Minhashing

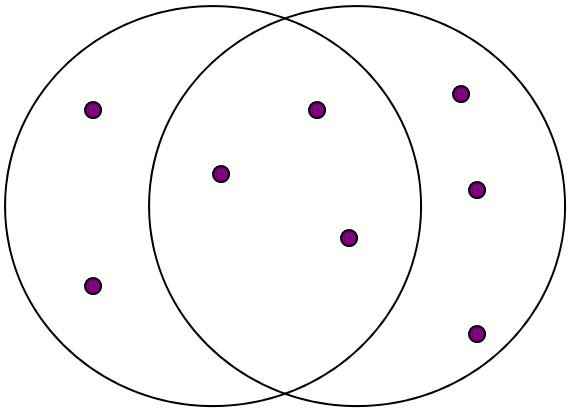
Jaccard Similarity Measure
Constructing Signatures

Jaccard Similarity

- The *Jaccard similarity* of two sets is the size of their intersection divided by the size of their union.
- $Sim(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$.

雅卡尔指数（英语：Jaccard index），又称为并交比（Intersection over Union）、雅卡尔相似系数（Jaccard similarity coefficient），是用于比较样本集的相似性与多样性的统计量。雅卡尔系数能够量度有限样本集合的相似度，其定义为两个集合交集大小与并集大小之间的比例：

Example: Jaccard Similarity



3 in intersection.
8 in union.
Jaccard similarity
= 3/8

From Sets to Boolean Matrices

- Rows = elements of the universal set.
  - Example: the set of all k-shingles.
- Columns = sets.
- 1 in row *e* and column *S* if and only if *e* is a member of *S*.
- Column similarity is the Jaccard similarity of the sets of their rows with 1.
- Typical matrix is sparse.

Example: Column Similarity

C_1	C_2		
0	1		*
1	0		*
1	1	*	*
0	0		
1	1	*	*
0	1		*

可以想象成是顾客买了亚马逊的书，row是书的种类，col是不同的顾客。矩阵稀疏，因为每个人只会买少部分的书。Column Similarity就可以找出顾客的相似性

$Sim(C_1, C_2) =$

$2/5 = 0.4$

Four Types of Rows

- Given columns C_1 and C_2, rows may be classified as:

	C_1	C_2
a	1	1
b	1	0
c	0	1
d	0	0

- Also, *a* = # rows of type *a* , etc.
- Note $Sim(C_1, C_2) = a/(a + b + c)$.

Minhashing

https://my.oschina.net/keyven/blog/628898
在经过随机行打乱后，两个集合的最小哈希值相等的概率等于这两个集合的Jaccard相似度

- Imagine the rows permuted randomly.
- Define *minhash function* $h(C)$ = the number of the first (in the permuted order) row in which column C has 1.
- Use several (e.g., 100) independent hash functions to create a signature for each column.
- The signatures can be displayed in another matrix – the *signature matrix* – whose columns represent the sets and the rows represent the minhash values, in order for that column.

https://blog.csdn.net/liujan511536/article/details/47729721
为了计算最小哈希，首先对特征矩阵的行进行打乱（也即随机调换行与行之间的位置），这个打乱是随机的。然后某一列的最小哈希值就等于打乱后的这一列第一个值为1的行所在的行号（不明白的直接看例子），行号从0开始。

Minhashing Example

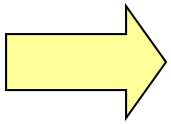
签名矩阵比特征矩阵小很多 https://blog.csdn.net/liujan511536/article/details/47729721

被打乱的顺序，按照这个顺序找第一个不为零的数

1	4	3
3	2	4
7	1	7
6	3	6
2	6	1
5	7	2
4	5	5

Input matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0



Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

明显可以看出来，矩阵已经被压缩了

Surprising Property

- The probability (over all permutations of the rows) that $h(C_1) = h(C_2)$ is the same as $Sim(C_1, C_2)$.
- Both are $a/(a+b+c)$!
- Why?
  - Look down the permuted columns C_1 and C_2 until we see a 1.
  - If it's a type-a row, then $h(C_1) = h(C_2)$. If a type-b or type-c row, then not.

Similarity for Signatures

- The *similarity of signatures* is the fraction of the minhash functions in which they agree.
  - Thinking of signatures as columns of integers, the similarity of signatures is the fraction of rows in which they agree.
- Thus, the expected similarity of two signatures equals the Jaccard similarity of the columns or sets that the signatures represent.
  - And the longer the signatures, the smaller will be the expected error.

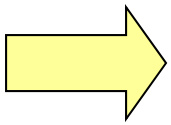
几百次的minhash之后，这个ture jaccard和two signature 来衡量相似度的值是一样的，with 非常小的error.见17页ppt,最上

Min Hashing – Example

Input matrix

1	4	3
3	2	4
7	1	7
6	3	6
2	6	1
5	7	2
4	5	5

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0



Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

	1-3	2-4	1-2
Col/Col	0.75	0.75	0
Sig/Sig	0.67	1.00	0

就是2/3

Implementation of Minhashing

- Suppose 1 billion rows.
- Hard to pick a random permutation of 1…billion.
- Representing a random permutation requires 1 billion entries.
- Accessing rows in permuted order leads to thrashing.

Implementation – (2)

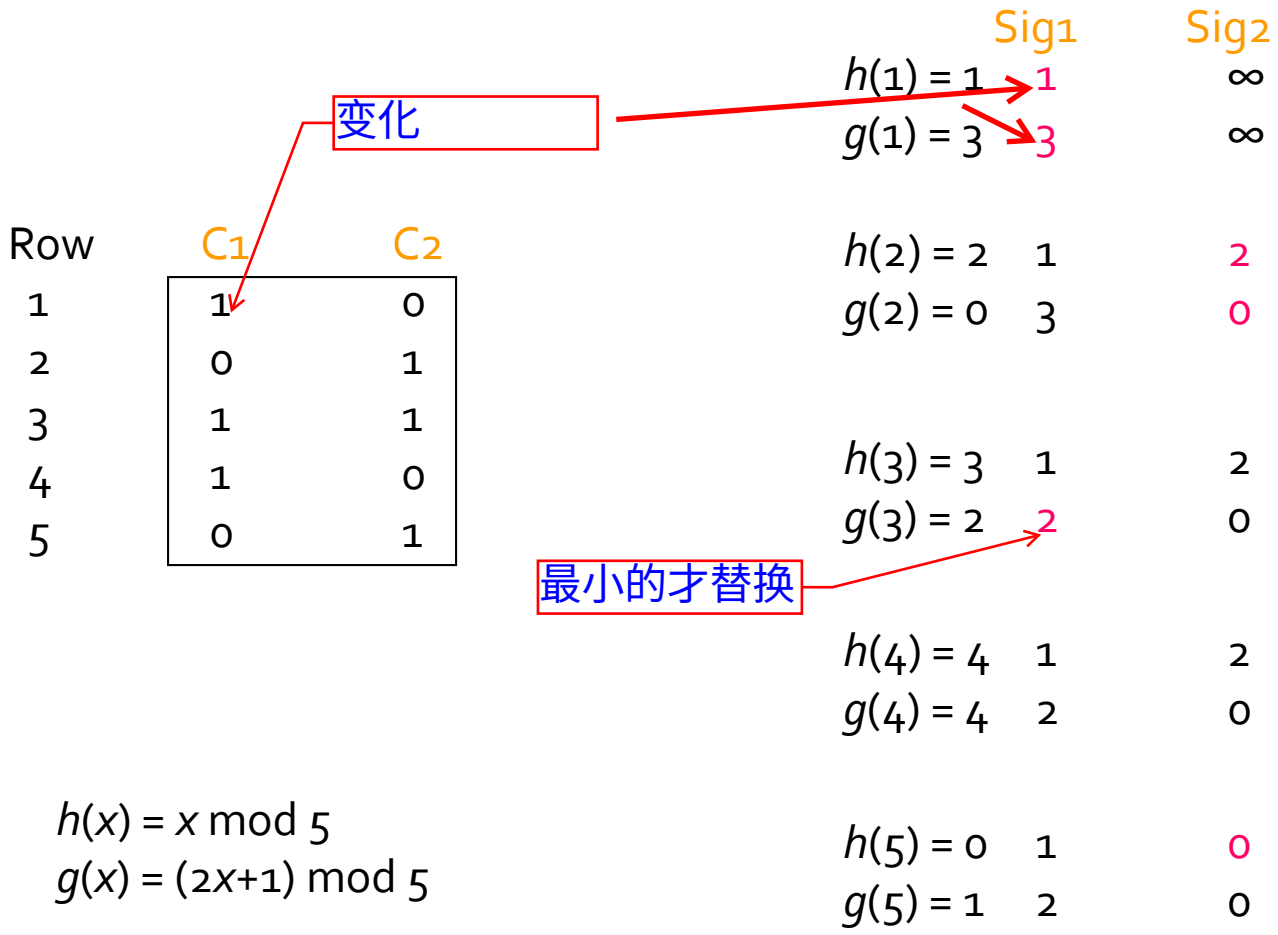
- A good approximation to permuting rows: pick, say, 100 hash functions.
- For each column c and each hash function h_i, keep a "slot" $M(i, c)$.
- Intent: $M(i, c)$ will become the smallest value of $h_i(r)$ for which column c has 1 in row r.
  - I.e., $h_i(r)$ gives order of rows for i^{th} permutation.

Implementation – (3)

for each row *r* **do begin**
  for each hash function h_i **do**
    compute $h_i(r)$;
  for each column *c*
    if c has 1 in row *r*
      for each hash function h_i **do**
        if $h_i(r)$ is smaller than $M(i, c)$ **then**
          $M(i, c) := h_i(r)$;
end;

https://blog.csdn.net/liujan511536/article/details/47729721

Example



$h(x) = x \bmod 5$

$g(x) = (2x+1) \bmod 5$

Implementation – (4)

- Often, data is given by column, not row.
  - Example: columns = documents, rows = shingles.
- If so, sort matrix once so it is by row.

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Start with a list of row column pairs where the ones are.
Initially sort it by column, and sort these pairs by row.
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