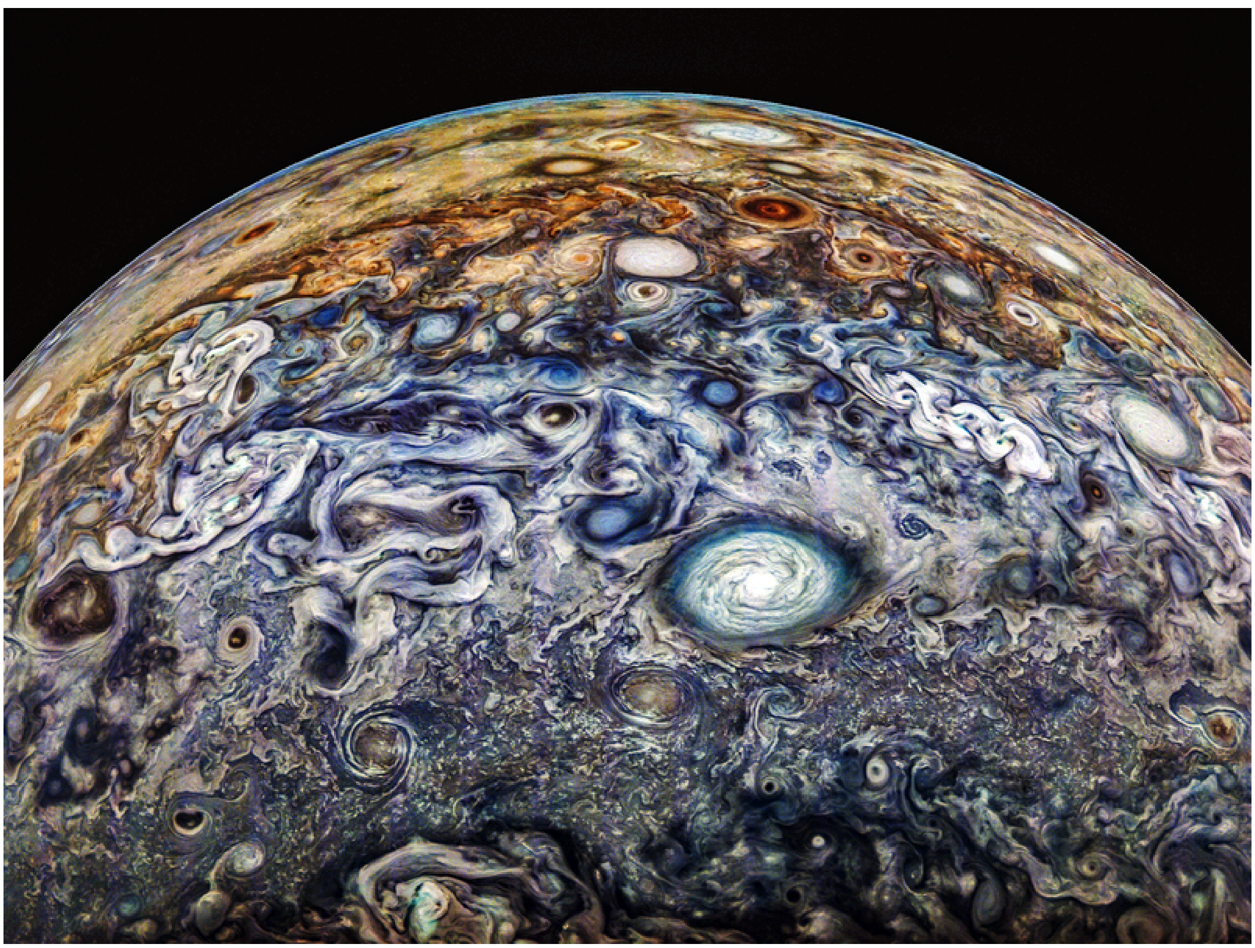
# N-Body Simulation Integrating Atmospheric and Orbital Dynamics for Optimizing Spacecraft Launch Trajectories

Team members: Ines Chellali (13518216); Hidde Poel (12402826); Kangzhi Qin (13496417); Andrei Stoian (13757318)
Team number and team name: team 10
URL of github repository: https://github.com/HiddePoel/Project-Computational-Science



https://www.smithsonianmag.com/smart-news/check-out-the-stunning-new-images-of-jupiter-from-nasas-juno-spacecraft-180985417

**Summary**
"How can we determine when, and in what direction, we launch a spacecraft from a set launch location such that there are no collisions with satellites while also reducing fuel consumption by performing a gravitational assist around Mars?"
To answer this, analytical two-body simulations were implemented to propagate all satellites, while using velocity verlet for the solar system as a whole. We implemented Hohmann transfers and gravitational assists analytically to describe a fuel efficient path to Jupiter around Mars.
As hypothesized, the chance to collide with a satellite is very low, since the shortest distance from the launch axis to the closest satellite ranged from ~20 km to ~350 km, averaging at ~100 km. For pathfinding, we found that a direct Hohmann transfer was viable. However, within the short simulation period of one week, Mars gravity assist proved inefficient to go to Jupiter.

**Details and results**

In order to find an efficient path from Earth to Jupiter, while taking into account the many satellites hovering Earth's atmosphere, we first needed to find an appropriate 'opening' in the sky for our rocket to exit Earth's orbit without hitting any satellites. For this we first obtained TLE (two line element) data of 10,744 satellites and translated the TLE data into 3D arrays of their position and velocity using the Python library SGP4. We then implemented an analytical solution for the two-body problem, this method provides exact position and velocity updates of the satellites using Keplerian orbital mechanics, eliminating numerical integration errors. This is run alongside the n-body simulation of the solar system which is implemented with velocity verlet (Appendix 3). We set our launch site right below the geostationary 'GOES 16' satellite so we could calculate the launch normal to avoid having to keep track of earth's rotation. At every time step the angle between the launch normal and all other satellites are calculated, the smallest of which determines the radius of the largest satellite free cylinder above the launch site, thus we have the shortest distance between the closest satellite and the launch axis. The radius to our surprise fluctuated quite a lot, but never got smaller than 20km as seen in the large dip in *fig 1*. We suspect this dip is caused by a slow moving satellite whose appearance is periodic according to *fig 2*.

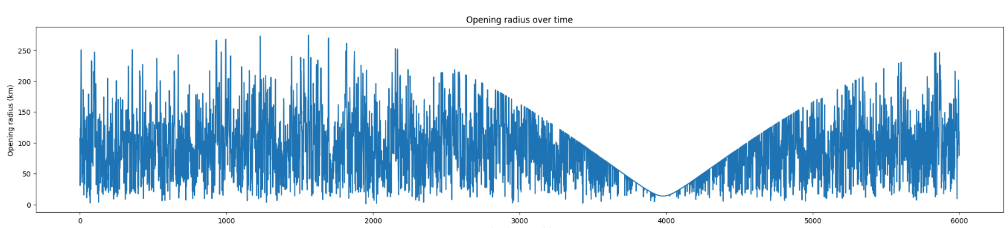



Figure 1. Satellite opening radius over 5 hours.

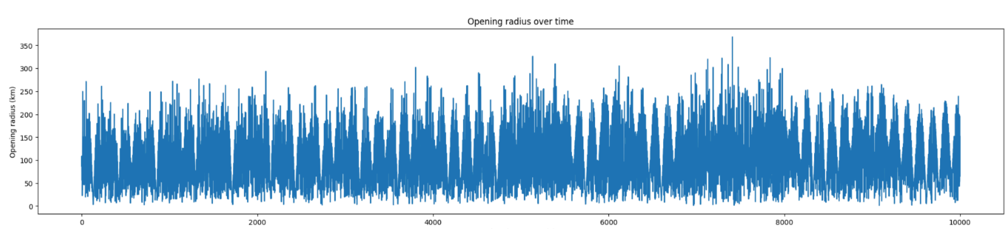



Figure 2. Satellite opening radius over 12 days.

Once a list of appropriate openings was found, we began searching for paths to Jupiter. While researching potential path-finding techniques, we found the Hohmann transfer orbit (HTO) technique (Figure 3), often used for its simplicity and computational efficiency. An HTO involves transferring a body (in this case a spacecraft) from one orbit to another and calculating the velocity vectors of this transfer. All calculations for a HTO can be found in Appendix 1.

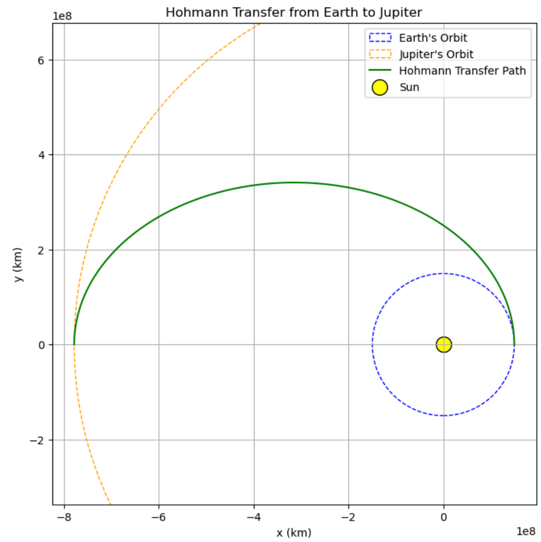



Figure 3. Hohmann transfer orbit (HTO) (green line) from Earth's orbit (blue line) to Jupiter's orbit (orange line).

Although this technique is very popular, we decided it was too simplistic, potentially resulting in an inefficient path to Jupiter. For this reason, we implemented a gravitational assist technique, also used by NASA in their mission to send a rocket to Jupiter. The calculations are given in Appendix 2. A gravitational assist entails using a third planet - for example Mars - as a way to redirect the planet to the target planet (Figure 4a) - Jupiter - while also increasing the velocity, therefore making it more efficient fuel-wise. We first looked for the closest planet between Earth and Jupiter at the time of the satellite opening, and calculated whether the difference in the angle between the direct path to Jupiter and an initial path to the assist planet was less than 45°. If this angle was more than 45°, we would perform a direct HTO to Jupiter as this path would result more efficiently. However, if the angle was less than 45°, we would first do a gravitational assist to the assist planet and use it to redirect the spacecraft to Jupiter with an increased velocity. In a gravitational assist, the distance at which the rocket approaches the assist planet affects the deflection angle - the degree to which the trajectory of the spacecraft is altered (Figure 4b), therefore we also needed to find the right angle at which to release the spacecraft from the assist planet. If we found an appropriate angle of deflection, we would perform a second HTO from the assist planet to Jupiter, therefore finding a more efficient path.

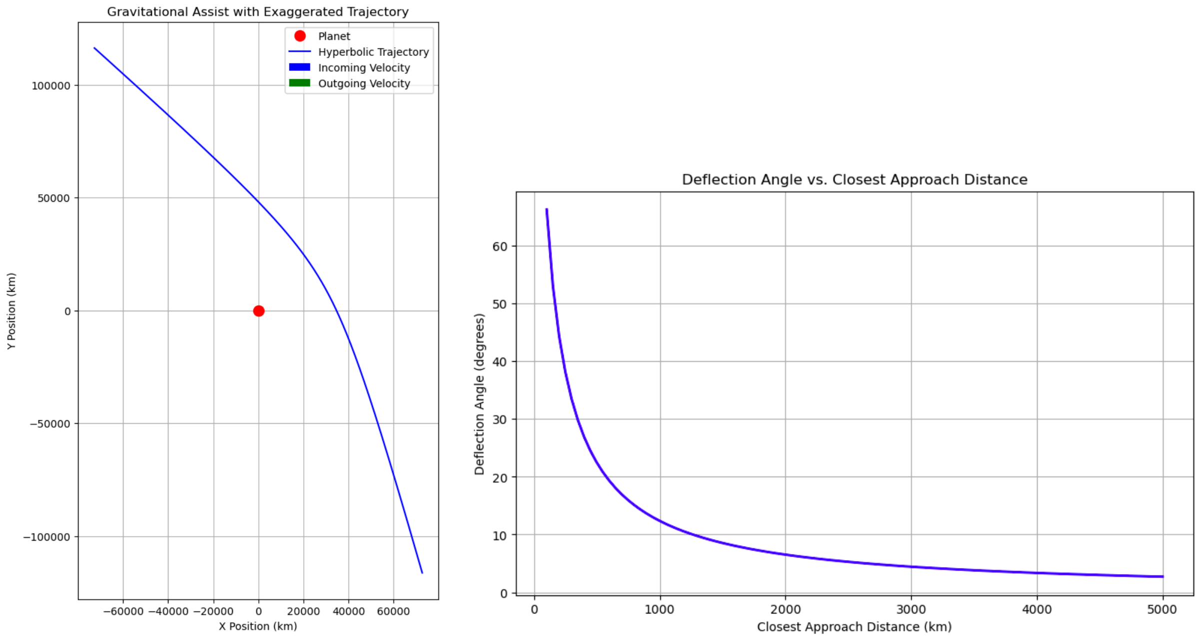



Figure 4. a) Figure showing a gravitational assist around a planet (red dot) showing the deflection angle effect on the trajectory of the spacecraft (blue line). b) Graph showing the negative effect of the spacecraft's approach distance to the planet on the deflection angle.

Within the time frame that we looked at and during the appropriate satellite opening times, we were not able to find a path that would include a gravitational assist to increase the efficiency of said path. Therefore, we assumed a direct HTO from Earth to Jupiter. Future work includes extending the simulation period to find possible gravity assists by Mars and other planets, making our goal of going to Jupiter a more realistic task.

**Contribution**

All members worked in tandem to review peers, write the reports and make the poster.
Ines: worked on finding TLE satellite data, planet data and initialized their position. Also worked on path finding and producing some figures.
Andrei: General code structuring; verlet; helped research, validate and optimise the two body solution; produced the figures on satellite openings; calculate launch normal.
Kangzhi: Wrote analytical two body update and launch opening calculation methods. Wrote functions to help the path finding.
Hidde: Downloading and parsing solar system data, visualizing the solar system data, also worked on pathfinding.

Git fame. (31-1-2025    19:21)

```
PS C:\Users\hidde\uva\projectComputationalScience> git fame --incl '\.py$|\.ipynb$'
Processing: 100%|
Total commits: 68
Total ctimes: 187
Total files: 26
Total loc: 3057
| Author           |   loc |   coms |   fils |  distribution   |
|:-----------------|------:|-------:|-------:|:----------------|
| AndreiRaduStoian |  1309 |     31 |     12 | 42.8/45.6/46.2  |
| ineschellali     |   746 |     10 |      3 | 24.4/14.7/11.5  |
| Kangzhi Qin      |   515 |      7 |      5 | 16.8/10.3/19.2  |
| HiddePoel        |   487 |     19 |      6 | 15.9/27.9/23.1  |
| Hidde Poel       |     0 |      1 |      0 | 0.0/ 1.5/ 0.0   |
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**Appendix A**

Calculate Semi-Major axis

$$a_{transfer} = \frac{r_{p1} + r_{p2}}{2}$$

Calculate the initial orbit velocity of planet $x$ (planet from which we are leaving) and planet $y$ (planet where we are going):

$$v_{\text{initial x}} = \sqrt{\frac{\mu}{r_{px}}}$$

Calculate velocity for transfer of orbit of planet $x$ and $y$:

$$v_{\text{transfer x}} = \sqrt{\frac{2}{r_{px}} - \frac{1}{a_{transfer}}}$$

Calculate total increase in velocity to exit planet $x$ and enter planet $y$

$$\Delta v_x = v_{\text{transfer x}} - v_{\text{initial x}}$$

Calculate total increase in velocity

$$Total\Delta v = \Delta v_x + \Delta v_y$$

Appendix 1. Calculations of Hohmann Transfer orbit. Inputs: radius of planet 1 and planet 2 around the Sun, and the gravitational parameter (mu) of the Sun.

Deflection angle:

$$\theta = 2\arcsin\left(\frac{1}{1 + \frac{r_{assist} \cdot v_{in}^2}{\mu_{assist}}}\right.$$

Velocity post-assist:

$$v_{\text{post-assist}} = v_{in} + v_{planet}$$

Appendix 2. Calculations of a gravitational assist. Inputs necessary are the radius, velocity and the gravitational parameter of the assist planet around the Sun and the velocity of the spacecraft when it reaches the assist planet.

$$X(t+\Delta t) = X(t) + V(t)\Delta t + \frac{1}{2}a(t)\Delta t^2$$

$$a(t+\Delta t) = f(X(t+\Delta t))$$

$$V(t+\Delta t) = V(t) + \frac{a(t) + a(t+\Delta t)}{2}\Delta t$$

Appendix 3. Velocity Verlet functions where X(t) is the position of the bodies at time t and similarly a(t), V(t) and f(t) being acceleration, velocity and force respectively.

1. **Eccentricity Vector Calculation**

$$\mathbf{e} = \frac{\mathbf{V}_0 \times \mathbf{h}}{\mu} - \frac{\mathbf{R}_0}{\|\mathbf{R}_0\|}$$

2. **Kepler's Equation**

$$M = E - e\sin(E)$$

3. **Newton-Raphson Iterative Method for Solving Kepler's Equation**

$$E_{\text{new}} = E_{\text{old}} - \frac{E_{\text{old}} - e\sin(E_{\text{old}}) - M}{1 - e\cos(E_{\text{old}})}$$

4. **True Anomaly from Eccentric Anomaly**

$$\nu = 2\arctan 2\left(\sqrt{1+e}\sin\left(\frac{E}{2}\right), \sqrt{1-e}\cos\left(\frac{E}{2}\right)\right)$$

5. **Vis-Viva Equation**

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

6. **Orbital Radius at True Anomaly**

$$r = \frac{a(1 - e^2)}{1 + e\cos(\nu)}$$

7. **Rotation Matrices for Coordinate Transformation**

$$R_z(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

8. **Combined Rotation Transformation**

$$\mathbf{r}_{\text{inertial}} = R_z(\Omega) \times R_x(i) \times R_z(\omega) \times \mathbf{r}_{\text{orbital}}$$

9. **Position Distribution Based on Center of Mass**

$$\mathbf{r}_1 = -\left(\frac{m_2}{m_1 + m_2}\right) \times \mathbf{R}_{\text{new}}$$

$$\mathbf{r}_2 = \left(\frac{m_1}{m_1 + m_2}\right) \times \mathbf{R}_{\text{new}}$$

10. **Velocity Distribution Based on Center of Mass**

$$\mathbf{v}_1 = -\left(\frac{m_2}{m_1 + m_2}\right) \times \mathbf{V}_{\text{new}}$$

$$\mathbf{v}_2 = \left(\frac{m_1}{m_1 + m_2}\right) \times \mathbf{V}_{\text{new}}$$

Appendix 4. Equations used in the two-body analytical update function. Calculate orbital elements, solve Kepler's equation, determine anomalies, perform coordinate rotations, and distribute positions and velocities.

**Appendix B**

**Peer review of team 15 by team 10**

Project proposal peer review
Submitted by team 10: <Ines Chellali; Hidde Poel; Kangzhi Qin; Andrei Stoian>
To review the project titled: <How the parameters of different rocket engines impact the results of rockets.>
From team 15: <Hans van Breda, Rik Heurter, Mark Jansen, George Petropoulos>

After reading the project proposal thoroughly, using the guidelines on what to review from the initial lecture slide titled “Peer review - Use the template from Canvas!”, we make the following constructive remarks (min 50, max 200 words):

1. The current research question is somewhat broad. If you could zoom in / split it up into specific performance metrics, it may help you develop hypotheses to keep perspective during the project.

2. To provide more clarity on your approach, consider specifying the sources from where you will obtain the known results for validation, and expanding on the process of how you will use these known results to validate your model.
3. It would be helpful to elaborate on which specific engine parameters you plan to focus on, and whether there is a priority among them, including the reasoning behind these choices.
4. Possibly incorporating some existing research as references could support your research questions.

We think these adjustments can help you in executing your proposal.

**Peer review of team 11 by team 10**

Project proposal peer review
Submitted by team 10: <Ines Chellali; Hidde Poel; Kangzhi Qin; Andrei Stoian>
To review the project titled: <Investigating the impact of pheromones on ant foraging speed in a simulated maze>
From team 11: < Kevin Guan, Ivar de Jong, Lydia Shumskaya, Anthony Hessing>

After reading the project proposal thoroughly, using the guidelines on what to review from the initial lecture slide titled “Peer review - Use the template from Canvas!”, we make the following constructive remarks (min 50, max 200 words):

1. It would be helpful to provide a more detailed explanation of how you will quantify maze difficulty, specifically how the number and length of correct paths contribute to it. Also, considering other factors such as the average degree of the nodes might offer a more comprehensive assessment of maze complexity.
2. The description of the model is clear and concise, but to emphasize novelty it might be a good idea to state how this project's model will differ/extend on the prior ones you mentioned.

3. Regarding model validation, maybe be more specific on which statistical methods you plan to use and the reason for choosing them.
4. For the division of work, assigning specific sub-components to individual team members to be responsible for might make it more efficient to work as a team.
5. Adding “testing and iterating the model” into your Week 2 timeline might help to provide a clearer picture of the necessary steps.

We think these adjustments could refine your project and improve its execution.

**Peer review of team 10 by team 9**

Project proposal peer review
Submitted by team: 9, Nicolai Albrecht, Timon Jašarević, Stefano Jonjić, Marijn Oude Groeneger
To review the project titled: N-Body Simulation Integrating Atmospheric and Orbital Dynamics for Optimizing Spacecraft Launch Trajectories
From team: 10, Ines Chellali, Hidde Poel, Kangzhi Qin, Andrei Stoian
After reading the project proposal thoroughly, using the guidelines on what to review from the initial lecture slide titled “Peer review - Use the template from Canvas!”, we make the following constructive remarks:
Very good, all-round proposal. You mentioned various sources for model validation; maybe you could consider including contingency plans if the model results deviate significantly from observations.
For the division of work, you could further improve this by adding names to who is responsible for
each task being carried out. In your current plan, all the work is divided over all team members equally. Having someone responsible for controlling one section of the work could help bring more
efficiency and even avoid complications or last minute stress later in the project.

**Peer review of team 10 by team 8**

Project proposal peer review
Submitted by team: Joel Shefer, Thijs Spoor, David Kraakman
To review the project titled: N-Body Simulation Integrating Atmospheric and Orbital Dynamics for Optimizing Spacecraft Launch Trajectories
From team: Ines Chellali, Hidde Poel, Kangzhi Qin, Andrei Stoian

After reading the project proposal thoroughly, using the guidelines on what to review from the initial lecture slide titled “Peer review - Use the template from Canvas!”, we make the following constructive remarks (min 50, max 200 words):
The proposal you have seems interesting, but quite ambitious for only 4 weeks. Perhaps it would be better to mainly focus on the "temporary satellite-free zone" part, which, in our opinion, is also motivated better in your proposal. If your wish is to still also incorporate the "optimal

trajectory" part in the project, then we suggest substantiating your choices better (e.g., why Jupiter?; In what time frame are you looking for satellite-free zones?; Does the type of rocket make a difference for the optimal trajectory?; What launch location will you use?). For your numerical methods, we would suggest looking into some way to model drag due to Earth's atmosphere. The gravitational pull of satellites among themselves is probably negligible; you could also opt to not simulate these interactions, resulting in way cheaper computation. Probably making the usage of a Barnes-Hut redundant. We would also suggest assigning a coordinator for every part of the project instead of stating that every part will be done by every member. A final remark is to keep the word limit per section in mind (especially for your division of work plan).