Craters on the moon

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R.Q. How does the angular momentum of an impactor affect the length of crater rays?

1 Introduction

Testing the affects of angular momentum on the length of rays coming out of the crater. I will be testing this by tying a mass to the end of a pendulum and swinging it into the surface.

1.1 Background research [1][2]

Angular momentum is a measure of an object around an axis. Linear momentum $\boldsymbol{p}$ is defined as the mass $\boldsymbol{m}$ of an object multiplied by the velocity $\boldsymbol{v}$ of the object

$$p = m \times v$$

To find the final velocity of the pendulum, we know that the initial potential energy will get converted into kinetic energy.

$$PE_i = KE_f$$
$$mgh = \frac{1}{2}mv^2$$

where $\boldsymbol{m}$ is the mass of the impactor, $\boldsymbol{g}$ is the gravitational acceleration, $\boldsymbol{h}$ is the height of the initial starting position of the pendulum, and $\boldsymbol{v}$ is the final velocity of the pendulum. Solving for $\boldsymbol{v}$, we get

$$mgh = \frac{1}{2}mv^2$$
$$gh = \frac{1}{2}v^2$$
$$2gh = v^2$$
$$\sqrt{2gh} = v$$

Substituting h with $(l - \cos\theta \cdot l)$ gives us the equation for the final velocity of the pendulum.

$$v = \sqrt{2g(l - l\cos\theta)}$$

Using this, we get the following equations:

$$v = \sqrt{2g(l - l\cos\theta)}$$
$$p = m \cdot \sqrt{2g(l - l\cos\theta)}$$
$$KE_f = mg(l - l\cos\theta)$$

1.2 Hypothesis

I believe that as the angular momentum of the impactor increases, so will the length of the rays coming out of the crater. This is due to the fact that as the impactor hits the surface of an object, the energy from the impactor will be transferred into the object. This would mean that as more energy is put into the impactor, specifically as angular momentum, more energy will be transferred into the surface of the object causing parts of the surface to break apart, creating rays coming out of the crater.

2 Method

2.1 Variables

I.V. The angle (θ) that the impactor is released on the pendulum

D.V. The length (cm) of the longest ray coming out of the crater

Control variable	Why should it be controlled	How can it be controlled
shape of the impactor	The shape of the impactor can affect the formation of the crater on impact.	Use spherical impactor
mass of the impactor	The mass of the impactor can unwantingly affect the momentum of the impactor.	Use same mass impactor
material of the impactor	The material of the impactor can affect the formation of the crater and can have undesirable affects on results.	Use the same brass impactor
length of pendulum	The length of the pendulum can change the velocity of the impactor at impact.	Use same length of string
surface material	The surface that's getting hit can affect the properties of the crater.	Use same flour for all tests

2.2 Safety precautions

- Be careful while handling the flour as it can easily go everywhere
- Don't inhale the flour
- Clear the area so nothing gets covered in flour or gets hit by the pendulum

2.3 Materials

- Pendulum stand
- 1.5 - 2 meters of string
- Spherical mass
- A tray 5 - 10cm deep
- 1 - 2 bags of flour
- Ruler
- Protractor

2.4 Procedure

1. Pour the flour into the tray until the tray is nearly full
2. Setup up the pendulum stand on a high surface like on a table so that there is roughly 1-1.5m to the ground
3. Tie one end of the string to the mass and the other end to the pendulum stand
4. Measure the length of string from the axis of rotation to the impactor and the mass of the impactor
5. Drop the mass from 5°
6. Measure the distance the furthest ray went from the crater and write it down
7. Repeat steps 5 and 6 with angles 10°, 15°, 20°, 25°, 30°, 35°, 40°

Figure 1: Example of setup used

3 Results

3.1 Raw data

Mass of impactor: 100g, length of pendulum: 1.37m

	Ray length (cm)		
Angle	Trial 1	Trial 2	Trial 3
5°	1.5	1.5	1.6
10°	2.3	2.6	2.4
15°	3.0	2.8	3.1
20°	3.4	3.6	3.8
25°	6.0	5.8	6.1
30°	12	11.2	11.4
35°	12.5	12.8	13.1
40°	14	14.4	14.2

3.2 Calculations

Using the equations we got earlier, we can substitute in the values for mass, pendulum length, angle, and gravitational acceleration. The equation for calculating momentum:

$$p = m \cdot \sqrt{2g(l - l\cos\theta)}$$

Using the an angle of 5° as an example, we can substitute in our values for mass, length, angle, and gravitational acceleration.

$$\begin{aligned}
p &= (0.1)\sqrt{2(9.81)(1.37 - (1.37)\cos(5))} \\
p &= (0.1)\sqrt{0.102} \\
p &= 0.03198 kg\frac{m}{s}
\end{aligned}$$

This means the momentum carried by the impactor at impact was approximately 0.03198 $kg\frac{m}{s}$. To calculate the kinetic energy of the pendulum, we use the equation:

$$KE_f = mg(l - l\cos\theta)$$

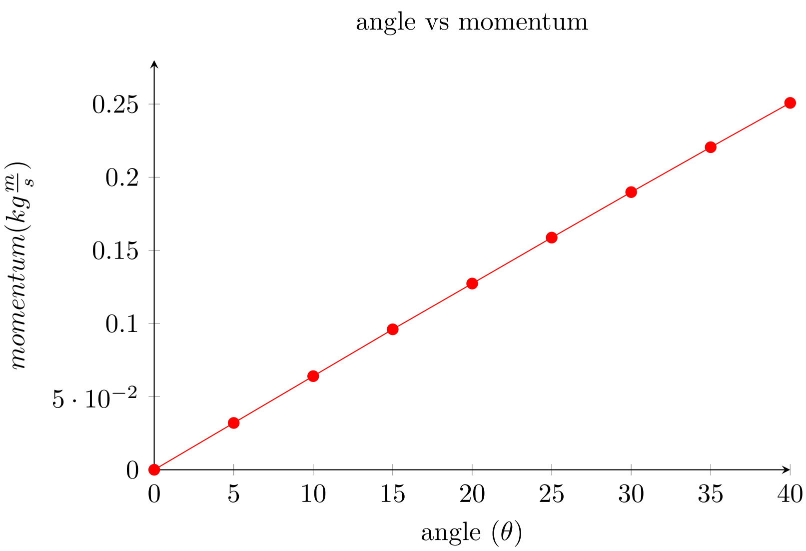
Substituting in the values for mass, length, angle, and gravitational acceleration at 5°, we get:

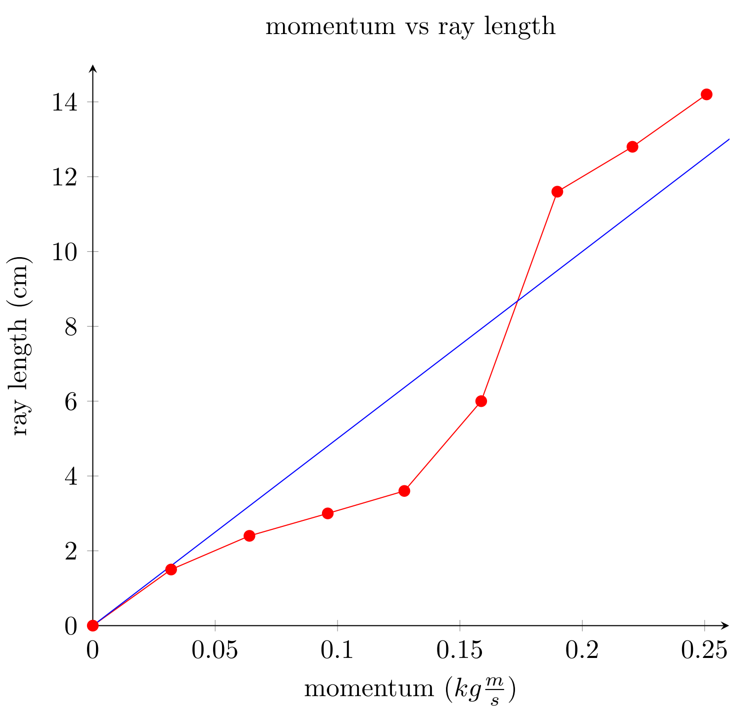
$$\begin{aligned}
KE_f &= (0.1)(9.81)(1.37 - (1.37)\cos 5) \\
KE_f &= (0.1)(0.051) \\
KE_f &= 0.00511J
\end{aligned}$$

This means that at 5°, the impactor had a kinetic energy of $0.00511J$ and had a momentum of 0.03198 $kg\frac{m}{s}$.

Angle (degrees)	Ray length (cm)	Angular momentum (kg m/s)	Kinetic energy (J)
0	0	0	0
5	1.5	0.03198	0.00511
10	2.4	0.06390	0.02042
15	3.0	0.09570	0.04579
20	3.6	0.12732	0.08105
25	6.0	0.15869	0.12592
30	11.6	0.18977	0.18006
35	12.8	0.22048	0.24305
40	14.2	0.25077	0.31443

3.3 Processed data





4 Discussion

From my data, I can see that as the angular momentum of the mass increased, so did the length of the rays coming out of the crater. This is because as the angular momentum of the mass increases, so does the amount of kinetic energy that is carried by the impactor. When the mass impacts the surface of the flour, the kinetic energy is transferred into the surface, causing flour to gain kinetic energy, creating rays.

The kinetic energy of the impactor can be calculated using the formula $KE = mg(l - l\cos\theta)$ where m is the mass of the impactor, l is the length of the pendulum and θ is the starting angle of the pendulum. From this we can see as the starting angle of the pendulum increases, so will the kinetic energy of the impactor at impact. This is because as the pendulum is raised higher, it will have more potential energy that gets converted into kinetic energy. This kinetic energy then will get transferred into the surface of the flour, causing bits of flour to gain kinetic energy, creating rays. From my data, we can see that this lines up with the test results. As the momentum of the impactor increased, so did the length of the rays. Though not perfectly, we can see a near linear relationship between the momentum of the impactor with the length of the rays.

5 Conclusion

In conclusion, as the angular momentum of the impactor increases, so will the length of rays coming out of the crater. This supports my original hypothesis *"I believe that as the angular momentum of the impactor increases, so will the length of the rays coming out of the crater"*. My results answer my research question as they show a correlation between the angular momentum of the impactor and the length of rays coming out of the crater. My results show that as the momentum of the impactor increases, so will the length of the rays.

The energy that is carried by the impactor is proportional to the angular momentum, meaning that as the angular momentum increases, so will the energy being transferred into the flour. As this kinetic energy is transferred into the flour it causes parts of the surface to break up, creating rays. As more kinetic energy is transferred into the flour, it will cause more energy to be put into the rays, causing them to travel further. This agrees with my test results. As the starting angle of the pendulum increased the average length of the rays also went up. This is because when the starting angle of the pendulum gets increased, so does the potential energy that gets converted into kinetic energy. This means that the impact will transfer more energy into the flour, causing longer rays.

However, these results do not directly compare to real craters on the moon. Most craters on the moon are circular and aren't longer in one direction. This is because of the high impact velocities of meteors on the moon. They impact the moon at such high velocities that the surface of the moon doesn't actually move out of the way but rather melts the surface due to the high pressure from the impact. The crater is actually made from material flying out in all directions. Usually, the crater is much larger than the meteorite that hits the surface as the impact is similar to an explosion.

6 Evaluation

Weakness/limitation	Impact on result	Suggested improvement
After raising the pendulum to around 25°, the momentum carried by the mass was too great for the flour to stop it so it would push the flour out of the tray.	It caused the length of the rays after 25° to be much greater than that before 25°.	Use a longer tray and use a material stronger than flour so it would be able to absorb the kinetic energy of the mass better.
It was hard to control the surface of the flour as it would change after each trial.	It would cause different results depending on how the surface of the flour was shaped.	Make the surface of the flour as flat as possible after each trial.
It was hard to accurately measure the length of the rays as sometimes it would be hard to see where the rays ended.	Would make the results less accurate.	Recording each impact in slow motion would make it easier to see how the rays formed.

References

[1] Tim Kirk. *Physics for the IB diploma.* Oxford IB study guides. Oxford Univeristy, 2012. ISBN: 9780198390039.

[2] Vanderbilt Univeristy. *Astronomy 201: Angular Momentum.* URL: https://www.vanderbilt.edu/AnS/physics/astrocourses/ast201/angular_momentum.html. accessed: 28.01.2023.