微積分

$\boxed{1}$

以下の問いに答えよ.

(i) 極限 $\displaystyle \lim_{n \to \infty} \frac{1}{n} \sin \frac{n\pi}{3}$ を求めよ.

(ii) 次の不定積分を求めよ.

$$\int \frac{x+1}{x^3 - 1} dx$$

(iii) 関数 $f(x, y) = \sin x + \sin y + \sin(x + y) \ (x > 0, y > 0, x + y < 2\pi)$ の極値を全て求めよ.

(iv) $D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leqq x + y \leqq 2, -1 \leqq x - y \leqq 1\}$ として，次の積分の値を求めよ.

$$\iint_D xy\,dxdy$$

(v) $\displaystyle \sqrt{\frac{1004}{1000}}$ の近似値を小数第 6 位まで求めよ.

An English Translation:

Calculus

$$\boxed{1}$$

Answer the following questions.

(i) Find the limit
$$\lim_{n\to\infty} \frac{1}{n} \sin \frac{n\pi}{3}.$$

(ii) Find the following indefinite integral
$$\int \frac{x+1}{x^3 - 1} dx.$$

(iii) Find all the extreme values of the function $f(x,y) = \sin x + \sin y + \sin(x+y)$ $(x > 0, y > 0, x + y < 2\pi)$.

(iv) Let $D = \{(x,y) \in \mathbb{R}^2 \mid 0 \leqq x + y \leqq 2, -1 \leqq x - y \leqq 1\}$. Find the value of the integral
$$\iint_D xy\,dx\,dy.$$

(v) Find the approximate value of $\sqrt{\dfrac{1004}{1000}}$ to six decimal places.