COMP424 Assignment 4

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Question 1: Health Behaviours

a.

$$P(D = 1 \mid C = 1) = \frac{28 + 8}{44 + 28 + 84 + 8} = \frac{9}{41}$$

$$P(D = 1 \mid C = 0) = \frac{13 + 124}{10 + 13 + 689 + 124} = \frac{137}{836}$$

b.

$$P(D = 1 \mid \mathrm{do}(C = 1)) = \sum_h P(D = 1 \mid C = 1, H = h)P(H = h)$$

$$= P(D = 1 \mid C = 1, H = 1)P(H = 1) + P(D = 1 \mid C = 1, H = 0)P(H = 0)$$

$$= (\frac{8}{92} * \frac{1}{2}) + (\frac{28}{72} * \frac{1}{2}) = \frac{197}{828}$$

$$P(D = 1 \mid \mathrm{do}(C = 0)) = \sum_h P(D = 1 \mid C = 0, H = h)P(H = h)$$

$$= P(D = 1 \mid C = 0, H = 1)P(H = 1) + P(D = 1 \mid C = 0, H = 0)P(H = 0)$$

$$= (\frac{124}{813} * \frac{1}{2}) + (\frac{13}{23} * \frac{1}{2}) = \frac{13421}{37398}$$

$$RRR = 1 - \frac{P(D = 1 \mid \mathrm{do}(C = 1))}{P(D = 1 \mid \mathrm{do}(C = 0))}$$

$$= 1 - \frac{\frac{197}{828}}{\frac{13421}{37398}} = \frac{27139}{80526}$$

Question 2: Fire Hazard

a. We know the following:

$$\begin{aligned}
P(F) &= P(R=F)*P(O=F)*P(F=T \mid R=F,O=F)\\
&+P(R=F)*P(O=T)*P(F=T \mid R=F,O=T)\\
&+P(R=T)*P(O=F)*P(F=T \mid R=T,O=F)\\
&+P(R=T)*P(O=T)*P(F=T \mid R=T,O=T)\\
&=(0.25*0.15*0.005)+(0.25*0.85*0.02)\\
&+(0.75*0.15*0.005)+(0.75*0.85*0.01)\\
&=0.011375
\end{aligned}$$

Now, we calculate expected utility with and without insurance:

$$EU_{\text{insurance}} = (-1000*0.011375)+(-50*0.988625) = -60.80625$$
$$EU_{\text{no insurance}} = (-50000*0.011375)+(0*0.988625) = -568.75$$

Finally, we calculate the optimal price the company should charge to break even:

$$\begin{aligned}
Pr_{\text{optimal}} &= EU_{\text{insurance}} - EU_{\text{no insurance}}\\
&= -60.80625-(-568.75)\\
&\approx 507.94
\end{aligned}$$

b. First, we know the following:

$$P(F=T \mid R=F,O=T) = 0.02$$

$$\begin{aligned}
P(F=F \mid R=F,O=T) &= 1-P(F=T \mid R=F,O=T)\\
&= 1-0.02 = 0.98
\end{aligned}$$

Now, we calculate expected utility with and without insurance:

$$EU_{\text{insurance}} = (-1000*0.02)+(-50*0.98) = -69$$
$$EU_{\text{no insurance}} = (-50000*0.02)+(0*0.98) = -100$$

Finally, we calculate the optimal price the company should charge:

$$\begin{aligned}
Pr_{\text{optimal}} &= EU_{\text{insurance}} - EU_{\text{no insurance}}\\
&= -69-(-100)\\
&= 31
\end{aligned}$$

c. First, we know the following:

$$P(F = T \mid R = T)$$

$$= P(O = T) {*} P(F = T \mid R = T, O = T) + P(O = F) {*} P(F = T \mid R = T, O = F)$$

$$= (0.85 * 0.01) + (0.15 * 0.005)$$

$$= 0.00925$$

$$P(F = F \mid R = T) = 1 - P(F = T \mid R = T)$$

$$= 1 - 0.00925 = 0.99075$$

Now, we calculate expected utility with and without insurance:

$$EU_{\text{insurance}} = (-1000 * 0.00925) + (-50 * 0.99075) = -58.7875$$

$$EU_{\text{no insurance}} = (-50000 * 0.00925) + (0 * 0.99075) = -462.5$$

Finally, we calculate the optimal price the company should charge:

$$Pr_{\text{optimal}} = EU_{\text{insurance}} - EU_{\text{no insurance}}$$

$$= -58.7875 - (-462.5)$$

$$\approx 403.71$$

d. We calculate expected utility with the new insurance:

$$EU_{\text{insurance}_2} = (-1000 {*} 0.011375 {*} 0.75) + (-50000 {*} 0.011375 {*} 0.25) + (-50 {*} 0.988625)$$

$$= -72.18125$$

We now calculate the price the company should pay for the new insurance to make it competitive (we use $EU_{\text{no insurance}}$ from part a):

$$Pr_{\text{optimal}} = EU_{\text{insurance}_2} - EU_{\text{no insurance}}$$

$$= -72.18125 - (-568.75)$$

$$\approx 496.57$$

Question 3: Bandits

a. We first calculate the action-reward pairs for $t = 3$ and $t = 4$.

$t = 3$:

$$A_3 = ((3-1) \mod 6) + 1 = 3$$

$$R_3 = 2\cos\left[\frac{\pi}{6}(3-1)\right] = 1$$

$t = 4$:

$$A_4 = ((4-1) \mod 6) + 1 = 4$$

$$R_4 = 2\cos\left[\frac{\pi}{6}(4-1)\right] = 0$$

At $t = 0$, $Q(0)$ is given by $\{1, 2, 2, 1, 0, 3\}$. At $t = 1$, Q_1 is updated to

$$Q_1 = \frac{2}{1} = 2$$

, and thus $Q(1)$ is given by $\{2, 2, 2, 1, 0, 3\}$.

At $t = 2$, Q_2 is updated to

$$Q_2 = \frac{\sqrt{3}}{1} = \sqrt{3}$$

, and thus $Q(2)$ is given by $\{2, \sqrt{3}, 2, 1, 0, 3\}$.

At $t = 3$, Q_3 is updated to

$$Q_3 = \frac{1}{1} = 1$$

, and thus $Q(3)$ is given by $\{2, \sqrt{3}, 1, 1, 0, 3\}$.

At $t = 4$, Q_4 is updated to

$$Q_4 = \frac{0}{1} = 0$$

, and thus $Q(4)$ is given by $\{2, \sqrt{3}, 1, 0, 0, 3\}$.

b. At $t = 1$, we know that $Q(0)$ is given by $\{1, 2, 2, 1, 0, 3\}$. Therefore, the best arm according to current estimates would be arm 6 (with a value of 3). However, arm 1 was chosen, therefore, it can be concluded with certainty that a random action was selected.

At $t = 2$, we know that $Q(1)$ is given by $\{2, 2, 2, 1, 0, 3\}$. Therefore, the best arm according to current estimates would be arm 6 (with a value of 3). However, arm 2 was chosen, therefore, it can be concluded with certainty that a random action was selected.

At $t = 3$, we know that $Q(2)$ is given by $\{2, \sqrt{3}, 2, 1, 0, 3\}$. Therefore, the best arm according to current estimates would be arm 6 (with a value of 3). However, arm 3 was chosen, therefore, it can be concluded with certainty that a random action was selected.

At $t = 4$, we know that $Q(3)$ is given by $\{2, \sqrt{3}, 1, 1, 0, 3\}$. Therefore, the best arm according to current estimates would be arm 6 (with a value of 3). However, arm 4 was chosen, therefore, it can be concluded with certainty that a random action was selected.