PageRank: Matrix Formulation

- **Stochastic adjacency matrix M**
  - Let page i has d_i out-links
  - If $i \to j$, then $M_{ji} = \frac{1}{d_i}$ else $M_{ji} = 0$
    - M is a **column stochastic matrix**
      - Columns sum to **1**
- **Rank vector r:** vector with an entry per page
  - r_i is the importance score of page i
  - $\sum_i r_i = 1$
- **The flow equations can be written**

$$r_j = \sum_{i \to j} \frac{r_i}{\mathrm{d_i}}$$

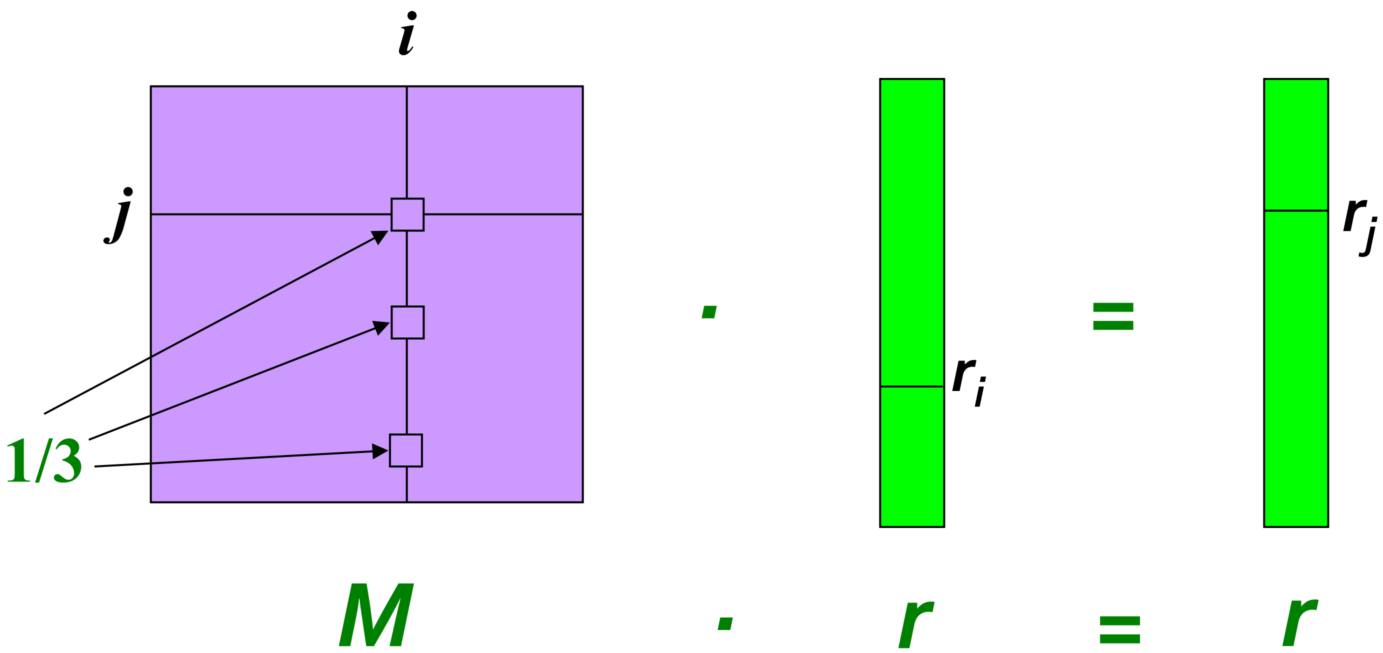
$$r = M \cdot r$$

Example

- **Remember the flow equation:** $r_j = \sum_{i \to j} \frac{r_i}{\mathrm{d_i}}$
- **Flow equation in the matrix form**

$$\boldsymbol{M \cdot r = r}$$

  - **Suppose page *i* links to 3 pages, including *j***



Eigenvector Formulation

NOTE: *x* is an eigenvector with the corresponding eigenvalue **λ** if:

$$Ax = \lambda x$$

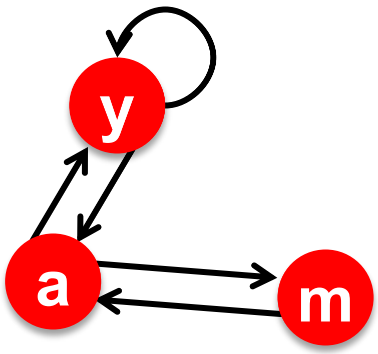
- **The flow equations can be written**

$$r = M \cdot r$$

- So the **rank vector** ***r*** is an **eigenvector** of the stochastic web matrix ***M***
  - In fact, its first or principal eigenvector, with corresponding eigenvalue ***1***
    - Largest eigenvalue of ***M*** is **1** since ***M*** is column stochastic
      - ***Why?*** *We know* ***r*** *is unit length and each column of* ***M*** *sums to one, so* $\boldsymbol{Mr} \leq \mathbf{1}$

- **We can now efficiently solve for *r*!**
  The method is called Power iteration

Example: Flow Equations & M



	y	a	m
y	½	½	0
a	½	0	1
m	0	½	0

$$\boldsymbol{r = M \cdot r}$$

$$r_y = r_y/2 + r_a/2$$
$$r_a = r_y/2 + r_m$$
$$r_m = r_a/2$$

$$\begin{bmatrix} y \\ a \\ m \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} y \\ a \\ m \end{bmatrix}$$