

Policy Gradients

CS 294-112: Deep Reinforcement Learning

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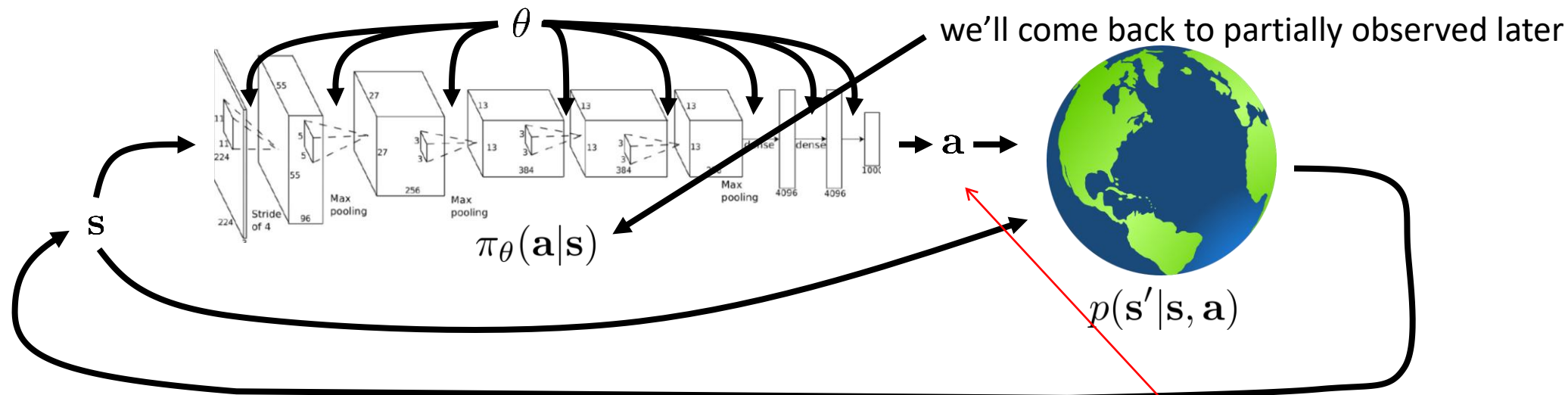
Class Notes

1. Homework 1 due today (11:59 pm)!
 - Don't be late!
2. Remember to start forming final project groups

Today's Lecture

1. The policy gradient algorithm
2. What does the policy gradient do?
3. Basic variance reduction: causality
4. Basic variance reduction: baselines
5. Policy gradient examples
 - Goals:
 - Understand policy gradient reinforcement learning
 - Understand practical considerations for policy gradients

The goal of reinforcement learning



$$\underbrace{p_\theta(\mathbf{s}_1, \mathbf{a}_1, \dots, \mathbf{s}_T, \mathbf{a}_T)}_{p_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^T \pi_\theta(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

$$\theta^\star = \arg \max_{\theta} E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

离散的话 : softmax 搞出 categorical distribution over the action.
连续的话 : NN 输出 Gaussian Distribution 的 mean and variance

The goal of reinforcement learning

$$\theta^* = \arg \max_{\theta} E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$\theta^* = \arg \max_{\theta} E_{(\mathbf{s}, \mathbf{a}) \sim p_{\theta}(\mathbf{s}, \mathbf{a})} [r(\mathbf{s}, \mathbf{a})]$$

infinite horizon case

$$\theta^* = \arg \max_{\theta} \sum_{t=1}^T E_{(\mathbf{s}_t, \mathbf{a}_t) \sim p_{\theta}(\mathbf{s}_t, \mathbf{a}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)]$$

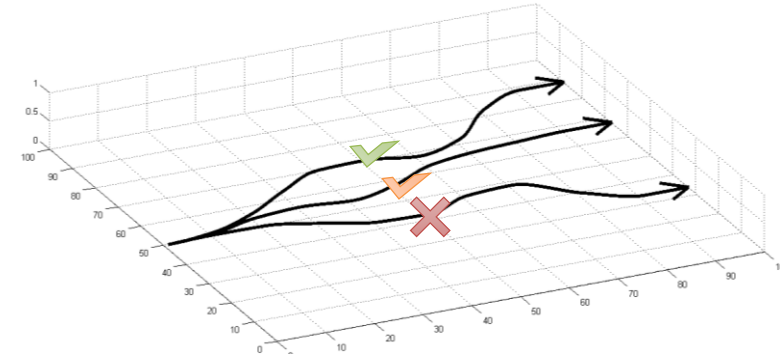
finite horizon case

Evaluating the objective

$$\theta^* = \arg \max_{\theta} \underbrace{E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]}_{J(\theta)}$$

$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right] \approx \frac{1}{N} \sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

sum over samples from π_{θ}



Direct policy differentiation

$$\theta^* = \arg \max_{\theta} \underbrace{E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]}_{J(\theta)}$$

a convenient identity

$$\underline{\pi_{\theta}(\tau) \nabla_{\theta} \log \pi_{\theta}(\tau)} = \pi_{\theta}(\tau) \frac{\nabla_{\theta} \pi_{\theta}(\tau)}{\pi_{\theta}(\tau)} = \underline{\nabla_{\theta} \pi_{\theta}(\tau)}$$

$$J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[\underbrace{r(\tau)}_{\sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t)} \right] = \int \pi_{\theta}(\tau) r(\tau) d\tau$$

$$\nabla_{\theta} J(\theta) = \int \underline{\nabla_{\theta} \pi_{\theta}(\tau)} r(\tau) d\tau = \int \underline{\pi_{\theta}(\tau) \nabla_{\theta} \log \pi_{\theta}(\tau)} r(\tau) d\tau = E_{\tau \sim \pi_{\theta}(\tau)} [\nabla_{\theta} \log \pi_{\theta}(\tau) r(\tau)]$$

Direct policy differentiation

$$\begin{aligned}
 \theta^* &= \arg \max_{\theta} J(\theta) \\
 J(\theta) &= E_{\tau \sim \pi_{\theta}(\tau)}[r(\tau)] \\
 \nabla_{\theta} J(\theta) &= E_{\tau \sim \pi_{\theta}(\tau)}[\nabla_{\theta} \log \pi_{\theta}(\tau) r(\tau)]
 \end{aligned}$$

log of both sides

$$\underbrace{\pi_{\theta}(\mathbf{s}_1, \mathbf{a}_1, \dots, \mathbf{s}_T, \mathbf{a}_T)}_{\pi_{\theta}(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

$$\log \pi_{\theta}(\tau) = \log p(\mathbf{s}_1) + \sum_{t=1}^T \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) + \log p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

$$\nabla_{\theta} \left[\log p(\mathbf{s}_1) + \sum_{t=1}^T \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) + \log p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t) \right]$$

$$\nabla_{\theta} J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[\left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \right) \left(\sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$$

Evaluating the policy gradient

recall: $J(\theta) = E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right] \approx \frac{1}{N} \sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$

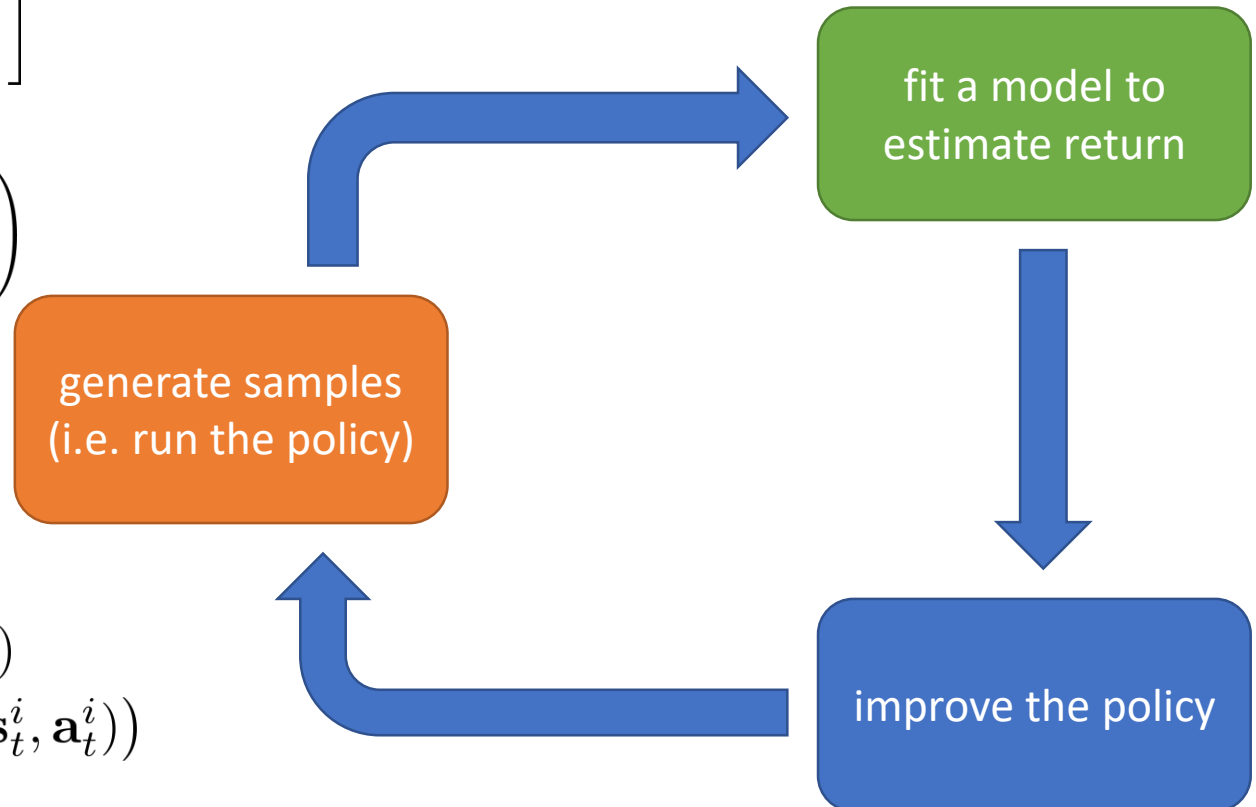
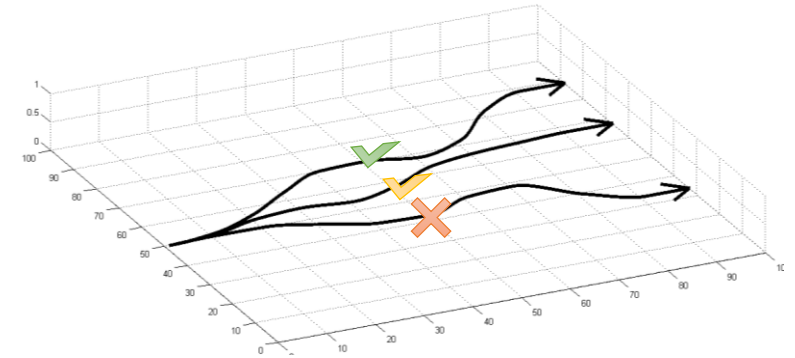
$$\nabla_{\theta} J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[\left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \right) \left(\sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^T r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$ (run the policy)
2. $\nabla_{\theta} J(\theta) \approx \sum_i \left(\sum_t \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t^i | \mathbf{s}_t^i) \right) \left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i) \right)$
3. $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$



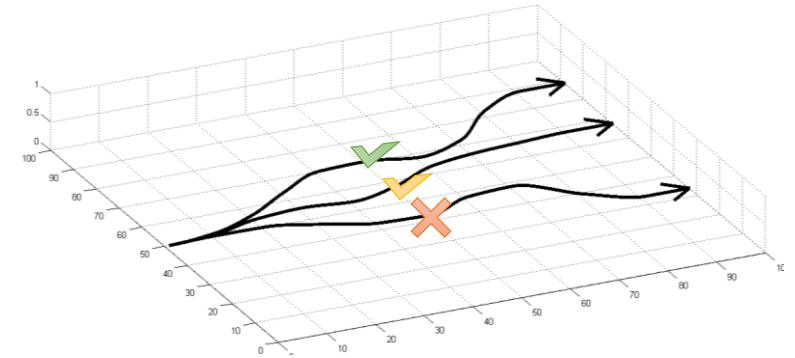
Evaluating the policy gradient

recall: $J(\theta) = E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right] \approx \frac{1}{N} \sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$

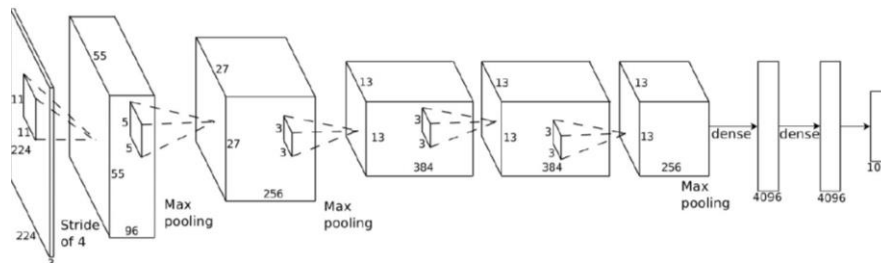
$$\nabla_{\theta} J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[\left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \right) \left(\sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^T r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

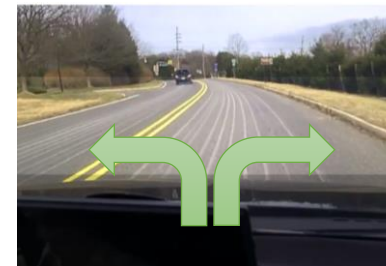
what is this?



\mathbf{s}_t



$\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$

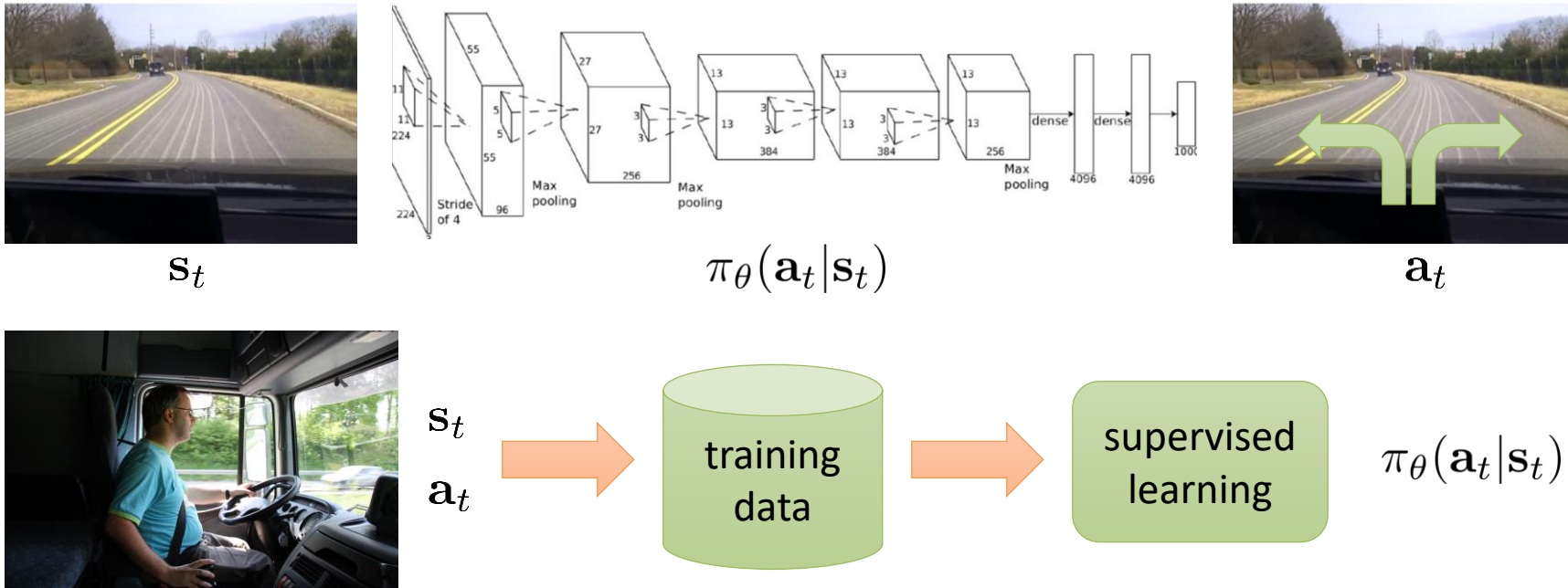


\mathbf{a}_t

Comparison to maximum likelihood

policy gradient:
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^T r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

maximum likelihood:
$$\nabla_{\theta} J_{\text{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right)$$



Example: Gaussian policies

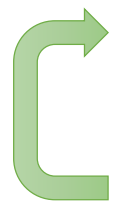
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^T r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

example: $\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) = \mathcal{N}(f_{\text{neural network}}(\mathbf{s}_t); \Sigma)$

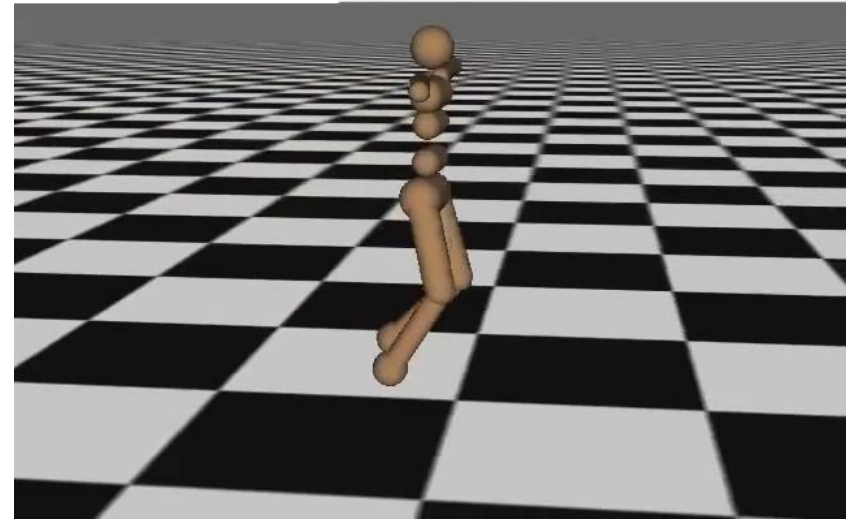
$$\log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) = -\frac{1}{2} \|f(\mathbf{s}_t) - \mathbf{a}_t\|_{\Sigma}^2 + \text{const}$$

$$\nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) = -\frac{1}{2} \Sigma^{-1} (f(\mathbf{s}_t) - \mathbf{a}_t) \frac{df}{d\theta}$$

REINFORCE algorithm:

- 
1. sample $\{\tau^i\}$ from $\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$ (run it on the robot)
 2. $\nabla_{\theta} J(\theta) \approx \sum_i \left(\sum_t \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t^i | \mathbf{s}_t^i) \right) \left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i) \right)$
 3. $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$

Iteration 2000



What did we just do?

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^T r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \underbrace{\nabla_{\theta} \log \pi_{\theta}(\tau_i) r(\tau_i)}_{\sum_{t=1}^T \nabla_{\theta} \log_{\theta} \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t})}$$

maximum likelihood: $\nabla_{\theta} J_{\text{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^N \nabla_{\theta} \log \pi_{\theta}(\tau_i)$

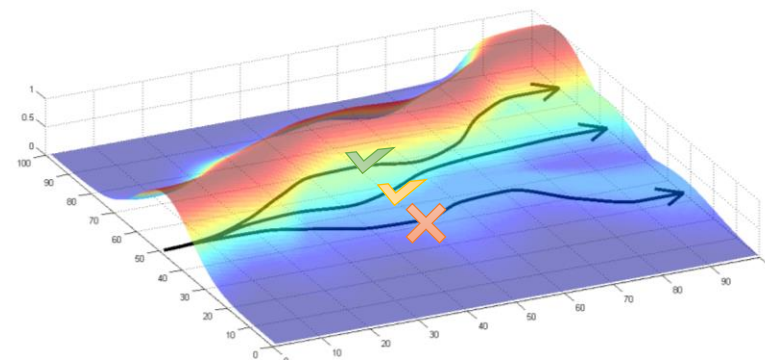
good stuff is made more likely

bad stuff is made less likely

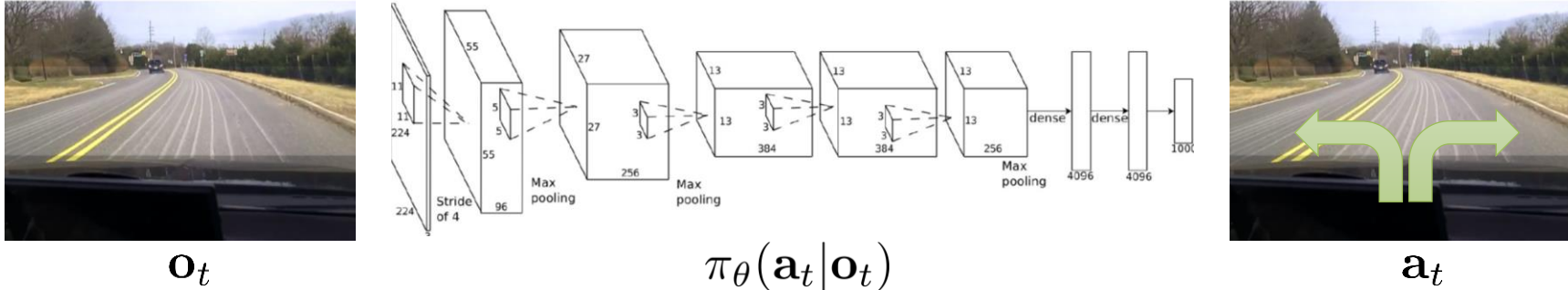
simply formalizes the notion of “trial and error”!

REINFORCE algorithm:

- 1. sample $\{\tau^i\}$ from $\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$ (run it on the robot)
- 2. $\nabla_{\theta} J(\theta) \approx \sum_i \left(\sum_t \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t^i | \mathbf{s}_t^i) \right) \left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i) \right)$
- 3. $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$



Partial observability



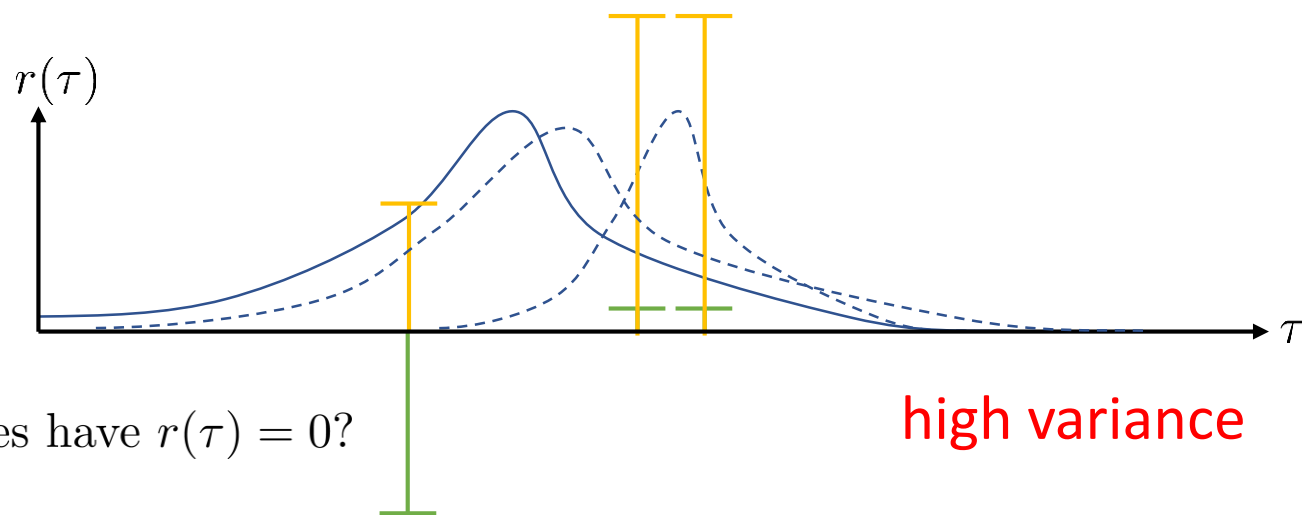
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{o}_{i,t}) \right) \left(\sum_{t=1}^T r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

Markov property is not actually used!

Can use policy gradient in partially observed MDPs without modification

What is wrong with the policy gradient?

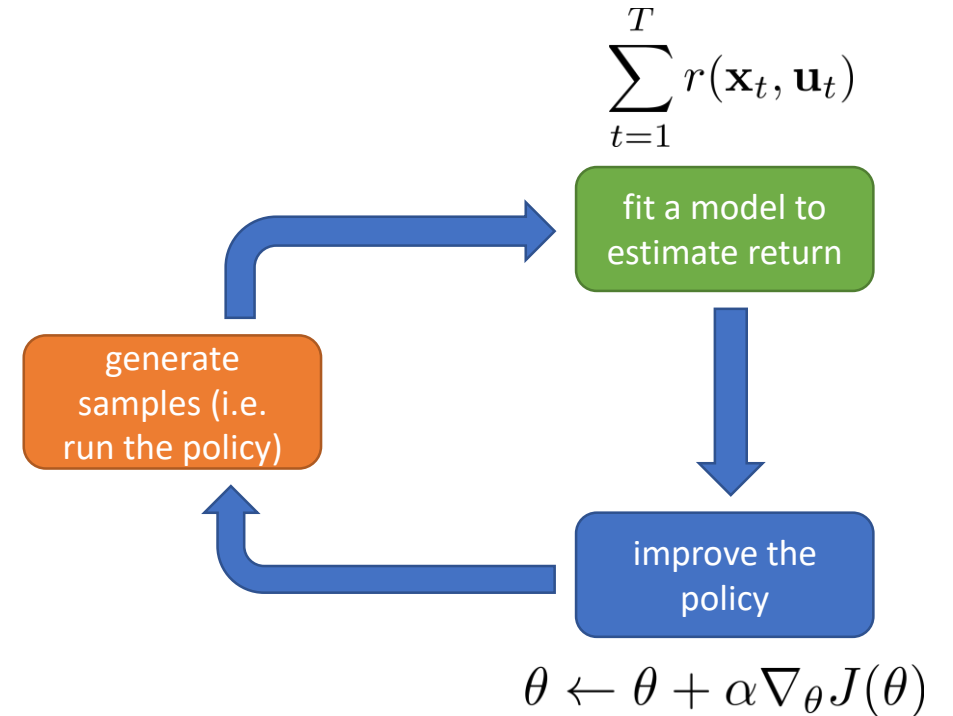
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \nabla_{\theta} \log \pi_{\theta}(\tau) r(\tau)$$



even worse: what if the two “good” samples have $r(\tau) = 0$?

Review

- Evaluating the RL objective
 - Generate samples
- Evaluating the policy gradient
 - Log-gradient trick
 - Generate samples
- Understanding the policy gradient
 - Formalization of trial-and-error
- Partial observability
 - Works just fine
- What is wrong with policy gradient?



Break

Reducing variance

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^T r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

Causality: policy at time t' cannot affect reward at time t when $t < t'$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \underbrace{\left(\sum_{t'=t}^T r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)}_{\text{"reward to go"}}$$

$\hat{Q}_{i,t}$

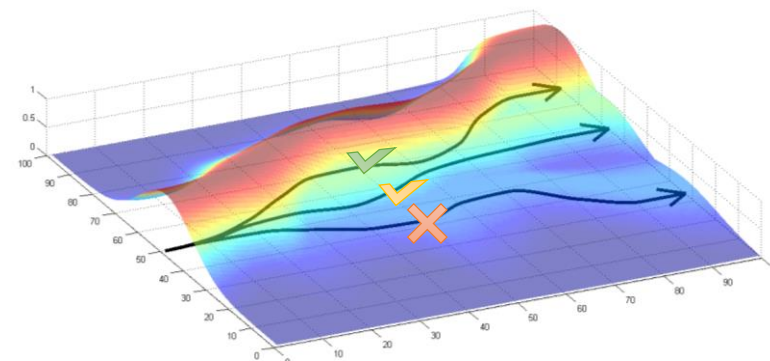
Baselines

a convenient identity

$$\pi_{\theta}(\tau) \nabla_{\theta} \log \pi_{\theta}(\tau) = \nabla_{\theta} \pi_{\theta}(\tau)$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \nabla_{\theta} \log \pi_{\theta}(\tau) [r(\tau) - b]$$

$$b = \frac{1}{N} \sum_{i=1}^N r(\tau) \quad \text{but... are we *allowed* to do that??}$$



$$E[\nabla_{\theta} \log \pi_{\theta}(\tau) b] = \int \pi_{\theta}(\tau) \nabla_{\theta} \log \pi_{\theta}(\tau) b d\tau = \int \nabla_{\theta} \pi_{\theta}(\tau) b d\tau = b \nabla_{\theta} \int \pi_{\theta}(\tau) d\tau = b \nabla_{\theta} 1 = 0$$

subtracting a baseline is *unbiased* in expectation!

average reward is *not* the best baseline, but it's pretty good!

Analyzing variance

can we write down the variance?

$$\text{Var}[x] = E[x^2] - E[x]^2$$

$$\nabla_{\theta} J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} [\nabla_{\theta} \log \pi_{\theta}(\tau) (r(\tau) - b)]$$

$$\text{Var} = E_{\tau \sim \pi_{\theta}(\tau)} [(\nabla_{\theta} \log \pi_{\theta}(\tau) (r(\tau) - b))^2] - \underbrace{E_{\tau \sim \pi_{\theta}(\tau)} [\nabla_{\theta} \log \pi_{\theta}(\tau) (r(\tau) - b)]^2}_{\text{this bit is just } E_{\tau \sim \pi_{\theta}(\tau)} [\nabla_{\theta} \log \pi_{\theta}(\tau) r(\tau)]}$$

(baselines are unbiased in expectation)

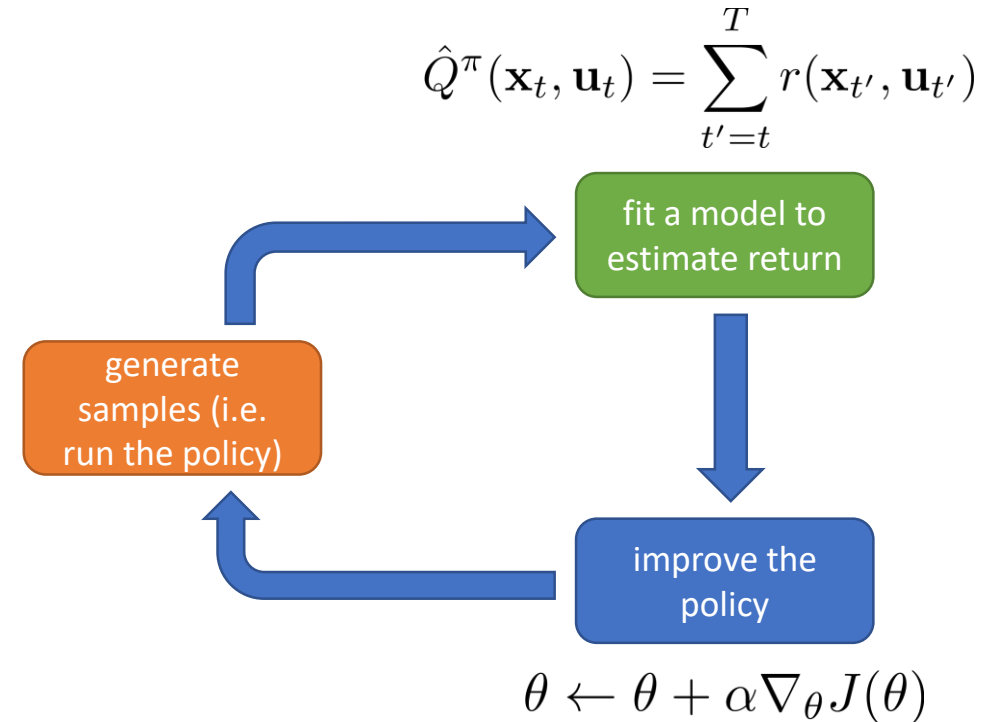
$$\begin{aligned} \frac{d\text{Var}}{db} &= \frac{d}{db} E[g(\tau)^2 (r(\tau) - b)^2] = \frac{d}{db} (E[\cancel{g(\tau)^2 r(\tau)^2}] - 2E[g(\tau)^2 r(\tau) b] + b^2 E[g(\tau)^2]) \\ &= -2E[g(\tau)^2 r(\tau)] + 2bE[g(\tau)^2] = 0 \end{aligned}$$

$$b = \frac{E[g(\tau)^2 r(\tau)]}{E[g(\tau)^2]}$$

← This is just expected reward, but weighted by gradient magnitudes!

Review

- The high variance of policy gradient
- Exploiting causality
 - Future doesn't affect the past
- Baselines
 - Unbiased!
- Analyzing variance
 - Can derive optimal baselines



Policy gradient is on-policy

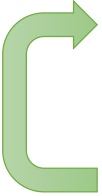
$$\theta^* = \arg \max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)}[r(\tau)]$$

$$\nabla_{\theta} J(\theta) = \underbrace{E_{\tau \sim \pi_{\theta}(\tau)}[\nabla_{\theta} \log \pi_{\theta}(\tau) r(\tau)]}_{\text{this is trouble...}}$$

- Neural networks change only a little bit with each gradient step
- On-policy learning can be extremely inefficient!

REINFORCE algorithm:

- 
1. sample $\{\tau^i\}$ from $\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
 2. $\nabla_{\theta} J(\theta) \approx \sum_i (\sum_t \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t^i|\mathbf{s}_t^i)) (\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i))$
 3. $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$

can't just skip this!



Off-policy learning & importance sampling

$$\theta^* = \arg \max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)}[r(\tau)]$$

what if we don't have samples from $\pi_{\theta}(\tau)$?

(we have samples from some $\bar{\pi}(\tau)$ instead)

$$J(\theta) = E_{\tau \sim \bar{\pi}(\tau)} \left[\frac{\pi_{\theta}(\tau)}{\bar{\pi}(\tau)} r(\tau) \right]$$

$$\pi_{\theta}(\tau) = p(\mathbf{s}_1) \prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

$$\frac{\pi_{\theta}(\tau)}{\bar{\pi}(\tau)} = \frac{\cancel{p(\mathbf{s}_1)} \prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \cancel{p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)}}{\cancel{p(\mathbf{s}_1)} \prod_{t=1}^T \bar{\pi}(\mathbf{a}_t | \mathbf{s}_t) \cancel{p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)}} = \frac{\prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}{\prod_{t=1}^T \bar{\pi}(\mathbf{a}_t | \mathbf{s}_t)}$$

importance sampling

$$\begin{aligned} E_{x \sim p(x)}[f(x)] &= \int p(x) f(x) dx \\ &= \int \frac{q(x)}{q(x)} p(x) f(x) dx \\ &= \int q(x) \frac{p(x)}{q(x)} f(x) dx \\ &= E_{x \sim q(x)} \left[\frac{p(x)}{q(x)} f(x) \right] \end{aligned}$$

Deriving the policy gradient with IS

$$\theta^* = \arg \max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)}[r(\tau)]$$

a convenient identity

$$\pi_{\theta}(\tau) \nabla_{\theta} \log \pi_{\theta}(\tau) = \nabla_{\theta} \pi_{\theta}(\tau)$$

can we estimate the value of some *new* parameters θ' ?

$$J(\theta') = E_{\tau \sim \pi_{\theta}(\tau)} \left[\frac{\pi_{\theta'}(\tau)}{\pi_{\theta}(\tau)} r(\tau) \right]$$

the only bit that depends on θ'

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim \pi_{\theta}(\tau)} \left[\frac{\nabla_{\theta'} \pi_{\theta'}(\tau)}{\pi_{\theta}(\tau)} r(\tau) \right] = E_{\tau \sim \pi_{\theta}(\tau)} \left[\frac{\cancel{\pi_{\theta'}(\tau)}}{\cancel{\pi_{\theta}(\tau)}} \nabla_{\theta'} \log \pi_{\theta'}(\tau) r(\tau) \right]$$

now estimate locally, at $\theta = \theta'$: $\nabla_{\theta} J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)}[\nabla_{\theta} \log \pi_{\theta}(\tau) r(\tau)]$

The off-policy policy gradient

$$\theta^* = \arg \max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)}[r(\tau)]$$

$$\frac{\pi_{\theta'}(\tau)}{\pi_{\theta}(\tau)} = \frac{\prod_{t=1}^T \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t)}{\prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}$$

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim \pi_{\theta}(\tau)} \left[\frac{\pi_{\theta'}(\tau)}{\pi_{\theta}(\tau)} \nabla_{\theta'} \log \pi_{\theta'}(\tau) r(\tau) \right] \quad \text{when } \theta \neq \theta'$$

$$= E_{\tau \sim \pi_{\theta}(\tau)} \left[\left(\prod_{t=1}^T \frac{\pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t)}{\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)} \right) \left(\sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) \right) \left(\sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t) \right) \right] \quad \text{what about causality?}$$

$$= E_{\tau \sim \pi_{\theta}(\tau)} \left[\sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) \underbrace{\left(\prod_{t'=1}^t \frac{\pi_{\theta'}(\mathbf{a}_{t'} | \mathbf{s}_{t'})}{\pi_{\theta}(\mathbf{a}_{t'} | \mathbf{s}_{t'})} \right)}_{\text{future actions don't affect current weight}} \left(\sum_{t'=t}^T r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \left(\prod_{t''=t}^{t'} \frac{\pi_{\theta'}(\mathbf{a}_{t''} | \mathbf{s}_{t''})}{\pi_{\theta}(\mathbf{a}_{t''} | \mathbf{s}_{t''})} \right) \right) \right]$$

future actions don't affect current weight

if we ignore this, we get
a policy iteration algorithm
(more on this in a later lecture)

A first-order approximation for IS (preview)

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim \pi_{\theta}(\tau)} \left[\sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) \underbrace{\left(\prod_{t'=1}^t \frac{\pi_{\theta'}(\mathbf{a}_{t'} | \mathbf{s}_{t'})}{\pi_{\theta}(\mathbf{a}_{t'} | \mathbf{s}_{t'})} \right)}_{\text{exponential in } T} \left(\sum_{t'=t}^T r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \right) \right]$$

let's write the objective a bit differently...

$$\theta^* = \arg \max_{\theta} \sum_{t=1}^T \underbrace{E_{(\mathbf{s}_t, \mathbf{a}_t) \sim p_{\theta}(\mathbf{s}_t, \mathbf{a}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)]}_{\text{expectation under state-action marginal}}$$


$$J(\theta) = \sum_{t=1}^T E_{(\mathbf{s}_t, \mathbf{a}_t) \sim p_{\theta}(\mathbf{s}_t, \mathbf{a}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)] = \sum_{t=1}^T E_{\mathbf{s}_t \sim p_{\theta}(\mathbf{s}_t)} [E_{\mathbf{a}_t \sim \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)]]$$

$$J(\theta') = \sum_{t=1}^T E_{\mathbf{s}_t \sim p_{\theta}(\mathbf{s}_t)} \left[\cancel{\frac{p_{\theta'}(\mathbf{s}_t)}{p_{\theta}(\mathbf{s}_t)}} E_{\mathbf{a}_t \sim \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)} \left[\frac{\pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t)}{\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)} r(\mathbf{s}_t, \mathbf{a}_t) \right] \right] \quad \text{We'll see why this is reasonable later in the course!}$$

ignore this part

Policy gradient with automatic differentiation

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}$$

 pretty inefficient to compute these explicitly!


How can we compute policy gradients with automatic differentiation?

We need a graph such that its gradient is the policy gradient!

maximum likelihood: $\nabla_{\theta} J_{\text{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t})$ $J_{\text{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t})$

Just implement “pseudo-loss” as a weighted maximum likelihood:

$$\tilde{J}(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}$$

 cross entropy (discrete) or squared error (Gaussian)

Policy gradient with automatic differentiation

Pseudocode example (with discrete actions):

Maximum likelihood:

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# Build the graph:
logits = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = tf.nn.softmax_cross_entropy_with_logits(labels=actions, logits=logits)
loss = tf.reduce_mean(negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

Policy gradient with automatic differentiation

Pseudocode example (with discrete actions):

Policy gradient:

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# q_values - (N*T) x 1 tensor of estimated state-action values
# Build the graph:
logits = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = tf.nn.softmax_cross_entropy_with_logits(labels=actions, logits=logits)
weighted_negative_likelihoods = tf.multiply(negative_likelihoods, q_values)
loss = tf.reduce_mean(weighted_negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

$$\tilde{J}(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}$$

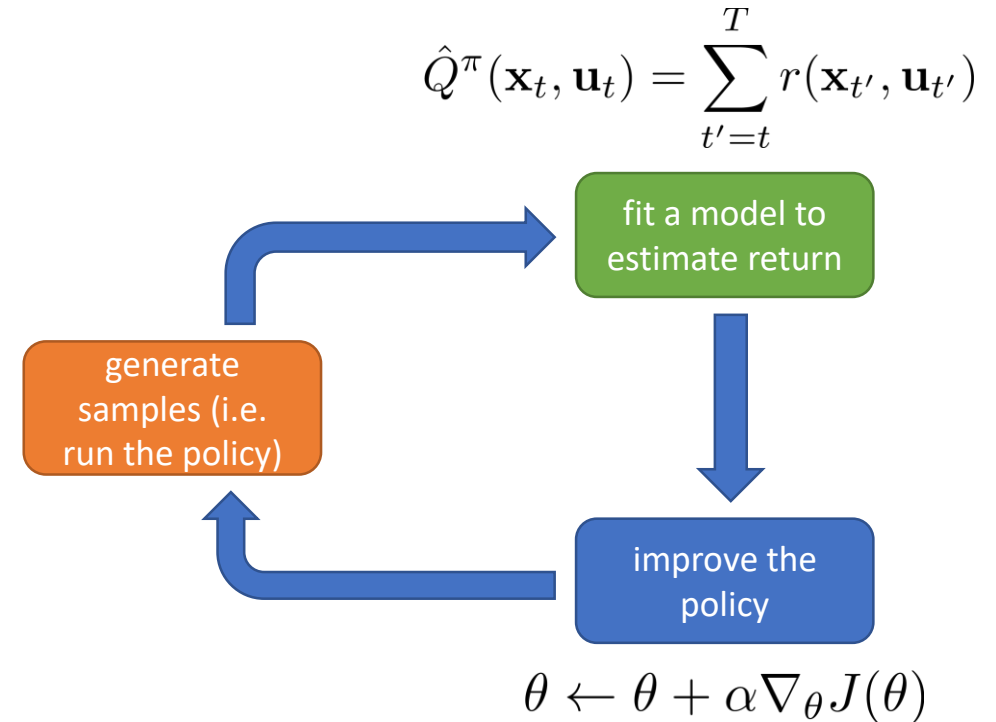
q_values

Policy gradient in practice

- Remember that the gradient has high variance
 - This isn't the same as supervised learning!
 - Gradients will be really noisy!
- Consider using much larger batches
- Tweaking learning rates is very hard
 - Adaptive step size rules like ADAM can be OK-ish
 - We'll learn about policy gradient-specific learning rate adjustment methods later!

Review

- Policy gradient is on-policy
- Can derive off-policy variant
 - Use importance sampling
 - Exponential scaling in T
 - Can ignore state portion (approximation)
- Can implement with automatic differentiation – need to know what to backpropagate
- Practical considerations: batch size, learning rates, optimizers



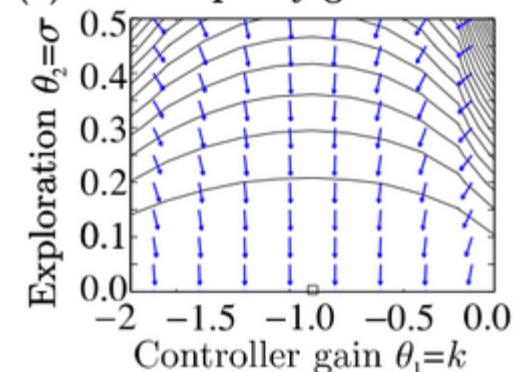
What *else* is wrong with the policy gradient?



$$r(s_t, a_t) = -s_t^2 - a_t^2$$

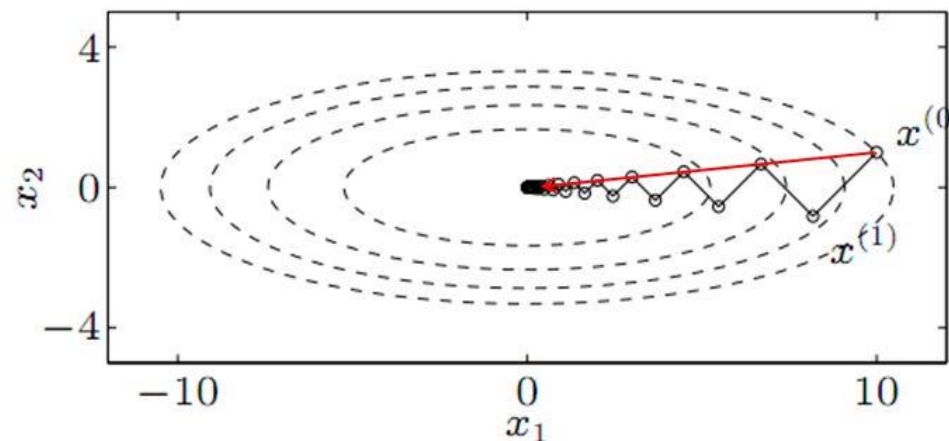
$$\log \pi_\theta(a_t|s_t) = -\frac{1}{2\sigma^2}(ks_t - a_t)^2 + \text{const} \quad \theta = (k, \sigma)$$

(a) 'Vanilla' policy gradients



(image from Peters & Schaal 2008)

Essentially the same problem as this:



Covariant/natural policy gradient

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta) \quad \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$$

some parameters change probabilities a lot more than others!

$$\theta' \leftarrow \arg \max_{\theta'} (\theta' - \theta)^T \nabla_{\theta} J(\theta) \text{ s.t. } \underline{\|\theta' - \theta\|^2 \leq \epsilon}$$

controls how far we go

can we *rescale* the gradient so this doesn't happen?

$$\theta' \leftarrow \arg \max_{\theta'} (\theta' - \theta)^T \nabla_{\theta} J(\theta) \text{ s.t. } \underline{D(\pi_{\theta'}, \pi_{\theta}) \leq \epsilon}$$

parameterization-independent divergence measure

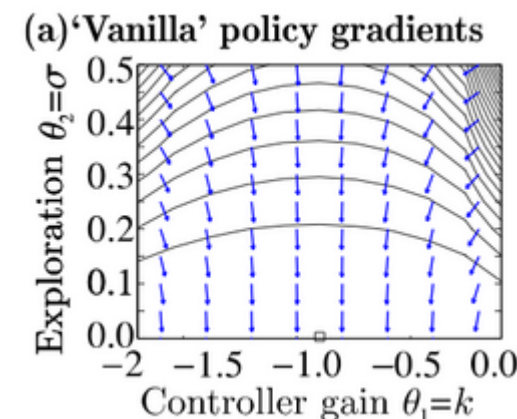
usually KL-divergence: $D_{\text{KL}}(\pi_{\theta'} \| \pi_{\theta}) = E_{\pi_{\theta'}} [\log \pi_{\theta} - \log \pi_{\theta'}]$

$$D_{\text{KL}}(\pi_{\theta'} \| \pi_{\theta}) \approx (\theta' - \theta)^T \underline{\mathbf{F}} (\theta' - \theta)$$

Fisher-information matrix

$$\mathbf{F} = E_{\pi_{\theta}} [\log \pi_{\theta}(\mathbf{a} | \mathbf{s}) \log \pi_{\theta}(\mathbf{a} | \mathbf{s})^T]$$

can estimate with samples



Covariant/natural policy gradient

$$D_{\text{KL}}(\pi_{\theta'} \parallel \pi_{\theta}) \approx (\theta' - \theta)^T \mathbf{F} (\theta' - \theta)$$

$$\mathbf{F} = E_{\pi_{\theta}} [\log \pi_{\theta}(\mathbf{a}|\mathbf{s}) \log \pi_{\theta}(\mathbf{a}|\mathbf{s})^T]$$

$$\theta' \leftarrow \arg \max_{\theta'} (\theta' - \theta)^T \nabla_{\theta} J(\theta) \text{ s.t. } D(\pi_{\theta'}, \pi_{\theta}) \leq \epsilon$$

$$\theta \leftarrow \theta + \alpha \mathbf{F}^{-1} \nabla_{\theta} J(\theta)$$

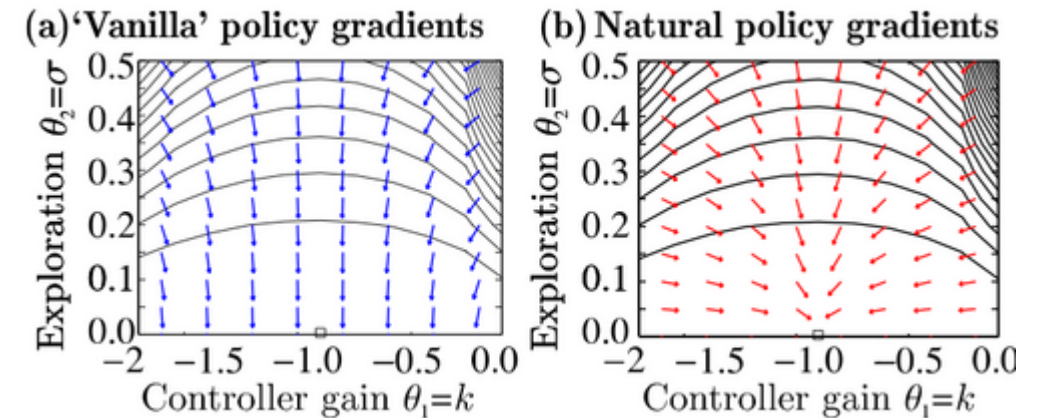
natural gradient: pick α

trust region policy optimization: pick ϵ

can solve for optimal α while solving $\mathbf{F}^{-1} \nabla_{\theta} J(\theta)$

conjugate gradient works well for this

see Schulman, L., Moritz, Jordan, Abbeel (2015) Trust region policy optimization



(figure from Peters & Schaal 2008)

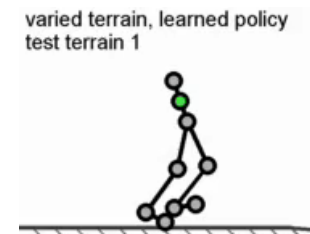
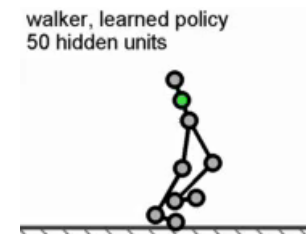
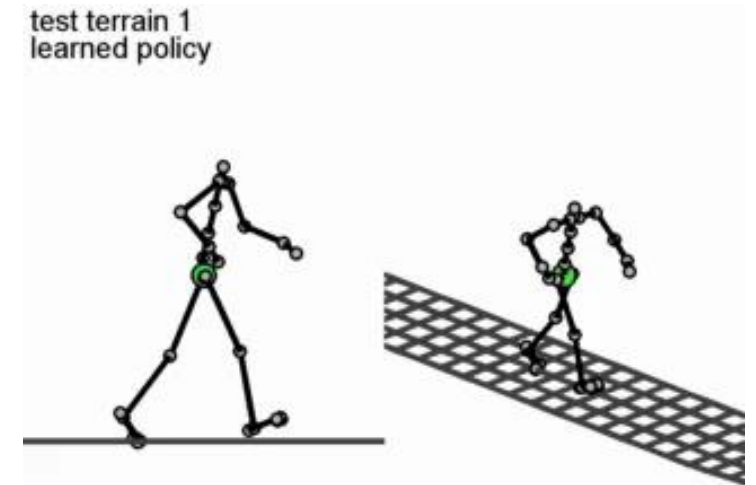
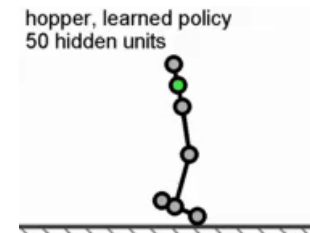
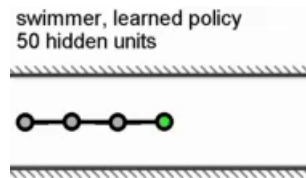
Advanced policy gradient topics

- What more is there?
- Next time: introduce value functions and Q-functions
- Later in the class: natural gradient and automatic step size adjustment

Example: policy gradient with importance sampling

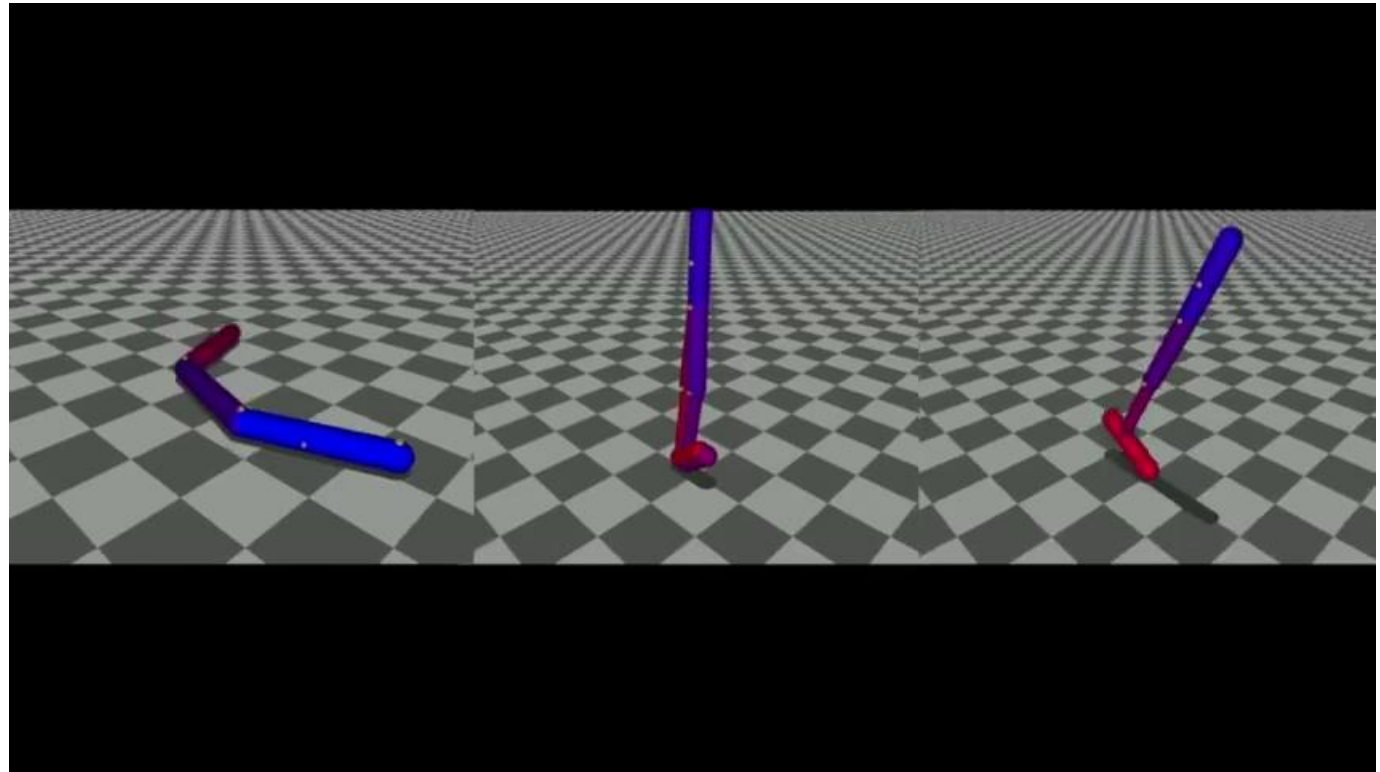
$$\nabla_{\theta'} J(\theta') = E_{\tau \sim \pi_{\theta}(\tau)} \left[\sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) \left(\prod_{t'=1}^t \frac{\pi_{\theta'}(\mathbf{a}_{t'} | \mathbf{s}_{t'})}{\pi_{\theta}(\mathbf{a}_{t'} | \mathbf{s}_{t'})} \right) \left(\sum_{t'=t}^T r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \right) \right]$$

- Incorporate example demonstrations using importance sampling
- Neural network policies



Example: trust region policy optimization

- Natural gradient with automatic step adjustment
- Discrete and continuous actions
- Code available (see Duan et al. '16)



Policy gradients suggested readings

- Classic papers
 - Williams (1992). Simple statistical gradient-following algorithms for connectionist reinforcement learning: introduces REINFORCE algorithm
 - Baxter & Bartlett (2001). Infinite-horizon policy-gradient estimation: temporally decomposed policy gradient (not the first paper on this! see actor-critic section later)
 - Peters & Schaal (2008). Reinforcement learning of motor skills with policy gradients: very accessible overview of optimal baselines and natural gradient
- Deep reinforcement learning policy gradient papers
 - Levine & Koltun (2013). Guided policy search: deep RL with importance sampled policy gradient (unrelated to later discussion of guided policy search)
 - Schulman, L., Moritz, Jordan, Abbeel (2015). Trust region policy optimization: deep RL with natural policy gradient and adaptive step size
 - Schulman, Wolski, Dhariwal, Radford, Klimov (2017). Proximal policy optimization algorithms: deep RL with importance sampled policy gradient