コンピュータ科学特別講義Ⅳ

Parallel Algorithm Design（#6）

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June 22, 2018

Please download handouts before class from
http://www.pdsl.jp/class/utyo2018/

Graduate School of Information Science, Nagoya Univ.
Parallel & Distributed Systems Lab.

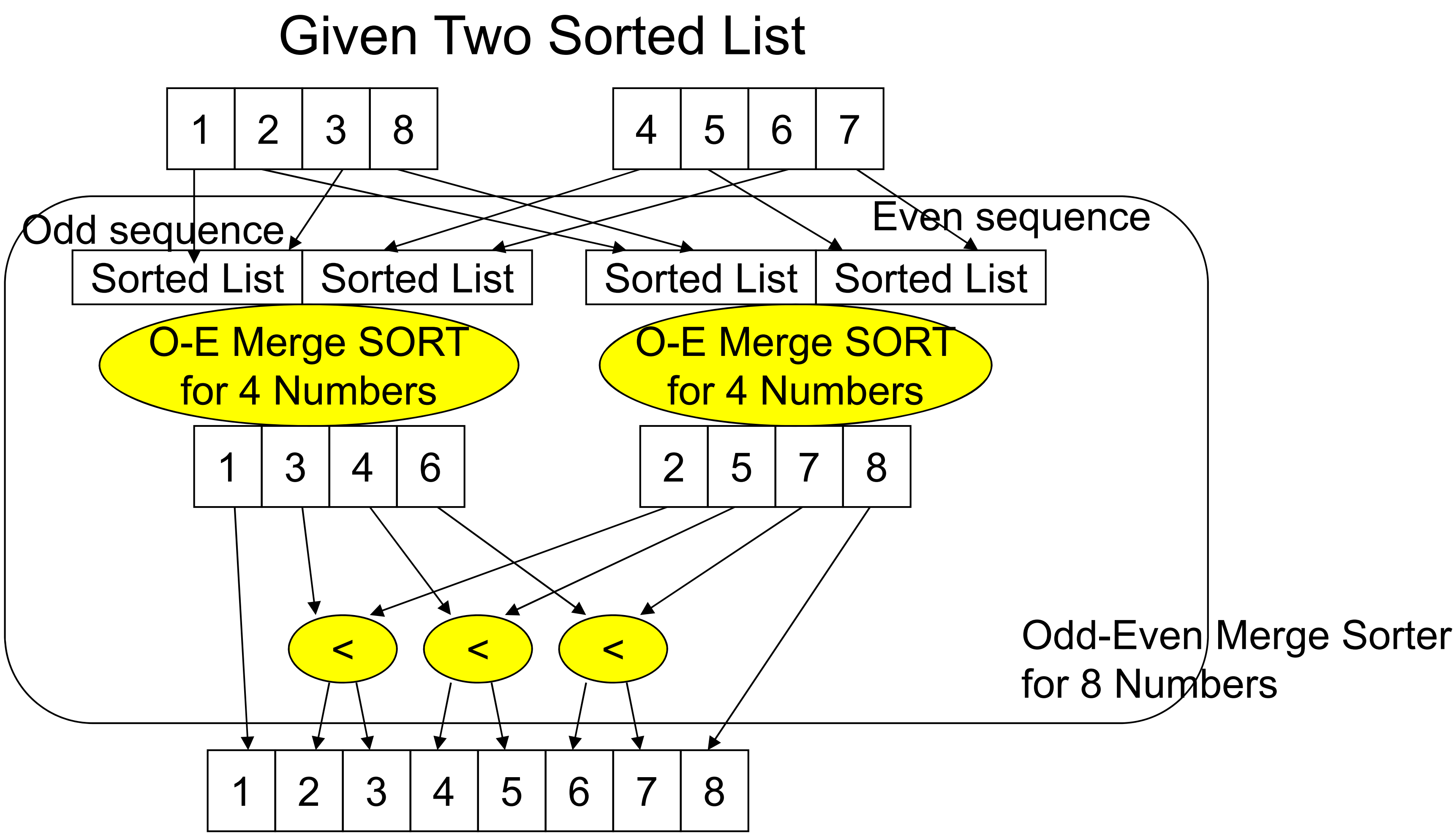
Contents of This Class

- Our Target
  - Understand Systems and Algorithms on “Multi-Core” processors
- Schedule (Tentative)
  - #1 April 6 (=Today) What’s “Multi-Core”?
  - #2 April 13 : Parallel Programming Languages (Ex. 1)
  - April 20, 27, May 4, 11, 18: NO CLASS
  - #3 May 25 : Parallel Algorithm Design
  - #4 June 1 (Fri) : Laws on Multi-Core
  - #5 June 8 : Examples of Parallel Algorithms (1) (Ex. 2)
  - June 15: NO CLASS
  - #6 June 22 : Examples of Parallel Algorithms (2)
  - #7 June 29 : Examples of Parallel Algorithms (3)
  - #8 July 6 : Examples of Parallel Algorithms (4)
  - #9 July 13 : Examples of Parallel Algorithms (5) (Ex. 3)
  - (July 20)
  - If you want to graduate in August, ask Edahiro asap.

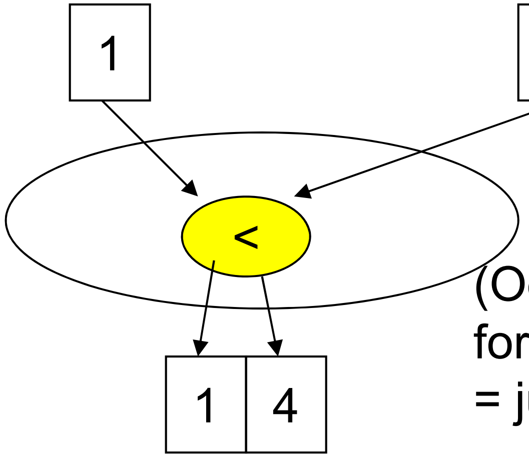
Sorting (2)

- Given a sequence of n numbers, reorder the numbers in increasing (decreasing) order
- Theoretical algorithms

Odd-Even Merge Sort: Basic Idea
(for 8 Numbers)

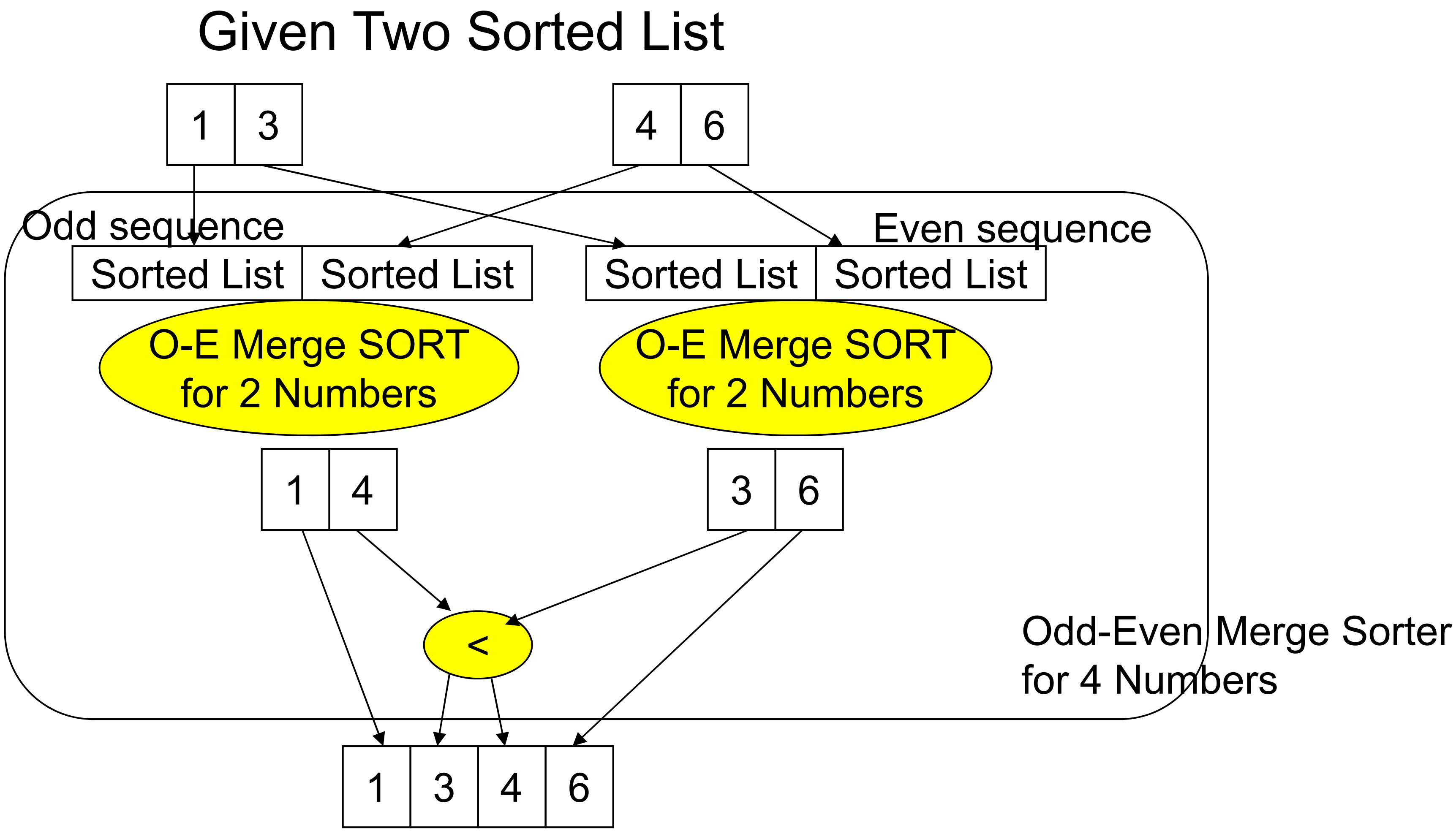


(Odd-Even Merge) Sorter for 2 Numbers (Just a Comparator)

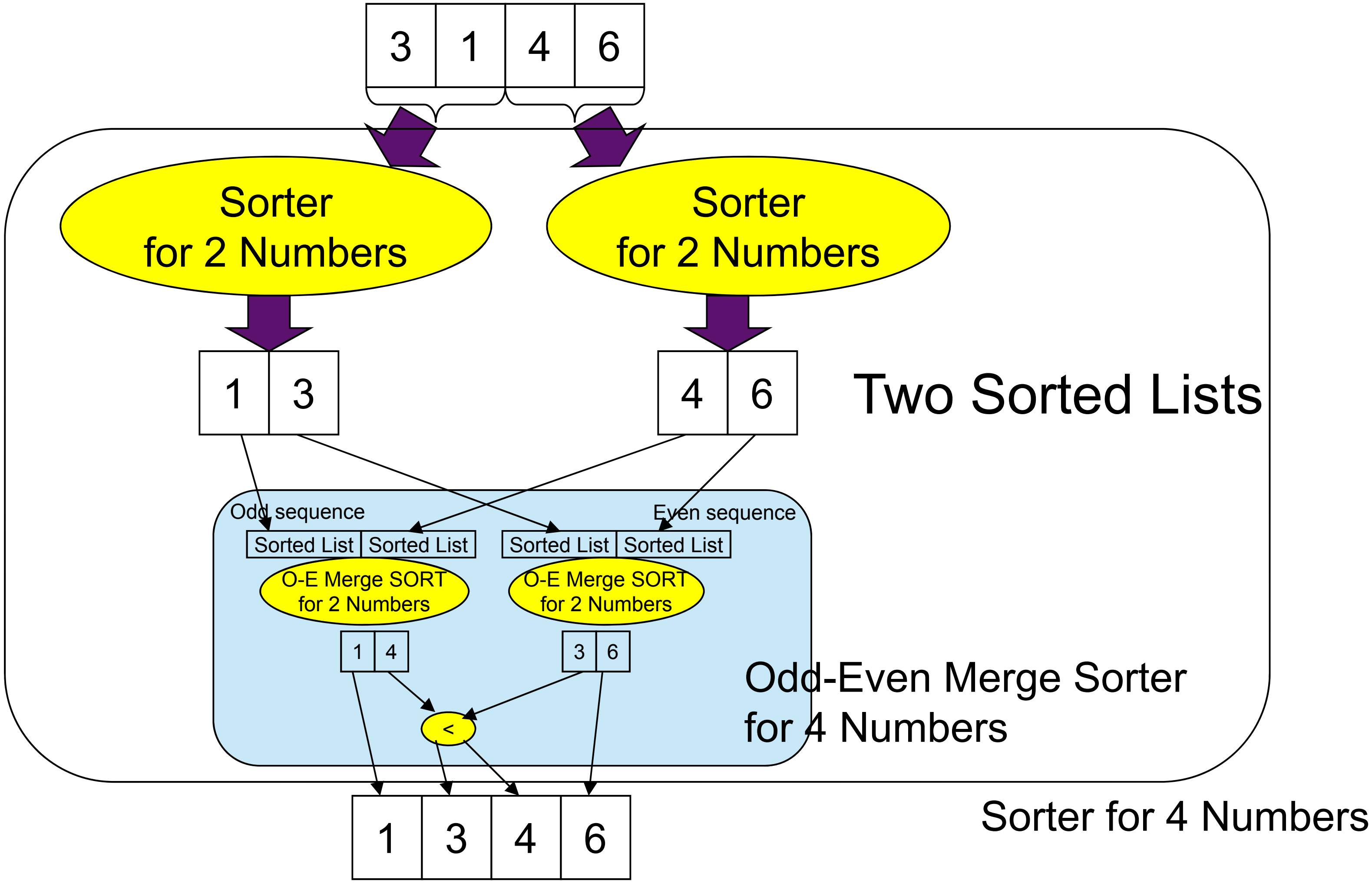


(Odd-Even Merge) Sorter
for 2 Numbers
= just a comparator

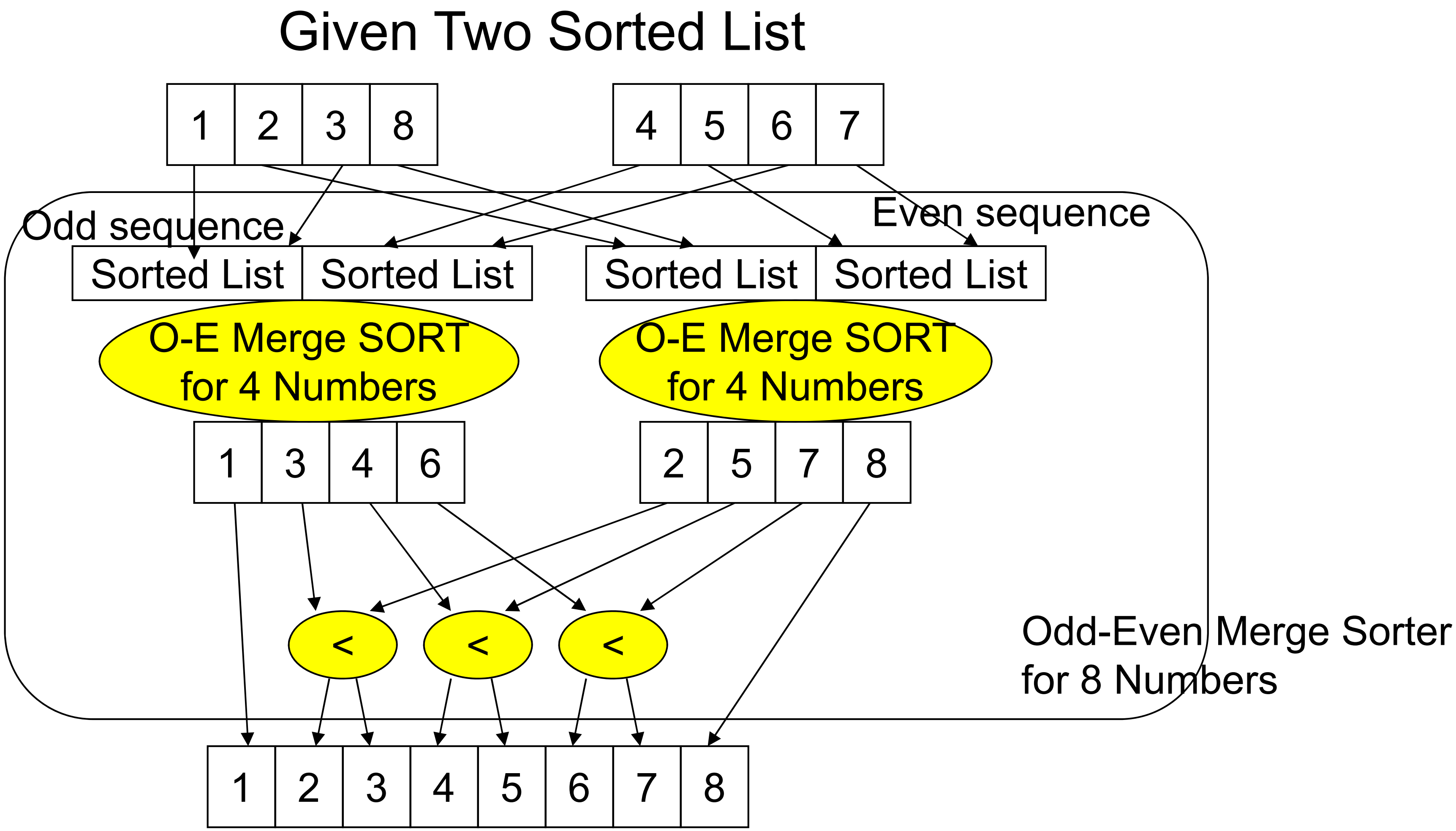
Odd-Even Merge Sorter for 4 Numbers



Sorter for 4 Numbers



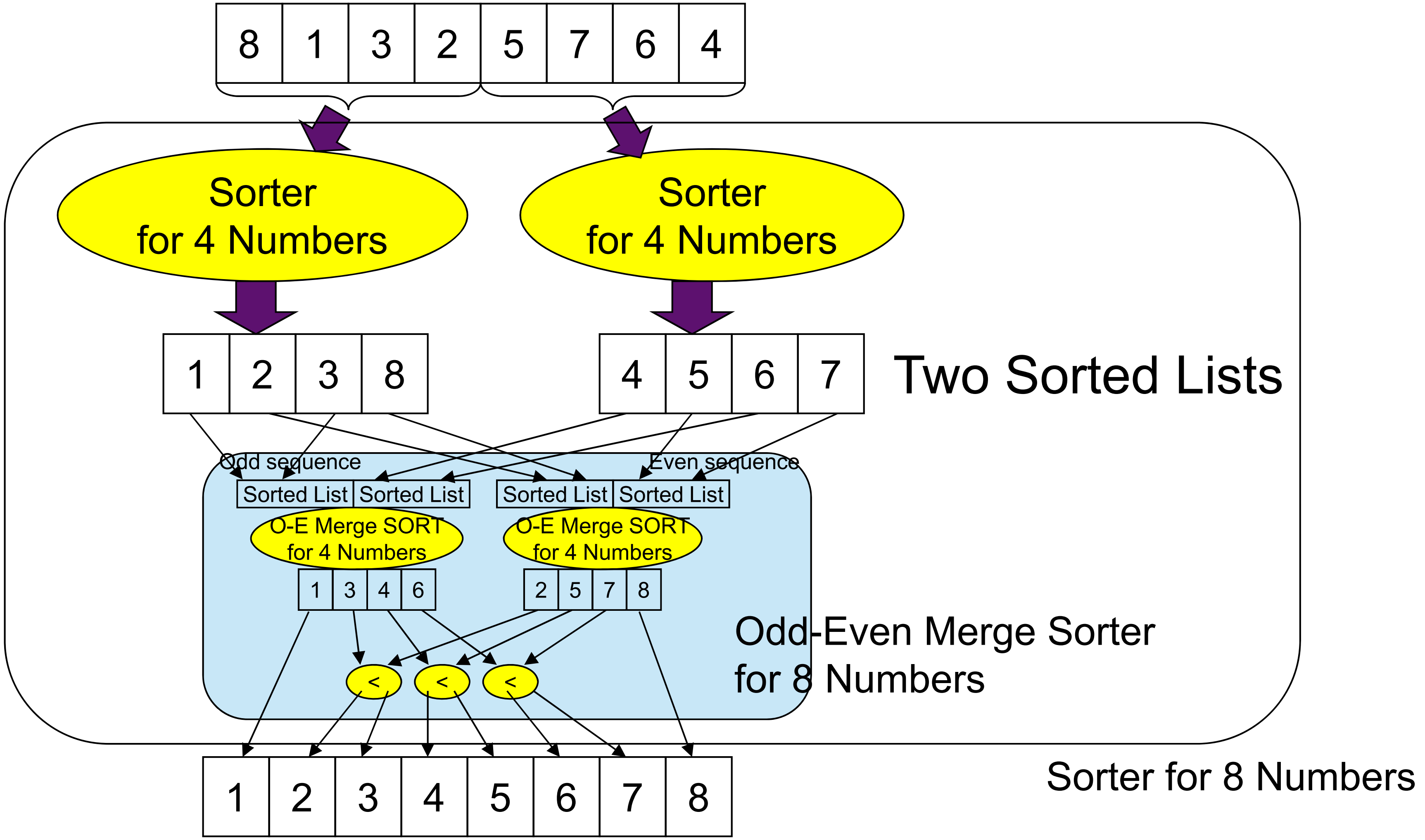
Odd-Even Merge Sorter for 8 Numbers



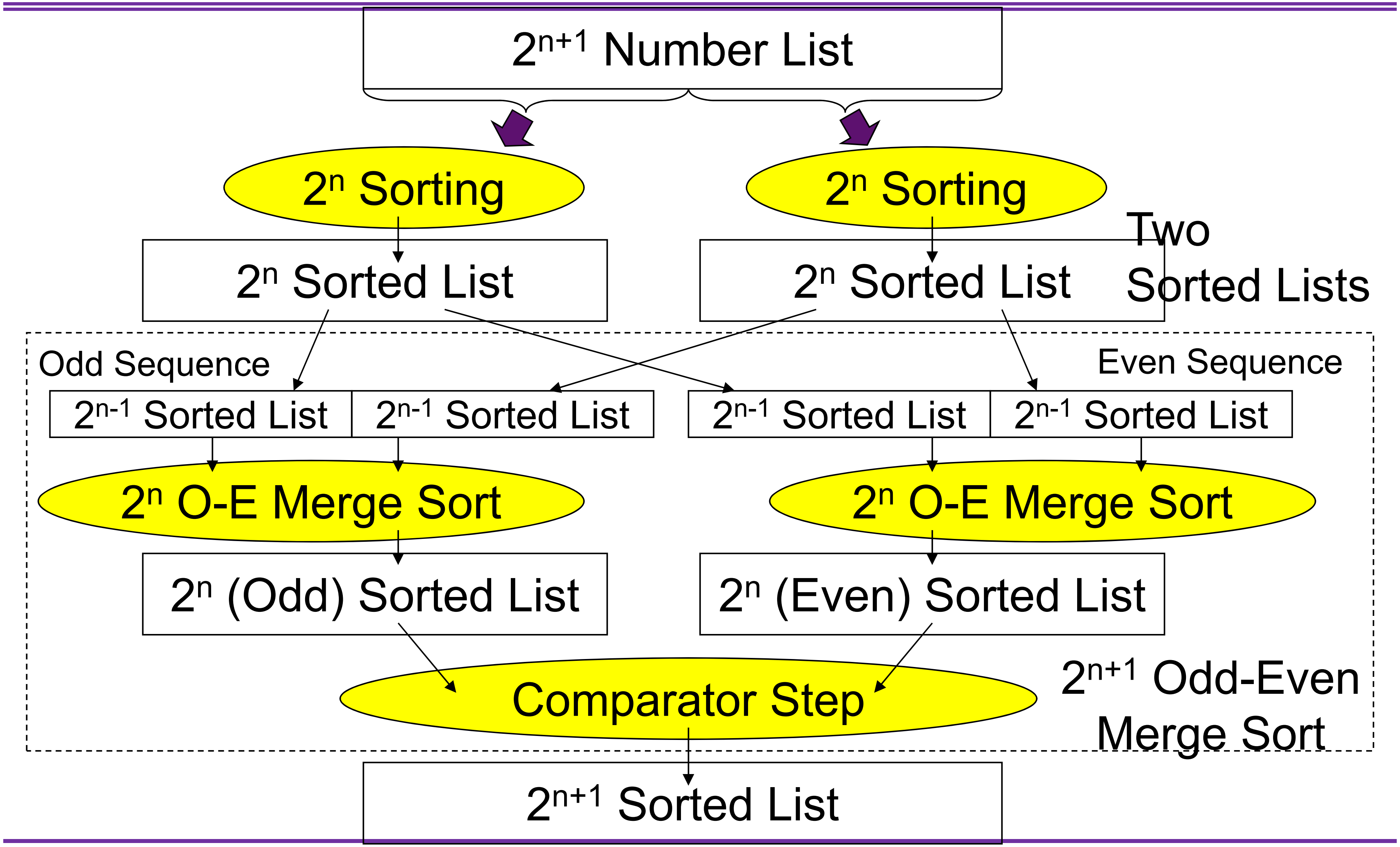
Given Two Sorted List

Odd-Even Merge Sorter for 8 Numbers

Sorter for 8 Numbers

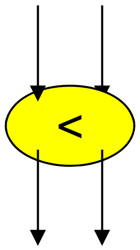


For 2^{n+1} Sorting

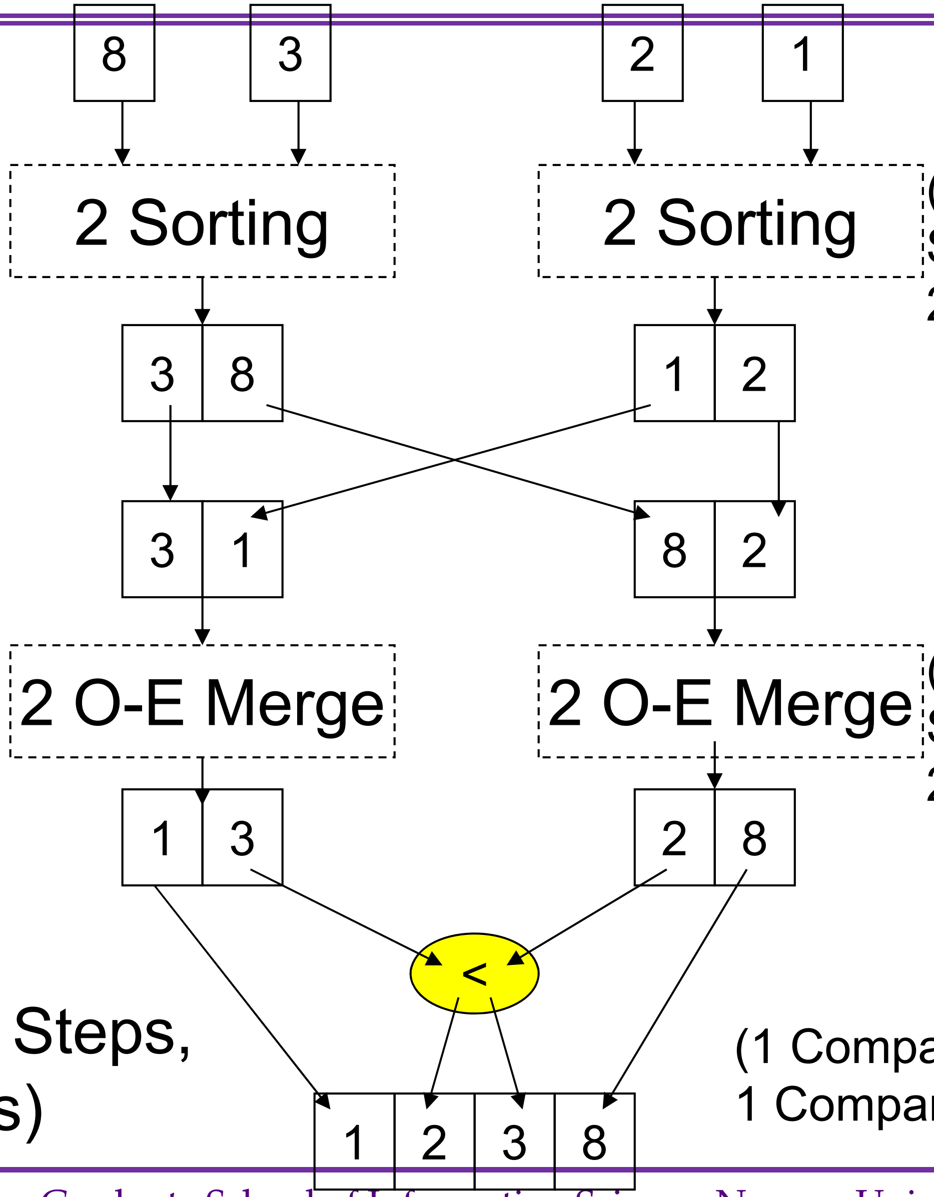


(4 Sorting)

(2 Sorting)



(1 Comparator Step,
1 Comparator)



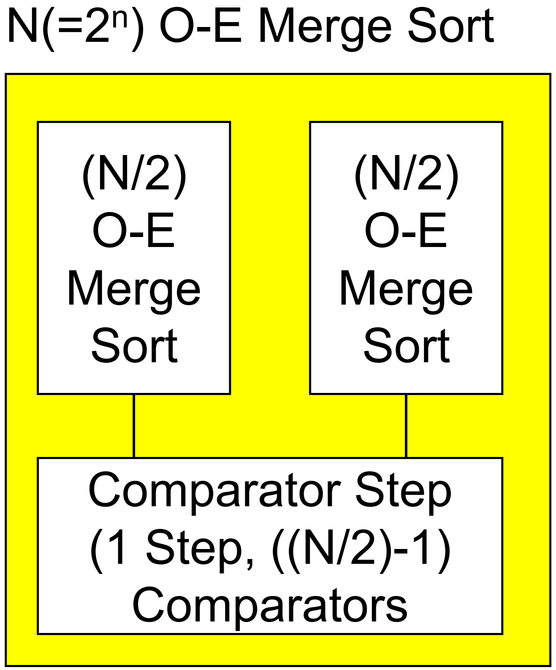
(1 Comparator Step,
2 Comparators)

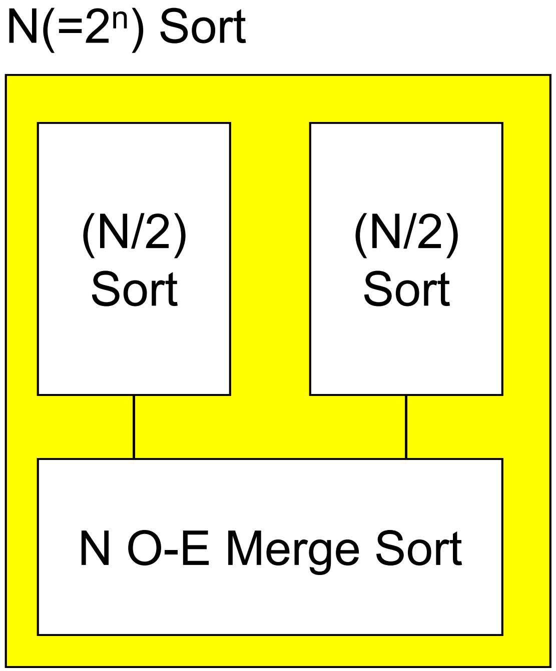
(1 Comparator Step,
2 Comparators)

(1 Comparator Step,
1 Comparator)

(3 Comparator Steps,
5 Comparators)

Analysis: # of Comparator Steps and # of Comparators





# of Comparator Step: S'_n

$S'_n = S'_{n-1} + 1$

# of Comparators: C'_n

$C'_n = 2*C'_{n-1} + (N/2) - 1$

# of Comparator Step: S_n

$S_n = S_{n-1} + S'_n$

# of Comparators: C_n

$C_n = 2*C_{n-1} + C'_n$

Analysis: # of Comparator Steps and # of Comparators

1. How many Comparator Steps of Odd-Even Merge Sorter for N Numbers?

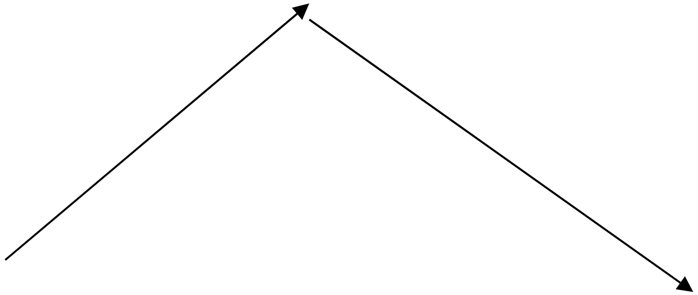
2. How many Comparator Steps of N-Sorter?

3. How many Comparators (Asymptotic Order) in each Comparator Step of N-Sorter?

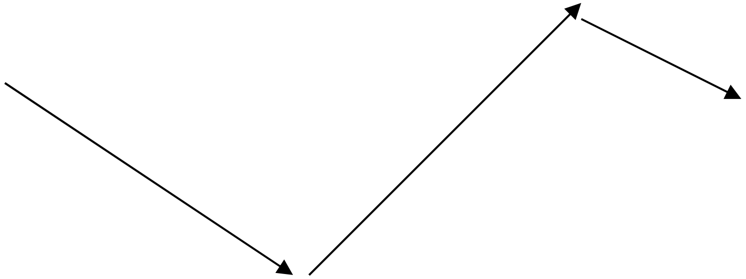
4. How many Comparators (Asymptotic Order) in N-Sorter?

Bitonic Merge Sort: Basic Idea

- Bitonic Sequence
  - Connect two monotonic sequence (increasing and decreasing)
  - Shifted bitonic sequence is also bitonic.



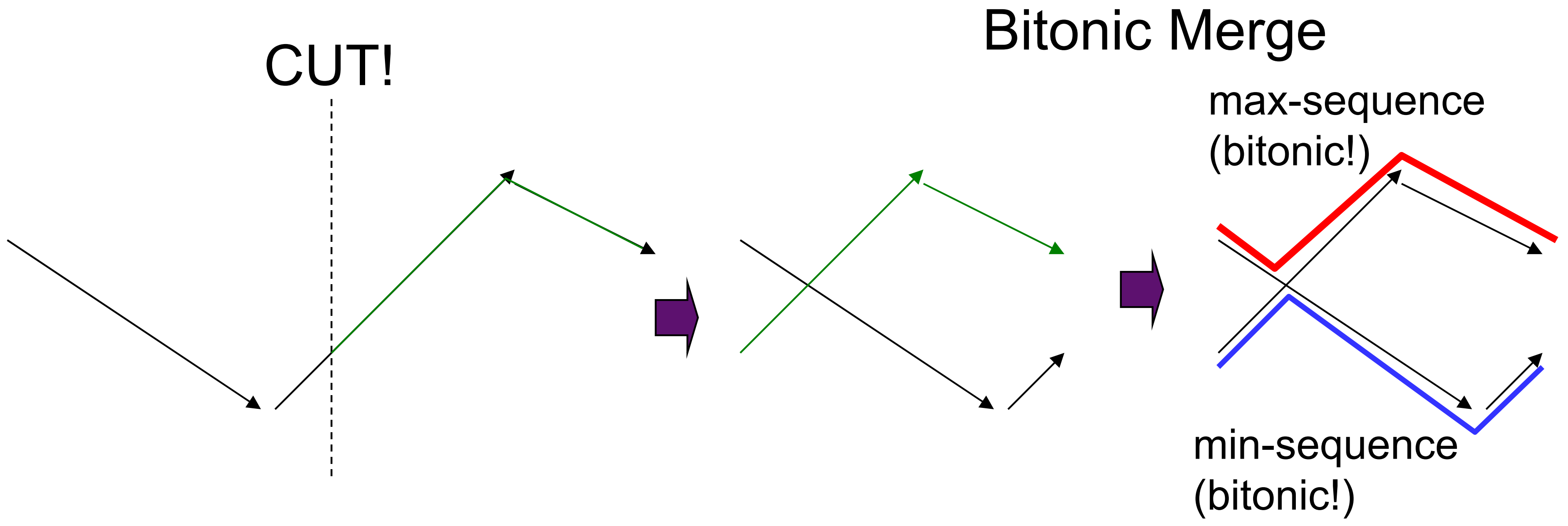
Bitonic Sequence (1)



Bitonic Sequence (2)

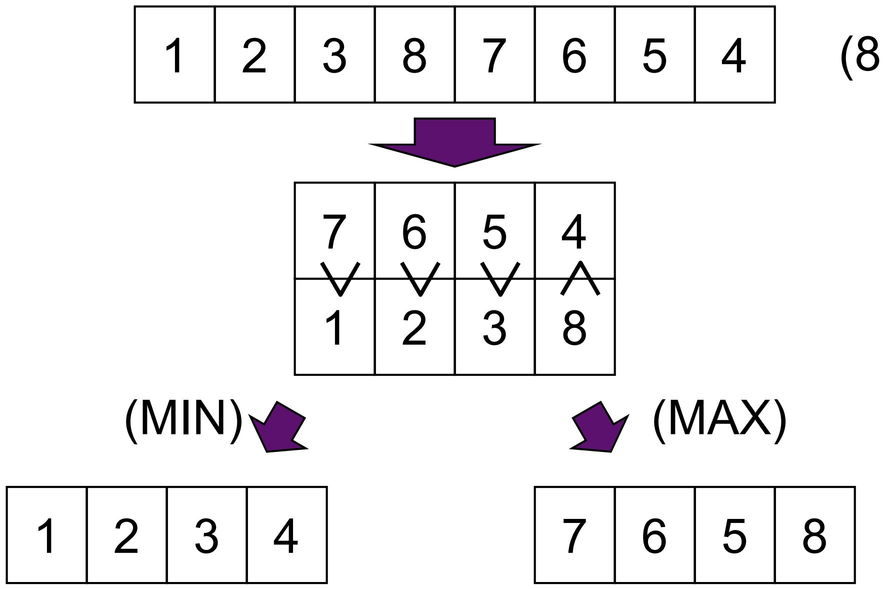
Bitonic Merge Sort: Basic Idea

- Take Two Bitonic Sequence
  - Max- and Min-Sequences are also bitonic.



Sorting is Done by Bitonic Merges Recursively

Bitonic Merge Sort: Basic Idea
(for 8 Numbers)



1 2 3 8 7 6 5 4 (8-Bitonic)

7 6 5 4

1 2 3 8

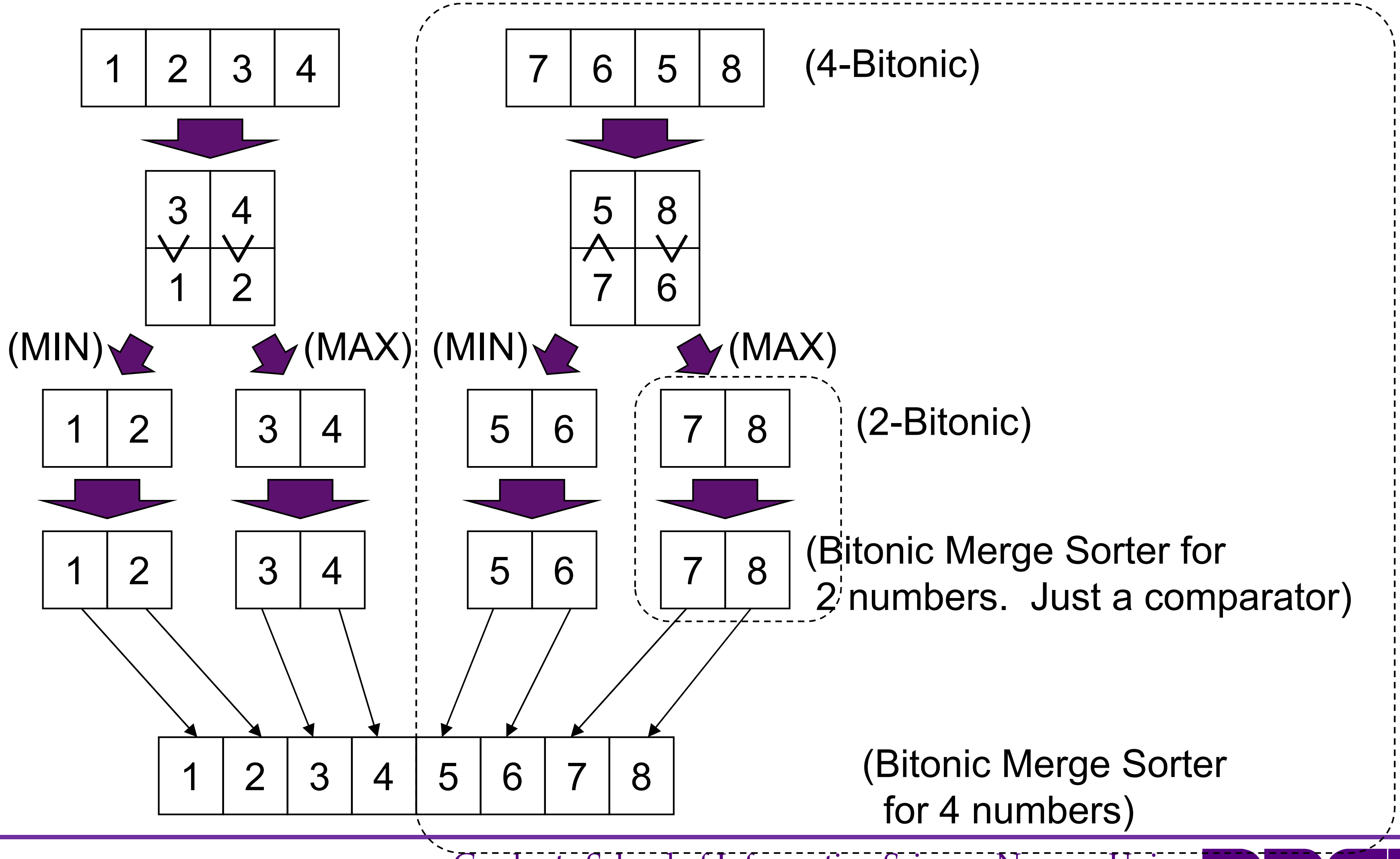
(MIN) (MAX)

1 2 3 4

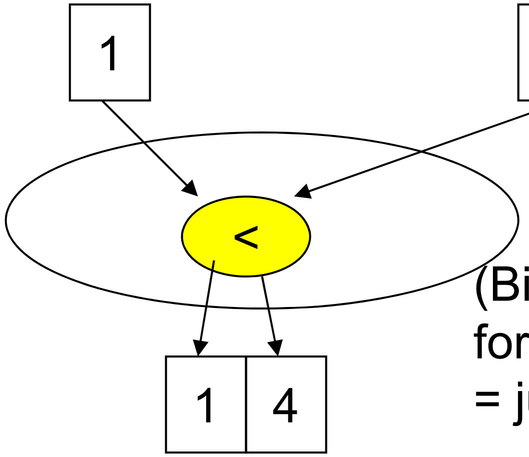
7 6 5 8 (4-Bitonic)

(to next page)

Bitonic Merge Sort: Basic Idea
(for 8 Numbers)

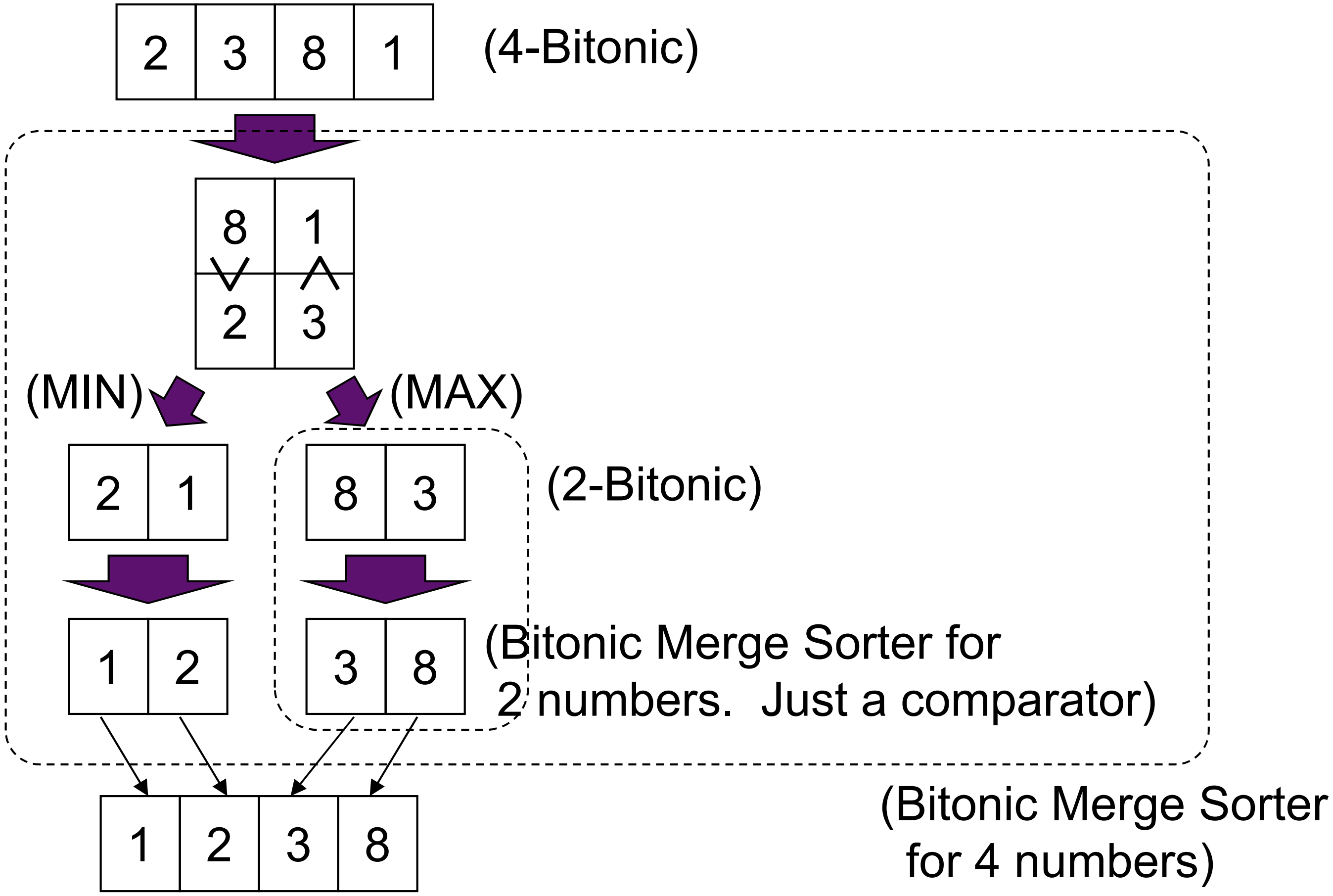


(Bitonic Merge) Sorter for 2 Numbers (Just a Comparator)



(Bitonic Merge) Sorter
for 2 Numbers
= just a comparator

Bitonic Merge Sorter for 4 Numbers



(4-Bitonic)

(MIN)

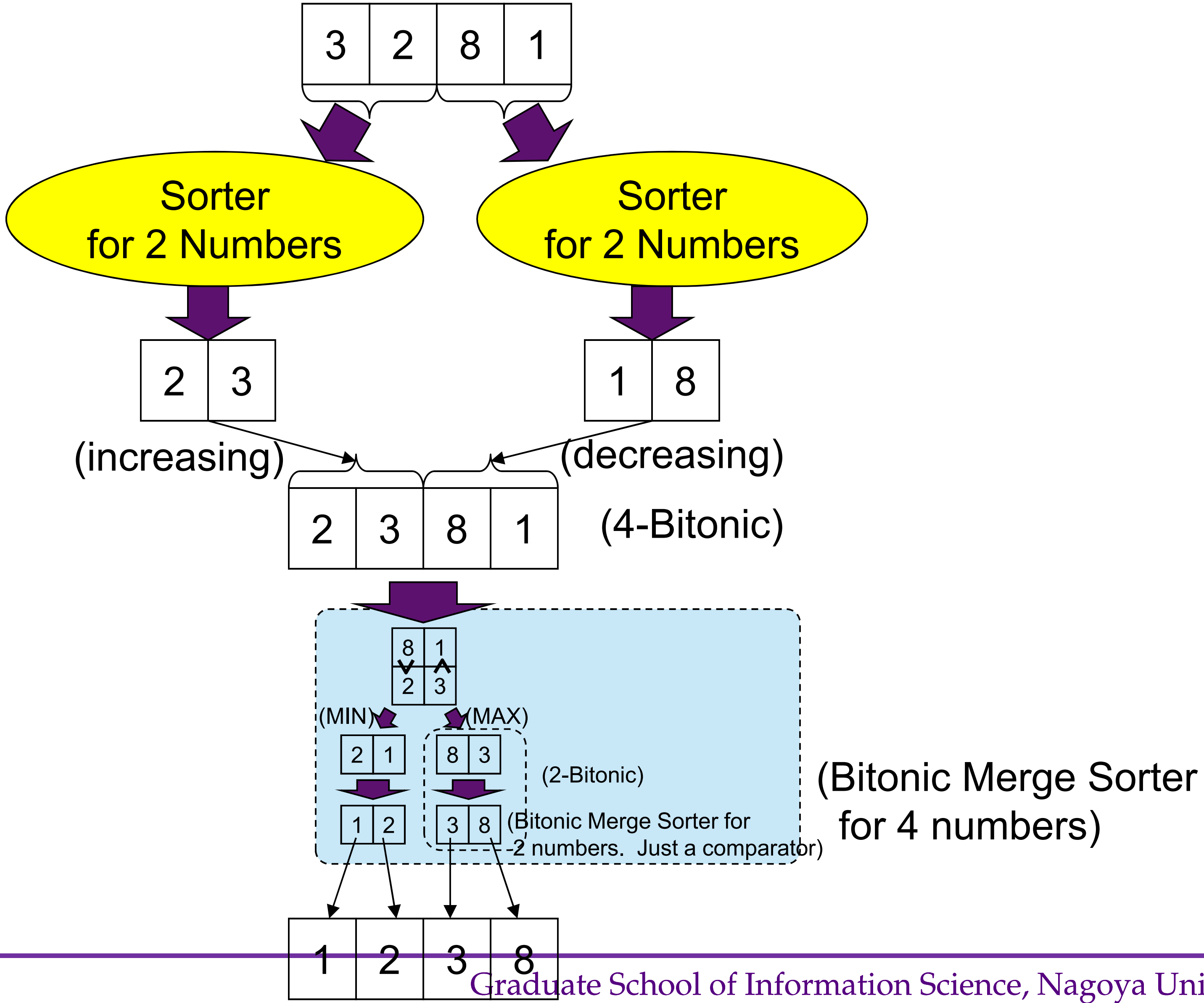
(MAX)

(2-Bitonic)

(Bitonic Merge Sorter for
2 numbers. Just a comparator)

(Bitonic Merge Sorter
for 4 numbers)

Sorter for 4 Numbers



Sorter for 2 Numbers

Sorter for 2 Numbers

(increasing)

(decreasing)

(4-Bitonic)

(MIN)

(MAX)

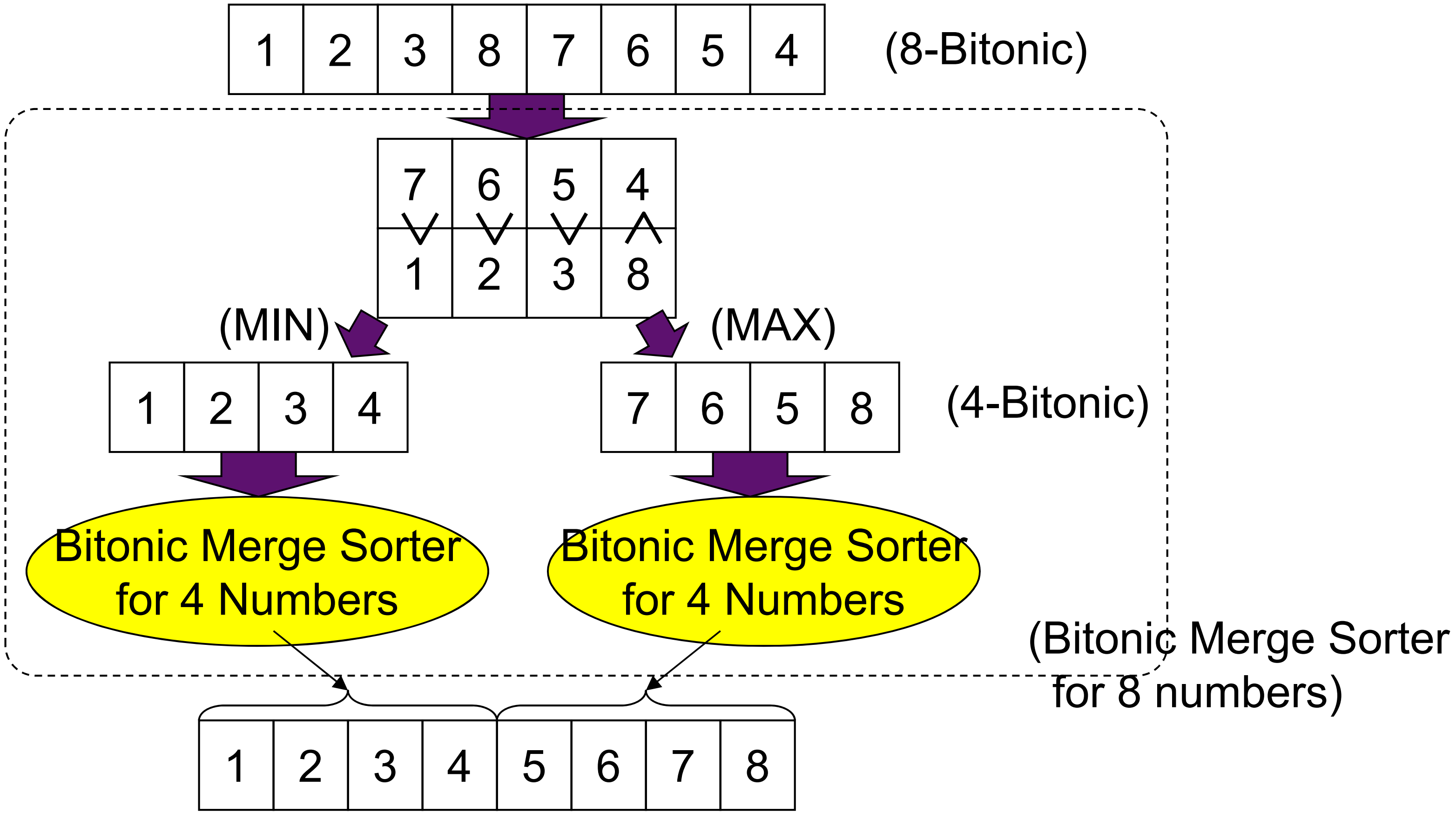
(2-Bitonic)

(Bitonic Merge Sorter for 2 numbers. Just a comparator)

(Bitonic Merge Sorter for 4 numbers)

Bitonic Merge Sorter for 8 Numbers

Given Two Sorted List



1	2	3	8	7	6	5	4

(8-Bitonic)

7	6	5	4
∨	∨	∨	∧
1	2	3	8

(MIN)

(MAX)

1	2	3	4

7	6	5	8

(4-Bitonic)

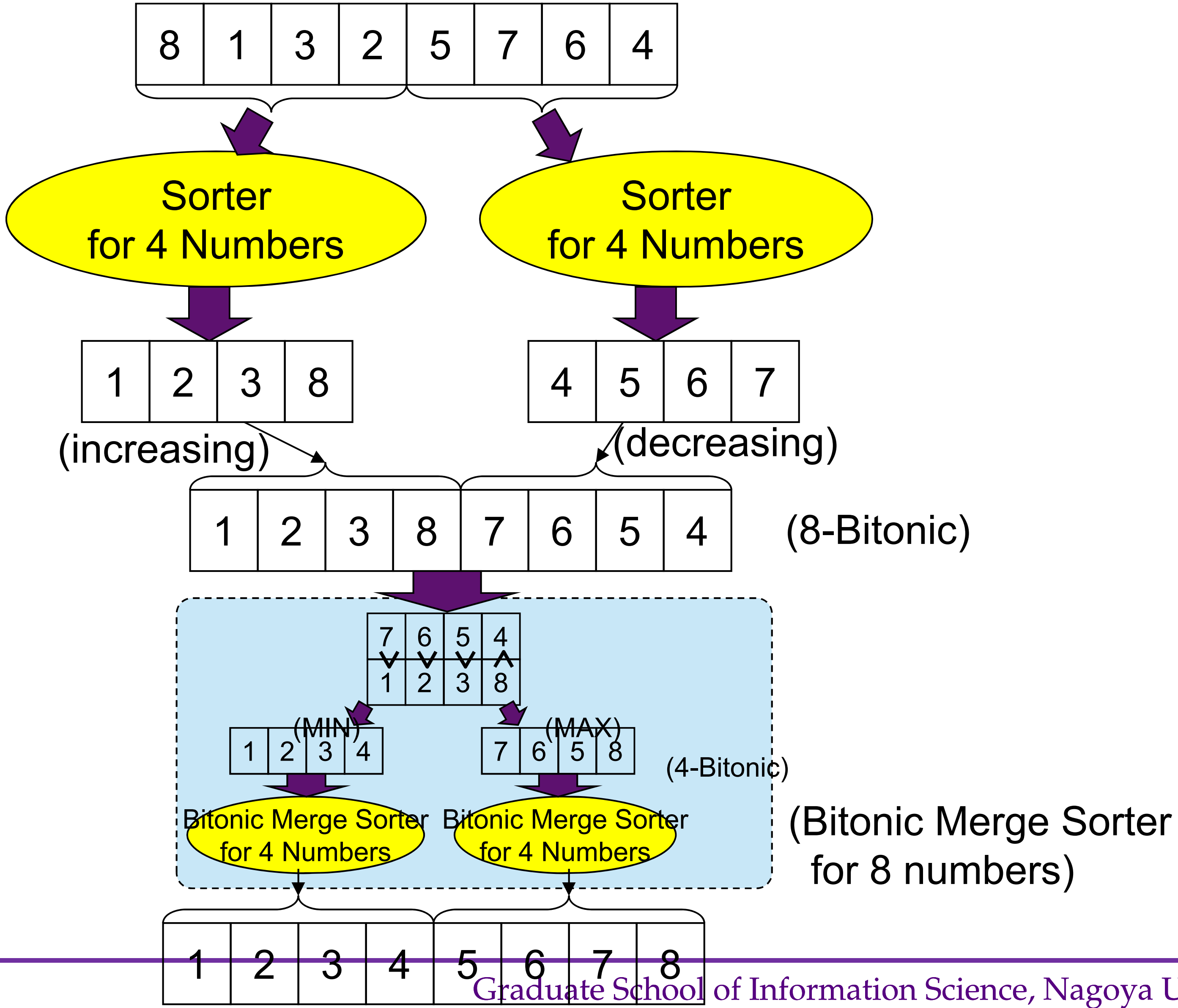
Bitonic Merge Sorter for 4 Numbers

Bitonic Merge Sorter for 4 Numbers

(Bitonic Merge Sorter for 8 numbers)

1	2	3	4	5	6	7	8

Sorter for 8 Numbers



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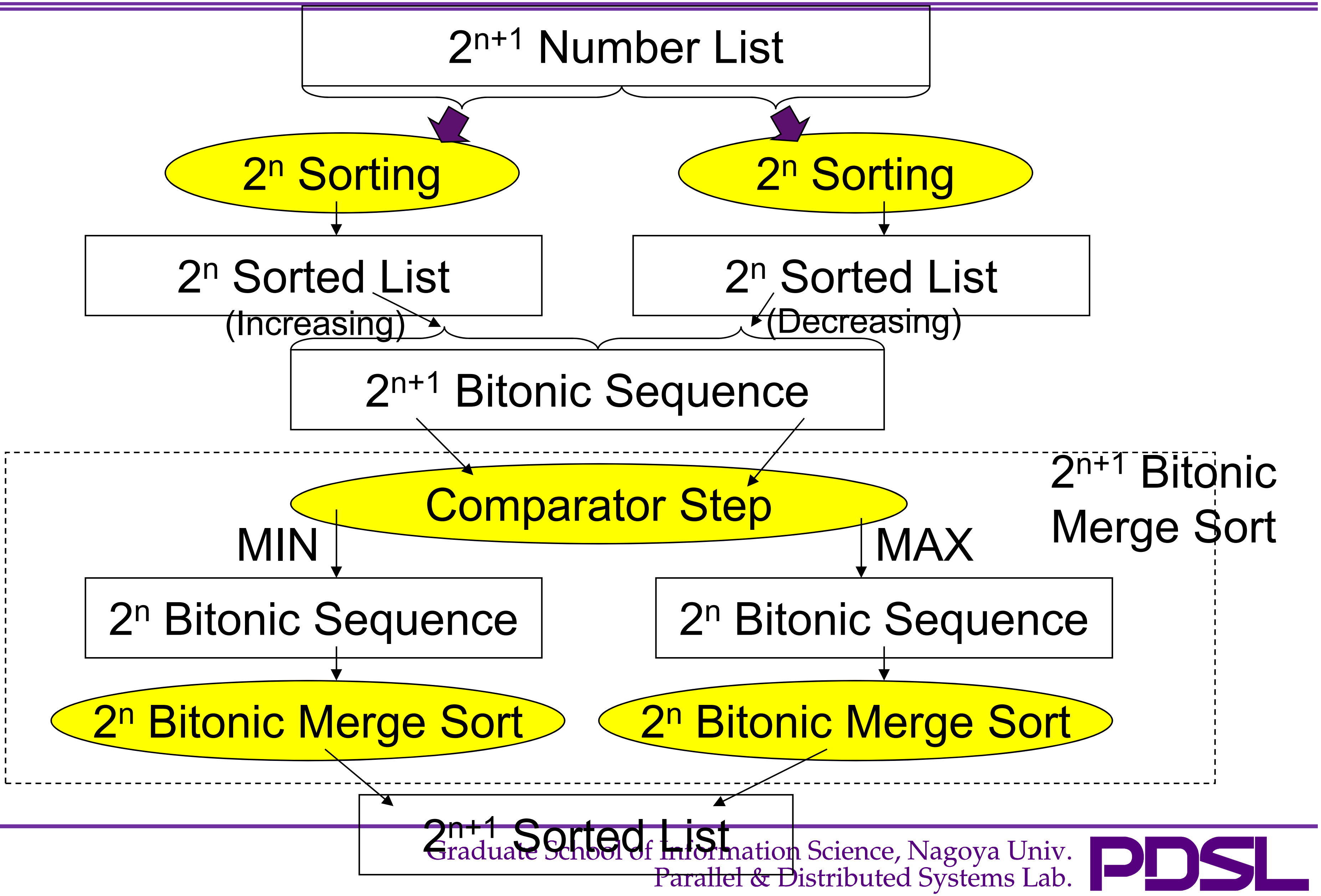

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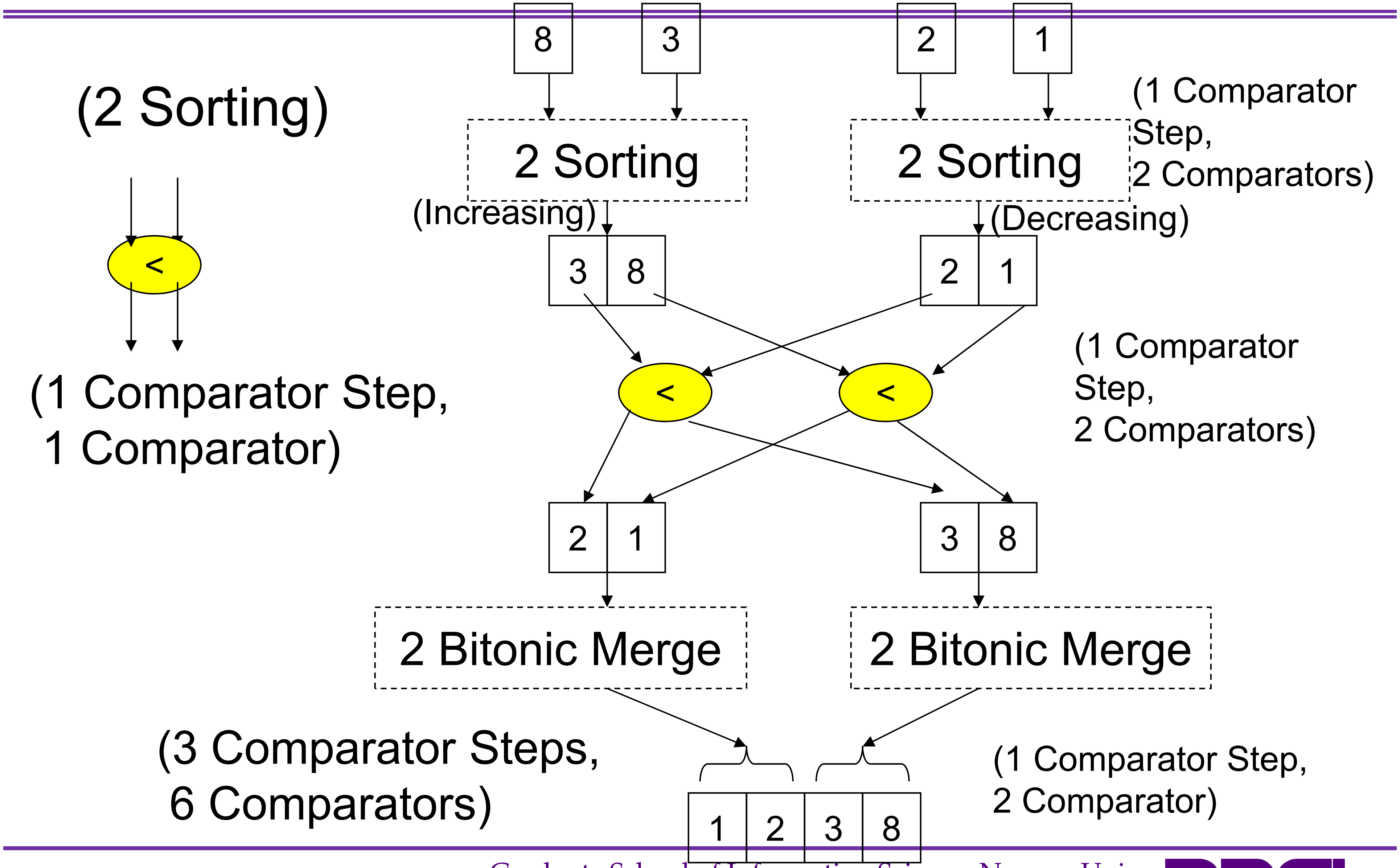


(2 Sorting)

(1 Comparator Step,
1 Comparator)

(1 Comparator Step,
2 Comparators)

(Increasing)

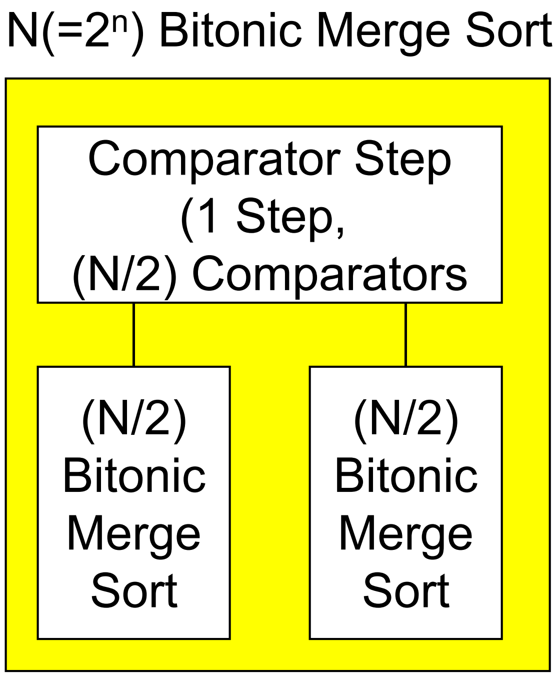
(Decreasing)

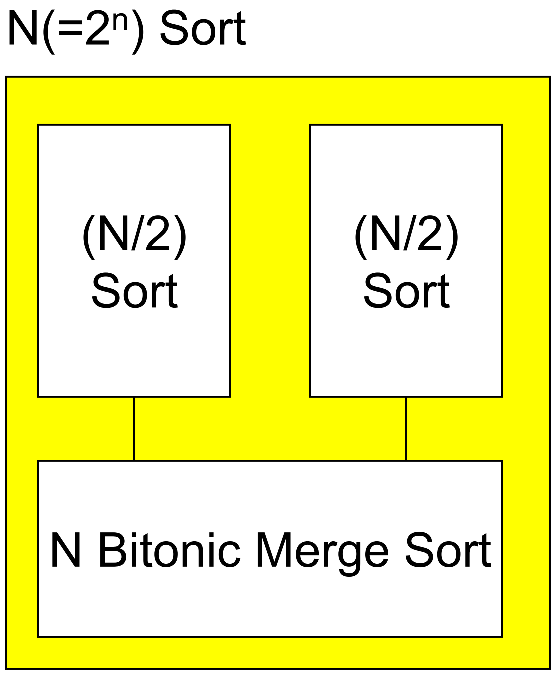
(1 Comparator Step,
2 Comparators)

(3 Comparator Steps,
6 Comparators)

(1 Comparator Step,
2 Comparator)

Analysis: # of Comparator Steps and # of Comparators





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