CH3-1

7/ 使用第1行餘因子展開

$$\begin{vmatrix} 5 & 3 & 0 & 6 \\ 4 & 6 & 4 & 12 \\ 0 & 2 & -3 & 4 \\ 0 & 1 & -2 & 2 \end{vmatrix} = 5\begin{vmatrix} 6 & 4 & 12 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix} - 4\begin{vmatrix} 3 & 0 & 6 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 6 & 4 & 12 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix} = 6\begin{vmatrix} -3 & 4 \\ -2 & 2 \end{vmatrix} - 2\begin{vmatrix} 4 & 12 \\ -2 & 2 \end{vmatrix} + 1\begin{vmatrix} 4 & 12 \\ -3 & 4 \end{vmatrix}$$

$$= 6\times 2 - 2\times 32 + 1\times 52 = 0.$$

$$\begin{vmatrix} 3 & 0 & 6 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix} = 3\begin{vmatrix} -3 & 4 \\ -2 & 2 \end{vmatrix} - 0\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} + 6\begin{vmatrix} 2 & -3 \\ 1 & -2 \end{vmatrix}$$

$$= 3\times 2 - 0 + 6\times(-1) = 0.$$

$$\therefore \begin{vmatrix} 5 & 3 & 0 & 6 \\ 4 & 6 & 4 & 12 \\ 0 & 2 & -3 & 4 \\ 0 & 1 & -2 & 2 \end{vmatrix} = 5\cdot(0) - 4\cdot(0) = 0 \ \#$$

CH3-1

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$$\begin{vmatrix} 5 & 8 & -4 & 2 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -1 \end{vmatrix} = 5\cdot 0\cdot 2\cdot(-1) = 0 \ \#$$

CH3-2

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① $(-3)R_1 + R_3 \rightarrow R_3$

$R_1 + R_4 \rightarrow R_4$

② R_2 提出(-3)

$$\begin{vmatrix} 4 & -7 & 9 & 1 \\ 6 & 2 & 7 & 0 \\ -9 & 27 & -30 & 0 \\ 4 & 0 & 13 & 0 \end{vmatrix} = (-1)\cdot\begin{vmatrix} 6 & 2 & 7 \\ -9 & 27 & -30 \\ 4 & 0 & 13 \end{vmatrix} = 3\cdot\begin{vmatrix} 6 & 2 & 7 \\ 3 & -9 & 10 \\ 4 & 0 & 13 \end{vmatrix}$$

③ $-2R_2 + R_1 \rightarrow R_1$

$$= 3\cdot\begin{vmatrix} 0 & 20 & -13 \\ 3 & -9 & 10 \\ 4 & 0 & 13 \end{vmatrix} = 3\times\left[(-3)\cdot\begin{vmatrix} 20 & -13 \\ 0 & 13 \end{vmatrix} + 4\cdot\begin{vmatrix} 20 & -13 \\ -9 & 10 \end{vmatrix}\right]$$

$$= 3\times[(-3)\cdot 260 + 4\cdot 83]$$

$$= -1344 \ \#$$

CH3-2

48/ 令B是A的第i列乘以c所得到的矩陣

則

$$\det(B) = \det\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ c\cdot a_{i1} & \cdots & c\,a_{in} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} = c\,a_{i1}C_{i1} + \cdots + c\,a_{in}C_{in}$$

$$= c(a_{i1}C_{i1} + \cdots + a_{in}C_{in})$$

$$= c\cdot\det(A) \ \#$$

CH3-3.

57/ (a) $|A^2| = |A|^2 = (-5)^2 = 25$ #

(b) $|B^2| = |B|^2 = 3^2 = 9.$ #

(c) $|A^3| = |A|^3 = (-5)^3 = -125$ #

(d) $|B^4| = |B|^4 = (3)^4 = 81$ #

59/

$AB = I \Rightarrow |AB| = |A||B| = |I| = 1$, $\because |A||B| = 1$

$\therefore |A| \neq 0$, 且 $|B| \neq 0$ #

CH3-3.

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$$[A \vdots I] = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & \vdots & 1 & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & \vdots & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} — R_1 \\ — R_2 \\ — R_3 \end{array}$$

① $\sqrt{2}R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & -1 & \vdots & \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & \vdots & 0 & 0 & 1 \end{bmatrix}$$

② $-1/\sqrt{2}R_1 + R_3 \to R_3$.

$$\begin{bmatrix} 1 & 0 & -1 & \vdots & \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 2/\sqrt{2} & \vdots & -1 & 0 & 1 \end{bmatrix}$$

③ $\frac{\sqrt{2}}{2}R_3 \to R_3$.

$$\begin{bmatrix} 1 & 0 & -1 & \vdots & \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 1 & \vdots & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$$

④ $R_3 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 1 & \vdots & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} = [I \vdots A^{-1}]$$

$$\therefore A^{-1} = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} = A^T$$ ⇒ 可逆矩陣 A 為正交矩陣 #

80/

若 $A^T = A^{-1}$，則 $|A^T| = |A^{-1}|$

$$|I| = |AA^{-1}| = |A||A^{-1}| = |A||A^T| = |A||A| = |A|^2 = 1$$

$\Rightarrow |A| = \pm 1$ #

CH3-4.

6/

$$A\text{的餘因子矩陣}\begin{bmatrix} \begin{vmatrix} 2 & 3 \\ -1 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ -1 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & -1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ -1 & -2 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\therefore \operatorname{adj}(A) = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \qquad \because \det(A)=0$$

$\Rightarrow$ 矩陣A沒有反矩陣 #

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係數矩陣 $A=\begin{bmatrix} 4 & -1 & -1 \\ 2 & 2 & 3 \\ 5 & -2 & -2 \end{bmatrix}$, $|A|=3$, $\because |A|\neq 0$, $\therefore$ 可使用 Cramer's Rule.

$$A_1 = \begin{array}{c} \begin{matrix} b & x_2 & x_3 \end{matrix} \\ \begin{bmatrix} 1 & -1 & -1 \\ 10 & 2 & 3 \\ -1 & -2 & -2 \end{bmatrix} \end{array}, \quad |A_1| = 3. \qquad \therefore x_1 = \frac{|A_1|}{|A|} = \frac{3}{3} = 1$$

$$A_2 = \begin{array}{c} \begin{matrix} x_1 & b & x_3 \end{matrix} \\ \begin{bmatrix} 4 & 1 & -1 \\ 2 & 10 & 3 \\ 5 & -1 & -2 \end{bmatrix} \end{array}, \quad |A_2| = 3. \qquad x_2 = \frac{|A_2|}{|A|} = \frac{3}{3} = 1$$

$$A_3 = \begin{array}{c} \begin{matrix} x_1 & x_2 & b \end{matrix} \\ \begin{bmatrix} 4 & -1 & 1 \\ 2 & 2 & 10 \\ 5 & -2 & -1 \end{bmatrix} \end{array}, \quad |A_3| = 6. \qquad x_3 = \frac{|A_3|}{|A|} = \frac{6}{3} = 2$$

#

CH3-4

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$$\det\begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{bmatrix} = \det\begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 2 & 1 \end{bmatrix} = 0$$

∴ 此四點是共平面 #

Review

34/

(a)

① $3R_1+R_2\to R_2$, $-2R_1+R_3\to R_3$

② $R_2\leftrightarrow R_3$, $-R_2\to R_2$

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ -3 & 1 & -2 & 1 \\ 2 & 3 & -1 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 7 & 1 & 13 \\ 0 & -1 & -3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & 3 & -1 \\ 0 & 7 & 1 & 13 \end{bmatrix}$$

③ $-7R_2+R_3\to R_3$

④ $-R_3\to R_3$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & -20 & 20 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

∴ $x_3=-1$

$x_2=2$

$x_1=1$ #

(b)

① $-2R_1+R_1\to R_1$, $-7R_2+R_3\to R_3$

② $\frac{-1}{20}R_3\to R_3$

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & 3 & -1 \\ 0 & 7 & 1 & 13 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -5 & 6 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & -20 & 20 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -5 & 6 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

③ $5R_3+R_1\to R_1$, $-3R_3+R_2\to R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

∴ $x_3=-1$

$x_2=2$

$x_1=1$

(9) Cramer's Rule.

$$A=\begin{bmatrix} 1 & 2 & 1 \\ -3 & 1 & -2 \\ 2 & 3 & -1 \end{bmatrix},\ |A|=-20.$$

$$A_1=\begin{bmatrix} 4 & 2 & 1 \\ 1 & 1 & -2 \\ 9 & 3 & -1 \end{bmatrix},\ |A_1|=-20.$$

$$A_2=\begin{bmatrix} 1 & 4 & 1 \\ -3 & 1 & 2 \\ 2 & 9 & 1 \end{bmatrix},\ |A_2|=-40.$$

$$A_3=\begin{bmatrix} 1 & 2 & 4 \\ -3 & 1 & 1 \\ 2 & 3 & 9 \end{bmatrix},\ |A_3|=20.$$

$$\therefore\ X_1=\frac{-20}{-20}=1,\ X_2=\frac{-40}{-20}=2,\ X_3=\frac{20}{-20}=-1\ \#.$$

Cumulative Test.

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$$a\begin{bmatrix}1\\0\\1\end{bmatrix}+b\begin{bmatrix}1\\1\\0\end{bmatrix}+c\begin{bmatrix}0\\1\\1\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}.$$

① $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 3 \end{bmatrix}$

② $-R_1+R_3 \rightarrow R_3$ $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & 1 & 2 \end{bmatrix}$

③ $R_2+R_3 \rightarrow R_3.$ $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$

$\therefore\ c=2,\ b=0,\ a=1\ \#.$