or not is a decision that can greatly impact an athletes life, athletic councils need to make sure that the ratio of accurate positives to false positives is great enough to be fairly sure that most athletes with a positive result are actually doping.

With current estimates of 1 in 10 athletes doping, the probability of an athlete with a positive result is actually doping is just 1 in 3. This means that most athletes that get a positive test result are not actually doping. With this number of athletes who aren't actually doping getting accused of doping, it makes no sense to be using these doping tests. The test is fairly unreliable with an accuracy to correctly identify a doping athlete from a positive test result of just $\frac{1}{3}$. However, as the proportion of athletes doping changes, so does the ratio of accurate positives results to false positives. The probability of a positive result given an athlete is actual doping, $P(T|D)$, is given by $\frac{P(T \cap D)}{P(T)}$ where $P(T)$ is the probability of a positive result and $P(D)$ is the chance of an athlete doping. To see how $\frac{P(T \cap D)}{P(T)}$ changes depends on changes in $P(D)$, we can do the following knowing $P(T|D) = 0.9$ and $P(T|D') = 0.2$

$$P(D|T) = \frac{P(T \cap D)}{P(T)}$$

$$P(D|T) = \frac{P(T \cap D)}{P(T \cap D) + P(T \cap D')}$$

$$P(D|T) = \frac{P(D) \cdot P(T|D)}{P(D) \cdot P(T|D) + P(D') \cdot P(T|D')}$$

$$P(D|T) = \frac{P(D) \cdot 0.9}{P(D) \cdot 0.9 + (1 - P(D)) \cdot 0.2}$$

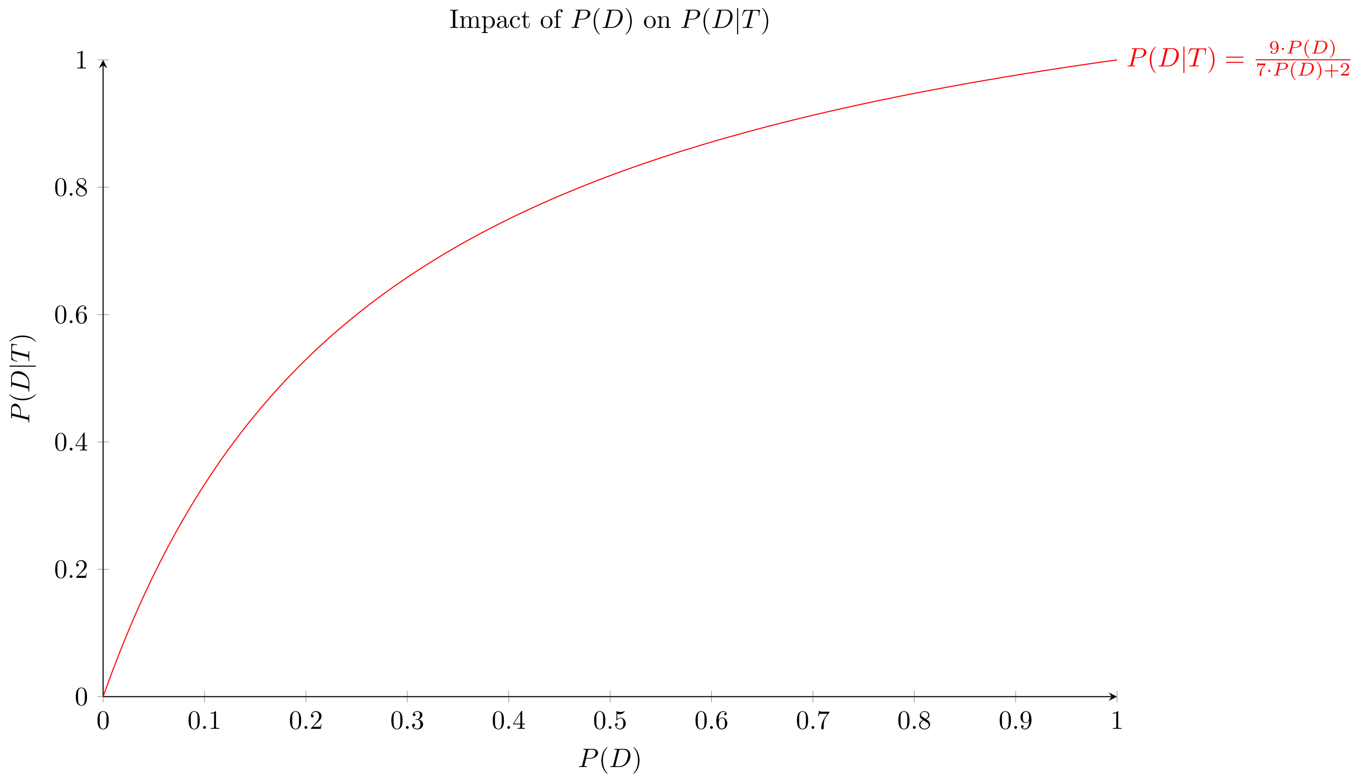
$$P(D|T) = \frac{0.9 \cdot P(D)}{P(D) \cdot 0.9 - P(D) \cdot 0.2 + 0.2}$$

$$P(D|T) = \frac{0.9 \cdot P(D)}{P(D) \cdot (0.9 - 0.2) + 0.2}$$

$$P(D|T) = \frac{0.9 \cdot P(D)}{0.7 \cdot P(D) + 0.2}$$

$$P(D|T) = \frac{9 \cdot P(D)}{7 \cdot P(D) + 2}$$

Graphing the equation we get the following:



From the graph, we can see that the equation continues to increase non linearly until it reaches 1 when the probability of an athlete doping is 1. This means as more people dope, then the chance of a person having a positive test result actually doping will also increase. The slope of the line decreases from $\frac{9}{2}$ at $P(D) = 0$ to $\frac{2}{9}$ at $P(D) = 1$ and always stays positive. As the proportion of athletes doping increases, so does the probability that an athletes whose medical shows doping is actually doping.

This shows that for most probabilities that an athlete is doping, the test is fairly unreliable and shouldn't be relied on to be a decision maker on whether or not an athlete should be disqualified as many non-doping athletes will be accused of doping, which will ruin their career's as an athlete. The test is fairly inaccurate, giving a high chance of giving a wrong result and of incorrectly identifying someone as doping.

In conclusion, doping in sports is a big problem and we need to find ways to effectively test athletes who are doping in competitions. With the effectiveness of our current medical tests, we shouldn't be disqualifying athletes on a positive test result as it would lead to more athletes who aren't actually doping getting disqualified. Accusing an athlete of doping is a big deal that can impact an athletes whole career, and at the current state of doping tests, we shouldn't be disqualifying athletes just on a positive test result. Doping tests still need to be improved and further developed until they can be used effectively, and until then, use of them should be limited to make sure as few athletes are falsely accused as possible.