平成１６年度　東京大学大学院

数理科学研究科　数理科学専攻　修士課程

英　語　（筆 記 試 験）

平成１５年　９月　１日（月）
１０：００　～　１２：００

問題は全部で２題ある。すべての問題に解答すること。

（１）　解答しようとする各問ごとに解答用紙を１枚使用すること。
各解答用紙の所定欄に各自の**氏名**、**受験番号**と解答する**問題の番号**を記入すること。

（２）　試験終了後に提出するものは、２枚の答案用紙である。着手した答案が２枚にみたない場合には、氏名と受験番号のみを記した白紙答案を補い、２枚とすること。
指示に反したもの、**提出答案用紙が2枚でないものは無効**とする。

（３）　解答用紙の裏面を使用する場合は、表面右下に「裏面使用」と明記すること。

E. 第 1 問 次の文章は，オランダの画家 M. C. Escher (1898–1972) の作品 “Print Gallery” （『画廊』）について書かれたものの一部である．この文章の下線部 (A) 及び (B) を和訳せよ．

As a teenager, number theorist Hendrik Lenstra was fascinated by the mathematical themes of M.C. Escher’s artwork. A few years later, however, he lost his early enthusiasm for the Dutch artist, finding real mathematics “much more exciting.”

Today, Lenstra is once again an Escher enthusiast. He owns more than a dozen books about the artist, two documentary videos, and an assortment of Escher ties, and is in the process of acquiring an original print of Escher’s “Print Gallery,” a well-known work for which Lenstra now has a particular affection.

“I came to realize that there is much more mathematics in Escher’s work than first meets the eye,” says Lenstra, who holds joint positions at the University of California, Berkeley, and the Universiteit Leiden, in the Netherlands.

(A) Using the theory of elliptic curves, Lenstra has shown that the distortion of the quayside scene depicted in “Print Gallery” can be described by a complex exponential function. This quirky finding has been featured in The New York Times, on Dutch television, and in several Dutch newspapers.

Lenstra’s project began two and a half years ago on a Continental Airlines flight from New Jersey to Amsterdam. Browsing through the airline magazine, Lenstra spotted a picture of “Print Gallery” and was struck by a seeming flaw in its construction.

The lithograph depicts a view, through a row of arching windows, of a man looking at a picture on the wall of a gallery. In the picture, a row of Mediterranean-style buildings along a quay looms larger and larger until it extends right out of the picture frame and curves around to include the gallery and the man within it. The picture continuously expands in scale as the eye moves clockwise about the center. At the same time, the lines of the picture curve, as if someone had reached into the center and pulled it outward with a twist of the wrist.

But the vision is incomplete: Smack in the center of the picture is a large, circular patch that Escher left blank. Lenstra was bothered by the apparent blemish in the otherwise consistent structure of the picture. With many hours still to kill on his flight, he formulated two precise mathematical questions.

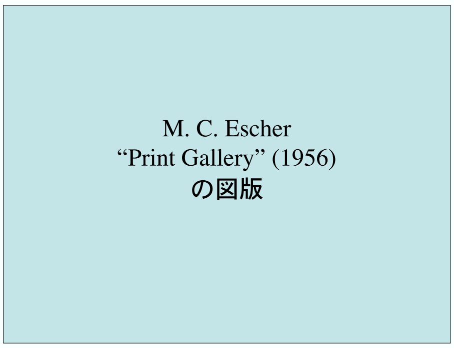
“First,” he says, “I wondered if, when you try to continue the arcs and lines, there is a mathematical problem you cannot solve.” His second question was: “What is the overall mathematical structure of this picture?”

(B) Readers might be surprised that Lenstra would have expected Escher’s lithograph to have a simple and consistent mathematical structure. Although fascinated by visual mathematical concepts, Escher had only a high school education in mathematics and little interest in its formalities. But to Lenstra, it was immediately clear that the mathematics was there. “It’s clear when you look at ‘Print Gallery’ that some transformation is being used,” he says. “And transformations belong to mathematics, so it was pretty evident that at least the question of how I, as a mathematician, would make the print, made sense.”

A few days after his arrival in Holland, Lenstra took the first step toward answering his questions. He consulted his copy of The Magic Mirror of M.C. Escher, a book by Hans de Rijk (written under the pen name Bruno Ernst). De Rijk was a friend of Escher’s and had visited him several times during the creation of “Print Gallery.” In the book, which was authorized and corrected by Escher, de Rijk described Escher’s method in detail.

（注）quayside: 波止場の縁　quirky: とっぴな　lithograph: 石版画

(Sara Robinson, “M.C.Escher: More Mathematics Than Meets the Eye,” SIAM News, Volume 35, Number 8, October 2002, より)



“Print Gallery” by M. C. Escher, 1956.

E. 第２問 次の文章を読み下線部分を英訳せよ。

n 項列ベクトルの空間 $\mathbf{K}^n$ には、n 個のベクトルがあって、任意のベクトルは、これらの n 個のベクトルの線型結合として表された（第２章 §3）．しかし、多項式全体の空間や、連続函数全体の空間の場合には、このような有限個のベクトルは見出すことができない．空間が非常に「大きい」のである．空間の大きさを測るために、われわれは次元の概念を必要とする．

$\mathbf{K}$ 上の線型空間 $\mathbf{V}$ を固定する．

$\mathbf{V}$ のベクトル $\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_k}$ に対し、

$$c_1\mathbf{a_1} + c_2\mathbf{a_2} + \cdots + c_k\mathbf{a_k}, \quad c_i \in \mathbf{K} \quad (i = 1, 2, \cdots, k)$$

の形のベクトルを、$\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_k}$ の**線型結合**と言う．ベクトル $\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_k}$ のあいだの関係

$$c_1\mathbf{a_1} + c_2\mathbf{a_2} + \cdots + c_k\mathbf{a_k} = \mathbf{o}$$

を**線型関係**と言う．$\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_k}$ がどんなベクトルであっても、線型関係はかならず存在する．それは $c_1 = c_2 = \cdots = c_k = 0$ としたもので、**自明な**線型関係と呼ばれる. 自明でない線型関係はいつでも存在するとは限らない.

$\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_k}$ のあいだに自明でない線型関係が存在するとき、$\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_k}$ は**線型従属**であると言い、自明でない線型関係が存在しないとき、$\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_k}$ は**線型独立**であると言う.

$\mathbf{K}^n$ における単位ベクトル $\mathbf{e_1}, \mathbf{e_2}, \cdots, \mathbf{e_n}$ は線型独立である．また、二個ないし三個の幾何ベクトルに対する線型独立性の概念（第１章 §1 参照）が、いま定義した線型独立性の定義に合っていることは容易に分かる．

(齋藤正彦著「線型代数入門」、東京大学出版会、より)