平成２１年度　東京大学大学院

数理科学研究科　数理科学専攻　修士課程

英　語　（筆記試験）

平成２０年　９月１日（月）
１０：００　～　１２：００

問題は全部で２題ある．２題とも解答すること．

（１）　解答しようとする各問ごとに解答用紙を１枚使用すること．
各解答用紙の所定欄に各自の**氏名**，**受験番号**と解答する**問題の番号**を記入すること．

（２）　草稿用紙の上部に各自の**受験番号**を明記すること．ただし氏名を記入してはならない．

（３）　試験終了後に提出するものは，１題につき１枚，計**２枚の答案**，および**草稿用紙**である．着手した問題数が２題にみたない場合でも，氏名と受験番号のみを記入した白紙の答案を補い，２枚とすること．
指示に反したもの，**答案が２枚でないものは無効**とする．

（４）　解答用紙の裏面を使用する場合は，表面の右下に「裏面使用」と明記すること．

E 第 1 問

次の英文の下線部を和訳せよ.

(出典：Timothy Gowers, "Mathematics", Oxford University Press, 2002 の Preface より)

Early in the 20th century, the great mathematician David Hilbert noticed that a number of important mathematical arguments were structurally similar. In fact, he realized that at an appropriate level of generality they could be regarded as the same. This observation, and others like it, gave rise to a new branch of mathematics, and one of its central concepts was named after Hilbert. The notion of a Hilbert space sheds light on so much of modern mathematics, from number theory to quantum mechanics, that if you do not know at least the rudiments of Hilbert space theory then you cannot claim to be a well-educated mathematician.

What, then, is a Hilbert space? In a typical university mathematics course it is defined as a complete inner-product space. Students attending such a course are expected to know, from previous courses, that an inner-product space is a vector space equipped with an inner product, and that a space is complete if every Cauchy sequence in it converges. Of course, for those definitions to make sense, the students also need to know the definitions of vector space, inner product, Cauchy sequence and convergence. To give just one of them (not the longest): a Cauchy sequence is a sequence x_1, x_2, $x_3, \ldots$ such that for every positive number ϵ there exists an integer N such that for any two integers p and q greater than N the distance from x_p to x_q is at most ϵ.

In short, to have any hope of understanding what a Hilbert space is, you must learn and digest a whole hierarchy of lower-level concepts first. Not surprisingly, this takes time and effort. Since the same is true of many of the most important mathematical ideas, there is a severe limit to what can be achieved by any book that attempts to offer an accessible introduction to mathematics, especially if it is to be very short.

E 第 2 問

次の文章を英訳せよ.

(出典：小平邦彦「複素解析」より一部改変)

$D \subset \mathbf{C}$ をある領域とし, $f(z)$ を D で定義された複素変数 z の関数とする. $f(z)$ の微分係数の定義は形式的には実変数の関数の場合と全く同様である. すなわち, z を D に属する一つの点とし, 正の実数 ρ を z の ρ 近傍:

$$U_\rho(z) = \{\zeta \,|\, |\zeta - z| < \rho\}$$

が D に含まれるように定めれば, $(f(z+h) - f(z))/h$ は複素変数 h, $0 < |h| < \rho$, の関数となる. このとき極限

$$\lim_{h \to 0} \frac{f(z+h) - f(z)}{h}$$

が存在するならば, 関数 $f(z)$ は点 z で微分可能であるという. そしてこの極限を $f(z)$ の z における微分係数といい, $f'(z)$ で表す.