COMP424 Assignment 2

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Question 1: Planning in STRIPS [30]

a. The Initial state description is given by:

   At(Monkey, A) ∧ At(Bananas, B) ∧ At(Box, C) ∧ Height(Monkey, Low) ∧ Height(Box, Low) ∧ Height(Box, High)

b. The six action schemas are given by:

   Go(x,y)

   Preconditions = At(Monkey, x) ∧ Height (Monkey, Low)

   Effect = At(Monkey, y) ∧ ¬At(Monkey, x)

   Push(b, x, y)

   Preconditions = At(Monkey, x) ∧ At (b, x) ∧ Height (Monkey, Low) ∧ Height(b, Low)

   Effect = At(b, y) ∧ ¬At(b, x) ∧ At(Monkey, y) ∧ ¬At(Monkey, x)

   ClimbUp(x, b)

   Preconditions = At(Monkey, x) ∧ At (b, x) ∧ Height(Monkey, Low) ∧ Height(b, Low)

   Effect = On(Monkey, b) ∧ ¬Height(Monkey, Low) ∧ Height(Monkey, High)

   ClimbDown(b, x)

   Preconditions = On(Monkey, b) ∧ At(Monkey, x) ∧ At (b, x) ∧ Height(Monkey, High) ∧ Height(b, Low)

   Effect = ¬On(Monkey, b) ∧ ¬Height(Monkey, High) ∧ Height(Monkey, Low)

   Grasp(x, b, h)

   Preconditions = At(Monkey, x) ∧ At(b, x) ∧ Height(Monkey, h) ∧ Height (b, h)

Effect = Holds(b, Monkey) ∧ ¬At(b, x) ¬Height(b, h)

Ungrasp(x, b, h)

Preconditions = At(Monkey, x) ∧ Height(Monkey, h) ∧ Holds(b, Monkey)

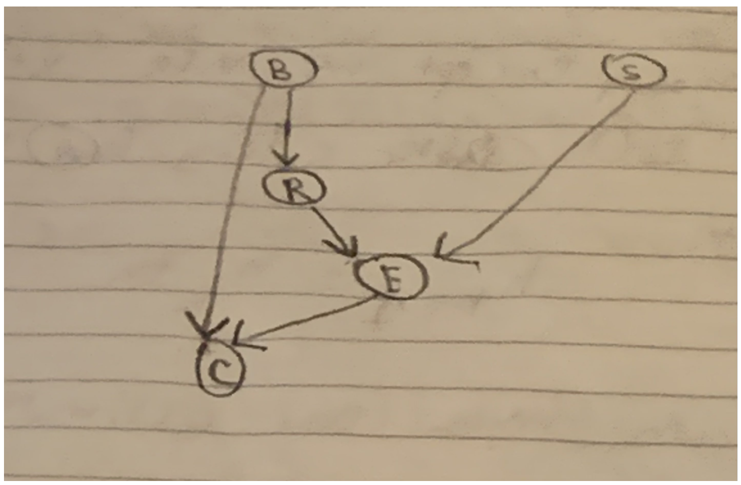
Effect = At(b, x) ∧ Height (b, h) ∧ ¬Holds(b, Monkey)

c. The goal is given by:

Holds(Bananas, Monkey) ∧ At(Box, C)

Question 2: Designing a Bayesian Network [30]

a. The Bayesian network for this domain is shown below:



b. The conditional probability table associated with R is given by:

$P(R \mid B)$		
	$R = 1$	$R = 0$
$B = 0$	y	$1 - y$
$B = 1$	x	$1 - x$

c. The conditional probability table associated with E is given by:

$P(E \mid R, S)$		
	$E = 1$	$E = 0$
$R = 0, S = 0$	0	1
$R = 0, S = 1$	0.5	0.5
$R = 1, S = 0$	1	0
$R = 1, S = 1$	1	0

d. We are looking for $P(R=1 \mid C=1) = \frac{P(R=1,C=1)}{P(C=1)}$. Now, we know the following:

$$\frac{P(R=1,C=1)}{P(C=1)} = \frac{\sum_{b,s,e} P(B)P(S)P(R=1 \mid B)P(E \mid R=1,S)P(C=1 \mid B,E)}{\sum_{b,s,r,e} P(B)P(S)P(R \mid B)P(E \mid R,S)P(C=1 \mid B,E)}$$

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Question 3: Inference in Bayesian Networks [40]

a. We calculate $P(a, \neg r)$ as follows:

$$P(a, \neg r) = \sum_{B,T,S} P(\neg r)P(B)P(T \mid \neg r, B, a)P(a \mid B)P(S \mid a)$$

Factoring out $P(\neg r)$:

$$= P(\neg r) \sum_{B,T,S} P(B)P(T \mid \neg r, B, a)P(a \mid B)P(S \mid a)$$

Expanding for the random variable S:

$$= P(\neg r)(P(s \mid a) + P(\neg s \mid a))(\sum_{B,T} P(B)P(T \mid \neg r, B, a)P(a \mid B))$$

We know that $P(s \mid a) + P(\neg s \mid a) = 1$:

$$= P(\neg r) \sum_{B,T} P(B)P(T \mid \neg r, B, a)P(a \mid B)$$

Expanding the sum:

$$= P(\neg r)[(P(b)P(t \mid \neg r, b, a)P(a \mid b)) + (P(b)P(\neg t \mid \neg r, b, a)P(a \mid b))$$

$$+(P(\neg b)P(t \mid \neg r, \neg b, a)P(a \mid \neg b)) + (P(\neg b)P(\neg t \mid \neg r, \neg b, a)P(a \mid \neg b))]$$

Plugging in the values:

$$= 0.9[(0.5*0.60*0.15)+(0.5*0.4*0.15)+(0.5*0.65*0.05)+(0.5*0.35*0.05)]$$

$$= 0.09$$

b. We calculate $P(b, a)$ as follows:

$$P(b, a) = \sum_{R,T,S} P(R)P(b)P(T \mid R, b, a)P(a \mid b)P(S \mid a)$$

Factoring out $P(b)$ and $P(a \mid b)$:

$$= P(b)P(a \mid b) \sum_{R,T,S} P(R)P(T \mid R, b, a)P(S \mid a)$$

Expanding for the random variable S:

$$= P(b)P(a \mid b)(P(s \mid a) + P(\neg s \mid a))(\sum_{R,T} P(R)P(T \mid R, b, a))$$

We know that $P(s \mid a) + P(\neg s \mid a) = 1$:

$$= P(b)P(a \mid b) \sum_{R,T} P(R)P(T \mid R, b, a)$$

Expanding the sum:

$$= P(b)P(a \mid b)[(P(r)P(t \mid r, b, a)) + (P(r)P(\neg t \mid r, b, a))$$
$$+(P(\neg r)P(t \mid \neg r, b, a)) + (P(\neg r)P(\neg t \mid \neg r, b, a))]$$

Plugging in the values:

$$= 0.5 * 0.15[(0.1 * 0.95) + (0.1 * 0.05) + (0.9 * 0.6) + (0.9 * 0.4)]$$
$$= 0.075$$

c. We calculate $P(t \mid b)$ as follows:

$$P(t \mid b) = \frac{P(t, b)}{P(b)}$$

We first calculate $P(t, b)$:

$$P(t, b) = \sum_{R,A,S} P(R)P(b)P(t \mid R, b, A)P(A \mid b)P(S \mid A)$$

Factoring out $P(b)$:

$$= P(b) \sum_{R,A,S} P(R)P(t \mid R, b, A)P(A \mid b)P(S \mid A)$$

We use the following table to calculate the sum:

	$P(R)$	$P(t \mid R, b, A)$	$P(A \mid b)$	$P(S \mid A)$	Π
$r \quad a \quad s$	0.1	0.95	0.15	0.75	0.0106875
$r \quad a \quad \neg s$	0.1	0.95	0.15	0.25	0.0035625
$r \quad \neg a \quad s$	0.1	0.8	0.85	0.05	0.0034
$r \quad \neg a \quad \neg s$	0.1	0.8	0.85	0.95	0.0646
$\neg r \quad a \quad s$	0.9	0.6	0.15	0.75	0.06075
$\neg r \quad a \quad \neg s$	0.9	0.6	0.15	0.25	0.02025
$\neg r \quad \neg a \quad s$	0.9	0.3	0.85	0.05	0.011475
$\neg r \quad \neg a \quad \neg s$	0.9	0.3	0.85	0.95	0.218025

We therefore know that:

$$P(t,b) = P(b)(0.0106875 + 0.0035625 + 0.0034 + 0.0646$$

$$+0.06075 + 0.02025 + 0.011475 + 0.218025)$$

$$= P(b)(0.39275)$$

We now know that

$$P(t \mid b) = \frac{P(t,b)}{P(b)} = \frac{P(b)(0.39275)}{P(b)} = 0.39275$$

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d. We calculate $P(r \mid \neg t, \neg s)$ as follows:

$$P(r \mid \neg t, \neg s) = \frac{P(r, \neg t, \neg s)}{P(\neg t, \neg s)}$$

. We first calculate $P(r, \neg t, \neg s)$ as:

$$P(r, \neg t, \neg s) = \sum_{B,A} P(r)P(B)P(\neg t \mid r, B, A)P(A \mid B)P(\neg s \mid A)$$

Factoring out $P(r)$:

$$= P(r, \neg t, \neg s) = P(r) \sum_{B,A} P(B)P(\neg t \mid r, B, A)P(A \mid B)P(\neg s \mid A)$$

Expanding the sum:

$$= P(r)[(P(b)P(\neg t \mid r, b, a)P(a \mid b)P(\neg s \mid a)) + (P(b)P(\neg t \mid r, b, \neg a)P(\neg a \mid b)P(\neg s \mid \neg a))$$

$$+(P(\neg b)P(\neg t \mid r, \neg b, a)P(a \mid \neg b)P(\neg s \mid a)) + (P(\neg b)P(\neg t \mid r, \neg b, \neg a)P(\neg a \mid \neg b)P(\neg s \mid \neg a))]$$

We therefore know that

$$P(r, \neg t, \neg s) = 0.1[(0.5 * 0.05 * 0.15 * 0.25) + (0.5 * 0.2 * 0.85 * 0.95)$$

$$+(0.5 * 0.1 * 0.95 * 0.25) + (0.5 * 0.3 * 0.95 * 0.95)]$$

$$= 0.1 * 0.2289375 = 0.02289375$$

We now calculate $P(\neg t, \neg s)$ as:

$$P(\neg t, \neg s) = \sum_{R,B,A} P(R)P(B)P(\neg t \mid R, B, A)P(A \mid B)P(\neg s \mid A)$$

We use the following table to calculate the sum:

	$P(R) * P(B)$	$P(\neg t \mid R, B, A)$	$P(A \mid B)$	$P(\neg s \mid A)$	Π
r b a	0.05	0.05	0.15	0.25	0.00009375
r b $\neg a$	0.05	0.2	0.85	0.95	0.008075
r $\neg b$ a	0.45	0.1	0.05	0.25	0.0005625
r $\neg b$ $\neg a$	0.45	0.3	0.95	0.95	0.1218375
$\neg r$ b a	0.05	0.4	0.15	0.25	0.00075
$\neg r$ b $\neg a$	0.05	0.7	0.85	0.95	0.0282625
$\neg r$ $\neg b$ a	0.45	0.35	0.05	0.25	0.00196875
$\neg r$ $\neg b$ $\neg a$	0.45	0.95	0.95	0.95	0.38581875

We therefore know that

$$P(\neg t, \neg s) = 0.00009375 + 0.008075 + 0.0005625 + 0.1218375$$

$$+0.00075 + 0.0282625 + 0.00196875 + 0.38581875 = 0.54736875$$

Finally, we know that

$$P(r \mid \neg t, \neg s) = \frac{P(r, \neg t, \neg s)}{P(\neg t, \neg s)} = \frac{0.02289375}{0.54736875} = .0418250951$$