McGill

Dec 2020

Final Examination

Theory of Computation

COMP 330 SEC 001

6:30pm-Dec 11 to 6:30pm-Dec 14, 2020

EXAMINER:	Hamed Hatami	ASSOC. EXAMINER:	

STUDENT NAME:		**McGILL ID:**									

INSTRUCTIONS

CLOSED BOOK ☐ OPEN BOOK ☒

- Submit a pdf file - either typed, or a clear and readable scan - of your solution.
- You can use the textbook, and lecture notes, but you are not allowed to search the internet for solutions to these questions.

1	2	3	4	5	6	7	8	**Total**
/10	/10	/10	/15	/10	/15	/10	/20	**/100**

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1. True or False? (Prove that your answer is correct). If L is a regular language and $A \subseteq L$, then A is decidable.

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2. True or False? (Prove that your answer is correct). If L_1 is decidable and L_2 is Turing recognizable, then $L_1 \cap L_2$ is decidable.

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3. True or False? (Prove that your answer is correct). If L_1 and L_2 are in NP, then $L_1 \cap L_2$ is in NP.

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4. Describe the language of the following TM over the alphabet $\{0, 1, 2\}$: There are three states q_{start}, q_{accept}, and q_{reject}. Three arrows from q_{start} to itself with labels $\underline{1 \to 0, R}$ and $\underline{0 \to 1, L}$, and $\underline{2 \to 2, L}$. There is also one arrow from q_{start} to q_{accept} with label $\underline{\sqcup \to \sqcup, R}$.

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5. Rigorously establish the decidability or undecidability of the following language:

$$L = \{\langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TM's and } L(M_1) \subseteq L(M_2)\}.$$

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6. Let us call a deterministic Turing Machine M super-fast if there exists a constant c (here c can depend on M but it does not depend on the input) such that the following holds: On every input w, the TM M halts after at most c steps. Rigorously establish the decidability or undecidability of the following languages:

$$L = \{\langle M \rangle \mid M \text{ is a super-fast TM}\}.$$

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7. Let L be the set of all $\langle p \rangle$ such that p is a multivariate polynomial with integer coefficients that evaluates to zero for some assignment of positive integers to its variables. For example $\langle x_1^2 + x_2^2 - 5 \rangle \in L$ as it evaluates to 0 if we set $x_1 = 1$ and $x_2 = 2$. On the other hand $\langle x_1^2 - 5 \rangle$ is not in L. Prove that L can be decided by an *oracle* Turing Machine that has access to an oracle for

$$\text{Halt}_{\text{TM}} = \{\langle M, w \rangle \mid M \text{ is a TM that halts on input } w\}.$$

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8. Consider the following language:

$$L = \{\langle M, w \rangle | M \text{ is a TM and } L(M) = \{w\}\}.$$

   (a) Is L Turing recognizable? (Prove your answer)

   (b) Is the complement of L Turing recognizable?(Prove your answer)