COMP424 Assignment 2

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Question 1: Constraint satisfaction [25]

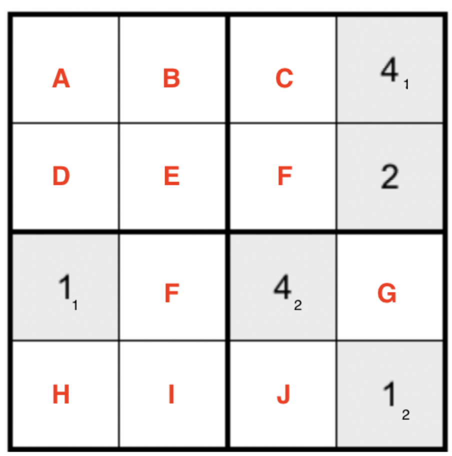
(a) Formulating the problem as a CSP, we define the variables, their domains, and the constraints as following:

Variables: The numbers assigned in each square. There are 16 of these.

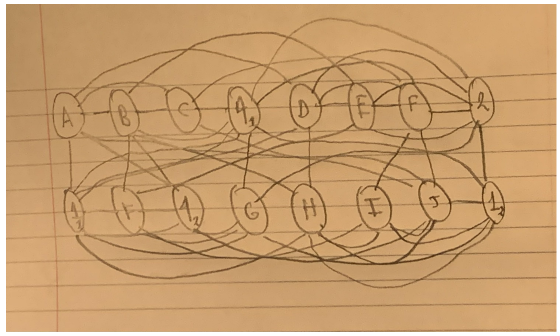
Domain: $\{1, 2, 3, 4\}$

Constraints: Each row, column and 2x2 square rooted in one of the 4 corners should contain the digits “1”, “2”, “3”, “4” exactly once.

To draw the constraint graph, we first label the configuration as follows:



The constraint graph is then given below:



(b) The first ten steps of backtracking search on this problem are shown below:

Step 1: Placing a 1 in position (1,1) fails (there exists a 1 in the same row). Backtrack. Placing a 2 in position (1,1) succeeds.

Step 2: Placing a 1 in position (2,1) fails (there exists a 1 in the same row). Backtrack. Placing a 2 in position (2,1) fails (there exists a 2 in the same row). Backtrack. Placing a 3 in position (2,1) succeeds.

Step 3: Placing a 1 in position (4,1) fails (there exists a 1 in the same row). Backtrack. Placing a 2 in position (4,1) fails (there exists a 2 in the same row). Backtrack. Placing a 3 in position (4,1) fails (there exists a 3 in the same row). Backtrack. Placing a 4 in position (4,1) succeeds.

Step 4: Placing a 1 in position (1,2) succeeds.

Step 5: Placing a 1 in position (2,2) fails (there exists a 1 in the same row). Backtrack. Placing a 2 in position (2,2) fails (there exists a 2 in the same square). Backtrack. Placing a 3 in position (2,2) fails (there exists a 3 in the same column). Backtrack. Placing a 4 in position (2,2) succeeds.

Step 6: Placing a 1 in position (3,2) fails (there exists a 1 in the same row). Backtrack. Placing a 2 in position (3,2) succeeds.

Step 7: Placing a 1 in position (4,2) fails (there exists a 1 in the same row). Backtrack. Placing a 2 in position (4,2) fails (there exists a 2 in the same row). Backtrack. Placing a 3 in position (4,2) succeeds.

Step 8: Placing a 1 in position (1,3) fails (there exists a 1 in the same column). Backtrack. Placing a 2 in position (1,3) fails (there exists a 2 in the same column). Backtrack. Placing a 3 in position (1,3) succeeds.

Step 9: Placing a 1 in position (2,3) succeeds.

Step 10: Placing a 1 in position (4,3) fails (there exists a 1 in the same row). Backtrack. Placing a 2 in position (4,3) succeeds.

(c) The first ten steps of backtracking search on this problem with one-step forward checking are shown below:

Step 1: Placing a 1 in position (1,1) fails (there exists a 1 in the same row). Backtrack. Placing a 2 in position (1,1) succeeds. Do not allow any other 2's in this row, column, and square.

Step 2: Placing a 1 in position (2,1) fails (there exists a 1 in the same row). Backtrack. Placing a 3 in position (2,1) succeeds. Do not allow any other 3's in this row, column, and square.

Step 3: Placing a 1 in position (4,1) fails (there exists a 1 in the same row). Backtrack. Placing a 4 in position (4,1) succeeds. Do not allow any other 4's in this row, column, and square.

Step 4: Placing a 1 in position (1,2) succeeds. Do not allow any other 1's in this row, column, and square.

Step 5: Placing a 4 in position (2,2) succeeds. Do not allow any other 4's in this row, column, and square.

Step 6: Placing a 2 in position (3,2) succeeds. Do not allow any other 2's in this row, column, and square.

Step 7: Placing a 3 in position (4,2) succeeds. Do not allow any other 3's in this row, column, and square.

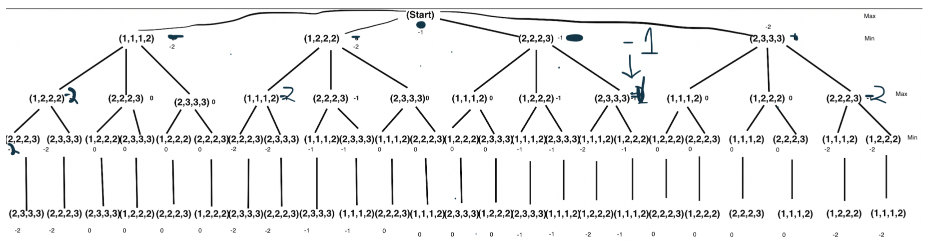
Step 8: Placing a 3 in position (1,3) succeeds. Do not allow any other 3's in this row, column, and square.

Step 9: Placing a 1 in position (2,3) succeeds. Do not allow any other 1's in this row, column, and square.

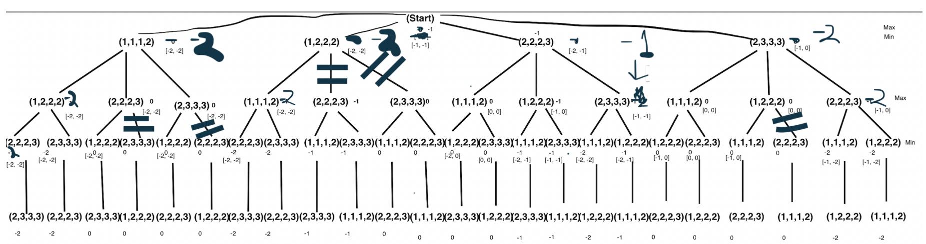
Step 10: Placing a 2 in position (4,3) succeeds. Do not allow any other 2's in this row, column, and square.

Question 2: Search and Game Playing [30]

(a) The search tree used for Minimax with evaluations at each node is shown below:



(b) The search tree used for alpha-beta pruning with evaluations given within square brackets [] at each node is shown below:



Because several branches are pruned, there is an advantage to using alpha-beta pruning.

Question 3: Propositional Logic [20]

(a) i. We know that the following is true:

$$\neg(A \land B) \lor \neg(C \land \neg B)$$

$$\equiv (\neg A \lor \neg B) \lor (\neg C \lor B) \quad \text{By De Morgan's Laws}$$

$$\equiv \neg A \lor \neg C \lor (\neg B \lor B)$$

$$\equiv \neg A \lor \neg C \lor 1$$

$$\equiv 1$$

.

Therefore, $\neg(A \land B) \lor \neg(C \land \neg B)$ is true in all $2^3 = 8$ models.

ii. The truth table for $A \Rightarrow (A \wedge B) \wedge \neg B \vee \neg C$ is given by:

A	B	C	$A \Rightarrow (A \wedge B) \wedge \neg B \vee \neg C$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

. Therefore, $A \Rightarrow (A \wedge B) \wedge \neg B \vee \neg C$ is true in 5 models.

iii. The truth table for $\neg(A \Rightarrow B \wedge C \wedge D) \vee (B \Rightarrow \neg C)$ is given by:

A	B	C	D	$\neg(A \Rightarrow B \wedge C \wedge D) \vee (B \Rightarrow \neg C)$
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

. Therefore, $\neg(A \Rightarrow B \wedge C \wedge D) \vee (B \Rightarrow \neg C)$ is true in 13 models.

(b) i. The truth table for $((A \vee B) \Rightarrow C) \Rightarrow (A \Rightarrow C) \wedge (B \Rightarrow C)$ is given by:

A	B	C	$((A \vee B) \Rightarrow C) \Rightarrow (A \Rightarrow C) \wedge (B \Rightarrow C)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

. Therefore, $((A \lor B) \Rightarrow C) \Rightarrow (A \Rightarrow C) \land (B \Rightarrow C)$ is valid.

ii. The truth table for $((A \Rightarrow B) \land (A \lor C)) \Rightarrow (B \lor C)$ is given by:

A	B	C	$((A \Rightarrow B) \land (A \lor C)) \Rightarrow (B \lor C)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

. Therefore, $((A \Rightarrow B) \land (A \lor C)) \Rightarrow (B \lor C)$ is valid.

Question 4: First Order Logic [30]

(a) Let x and y represent predicates and z represent occupations in the following assertions.

i. Emily is either a surgeon or a lawyer.:

$$\text{Occupation(Emily, Surgeon)} \lor \text{Occupation(Emily, Lawyer)}$$

ii. Joe is an actor, but he also holds another job.:

$$\text{Occupation(Joe, Actor)} \land (\exists z : \text{Occupation(Joe, } z) \land \neg (z = \text{Actor}))$$

iii. All surgeons are doctors.:

$$\forall x : \text{Occupation}(x, \text{Surgeon}) \Rightarrow \text{Occupation}(x, \text{Doctor})$$

iv. Joe does not have a lawyer.:

$$\forall x : \text{Occupation}(x, \text{Lawyer}) \Rightarrow \neg \text{Customer(Joe, } x)$$

v. Emily has a boss who is a lawyer.:

$$\exists x : \text{Occupation}(x, \text{Lawyer}) \ \land \ \text{Boss}(x, \text{Emily})$$

vi. There exists a lawyer all of whose customers are doctors.:

$$\exists x \forall y : \text{Occupation}(x, \text{Lawyer}) \land (\ \text{Customer}(y, x) \Rightarrow \text{Occupation}(y, \text{Doctor}))$$

vii. Every surgeon has a lawyer.:

$$\forall x : \text{Occupation}(x, \text{Surgeon}) \Rightarrow (\exists y : \text{Occupation}(y, \text{Lawyer}) \land \text{Customer}(x, y))$$

(b) A model that satisfies all the clauses is given by:

$$m = \{\text{Occupation(Emily, Surgeon), Occupation(Joe, Actor),}$$
$$\text{Occupation(Joe, Lawyer), Occupation(Emily, Doctor),}$$
$$\text{Boss(Joe, Emily), Customer(Emily, Joe)}\}$$