Transistors for Microprocessors

Mathematical modeling - C and D report

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1 Introduction

A microprocessor (also known as a Central Processing Unit or CPU) is an integrated circuit consisting of many transistors on a single chip. In 1964 Gordon Moore, one of the founders of the Intel Corporation, predicted that the number of transistors in a chip would grow exponentially by doubling every 18 months. The first microprocessor was the Intel 4004, introduced in 1971, with 2,250 transistors. It was not very powerful, but it could add and subtract and it enabled one of the first portable electronic calculators to come into production. Having everything on one chip, rather than components with transistors wired one at a time made microprocessors revolutionary.

The table contains data on the year of release and the number of transistors for Intel microprocessors from 1971 to 2010:

Processor	**Date**	**Transistors**
4004	1971	2,250
8008	1972	2,500
8080	1974	5,000
8086	1978	29,000
296	1982	120,000
386	1985	275,000
486 DX	1989	1,180,000
Pentium	1993	3,100,000
Pentium II	1997	7,500,000
Pentium III	1999	24,000,000
Pentium 4	2000	42,000,000
Itanium 2	2003	220,000,000
Itanium 2 w/9MB cache	2004	592,000,000
Dual-core Itanium 2	2006	1,700,000,000
Six-core Xeon 7400	2008	1,900,000,000
8-core Xeon Nehalem-EX	2010	2,300,000,000

Task: *Create a suitable exponential model for the data and use it to reflect on the validity of the claim made by Gordon Moore in 1965. Write a report on your findings.*

2 Calculations

To find an exponential model to best fit the data, one can first linearize the data and use a linear regression model to find the best-fit equation for the data.

An exponential equation has the following form:

$$y = \alpha \cdot \beta^{\frac{1}{k}x}$$

where α is the starting value and y increases by a factor of β every k units of time.

Since Moore's law states that the number of transistors in microprocessors doubles every 18 months or 1.5 years, we can keep the value of k constant at 1.5 and see how close the value of β is to 2, either verifying or disproving Gordon Moore's statement.

$$y = \alpha \cdot \beta^{\frac{2x}{3}}$$

To linearize this exponential, we can take the natural log on both sides and simplify:

$$\begin{aligned}
y &= \alpha \cdot \beta^{\frac{2x}{3}} \\
\ln y &= \ln \alpha \cdot \beta^{\frac{2x}{3}} \\
\ln y &= \ln \alpha + \ln \beta^{\frac{2x}{3}} \\
\ln y &= \ln \alpha + \frac{2x}{3} \ln \beta \\
\ln y &= (\frac{2}{3} \ln \beta)x + \ln \alpha \\
Y &= mx + b
\end{aligned}$$

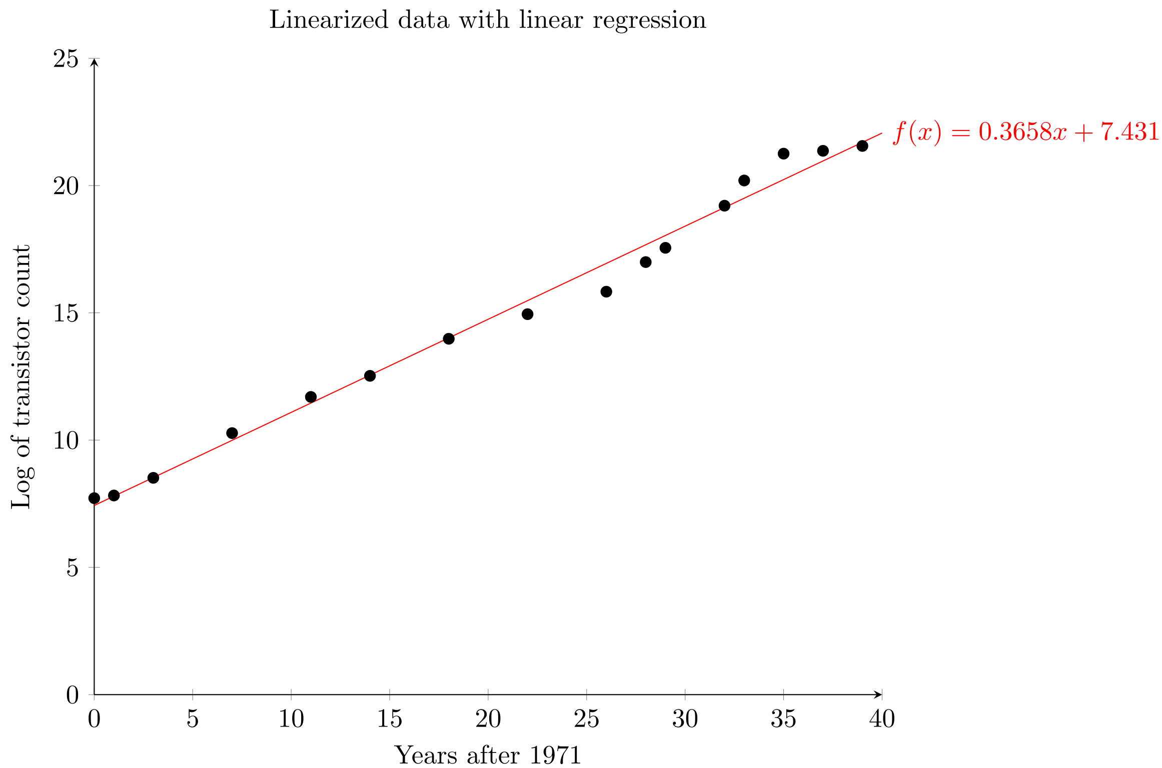
where $m = \frac{2}{3} \ln \beta$, $Y = \ln y$, and $b = \ln \alpha$. To retrieve the values of α and β in terms of m and b, we can solve for the variables, obtaining:

$$\begin{aligned}
\beta &= e^{m \cdot \frac{3}{2}} \\
\alpha &= e^{b}
\end{aligned}$$

To linearize the data, we need to take the natural log of the number of the transistors, the y in the equation, to get the data into the form $Y = mx + b$. We also should start the year at 0 instead of 1971 so that the starting number of transistors starts $x = 0$. Doing these changes gives us the following data:

0	7.7186855
1	7.8240460
3	8.5171932
7	10.2750511
11	11.6952470
14	12.5245264
18	13.9810250
22	14.9469127
26	15.8304136
28	16.9935644
29	17.5531802
32	19.2091381
33	20.1990172
35	21.2538941
37	21.3651197
39	21.5561750

We can now do a linear regression on this data as it's in the form $y = mx + c$. Inputting these values into a linear regression model gives us the equation $Y = 0.3658x + 7.431$ with an R^2 value of 0.9886 meaning it's a fairly good fit to the data. Plotting this gives:



From the equation $Y = 0.3658x + 7.431$ we get values of m, and b being:

$$m = 0.3658$$
$$b = 7.431$$

To convert this linearized version of our data back into the original exponential form, we can convert the values of m and b into α and β using the equations we get previously:

$$\beta = e^{m \cdot \frac{3}{2}}$$
$$\alpha = e^{b}$$

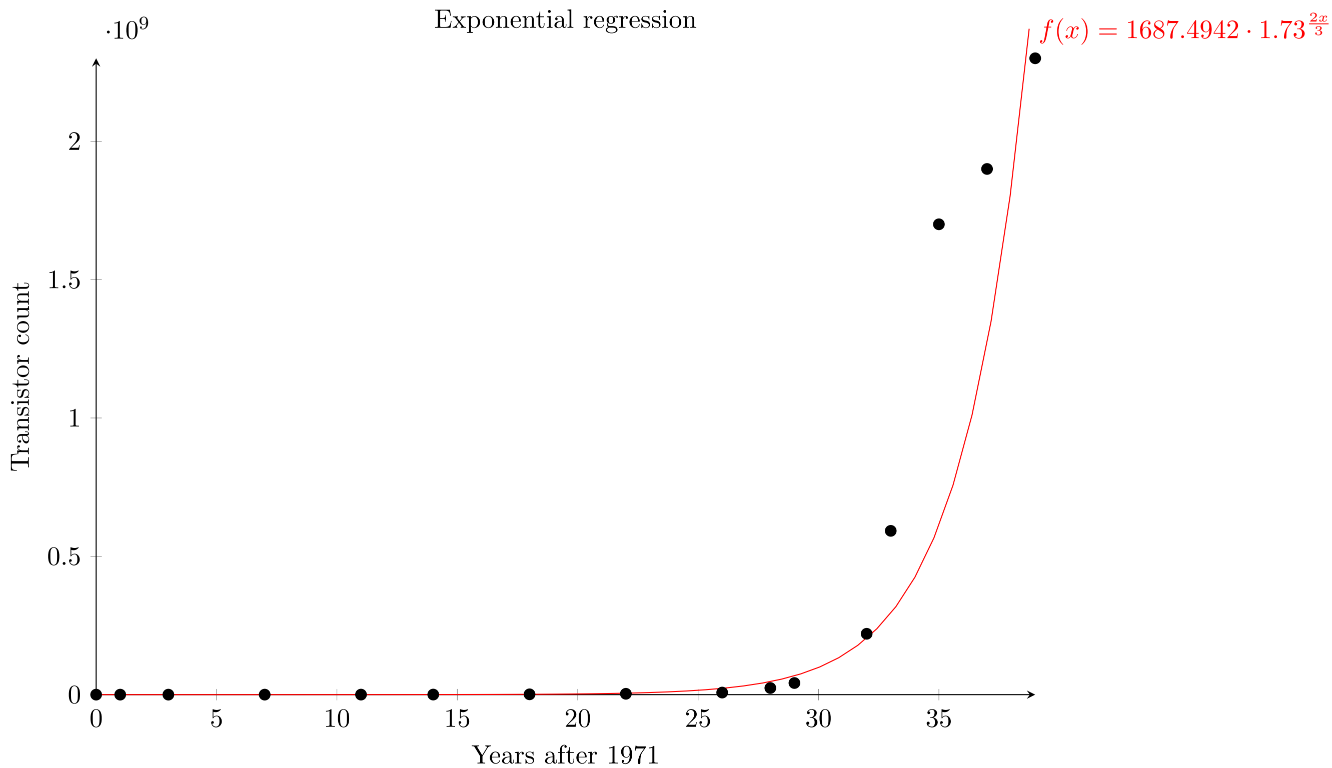
Using these equations we get the following values of α and β:

$$\alpha = e^{b} \qquad \beta = e^{m \cdot \frac{3}{2}}$$
$$\alpha = e^{(7.431)} \qquad \beta = e^{(0.3658) \cdot \frac{3}{2}}$$
$$\alpha = 1687.4942 \qquad \beta = e^{0.5487}$$
$$\beta = 1.7310$$

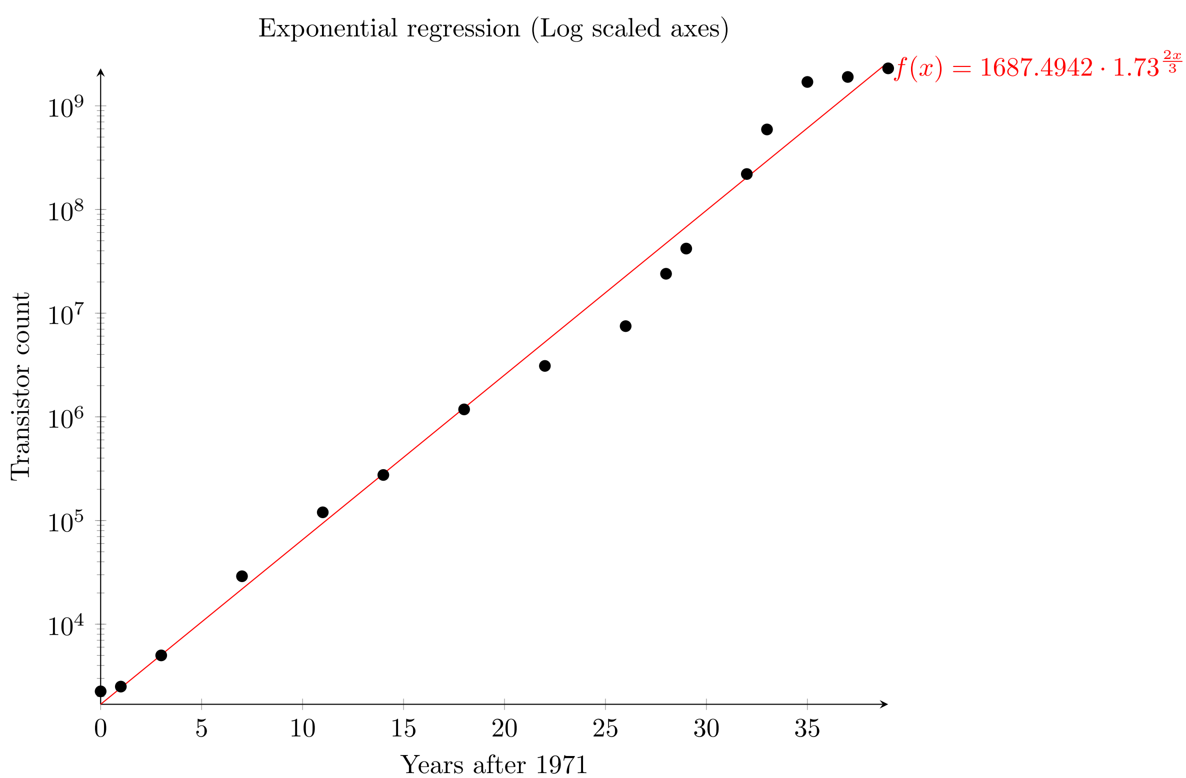
Using these values, we can get the exponential equation for the original data:

$$y = 1687.4942 \cdot 1.73^{\frac{2x}{3}}$$

Plotting this against the original gives the following:



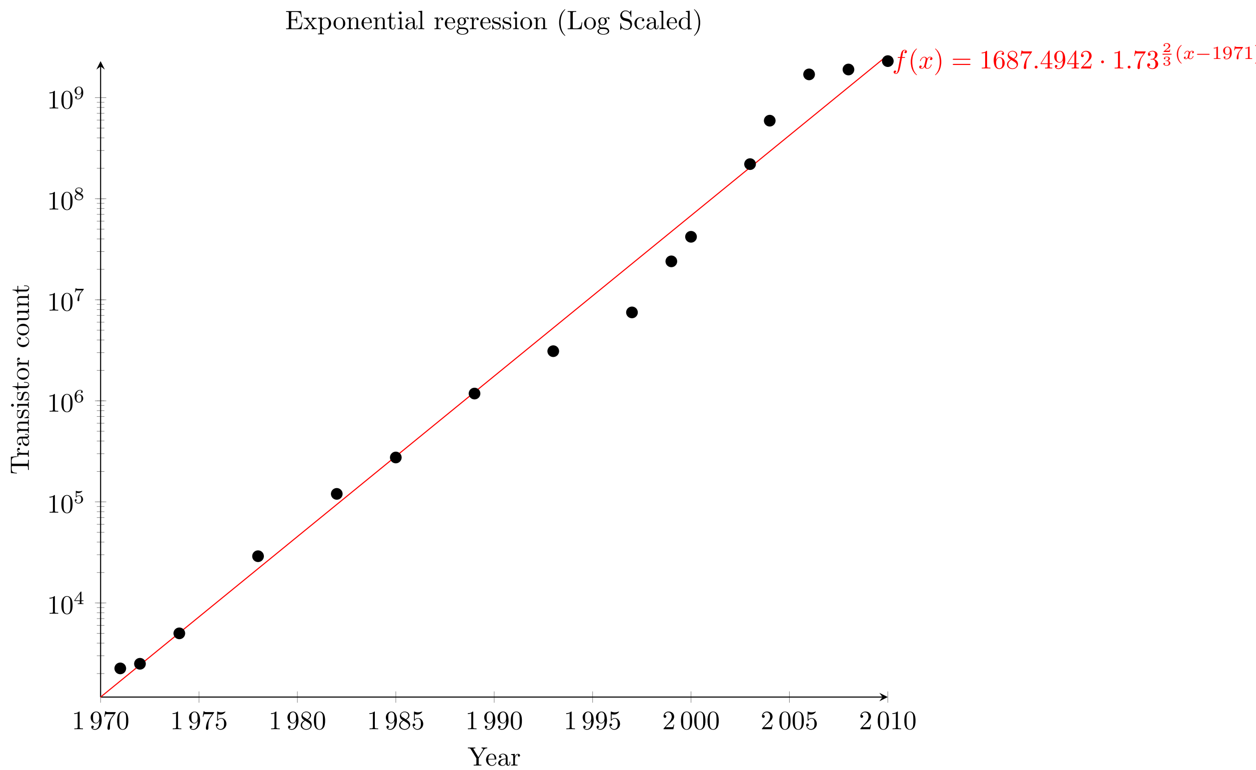
With logarithmically scaled axes:



To convert this back to standard years, we can transform the equation through a linear translation of 1971 units in the positive x direction giving the equation:

$$y = 1687.4942 \cdot 1.73^{\frac{2}{3}(x-1971)}$$

and the graph:



3 Analysis

The exponential equation we got that best modeled the data was:

$$y = 1687.4942 \cdot 1.73^{\frac{2}{3}(x-1971)}$$

This equation says that at the first data point in the year 1971, the number of transistors was 1971 and on average increased by a factor of 1.73 every 18 months or 1.5 years. Gordon Moore predicted that the number of transistors would double every 18 months, meaning a factor of 2 every 1.5 years. Using a factor of 2 instead of 1.73 with the same starting value gives a transistor count 2 magnitudes larger than the actual value in 2010 at the last data point. This shows that Moore's prediction of the number of transistors doubling every 18 months was fairly inaccurate.

To see how many years on average it took for the number of transistors to double, we can change the base of the exponential to 2 by multiplying the exponent by $\frac{\ln 1.731}{\ln 2}$ and the coefficient of x will be the number of years it takes on average for the number of transistors to double. This gives the new equation:

$$y = 1687.4942 \cdot 2^{\frac{\ln 1.731}{\ln 2}\frac{2}{3}(x-1971)}$$
$$y = 1687.4942 \cdot 2^{0.5277(x-1971)}$$
$$y = 1687.4942 \cdot 2^{\frac{(x-1971)}{1.895}}$$

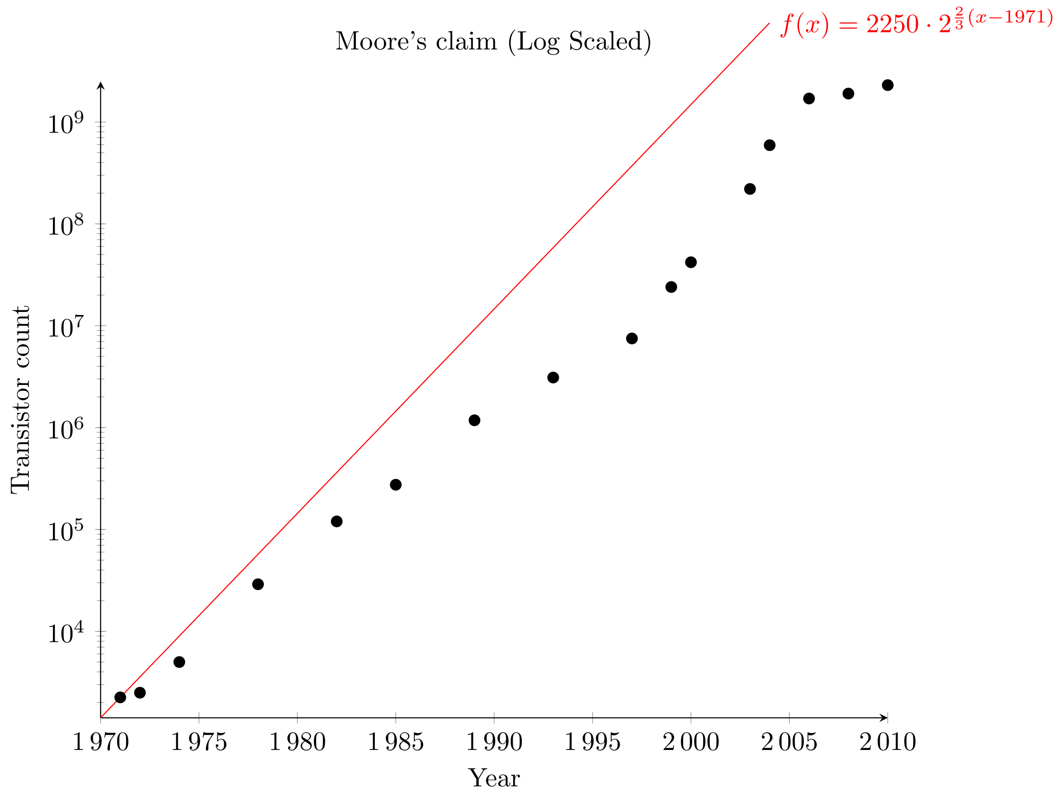
This means that on average, the number of transistors doubles every 1.895 years or 22.75 months. This is fairly different from Moore's prediction of 18 months, which reaffirms that his prediction is fairly inaccurate.

If we were to instead of using 18 months as our base for our regression, and instead just do a standard best-fit model to the data, we'd get the following:

$$y = 1687.49 \cdot 1.73^{\frac{2}{3}(x-1971)}$$
$$y = 1687.49 \cdot (1.73^{\frac{2}{3}})^{x-1971}$$
$$y = 1687.49 \cdot 1.44^{(x-1971)}$$

This shows us that on average, the number of transistors in microprocessors will increase by a factor of 1.44 each year.

Graphing Moore's claim, we get the following graph:



As can be seen, Moore's claim drastically overestimates the number of transistors and overestimates the rate of change of the number of transistors. However, for certain parts of the graph, Moore's claim does work such as from 1971 to 1980. In fact, from 1997 to 2006, Moore's prediction underestimates the growth rate of the number of transistors per year. This is due to large developments in transistors, such as a sharper decrease in size, a lowering of costs, and an overall increase in funding and development. However, the number of transistors in microprocessors is bound to have physical limitations due to the size of the transistors. Modern transistors are generally around 2-3nm in size. At that scale, semiconductors can't properly control electrons to due quantum effects: quantum tunneling and quantum uncertainty. At smaller sizes, these effects are amplified and make creating semiconductors and transistors at that size much harder. This means that to increase the number of transistors, the physical semiconductor die size needs to be increased which can lead to other issues such as costs and an increase in chip defects.

4 Conclusion

The equation that best fit the data was

$$y = 1687.49 \cdot 1.73^{\frac{2}{3}(x-1971)}$$

or

$$y = 1687.49 \cdot 2^{\frac{(x-1971)}{1.895}}$$

or

$$y = 1687.49 \cdot 1.44^{(x-1971)}$$

which doesn't follow Gordon Moore's prediction that the number of transistors should double every 18 months. From our calculations, we see that the more accurate factor is around 1.73 every 18 months and doubles every 22.75 months. Even though Gordon Moore was right in it being an exponential relationship, he was incorrect in predicting the exact exponential relation. From the data, we see that on average, the amount of transistors will increase by a factor of 1.44 each year. Having a true exponential relation between two physical variables is impossible as the number will blow up to infinity very quickly, leading to unrealistic results.