n 個の観測値 $(\boldsymbol{x_1}, y1), \ldots, (\boldsymbol{x_n}, y_n)$

$$y_i = \boldsymbol{x_i^T}\boldsymbol{\beta} + \varepsilon_i \tag{1}$$

$$\boldsymbol{x_i} = (x_{1i}, \ldots, x_{pi}) \tag{2}$$

$$\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)^T \tag{3}$$

$$y_i = x_{i1}\beta_1 + \cdots + x_{ip}\beta_p + \varepsilon_i \tag{4}$$

$$\boldsymbol{y} = (y_1, \ldots, y_n)^T \tag{5}$$

$$\boldsymbol{X} = (\boldsymbol{x_1}, \ldots, \boldsymbol{x_n})^T = \begin{bmatrix} x_{11} & \cdots & x_{p1} \\ \vdots & & \vdots \\ x_{1n} & \cdots & x_{pn} \end{bmatrix} \tag{6}$$

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{7}$$

$$\boldsymbol{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_n)^T \tag{8}$$

$$E[\boldsymbol{\varepsilon}] = \mathbf{0} \tag{9}$$

$$V[\boldsymbol{\varepsilon}] = \sigma^2\boldsymbol{I} \tag{10}$$

$$rank\boldsymbol{X} = p \tag{11}$$

$$y = \boldsymbol{x}\beta \tag{12}$$

$$\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_{n-1} \quad \varepsilon_n \quad \varepsilon \tag{13}$$

$$x_1 \quad x_2 \quad x_{n-1} \quad x_n \quad x \quad \mathcal{M}(\boldsymbol{X}) \tag{14}$$

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X^T X})^{-1} \boldsymbol{X^T y} \tag{15}$$

$$\begin{aligned} \boldsymbol{e} &= \boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}} && (16) \\ &= [\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T}]\boldsymbol{y} && (17) \\ &= [\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T}]\boldsymbol{\varepsilon} && (18) \end{aligned}$$

$$\|\boldsymbol{e}\|^{\boldsymbol{2}} = \boldsymbol{y^T}[\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T}]\boldsymbol{y} \tag{19}$$

$$E[\widehat{\boldsymbol{\beta}}] = \boldsymbol{\beta} \tag{20}$$

$$V[\widehat{\boldsymbol{\beta}}] \leq V[\boldsymbol{b}] \tag{21}$$

$$V[\widehat{\boldsymbol{\beta}}] = \sigma^2 (\boldsymbol{X^T X})^{-\boldsymbol{1}} \tag{22}$$

$$\boldsymbol{D_n}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \xrightarrow[n \to \infty]{\mathrm{d}} N(\boldsymbol{0}, \sigma^2 \boldsymbol{R_\infty^{-1}}) \tag{23}$$

$$\boldsymbol{A_n} = (a_{ij}) = \boldsymbol{X^T X} \tag{24}$$

$$\boldsymbol{D_n} = diag(\sqrt{a_{11}^n}, \ldots, \sqrt{a_{pp}^n}) \tag{25}$$

$$\boldsymbol{R_n} = \boldsymbol{D_n^{-1} A_n D_n^{-1}} \longrightarrow \boldsymbol{R_\infty} \tag{26}$$

$$s^2 = \frac{\boldsymbol{e^T e}}{n - p} \tag{27}$$

$$\hat{\sigma}^2 = \frac{\boldsymbol{e^T e}}{n} \tag{28}$$

$$E[\hat{\sigma}^2] = \frac{n-p}{n}\sigma^2 \to \sigma^2 \tag{29}$$

$$V[s^2] = \frac{2\sigma^4}{(n-p)^2} tr \bar{P}_X = \frac{2\sigma^4}{n-p} > \frac{2\sigma^4}{n} = V[\hat{\sigma}^2] \tag{30}$$

$$\bar{P}_X = I - X(X^T X)^{-1} X^T \tag{31}$$

$$\Pr\{|X - \mu_X| > k\sigma_X\} \leq \frac{1}{k^2} \quad (k > 0) \tag{32}$$