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Chapter 3 - Linear Regression

- Load Datasets
- 3.1 Simple Linear Regression
- 3.2 Multiple Linear Regression
- 3.3 Other Considerations in the Regression Model

In [39]:
```
# %load ../standard_import.txt
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import axes3d
import seaborn as sns

from sklearn.preprocessing import scale
import sklearn.linear_model as skl_lm
from sklearn.metrics import mean_squared_error, r2_score
import statsmodels.api as sm
import statsmodels.formula.api as smf

%matplotlib inline
plt.style.use('seaborn-white')
```

Load Datasets

Datasets available on http://www-bcf.usc.edu/~gareth/ISL/data.html (http://www-bcf.usc.edu/~gareth/ISL/data.html)

In [2]:
```
advertising = pd.read_csv('Data/Advertising.csv', usecols=[1,2,3,4])
advertising.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 200 entries, 0 to 199
Data columns (total 4 columns):
TV           200 non-null float64
Radio        200 non-null float64
Newspaper    200 non-null float64
Sales        200 non-null float64
dtypes: float64(4)
memory usage: 6.3 KB
```

In [3]:
```
credit = pd.read_csv('Data/Credit.csv', usecols=list(range(1,12)))
credit['Student2'] = credit.Student.map({'No':0, 'Yes':1})
credit.head(3)
```

Out[3]:

	Income	Limit	Rating	Cards	Age	Education	Gender	Student	Married	Ethni
0	14.891	3606	283	2	34	11	Male	No	Yes	Cauca
1	106.025	6645	483	3	82	15	Female	Yes	Yes	Asian
2	104.593	7075	514	4	71	11	Male	No	No	Asian

2	104.593	7075	514	4	71	11	Male	No	No	Asian

In [6]:
```
auto = pd.read_csv('Data/Auto.csv', na_values='?').dropna()
auto.info()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 392 entries, 0 to 396
Data columns (total 9 columns):
mpg             392 non-null float64
cylinders       392 non-null int64
displacement    392 non-null float64
horsepower      392 non-null float64
weight          392 non-null int64
acceleration    392 non-null float64
year            392 non-null int64
origin          392 non-null int64
name            392 non-null object
dtypes: float64(4), int64(4), object(1)
memory usage: 30.6+ KB
```

3.1 Simple Linear Regression

Figure 3.1 - Least squares fit

In [7]:
```
sns.regplot(advertising.TV, advertising.Sales, order=1, ci=None, scatte
r_kws={'color':'r', 's':9})
plt.xlim(-10,310)
plt.ylim(ymin=0);
```

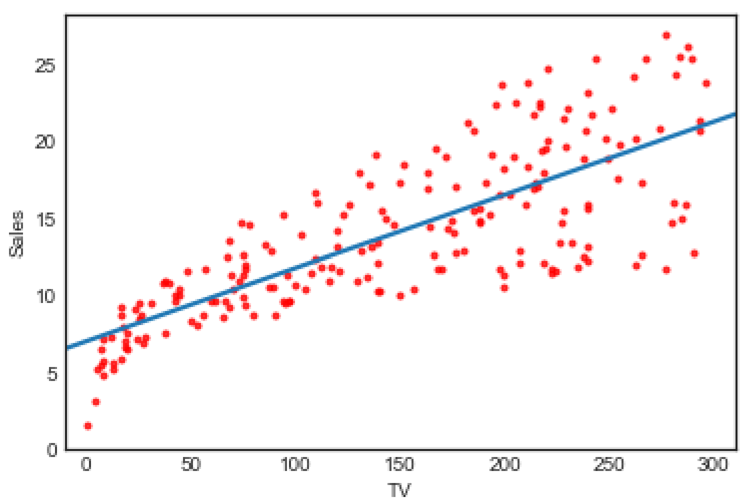


Figure 3.2 - Regression coefficients - RSS

Note that the text in the book describes the coefficients based on uncentered data, whereas the plot shows the model based on centered data. The latter is visually more appealing for explaining the concept of a minimum RSS. I think that, in order not to confuse the reader, the values on the axis of the B0 coefficients have been changed to correspond with the text. The axes on the plots below are unaltered.

In [8]:
```
# Regression coefficients (Ordinary Least Squares)
regr = skl_lm.LinearRegression()

X = scale(advertising.TV, with_mean=True, with_std=False).reshape(-1,1)
y = advertising.Sales

regr.fit(X,y)
print(regr.intercept_)
print(regr.coef_)
```

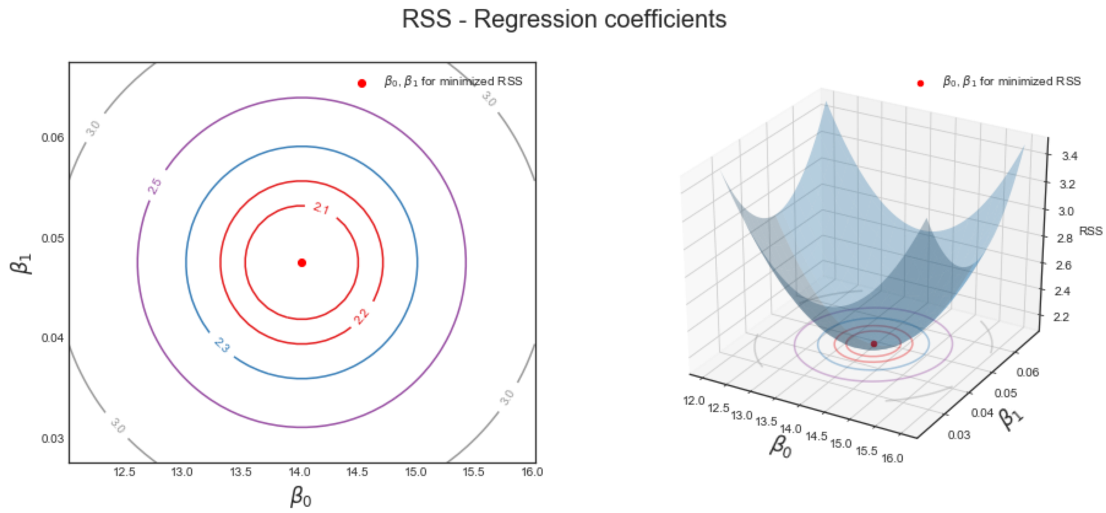
```
14.0225
[ 0.04753664]
```

In [9]:
```
# Create grid coordinates for plotting
B0 = np.linspace(regr.intercept_-2, regr.intercept_+2, 50)
B1 = np.linspace(regr.coef_-0.02, regr.coef_+0.02, 50)
xx, yy = np.meshgrid(B0, B1, indexing='xy')
Z = np.zeros((B0.size,B1.size))

# Calculate Z-values (RSS) based on grid of coefficients
for (i,j),v in np.ndenumerate(Z):
    Z[i,j] =((y - (xx[i,j]+X.ravel()*yy[i,j]))**2).sum()/1000

# Minimized RSS
min_RSS = r'$\beta_0$, $\beta_1$ for minimized RSS'
min_rss = np.sum((regr.intercept_+regr.coef_*X - y.values.reshape(-1,1
))**2)/1000
min_rss
```

Out[9]: 2.1025305831313514

In [10]:
```
fig = plt.figure(figsize=(15,6))
fig.suptitle('RSS - Regression coefficients', fontsize=20)

ax1 = fig.add_subplot(121)
ax2 = fig.add_subplot(122, projection='3d')

# Left plot
CS = ax1.contour(xx, yy, Z, cmap=plt.cm.Set1, levels=[2.15, 2.2, 2.3,
2.5, 3])
ax1.scatter(regr.intercept_, regr.coef_[0], c='r', label=min_RSS)
ax1.clabel(CS, inline=True, fontsize=10, fmt='%1.1f')

# Right plot
ax2.plot_surface(xx, yy, Z, rstride=3, cstride=3, alpha=0.3)
ax2.contour(xx, yy, Z, zdir='z', offset=Z.min(), cmap=plt.cm.Set1,
            alpha=0.4, levels=[2.15, 2.2, 2.3, 2.5, 3])
ax2.scatter3D(regr.intercept_, regr.coef_[0], min_rss, c='r', label=min
_RSS)
ax2.set_zlabel('RSS')
ax2.set_zlim(Z.min(),Z.max())
ax2.set_ylim(0.02,0.07)

# settings common to both plots
for ax in fig.axes:
    ax.set_xlabel(r'$\beta_0$', fontsize=17)
```

```
    ax.set_ylabel(r'$\beta_1$', fontsize=17)
    ax.set_yticks([0.03,0.04,0.05,0.06])
    ax.legend()
```



Confidence interval on page 67 & Table 3.1 & 3.2 - Statsmodels

In [11]:
```
est = smf.ols('Sales ~ TV', advertising).fit()
est.summary().tables[1]
```

Out[11]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	7.0326	0.458	15.360	0.000	6.130	7.935
TV	0.0475	0.003	17.668	0.000	0.042	0.053

In [12]:
```
# RSS with regression coefficients
((advertising.Sales - (est.params[0] + est.params[1]*advertising.TV))**
2).sum()/1000
```

Out[12]: 2.1025305831313514

Table 3.1 & 3.2 - Scikit-learn

In [13]:
```
regr = skl_lm.LinearRegression()

X = advertising.TV.values.reshape(-1,1)
y = advertising.Sales

regr.fit(X,y)
print(regr.intercept_)
print(regr.coef_)
```

```
7.03259354913
[ 0.04753664]
```

In [14]:
```
Sales_pred = regr.predict(X)
r2_score(y, Sales_pred)
```

Out[14]: 0.611875050850071

3.2 Multiple Linear Regression

Table 3.3 - Statsmodels

In [15]:
```
est = smf.ols('Sales ~ Radio', advertising).fit()
est.summary().tables[1]
```

Out[15]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	9.3116	0.563	16.542	0.000	8.202	10.422
Radio	0.2025	0.020	9.921	0.000	0.162	0.243

In [16]:
```
est = smf.ols('Sales ~ Newspaper', advertising).fit()
est.summary().tables[1]
```

Out[16]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	12.3514	0.621	19.876	0.000	11.126	13.577
Newspaper	0.0547	0.017	3.300	0.001	0.022	0.087

Table 3.4 & 3.6 - Statsmodels

In [17]:
```
est = smf.ols('Sales ~ TV + Radio + Newspaper', advertising).fit()
est.summary()
```

Out[17]:

Dep. Variable:	Sales	**R-squared:**	0.897
Model:	OLS	**Adj. R-squared:**	0.896
Method:	Least Squares	**F-statistic:**	570.3
Date:	Tue, 09 Jan 2018	**Prob (F-statistic):**	1.58e-96
Time:	23:14:15	**Log-Likelihood:**	-386.18
No. Observations:	200	**AIC:**	780.4
Df Residuals:	196	**BIC:**	793.6
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	2.9389	0.312	9.422	0.000	2.324	3.554
TV	0.0458	0.001	32.809	0.000	0.043	0.049

Radio	0.1885	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.0010	0.006	-0.177	0.860	-0.013	0.011

Omnibus:	60.414	**Durbin-Watson:**	2.084
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	151.241
Skew:	-1.327	**Prob(JB):**	1.44e-33
Kurtosis:	6.332	**Cond. No.**	454.

Table 3.5 - Correlation Matrix

In [18]:
```
advertising.corr()
```

Out[18]:

	TV	Radio	Newspaper	Sales
TV	1.000000	0.054809	0.056648	0.782224
Radio	0.054809	1.000000	0.354104	0.576223
Newspaper	0.056648	0.354104	1.000000	0.228299
Sales	0.782224	0.576223	0.228299	1.000000

Figure 3.5 - Multiple Linear Regression

In [19]:
```
regr = skl_lm.LinearRegression()

X = advertising[['Radio', 'TV']].as_matrix()
y = advertising.Sales

regr.fit(X,y)
print(regr.coef_)
print(regr.intercept_)
```

```
[ 0.18799423  0.04575482]
2.92109991241
```

In [20]:
```
# What are the min/max values of Radio & TV?
# Use these values to set up the grid for plotting.
advertising[['Radio', 'TV']].describe()
```

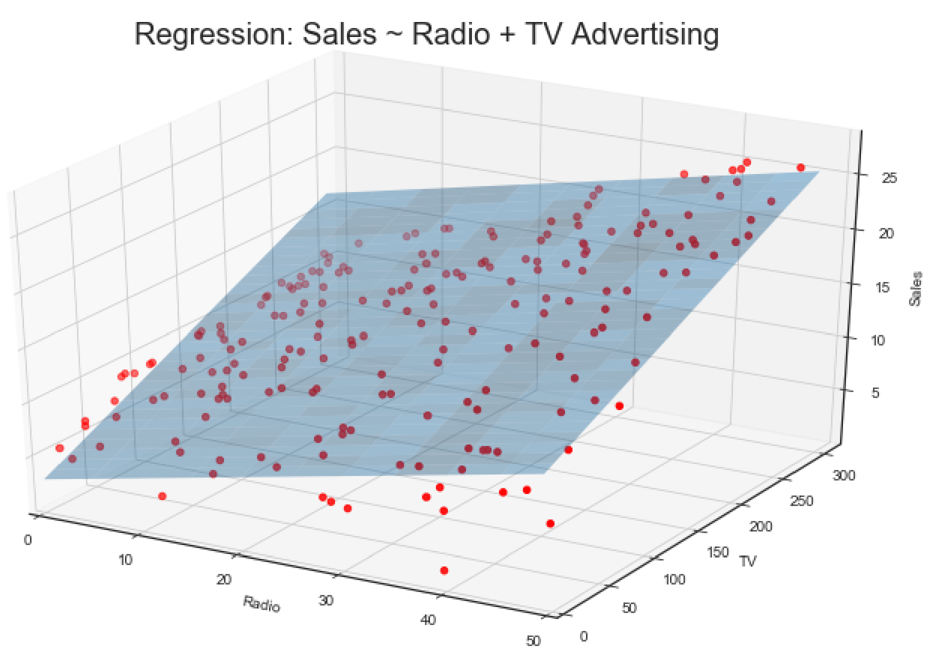
Out[20]:

	Radio	TV
count	200.000000	200.000000
mean	23.264000	147.042500
std	14.846809	85.854236
min	0.000000	0.700000
25%	9.975000	74.375000

50%	22.900000	149.750000
75%	36.525000	218.825000
max	49.600000	296.400000

In [21]:
```
# Create a coordinate grid
Radio = np.arange(0,50)
TV = np.arange(0,300)

B1, B2 = np.meshgrid(Radio, TV, indexing='xy')
Z = np.zeros((TV.size, Radio.size))

for (i,j),v in np.ndenumerate(Z):
        Z[i,j] =(regr.intercept_ + B1[i,j]*regr.coef_[0] + B2[i,j]*regr
.coef_[1])
```

In [22]:
```
# Create plot
fig = plt.figure(figsize=(10,6))
fig.suptitle('Regression: Sales ~ Radio + TV Advertising', fontsize=20)

ax = axes3d.Axes3D(fig)

ax.plot_surface(B1, B2, Z, rstride=10, cstride=5, alpha=0.4)
ax.scatter3D(advertising.Radio, advertising.TV, advertising.Sales, c=
'r')

ax.set_xlabel('Radio')
ax.set_xlim(0,50)
ax.set_ylabel('TV')
ax.set_ylim(ymin=0)
ax.set_zlabel('Sales');
```



3.3 Other Considerations in the Regression Model

3.3 Other Considerations in the Regression Model

Figure 3.6

In [23]:
```
sns.pairplot(credit[['Balance','Age','Cards','Education','Income','Limit','Rating']]);
```

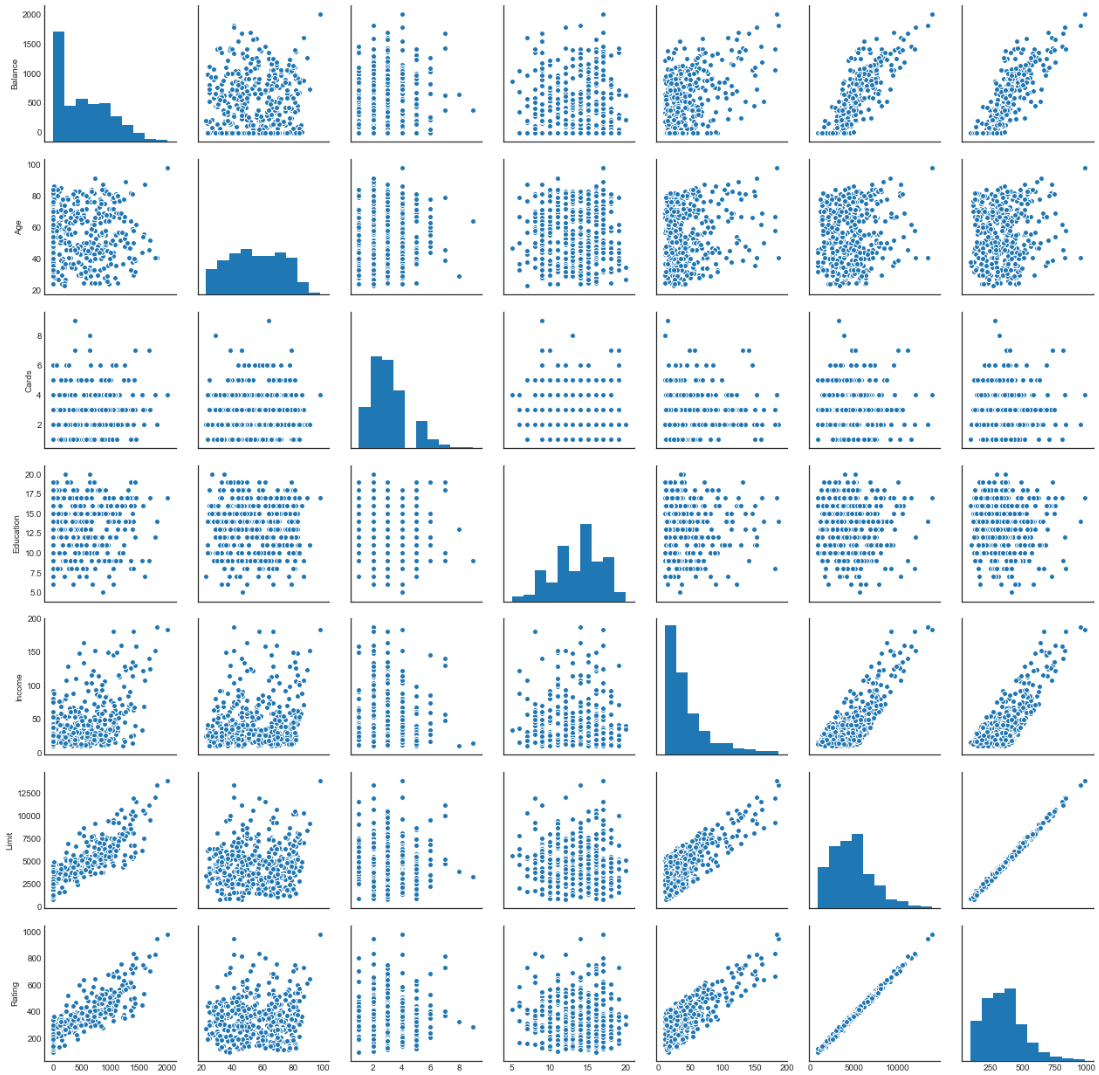


Table 3.7

In [24]:
```
est = smf.ols('Balance ~ Gender', credit).fit()
est.summary().tables[1]
```

Out[24]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	509.8031	33.128	15.389	0.000	444.675	574.931
Gender[T.Female]	19.7331	46.051	0.429	0.669	-70.801	110.267

Table 3.8

In [25]:
```
est = smf.ols('Balance ~ Ethnicity', credit).fit()
est.summary().tables[1]
```

Out[25]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	531.0000	46.319	11.464	0.000	439.939	622.061
Ethnicity[T.Asian]	-18.6863	65.021	-0.287	0.774	-146.515	109.142
Ethnicity[T.Caucasian]	-12.5025	56.681	-0.221	0.826	-123.935	98.930

Table 3.9 - Interaction Variables

In [26]:
```
est = smf.ols('Sales ~ TV + Radio + TV*Radio', advertising).fit()
est.summary().tables[1]
```

Out[26]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	6.7502	0.248	27.233	0.000	6.261	7.239
TV	0.0191	0.002	12.699	0.000	0.016	0.022
Radio	0.0289	0.009	3.241	0.001	0.011	0.046
TV:Radio	0.0011	5.24e-05	20.727	0.000	0.001	0.001

Figure 3.7 - Interaction between qualitative and quantative variables

In [27]:
```
est1 = smf.ols('Balance ~ Income + Student2', credit).fit()
regr1 = est1.params
est2 = smf.ols('Balance ~ Income + Income*Student2', credit).fit()
regr2 = est2.params

print('Regression 1 - without interaction term')
print(regr1)
print('\nRegression 2 - with interaction term')
print(regr2)
```

```
Regression 1 - without interaction term
Intercept    211.142964
Income         5.984336
Student2     382.670539
dtype: float64

Regression 2 - with interaction term
Intercept          200.623153
Income               6.218169
Student2           476.675843
Income:Student2     -1.999151
dtype: float64
```

In [28]:
```
# Income (x-axis)
```

```
income = np.linspace(0,150)

# Balance without interaction term (y-axis)
student1 = np.linspace(regr1['Intercept']+regr1['Student2'],
                       regr1['Intercept']+regr1['Student2']+150*regr1[
'Income'])
non_student1 =  np.linspace(regr1['Intercept'], regr1['Intercept']+150*
regr1['Income'])

# Balance with iteraction term (y-axis)
student2 = np.linspace(regr2['Intercept']+regr2['Student2'],
                       regr2['Intercept']+regr2['Student2']+
                       150*(regr2['Income']+regr2['Income:Student2']))
non_student2 =  np.linspace(regr2['Intercept'], regr2['Intercept']+150*
regr2['Income'])

# Create plot
fig, (ax1,ax2) = plt.subplots(1,2, figsize=(12,5))
ax1.plot(income, student1, 'r', income, non_student1, 'k')
ax2.plot(income, student2, 'r', income, non_student2, 'k')

for ax in fig.axes:
    ax.legend(['student', 'non-student'], loc=2)
    ax.set_xlabel('Income')
    ax.set_ylabel('Balance')
    ax.set_ylim(ymax=1550)
```

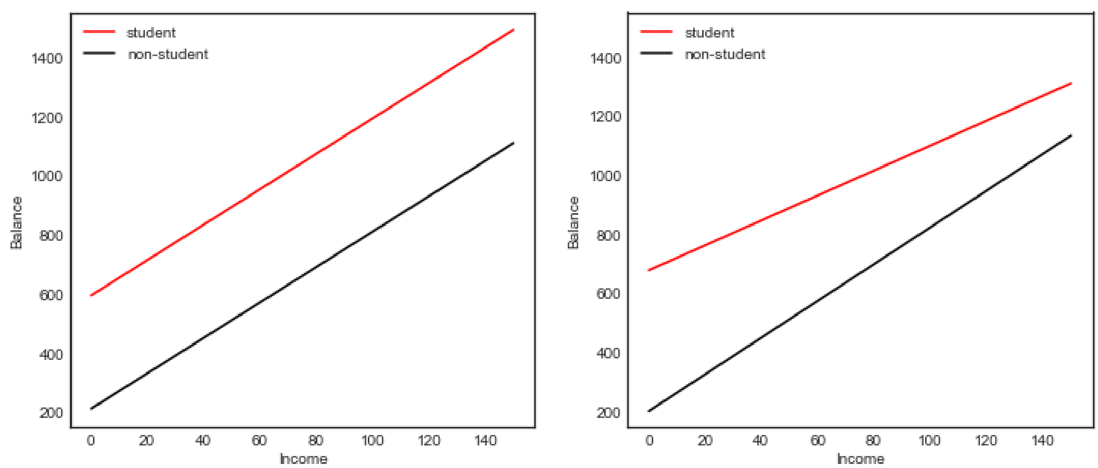


Figure 3.8 - Non-linear relationships

In [29]:
```
# With Seaborn's regplot() you can easily plot higher order polynomial
s.
plt.scatter(auto.horsepower, auto.mpg, facecolors='None', edgecolors=
'k', alpha=.5)
sns.regplot(auto.horsepower, auto.mpg, ci=None, label='Linear', scatter
=False, color='orange')
sns.regplot(auto.horsepower, auto.mpg, ci=None, label='Degree 2', order
=2, scatter=False, color='lightblue')
sns.regplot(auto.horsepower, auto.mpg, ci=None, label='Degree 5', order
=5, scatter=False, color='g')
plt.legend()
plt.ylim(5,55)
```

```
plt.xlim(40,240);
```

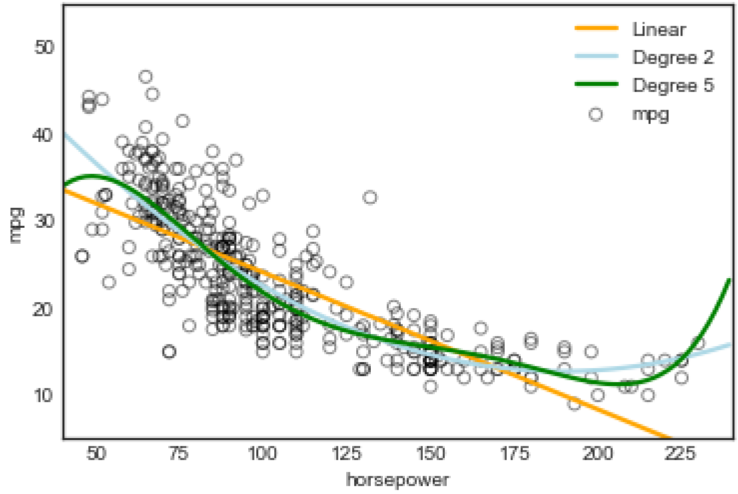


Table 3.10

In [30]:
```
auto['horsepower2'] = auto.horsepower**2
auto.head(3)
```

Out[30]:

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin	n
0	18.0	8	307.0	130.0	3504	12.0	70	1	c c n
1	15.0	8	350.0	165.0	3693	11.5	70	1	b s 3
2	18.0	8	318.0	150.0	3436	11.0	70	1	p s

In [31]:
```
est = smf.ols('mpg ~ horsepower + horsepower2', auto).fit()
est.summary().tables[1]
```

Out[31]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	56.9001	1.800	31.604	0.000	53.360	60.440
horsepower	-0.4662	0.031	-14.978	0.000	-0.527	-0.405
horsepower2	0.0012	0.000	10.080	0.000	0.001	0.001

Figure 3.9

In [32]:
```
regr = skl_lm.LinearRegression()

# Linear fit
X = auto.horsepower.values.reshape(-1,1)
```

```
y = auto.mpg
regr.fit(X, y)

auto['pred1'] = regr.predict(X)
auto['resid1'] = auto.mpg - auto.pred1

# Quadratic fit
X2 = auto[['horsepower', 'horsepower2']].as_matrix()
regr.fit(X2, y)

auto['pred2'] = regr.predict(X2)
auto['resid2'] = auto.mpg - auto.pred2
```

In [33]:
```
fig, (ax1,ax2) = plt.subplots(1,2, figsize=(12,5))

# Left plot
sns.regplot(auto.pred1, auto.resid1, lowess=True,
            ax=ax1, line_kws={'color':'r', 'lw':1},
            scatter_kws={'facecolors':'None', 'edgecolors':'k', 'alpha'
:0.5})
ax1.hlines(0,xmin=ax1.xaxis.get_data_interval()[0],
           xmax=ax1.xaxis.get_data_interval()[1], linestyles='dotted')
ax1.set_title('Residual Plot for Linear Fit')

# Right plot
sns.regplot(auto.pred2, auto.resid2, lowess=True,
            line_kws={'color':'r', 'lw':1}, ax=ax2,
            scatter_kws={'facecolors':'None', 'edgecolors':'k', 'alpha'
:0.5})
ax2.hlines(0,xmin=ax2.xaxis.get_data_interval()[0],
           xmax=ax2.xaxis.get_data_interval()[1], linestyles='dotted')
ax2.set_title('Residual Plot for Quadratic Fit')

for ax in fig.axes:
    ax.set_xlabel('Fitted values')
    ax.set_ylabel('Residuals')
```

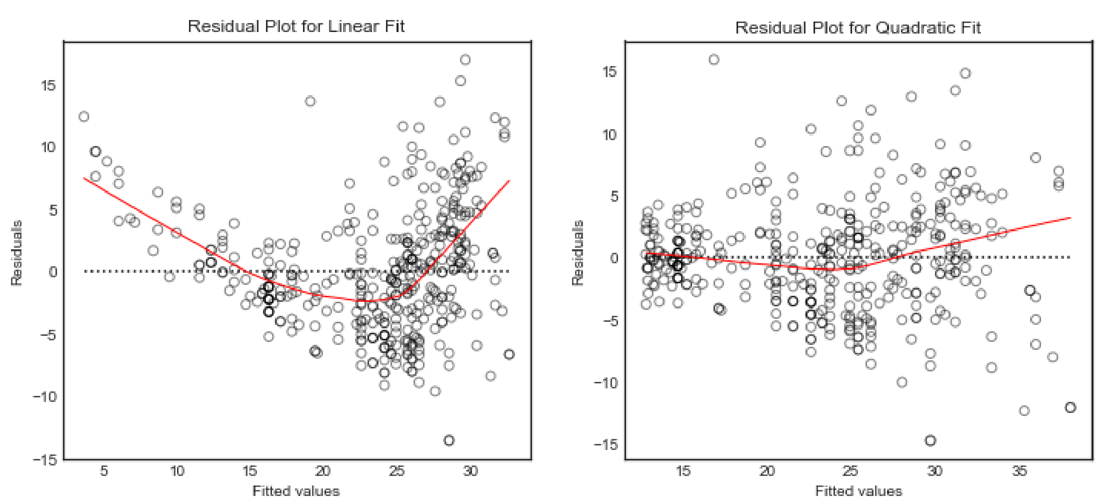


Figure 3.14

In [34]:
```
fig, (ax1,ax2) = plt.subplots(1,2, figsize=(12,5))
```

```python
# Left plot
ax1.scatter(credit.Limit, credit.Age, facecolor='None', edgecolor='r')
ax1.set_ylabel('Age')

# Right plot
ax2.scatter(credit.Limit, credit.Rating, facecolor='None', edgecolor=
'r')
ax2.set_ylabel('Rating')

for ax in fig.axes:
    ax.set_xlabel('Limit')
    ax.set_xticks([2000,4000,6000,8000,12000])
```

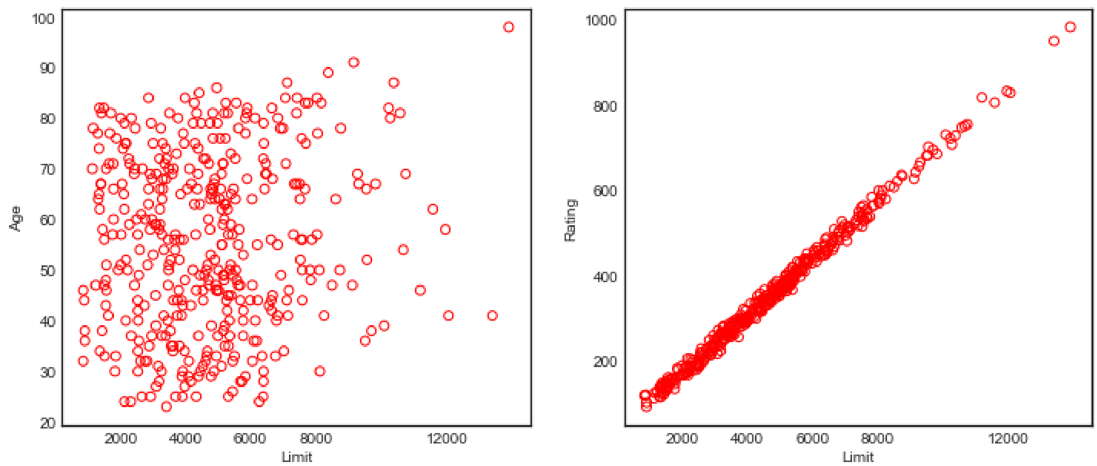


Figure 3.15

In [35]:
```python
y = credit.Balance

# Regression for left plot
X = credit[['Age', 'Limit']].as_matrix()
regr1 = skl_lm.LinearRegression()
regr1.fit(scale(X.astype('float'), with_std=False), y)
print('Age/Limit\n',regr1.intercept_)
print(regr1.coef_)

# Regression for right plot
X2 = credit[['Rating', 'Limit']].as_matrix()
regr2 = skl_lm.LinearRegression()
regr2.fit(scale(X2.astype('float'), with_std=False), y)
print('\nRating/Limit\n',regr2.intercept_)
print(regr2.coef_)
```

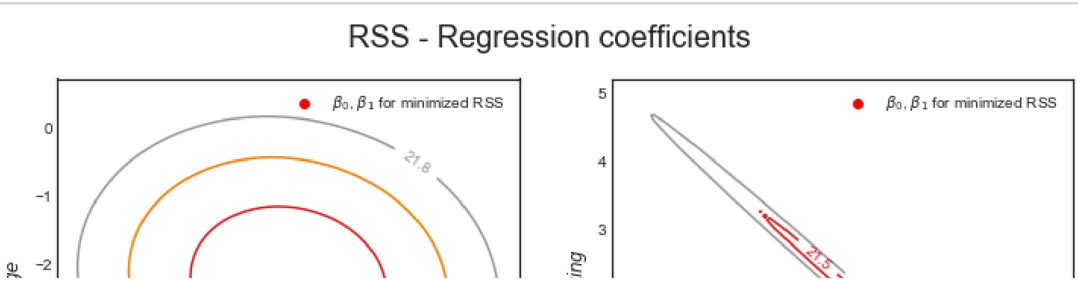
```
Age/Limit
 520.015
[-2.29148553  0.17336497]

Rating/Limit
 520.015
[ 2.20167217  0.02451438]
```

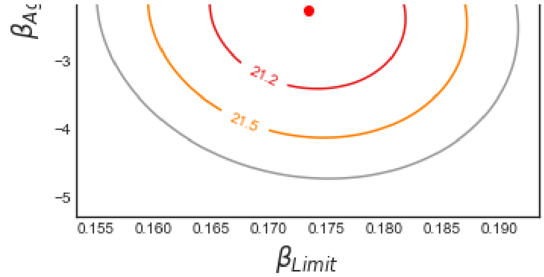
In [36]:
```python
# Create grid coordinates for plotting
```

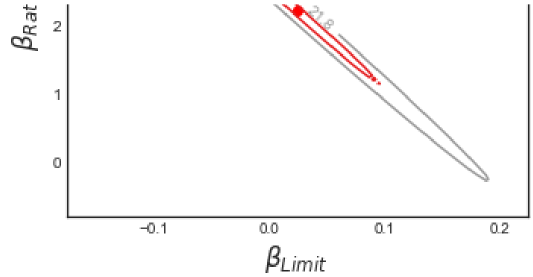
```
B_Age = np.linspace(regr1.coef_[0]-3, regr1.coef_[0]+3, 100)
B_Limit = np.linspace(regr1.coef_[1]-0.02, regr1.coef_[1]+0.02, 100)

B_Rating = np.linspace(regr2.coef_[0]-3, regr2.coef_[0]+3, 100)
B_Limit2 = np.linspace(regr2.coef_[1]-0.2, regr2.coef_[1]+0.2, 100)

X1, Y1 = np.meshgrid(B_Limit, B_Age, indexing='xy')
X2, Y2 = np.meshgrid(B_Limit2, B_Rating, indexing='xy')
Z1 = np.zeros((B_Age.size,B_Limit.size))
Z2 = np.zeros((B_Rating.size,B_Limit2.size))

Limit_scaled = scale(credit.Limit.astype('float'), with_std=False)
Age_scaled = scale(credit.Age.astype('float'), with_std=False)
Rating_scaled = scale(credit.Rating.astype('float'), with_std=False)

# Calculate Z-values (RSS) based on grid of coefficients
for (i,j),v in np.ndenumerate(Z1):
    Z1[i,j] =((y - (regr1.intercept_ + X1[i,j]*Limit_scaled +
                    Y1[i,j]*Age_scaled))**2).sum()/1000000

for (i,j),v in np.ndenumerate(Z2):
    Z2[i,j] =((y - (regr2.intercept_ + X2[i,j]*Limit_scaled +
                    Y2[i,j]*Rating_scaled))**2).sum()/1000000
```

In [37]:

```
fig = plt.figure(figsize=(12,5))
fig.suptitle('RSS - Regression coefficients', fontsize=20)

ax1 = fig.add_subplot(121)
ax2 = fig.add_subplot(122)

min_RSS = r'$\beta_0$, $\beta_1$ for minimized RSS'

# Left plot
CS = ax1.contour(X1, Y1, Z1, cmap=plt.cm.Set1, levels=[21.25, 21.5, 21.
8])
ax1.scatter(regr1.coef_[1], regr1.coef_[0], c='r', label=min_RSS)
ax1.clabel(CS, inline=True, fontsize=10, fmt='%1.1f')
ax1.set_ylabel(r'$\beta_{Age}$', fontsize=17)

# Right plot
CS = ax2.contour(X2, Y2, Z2, cmap=plt.cm.Set1, levels=[21.5, 21.8])
ax2.scatter(regr2.coef_[1], regr2.coef_[0], c='r', label=min_RSS)
ax2.clabel(CS, inline=True, fontsize=10, fmt='%1.1f')
ax2.set_ylabel(r'$\beta_{Rating}$', fontsize=17)
ax2.set_xticks([-0.1, 0, 0.1, 0.2])

for ax in fig.axes:
    ax.set_xlabel(r'$\beta_{Limit}$', fontsize=17)
    ax.legend()
```







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In [38]:
```
est_Age = smf.ols('Age ~ Rating + Limit', credit).fit()
est_Rating = smf.ols('Rating ~ Age + Limit', credit).fit()
est_Limit = smf.ols('Limit ~ Age + Rating', credit).fit()

print(1/(1-est_Age.rsquared))
print(1/(1-est_Rating.rsquared))
print(1/(1-est_Limit.rsquared))
```

```
1.01138468607
160.668300959
160.592879786
```