Faculty of Actuaries **Institute of Actuaries**

EXAMINATION

April 2005

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M Flaherty
Chairman of the Board of Examiners

15 June 2005

1 The profit vector is the vector of expected end-year profits for policies which are still in force at the start of each year.

The profit signature is the vector of expected end-year profits allowing for survivorship from the start of the contract.

2 (a) $$\ddot{a}_{50:\overline{20}|} = \frac{1 - A_{50:\overline{20}|}}{d}$$

(b) $$A_{50:\overline{20}|} = A_{50} + v^{20}\,{}_{20}p_{50}(1 - A_{70})$$

$$= 0.32907 + 0.45639 \times \frac{8054.0544}{9712.0728}(1 - 0.60097)$$

$$= 0.480093$$

$$\ddot{a}_{50:\overline{20}|} = \frac{1 - 0.480093}{d} = 13.5176$$

3 (a) $$({}_tV' + OP - e_t)(1 + i) = q_{x+t}(S) + p_{x+t}({}_{t+1}V')$$

where

${}_tV'$ = gross premium provision at time t

OP = office premium

e_t = expenses incurred at time t

i = interest rate in premium/valuation basis

S = sum assured

p_{x+t} is the probability that a life aged $x + t$ survives one year on the premium/valuation mortality basis

q_{x+t} is the probability that a life aged $x + t$ dies within one year on the premium/valuation mortality basis

(b) Income (opening provision plus interest on excess of premium over expense, and provision) equals outgo (death claims and closing provision for survivors) if assumptions are borne out.

4 The value of a pension of £1 p.a. is

$\ddot{a}^{(12)}_{\overline{10}|} + {}_{10|}\ddot{a}^{(12)}_{60}$ where first term is an annuity certain

$$\ddot{a}^{(12)}_{\overline{10}|} = \frac{1-v^{10}}{d^{(12)}} @ 6\% = \frac{1-0.55839}{0.058128} = 7.59720$$

$${}_{10|}\ddot{a}^{(12)}_{60} = \ddot{a}^{(12)}_{60} - \ddot{a}^{(12)}_{60:\overline{10}|} = v^{10}\,{}_{10}p_{60}\ddot{a}^{(12)}_{70}$$

$${}_{10}p_{60} = \frac{8054.0544}{9287.2164} = 0.867219$$

$$\ddot{a}^{(12)}_{70} = \ddot{a}_{70} - 11/24 = 9.140 - 11/24 = 8.682$$

So value of a pension of £1 p.a. is

$7.59720 + v^{10} \times 0.867219 \times 8.682 = 11.801$

So annuity purchased by £200,000 is 200000/11.801 = £16,948

5 The present value is $\int_0^{20} 2000.e^{-\delta t}\, {}_t p_{40}^{\overline{ii}} dt$ where $\delta = \ln(1.04)$

$${}_t p_{40}^{\overline{ii}} = \exp\left(-\int_0^t (\rho + \nu) ds\right)$$

$$= \exp(-.05t)$$

So value is

$$2000 \int_0^{20} e^{-\delta t} e^{-5\% t} dt \text{ where } \delta = \ln(1.04)$$

$$= 2000 \left[\frac{e^{-t(.05+\ln(1.04))}}{-(.05+\ln(1.04))} \right]_0^{20}$$

$$= 18,653$$

6 Require to calculate ${}_{14½} p_{45½} = {}_{½} p_{45½} \cdot {}_{14} p_{46}$

$${}_{14} p_{46} = \frac{l_{60}}{l_{46}} = \frac{86714}{95266} = 0.91023$$

(a) Assume deaths uniformly distributed so ${}_t p_x . \mu_{x+t}$ constant

Then $${}_{½} q_{45½} = \frac{(1-½) q_{45}}{(1-½ q_{45})} = \frac{½0.00266}{(1-½.00266)} = .001332$$

So $${}_{14½} p_{45½} = (1-.001332) \times 0.91023 = 0.909018$$

(b) Assume that force of mortality is constant across year of age 45 to 46

$${}_{½} p_{45½} = e^{-½\mu_{45}}$$

$$\mu_{45} = -\ln(1-q_{45}) = -\ln(1-0.00266) = 0.002664$$

$${}_{½} p_{45½} = e^{-½0.002664} = 0.998669$$

So $${}_{14½} p_{45½} = 0.998669 \times 0.91023 = 0.909018$$

7 Define a random variable T_{xy}, the lifetime of the joint life status

The expected value at a rate of interest i is

$$\bar{a}_{xy} = E(\bar{a}_{\overline{T_{xy}}|})$$

$$= E\left(\frac{1-v^{T_{xy}}}{\delta}\right)$$

$$= \frac{1-E(v^{T_{xy}})}{\delta}$$

$$= \frac{1-\bar{A}_{xy}}{\delta}$$

The variance is

$$\text{var}\left(\frac{1-v^{T_{xy}}}{\delta}\right)$$

$$= \frac{1}{\delta^2}\text{var}(v^{T_{xy}})$$

$$= \frac{1}{\delta^2}({}^2\bar{A}_{xy} - (\bar{A}_{xy})^2)$$

where ${}^2\bar{A}_{xy}$ is at $(1+i)^2 - 1$

8 Past Service

$$\frac{10}{80}20000\sum_{t=0}^{29}\frac{i_{35+t}}{l_{35}}\frac{v^{35+t+½}}{v^{35}}\frac{z_{35+t+½}}{s_{34}}\bar{a}_{35+t+½}$$

or

$$\frac{10}{80}20000\frac{{}^zM_{35}^{ia}}{{}^sD_{35}}$$

Future Service

$$\frac{10}{80}20000\frac{{}^{z}M_{35}^{ia}}{{}^{s}D_{35}}+\frac{1}{80}20000\frac{{}^{z}\overline{R}_{45}^{ia}}{{}^{s}D_{35}}$$

9

- Insurance works on the basis of pooling independent homogeneous risks
- The central limit theorem then implies that profit can be defined as a random variable having a normal distribution.
- Life insurance risks are usually independent
- Risk classification ensures that the risks are homogeneous
- Lives are divided by risk factors
- More factors implies better homogeneity
- But the collection of more factors is restricted by
  - The cost of obtaining data
  - Problems with accuracy of information
  - The significance of the factors
  - The desires of the marketing department

10

	Males		***Females***		***Male***	***Female***	***Total***	***Total***	***Female***	***Total***
Age band	*Exposed to risk*	*Observed Mortality rate*	*Exposed to risk*	*Observed Mortality rate*	*Actual deaths*	*Actual deaths*	*Actual deaths*	*Exposed to risk*	*Expected deaths using total mortality rates*	*Expected deaths using female rates*
20–29	125000	0.00356	100000	0.00125	445	125	570	225000	253.333333	281.25
30–39	200000	0.00689	250000	0.00265	1378	662.5	2040.5	450000	1133.61111	1192.5
40–49	100000	0.00989	200000	0.00465	989	930	1919	300000	1279.33333	1395
50–59	90000	0.01233	150000	0.00685	1109.7	1027.5	2137.2	240000	1335.75	1644
					3921.7	2745	6666.7	1215000	4002.02778	4512.75
									Direct	0.003714
									Indirect	0.003764

11 Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25}|} = 155.124P$$

$$\ddot{a}^{(12)}_{[40]:\overline{25}|} = \ddot{a}_{[40]:\overline{25}|} - \frac{11}{24}(1 - {}_{25}p_{[40]}v^{25})$$

$$= 13.290 - \frac{11}{24}\left(1 - (1.06)^{-25} \times \frac{8821.2612}{9854.3036}\right) = 12.927$$

EPV of benefits:

$$\frac{100,000}{(1+b)} \times (1.06)^{1/2}\{q_{[40]}(1+b)v \ \ + {}_1|q_{[40]}(1+b)^2v^2$$

$$+ .... + {}_{24}|q_{[40]}(1+b)^{25}v^{25}\} + 100,000\,{}_{25}p_{[40]}(1+b)^{25}v^{25}$$

where b = 0.0192308

$$= \frac{100,000}{(1+b)} \times (1.06)^{1/2} A^1_{[40]:\overline{25}|} @ i' + 100,000 \times \frac{D_{65}}{D_{[40]}} @ i'$$

$$= \frac{100,000}{1.0192308} \times (1.06)^{1/2} \times (.38896 - .33579) + 100,000 \times .33579 = 38949.90$$

where $i' = \frac{1.06}{1+b} - 1 = 0.04$

EPV of expenses:

$$.875 \times 12P + 175 + 0.025 \times 12 \times P(\ddot{a}^{(12)}_{[40]:\overline{25}|} - \ddot{a}^{(12)}_{[40]:\overline{1}|}) + 65[\ddot{a}_{[40]:\overline{25}|} - 1] = 14.086P + 973.85$$

$$\ddot{a}^{(12)}_{[40]:\overline{1}|} = \ddot{a}_{[40]:\overline{1}|} - \frac{11}{24}(1 - \ \ p_{[40]}v\ \ ) = 1 - \frac{11}{24}\left(1 - (1.06)^{-1} \times \frac{9846.5384}{9854.3036}\right) = 0.974$$

EPV of claim expense:

$.025 \times 38949.9 = 973.748$

Equation of value gives $155.124P = 38949.9 + 14.086P + 973.85 + 973.75$

and $P = £289.98$

12 (i) $\bar{A}^{1}_{x:\overline{n}|} = \sum_{t=0}^{n-1} {}_{t|}\bar{A}^{1}_{x:\overline{1}|}$

$$= \sum_{t=0}^{n-1} v^t \, {}_t p_x \bar{A}^{1}_{x+t:\overline{1}|}$$

$$\bar{A}^{1}_{x+t:\overline{1}|} = \int_0^1 v^s \, {}_s p_{x+t} \mu_{x+t+s} ds$$

Assuming a uniform distribution of deaths, then ${}_s p_{x+t} \mu_{x+t+s} = q_{x+t}$

$$\bar{A}^{1}_{x+t:\overline{1}|} = \int_0^1 v^s q_{x+t} ds = q_{x+t} \int_0^1 v^s ds$$

$$= q_{x+t} \frac{iv}{\delta}$$

$$\bar{A}^{1}_{x:\overline{n}|} = \sum_{t=0}^{n-1} v^t \cdot {}_t p_x \cdot q_{x+t} \frac{iv}{\delta}$$

$$= \frac{i}{\delta} \sum_{t=0}^{n-1} v^{t+1} \cdot {}_t p_x \cdot q_{x+t}$$

$$= \frac{i}{\delta} A^{1}_{x:\overline{n}|}$$

(ii) $$\text{var}(\bar{A}^1_{x:\overline{n}|}) = \text{var}(\frac{i}{\delta} A^1_{x:\overline{n}|}) = \left(\frac{i}{\delta}\right)^2 \text{var}(A^1_{x:\overline{n}|})$$

$$= \left(\frac{i}{\delta}\right)^2 ({}^2A^1_{x:\overline{n}|} - (A^1_{x:\overline{n}|})^2)$$

$$A^1_{[40]:\overline{30}|} = A_{[40]} - v^{30} \cdot {}_{30}p_{[40]} \cdot A_{70}$$

$$= 0.23041 - v^{30} \frac{8054.0544}{9854.3036} 0.60097 = 0.078970$$

$${}^2A^1_{[40]:\overline{30}|} = {}^2A_{[40]} - v^{30} \cdot {}_{30}p_{[40]} \cdot {}^2A_{70}$$

$$= 0.06775 - v^{30} \frac{8054.0544}{9854.3036} 0.38975 = 0.037469$$

where v = 1/1.0816

$$\text{var}(\bar{A}^1_{x:\overline{n}|}) = \left(\frac{0.04}{\ln(1.04)}\right)^2 (0.037469 - (0.078970)^2) = 0.032486$$

$$\text{Expected value} = \frac{i}{\delta} A^1_{[40]:\overline{30}|} = \frac{0.04}{\ln(1.04)} 0.078970 = 0.080539$$

13

Annual premium	1000.00	Allocation % (1st yr)	50.0%
Risk discount rate	8.0%	Allocation % (2nd yr +)	102.50%
Interest on investments	6.0%	Man charge	0.50%
Interest on sterling provisions	4.0%	B/O spread	5.0%
Minimum death benefit	4000.00		

	£	% prm	Total
Initial expense	150	20.0%	350
Renewal expense	50	2.5%	75

(i) Multiple decrement table

x	q_x^d	q_x^s
40	0.000788	0.10
41	0.000962	0.05
42	0.001104	0.05
43	0.001208	0.05

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$
40	0.000749	0.09996	0.899291	1.000000
41	0.000938	0.04998	0.949086	0.899291
42	0.001076	0.04997	0.948951	0.853504
43	0.001178	0.04997	0.948852	0.809934

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
value of units at start of year	0.000	500.983	1555.400	2667.495
alloc	500.000	1025.000	1025.000	1025.000
B/O	25	51.25	51.25	51.25
interest	28.500	88.484	151.749	218.475
management charge	2.518	7.816	13.404	19.299
value of units at year end	500.983	1555.400	2667.495	3840.421

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
unallocated premium	500.000	–25.000	–25.000	–25.000
B/O spread	25.000	51.250	51.250	51.250
expenses	350.000	75.000	75.000	75.000
interest	7.000	–1.950	–1.950	–1.950
man charge	2.518	7.816	13.404	19.299
extra death benefit	2.619	2.293	1.434	0.188
end of year cashflow	181.898	–45.177	–38.730	–31.589
probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925926	0.85733882	0.793832241	0.735029853
expected p.v. of profit	88.54607934			
premium signature	1000	832.67667	731.74245	642.95174
expected p.v. of premiums	3207.370861			
profit margin	2.76%			

(ii)

(a)

To calculate the expected provisions at the end of each year we have (utilising the end of year cashflow figures and decrement tables in (i) above):

$$_3V = \frac{-31.589}{1.04} = 30.374$$

$$_2V \times (1.04) - (ap)_{42} \times {_3V} = -38.73 \Rightarrow {_2V} = 64.9552$$

$$_1V \times (1.04) - (ap)_{41} \times {_2V} = -45.177 \Rightarrow {_1V} = 102.7164$$

These need to be adjusted as the question asks for the values in respect of the beginning of the year. Thus we have:

Year 3 $30.374(ap)_{42} = 28.823$
Year 2 $64.9552(ap)_{41} = 61.648$
Year 1 $102.7164(ap)_{40} = 92.372$

(b)

Based on the expected provisions calculated in (a) above, the cash flow for years 2, 3 and 4 will be zeroised whilst year 1 will become:

181.898 – 92.372 = 89.526

Hence the table blow can now be completed for the revised profit margin.

revised end of year cash flow	89.526	0	0	0
probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925926	0.85733882	0.793832241	0.735029853
expected p.v. of profit	82.89461768			
profit margin	2.58%			

14 (i) The death strain at risk for a policy for year $t + 1$ ($t = 0, 1, 2\ldots$) is the excess of the sum assured (i.e. the present value at time $t + 1$ of all benefits payable on death during the year $t + 1$) over the end of year provision.

i.e. DSAR for year $t + 1 = S - {}_{t+1}V$

The "expected death strain" for year $t + 1$ ($t = 0, 1, 2\ldots$) is the amount that the life insurance company expects to pay extra to the end of year provision for the policy.

i.e. EDS for year $t + 1 = q(S - {}_{t+1}V)$

The "actual death strain" for year $t + 1$ ($t = 0, 1, 2\ldots$) is the observed value at $t+1$ of the death strain random variable

i.e. ADS for year $t + 1 = (S - {}_{t+1}V)$ if the life died in the year t to $t + 1 = 0$ if the life survived to $t + 1$

(ii) Annual premium for pure endowment with £75,000 sum assured given by:

$$P^{PE} = \frac{75,000}{\ddot{a}_{45:\overline{15|}}} \times \frac{D_{60}}{D_{45}} = \frac{75,000}{11.386} \times \frac{882.85}{1677.97} = 3465.71$$

Annual premium for term assurance with £150,000 sum assured given by:

$$P^{TA} = P^{EA} - 2P^{PE} = \frac{150,000 A_{45:\overline{15|}}}{\ddot{a}_{45:\overline{15|}}} - 2P^{PE}$$

$$= \frac{150,000 \times 0.56206}{11.386} - 2 \times 3465.71 = 473.20$$

Provisions at the end of the third year:

for pure endowment with £75,000 sum assured given by:

$${}_3V^{PE} = 75,000 \times \frac{D_{60}}{D_{48}} - P^{PE} \ddot{a}_{48:\overline{12|}}$$

$$= 75,000 \times \frac{882.85}{1484.43} - 3465.71 \times 9.613 = 11289.63$$

for term assurance with £150,000 sum assured given by:

$${}_3V^{TA} = {}_3V^{EA} - {}_3V^{PE}$$

$$= 150,000 A_{48:\overline{12|}} - (2 \times 3465.71 + 473.20) \ddot{a}_{48:\overline{12|}} - 2 \times 11289.63$$

$$= 150,000 \times 0.63025 - 7,404.62 \times 9.613 - 22,579.26$$

$$= 777.63$$

for temporary immediate annuity paying an annual benefit of £25,000 given by:

$${}_3V^{IA} = 25,000 a_{58:\overline{2|}}$$

$$= 25,000(\ddot{a}_{58:\overline{3|}} - 1)$$

$$= 25,000(\ddot{a}_{58} - v^3 \, {}_3p_{58} \ddot{a}_{61} - 1)$$

$$= 25,000\left(16.356 - (1.04)^{-3} \frac{9802.048}{9864.803} \times 15.254 - 1\right) = 47,037.91$$

Sums at risk:

Pure endowment: DSAR = 0 – 11,289.63 = –11,289.63

Term assurance: DSAR = 150,000 – 777.63 = 149,222.37

Immediate annuity: DSAR = –(47,037.91 + 25,000) = –72,037.91

Mortality profit = EDS – ADS

For term assurance

$$EDS = 4985 \times q_{47} \times 149,222.37 = 4985 \times .001802 \times 149,222.37 = 1,340,460.07$$

$$ADS = 8 \times 149,222.37 = 1,193,778.96$$

mortality profit = 146,681.11

For pure endowment

$$EDS = 1995 \times q_{47} \times -11,289.63 = 1995 \times .001802 \times -11,289.63 = -40,586.11$$

$$ADS = 1 \times -11,289.63 = -11,289.63$$

mortality profit = –29,296.48

For immediate annuity

$$EDS = 995 \times q_{57} \times -72,037.91 = 995 \times .001558 \times -72,037.91 = -111,673.89$$

$$ADS = 1 \times -72,037.91 = -72,037.91$$

mortality profit = –39,635.98

Hence, total mortality profit = £77,748.65

END OF EXAMINERS' REPORT

Faculty of Actuaries **Institute of Actuaries**

EXAMINATION

September 2005

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Core Technical

EXAMINERS' REPORT

Introduction

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M Flaherty
Chairman of the Board of Examiners

15 November 2005

In general, this examination was done well by students who were well prepared. Several questions gave difficulties particularly Question 7 and 12(ii) the latter one being very challenging. To help students comments are attached to those questions where particular points are of relevance. Absence of comments can be indicate that the particular question was generally done well.

1 Adverse selection is the manner in which lives form part of a group, which acts against a controlled process of selecting the lives with respect to some characteristic that affects mortality or morbidity.

An example is where a life insurance company does not distinguish between smokers and non-smokers in proposals for term assurance cover. A greater proportion of smokers are likely to select this company in preference to a company that charges different rates to smokers and non-smokers. This would be adverse to the company's selection process, if the company had assumed that its proportion of smokers was similar to that in the general population.

Other examples were credited.

2 Occupation can have several direct effects on mortality and morbidity. Occupation determines a person's environment for 40 or more hours each week. The environment may be rural or urban, the occupation may involve exposure to harmful substances e.g. chemicals, or to potentially dangerous situations e.g. working at heights. Much of this is moderated by health and safety at work regulations.

Some occupations are more healthy by their very nature e.g. bus drivers have a sedentary and stressful occupation while bus conductors are more active and less stressed. Some work environments e.g. pubs, give exposure to a less healthy lifestyle.

Some occupations by their very nature attract more healthy workers. This may be accentuated by health checks made on appointment or by the need to pass regular health checks e.g. airline pilots. Some occupations can attract less healthy workers, for example, former miners who have left the mining industry as a result of ill health and then chosen to sell newspapers. This will inflate the mortality rates of newspaper sellers.

A person's occupation largely determines their income, which permits them to adopt a particular lifestyle e.g. content and pattern of diet, quality of housing. This effect can be positive or negative e.g. over-indulgence.

Other appropriate examples were credited.

3 As t increases, μ_t increases, but ${}_t p_0$ decreases. At t = 80 approximately, the decrease in ${}_t p_0$ is greater than the increase in μ_t, hence $f_0(t) = {}_t p_0 \mu_t$ decreases.

A deceptively straightforward answer which many students struggled to find. The key point is to compare the 2 parameters as shown.

4 $\quad {}_{1.75}p_{45.5} = {}_{0.5}p_{45.5} * p_{46} * {}_{0.25}p_{47}$

$$= \frac{1-q_{45}}{1-0.5q_{45}} * (1-q_{46}) * (1-0.25q_{47})$$

$$= \frac{1-0.001465}{1-0.5*0.001465} * (1-0.001622) * (1-0.25*0.001802)$$

$$= 0.999267 * 0.998378 * 0.99955 = 0.997197$$

5 (a) The required probability is

$$1 - e^{-\int_0^{1.25} 0.015dt} = 1 - e^{-0.01875} = 0.018575$$

(b) The curtate expectation is

$$\sum_{k=1}^{\infty} {}_k p_{20} = \sum_{k=1}^{\infty} e^{-\int_0^k 0.015dt} = \sum_{k=1}^{\infty} e^{-0.015k} = \frac{e^{-0.015}}{1-e^{-0.015}} = 66.168.$$

6 $\ddot{a}^{(12)}_{60:50:\overline{20|}}$ is the present value of 1 p.a. payable monthly in advance while two lives aged 60 and 50 are both still alive, for a maximum period of 20 years.

$$\ddot{a}^{(12)}_{60:50:\overline{20|}} = \ddot{a}^{(12)}_{60:50} - v^{20}\,{}_{20}p_{60:50}\ddot{a}^{(12)}_{80:70}$$

$$= (\ddot{a}_{60:50} - 11/24) - v^{20}\,{}_{20}p_{60:50}(\ddot{a}_{80:70} - 11/24)$$

$$= (15.161 - 0.458) - v^{20}\frac{6953.536}{9826.131}\frac{9392.621}{9952.697}(6.876 - 0.458) = 12.747$$

7 $$\text{EPV} = £10,000 \int_0^{45} {}_t p_{20}^{hh} * \sigma_{20+t} * {}_1 p_{20+t}^{\overline{ss}} * \left(\int_0^{44-t} e^{-\delta(t+u+1)} {}_u p_{21+t}^{\overline{ss}} du \right) dt$$

where δ is the force of interest

${}_t p_{20}^{hh}$ is the probability of a healthy life aged 20 being healthy at age $20+t$

${}_1 p_{20+t}^{\overline{ss}}$ is the probability that a life who is sick at age $20+t$ is sick continuously for one year thereafter

${}_u p_{21+t}^{\overline{ss}}$ is the probability that a life who is sick at age $21+t$ is still sick at age $21+t+u$

This question was not done well and few students obtained the whole result. Partial credits were given for correct portions. There were other potentially correct approaches which were credited provided proper definitions of symbols given.

8 $$\text{Premium} = 20,000\left(\ddot{a}_{\overline{5}|}^{(12)} + \frac{D_{70}}{D_{65}}\ddot{a}_{70}^{(12)}\right) + 10,000\left[\left(1-\frac{l_{70}}{l_{65}}\right)\frac{D_{67}}{D_{62}}\ddot{a}_{67}^{(12)} + \frac{l_{70}}{l_{65}}\frac{D_{67}}{D_{62}}\ddot{a}_{70|67}^{(12)}\right]$$

$$\ddot{a}_{\overline{5}|}^{(12)} = 4.5477$$

$$\frac{D_{70}}{D_{65}} = v^5 \frac{9238.134}{9647.797} = 0.787027$$

$$\ddot{a}_{70}^{(12)} = 11.562 - 0.458 = 11.104$$

$$\frac{l_{70}}{l_{65}} = 0.957538$$

$$\ddot{a}_{67}^{(12)} = 14.111 - 0.458 = 13.653$$

$$\frac{D_{67}}{D_{62}} = v^5 \frac{9605.483}{9804.173} = 0.80527$$

$$\ddot{a}_{70|67}^{(12)} = \ddot{a}_{67}^{(12)} - \ddot{a}_{70:67}^{(12))} = (14.111 - 0.458) - (10.233 - 0.458) = 3.878$$

Premium = 265,736.96 + 34,570.77 = £300,308 to nearer £

9 (i) $10{,}000\ddot{a}_{\overline{\max\left(K_{60}^{A}+1,K_{60}^{B}+1\right)}|} - P$, where *A* and *B* refer to the first and second lives and *P* is the single premium.

(ii) $P = 10{,}000\left(\ddot{a}_{60}^{A} + \ddot{a}_{60}^{B} - \ddot{a}_{60}^{A} * \ddot{a}_{60}^{B}\right)$

$= 10{,}000*\left(15.632 + 16.652 - 14.09\right) = £181{,}940$

Variance $= \dfrac{10^8}{d^2}\left({}^{2}A_{\overline{60^A:60^B}} - \left(A_{\overline{60^A:60^B}}\right)^2\right)$

$= \dfrac{10^8}{0.038462^2}*\left(1 - 0.075444*11.957 - \left(1 - 0.038462*18.194\right)^2\right)$

standard deviation = 22,936.7

10 (i) The gross future loss random variable is

$100{,}000\left(1 + bK_{20+1}\right)v^{T_{20}} + I + e\ddot{a}_{\overline{K_{20}+1}|} + fv^{T_{20}} - Ga_{\overline{K_{20}+1}|}$

where *b* is the annual rate of bonus
I is the initial expense
e is the annual renewal expense and
f is the claim expense
G is the gross annual premium

(ii) The premium is given by

$G\ddot{a}_{[20]} = 100{,}000\bar{A}_{[20]} + 3{,}000\left(I\bar{A}\right)_{[20]} + 200 + 0.05Ga_{[20]}$

$\Rightarrow G*16.877 = 100{,}000*1.06^{0.5}*0.04472 + 3{,}000*1.06^{0.5}*2.00874$
$+200 + 0.05G*(16.877-1)\,1] \Rightarrow 16.083G = 4604.206 + 6204.373 + 200$

$\Rightarrow G = £684.49$

(iii) The required provision is

$$110{,}000\bar{A}_{23} + 4{,}000(I\bar{A})_{23} - 0.95 * 684.48 * \ddot{a}_{23}$$

$$= 110{,}000 * 1.04^{0.5} * 0.12469 + 4{,}000 * 1.04^{0.5} * 6.09644 - 0.95 * 684.49 * 22.758$$

$$= 13{,}987.528 + 24{,}868.693 - 14{,}798.742$$

$$= £24{,}057.48$$

11 Unit fund

Year	*1*	*2*	*3*
Fund at the start of the year	0	8611.988	17796.67
Premium	10000	10000	10000
Allocation to units	8075	8075	8075
Interest	646	1334.959	2069.734
Management charge	109.0123	225.274	349.268
Fund at the end of the year	8611.988	17796.67	27592.14

Non-unit fund before provisions

Year	*1*	*2*	*3*
Premium margin	1925	1925	1925
Expenses	600	100	100
Interest	53	73	73
Death cost	115.156	24.995	0
Maturity cost	0	0	4138.821
Management charge	109.013	225.274	349.268
Profit	1371.857	2098.28	–1891.55

Provision required at the start of year 3 = (1891.55 – 4138.821 (1 – p_{64})) / 1.04
= 1768.192

Reduced profit at the end of year 2 = 2098.28 – 1768.192*p_{63} = 350.146

Revised profit vector: $(1371.857, 350.146, 0)$

$p_{62} = 0.989888$

${}_2p_{62} = 0.978659$

Net Present Value = $\dfrac{1371.857}{1.15} + \dfrac{350.146 * p_{62}}{1.15^2} = 1455.003$

Present value of premiums = $10000 * \left(1 + \dfrac{p_{62}}{1.15} + \dfrac{{}_2p_{62}}{1.15^2}\right) = 26007.788$

Profit margin = $\dfrac{1455.003}{26007.788} = 5.59\%$

Most students completed the tables satisfactorily in this question but struggled to get the revised profit vectors. Very few produced a complete result.

12 (i) Let P be the annual premium.

$$0.95 * P * \ddot{a}_{\overline{50^m:50^f}} = 1000 + 200000 * \bar{A}_{\overline{50^m:50^f}}$$

$$\ddot{a}_{\overline{50^m:50^f}} = \ddot{a}_{50^m} + \ddot{a}_{50^f} - \ddot{a}_{50^m:50^f} = 18.843 + 19.539 - 17.688 = 20.694$$

$$\bar{A}_{\overline{50^m:50^f}} = 1.04^{0.5} * A_{\overline{50^m:50^f}} = 1.04^{0.5} * (1 - d * \ddot{a}_{\overline{50^m:50^f}})$$

$$= 1.04^{0.5} * (1 - 0.038462 * 20.694) = 0.208109$$

$$\therefore 0.95 * P * 20.694 = 1000 + 200000 * 0.208109$$

$$\Rightarrow P = £2{,}168.02\,.$$

(ii) From part (i) the net premium is:

$$200000 * (1.04)^{0.5} * \left(\frac{1}{\ddot{a}_{\overline{50^m:50^f}}} - d\right) \text{ at } 4\%$$

$$= 200000 * (1.04)^{0.5} * \left(\frac{1}{20.694} - \frac{.04}{1.04}\right)$$

$$= 2011.39$$

We require 3 provisions at end of 5th policy year

Both lives alive

$$200000*(1.04)^{0.5}*\left(1-\frac{\ddot{a}_{\overline{55^m:55^f}}}{\ddot{a}_{\overline{50^m:50^f}}}\right)$$

$$= \ 200000*(1.04)^{0.5}\left(1-\frac{17.364+18.210-16.016}{18.843+19.539-17.688}\right)$$

= 11196.46

Male only alive

$$200000\,\bar{A}_{55^m} - 2011.39\,\ddot{a}_{55^m}$$

$$= \ 200000*(1.04)^{0.5}*\left(1-\frac{.04}{1.04}*17.364\right)-2011.39*17.364$$

= 32820.60

Female only alive

$$200000\,\bar{A}_{55^f} - 2011.39\,\ddot{a}_{55^f}$$

$$= \ 200000*(1.04)^{0.5}*\left(1-\frac{.04}{1.04}*18.210\right)-2011.39*18.210$$

= 24482.39

Mortality Profit Loss

= Expected Death Strain – Actual Death Strain

In this case there are 4 components:

(a) Both lives die during 2004 – no actual claims

Result

$$= \ (4900*q_{54^m}*q_{54^f}-0)(200000*1.04^{0.5}-11196.46)$$

= (4900 * 0.000986 * 0.000912) (192764.32)

= 849.37

(b) Female alive at begin 2004, death during 2004 – 1 actual claim

Result

$$= \ (100*q_{54^f}-1)(200000*1.04^{0.5}-24482.39)$$

= (100 * 0.000912 – 1) (179478.39)

= – 163109.96

(c) Both lives alive beginning 2004, males only die during 2004 -1 actual claim. Here the "claim cost" is the change in provision from joint lives to female only surviving i.e.

Result = $(4900 * q_{54^m} * q_{54^f} - 1)(24482.39 - 11196.46)$

= (4900 * 0.000986 * 0.999088 – 1) (13285.93)

= 50845.17

(d) Both lives alive beginning 2004, females only die during 2004 – no actual claims. Claim cost change in provision from joint lives to male only surviving

Result = $(4900 * p_{54^m} * q_{54^f} - 0)(32820.611 - 11196.46)$

= (4900 * 0.999014 * 0.000912) (21624.14)

= 96538.66

Hence overall total = 849.37 – 163109.16 + 50845.17 + 96538.66

= – 14876.77

i.e. a mortality loss of 14877 when rounded.

For part (i) assuming renewal expenses did not include the first premium (answer £2162.62) was also fully acceptable.

Part (ii) was very challenging and very few students realised the extension of mortality profit/loss extended to joint life contracts involved reserve change costs on first death. Most just considered the first 2 components of the answer and in many cases failed to correctly cost this part. A few exceptional students did manage to reach the final result.

13 (i) Define a service table:

l_{26+t} = no. of members aged 26 + t last birthday

r_{26+t} = no. of members who retire age 26 + t last birthday

s_{x+t} / s_x = ratio of earnings in the year of age $x + t$ to $x + t + 1$ to the earnings in the year of age x to $x + 1$

Define $z_{26+t} = \frac{1}{3}\left(s_{26+t-3} + s_{26+t-2} + s_{26+t-1}\right); \bar{a}^r_{26+t}$ = value of annuity of 1 p.a. to a retiree aged exactly 26 + t.

Past service:

Assume that retirements take place uniformly over the year of age between 60 and 65. Retirement for those who attain age 65 takes place at exact age 65.

Consider retirement between ages 26 + t and 26 + t + 1, $34 \le t \le 38$.

The present value of the retirement benefits related to past service:

$$\frac{50000*5}{60}\frac{z_{26+t+\frac{1}{2}}}{s_{25.25}}\frac{v^{26+t+\frac{1}{2}}}{v^{26}}\frac{r_{26+t}}{l_{26}}\bar{a}^r_{26+t+\frac{1}{2}} = \frac{50000*5}{60}\frac{{}^zC^{ra}_{26+t}}{{}^sD_{26}}$$

where ${}^zC^{ra}_{26+t} = z_{26+t+\frac{1}{2}}v^{26+t+\frac{1}{2}}r_{26+t}\bar{a}^r_{26+t+\frac{1}{2}}$

and ${}^sD_{26} = s_{25.25}v^{26}l_{26}$

For retirement at age 65, the present value of the benefits is:

$$\frac{50000*5}{60}\frac{z_{65}}{s_{25.25}}\frac{v^{65}}{v^{26}}\frac{r_{65}}{l_{26}}\bar{a}^r_{65} = \frac{50000*5}{60}\frac{{}^zC^{ra}_{65}}{{}^sD_{26}}$$

where ${}^zC^{ra}_{65} = z_{65}v^{65}r_{65}\bar{a}^r_{65}$

Summing over all ages, the value is:

$$\frac{50000*5}{60}\frac{{}^zM^{ra}_{60}}{{}^sD_{26}}$$

where ${}^zM^{ra}_{60} = \sum_{t=34}^{39} {}^zC^{ra}_{26+t}$

Future service:

Assume that retirements take place uniformly over the year of age, between ages 60 and 65. Retirement at 65 takes place at exactly age 65.

If retirement takes place between ages 60 and 61, the number of future years service to count is 34. If retirement takes place at age 61 or after, the number of future years service to count is 35.

For retirement between ages 60 and 61, the present value of the retirement benefits is:

$$\frac{34*50000}{60}\frac{z_{60+\frac{1}{2}}}{s_{25.25}}\frac{v^{60+\frac{1}{2}}}{v^{26}}\frac{r_{60}}{l_{26}}\bar{a}^{i}_{60+\frac{1}{2}} = \frac{34*50000}{60}\frac{{}^{z}C^{ra}_{60}}{{}^{s}D_{26}}$$

For retirement at later years, the formula is similar to the above, with 35 in place of 34.

Adding all these together gives:

$$\frac{50000}{60\,{}^{s}D_{26}}\left[34\,{}^{z}C^{ra}_{60} + 35({}^{z}C^{ra}_{61} + \ldots + {}^{z}C^{ra}_{65})\right]$$

$$= \frac{50000}{60\,{}^{s}D_{26}}\,{}^{z}\bar{M}^{ra}_{60}$$

where $\displaystyle {}^{z}\bar{M}^{ra}_{60} = \sum_{t=0}^{5}\left(35*\,{}^{z}C^{ra}_{60+t} - {}^{z}C^{ra}_{60}\right)$

(ii) Define a service table, with l_{26+t} and s_{x+t}/s_x defined as in part (i). In addition, define d_{26+t} as the number of members dying age $26+t$ last birthday.

Assume that deaths take place on average in the middle of the year of age.

The present value of the death benefit, for death between ages $26+t$ and $26+t+1$, is

$$50000*4*\frac{s_{26.25+t}}{s_{25.25}}\frac{v^{26+t+\frac{1}{2}}}{v^{26}}\frac{d_{26+t}}{l_{26}} = 50000*4*\frac{{}^{s}C^{d}_{26+t}}{{}^{s}D_{26}}$$

where ${}^{s}C^{d}_{26+t} = s_{26.25+t}v^{26+t+\frac{1}{2}}d_{26+t}$

Adding the present value of benefits for all possible years of death gives

$$50000*4*\sum_{t=0}^{38}\frac{{}^{s}C^{d}_{26+t}}{{}^{s}D_{26}} = 200000*\frac{{}^{s}M^{d}_{26}}{{}^{s}D_{26}}$$

where $\displaystyle {}^{s}M^{d}_{26} = \sum_{t=0}^{38}{}^{s}C^{d}_{26+t}$

Examiners felt that this question was quite simple provided students constructed proper definitions and followed them through logically allowing of course for the adjusted salary scale. The above answer is one of a number possible and full credit was given for credible alternatives.

Many students, however struggled with this question despite these remarks.

END OF EXAMINERS' REPORT

EXAMINATION

April 2006

Subject CT5 — Contingencies
Core Technical

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M Flaherty
Chairman of the Board of Examiners

June 2006

Comments

No comments are given.

1 $$10{,}000\int_0^\infty e^{-\delta t}\left({}_t p_x^{aa}\mu_{x+t} + {}_t p_x^{ar}\nu_{x+t}\right)dt$$

2 (i) The super compound bonus method is a method of allocating bonuses (mostly these days on an annual basis) under which two bonus rates are declared each year. The first rate, usually the lower, is applied to the basic sum assured and the second rate is applied to the bonuses already declared.

(ii) The sum assured and bonuses increase more slowly than under other methods for the same ultimate benefit, enabling the office to retain surplus for longer and thereby providing greater investment freedom.

3 (a) $\mu_{65:60} = \mu_{65} + \mu_{60} = 0.005543 + 0.002266 = 0.007809$

(b) $${}_5 p_{65:60} = \frac{l_{70}}{l_{65}}.\frac{l_{65}}{l_{60}} = \frac{9238.134}{9647.797}.\frac{9647.797}{9826.131} = 0.940160$$

(c) $${}_2 q^{\ 1}_{65:65} = ½.{}_2 q_{65:65} = ½.(1 - {}_2 p_{65:65})$$

$$= ½(1 - {}_2 p_{65}\cdot{}_2 p_{65}) = ½\left(1 - \frac{9521.065}{9647.797}.\frac{9521.065}{9647.797}\right) = 0.013050$$

4 Overhead expenses are those that in the short term do not vary with the amount of business.

An example of an overhead expense is the cost of the company's premises (as the sale of an extra policy now will have no impact on these costs).

Direct expenses are those that do vary with the amount of business.
An example of a direct expense is commission payment to a direct salesman (as the sale of an extra policy now will have an impact on these costs).

5 The expected share of the fund is

$$\frac{10,000(1.04)^{25}.A^{1}_{[30]:\overline{25}|}}{\cdot_{25}p_{[30]}}$$

$$=\frac{10,000(1.04^{25}(A_{[30]}-v^{25}\cdot_{25}p_{[30]}.A_{55}))}{_{25}p_{[30]}}$$

$$=10,000\left[\frac{2.66584(0.16011-0.37512x\,{}^{9557.8179}\!/\!_{9923.7497}\,x0.38950)}{\dfrac{9557.8179}{9923.7497}}\right]$$

$$=536.65$$

6 The insurer should expect to find:

Time selection — the experience would be different in different time periods; in developed economies mortality has tended to improve with time.

Class selection — The insurer may price policies differently depending on fixed factors such as age/sex. Also different groups of recipients may have different mortality based on factors such as occupation.

Temporary Initial Selection — if there is no evidence of health required then there is an expectation that poor lives would be likely to take out the insurance and in the short term the experience would be adverse. This effect should reduce with duration. Conversely, if there are medical questions on the application form then we would expect to see some form of self selection and mortality experience would be better in the short term.

This is also evidence of adverse selection as highlighted above.

Spurious selection — If there is no evidence of health required then the duration effect would be confounded by the differential mortality experience of withdrawals, as those lives withdrawing would be expected to have lighter mortality.

7 (i) $$\frac{1}{{}_s p_{x+t}} \times \frac{\partial}{\partial t} {}_s p_{x+t} = \frac{\partial}{\partial t} \ln({}_s p_{x+t}) = \frac{\partial}{\partial t} (\ln l_{x+t+s} - \ln l_{x+t})$$

$$= -\mu_{x+t+s} + \mu_{x+t}$$

Multiplying through by ${}_s p_{x+t}$ gives the required result.

(ii) Now

$$ {}_t\bar{V}_x = \bar{A}_{x+t} - \bar{P}_x \times \bar{a}_{x+t} = 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}$$

$$\frac{\partial}{\partial t} \bar{a}_{x+t} = \frac{\partial}{\partial t} \int_0^\infty e^{-\delta s} {}_s p_{x+t} ds = \int_0^\infty e^{-\delta s} \frac{\partial}{\partial t} {}_s p_{x+t} ds$$

$$= \int_0^\infty e^{-\delta s} {}_s p_{x+t} (\mu_{x+t} - \mu_{x+t+s}) ds = \mu_{x+t} \bar{a}_{x+t} - \bar{A}_{x+t}$$

$$\Rightarrow \frac{\partial}{\partial t} {}_t\bar{V}_x = \frac{-(\mu_{x+t}\bar{a}_{x+t} - \bar{A}_{x+t})}{\bar{a}_x} = -\mu_{x+t}(1 - {}_t\bar{V}_x) + \frac{(1 - \delta\bar{a}_{x+t})}{\bar{a}_x}$$

$$= -\mu_{x+t}(1 - {}_t\bar{V}_x) + \delta\left(1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}\right) - \delta + \frac{1}{\bar{a}_x}$$

$$= -(1 - {}_t\bar{V}_x)\mu_{x+t} + \delta\,{}_t\bar{V}_x + \left(\frac{1 - \delta\bar{a}_x}{\bar{a}_x}\right)$$

$$= -(1 - {}_t\bar{V}_x)\mu_{x+t} + \delta\,{}_t\bar{V}_x + \bar{P}_x$$

8 (i) Expected present value:

$$\ddot{a}_{\overline{70:67}} = \ddot{a}_{70} + \ddot{a}_{67} - \ddot{a}_{70:67}$$

$$= 11.562 + 14.111 - 10.233$$

$$= 15.440$$

(ii) Variance:

$$\frac{1}{d^2}\left\{ {}^2A_{\overline{xy}} - (A_{\overline{xy}})^2 \right\} = \frac{1}{d^2}\left\{ (1-(1-v^2).{}^2\ddot{a}_{\overline{xy}}) - (1-d.\ddot{a}_{\overline{xy}})^2 \right\}$$

where normal functions are at a rate of interest i and functions with a left superscript are at a rate of interest i^2+2i.

The expression $(1\text{-}v^2)$ in the right hand side of the above equation can also be expressed as ${}^2d$.

9 (i) The expected present value
of 1 per annum
payable continuously
until the second death
of 2 lives
currently age x and y
for a maximum n years

(ii) $\bar{a}_{\overline{xy}:\overline{n}|} = E(\bar{a}_{\overline{\min(\max(T_x,T_y),n)}|})$

T_x and T_y are random variables which measures the complete lifetime of two lives aged x and y

$$E(\bar{a}_{\overline{\min(\max(T_x,T_y),n)}|}) = E\left(\frac{1-v^{\min(\max(T_x,T_y),n)}}{\delta}\right)$$

$$= \frac{1-E(v^{\min(\max(T_x,T_y),n)})}{\delta}$$

$$= \frac{1-\bar{A}_{\overline{xy}:\overline{n}|}}{\delta}$$

10 (i) Let P be the net premium for the policy payable annually in advance. Then, equation of value becomes:

$$P\ddot{a}_{45:\overline{15}|} = 10,000(A_{45:\overline{20}|} + v^{20}\,{}_{20}p_{45})$$

$$11.386P = 10,000(0.46998 + 0.41075)$$

$$\Rightarrow P = £773.52$$

Net premium reserve at the end of the 13th policy year is

$${}_{13}V = 10,000(A_{58:\overline{7}|} + v^{7}\,{}_{7}p_{58}) - P\ddot{a}_{58:\overline{2}|}$$

$$= 10,000(0.76516 + 0.71209) - 773.52 \times 1.955$$

$$= 14,772.48 - 1,512.23 = 13,260.25$$

Death strain at risk per policy = 10,000 – 13,260.25 = –3,260.25

$$EDS = 199q_{57} \times -3,260.25 = 199 \times 0.00565 \times -3,260.25 = -3,665.66$$
$$ADS = 4 \times -3,260.25 = -13,041.00$$

mortality profit = –3,665.66 + 13,041.00 = £9,375.34

(ii) The death strain at risk is negative. Hence, the life insurance company makes money on early deaths.

More people die than expected during the year considered so the company makes a mortality profit.

11 (i) $$1,000.n.\frac{M_x^r}{D_x} + 1,000.\frac{\overline{R}_x^r}{D_x}$$

Where $D_x = v^x l_x$

$$C_x^r = v^{x+½} r_x \text{ for } x < 65$$
$$C_{65}^r = v^{65} r_{65}$$

$$M_x^r = \sum_{t=0}^{65-x} C_{x+t}^r$$

$$\overline{M}_x^r = M_x^r - ½C_x^r \text{ for } x < 65$$

$$\overline{R}_x^r = \sum_{t=0}^{64-x} \overline{M}_{x+t}^r$$

(ii) $1,000.10.\dfrac{782}{7,874} + 1000.\dfrac{25,502}{7,874} = 4,231.902$

(iii) Given contribution of C then

$$C.\frac{\overline{N}_x}{D_x} = 4,231.902$$

$$\overline{N}_{30} = 90684,\ D_{30} = 7874$$

Therefore $C = £367.45$

12 Definitions:

(i) (a) Crude Mortality Rate — the ratio of the total number of deaths in a category to the total exposed to risk in the same category.

(b) Directly Standardised Mortality Rate — the mortality rate of a category weighted according to a standard population.

(c) Indirectly Standardised Mortality Rate — an approximation to the directly standardised mortality rate being the crude rate for the standard population multiplied by the ratio of actual to expected deaths for the region.

This is the same as the crude rate for the local population multiplied by the Area Comparability Factor.

(ii) (a) Calculations.

	England and Wales		Tyne and Wear	
	Population	*Number of births*	*Population*	*Number of births*
Total	10,845,000	595,000	227,000	11,000

Crude birth rate: England and Wales 595,000/10,845,000 = 5.49%

Tyne and Wear: 11,000/227,000 = 4.85%

	England and Wales		Tyne and Wear	
	Population		*Fertility rate*	*Expected number of births*
Under 25	3,149,000		0.0563	177,408
25–35	3,769,000		0.0811	305,595
35+	3,927,000		0.0122	47,890
Total	10,845,000			530,893

Directly standardised rate: 530,893/10,845,000 = 4.90%

	England and Wales		Tyne and Wear	
	Fertility rate		*Population*	*Expected Births*
Under 25	0.0486		71,000	3,450
25–35	0.0899		74,000	6,656
35+	0.0262		82,000	2,151
Total			227,000	12,256

Indirectly standardised rate: 5.49%/(12,256/11,000) = 4.93%

(b) The indirectly standardised rate does not require local records of births to be analysed by age grouping.

The standardised rates are similar so the approximation is acceptable.

Both standardised rates are higher than the crude rate, showing that the reason for the low cruder rate compared to the national population is due to population distribution by age.

Both standardised rates are below the crude rate for England and Wales so the birth rate of Tyne and Wear is lower, even allowing for the age distribution.

13 (i) Let P denote the monthly premium for the contract. Then

EPV of premiums =

$$12P\ddot{a}^{(12)}_{[35]:\overline{30}|} = 12\left(\ddot{a}_{[35]:\overline{30}|} - \frac{11}{24}(1 - v^{30}\ {}_{30}p_{[35]})\right)$$

$$= 12P\left(17.631 - \frac{11}{24}\left(1 - \frac{689.23}{2507.02}\right)\right) = 207.5841P$$

EPV of benefits and expenses =

$$(245,000 + 300)A_{[35]:\overline{30}|} + 5000\left(IA\right)^{1}_{[35]:\overline{30}|} + (155,000 - 150)v^{30}\ {}_{30}p_{[35]}$$

$$+0.03 \times 12P\ddot{a}^{(12)}_{[35]:\overline{30}|} - 0.03P + 250 + 0.5 \times 12P$$

where

$$\left(IA\right)^{1}_{[35]:\overline{30}|} = \left(IA\right)_{[35]} - v^{30}\ {}_{30}p_{[35]}\left((IA)_{65} + 30A_{65}\right) =$$

$$7.47005 - \frac{689.23}{2507.02}\left(7.89442 + 30 \times 0.52786\right) = 0.946137$$

EPV of benefits and expenses =

$$245,300 \times 0.32187 + 5,000 \times 0.946137 + 154,850 \times \frac{689.23}{2507.02}$$

$$0.03 \times 12P \times 17.298675 - 0.03P + 250 + 0.5 \times 12P$$

Equating EPV of premiums with EPV of benefits and expenses gives

$$207.5841P = 78,954.711 + 4,730.685 + 42,571.366 + 6.227523P - 0.03P + 250 + 6P$$

$$\Rightarrow P = \frac{126,506.762}{195.3866} = £647.47$$

(ii) (a)

$$ {}_{25}V^{\text{retrospective}} = \frac{(1+i)^{25}}{{}_{25}p_{[35]}}\left(0.97\times 12P\ddot{a}^{(12)}_{[35]:\overline{25}|} - 245,300A^{1}_{[35]:\overline{25}|} - 5,000\left(IA\right)^{1}_{[35]:\overline{25}|} + 0.03P - 250 - 0.5\times 12P\right) $$

where

$$ \left(IA\right)^{1}_{[35]:\overline{25}|} = \left(IA\right)_{[35]} - v^{25}\,{}_{25}p_{[35]}\left(\left(IA\right)_{60} + 25A_{60}\right) $$

$$ = 7.47005 - \frac{882.85}{2507.02}\left(8.36234 + 25\times 0.4564\right) = 0.507198 $$

$$ A^{1}_{[35]:\overline{25}|} = A_{[35]:\overline{25}|} - v^{25}\,{}_{25}p_{[35]} = 0.3835 - 0.35215 = 0.03135 $$

$$ \ddot{a}^{(12)}_{[35]:\overline{25}|} = \ddot{a}_{[35]:\overline{25}|} - \frac{11}{24}(1 - v^{25}\,{}_{25}p_{[35]}) = 16.029 - 0.29693 = 15.73207 $$

$$ {}_{25}V^{\text{retrospective}} = 2.83969(11.64P\times 15.73207 - 245,300\times 0.03135 - 5000\times 0.507198 + 0.03P - 250 - 6P) $$

$$ = 2.83969(177.151295P - 10476.145) = £295,963.86 $$

(b) Surrender value would be the same i.e. ${}_{25}V^{\text{retrospective}} = {}_{25}V^{\text{prospective}}$ at 4% per annum rate of interest as the equality of bases ensures that the prospective and retrospective reserves of any policy at any given time t should be equal.

14 (i) Let P be the annual premium required to meet the company's profit criteria. Then:

(a) Multiple decrement table

Here not all decrements are uniform as whilst deaths can be assumed to be uniformly distributed over the year, surrenders occur only at the year end.

Hence:

$(aq)_x^d = q_x^d$ and $(aq)_x^w = q_x^w(1-q_w^d)$

x	q_x^d	q_x^w	$(aq)_x^d$	$(aq)_x^w$	$(ap)_x$	${}_{t-1}(ap)_x$
45	0.001465	0.05	0.001465	0.049927	0.948608	1
46	0.001622	0.05	0.001622	0.049919	0.948459	0.948608
47	0.001802	0.05	0.001802	0.049910	0.948288	0.899716

(b) Unit fund cashflows (per policy at start of year)

	Year 1	*Year 2*	*Year 3*
Value of units at start of year	0	0.347379P	1.405063P
Allocation	0.35P	1.05P	1.05P
Bid/offer	0.0175P	0.0525P	0.0525P
Interest	0.016625P	0.067244P	0.120128P
Management charge	0.001746P	0.007061P	0.012613P
Value of units at start of year	0.347379P	1.405063P	2.510077P

(c) Non-unit fund cashflows

	Year 1	*Year 2*	*Year 3*
Unallocated premium	$0.65P$	$-0.05P$	$-0.05P$
Bid/offer	$0.0175P$	$0.0525P$	$0.0525P$
Expenses	$0.2P+250$	$0.025P+50$	$0.025P+50$
Interest	$0.0187P-10$	$-0.0009P-2$	$-0.0009P-2$
Management charge	$0.001746P$	$0.007061P$	$0.012613P$
End of year cashflows	$0.487946P-260$	$-0.016339P-52$	$-0.010787P-52$

Probability in force	1	0.948608	0.899716
Discount factor	0.925926	0.857339	0.793832
Expected present value of profit	$0.430809P-320.170863$		

NPV = $.15P = 0.430809P – 320.170863 \Rightarrow P$ = £1140.17

(ii) Later expected future negative cashflows should be reduced to zero by establishing reserves in the non-unit fund at earlier durations so that the company does not expect to have to input further money in the future. The expected non-unit fund cashflows derived in (i) are negative in years 2and 3 so need to be eliminated.

END OF EXAMINERS' REPORT

EXAMINATION

September 2006

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M A Stocker
Chairman of the Board of Examiners

November 2006

Comments

No comments are given

1 If funds chose at random which annuities to insure and which to self-insure, we would expect approximately the same mortality experience in both groups. The self-insured experience is heavier, meaning their lives are somehow below standard on average.

The most likely explanation is that the pension funds make informed decisions based on the health or reason for retirement of the pensioners. Those retiring early due to ill-health or those known to have poor health are retained for payment directly by the fund. That should be cheaper than paying a premium to the insurer based on normal mortality for these lives. The remainder of the lives, known to be on reasonable health are insured.

This is adverse selection.

Sensible comments regarding other forms of selection are also acceptable.

2 (i)

$$EPV = 200,000\int_0^{20} e^{-\delta t}\,{}_tp_{40}^{HH}\sigma_{40+t}dt = 200,000\int_0^{20} e^{-\delta t}\,{}_tp_{40}^{\overline{HH}}\sigma_{40+t}dt$$

$$= 200,000\int_0^{20} e^{-\delta t}e^{-\int_0^t(\mu_{40+r}+\sigma_{40+r})dr}\sigma_{40+t}dt$$

where:

${}_tp_{40}^{HH}$ is the probability that the healthy life aged 40is healthy at age 40+t

${}_tp_{40}^{\overline{HH}}$ is the probability that the healthy life aged 40 is healthy at all points up to age 40+t (These 2 probabilities are the same for this model)

$\delta = \ln(1.08) = 0.076961$

(ii)

From $$EPV = 200,000\int_0^{20} e^{-\delta t}e^{-\int_0^t(\mu_{40+r}+\sigma_{40+r})dr}\sigma_{40+t}dt$$

$$EPV = 200,000\int_0^{20} e^{-(0.076961)t}e^{-\int_0^t\{(0.01)+(0.02)\}dr}(0.02)dt$$

$$= 200,000\int_0^{20} e^{-(0.076961)t}e^{-(0.03)t}(0.02)dt = 200,000\int_0^{20} e^{-(0.106961)t}(0.02)dt$$

$$= -\frac{(200,000)(0.02)}{0.106961}\left[e^{-(0.106961)t}\right]_0^{20} = 37,396.79[1-0.11775] = 32,993.32$$

3 $\bar{A}^{\ \ 1}_{70:\overline{1}|} = \int_0^1 e^{-\delta_{.075}t}\ {}_tp_{70}\mu_{70+t}dt$ in the general case.

Here, assuming μ is constant for 0 < *t* < 1, we get

$\mu = -\ln(p_{70}) = -\ln(1 - .03930) = 0.040093$
${}_tp_{70} = \exp(-\mu t) = \exp(-.040093t)$
$\delta_{.075} = \ln(1.075) = 0.07232$

$$\bar{A}^{\ \ 1}_{70:\overline{1}|} = \int_0^1 e^{-0.07232t}e^{-0.040093t}(0.040093)dt$$
$$= \frac{-(0.040093)}{(0.07232+0.040093)}\left[e^{-(0.07232+0.040093)t}\right]_0^1$$
$$= (-0.35610)(0.89368-1) = 0.0379$$

4 EPV of benefits:

$$20,000a^{(12)}_{65|60} = 20,000(a^{(12)}_{60} - a^{(12)}_{65:60}) = 20,000(a_{60} - a_{65:60}) = 20,000(15.652-11.682)$$
$$= 79,400$$

EPV premiums:

(The premium term will be the joint lifetime of the two lives because if his death is first the annuity commences or if her death is first, there will never be any annuity.)

Let *P* be the monthly premium

$$12P\ddot{a}^{(12)}_{65:60} = 12P(\ddot{a}_{65:60} - \tfrac{11}{24}) = 12P(12.682-0.458) = 146.688P$$

Equation of value allowing for expenses:

$1.015(79{,}400) = (1 - 0.05)(146.688P) \Rightarrow 80{,}591 = 139.3536P \Rightarrow P$
$= 578.32$ per month

5 (i) This is the present value of a joint life annuity of amount 1 per annum payable continuously until the first death of 2 lives (x) and (y).

(ii) $E[g(T)] = \int_0^\infty {}_tp_{xy}\mu_{x+t:y+t}\bar{a}_{\overline{t}|}dt$ or $E[g(T)] = \int_0^\infty {}_tp_{xy}e^{-\delta t}dt$

(iii) $Var[g(T)] = \dfrac{{}^2\bar{A}_{xy} - (\bar{A}_{xy})^2}{\delta^2}$ where ${}^2\bar{A}_{xy}$ indicates that the function is to be evaluated at force of interest 2δ.

6 (i) EPV past service benefits:

$$40,000 \frac{20}{80} \frac{({}^zM_{55}^{ia} + {}^zM_{55}^{ra})}{s_{54}D_{55}} = 40,000 \frac{20}{80} \frac{(34,048 + 128,026)}{(9.745)(1,389)} = 119,737$$

EPV future service benefits:

$$\frac{40,000}{80} \frac{({}^z\bar{R}_{55}^{ia} + {}^z\bar{R}_{55}^{ra})}{s_{54}D_{55}} = \frac{40,000}{80} \frac{(163,063 + 963,869)}{(9.745)(1,389)} = 41,628$$

EPV total pension benefits = 119,737 + 41,628 = £161,365

(ii) $(k)(40,000)\dfrac{{}^s\bar{N}_{55}}{s_{54}D_{55}} = 41,628 \Rightarrow (k)(40,000)\dfrac{88,615}{(9.745)(1,389)} = 41,628 \Rightarrow k = .159$

i.e. 15.9% salary per annum

7 ACF $= \dfrac{\sum_x {}^sE_{x,t}^c \, {}^sm_{x,t}}{\sum_x {}^sE_{x,t}^c} \Bigg/ \dfrac{\sum_x E_{x,t}^c \, {}^sm_{x,t}}{\sum_x E_{x,t}^c}$

Here

	${}^sE_{x,t}^c$	${}^sE_{x,t}^c \, {}^sm_{x,t}$	$E_{x,t}^c$
Age-group	Population	Deaths	Population
0–19	2,900,000	580	800,000
20–44	3,500,000	2,450	1,000,000
45–69	2,900,000	20,300	900,000
70 and over	700,000	49,000	300,000
Total	10,000,000	72,330	3,000,000

leading to

${}^sm_{x,t}$	$E_{x,t}^c \, {}^sm_{x,t}$
0.0002	160
0.0007	700
0.007	6,300
0.07	21,000
	28,160

$$\text{ACF} \quad = \quad \frac{72,330}{10,000,000} \Big/ \frac{28,160}{3,000,000} = \quad (0.007233/\ 0.0093867) = 0.77056$$

Indirectly standardised mortality rate = (ACF)*(Province crude rate)

$$= (0.77056)\left(\frac{25,344}{3,000,000}\right) = (0.77056)(0.008448) = 0.00651$$

8 (i)

$${}_t\overline{V} = {}_{n-t}p_{x+t}e^{-\delta(n-t)}$$

$$\frac{\partial}{\partial t}\,{}_t\overline{V} = \frac{\partial}{\partial t}({}_{n-t}p_{x+t}e^{-\delta(n-t)}) = \{e^{-\delta(n-t)}\frac{\partial}{\partial t}({}_{n-t}p_{x+t})\} + \{{}_{n-t}p_{x+t}\frac{\partial}{\partial t}(e^{-\delta(n-t)})\}$$

$$\frac{1}{{}_{n-t}p_{x+t}}\frac{\partial}{\partial t}({}_{n-t}p_{x+t}) = \frac{\partial}{\partial t}\ln({}_{n-t}p_{x+t}) = \frac{\partial}{\partial t}\ln\left(\frac{l_{x+n}}{l_{x+t}}\right) = \frac{\partial}{\partial t}\{\ln(l_{x+n}) - \ln(l_{x+t})\} = \mu_{x+t}$$

$$\Rightarrow \frac{\partial}{\partial t}({}_{n-t}p_{x+t}) = (\mu_{x+t})({}_{n-t}p_{x+t})$$

$$\frac{\partial}{\partial t}(e^{-\delta(n-t)}) = \delta e^{-\delta(n-t)}$$

$$\Rightarrow \frac{\partial}{\partial t}\,{}_t\overline{V} = \{e^{-\delta(n-t)}(\mu_{x+t})({}_{n-t}p_{x+t})\} + \{{}_{n-t}p_{x+t}\delta e^{-\delta(n-t)}\} = {}_{n-t}p_{x+t}e^{-\delta(n-t)}(\mu_{x+t} + \delta)$$

$$\Rightarrow \frac{\partial}{\partial t}\,{}_t\overline{V} = (\mu_{x+t} + \delta)\,{}_t\overline{V}$$

(ii) The change in reserve at time t consists of the interest earned and the release of reserves from the deaths.

(*The release may be more easily seen if the last line of (i) is rewritten as:* $\frac{\partial}{\partial t}\,{}_t\overline{V} = \delta\,{}_t\overline{V} - \mu_{x+t}(0 - {}_t\overline{V})$ *where the pure endowment has zero death benefit.)*

(iii) ${}_n\overline{V} = 1$.

9 (i) Survival table

x	q_x	p_x	$_{t-1}p_x$
60	0.008022	0.99198	1
61	0.009009	0.99099	0.991978
62	0.010112	0.98989	0.983041

Unit fund

	Value of units at start of year	*Allocation*	*Bid/offer*	*Interest*	*Management charge*	*Value of units at end of year*
Year 1	0.00	4,250.00	212.50	242.25	32.10	4,247.65
Year 2	4,247.65	5,200.00	260.00	551.26	73.04	9,665.87
Year 3	9,665.87	5,200.00	260.00	876.35	116.12	15,366.10

Non-unit fund

	Unallocated premium	*Bid/offer*	*Expenses*	*Interest*	*Management charge*	*Extra death benefit*	*End of year cashflows*
Year 1	750.00	212.50	600.00	14.50	32.10	126.37	282.73
Year 2	-200.00	260.00	100.00	-1.60	73.04	93.10	-61.66
Year 3	-200.00	260.00	100.00	-1.60	116.12	46.86	27.66

	Non-unit fund cash flow (profit vector)	*Probability in force at start of year*	*Profit signature*	*Discount factor*	*Expected present value of profit*
Year 1	282.73	1	282.73	0.909091	257.03
Year 2	-61.66	0.991978	-61.16	0.826446	-50.55
Year 3	27.66	0.983041	27.19	0.751315	20.43

Total NPV 226.91

Expected NPV = 226.91

(ii) The NPV would decrease. Holding reserves would delay the emergence of some of the Year 1 cash flow, and as the non-unit fund earns 4%, well below the risk discount rate, the NPV would reduce.

10 (i) The 2 deaths were 70 and 69 respectively at 1/1/2005. The reserves for these policies at 31/12/2005 were

$$ {}_{26}V = 12,000\left(1-\frac{\ddot{a}_{71}}{\ddot{a}_{45}}\right) = 12,000\left(1-\frac{9.998}{18.823}\right) = 5,626.10 \text{ and} $$

$$ {}_{24}V = 10,000\left(1-\frac{\ddot{a}_{70}}{\ddot{a}_{46}}\right) = 10,000\left(1-\frac{10.375}{18.563}\right) = 4,410.92 $$

Total death strain at risk, sorted by age at 1/1/2005:

Age 69: 500,000 - (175,000 + 4,410.92) = 320,589.08
Age 70: 400,000 - (150,000 + 5,626.10) = 244,373.90

Expected death strain:

$(q_{69})(320,589.08) + (q_{70})(244,373.90)$
= (0.022226)(320,589.08) + (0.024783)(244,373.90)
= 7,125.41 + 6,056.32
= 13,181.73

Actual death strain:

(12,000 - 5,626.10)+(10,000 - 4,410.92) = 6,373.90+5,589.08 = 11,962.98

Mortality profit = EDS – ADS = 13,181.73-11,962.98 = 1,218.75 profit

(ii) (a) Expected claims:

$(q_{69})(500,000)+(q_{70})(400,000)$
= (0.022226)(500,000) + (0.024783)(400,000)
= 11,113 + 9,913.2 = 21,026.20

Actual claims:

12,000 + 10,000 = 22,000

(b) Actual claims were higher than expected claims but the company still made a mortality profit. This can only have occurred because the deaths were disproportionately concentrated on lower DSAR lives (policies more mature on average). (This can be seen by comparing the ratio of reserves to sum assured for the death claim policies with the corresponding ratio for the full portfolio.)

11 (i) $$g(T) = \begin{cases} 5{,}000v^2\bar{a}_{\overline{T_{63}-2}|} & \text{if } T_{63} \geq 2 \quad (\text{or } 5{,}000(\bar{a}_{\overline{T_{63}}|} - \bar{a}_{\overline{2}|})) \\ 0 & \text{if } T_{63} < 2 \end{cases}$$

(ii)

$$E[g(T)] = (100)(5{,}000)v^2 \, {}_2p_{63}\bar{a}_{65} = (500{,}000)(0.92456)(0.992617)(14.871 - 0.5)$$
$$= (500{,}000)(13.1887) = 6{,}594{,}350$$

(iii) $Var[g(T)] = E[g(T)^2] - E[g(T)]^2$

For £1 of annuity:

$$E[g(T)^2] = \int_2^\infty {}_tp_{63}\mu_{63+t}[v^2\bar{a}_{\overline{t-2}|}]^2 dt$$

Let $t = r + 2 \Rightarrow$

$$E[g(T)^2] = \int_0^\infty {}_{r+2}p_{63}\mu_{63+r+2}[v^2\bar{a}_{\overline{r}|}]^2 dr$$
$$= \int_0^\infty {}_rp_{65} \, {}_2p_{63}\mu_{65+r}v^4\left[\frac{1-v^r}{\delta}\right]^2 dr$$
$$= \frac{{}_2p_{63}v^4}{\delta^2}\int_0^\infty {}_rp_{65}\mu_{65+r}[1 - 2v^r + v^{2r}]dr$$
$$= \frac{{}_2p_{63}v^4}{\delta^2}[1 - 2\bar{A}_{65} + {}^2\bar{A}_{65}]$$

where

$$\bar{A}_{65} = (1.04)^{0.5}(1 - d\ddot{a}_{65}) = 1.019804\{1 - \left(\frac{0.04}{1.04}\right)(14.871)\} = 0.436515$$

and ${}^2\bar{A}_{65} = (1.04)({}^2A_{65}) = (1.04)(0.20847) = 0.21681$

$$\therefore E[g(T)^2] = \frac{(0.992617)(0.85480)}{(0.039221)^2}[1 - (2)(0.436515) + (0.21681)] = 189.622$$

$$Var[g(T)] = 189.622 - (13.1887)^2 = 15.680$$

For annuity of 5,000 we need to increase by $5{,}000^2$ and for 100 (independent) lives we need to multiply by 100.

Total variance = $(15.680)(5{,}000^2)(100) = 39{,}200{,}000{,}000 = (197{,}999)^2$

12 EPV benefits:

$$110{,}000 A^{\;1}_{[50]:\overline{10}|} - 10{,}000(IA)^{\;1}_{[50]:\overline{10}|} \text{ (functions @ 6\% p.a.)}$$

$$= 110{,}000\{A_{[50]} - v^{10}\,{}_{10}p_{[50]}A_{60}\} - 10{,}000\{(IA)_{[50]} - v^{10}\,{}_{10}p_{[50]}(10A_{60} + (IA)_{60})\}$$
$$= 110{,}000A_{[50]} - 10{,}000(IA)_{[50]} + v^{10}\,{}_{10}p_{[50]}\{10{,}000((IA)_{60} - A_{60})\}$$
$$= (110{,}000)(0.20463) - (10{,}000)(4.84789) + (0.55839)(0.95684)\{10{,}000(5.46572 - 0.32692)\}$$
$$= 22{,}509.30 - 48{,}478.90 + 27{,}456.09 = 1{,}486.49$$

EPV gross premiums

Let P be annual premium

$$P\ddot{a}^{6\%}_{[50]:\overline{10}|} = 7.698P$$

EPV expenses

$$200 + 0.25P + 0.02Pa^{6\%}_{[50]:\overline{9}|} + 50a^{4\%}_{[50]:\overline{9}|} + 200A^{\;1\;\;4\%}_{[50]:\overline{10}|}$$
$$= 150 + 0.23P + 0.02P\ddot{a}^{6\%}_{[50]:\overline{10}|} + 50\ddot{a}^{4\%}_{[50]:\overline{10}|} + 200(A_{[50]} - v^{10}_{(4\%)}\,{}_{10}p_{[50]}A^{4\%}_{60})$$
$$= 150 + 0.23P + 0.02P(7.698) + (50)(8.318) + 200(0.32868 - (0.67556)(0.95684)(0.45640))$$
$$= 150 + 415.90 + 6.73 + P(.23 + 0.15396) = 572.63 + 0.38396P$$

Equation of value:

$7.698P$ = 1,486.49 + 572.63 + $0.38396P \Rightarrow 7.31404P$ = 2,059.12 so P = 281.53 p.a.

(ii) If $K_{59} \geq 1$ $\;GFLRV = 50(1.01923)^9 - 0.98(281.53)$ else (i.e. $K_{59} = 0$)

$$GFLRV = 10{,}000v_{.06} + 200(1.01923)^9 v_{.04}$$
$$+50(1.01923)^9 - 0.98(281.53)$$

or

$$GFLRV = 10{,}000v_{.06} + 200(1.01923)^{10} v_{.06}$$
$$+50(1.01923)^9 - 0.98(281.53)$$

(iii)

$$
\begin{aligned}
{}_9V &= 10{,}000 q_{59} v_{.06} + 200(1.01923)^9 q_{59} v_{.04} \\
&\quad +50(1.01923)^9 - 0.98(281.53) \\
&= (10{,}000)(0.007140)(0.94340) + (237.40)((0.007140)(0.96154) + 59.35 - 275.90 \\
&= 67.36 + 1.63 + 59.35 - 275.90 = -147.56
\end{aligned}
$$

(iv) The reserve is negative. The expected future income exceeds expected future outgo, because past outgo exceeded past income, meaning the office needs a net inflow in the last year to recoup previous losses. However, it is at risk of the policy lapsing, and never getting this net inflow.

END OF EXAMINERS' REPORT

EXAMINATION

April 2007

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M A Stocker
Chairman of the Board of Examiners

June 2007

Comments

Comments, where applicable, are given in the solutions that follow.

1 (i) $_{5|10}q_{[52]} = (l_{57} - l_{67}) / l_{[52]} = (9467.2906 - 8557.0118) / 9652.6965 = 0.094303$

(ii) $p_{[50]:[60]} = p_{[50]} \times p_{[60]} = (1 - 0.001971) \times (1 - 0.005774) = 0.992266$

2 Class selection — different classes of members experience different mortality rates. e.g. works versus staff. Alternatively ill-health retirements, other early retirement and normal retirements experience different mortality

Temporary Initial selection — employee turnover rates vary with duration of employment, recent joiners are most likely to leave.

Time selection — turnover rates vary with economic conditions.

Other answers given credit if properly defined with pension fund specific examples.

3
$$
\begin{aligned}
EPV &= 100e^{-\int_{55}^{65}(0.02+0.03+0.04)dx} \\
&= 100e^{-0.09*[65-55]} \\
&= 100e^{-0.9} \\
&= 40.66
\end{aligned}
$$

4 (i) The conditions are:

- The retrospective and prospective reserves are calculated on the same basis.
- The basis is the same as the basis used to calculate the premiums used in the reserve calculation.

(ii) Two reasons are:

- The assumptions used for the retrospective calculation (for which the experienced conditions over the duration of the contract up to the valuation date are used) are not generally appropriate for the prospective calculation (for which the assumptions considered suitable for the remainder of the policy term are used).
- The assumptions considered appropriate at the time the premium was calculated may not be appropriate for the retrospective or prospective reserve some years later.

5 Value of death benefit:

$$1000 * \int_0^\infty 0.05 * \exp(-\int_0^t 0.09 ds) dt =$$

$$1000 * \int_0^\infty 0.05 * e^{-0.09t} dt = 1000 * 0.05 / 0.09$$

$$= 555.56$$

Value of survival benefit every 5th year:

$500*(e^{-0.45} + e^{-0.9} + e^{-1.35} + \ldots\ldots\ldots)$

$= 500*e^{-0.45}/(1 - e^{-0.45}) = 500*0.63763/0.36237$

$= 879.81$

Value of premiums:

$P*(1 + e^{-.09} + e^{-.18} + e^{-.27} + \ldots\ldots)$

$= P*(1/1 - e^{-.09}) = 11.619*P$

Hence

$11.619*P = 555.56+879.81$

$P = 123.54$

6
$$4 \times 25,000 \times \frac{\sum_{t=0}^{29} s_{35+t+1} d_{35+t} v^{35+t+0.5}}{s_{36} l_{35} v^{35}}$$

definitions:

s_x — salary in year to age x
d_x — number of deaths in year of age x to $x+1$
l_x — number of lives alive at age x exact

Other schemes given credit if properly defined.

7 Present value

$$10000A^{1}_{[50]:\overline{10}} = 10000(A_{[50]} - v^{10}\, {}^{l_{60}}\!\big/_{l_{[50]}}\, A_{60})$$

$$= 10000(0.32868 - v^{10}\, {}^{9287.2164}\!\big/_{9706.0977}\, 0.45640) = 336.60$$

Variance

$$= 10000^2({}^2A^{1}_{[50]:\overline{10}} - (A^{1}_{[50]:\overline{10}})^2)$$

$$= 10000^2(({}^2A_{[50]} - v^{20}\, {}^{l_{60}}\!\big/_{l_{[50]}}\, {}^2A_{60}) - (336.66/10000)^2)$$

$$= \ 10000^2((0.13017 - v^{20}\, {}^{9287.2164}\!\big/_{9706.0977}\, 0.23723) - 0.033666^2) = 2543992$$

The function with the 2 suffix is calculated at rate i^2+2i i.e 8.16% in this case.

8 (i) The standardised mortality ratio is the ratio of the indirectly standardised mortality rate to the crude mortality rate in the standard population.

$$\text{SMR} \ = \ \frac{\sum_x \mathrm{E}^{\mathrm{c}}_{\mathrm{x,t}} m_{x,t}}{\sum_x \mathrm{E}^{\mathrm{c}}_{\mathrm{x,t}}\, {}^{s}m_{x,t}}$$

$E^{c}_{x,t}$ = central exposed to risk in population being studied between age x and age $x+t$

$m_{x,t}$ = central mortality rate in population being studied for ages x to $x+t$

${}^{s}m_{x,t}$ = central mortality rate in standard population for ages x to $x+t$

(ii) SMR = (2 + 3 + 6 + 50) / (25 × 12/300 + 35 × 10/275 + 100 × 9 / 200
+ 500 × 8 / 175)
= 2.058

As the SMR is greater than 1, Giro experiences heavier mortality than Actuaria

9 Equation of present value:

$$\text{Purchase price} = 15{,}000\ddot{a}^{(12)}_{\overline{5|}} + 7500\ {}_{5|}\ddot{a}^{(12)}_{60} + 7500\ {}_{5|}\ddot{a}^{(12)}_{\overline{60:55}}$$

$$+7500v^5(1-{}_5p_{60})\,{}_5p_{55}\,\ddot{a}^{(12)}_{60} + 7500v^5(1-{}_5p_{55})\,{}_5p_{60}\,\ddot{a}^{(12)}_{65}$$

$$=15{,}000(1-v^5)/d^{(12)} + 7500\,v^5\, {}^{l_{65}}\!/_{l_{60}}\,(\ddot{a}_{65} - 11/24) + 7500v^5$$

$$ {}^{l_{65}l_{60}}\!/_{l_{60}l_{55}}\,(\ddot{a}_{65} + \ddot{a}_{60} - \ddot{a}_{65:60} - 11/24)$$

$$+7500v^5[(1-l_{65}/l_{60})l_{60}/l_{55})(\ddot{a}_{60}-11/24) + (1-l_{60}/l_{55})l_{65}/l_{60})(\ddot{a}_{65}-11/24)]$$

$$=15000\times(1-0.82193)/0.039157 + 7500\times 0.82193\times {}^{9647.797}\!/_{9826.131}\times(13.666-11/24)$$

$$+7500\times 0.82193\times {}^{9647.797}\!/_{9826.131}\times {}^{9848.431}\!/_{9917.623}\times(13.666+16.652-12.682-11/24)$$

$$+7500\times 0.82193\times(1-9647.797/9826.131)\times 9848.431/9917.623\times(16.652-11/24)$$
$$+7500\times 0.82193\times(1-9848.431/9917.623)\times 9647.797/9826.131\times(13.666-11/24)$$
$$= 68213.86 + 79940.67 + 103244.12 + 1799.09 + 557.72$$

= £253755 to nearest £

The following is an alternative derivation of the formula for the purchase price above.

$$15{,}000\ddot{a}^{(12)}_{\overline{5|}} + 15{,}000v^5\,{}_5p_{60}(1-{}_5p_{55})\ddot{a}^{(12)}_{65} + 7{,}500v^5\,{}_5p_{55}(1-{}_5p_{60})\ddot{a}^{(12)}_{60}$$
$$+v^5\,{}_5p_{60}\,{}_5p_{55}(15{,}000\ddot{a}^{(12)}_{65} + 7{,}500[\ddot{a}^{(12)}_{60} - \ddot{a}^{(12)}_{65:60}])$$

10 (i) $X - Y$ is the present value of a deferred whole of life assurance with a sum assured of 1 payable at the end of the year of death of a life now aged x provided the life dies after age $x + n$.

(ii) $X = v^{k+1}$ all k $\quad Y = \begin{cases} v^{k+1} & 0 \le k < n \\ 0 & k \ge n \end{cases}$

$\text{Cov}(X, Y) = E[XY] - E[X]\,E[Y]$

Now $E[XY] = \sum_{k=0}^{k=n-1} (v^{k+1})^2 \; P[K_x = k] + \sum_{k=n}^{k=\infty} v^{k+1} \times 0 \times P[K_x = k]$

$= \sum_{k=0}^{k=n-1} (v^2)^{k+1} \; P[K_x = k]$

$= {}^2A^1_{x:\overline{n}|}$

Where ${}^2A$ is determined using a discount function v^2 , i.e. using an interest rate

$i^* = (1 + i)^2 - 1 = 2i + i^2$

Then: $\mathrm{Cov}(X, Y) = {}^2A^1_{x:\overline{n}|} - A_x \, . \, A^1_{x:\overline{n}|}$

Now: $\mathrm{Var}(X - Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) - 2\,\mathrm{Cov}(X, Y)$

$= ({}^2A_x - (A_x)^2) + ({}^2A^1_{x:\overline{n}|} - (A^1_{x:\overline{n}|})^2) - 2({}^2A^1_{x:\overline{n}|} - A_x \, . A^1_{x:\overline{n}|})$

$= ({}^2A_x + {}^2A^1_{x:\overline{n}|} - 2\,{}^2A^1_{x:\overline{n}|}) - ((A_x)^2 + (A^1_{x:\overline{n}|})^2) - 2A^1_{x:\overline{n}|}\, A_x)$

$= {}^2A_x - {}^2A^1_{x:\overline{n}|} - (A_x - A^1_{x:\overline{n}|})^2$

$= {}^2A_x - {}^2A^1_{x:\overline{n}|} - ({}_{n|}A_x)^2$

The Examiners regret that two typographical errors occurred in the question wording set in the Examination:

- *In line 2 of 10(ii) the symbol shown as ${}^2A^1_x$ should have been ${}^2A^1_{x:\overline{n}|}$.*
- *In the same line the function on the left hand side of the equation should have read Cov(X,Y) and not have included in the brackets 2 assurance functions (which as erroneously stated would have equated to zero).*

In the event this question was done well despite the errors. The majority of students attempting the question noticed the first error as obvious and adjusted accordingly. The second error was rarely noticed by students who often went on to produce an otherwise good proof.

The question has been corrected for publication. The Examiners wish to sincerely apologise for these errors and wish to assure students that the marking system was sympathetically adjusted to meet the circumstances.

11 (i) It is a principle of prudent financial management that once sold and funded at outset a product should be self-supporting. Many products produce profit signatures that usually have a single financing phase. However, some products, particularly those with substantial expected outgo at later policy durations, can give profit signatures which have more than one financing phase. In such cases these later negative cashflows should be reduced to zero by establishing reserves in the non-unit fund at earlier durations. These reserves are funded by reducing earlier positive cashflows.

(ii) The reserves required at the end of year 2 and year 1 are:

$$_2V = \frac{20.15}{1.05} = 19.190$$

$$_1V = \frac{1}{1.05}(30.18 + p_{51} \times 19.190) = \frac{1}{1.05}(30.18 + 0.99719 \times 19.190) = 46.968$$

(iii) Before zeroisation, the net present value (based on a risk discount rate of 8%) is:

$$NPV = -\frac{95.21}{1.08} - \frac{p_{50} \times 30.18}{1.08^2} - \frac{{}_2p_{50} \times 20.15}{1.08^3} + \frac{{}_3p_{50} \times 77.15}{1.08^4} + \frac{{}_4p_{50} \times 120.29}{1.08^5}$$

$$= -\frac{95.21}{1.08} - \frac{0.99749 \times 30.18}{1.08^2} - \frac{0.99469 \times 20.15}{1.08^3} + \frac{0.99155 \times 77.15}{1.08^4} + \frac{0.98804 \times 120.29}{1.08^5}$$

$$= -88.157 - 25.810 - 15.911 + 56.228 + 80.888 = 7.238$$

After zeroisation, the profit in year 1 becomes:

Profit in year 1 $= -95.21 - p_{50} \times {}_1V = -95.21 - 0.99749 \times 46.968 = -142.06$

So profit vector will become (-142.06, 0, 0, 77.15, 120.29)

And *NPV* after zeroisation will be:

$$NPV = -\frac{142.06}{1.08} + 0 + 0 + \frac{{}_3p_{50} \times 77.15}{1.08^4} + \frac{{}_4p_{50} \times 120.29}{1.08^5}$$

$$= -\frac{142.06}{1.08} + 0 + 0 + \frac{0.99155 \times 77.15}{1.08^4} + \frac{0.98804 \times 120.29}{1.08^5}$$

$$= -131.537 + 56.228 + 80.888 = 5.579$$

As expected, the NPV after zeroisation is smaller because the emergence of the profits has been deferred and the risk discount rate is greater than the accumulation rate.

12 (i) Net premium per policy is P where $P\ddot{a}_{30:\overline{20}|} = 75,000A^{1}_{30:\overline{25}|}$

$$P = \frac{75,000\left(A_{30} - v^{25}\,{}_{25}p_{30}A_{55}\right)}{\ddot{a}_{30} - v^{20}\,{}_{20}p_{30}\ddot{a}_{50}}$$

$$= \frac{75,000\left(0.16023 - 1.04^{-25}\dfrac{9557.8179}{9925.2094}0.38950\right)}{21.834 - 1.04^{-20}\dfrac{9712.0728}{9925.2094}17.444}$$

$$= 75,000\frac{(0.16023 - 0.14070)}{(21.834 - 7.7903)} = £104.30$$

(ii) Net premium reserve per policy at the end of the 20th year

$$= 75,000A^{1}_{50:\overline{5}|} - 0 = 75,000\left(A_{50} - v^{5}\,{}_{5}p_{50}A_{55}\right)$$

$$= 75,000\left(0.32907 - 1.04^{-5}\frac{9557.8179}{9712.0728}0.38950\right) = 75,000 \times 0.014014 = £1051.06$$

Net premium reserve per policy at the start of the 20th year

$$= \frac{Sq_{49} + {}_{20}Vp_{49}}{1+i} - P$$

$$= \frac{75,000q_{49} + 1051.06p_{49}}{1.04} - 104.30$$

$$= \frac{75,000 \times 0.002241 + 1051.06 \times 0.997759}{1.04} - 104.30$$

$$= 1065.68$$

(iii) Death strain at risk = 75,000 – 1051 = 73,949

EDS = $738q_{49} \times 73{,}949 = 122,301$

ADS = $2 \times 73{,}949 = 147,898$

Mortality profit = 122,301 – 147,898 = -£25,597 (i.e. a loss)

13 (i) Let P be the monthly premium for the contract with simple bonus. Then equation of value (at 4% p.a. interest) is:

$$12P(.95\ddot{a}^{(12)}_{[30]:\overline{35}|}) - 5.95P = (48,000 + 250)\bar{A}_{[30]} + 2,000(I\bar{A})_{[30]} + 300$$

where $\ddot{a}^{(12)}_{[30]:\overline{35}|} = \ddot{a}_{[30]:\overline{35}|} - \frac{11}{24}\left(1 - v^{35}\,{}_{35}p_{[30]}\right) = 19.072 - \frac{11}{24}\left(1 - 1.04^{-35}\frac{8821.2612}{9923.7497}\right)$

$= 18.7169$

Therefore:

$$12P(.95 \times 18.7169) - 5.95P = (48,000 + 250) \times 1.04^{0.5} \times 0.16011 + 2,000 \times 1.04^{0.5} \times 6.91644 + 300$$

i.e.

$$207.42266P = 7,878.299 + 14,106.825 + 300$$

$$\Rightarrow P = \frac{22,285.124}{207.42266} = £107.44$$

(ii) Let P' be the monthly premium for the contract with compound bonus. Then equation of value (at 4% p.a. interest) is:

$$12P'(.95\ddot{a}^{(12)}_{[30]:\overline{35}|}) - 5.95P' = 50,000\left[v^{0.5}q_{[x]} + v^{1.5}p_{[x]}q_{[x]+1}(1.04) + \ldots\right] + 250\bar{A}^{@4\%}_{[30]} + 300$$

$$= \frac{50,000}{1.04}\left[v^{0.5} \times 1.04q_{[x]} + v^{1.5} \times 1.04^2 p_{[x]}q_{[x]+1} + \ldots\right] + 250\bar{A}^{@4\%}_{[30]} + 300$$

$$= \frac{50,000}{(1.04)^{0.5}}\left[v \times 1.04q_{[x]} + v^2 \times 1.04^2 p_{[x]}q_{[x]+1} + \ldots\right] + 250\bar{A}^{@4\%}_{[30]} + 300$$

$$= \frac{50,000}{(1.04)^{0.5}} A^{@0\%}_{[30]} + 250\bar{A}^{@4\%}_{[30]} + 300$$

where $A^{@0\%}_{[30]} = 1$

$$\Rightarrow 12P'(.95 \times 18.7169) - 5.95P' = \frac{50,000}{(1.04)^{0.5}} + 250 \times 1.04^{0.5} \times 0.16011 + 300$$

$$207.42266P' = 49,029.034 + 40.820 + 300$$

$$\Rightarrow P' = \frac{49,369.854}{207.42266} = £238.02$$

14 Multiple decrement table:

X	$q^d_{[x]} = (aq)^d_{[x]}$	$q^s_{[x]}$	$(aq)^s_{[x]} = q^s_{[x]}\left(1-(aq)^d_{[x]}\right)$
61	0.006433	0.05	0.04968
62	0.009696	0.05	0.04952
63	0.011344	0.05	0.04943
64	0.012716	—	—

T	$(ap)_{[61]+t-1}$	${}_{t-1}(ap)_{[61]}$
1	0.943887	1
2	0.940784	0.94389
3	0.939226	0.88799
4	0.987284	0.83403

Let P be the annual premium payable. Then equation value is:

$$P\ddot{a}_{[61]:\overline{4}|} = 100,000A_{[61]:\overline{4}|} + (50 + 0.025P)(\ddot{a}_{[61]:\overline{4}|} - 1) + 500$$

$$\Rightarrow P(0.975\ddot{a}_{[61]:\overline{4}|} + 0.025) = 100,000A_{[61]:\overline{4}|} + 50(\ddot{a}_{[61]:\overline{4}|} - 1) + 500$$

$$\Rightarrow P(0.975 \times 3.730 + 0.025) = 100,000 \times 0.85654 + 50 \times 2.730 + 500$$

$$P = \frac{85,654 + 636.5}{3.66175} = 23,565.37$$

Reserves required on the policy per unit sum assured are:

$${}_1V_{61:\overline{4}|} = 1 - \frac{\ddot{a}_{62:\overline{3}|}}{\ddot{a}_{61:\overline{4}|}} = 1 - \frac{2.857}{3.722} = 0.23240$$

$${}_2V_{61:\overline{4}|} = 1 - \frac{\ddot{a}_{63:\overline{2}|}}{\ddot{a}_{61:\overline{4}|}} = 1 - \frac{1.951}{3.722} = 0.47582$$

$${}_3V_{61:\overline{4}|} = 1 - \frac{\ddot{a}_{64:\overline{1}|}}{\ddot{a}_{61:\overline{4}|}} = 1 - \frac{1.000}{3.722} = 0.73133$$

Year t	*Prem*	*Expense*	*Opening reserve*	*Interest*	*Death Claim*	*Surr Claim*	*Mat Claim*	*Closing reserve*	*Profit vector*
1	23565.4	500	0	1153.3	643.3	1170.7	0	21935.9	468.8
2	23565.4	639.1	23240.0	2308.3	969.6	2333.9	0	44764.4	406.7
3	23565.4	639.1	47582.0	3525.4	1134.4	3494.5	0	68688.4	716.4
4	23565.4	639.1	73133.0	4803.0	1271.6	0	98728.4	0	862.3

Year t	*Profit signature*	*Discount factor*	*NPV of profit signature*
1	468.8	.92593	434.1
2	383.9	.85734	329.1
3	636.2	.79383	505.0
4	719.2	.73503	528.6

NPV of profit signature = £1,796.8

Year t	*Premium*	${}_{t-1}p_{[61]}$	*Discount factor*	*NPV of premium*
1	23565.4	1	1	23565.4
2	23565.4	0.94389	.92593	20595.6
3	23565.4	0.88799	.85734	17940.6
4	23565.4	0.83403	.79383	15602.1

NPV of premiums = £77,703.7

Profit margin = $\dfrac{1,796.8}{77,703.7} = 0.0231$ i.e.   2.31%

END OF EXAMINERS' REPORT

EXAMINATION

September 2007

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M A Stocker
Chairman of the Board of Examiners

December 2007

1

$$({}_tV_x + P_x)(1+i) = q_{x+t} + p_{x+t}({}_{t+1}V_x)$$
$$\Rightarrow (0.468 + 0.017)(1.03) = 0.024 + (0.976)({}_{t+1}V_x)$$
$$\Rightarrow {}_{t+1}V_x = (0.49955 - 0.024)/0.976 = 0.487$$

2

$$p_{[x]} = ({}_{0.5}p_{[x]})({}_{0.5}p_{[x]+0.5}) = (1 - {}_{0.5}q_{[x]})(1 - {}_{0.5}q_{[x]+0.5})$$
$$= (1 - 0.33q_x)(1 - 0.5q_x) = (1 - 0.33(1 - p_x))(1 - 0.5(1 - p_x))$$
$$= (0.67 + 0.33p_x)(0.5 + 0.5p_x)$$
$$= 0.335 + 0.5p_x + 0.165p_x{}^2$$

3

(+1, -1, +1, +1, +1, -1, 0, -1, +1, -1, +1, +1)
→ (+1, -1, +1, +1, +1, -1, 0, -1, 0, 0, +1, +1)
→ (+1, -1, +1, +1, +1, -1, -1, 0, 0, 0, +1, +1)
→ (+1, -1, +1, +1, +1, -2, 0, 0, 0, 0, +1, +1)
→ (+1, -1, +1, +1, -1, 0, 0, 0, 0, 0, +1, +1)
→ (+1, -1, +1, 0, 0, 0, 0, 0, 0, 0, +1, +1)
→ (0, 0, +1, 0, 0, 0, 0, 0, 0, 0, +1, +1)

(Following not necessary for marks but to help explain)

All positives after last negative remain unchanged.

Consider last negative in year 10. The underlying cash flow that year, per policy in force at start of year 10, is

$$\frac{-1}{{}_9p_x} = (NUCF)_{10}\,.$$

The reserve needed at t = 9 to counter this is

$$\frac{-(NUCF)_{10}}{1+i} = -(NUCF)_{10} \text{since } i = 0\,.$$

This is funded from year 9 cash flows at a cost of

$$= (p_{x+8})(-(NUCF)_{10})$$

per policy in force at the start of year 9.

The change in year 9 cash flow is

$$(NUCF)_9 = -(p_{x+8})(-(NUCF)_{10}) = p_{x+8}(NUCF)_{10}$$

and the change in year 9 profit signature becomes

$$(PS)_9^* = {}_8p_x\, p_{x+8}(NUCF)_{10} = {}_9p_x(NUCF)_{10} = -1$$

resulting in a revised year 9 profit signature of +1-1=0.

The other results follow by repeating this step from year 9 towards year 1 wherever there are negative values in the profit signature.

4 $EPV = 1,000({}_{1/12}p_{65}\frac{1}{1.09^{1/12}} + {}_{2/12}p_{65}\frac{1.0039207}{1.09^{2/12}} + {}_{3/12}p_{65}\frac{1.0039207^2}{1.09^{3/12}} + +$

But $1.0039207^{12} = 1.048076$ and $\frac{1.048076}{1.09} = \frac{1}{1.04}$ leading to

$$EPV = \frac{1,000}{1.0039207}({}_{1/12}p_{65}\frac{1}{1.04^{1/12}} + {}_{2/12}p_{65}\frac{1}{1.04^{2/12}} + {}_{3/12}p_{65}\frac{1}{1.04^{3/12}} + +$$

$$= \frac{12,000}{1.0039207}a_{65}^{(12)} @ 4\%\text{p.a.}$$

EPV = 11,953.14 * (14.871 - 1+(11/24)) = 11,953.14 * 14.329 = 171,276

5 (i) Directly standardised mortality rate = $\dfrac{\sum_x {}^sE_{x,t}^c m_{x,t}}{\sum_x {}^sE_{x,t}^c}$.

Where:

${}^sE_{x,t}^c$: Central exposed to risk in standard population between ages *x* and *x*+*t*

$m_{x,t}$: central rate of mortality either observed or from a life table in population being studied for ages *x* to *x*+*t*

(ii) The main disadvantage is that it requires age-specific mortality rates, $m_{x,t}$, for the group / population in question, and these are often not available conveniently. To overcome this, indirect standardisation, which relies on easily available data, can be used.

6 $$ {}_{0.5|}q_{75} = {}_{0.5}p_{75}(q_{75.5}) = {}_{0.5}p_{75}[{}_{0.5}q_{75.5} + ({}_{0.5}p_{75.5})({}_{0.5}q_{76})] $$
$$ = [({}_{0.5}p_{75})({}_{0.5}q_{75.5}) + (p_{75})({}_{0.5}q_{76})] $$

(a) $UDD \Rightarrow {}_tq_x = (t)q_x,\ 0 \le t \le 1$

$$ {}_{0.5|}q_{75} = {}_{0.5}p_{75}(q_{75.5}) = {}_{0.5}p_{75}(1 - p_{75.5}) = {}_{0.5}p_{75}[1 - ({}_{0.5}p_{75.5})({}_{0.5}p_{76})] $$
$$ = {}_{0.5}p_{75} - (p_{75})({}_{0.5}p_{76}) = (1 - {}_{0.5}q_{75}) - (1 - q_{75})(1 - {}_{0.5}q_{76}) $$
$$ = (1 - (0.5)(.05) - (1 - 0.05)(1 - (0.5)(.06)) = 0.975 - (0.95)(0.97) = 0.0535 $$ or

using

$$ {}_{0.5|}q_{75} = [({}_{0.5}p_{75})({}_{0.5}q_{75.5}) + (p_{75})({}_{0.5}q_{76})] = [(1 - {}_{0.5}q_{75})({}_{0.5}q_{75.5}) + (1 - q_{75})({}_{0.5}q_{76})] $$
$$ = [((1 - (0.5)(.05))(\tfrac{(0.5)(.05)}{1-(0.5)(.05)}) + (1 - .05)(0.5)(.06)] = 0.025 + 0.0285 = 0.0535 $$

(b) Constant force of mortality $\Rightarrow {}_tp_{x+r} = e^{-\mu t} = (e^{-\mu})^t = (p_x)^t,\ 0 \le r + t \le 1$

$$ {}_{0.5|}q_{75} = {}_{0.5}p_{75}[1 - ({}_{0.5}p_{75.5})({}_{0.5}p_{76})] $$
$$ = (0.95)^{0.5}[1 - (0.95)^{0.5}(0.94)^{0.5}] $$
$$ = (0.974679)[1 - 0.944987] = 0.05362 $$

7 Policy A:

$$ {}_{10}V = 145,000A_{50} + 5,000(IA)_{50} - (NP)\ddot{a}_{50}^{(12)} $$ where NP from

$$ 95,000A_{[40]} + 5,000(IA)_{[40]} = (NP)\ddot{a}_{[40]}^{(12)} $$

$\Rightarrow$ NP = {(95,000)(0.23041) + (5,000)(7.95835)}/(20.009 - 0.458) = 3,154.86

and ${}_{10}V$ = {(145,000)(0.32907) + (5,000)(8.55929)} - (3,154.86)(17.444 - 0.458)
= 36,923.15

Policy B:

$$ {}_{10}V = 150,000A_{50} - (NP)\ddot{a}_{50}^{(12)} $$ where NP from

$$ 100,000A_{[40]} = (NP)\ddot{a}_{[40]}^{(12)} $$

$\Rightarrow$ NP = (100,000)(0.23041)/(20.009 - 0.458) = 1,178.51

and $_{10}V$ = $(150{,}000)(0.32907) - (1{,}178.51)(17.444 - 0.458)$
= $29{,}342.33$

8 (a) Class selection: groups with different permanent attributes having different mortality

e.g. sex, male and female rates differ at all ages

(b) Spurious selection: ascribing mortality differences to groups formed by factors which are not the true causes of these differences. The influence of some confounding factor has been ignored.

e.g. Regional mortality differences actually explained by the different composition of occupations in the different regions.

(c) Time selection: within a population, mortality varies over calendar time. The effect is usually noticed at all ages and usually rates become lighter over time

e.g. ELT12 male mortality vs. ELT15male

9 (i)

$$EPV = 20{,}000 a_{\overline{68:65}} = 20{,}000(a_{68}^{m} + a_{65}^{f} - a_{68:65})$$
$$= 20{,}000(11.412 + 13.871 - 10.112) = 20{,}000(15.171) = 303{,}420$$

(ii) The office loses money if PV of payments > 320,000
i.e. if $20{,}000\, a_{\overline{n}|} > 320{,}000$ or $a_{\overline{n}|} > 16$.

At 4% p.a., $a_{\overline{26}|} = 15.9828$ and $a_{\overline{27}|} = 16.3296$ so if the office makes the 27th payment under this annuity, it incurs a loss. It therefore makes a profit so long as both lives have died before this time, with probability $_{27}q_{\overline{68:65}}$

$$_{27}q_{\overline{68:65}} = ({}_{27}q_{68}^{m})({}_{27}q_{65}^{f}) = \left(1 - \frac{l_{95}^{m}}{l_{68}^{m}}\right)\left(1 - \frac{l_{92}^{f}}{l_{65}^{f}}\right)$$
$$= \left(1 - \frac{1{,}020.409}{9{,}440.717}\right)\left(1 - \frac{3{,}300.559}{9{,}703.708}\right) = (0.891914)(0.65987) = 0.5885$$

10 (i) $$g(T) = \begin{cases} 500,000v^{T_y} & T_y > T_x \\ 0 & T_y \le T_x \end{cases}$$

(ii) $$E[g(T)] = 500,000\int_0^\infty v^t(1-{}_tp_x)\,{}_tp_y\mu_{y+t}dt$$

(iii) Lifetime of (y). If (y) dies first, no benefit is possible and if (y) dies second, SA becomes payable immediately. (x)'s lifetime is irrelevant in this context. Premium could be payable for joint lifetime of (x) and (y) but this is shorter than (y) and therefore we use (y)'s lifetime.

11 (i) $$X = \begin{cases} v^n & K_x \ge n \\ 0 & K_x < n \end{cases} \qquad Y = \begin{cases} 0 & K_x \ge n \\ v^{K_x+1} & K_x < n \end{cases}$$

$$\Rightarrow XY = 0 \text{ for all } K_x$$

$$COV(X,Y) = E[XY] - E[X]E[Y] = 0 - (A_{x:\overline{n}|}^{\;\;\;1})(A^1_{x:\overline{n}|})$$

(ii) $$VAR(X+Y) = VAR(X) + VAR(Y) + 2COV(X,Y)$$

$$= {}^2A_{x:\overline{n}|}^{\;\;\;1} - (A_{x:\overline{n}|}^{\;\;\;1})^2 + {}^2A^1_{x:\overline{n}|} - (A^1_{x:\overline{n}|})^2 - 2(A_{x:\overline{n}|}^{\;\;\;1})(A^1_{x:\overline{n}|})$$
$$= \{{}^2A_{x:\overline{n}|}^{\;\;\;1} + {}^2A^1_{x:\overline{n}|}\} - \{(A_{x:\overline{n}|}^{\;\;\;1})^2 + (A^1_{x:\overline{n}|})^2 + 2(A_{x:\overline{n}|}^{\;\;\;1})(A^1_{x:\overline{n}|})\}$$
$$= \{{}^2A_{x:\overline{n}|}^{\;\;\;1} + {}^2A^1_{x:\overline{n}|}\} - \{(A_{x:\overline{n}|}^{\;\;\;1}) + (A^1_{x:\overline{n}|})\}^2$$
$$= {}^2A_{x:\overline{n}|} - (A_{x:\overline{n}|})^2$$

12 (i) $$P\ddot{a}_{[40]:\overline{20}|} = 75,000A_{[40]:\overline{20}|}^{\;\;\;\;\;\;1} = 75,000v^{20}\,{}_{20}p_{[40]}$$
$$\Rightarrow P(13.930) = (75,000)(0.45639)(0.94245)$$
$$\Rightarrow P = 32,259.45/13.93 = 2,315.83$$

Mortality profit = Expected Death Strain – Actual Death Strain

$$DSAR = 0 - {}_{15}V = -(75,000A_{55:\overline{5}|}^{\;\;\;\;1} - P\ddot{a}_{55:\overline{5}|})$$
$$= -(75,000v^5\,{}_5p_{55} - P\ddot{a}_{55:\overline{5}|})$$
$$= -\{(75,000)(0.82193)(0.97169) - (2,315.83)(4.585)\} = -49,281.51$$

EDS = (q_{54})(500)(-49,281.51) = (0.003976)(500)(-49,281.51) = -97,971.64
ADS = (3)(-49,281.51) = -147,844.53

Mortality Profit = $-$97,971.64 $-$ ($-$147,844.53) = 49,872.89 profit.

(ii) We expected $500q_{54}$ = 1.988 deaths. Actual deaths were 3. With pure endowments, the death strain is negative because no death claim is paid and there is a release of reserves to the company on death. In this case, more deaths than expected means this release of reserves is greater than required by the equation of equilibrium and the company therefore makes a profit.

13 (i) $P\ddot{a}_{30:\overline{35}|} = 200,600A_{30:\overline{35}|} - 400A_{30:\overline{35}|}^{\;\;\,1} + (0.02)P\ddot{a}_{30:\overline{35}|} - 0.02P + 300 + (0.5)(P)$

Expected present value of premiums:

$$P\ddot{a}_{30:\overline{35}|} = 15.150P$$

EPV of benefit and claim expenses:

$$A_{30:\overline{35}|} = 0.14246$$

$$A_{30:\overline{35}|}^{\;\;\,1} = v^{35}\,{}_{35}p_{30} = (0.13011)(0.88877) = 0.11563$$

$\Rightarrow$ EPV of benefits and claim expenses

= (200,600)(0.14246) $-$ (400)(0.11563)

= 28,577.48 $-$ 46.25 = 28,531.23

EPV of remaining expenses:

[(0.02)(15.150P)] $-$ 0.02P + 0.5P + 300 = 0.783P + 300

Equation of value:

15.150P = 28,531.23 + 300 + 0.783P $\Rightarrow$14.367P = 28,831.23

$\Rightarrow$ P = 2,006.77 per annum = 2,007 p.a.

(ii) GFLRV = $\begin{cases} 200,600v^{K_{55}+1} - (0.98)(2,007)(\ddot{a}_{\overline{K_{55}+1}|}) & K_{55} < 10 \\ 200,200v^{10} - (0.98)(2,007)(\ddot{a}_{\overline{10}|}) & K_{55} \geq 10 \end{cases}$

(iii) $${}_{25}V^{retro} = \frac{1}{v^{25}\,{}_{25}p_{30}}\{0.98P\ddot{a}_{30:\overline{25|}} - 0.48P - 300 - 200{,}600A^{1}_{30:\overline{25|}}\}$$

$$v^{25}\,{}_{25}p_{30} = (0.37512)(0.96298) = 0.36123$$

$$\ddot{a}_{30:\overline{25|}} = \ddot{a}_{30} - v^{25}\,{}_{25}p_{30}\ddot{a}_{55} = 21.834 - (0.36123)(15.873) = 16.100$$

$$A^{1}_{30:\overline{25|}} = A_{30} - v^{25}\,{}_{25}p_{30}A_{55} = 0.16023 - (0.36123)(0.38950) = 0.01953$$

$${}_{25}V^{retro} = \frac{1}{0.36123}\{[2{,}007][(0.98)(16.100) - (0.48)] - 300 - (200{,}600)(0.01953)\}$$

$$= \frac{1}{0.36123}\{30{,}703.09 - 300 - 3{,}917.72\} = 73{,}319.96$$

(iv) It would have been larger. At 6% both would be the same but

$$V^{retro}_{@4\%} < V^{retro}_{@6\%} = V^{pro}_{@6\%} < V^{pro}_{@4\%}$$

since retrospective reserves are accumulating premiums in excess of claims and expenses and lower interest leads to lower reserves but prospective reserves are meeting the excess of future benefits claims over future premiums and lower interest leads to higher reserves.

14 (i), (ii) and (iii)

Transition probabilities not given explicitly are

t	p^{HH}_{63+t}	p^{CC}_{63+t}	p^{DD}_{63+t}
0	0.94	0.75 (not needed)	1.00 (not needed)
1	0.91	0.67	1.00

Outcome	*PV of Cash flow (000's)*	*PV Ben*	*(PV Ben)^2*	*Prob.*	*Prob.*	*E[PVB]*	*E[PVB^2]*
HH	0	0.00	0.00	0.94*0.91	0.8554	0	0
HC	$60v^2$	49.59	2458.85	0.94*0.06	0.0564	2.79669	138.67905
HD	$100v^2$	82.64	6830.13	0.94*0.03	0.0282	2.33058	192.60979
CC	$60v$	54.55	2975.21	0.04*0.67	0.0268	1.46182	79.73554
CD	$60v+40v^2$	87.60	7674.34	0.04*0.33	0.0132	1.15636	101.30128
DD	$100v$	90.91	8264.46	0.02	0.02	1.81818	165.28926
Total				1		9.56364	677.61492

(iv) Mean $= (1{,}000)(9.56364) = 9{,}563.64$

Var. $= (1{,}000)^2\{(677.61492 - (9.56364)^2\}$

$= 586{,}151{,}710 = (24{,}210.57)^2$

(v) Var.(profit) = Var.(*SP* - *EPV*(bens)) = Var.(*EPV*(bens))

EPV(profit) = (10,000)(*SP* - 9,563.64) = 10,000*SP* - 95,636,400

Var.(profit) = Var.(*SP* - *EPV*(bens)) = Var.(*EPV*(bens))

For 10,000 independent policies,

Var.(profit) = (10,000)(586,151,710) = $(2{,}421{,}057)^2$

St. Dev.(profit) = 2,421,057

We need *SP* so Prob.(profit > 0) = 0.95

$$\Rightarrow \Pr.\left(\frac{\text{profit} - (EPV(\text{profit})}{\text{StDev(profit)}} > \frac{0 - (10{,}000SP - 95{,}636{,}400)}{2{,}421{,}057}\right) = 0.95$$

Assuming profit is normally distributed

$$\Rightarrow \Pr.\left(z > \frac{95{,}636{,}400 - 10{,}000SP}{2{,}421{,}057}\right) = 0.95 \Rightarrow \Phi\left(\frac{95{,}636{,}400 - 10{,}000SP}{2{,}421{,}057}\right) = 0.05$$

$$\Rightarrow \left(\frac{95{,}636{,}400 - 10{,}000SP}{2{,}421{,}057}\right) = \Phi^{-1}(0.05) = -1.6449$$

$$\Rightarrow SP = \left(\frac{95{,}636{,}400 + (1.6449)(2{,}421{,}057)}{10{,}000}\right) = 9{,}961.88 = 9{,}962$$

END OF EXAMINERS' REPORT

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

April 2008

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M A Stocker
Chairman of the Board of Examiners

June 2008

1 The probability that an ultimate life age 40 dies between 45 and 55 (all exact)

$$ {}_{5|10}q_{40} = \frac{(l_{45} - l_{55})}{l_{40}} = \frac{(9801.3123 - 9557.8179)}{9856.2863} = 0.024704 $$

2 The following are three types of guaranteed reversionary bonuses. The bonuses are usually allocated annually in arrears, following a valuation.

Simple – the rate of bonus each year is a percentage of the initial (basic) sum assured under the policy. The effect is that the sum assured increases linearly over the term of the policy.

Compound – the rate of bonus each year is a percentage of the initial (basic) sum assured and the bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy.

Super compound – two compound bonus rates are declared each year. The first rate (usually the lower) is applied to the initial (basic) sum assured. The second rate is applied to bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy. The sum assured usually increases more slowly than under a compound allocation in the earlier years and faster in the later years.

(Note: credit given if special reversionary bonus mentioned)

3 The expected cost of paying benefits usually increases as the life ages and the probability of a claim by death increases. In the final year the probability of payment is large, since the payment will be made if the life survives the term, and for most contracts the probability of survival is large.

Level premiums received in the early years of a contract are more than enough to pay the benefits that fall due in those early years, but in the later years, and in particular in the last year of an endowment assurance policy, the premiums are too small to pay for the benefits. It is therefore prudent for the premiums that are not required in the early years of the contract to be set aside, or reserved, to fund the shortfall in the later years of the contract.

If premiums received that were not required to pay benefits were spent by the company, perhaps by distributing to shareholders, then later in the contract the company may not be able to find the money to pay for the excess of the cost of benefits over the premiums received.

(Credit given for other valid points)

4 $Value =$

$$50,000\int_0^{20} v^t \cdot {}_t p_{40}^{hh} (\mu_{40+t} + \sigma_{40+t})dt$$

$$= 50,000\int_0^{20} e^{-\ln(1.05)t} \cdot {}_t p_{40}^{hh} 0.008dt$$

$${}_t p_{40}^{hh} = {}_t p_{40}^{\overline{hh}} = \exp(-\int_{40}^{40+t} (\mu_s + \sigma_s)ds)$$

$$= \exp(-\int_{40}^{40+t} 0.008ds)$$

$$= e^{-0.008t}$$

$Therefore, value =$

$$50,000 * 0.008\int_0^{20} e^{-\ln(1.05)t} . e^{-0.008t} dt$$

$$= 400\int_0^{20} e^{-0.05679t} dt$$

$$= 400 * [-e^{-.05679t} / .05679]_0^{20}$$

$$= 400 * (-5.65531 + 17.60873)$$

$$= 4781.4$$

5 (a) Define random variables T_x and T_y for the complete duration of life for the lives aged x and y.

Define a random variable $\overline{Z}$ for the value of the reversionary annuity, which has the following definition:

$\overline{Z} = \bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{T_x}|}$ if $T_y > T_x = 0$ otherwise

$\overline{Z} = \bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{T_{xy}}|}$ where T_{xy} is a random variable for the duration to the first death

$$\overline{Z} = \frac{(1 - v^{T_y})}{\delta} - \frac{(1 - v^{T_{xy}})}{\delta} = \frac{(v^{T_{xy}} - v^{T_y})}{\delta}$$

(b) $$E[\overline{Z}] = \frac{(E[v^{T_{xy}}] - E[v^{T_y}])}{\delta} = \frac{(\overline{A}_{xy} - \overline{A}_y)}{\delta}$$

6 The probability that the life age 25 survives 10 years
= 9894.4299 / 9953.6144 = 0.994054
The probability that the life age 30 survives 10 years
= 9856.2863 / 9925.2094 = 0.993056

There are four possible outcomes:

Outcome	*Expression for value*	*value*
Both survive	$V^{10}\times 0.994054\times 0.993056\times 15000/3$	3672.67
Only 25 survives	$V^{10}\times 0.994054\times(1-0.993056)\times 15000/2$	38.52
Only 30 survives	$V^{10}\times(1-0.994054)\times 0.993056\times 15000/2$	32.95
Neither survive	$V^{10}\times(1-0.994054)\times(1-0.993056)\times 15000$	0.46

Total value is 3744.60

7 Value of future service benefits

$$\frac{1}{60}.S.\frac{({}^{z}\overline{R}^{ra}_{40}+{}^{z}\overline{R}^{ia}_{40})}{{}^{s}D_{40}}=\frac{1}{60}.S.\frac{(2884260+887117)}{25059}=2.5S$$

Value of contributions of k% of future salary

$$\frac{k}{100}.S.\frac{{}^{s}\overline{N}_{40}}{{}^{s}D_{40}}=\frac{k}{100}.S.\frac{(363573)}{25059}=k/100*14.5S$$

Equating these values give $k = 17.3$

8 (i) The uniform distribution of deaths is consistent with an assumption that

$$_s q_x = s.q_x$$

$$\begin{aligned}
{}_{t-s}q_{x+s} &= (1 - {}_{t-s}p_{x+s}) \\
&= (1 - \frac{{}_t p_x}{{}_s p_x}) \\
&= (1 - \frac{(1 - {}_t q_x)}{(1 - {}_s q_x)}) \\
&= (1 - \frac{(1 - tq_x)}{(1 - sq_x)}) \\
&= \frac{(t-s).q_x}{(1 - s.q_x)}
\end{aligned}$$

(ii) (a) $_{0.5}q_{62.25} = 0.5q_{62}/(1 - 0.25q_{62}) = 0.5\times0.00355/(1 - 0.25*0.00355)$
$= 0.001777$

(b) The assumption of a constant force of mortality requires μ to be derived from the expression $p = e^{-\mu}$. p_{62}=0.99645 => μ_{62}=0.003556

$${}_{t-s}p_{x+s} = e^{-(t-s)\mu} = e^{-0.5x0.003556} = 0.998224$$
$${}_{t-s}q_{x+s} = 0.001776$$

9 (i) Occupation – as some occupations have a regional distribution

Housing – as quality of housing will be impacted by occupationally influenced income levels

Climate – different locations having different climates

Using location is a spurious form of class selection as it disguises the underlying causes

(ii) Actual deaths (location A) = 9
Actual deaths (location B) = 11

The calculation of the expected deaths is

		Location A		*Location B*	
Age	*Standard Mortality Rate*	*Initial Exposed to risk*	*Number of deaths*	*Initial Exposed to risk*	*Number of deaths*
60	0.01392	100	1.4	200	2.8
61	0.01560	175	2.7	150	2.3
62	0.01749	190	3.3	170	3.0
63	0.01965	210	4.1	100	2.0
Total			11.5		10.1

The SMRs are therefore

Location A = 9/11.5 = 0.78
Location B = 11/10.1 = 1.09

10 (a) wife
value = $5000(a_{55}^{(12)} - a_{60:55}^{(12)}) = 5000(18.210 - 14.756) = 17,270$
Note no effect of monthly payments

(b) grandson
value =
$2000(a_{\overline{8}|}^{(12)} - a_{60:55:\overline{8}|}^{(12)})$

$$a_{\overline{8}|}^{(12)} = \frac{(1-v^8)}{i^{(12)}} = 6.7327x\, 0.04 \big/ 0.039285 = 6.855$$

$$a_{60:55:\overline{8}|}^{(12)} = a_{60:55}^{(12)} - v^8\, {}_8p_{60} \cdot {}_8p_{55}\, a_{68:63}^{(12)}$$

$$= a_{60:55} + 11/24 - v^8\, {}_8p_{60} \cdot {}_8p_{55}(a_{68:63} + 11/24)$$

$$= (14.756 - 1) + 11/24 - v^8 \frac{9440.717}{9826.131}\frac{9775.888}{9917.623}(11.372 - 1 + 11/24)$$

$$= 6.721$$

Therefore value = 2000(6.855-6.721) = 268

Total value = 17270 + 268 = 17,538

11 (i) Let P be the annual premium. Then:

EPV of premiums:

$$P\ddot{a}_{[50]:\overline{10}|} = 7.698P$$

EPV of benefits:

$$\frac{75,000}{(1+b)} \times (1.06)^{1/2} \{q_{[50]}(1+b)v \ \ +_1|q_{[50]}(1+b)^2 v^2$$

$$+....+_9|q_{[50]}(1+b)^{10} v^{10}\} + 75,000\,{}_{10}p_{[50]}(1+b)^{10} v^{10}$$

where $b = 0.0192308$

$$= \frac{75,000}{(1+b)} \times (1.06)^{1/2} A^{1}_{[50]:\overline{10}|} @ i' + 75,000 \times {}_{10}p_{[50]} \times \frac{1}{(1+i')^{10}}$$

$$= \frac{75,000}{1.0192308} \times (1.06)^{1/2} \times (.68007 - .64641) + 75,000 \times .64641 = 2,550.091 + 48,480.75$$

$$= 51,030.84$$

where $i' = \dfrac{1.06}{1+b} - 1 = 0.04$

EPV of other expenses:

$$.5 \times P + 350 + 0.05 \times P(\ddot{a}_{[50]:\overline{10}|} - 1) = 0.8349P + 350$$

Equation of value gives $7.698P = 51{,}030.84 + 0.8349P + 350$

and $P = £7{,}486.54$

(ii) Gross premium prospective reserve (calculated at 6%) is given by:

EPV of benefits and expenses less EPV of premiums

EPV of benefits and expenses:

$$= \frac{82,494.3}{(1+b)} \times (1.06)^{1/2} A^{1}_{55:\overline{5}|} @ i' + 82,494.3 \times {}_5p_{55} \times \frac{1}{(1+i')^5} + 0.05P\ddot{a}_{55:\overline{5}|}$$

$$= \frac{82,494.3}{1.0192308} \times (1.06)^{1/2} \times (.82365 - .79866) @ i' + 82,494.3 \times 0.79866 + 0.05 \times 7486.54 \times 4.423$$

$$= 2,082.43 + 65,884.90 + 1,655.65 = 69,622.98$$

EPV of premiums:

$$P\ddot{a}_{55:\overline{5}|} = 4.423P = 33{,}112.97$$

=> Gross premium prospective reserve = £36,510.00

12 (i) (a) Annual premium for pure endowment with £50,000 sum assured given by:

$$P^{PE} = \frac{50{,}000}{\ddot{a}_{50:\overline{10}|}} \times {}_{10}p_{50} \times v^{10} = \frac{50{,}000}{8.314} \times 0.64601 = 3885.10$$

Annual premium for term assurance with £50,000 sum assured given by:

$$P^{TA} = P^{EA} - P^{PE} = \frac{50{,}000A_{50:\overline{10}|}}{\ddot{a}_{50:\overline{10}|}} - P^{PE}$$

$$= \frac{50{,}000 \times 0.68024}{8.314} - 3885.10 = 205.83$$

Reserves at the end of the second year:

for pure endowment with £50,000 sum assured given by:

$${}_2V^{PE} = 50{,}000 \times {}_8p_{52} \times v^8 - P^{PE}\ddot{a}_{52:\overline{8}|}$$

$$= 50{,}000 \times 0.70246 - 3885.10 \times 6.910 = 35123.0 - 26846.04 = 8276.96$$

for term assurance with £50,000 sum assured given by:

$${}_2V^{TA} = {}_2V^{EA} - {}_2V^{PE}$$

$$= 50{,}000A_{52:\overline{8}|} - (3885.1 + 205.83)\ddot{a}_{52:\overline{8}|} - 8276.96$$

$$= 50{,}000 \times 0.73424 - 4090.93 \times 6.91 - 8276.96$$

$$= 166.71$$

Sums at risk:

Pure endowment: DSAR = 0 − 8,276.96 = −8,276.96
Term assurance: DSAR = 50,000 – 166.71 = 49,833.29

(b) Mortality profit = EDS – ADS

For term assurance

$$EDS = 4995 \times q_{51} \times 49,833.29 = 4995 \times .002809 \times 49,833.29 = 699,208.65$$

$$ADS = 10 \times 49,833.29 = 498,332.90$$

mortality profit = 200,875.75

For pure endowment

$$EDS = 4995 \times q_{51} \times -8,276.96 = 4995 \times .002809 \times -8,276.96 = -116,133.65$$

$$ADS = 10 \times -8,276.96 = -82,769.60$$

mortality profit = −33,364.05

Hence, total mortality profit = £167,511.70

(ii) (a) The actual mortality profit would remain as that calculated in (i) (b).

(b) The variance of the benefits would be lower than that calculated in (i).

In this case, the company would not pay out benefits under both the PE and the TA but will definitely pay out one of the benefits. Under the scenario in (i), the company could pay out all the benefits (if all the TA policyholders die and the PE policyholders survive). Alternatively, they could pay out no benefits at all (if all the TA policyholders survive and the PE policyholders immediately die).

13

Annual premium	750.00	Allocation % (1st yr)	25.0%
Risk discount rate	8.5%	Allocation % (2nd yr +)	102.50%
Interest on investments	6.5%	Man charge	1.0%
Interest on sterling provisions	5.5%	B/O spread	5.0%
Minimum death benefit	3000.00		

	£	% prm	Total
Initial expense	150	10.0%	225
Renewal expense	65	2.5%	83.75

(i) Multiple decrement table

x	q_x^d	q_x^s
50	0.001971	0.1
51	0.002732	0.1
52	0.003152	0.1
53	0.003539	0.0

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	${}_{t-1}(ap)$
50	0.001971	0.09980	0.898226	1.000000
51	0.002732	0.09973	0.897541	0.898226
52	0.003152	0.09968	0.897163	0.806195
53	0.003539	0.00000	0.996461	0.723288

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
value of units at start of year	0.000	187.806	968.018	1790.635
alloc	187.500	768.750	768.750	768.750
B/O	9.375	38.4375	38.4375	38.4375
interest	11.578	59.678	110.392	163.862
management charge	1.897	9.778	18.087	26.848
value of units at year end	187.806	968.018	1790.635	2657.961

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
unallocated premium	562.500	−18.750	−18.750	−18.750
B/O spread	9.375	38.4375	38.4375	38.4375
expenses	225.000	83.750	83.750	83.750
interest	19.078	−3.523	−3.523	−3.523
man charge	1.897	9.778	18.087	26.848
extra death benefit	5.543	5.551	3.812	1.210
Extra maturity benefit	0.000	0.000	0.000	264.855
end of year cashflow	362.307	−63.359	−53.311	−306.804

probability in force	1	0.898226	0.806195	0.723288
discount factor	0.921659	0.849455	0.782908	0.721574
expected p.v. of profit	91.809			
premium signature	750.000	620.894	513.620	424.701
expected p.v. of premiums	2309.215			
profit margin	3.98%			

(ii) (a) To calculate the expected provisions at the end of each year we have (utilising the end of year cashflow figures and decrement tables in (i) above):

$$_3V = \frac{306.804}{1.055} = 290.809$$
$$_2V \times 1.055 - (ap)_{52} \times {}_3V = -53.311 \Rightarrow {}_2V = 297.833$$
$$_1V \times 1.055 - (ap)_{51} \times {}_2V = -63.359 \Rightarrow {}_1V = 313.437$$

These need to be adjusted as the question asks for the values in respect of the beginning of the year. Thus we have:

Year 3 $290.809(ap)_{52} = 260.903$
Year 2 $297.833(ap)_{51} = 267.318$
Year 1 $313.437(ap)_{50} = 281.538$

(b) Based on the expected provisions calculated in (a) above, the cash flow for years 2, 3 and 4 will be zeroised whilst year 1 will become:

$362.307 - 281.538 = 80.769$

Hence the table below can now be completed for the revised profit margin.

revised end of year cash flow	80.769	0	0	0
probability in force	1	0.898226	0.806195	0.723288
discount factor	0.921659	0.849455	0.782908	0.721574
expected p.v. of profit	74.442			
profit margin	3.22%			

END OF EXAMINERS' REPORT

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

September 2008

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R D Muckart
Chairman of the Board of Examiners

November 2008

1 Let t equal future lifetime. Lower quartile means that 25% of people have future lifetime less than t.

$$ {}_tq_{25} = 0.25 \Rightarrow \frac{l_{25+t}}{l_{25}} = 0.75 \qquad \frac{l_{25+t}}{98,797} = 0.75 \Rightarrow l_{25+t} = 74,098 $$

$$ l_{73} = 74,287 \quad l_{74} = 72,048, $$

So 73–25 = 48 is lower quartile future lifetime to nearest integer.

2 Profit margin = (EPV profit / EPV premiums)

EPV profit = $-250v + 150v^2 + 200v^3 = 38.72$

EPV premiums = $500(1 + {}_1p_{57}v + {}_2p_{57}v^2) = 500(1 + 0.8878125 + 0.7876546)$
= 1,337.73

Profit margin = 38.72/1,337.37 = 2.89%

3 £75,000 represents earnings from 1/7/2007 to 1/7/2008

i.e. from age 46.25 to 47.25

2008 is the period from age 46.75 to 47.75

2008 expected earnings = $75,000\dfrac{s_{46.75}}{s_{46.25}}$

$(s_{46} + 3s_{47})$ is a satisfactory alternative to the numerator above and $(3s_{46} + s_{47})$ a satisfactory alternative for the denominator.

Alternative: $75,000\{0.5 + 0.5(\dfrac{s_{47.25}}{s_{46.25}})\}$

4 (i) ${}_nq_{\overline{xy}}$

(ii) $A^{1}_{x:\overline{n}|}$

(iii) ${}_{n|m-n}q_x$

(iv) $\mu_{x+t:y+t}$ or $\mu_{x+t}+\mu_{y+t}$

(v) $\ddot{a}_{x:\overline{n}|}$

5 It is the accumulation of an n-year annuity due i.e. the expected fund per survivor after *n* years, from a group of people, initially aged *x*, who each put 1 at the start of each of the *n* years, if they are still alive, into a fund earning interest at rate *i* per annum.

$$\ddot{s}_{50:\overline{20}|}=\frac{\ddot{a}_{50:\overline{20}|}}{v^{20}\,{}_{20}p_{50}}$$
$$=\frac{1}{v^{20}\,{}_{20}p_{50}}(\ddot{a}_{50}-v^{20}\,{}_{20}p_{50}\ddot{a}_{70})$$
$$=\frac{1}{(0.45639)(0.82928)}(17.444)-10.375=35.715$$

6

x	$q_{[x]}$	$q_{[x-1]+1}$	$q_{[x-2]+2}$	$q_{[x-3]+3}$	q_x
62	**0.0045**	0.006	0.009	0.018	0.018
63	0.005	**0.006667**	0.01	0.02	0.02
64	0.0055	0.007333	**0.011**	0.022	0.022

EPV premiums = 30,000$\{1 + v^*p_{[62]} + v^{2*}\, p_{[62]}{}^*p_{[62]+1}\}$

= 30,000$\{1 + v^*(1 - 0.0045) + v^{2*}(1 - 0.0045)^*(1 - 0.006667)\}$

= (30,000)(2.781742) = 83,452.27

EPV benefits = 100,000$\{v^*q_{[62]} + v^{2*}p_{[62]}{}^*q_{[62]+1} + v^{3*}p_{[62]}{}^*p_{[62]+1}{}^*(q_{[62]+2} + p_{[62]+2})\}$ = 80,592.50

EPV profit = 83,452.27 - 80,592.50 = 2,859.77

Cash flow approach:

Year	*Premium*	*Interest*	*Death Cost*	*Maturity Cost*	*Profit Vector*	*Profit Signature*	*NPV*
1	30,000.00	2,250.00	450.00		31,800.00	31,800.00	29,581.40
2	30,000.00	2,250.00	666.67		31,583.33	31,441.21	27,207.10
3	30,000.00	2,250.00	1,100.00	98,900.00	-67,750.00	–66,995.49	–53,928.73
							2,859.77

7 (i) $$(aq)_x^{\alpha} = q_x^{\alpha}\{1-\tfrac{1}{2}(q_x^{\beta}+q_x^{\gamma})+\tfrac{1}{3}(q_x^{\beta}q_x^{\gamma})\}$$

(ii) $$(aq)_x^{\alpha} = \int_0^1 {}_t p_x^{\alpha}\, {}_t p_x^{\beta}\, {}_t p_x^{\gamma} \mu_{x+t}^{\alpha} dt$$

$${}_t p_x^{\alpha} = 1 - t^2 q_x^{\alpha} \Rightarrow {}_t p_x^{\alpha}\mu_{x+t}^{\alpha} = -\frac{d}{dt}\, {}_t p_x^{\alpha} = 2tq_x^{\alpha}$$

With β and γ uniformly distributed, then

$$(aq)_x^{\alpha} = \int_0^1 {}_t p_x^{\alpha}\, {}_t p_x^{\beta}\, {}_t p_x^{\gamma} \mu_{x+t}^{\alpha} dt = \int_0^1 2tq_x^{\alpha}(1-tq_x^{\beta})(1-tq_x^{\gamma})dt$$

$$= q_x^{\alpha}\int_0^1 \{2t - 2t^2(q_x^{\beta}+q_x^{\gamma}) + 2t^3(q_x^{\beta}q_x^{\gamma})\}1dt$$

$$= q_x^{\alpha}\{1-\frac{2}{3}(q_x^{\beta}+q_x^{\gamma})+\frac{2}{4}(q_x^{\beta}q_x^{\gamma})\}$$

8 (i) The Death Strain at risk per policy is
[0 – (payment due 31.12 + reserve @ 31.12] = $-25,000\ddot{a}_{66}$

Expected DS =
$-q_{65} * 1,000 * 25,000\ddot{a}_{66} = -(0.004681)(25,000,000)(14.494) = -1,696,160$

Actual DS = $-5 * 25,000\ddot{a}_{66} = -1,811,750$

Profit = EDS – ADS = -1,696,160 + 1,811,750 = 115,590 profit

(ii) We expected 4.681 deaths and had more than this with 5. There is no death benefit, just a release of reserves on death, so more deaths than expected leads to profit.

9 EPV of benefits: $\dfrac{40,000}{60}\dfrac{{}^{z}\overline{R}^{ra}_{35}+{}^{z}\overline{R}^{ia}_{35}}{{}^{s}D_{35}}=\dfrac{40,000}{60}\dfrac{3,524,390+1,187,407}{31,816}=98,730$

PV of contribution:

$$40,000\frac{(0.04)\,{}^{s}\overline{N}_{35}+(0.01)\,{}^{s}\overline{N}_{50}}{{}^{s}D_{35}}=40,000\frac{(0.04)502,836+(0.01)163,638}{31,816}=27,345$$

Employer's proportion = (98,730 - 27345)/27,345 = 2.61 times employee's contribution.

10 Variance of $\bar{a}_{\overline{T_x}|}=\dfrac{{}^{2}\overline{A}_x-(\overline{A}_x)^2}{\delta^2}$

$$\overline{A}_x=\int_0^\infty e^{-\delta t}\,{}_tp_x\mu_{x+t}dt=\int_0^\infty e^{-\delta t}e^{-\mu t}\mu dt=\mu\int_0^\infty e^{-t(\delta+\mu)}dt=-\frac{\mu}{\mu+\delta}(e^{-t}\Big|_0^\infty)=-\frac{\mu}{\mu+\delta}(0-1)$$
$$=\frac{\mu}{\mu+\delta}=\frac{0.02}{0.07}$$

Similarly, ${}^{2}\overline{A}_x=\displaystyle\int_0^\infty e^{-2\delta t}\,{}_tp_x\mu_{x+t}dt=\int_0^\infty e^{-\delta t}e^{-\mu t}\mu dt=\frac{\mu}{\mu+2\delta}=\frac{0.02}{0.12}$

Variance of $\bar{a}_{\overline{T_x}|}=\dfrac{\frac{0.02}{0.12}-(\frac{0.02}{0.07})^2}{0.05^2}=34.01$

Alternatively

Variance of $X=E[X^2]-\{E[X]\}^2$

Here X= $\bar{a}_{\overline{T_x}|}=\dfrac{1-e^{-\delta T_x}}{\delta}$

$$E[X]=\int_0^\infty\frac{1-e^{-\delta t}}{\delta}\,{}_tp_x\mu_{x+t}dt=\int_0^\infty\frac{1-e^{-\delta t}}{\delta}e^{-\mu t}\mu dt=\frac{1}{\delta}\int_0^\infty(e^{-\mu t}\mu)-(e^{-(\delta+\mu)t}\mu)dt$$
$$=\frac{1}{\delta}\{(-e^{-\mu t}\Big|_0^\infty)-(\frac{\mu}{\delta+\mu})(-e^{-(\delta+\mu)t}\Big|_0^\infty)\}=\frac{1}{\delta}\{(0-(-1))-(\frac{\mu}{\delta+\mu})(0-(-1))\}$$
$$=\frac{1}{\delta}\{1-(\frac{\mu}{\delta+\mu})\}=\frac{1}{\delta}\{\frac{\delta+\mu-\mu}{\delta+\mu}\}=\frac{1}{\delta+\mu}=\frac{1}{0.07}=14.2857$$

$$E[X^2] = \int_0^\infty \left(\frac{1-e^{-\delta t}}{\delta}\right)^2 {}_t p_x \mu_{x+t} dt = \int_0^\infty \left(\frac{1-2e^{-\delta t}+e^{-2\delta t}}{\delta^2}\right) e^{-\mu t} \mu dt$$

$$= \frac{1}{\delta^2} \int_0^\infty (e^{-\mu t}\mu) - (2e^{-(\delta+\mu)t}\mu) + (e^{-(2\delta+\mu)t}\mu) dt$$

$$= \frac{1}{\delta^2}\{(-e^{-\mu t}\Big|_0^\infty) - (2)\left(\frac{\mu}{\delta+\mu}\right)(-e^{-(\delta+\mu)t}\Big|_0^\infty) + \left(\frac{\mu}{2\delta+\mu}\right)(-e^{-(2\delta+\mu)t}\Big|_0^\infty)\}$$

$$= \frac{1}{\delta^2}\{(0-(-1)) - 2\left(\frac{\mu}{\delta+\mu}\right)(0-(-1)) + \left(\frac{\mu}{2\delta+\mu}\right)(0-(-1))\}$$

$$= \frac{1}{\delta^2}\{1 - \left(\frac{2\mu}{\delta+\mu}\right) + \left(\frac{\mu}{2\delta+\mu}\right)\} = \frac{1}{0.05^2}\{1 - \frac{0.04}{0.07} + \frac{0.02}{0.12}\} = 238.0952$$

Variance = $238.0952-(14.2857)^2 = 34.01$

11 Class selection

People with same age definition will have different underlying mortality due to particular permanent attributes, e.g. sex. The existence of such classes would be certainly found in these data: e.g. male / female smoker / non-smoker, people having different occupational and/or social backgrounds, etc.

Solution would be to subdivide the data according to the nature of the attribute.

Time selection

Where mortality is changing over calendar time, people of the same age could experience different levels of mortality at different times. This might well be a problem here, as data from as much as ten years apart are being combined.

Solution would be to subdivide the data into shorter time periods.

Temporary initial selection

Mortality changes with policy duration and the combination of subgroups of policyholders with different durations into a single sample will cause heterogeneity. Lives accepted for insurance have passed a medical screening process. The longer that has elapsed since screening (i.e. since entry) the greater the proportion of lives who may have developed impairments since the screening date and hence the higher the mortality. Mortality rates would then be expected to rise with policy duration, and hence result in heterogeneous data.

The solution would be to perform a select mortality investigation, that is one in which the data are subdivided by policy duration as well as by age.

Self selection

By purchasing a particular product type, policyholders are putting themselves in a particular group. People expecting lighter than normal mortality might purchase annuities and experience better mortality rates than, for example, term assurance buyers.

The solution would be to subdivide the data by product type.

12 (i) (a) 100,000{1 + (20)(0.045)) = 190,000

(b) $100{,}000(1.0384615)^{20} = 212{,}720$

(c) $100{,}000\{1 + 0.03\, s_{\overline{20|}}^{6\%}\} = 210{,}357$

(ii) EPV maturity benefits:

$$100{,}000 A_{[45]:\overline{20|}}^{\;\;\;\;\;\;1} = 100{,}000 v^{20}\,{}_{20}p_{[45]} \; @ \; \frac{1.08}{1.0384615} - 1 = 4\%$$

= 100,000*(0.45639)(8,821.2612/9,798.0837) = 0.41089*100,000 = 41,089

EPV death benefits:

$$\frac{100{,}000}{1.0384615} A^{1}_{[45]:\overline{20|}} @ 4\% = \frac{100{,}000}{1.0384615}\left(A_{[45]:\overline{20|}} - A_{[45]:\overline{20|}}^{\;\;\;\;\;\;1}\right)$$

$$= \frac{100{,}000}{1.0384615}(0.46982 - 0.41089)$$

= (100,000)(0.05893 / 1.0384615) = 0.05675*100,000 = 5,675

EPV total benefits = 41,089 + 5,675 = 46,764

(iii) Making appropriate adjustments to (ii)

(a) 1.0384615*5,675+41,089 = 46,982

(b) $(1.08)^{0.5}$*5,675+41,089 = 46,987

(c) = $(1.04)^{0.5}$*1.0384615*5,675 + 41,089 = 47,099

Alternatively, just starting a fresh for each condition:

(a) $A_{[45]:\overline{20|}}$ at 4%

(c) $\bar{A}_{[45]:\overline{20|}}$ at 4% = $\bar{A}^{1}_{[45]:\overline{20|}} + A_{[45]:\overline{20|}}^{\ \ \ \ 1} = \{(1.04)^{0.5} A^{1}_{[45]:\overline{20|}}\} + A_{[45]:\overline{20|}}^{\ \ \ \ 1}$

13 Let P be the monthly premium

$$12P\ddot{a}^{(12)}_{65:60:\overline{10|}} = 350 + 0.025P(12\ddot{a}^{(12)}_{65:60:\overline{10|}} - 1) + 100,000A^{\ \ 1}_{\overbrace{65:60}:\overline{10|}} + 20,000\,{}_{10|}\ddot{a}_{\overline{65:60}}:$$

$$\ddot{a}_{65:60:\overline{10|}} = \ddot{a}_{65:60} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}\ddot{a}_{75:70}$$

$$= 12.682 - (0.67556)(0.87120)(0.95372)(8.357)$$

$$= 12.682 - (0.56131)8.357 = 7.991$$

$$\ddot{a}^{(12)}_{65:60:\overline{10|}} = \ddot{a}_{65:60:\overline{10|}} - \frac{11}{24}(1 - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60})$$

$$= 7.991 - (0.458)(1 - 0.56131)$$

$$= 7.991 - 0.201 = 7.790$$

$$A^{\ \ 1}_{\overbrace{65:60}:\overline{10|}} = A_{65:60:\overline{10|}} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60} = 1 - d\ddot{a}_{65:60:\overline{10|}} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}$$

$$= 1 - \frac{0.04}{1.04}(7.991) - 0.56131 = 0.13134$$

$$A^{\ \ 1}_{\overbrace{65:60}}{}^{\overline{10|}} = A_{65:60} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}A_{75:70} = 1 - d\ddot{a}_{65:60} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}(1 - d\ddot{a}_{75:70})$$

or

$$= 1 - \frac{0.04}{1.04}(12.682) - (0.67556)(0.87120)(0.95372)(1 - \frac{0.04}{1.04}8.357$$

$$= 0.51223 - 0.38089 = 0.13134$$

$${}_{10|}\ddot{a}_{\overline{65:60}} = v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}\ddot{a}_{\overline{75:70}} + v^{10}\,{}_{10}p_{65}(1 - {}_{10}p_{60})\ddot{a}_{75} + v^{10}(1 - {}_{10}p_{65})\,{}_{10}p_{60}\ddot{a}_{70}$$

$$= v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}(\ddot{a}_{75} + \ddot{a}_{70} - \ddot{a}_{75:70}) + v^{10}\,{}_{10}p_{65}(1 - {}_{10}p_{60})\ddot{a}_{75} + v^{10}(1 - {}_{10}p_{65})\,{}_{10}p_{60}\ddot{a}_{70}$$

$$= (0.67556)(0.87120)(0.95372)(9.456 + 12.934 - 8.357)$$

$$+ (0.67556)(0.87120)(1 - 0.95372)(9.456)$$

$$+ (0.67556)(1 - 0.87120)(0.95372)(12.934)$$

$$= 7.877 + 0.258 + 1.073 = 9.208$$

or

$${}_{10|}\ddot{a}_{\overline{65:60}} = \ddot{a}_{\overline{65:60}} - \ddot{a}_{\overline{65:60:\overline{10|}}} = (\ddot{a}_{65} + \ddot{a}_{60} - \ddot{a}_{65:60}) - (\ddot{a}_{65:\overline{10|}} + \ddot{a}_{60:\overline{10|}} - \ddot{a}_{65:60:\overline{10|}})$$

$$= (\ddot{a}_{65} + \ddot{a}_{60} - \ddot{a}_{65:60}) - (\ddot{a}_{65} - v^{10}\,{}_{10}p_{65}\ddot{a}_{75} + \ddot{a}_{60} - v^{10}\,{}_{10}p_{60}\ddot{a}_{70} - \ddot{a}_{65:60} - v^{10}\,{}_{10}p_{65:60}\ddot{a}_{75:70}$$

$$= 13.666 + 16.652 - 12.682$$
$$-\{13.666 - (0.67556)(0.87120)(9.456)\}$$
$$-\{16.652 - (0.67556)(0.95372)(12.934)\}$$
$$+\{12.682 - (0.67556)(0.87120)(0.95372)(8.357)$$

$$= 17.636 - 8.101 - 8.319 + 7.991 = 17.636 - 8.429 = 9.207$$

$$12P(7.790) = 350 + 0.025P(12*7.790 - 1) + 100,000(0.13134) + 20,000*9.208$$
$$\Rightarrow P(93.480 - 2.312) = 350 + 13,134 + 184,160$$
$$\Rightarrow P(91.168) = 197,644 \Rightarrow P = 2,168 \ \textit{per month}$$

14 (i) GFLRV=

$$300 + 0.25P + 0.05P*a^{4\%}_{\overline{\min(K_{[55]},9)|}} + 50*a^{0\%}_{\overline{\min(K_{[55]},9)|}} - P*\ddot{a}^{4\%}_{\overline{\min(K_{[55]}+1,10)|}}$$
$$+(\textit{if } K_{[55]} < 10 \textit{ only})\ v^{T_{[55]}}(100,000 - 10,000*K_{[55]}) + 200$$

(ii)

$$P*\ddot{a}^{4\%}_{[55]:\overline{10|}} = 250 + 0.20P + 0.05P*\ddot{a}^{4\%}_{[55]:\overline{10|}} + 50\ddot{a}^{0\%}_{[55]:\overline{10|}}$$
$$+110,000\bar{A}^{1}_{[55]:\overline{10|}} - 10,000(I\bar{A})^{1}_{[55]:\overline{10|}} + 200*{}_{10}q_{[55]}$$

$$\ddot{a}^{4\%}_{[55]:\overline{10|}} = 8.228$$

$$\ddot{a}^{0\%}_{[55]:\overline{10|}} = (1 + e_{[55]}) - {}_{10}p_{[55]}(1 + e_{65})$$
$$= 26.037 - \left(\frac{8,821.2612}{9,545.9929}\right)17.645$$
$$= 26.037 - (0.92408)17.645$$
$$= 9.732$$

$$\bar{A}^{1}_{[55]:\overline{10|}} = (1.04)^{0.5}(A_{[55]\overline{10|}} - v^{10}\,{}_{10}p_{[55]})$$
$$= (1.04)^{0.5}(0.68354 - 0.67556*0.92408)$$
$$= 0.06044$$

$$(I\bar{A})^{1}_{[55]:\overline{10|}} = (1.04)^{0.5}[(IA)_{[55]} - v^{10}\,{}_{10}p_{[55]}(10A_{65} + (IA)_{65}]$$
$$= (1.04)^{0.5}[8.58908 - 0.67556 * 0.92408(10 * 0.52786 + 7.89442)]$$
$$= 0.37278$$

$$P * 8.228$$
$$= 250 + 0.20P + 0.05P * 8.228 + 50 * 9.732 + 110,000 * 0.06044$$
$$-10,000 * 0.37278 + 200 * (1 - 0.92408)$$
$$\Rightarrow P * 7.6166 = 250 + 486.60 + 6,648.40 - 3,727.80 + 15.18$$
$$\Rightarrow P = 3,672.38 / 7.6166 = 482.15$$

(iii) $${}_9V = q_{64}v^{0.5}[10,000 + 200 * (1.04)^{9.5}] - [0.95P - 50 * (1.04)^{9}]$$
$$= (0.012716)(0.980581)[10,000 + 290.30] - [0.95 * 482.15 - 71.17]$$
$$128.31 - 386.87 = -258.56$$

END OF EXAMINERS' REPORT

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

April 2009

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

R D Muckart
Chairman of the Board of Examiners

June 2009

Comments

Where relevant, comments for individual questions are given after each of the solutions that follow.

1 $_{5|10}q_{[40]+1}$ is the probability that a life now aged 41 exact and at the beginning of the second year of selection will die between the ages of 46 and 56 both exact.

Value is:

$$(l_{46}-l_{56})/l_{[40]+1}$$
$$=(9786.9534-9515.104)/9846.5384$$
$$=0.02761$$

2

$$l_x = 110 - x$$
$$\Rightarrow d_x = l_x - l_{x+1}$$
$$= (110-x)-(110-x-1)$$
$$= 1 \quad \text{for all } x$$

(i)
$$A^{1}_{40:\overline{20}|} = \left(\sum_{0}^{19} v^{t+1} * d_{40+t}\right)/l_{40}$$
$$= \left(\sum_{0}^{19} v^{t+1}\right)/l_{40}$$
$$= a_{\overline{20}|}/l_{40}$$
$$= 13.5903/(110-40)$$
$$= 0.19415$$

(ii)
$$A_{40:\overline{20}|} = A^{1}_{40:\overline{20}|} + v^{20} * l_{60}/l_{40}$$
$$= 0.19415 + 0.45639*(110-60)/(110-40)$$
$$= 0.52014$$

3 Assuming contributions are payable continuously we make the approximation that they are payable on average half-way through the year. The present value of contributions in the year t to $t+1$ is:

$$(0.04).\left(S\frac{s_{x+t}}{s_{x-1}}-5000\right).\frac{v^{x+t+0.5}}{v^{x}}.\frac{l_{x+t+0.5}}{l_x}$$

Define the following parameters and commutation functions:

$\dfrac{s_{x+t}}{s_x}$ represents the ratio of a member's earnings in the year of age $x+t$ to $x+t+1$ to their earnings in the year x to $x+1$.

$$D_{x+t} = v^{x+t} l_{x+t}$$

$$\overline{D}_{x+t} = v^{x+t+0.5} l_{x+t+0.5}$$

$$^{s}D_x = s_{x-1} v^x l_x$$

$$^{s}\overline{D}_{x+t} = s_{x+t} v^{x+t+0.5} l_{x+t+0.5}$$

$$\overline{N}_x = \sum_{t=0}^{t=NRA-x-1} \overline{D}_{x+t}$$

$$^{s}\overline{N}_x = \sum_{t=0}^{t=NRA-x-1} {}^{s}\overline{D}_{x+t}$$

Then the present value of all future contributions is:

$$(0.04).\left(S \frac{^{s}\overline{N}_x}{^{s}D_x} - 5000 \frac{\overline{N}_x}{D_x} \right)$$

4 (a) Different groups or classes of policyholders may have higher or lower lapse rates for all major risk factors (age, duration, gender etc.) than other classes. An example would be where a class of policyholders is defined as those who purchased their policies through a particular sales outlet (e.g. broker versus newspaper advertising).

(b) Lapse rates may vary by policy duration as well as age for shorter durations. At shorter durations lapse rates may be the result of "misguided" purchase by policyholder whereas at longer durations the policy has become more stable.

(c) Lapse rates vary with calendar time for all major risk factors, e.g. economic prosperity varies over time and this results in a similar variation in lapse rates.

Other valid comments were credited. Many students ignored lapses altogether attempting to answer the question from a mortality standpoint only. No credit was given for this.

5 **Assumptions**

- Equal forces in the multiple and single decrement tables
- Uniform distribution of all decrements across year of age

Then

$$(aq)_x^\beta = \int_0^1 {}_t(ap)_x.\mu_{x+t}^\beta.\frac{{}_tp_x^\beta}{{}_tp_x^\beta}.dt = \int_0^1 ({}_tp_x^\beta\mu_{x+t}^\beta).\frac{{}_t(ap)_x}{{}_tp_x^\beta}.dt$$

Our assumptions give us:

$${}_tp_x^\beta\mu_{x+t}^\beta = q_x^\beta$$

$$\frac{{}_t(ap)_x}{{}_tp_x^\beta} = {}_tp_x^\alpha = 1 - t.q_x^\alpha$$

Therefore:

$$(aq)_x^\beta = \int_0^1 q_x^\beta.(1 - t.q_x^\alpha).dt$$

$$= q_x^\beta\left[t - \frac{t^2}{2}q_x^\alpha\right]_0^1 = q_x^\beta\left(1 - \frac{1}{2}q_x^\alpha\right)$$

$$= \left(\frac{1}{3} + \frac{1}{4}q_x^\alpha\right)\left(1 - \frac{1}{2}q_x^\alpha\right) = \frac{1}{3} + \frac{1}{12}q_x^\alpha - \frac{1}{8}\left(q_x^\alpha\right)^2$$

Alternatively the solution can be expressed in terms of q_x^β:

$$= q_x^\beta\left(1 - \frac{1}{2}*4*\left(q_x^\beta - \frac{1}{3}\right)\right)$$

$$= q_x^\beta\left(\frac{5}{3} - 2q_x^\beta\right)$$

This question was essentially course bookwork plus a substitution. To gain good credit it was necessary to work though the solution as above.

6

$$E[T_{xy}] = \int_0^\infty t.{}_tp_x.{}_tp_y(\mu_{x+t} + \mu_{y+t})dt$$

$$= \int_0^\infty t.e^{-.02t}e^{-.03t}(0.02 + 0.03)dt$$

$$= 0.05\int_0^\infty t.e^{-.05t}dt$$

Integrating by parts:

$$= 0.05([-t.e^{-.05t}/.05]_0^\infty + 1/.05 * \int_0^\infty e^{-.05t}dt)$$
$$= 0.05(0 - 20/.05[e^{-.05t}]_0^\infty)$$
$$= 20$$

Alternatively:

$$E[T_{xy}] = \int_0^\infty {}_tp_x.{}_tp_y dt$$

$$= \int_0^\infty e^{-.02t}e^{-.03t}dt$$

$$= \int_0^\infty e^{-.05t}dt$$

$$= [-1/.05 * e^{-.05t}]_0^\infty$$
$$= 20$$

The alternative solution above in essence belongs to the Course CT4 but students who used this were given full credit. The first solution is that which applies to the CT5 Course.

7 Consider the continuous version

This would be

$$50000\int_0^\infty v^{t+5}(1 - {}_tp_{70}^m)\,{}_{t+5}p_{60}^f dt$$

$$= 50000v^5\,{}_5p_{60}^f\int_0^\infty v^t(1 - {}_tp_{70}^m)\,{}_tp_{65}^f dt$$

$$= 50000v^5\,{}_5p_{60}^f(\bar{a}_{65}^f - \bar{a}_{70:\,65}^{m\quad f})$$

The monthly annuity equivalent SP is:

$$= 50000 v^5 {}_5p_{60}^f (\ddot{a}_{65}^{(12)f} - \ddot{a}^{(12)m\ \ f}_{70:65})$$
$$= 50000 v^5 {}_5p_{60}^f (\ddot{a}_{65}^f - \ddot{a}^{m\ \ f}_{70:65}) \text{ (note the monthly adjustment cancels out)}$$
$$= 50000 * 0.82193 * 9703.708 / 9848.431 * (14.871 - 10.494)$$
$$= 177236$$

Other methods were credited. Students who developed the formulae without recourse to continuous functions were given full credit.

8 (i) Final salary – rate of salary at retirement

Final average salary – salary averaged over a fixed period (usually 3 to 5 years) before retirement

Career average salary – salary averaged over total service

.

(ii) $\frac{s_{x+t}}{s_x}$ represents the ratio of a member's earnings in the year of age $x+t$ to $x+t+1$ to their earnings in the year x to $x+1$.

$z_x = \frac{s_{x-1} + s_{x-2} + \ldots + s_{x-y}}{y}$ is defined as a y-year final average salary scale.

Other versions credited. Strictly speaking Final Salary is not an average but this caused no confusion and was fully credited

9 $$25000 \int_0^{10} e^{-\delta t} ({}_tp_{55}^{aa} \mu_{55+t} + {}_tp_{55}^{ai} \nu_{55+t}) dt$$
$$+ \int_0^{10} e^{-\delta t} (0.{}_tp_{55}^{aa} + 1000\, {}_tp_{55}^{ai}) dt$$

where:

δ= the force of interest

${}_tp_{55}^{aa}$ = the probability that an able life age 55 is able at age $55+t$

${}_tp_{55}^{ai}$ = the probability that an able life age 55 is ill at age 55 + t

10 This is the same as:

If y dies in 10 years, then 50000 is paid if x is alive, 200000 if x is dead,
If x dies in 10 years, then 100000 is paid

So the expected present value =

$$\int_0^{10} {}_t p_y \mu_{y+t} (50000\,{}_t p_x + 200000\,{}_t q_x).e^{-\delta t} dt$$

$$+100000\int_0^{10} {}_t p_x.\mu_{x+t}.e^{-\delta t} dt$$

$${}_t p_x = e^{-\int_0^t 0.02 dr} = e^{-.02t}$$

$${}_t p_y = e^{-\int_0^t 0.03 dr} = e^{-.03t}$$

Therefore value =

$$\int_0^{10} e^{-.03t} 0.03(50000e^{-.02t} + 200000(1-e^{-.02t})).e^{-.04t} dt$$

$$+100000\int_0^{10} e^{-.02t} 0.02.e^{-0.4t} dt$$

$$= 6000\int_0^{10} e^{-.07t} dt + 2000\int_0^{10} e^{-.06t} dt - 4500\int_0^{10} e^{-.09t} dt$$

$$= \frac{6000}{-0.07}\left[e^{-0.7} - 1\right] + \frac{2000}{-0.06}\left[e^{-0.6} - 1\right] - \frac{4500}{-0.09}\left[e^{-0.9} - 1\right]$$

$$= 28,518$$

11 (i) Annual premium for endowment with £75,000 sum assured given by:

$$P = \frac{75,000A_{[45]:\overline{20}|}}{\ddot{a}_{[45]:\overline{20}|}} = \frac{75,000 \times 0.46982}{13.785} = 2556.15$$

Reserves at the end of the eighth year:

for endowment with £75,000 sum assured is given by:

$$_8V = 75,000 \times A_{53:\overline{12}|} - 2556.15\ddot{a}_{53:\overline{12}|}$$

$$= 75,000 \times 0.63460 - 2556.15 \times 9.5 = 23,311.58$$

for temporary annuity paying an annual benefit of £18,000 is given by:

$$_8V = 18,000\ddot{a}_{53:\overline{12}|} = 18,000 \times 9.5 = 171,000.00$$

Death strain at risk:

Endowment:	DSAR = 75,000 − 23,311.58 = 51,688.42
Immediate annuity	DSAR = – 171,000.00

(ii) Mortality profit = EDS – ADS

For endowment assurance

$$EDS = (5000 - 65) \times q_{52} \times 51,688.42$$
$$= 4935 \times 0.003152 \times 51,688.42 = 804,019.58$$

$$ADS = 10 \times 51,688.42 = 516,884.20$$

mortality profit = 287,135.38

For immediate annuity

$$EDS = (2500 - 30) \times q_{52} \times -171,000.00$$
$$= 2470 \times 0.003152 \times -171,000.00 = -1,331,310.24$$

$$ADS = 5 \times -171,000.00 = -855,000.00$$

mortality profit = – 476,310.24

Hence, total mortality profit = 287,135.38 – 476,310.24 = –189,174.86
(i.e. a mortality loss)

12 (i) For a unit-linked life assurance contract, we have:

the ***unit fund*** that belongs to the policyholder. This fund keeps track of the premiums allocated to units and benefits payable from this fund to policyholders are denominated in these units. This fund is normally subject to unit fund charges.

the ***non-unit fund*** that belongs to the company. This fund keeps track of the premiums paid by the policyholder which are not allocated to units together with unit fund charges from the unit-fund. Company expenses will be charged to this fund together with any non-unit benefits payable to policyholders.

(ii) It is a principle of prudent financial management that once sold and funded at outset, a product should be self-supporting. However, some products can give profit signatures which have more than one financing phase. In such cases, reserves are required at earlier durations to eliminate future negative cash flows, so that the office does not expect to have to input further money in the future.

(iii)

Year t	$q_{[50]+t-1}$	$p_{[50]+t-1}$
1	0.001971	0.998029
2	0.002732	0.997268
3	0.003152	0.996848
4	0.003539	0.996461

$${}_3V = \frac{118.0}{1.055} = 111.85$$

$${}_2V \times 1.055 - p_{52} \times {}_3V = 136.2 \Rightarrow {}_2V = 234.78$$

$${}_1V \times 1.055 - p_{[50]+1} \times {}_2V = 152.0 \Rightarrow {}_1V = 366.01$$

13 Multiple decrement table constructed using $(aq)_x^d = q_x^d\left[1 - \tfrac{1}{2}(q_x^s + q_x^m) + \tfrac{1}{3} q_x^s \times q_x^m\right]$ etc. which assumes that the decrements in each single decrement table are uniformly distributed over each year of age

x	q_x^d	q_x^s	q_x^m	$(aq)_x^d$	$(aq)_x^s$	$(aq)_x^m$
40	0.0009370	0.10	0.05	0.0008683	0.0974547	0.0474781
41	0.0010140	0.10	0.05	0.0009396	0.0974510	0.0474763
42	0.0011040	0.10	0.05	0.0010230	0.0974466	0.0474742

Using an arbitrary radix of 1,000,000, we can construct the following multiple decrement table

X	$(al)_x$	$(ad)_x^d$	$(ad)_x^s$	$(ad)_x^m$
40	1,000,000	868.3	97,454.7	47,478.1
41	854,198.9	802.6	83,242.5	40,554.2
42	729,599.6	746.4	71,097.0	34,637.2
43	623,119.0			

Let P be the annual premium for the contract.

Then equation of value gives:

PV of premiums = PV of death benefits + PV of surrender benefits + PV of survival benefits + PV of expenses

PV of premiums

$$= P\left(1 + \frac{854{,}198.9}{1{,}000{,}000} \times v_{0.05} + \frac{729{,}599.6}{1{,}000{,}000} \times v_{0.05}^2\right)$$
$$= P\left(1 + 0.813523 + 0.661768\right) = 2.475291P$$

PV of expenses = $0.005 \times 2.475291P = 0.0123765P$

PV of death benefits

$$= 15{,}000 \times (1.05)^{1/2} \times \left(\frac{868.3}{1{,}000{,}000} \times v_{0.05} + \frac{802.6}{1{,}000{,}000} \times v_{0.05}^2 + \frac{746.4}{1{,}000{,}000} \times v_{0.05}^3\right)$$
$$= 15{,}370.4262\left(0.00082695 + 0.00072798 + 0.00064477\right) = 33.8103$$

PV of withdrawal benefits =

$$= P\left(1 \times \frac{97{,}454.7}{1{,}000{,}000} v_{0.05} + 2 \times \frac{83{,}242.5}{1{,}000{,}000} v_{0.05}^2 + 3 \times \frac{71{,}097.0}{1{,}000{,}000} v_{0.05}^3\right) \times 1.05^{1/2}$$
$$= P\left(0.092814 + 0.1510068 + 0.1842488\right) \times 1.024695 = 0.4386408P$$

PV of marriage benefits =

$$= P\left(\frac{47,478.1}{1,000,000} \times \ddot{s}_{\overline{1}|}^{0.04} \times v_{0.05} + \frac{40,554.2}{1,000,000} \times \ddot{s}_{\overline{2}|}^{0.04} \times v_{0.05}^2 + \frac{34,637.2}{1,000,000} \times \ddot{s}_{\overline{3}|}^{0.04} \times v_{0.05}^3\right) \times \left(\frac{1.05}{1.04}\right)^{1/2}$$

$$= P\left(0.0470259 + 0.0780406 + 0.0971372\right) \times 1.0047962 = 0.2232694P$$

PV of survival benefits =

$$5000 \times \frac{623,119.0}{1,000,000} v_{0.05}^3 = 2691.3681$$

Equation of value becomes

2.475291P = 33.8103 + 0.4386408P + 0.2232694P + 2,691.3681 + 0.0123765P

=> P = 2725.1784/1.801004 = 1513.14

14 (i) Let P be the annual premium payable. Then equation of value gives:

PV of premiums = PV of benefits + PV of expenses

i.e.

$$P\ddot{a}_{[60]:\overline{5}|} = 10,000A_{[60]:\overline{5}|} + 400(IA)_{60:\overline{5}|} + 0.05P\ddot{a}_{[60]:\overline{5}|} + 0.55P \quad at \quad 6\%$$

where $(IA)_{[60]:\overline{5}|} = (IA)_{[60]} - \frac{l_{65}}{l_{[60]}} \times v_{0.06}^5 \left(5A_{65} + (IA)_{65}\right) + 5 \times \frac{l_{65}}{l_{[60]}} \times v_{0.06}^5$

$$= 5.4772 - 0.7116116(5 \times 0.40177 + 5.50985) + 5 \times 0.7116116 = 3.684864$$

and $\frac{l_{65}}{l_{[60]}} = \frac{8821.2612}{9263.1422}$

$$\Rightarrow P(0.95\ddot{a}_{[60]:\overline{5}|} - 0.55) = 10,000A_{[60]:\overline{5}|} + 400(IA)_{60:\overline{5}|}$$

$$\Rightarrow P(0.95 \times 4.398 - 0.55) = 10,000 \times 0.75104 + 400 \times 3.684864$$

$$P = \frac{8984.3456}{3.6281} = 2476.32$$

(ii) Reserves required on the policy at 4% interest are:

$$ {}_1V_{60:\overline{5|}} = 10,400A_{61:\overline{4|}} - NP\ddot{a}_{61:\overline{4|}} $$

$$ = 10,000\left(1-\frac{\ddot{a}_{61:\overline{4|}}}{\ddot{a}_{60:\overline{5|}}}\right) + 400A_{61:\overline{4|}} = 10,000\left(1-\frac{3.722}{4.550}\right) + 400\times 0.85685 = 2162.52 $$

$$ {}_2V_{60:\overline{5|}} = 10,000\left(1-\frac{\ddot{a}_{62:\overline{3|}}}{\ddot{a}_{60:\overline{5|}}}\right) + 800A_{62:\overline{3|}} = 10,000\left(1-\frac{2.857}{4.550}\right) + 800\times 0.89013 = 4432.98 $$

$$ {}_3V_{60:\overline{5|}} = 10,000\left(1-\frac{\ddot{a}_{63:\overline{2|}}}{\ddot{a}_{60:\overline{5|}}}\right) + 1200A_{63:\overline{2|}} = 10,000\left(1-\frac{1.951}{4.550}\right) + 1200\times 0.92498 = 6822.06 $$

$$ {}_4V_{60:\overline{5|}} = 10,000\left(1-\frac{\ddot{a}_{64:\overline{1|}}}{\ddot{a}_{60:\overline{5|}}}\right) + 1600A_{64:\overline{1|}} = 10,000\left(1-\frac{1.000}{4.550}\right) + 1600\times 0.96154 = 9340.66 $$

Year t	*Prem*	*Expense*	*Opening reserve*	*Interest*	*Death Claim*	*Mat Claim*	*Closing reserve*	*Profit vector*
1	2476.32	1485.79	0	69.34	83.43	0	2145.17	−1168.73
2	2476.32	123.82	2162.52	316.05	97.30	0	4393.04	340.73
3	2476.32	123.82	4432.98	474.98	113.25	0	6753.08	394.13
4	2476.32	123.82	6822.06	642.22	131.59	0	9234.20	450.99
5	2476.32	123.82	9340.66	818.52	152.59	11847.41	0	511.68

Year t	${}_{t-1}p$	*Profit signature*	*Discount factor*	*NPV of profit signature*
1	1.0	−1168.73	.91743	−1072.23
2	0.991978	338.00	.84168	284.49
3	0.983041	387.45	.77218	299.18
4	0.973101	438.85	.70843	310.89
5	0.962062	492.27	.64993	319.94

NPV of profit signature = £142.28

Year t	*Premium*	${}_{t-1}p$	*Discount factor*	*NPV of premium*
1	2476.32	1.0	1	2476.32
2	2476.32	0.991978	.91743	2253.63
3	2476.32	0.983041	.84168	2048.92
4	2476.32	0.973101	.77218	1860.73
5	2476.32	0.962062	.70843	1687.74

NPV of premiums = £10,327.34

Profit margin = $\dfrac{142.28}{10,327.34} = 0.0138$ *i.e.* 1.38%

END OF EXAMINERS' REPORT

Subject CT5 — Contingencies. Core Technical

September 2009 Examinations

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

R D Muckart
Chairman of the Board of Examiners

December 2009

Comments for individual questions are given with the solutions that follow.

1

$$
\begin{aligned}
{}_{20}q^{2}_{[45]:[45]} &= 1/2 * {}_{20}q_{[45]:[45]} \\
&= 1/2 * (1 - {}_{20}p_{[45]})^2 \\
&= 1/2 * (1 - l_{65}/l_{[45]})^2 \\
&= 1/2 * (1 - 8821.2612/9798.0837)^2 \\
&= .00497
\end{aligned}
$$

In general well this question was well done.

2

Define

${}_k(ap)_x$ = the probability that a life aged x is alive and not diagnosed as critically ill at time k

$(aq)^t_{x+k}$ = the probability that a life aged $x + k$ is diagnosed as critically ill in the following year

Then the value is

$$\sum_{k=0}^{n-1} v^{k+1}\, {}_k(ap)_x . (aq)^t_{x+k}$$

Where the benefit is payable at the end of the year of diagnosis

Students often failed to define symbols adequately. The continuous alternative was also fully acceptable

3

For contant force of mortality at age 72:

$$p_{72} = (1 - q_{72}) = .969268 = e^{-\int_0^1 \mu dt} = e^{-\mu}$$

Hence $\mu = -\ln(.969268) = .031214$

$$
\begin{aligned}
{}_{0.5}p_{72.25} &= e^{-\int_{0.25}^{0.75} .031214 dt} = e^{-.015607} \\
&= .984514
\end{aligned}
$$

$$
\begin{aligned}
{}_{0.5}q_{72.25} &= 1 - .984504 \\
&= .015486
\end{aligned}
$$

Generally well done. The alternative very quick answer of $1-(p_{72})^{1/2}$ *was fully acceptable*

4

(i) Crude rate = (104+127+132)/(121376+134292+133277)=0.000933

(ii)

Age	*Population*	q_x	*Expected Number of deaths*
40	121,376	0.000937	114
41	134,292	0.001014	136
42	133,277	0.001104	147

SMR = actual deaths / expected deaths

= (104+127+132) / (114+136+147) = 0.914

Generally well done.

5

$$\text{var}\left[\ddot{a}_{\overline{\min(K_x+1,n)}|}\right] = \text{var}\left[\frac{1-v^{\min(K_x+1,n)}}{d}\right]$$

$$= \frac{1}{d^2}\text{var}\left[v^{\min(K_x+1,n)}\right]$$

$$= \frac{1}{d^2}\left[{}^2A_{x:\overline{n}|} - (A_{x:\overline{n}|})^2\right] \text{ where } {}^2A_{x:\overline{n}|} \text{ is at rate } (1+i)^2-1$$

Straightforward bookwork where considerable information was given in Handbook. The Examiners were looking to see students knew how to derive the relationship. Generally well done.

6

a. $({}_tV' + GP\text{-}e_t)(1+i) = q_{x+t}S + p_{x+t}({}_{t+1}V')$

where $_tV'$ = gross premium reserve at time t

GP = office premium

e_t = renewal expenses incurred at time t

i = interest rate in premium/valuation basis

S = Sum Assured

q_{x+t} = probability life aged $x+t$ dies within one year on premium/valuation basis

p_{x+t} = probability life aged $x+t$ survives one year on premium/valuation basis

b. Income (opening reserve and excess of premium over renewal expenses) plus interest equals outgo (death claims and closing reserve for survivors) if assumptions are borne out.

Generally well done.

7

Direct expenses are those that vary with the amount of business written. Direct expenses are divided into:

Initial expenses

Renewal expenses

Termination expenses

Examples of each:

Initial expenses – those arising when the policy is issued e.g. initial commission

Renewal expenses – those arising regularly during the policy term e.g. renewal commission

Termination expenses – those arising when the policy terminates as a result of an insured contingency (e.g. death claim for a temporary life insurance policy)

Generally well done and other valid comments and examples were credited.

8

(i) Pensioners retiring at normal retirement age

Pensioners retiring before normal retirement age

Pensioners retiring before normal retirement age on the grounds of ill-health

(ii) Class selection – ill-health pensioners will have different mortality to other retirements.

Temporary initial selection – the difference between these classes will diminish with duration since retirement

Anti-Selection and Time Selection were credited provided they were properly justified. Generally well done.

9

(i) To set premium rates to ensure the probability of a profit is set at an acceptable level then the insurer takes advantage of the Central Limit Theorem while pooling risks which are independent and homogeneous.

Independence of risk usually follows naturally.

Homogeneity is ensured by careful underwriting. Risk groups are separated by the use of risk factors, such as age and sex.

The life assurance company uses responses to questions to allocate prospective customers to the appropriate risk group.

Enough questions should be asked to ensure that the variation between categories is smaller than the random variation that remains but in practice there will be limits on the number and type of questions that can be asked.

(ii) Equity – insurance is about pooling of risks and the use of genetic information reduces that pooling.

Ethics – use of genetic information could create an "underclass" of lives who are not able to obtain insurance products at an affordable price, given the results of their genetic tests.

In some countries legislation may prohibit genetic testing or there might be political or social reasons why it is avoided.

Generally part (i) was done poorly with students failing to appreciate the key points. Part (ii) was done better but in this case also most students failed to obtain all the main valid points.

10

Pension at retirement = 3 × 1000 = 3000

Annuity at retirement

$$\ddot{a}^{(12)}_{\overline{5|}} + v^5 \cdot {}_5p_{65}\ddot{a}^{(12)}_{70}$$

$$= a_{\overline{5|}} \cdot \frac{i}{d^{(12)}} + v^5 \cdot {}_5p_{65}\ddot{a}^{(12)}_{70}$$

$$= 4.4518 \times 1.021537 + 0.82193 * \frac{9238.134}{9647.797} . \left(11.562 - \tfrac{11}{24}\right) = 13.28659$$

Multiple decrement table

Use the formula

$$(aq)_x = q_x^{\alpha} + q_x^{\beta} - q_x^{\alpha}.q_x^{\beta}$$

To derive the following

$(aq)_{62}$= 0.021233, $(aq)_{63}$= 0.084295, $(aq)_{64}$= 0.062397

And ${}_3(ap)_{62}$ = (1 – 0.021233).(1–0.084295).(1–0.062397)= 0.840338

So value = 3000×13.28659×0.840338×$(1.04)^{-3}$ = 29778

A large proportion of students whilst understanding how to approach this question failed to calculate some or all of it correctly. In some cases certain parts were omitted or calculated wrongly. Credit was given where parts of the solution were correct.

11

Let EDS and ADS denote the expected and actual death strain in 2008. Then

$$EDS = \sum_i q_{60}\left[S_i - \left\{ S_i (A_{61:\overline{4|}} + \frac{l_{65}}{l_{61}} v^4) - P_i \ddot{a}_{61:\overline{4|}} \right\} \right]$$

where S_i is the death benefit per policy and the summation is over all policies in force at start of the year i.e. (where figures are in £000's)

$$EDS = q_{60}\left[\left(\sum S_i \right) - \left(\sum S_i \right)(A_{61:\overline{4|}} + \frac{l_{65}}{l_{61}} v^4) + \left(\sum P_i \right) \ddot{a}_{61:\overline{4|}} \right]$$

$$= 0.008022 \times \left\{ 6125 - 6125\left(0.85685 + \frac{8821.2612}{9212.7143} \times 0.854804 \right) + 440 \times 3.722 \right\}$$

$$= 0.008022 \times \ 6125 - 8623.75 \ = -20.045$$

The actual death strain is obtained by summation of the death strains at risk over the policies that become claims. Therefore

$$ADS = \sum_{claims}\left[S_i - \left\{ S_i (A_{61:\overline{4|}} + \frac{l_{65}}{l_{61}} v^4) - P_i \ddot{a}_{61:\overline{4|}} \right\} \right]$$

$$= \left(\sum_{claims} S_i \right) - \left(\sum_{claims} S_i \right)(A_{61:\overline{4|}} + \frac{l_{65}}{l_{61}} v^4) + \left(\sum_{claims} P_i \right) \ddot{a}_{61:\overline{4|}}$$

$$= 100 - \ 167.5333 - 26.054 \ = -41.479$$

Therefore, mortality profit = -20.045 + 41.479 = 21.434 (i.e. a profit of £21,434).

This question was very poorly done. Students failed to properly identify the data and the subtleties of a Pure Endowment contract.

12

(i) The expected present value of a **continuous assurance** for **a sum assured of 1000** calculated at a **force of interest** δ on **2 lives aged *x* and *y*** whereby the **sum is paid on the death of *x*** only **if life aged *x* dies after life aged *y*.**

(ii) For both parts (a) and (b):

for the life aged 30

$${}_tp_{30} = e^{-\int_0^t \mu_{30+r} dr} = e^{-\int_0^t 0.02 dr} = e^{-0.02t}$$

Similarly for the life 40

$${}_tp_{40} = e^{-0.03t}$$

$$\text{(a)}\ 1000\ \bar{A}^{\ 2}_{30:40} = 1000\int_0^\infty v^t\ {}_tp_{30}(1-{}_tp_{40})\mu_{30+t}\,dt$$

$$= 1000\int_0^\infty e^{-.05t} * e^{-.02t} * (1-e^{-.03t}) * .02dt$$

$$= 1000\int_0^\infty .02 * (e^{-.07t} - e^{-.1t})dt$$

$$= 1000[-.02/.07 * e^{-.07t} + .02/.1 * e^{-.1t}]_0^\infty$$

$$= 1000(.02/.07 - .02/.1)$$

$$= 85.714$$

(b) To calculate premium we need $\bar{a}_{\overline{30:40}}$

$$\bar{a}_{\overline{30:40}} = \int_0^\infty v^t * (1-(1-{}_tp_{30})(1-{}_tp_{40}))dt$$

$$= \int_0^\infty e^{-.05t}(e^{-.02t} + e^{-.03t} - e^{-.05t})dt$$

$$= \int_0^\infty (e^{-.07t} + e^{-.08t} - e^{-1.t})dt$$

$$= [-e^{-.07t}/.07 - e^{-.08t}/.08 + e^{-.1t}/.1]_0^\infty$$

$$= (1/.07 + 1/.08 - 1/.1)$$

$$= 16.786$$

So the required premium = $85.714/16.786$

$$= 5.11$$

(iii) If the life age 30 dies first the policy ceases without benefit yet the premium is expected to be maintained by the life aged 40 so long as they survive. There is no incentive to continue.

The sensible option would be to establish the premium paying period as ceasing on the death of the life aged 30.

A single premium is possible as an alternative if affordable.

In general terms this question was reasonably well done although a large number of students failed to obtain all of the required numerical solutions (the main error being failure to calculate the joint life last survivor annuity). In part (iii) a student who suggested a joint life first death approach was given credit although this is an expensive option.

13

(i) Let P be the monthly premium for the contract. Then:

EPV of premiums valued at rate i where i = 0.06 is:

$$12P\ddot{a}^{(12)}_{[30]:\overline{35}|} = 12P(\ddot{a}_{[30]:\overline{35}|} - \frac{11}{24}(1 - v^{35}\frac{l_{65}}{l_{[30]}}))$$

$$where\ v^{35}\frac{l_{65}}{l_{[30]}} = 0.13011 \times \frac{8821.2612}{9923.7497} = 0.11566$$

$$= 12P(15.152 - \frac{11}{24}(1 - .11566)) = 12P \times 14.74668 = 176.9601P$$

EPV of benefits valued at rate i where i = 0.06 is:

$$= 75,000A_{[30]:\overline{35}|} = 75,000 \times 0.14234 = 10,675.5$$

EPV of expenses not subject to inflation and therefore valued at rate i where i = 0.06 is:

$$0.025 \times 12P\ddot{a}^{(12)}_{[30]:\overline{35}|} - 0.025P + 250 + 0.5 \times 12P$$
$$= 250 + 10.399P$$

EPV of expenses subject to inflation and therefore valued at rate j where

$$1 + j = \frac{1.06}{1.0192308} = 1.04 \text{ is:}$$

$$75(\ddot{a}_{[30]:\overline{35}|} - 1) + 300A_{[30]:\overline{35}|} = 75 \times 18.072 + 300 \times 0.26647 = 1435.341$$

Equating *EPV* of premiums and *EPV* of benefits and expenses gives:

176.9601P = 10,675.5 + 250 + 10.399P + 1,435.341
=> P = 12,360.841/166.5611 = £74.21

(ii) Gross retrospective policy value is given by:

$V^{retrospective}$

$$= (1+i)^{30}\frac{l_{[30]}}{l_{60}}\left[\begin{array}{l} 12P\times 0.975\ddot{a}^{(12)}_{[30]:\overline{30|}} @ i\% + 0.025P - 12\times 0.5P - 250 - 75,000A^{1}_{[30]:\overline{30|}} @ i\% \\ \qquad -75(\ddot{a}^{@ j\%}_{[30]:\overline{30|}} - 1) - 300A^{1}_{[30]:\overline{30|}} @ j\% \end{array}\right]$$

where,

$$\frac{l_{[30]}}{l_{60}} = \frac{9923.7497}{9287.2164} = 1.06854$$

and at rate i = 0.06

$$\ddot{a}^{(12)}_{[30]:\overline{30|}} = \ddot{a}_{[30]:\overline{30|}} - \frac{11}{24}\left(1 - v^{30}\frac{l_{60}}{l_{[30]}}\right) = 14.437 - \frac{11}{24}(1 - 0.16294) = 14.0533$$

$$A^{1}_{[30]:\overline{30|}} = \left(A_{[30]:\overline{30|}} - v^{30}\frac{l_{60}}{l_{[30]}}\right) = (0.18283 - 0.16294) = 0.0198$$

and at rate j = 0.04

$$\ddot{a}_{[30]:\overline{30|}} = 17.759$$

$$A^{1}_{[30]:\overline{30|}} = A_{[30]:\overline{30|}} - v^{30}\frac{l_{60}}{l_{[30]}} = 0.31697 - 0.30832\times 0.93586 = 0.02843$$

$$\Rightarrow V^{retrospective}$$
$$= 6.13715\ 12,201.876 + 1.8553 - 445.26 - 250 - 1,491.75 - 1,256.925 - 8.529$$
$$= £53,707.84$$

Generally part (i) was done well. Students did however often struggle to reproduce part (ii) which is often the case with retrospective reserves.

In this case because the reserve basis matched the premium basis the retrospective reserve equalled the prospective reserve. If the student realised this, fully stated the fact and then calculated the prospective reserve full credit was given.

Minimal credit was however given if just a prospective reserve method was attempted without proper explanation.

14

(i) First calculate net premium NP and reserve ${}_tV_{57:\overline{3|}}$ for t = 1 and 2

$$NP\ddot{a}_{57:\overline{3}|} = 10000(A_{57:\overline{3}|} - v^3 \frac{l_{60}}{l_{57}}) + 0.5 \times 3 \times NP \times v^3 \frac{l_{60}}{l_{57}}$$

$$NP \times 2.870 = 8896.3 - 10000 \times 0.889 \times 9287.2164 / 9467.2906$$
$$+1.5NP \times 0.889 \times 9287.2164 / 9467.2906$$
$$= 175.394 + NP \times 1.308$$
$$\Rightarrow NP = 112.29$$

$${}_1V_{57:\overline{3}|} = (112.29 \times (1.04) - 10000 \times q_{57}) / (1 - q_{57})$$
$$= (116.782 - 56.50) / 0.99435$$
$$= 60.62$$

$${}_2V_{57:\overline{3}|} = ((112.29 + 60.62) \times (1.04) - 10000 \times q_{58}) / (1 - q_{58})$$
$$= (179.826 - 63.520) / 0.993648$$
$$= 117.05$$

The end 3rd year reserve needs to be 1.5 times the office premium to be calculated so as to meet the return guarantee.

We can complete the following table (denoting the office premium by *P*). Note as withdrawals are assumed at the end of the year the decrements of mortality and withdrawal are not dependent.

	Year 1 Age 57	*Year 2 Age 58*	*Year 3 Age 59*
80% AM92 q select	0.0033368	.004944	.005712
Withdrawal	.19933264	.0995056	0
In force factor begin year	1	.79733056	.7140497
Premium	*P*	*P*	*P*
Expenses	0.2*P*	0.05*P*	0.05*P*
Death Claims	33.368	49.440	57.120
Opening Reserve	0	60.62	117.05
Closing Reserve	48.334	104.824	1.4914*P*
Interest	.048*P*	.057*P*+3.6372	.057*P*+7.023
Profit vector	.848*P*–81.702	1.007*P*–90.007	–0.4844*P*+66.953
Profit signature	.848*P*–81.702	.8029*P*–71.7653	–0.3459*P*+47.808

Alternatively the Closing Reserve at End Year 3 can be taken as zero and an additional item termed "Maturity Value" can be shown in Year 3 only equal to 1.4914*P*.

To obtain 10% return the equation is:

$P \times [.848/(1.1) + 0.8029/(1.1)^2 - 0.3459/(1.1)^3] - [81.702/(1/1)$

$+ 71.7653/(1.1)^2 - 47.808/(1.1)^3] = 0$

$\Rightarrow 1.1746 \times P - 97.6659 = 0 \quad \Rightarrow P = 83.15$ say £83

(ii) The impact of increasing withdrawal rates depends primarily on the relationship between expenses, reserves and any surrender value. In this case there is no surrender value, a substantial reserve for a maturity benefit and low expenses.

In that scenario, increasing the lapse rates actually improves the return to the company as it retains a substantial premium and reserve with low expected death costs and returns nothing to the policyholder. This return comes earlier also and benefits from the high risk discount rate.

(iii) A revised office premium is now required say P'.

In this case a life who surrenders obtains $0.25P'$ at the end of year1 and $0.5P'$ at the end of year 2.

On the same parameters the present value of these 2 cash flow items are:

$P' \times [0.25 \times 0.19933264/(1.1)+0.5 \times 0.0995056 \times 0.79733056/(1.1)^2]$

$= 0.07809P'$

Hence from above with adjustment:

$1.1746 \times P' - 97.6659 - 0.07809P' = 0 \Rightarrow P' = 89.07$ say £89

Most students found this a very daunting question and overall performance was lower than expected. Certain comments are appropriate:

- *Because of the stated fact that withdrawals happened at the end of the year calculating dependent decrements was not necessary. Many students wasted much time attempting to perform this.*
- *Many students did not know how to calculate a net premium for this contract.*
- *The reserve process was very straightforward if done on a recursive basis (see question 6)*
- *Once these facts were realised the question was then a relatively simple manipulation of cash flows.*

Credit was given to students who gave some reasonable verbal explanation of what needed to be done even if calculations were incomplete.

END OF EXAMINERS' REPORT

EXAMINERS' REPORT

April 2010 Examinations

Subject CT5 — Contingencies
Core Technical

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

R D Muckart
Chairman of the Board of Examiners

July 2010

Comments

These are given in italics at the end of each question.

1 (i) The number of lives still alive at age $x + r$ out of l_x lives alive at age x subject to select mortality.

(ii) The probability that a life age x will die between age $x + n$ and $x + n + m$.

(iii) The number of lives that die between x and $(x + 1)$ out of l_x lives alive at x.

Question generally answered well.

2 Spurious selection occurs when mortality differences ascribed to groups are formed by factors which are not the true causes of these differences.

For example mortality differences by region may be put down to the actual class structure of the region itself whereas a differing varying mix of occupations region by region could be having a major effect. So Region is spurious and being confounded with occupation.

Another example might be in a company pension scheme which might be showing a significant change in mortality experience which could be viewed as change over time. However withdrawers from the scheme may be having an effect as their mortality could be different. To that degree Time Selection may be spurious.

Question generally answered well. Credit was given for a wide range of valid examples.

3 The Standardised mortality ratio is the ratio of actual deaths in the population divided by the expected number of deaths in the population if the population experienced standard mortality.

Actual number of deaths for Urbania = 130+145+173 = 448

Mortality rates in standard population are:

Age 60: 26,170 / 2,500,000 = 0.0104680
Age 61: 29,531 / 2,400,000 = 0.0123046
Age 62: 32,542 / 2,200,000 = 0.0147918

Expected number of deaths for Urbania

$= 0.010468 \times 10{,}000 + 0.0123046 \times 12{,}000 + 0.0147918 \times 11{,}000 = 415$

SMR = 448/415 = 107.95%

Question generally answered well.

4 $EPV = (10,000 - 100)A^{1}_{[50]:\overline{5|}} + 100(IA)^{1}_{[50]:\overline{5|}}$

$$= 9,900(A_{[50]} - v^5\,{}_5p_{[50]}A_{55}) + 100((IA)_{[50]} - v^5\,{}_5p_{[50]}(5A_{55} + (IA)_{55}))$$

$$= 9,900(0.32868 - v^5\frac{9557.8179}{9706.0977} * 0.38950)$$

$$+100 * \left(8.5639 - v^5\frac{9557.8179}{9706.0977}(5 * 0.38950 + 8.57976)\right)$$

$$= 132.96 + 4.34$$

$$= 137.30$$

Many students answered the question well. The most common error was the use of 10,000 as the multiplier before the temporary assurance function rather then 9,900.

5 $${}_tp_x = \exp(-\int_x^{x+t}\mu_s ds)$$

$$= \exp(-\int_x^{x+t}(e^{0.0002S} - 1)ds)$$

$$= \exp(-\int_x^{x+t}e^{0.0002S}ds + \int_x^{x+t}ds)$$

$$= \exp\left(-\frac{\left[e^{0.0002(x+t)} - e^{0.0002x}\right]}{0.0002} + t\right)$$

(i) Probability =

$$= \exp\left(-\frac{\left[e^{0.0002x70} - e^{0.0002x20}\right]}{0.0002} + 50\right)$$

$$= 0.6362$$

(ii) This is the probability that the life survives to 60 and then dies between 60 and 70

Probability $= {}_{40}p_{20}(1-{}_{10}p_{60})$

$$= \exp\left(-\frac{\left[e^{0.0002x60}-e^{0.0002x20}\right]}{0.0002}+(60-20)\right).\left(1-\exp\left(-\frac{\left[e^{0.0002x70}-e^{0.0002x60}\right]}{0.0002}+(70-60)\right)\right)$$

$= 0.725x(1-0.8773)$

$= 0.0889$

This question was answered poorly overall. It was an unusual representation of the μ_x function but other than that was a straight forward probability and integration question.

6 $p_{50} = 97{,}702 / 99{,}813 = 0.978850$
$p_{51} = 95{,}046 / 97{,}702 = 0.972815$

Uniform distribution of deaths

$$\frac{p_{50\;0.25}p_{51}}{{}_{0.5}p_{50}} = \frac{p_{50}(1-0.25(1-p_{51}))}{(1-0.5(1-p_{50}))} = \frac{0.978850*(1-0.25*(1-0.972815))}{(1-0.5*(1-0.978850))} = 0.982588$$

Constant force of mortality

$\mu_t = -\ln(p_t)$
$\mu_{50} = -\ln(0.978850) = 0.021377$
$\mu_{51} = -\ln(0.972815) = 0.027561$
${}_{0.5}p_{50} * {}_{0.25}p_{51} = e^{-0.5*0.021377} * e^{-0.25*0.027561} = 0.989368 * 0.993133 = 0.982574$

Generally answered well. A limited number of students used the Balducci Assumption as one of their answers. This is not in the CT5 Course whilst the above 2 methods clearly are. This method was however credited – solution not published as not in CT5

7 (i) *Age retirement benefit*

$$\frac{1}{60}40{,}000\frac{(20\,{}^{z}M_{55}^{ra} + {}^{z}\overline{R}_{55}^{ra})}{s_{54}D_{55}}$$

$$= \frac{1}{60} 40,000 \frac{(20 * 128,026 + 963,869)}{9.745 * 1,389}$$

$$= 173,584$$

(ii) *Contributions*

$$K 40,000. \frac{{}^{s}\overline{N}_{55}}{s_{54} D_{55}}$$

$$= K.40,000 x \frac{88,615}{9.745 * 1,389}$$

$$= 261,868K$$

Therefore K = 173,584 / 261,868 i.e. 66.3%

Most students answered reasonably well. Most common error was the wrong s_x function. Also some students included early retirement calculations which were not asked for.

Also students often did not include the past service benefits in the final contribution rate believing the final result would have been too high (the question however was quite specific on providing past benefits).

8 (i) $$\text{Fund} = 52 * \frac{1.04^{(66-21)} \bar{a}_{21:\overline{45}|}}{{}_{45}p_{21}}$$

$$\bar{a}_{21:\overline{45}|} = \ddot{a}_{21:\overline{45}|} - \frac{1}{2} * (1 - v^{45} * l_{66} / l_{21}) = \ddot{a}_{21:\overline{45}|} - \frac{1}{2} * \left(1 - 0.17120 * \frac{8695.6199}{9976.3909}\right)$$

$$= \ddot{a}_{21:\overline{45}|} - 0.42539$$

$$\ddot{a}_{21:\overline{45}|} = \ddot{a}_{21:\overline{44}|} + v^{44} * l_{65} / l_{21} = 21.045 + .17805 * \frac{8821.2612}{9976.3909} = 21.202$$

$$\Rightarrow \bar{a}_{21:\overline{45}|} = 20.777$$

$$\text{therefore fund } = \frac{52 * 1.04^{45} (20.777)}{\left(8695.6199 / 9976.3909\right)} = 7,240$$

(ii) Let annuity be £P per week. Then EPV of annuity at 66 is

$$52P(\bar{a}_{\overline{10|}} + \tfrac{2}{3} * v^{10}\,{}_{10}p_{66}.\bar{a}_{76})$$

$$= 52P\left[\frac{(1-v^{10})}{\ln(1.04)} + \tfrac{2}{3} * 0.675564 * \frac{6589.9258}{8695.6199}(8.169 - 0.5)\right]$$

$$= 52P\left[8.272 + 2.618\right]$$

$$= 566.26P$$

Therefore pension is given by

$$7{,}240 = 566.26P$$

$$P = 12.79$$

Many students struggled with this question and indeed a large number did not attempt it. As will be seen from the solution above the actuarial mathematics involved are relatively straightforward.

Note that 52.18 (i.e. 365.25/7) would have been an acceptable alternative to 52 as the multiplier which will of course have adjusted the answer slightly.

9 (i) We are looking to derive $(aq)_x^r$ in terms of σ_x and μ_x

Use the Kolmogorov equations (assuming the transition intensities are constant across a year age):

$$\frac{\partial}{\partial t}\,{}_t(aq)_x^r = \sigma e^{-(\sigma+\mu)t}$$

$$(aq)_x^r = \frac{\sigma}{(\sigma+\mu)}(1 - e^{-(\sigma+\mu)})$$

(ii) Similarly

$$(aq)_x^d = \frac{\mu}{(\sigma+\mu)}(1 - e^{-(\sigma+\mu)})$$

Note that:

$$1 - ((aq)_x^r + (aq)_x^d) = e^{-(\sigma+\mu)}$$

$$\Rightarrow \sigma + \mu = -\log(1 - ((aq)_x^r + (aq)_x^d))$$

So

$$(aq)_x^r = \frac{\sigma}{(-\log(1-((aq)_x^r+(aq)_x^d)))}((aq)_x^r+(aq)_x^d)$$

this can be rearranged to show

$$-\sigma = \frac{(aq)_x^r}{(aq)_x^r+(aq)_x^d}\log(1-((aq)_x^r+(aq)_x^d))$$

Given that:

$$q_x^r = 1-e^{-\sigma},$$

then

$$q_x^r = 1-\left[1-((aq)_x^r+(aq)_x^d)\right]^{(aq)_x^r \big/ ((aq)_x^r+(aq)_x^d)}$$

In general this was poorly answered with most students making a limited inroad to the question.

However, the question did not specify that constant forces must be assumed. So, a valid alternative to part (i) is:

$$(aq)_x^r = \int_0^1 {}_t(ap)_x\,\sigma_{x+t}\,dt = \int_0^1 \exp\left[-\int_0^t \left(\mu_{x+r}+\sigma_{x+r}\right)dr\right]\sigma_{x+t}\,dt$$

This makes no assumptions and provides an answer in the form asked for in the question, and so would merit full marks. If constant forces are assumed, the above expression will turn into the answer in the above solution.

For part (ii) a solution is only possible if some assumption is made. The following alternatives could be valid:

(1) Assume dependent decrements are uniformly distributed over the year of age

With this assumption, deaths occur on average at age $x+½$*, so:*

$$q_x^r = \frac{(ad)_x^r+\frac{1}{2}(ad)_x^d\times q_x^r}{(al)_x} = (aq)_x^r+\tfrac{1}{2}(aq)_x^d\times q_x^r \Rightarrow q_x^r = \frac{(aq)_x^r}{1-\frac{1}{2}(aq)_x^d}$$

(This is covered by the Core Reading in Unit 8 Section 10.1.3.)

(2) Assume independent decrements are uniformly distributed over the year of age

This leads to two simultaneous equations:

$$q_x^d = \frac{(aq)_x^d}{1-½q_x^r} \qquad \text{and} \qquad q_x^r = \frac{(aq)_x^r}{1-½q_x^d}$$

which results in a quadratic equation in q_x^r. *(This is covered by the Core Reading Unit 8 Section 10.1.6.)*

Whilst a full description has been given above to assist students, in reality those who successfully attempted this question did assume constant forces.

10 First calculate $(aq)_x^d$ and $(aq)_x^w$

Age (x)	*Number of employees* $(al)_x$	$(aq)_x^d$	$(aq)_x^w$
40	10,000	.00250	.01200
41	9,855	.00274	.01461
42	9,684		

From this table and relationship

$$q_x^d = (aq)_x^d / (1 - \frac{1}{2} * (aq)_x^w) \text{ and } q_x^w = (aq)_x^w / (1 - \frac{1}{2} * (aq)_x^d)$$

Calculate q_x^d and q_x^w

q_{40}^d =.00250/(1–.006) = .00252 and q_{41}^d = .00274/(1–.00731) = .00276

q_{40}^w =.01200/(1–.00125) = .01201 and q_{41}^w = .01461/(1–.00137) = .01463

Adjusting for the 75% multiplier of independent withdrawal decrements:

$$(aq)_{40}^d = .00252 * \left(1 - \frac{1}{2} * \frac{3}{4} * .01201\right) = .00251$$

$$(aq)_{41}^d = .00276 * \left(1 - \frac{1}{2} * \frac{3}{4} * .01463\right) = .00274$$

$$(aq)_{40}^w = .01201 * \frac{3}{4} * \left(1 - \frac{1}{2} * .00252\right) = .00900$$

$$(aq)_{41}^w = .01463 * \frac{3}{4} * \left(1 - \frac{1}{2} * .00276\right) = .01096$$

Using the above data the Table can now be reconstructed

Age (x)	*Number of employees* $(al)_x$	*Deaths* $(ad)_x^d$	*Withdrawals* $(ad)_x^w$
40	10,000	(10000*.00251)=25.1	10000*.00900=90.0
41	9,884.9	(9,884.9*.00274)=27.1	9,884.9*.01096=108.3
42	9749.5		

It should be noted that if more decimal places are used in the aq factors then the deaths at 40 become 25.0 so full credit was given for this answer also.

Because of the limited effect on the answer from the original table students were asked to show the result to 1 decimal place. Many failed to do so and were penalised accordingly.

11 (i) Policy value at duration *t* of an immediate annuity payable continuously at a rate of £1 per annum and secured by a single premium at age *x* is given by:

$$ {}_t\bar{V}_x = \bar{a}_{x+t} = \int_0^\infty e^{-\delta s} {}_s p_{x+t} ds $$

$$ \Rightarrow \frac{\partial}{\partial t} {}_t\bar{V}_x = \frac{\partial}{\partial t}\bar{a}_{x+t} = \frac{\partial}{\partial t}\int_0^\infty e^{-\delta s} {}_s p_{x+t} ds = \int_0^\infty e^{-\delta s} \frac{\partial}{\partial t} {}_s p_{x+t} ds $$

$$ \frac{1}{{}_s p_{x+t}} \times \frac{\partial}{\partial t} {}_s p_{x+t} = \frac{\partial}{\partial t}\ln({}_s p_{x+t}) = \frac{\partial}{\partial t}(\ln l_{x+t+s} - \ln l_{x+t}) = -\mu_{x+t+s} + \mu_{x+t} $$

$$ \Rightarrow \frac{\partial}{\partial t} {}_s p_{x+t} = {}_s p_{x+t}(-\mu_{x+t+s} + \mu_{x+t}) $$

$$ \Rightarrow \frac{\partial}{\partial t} {}_t\bar{V}_x = \int_0^\infty e^{-\delta s} {}_s p_{x+t}(\mu_{x+t} - \mu_{x+t+s}) ds $$

$$ = \mu_{x+t} \times \bar{a}_{x+t} - \int_0^\infty e^{-\delta s} {}_s p_{x+t} \times \mu_{x+t+s} ds $$

$$ = \mu_{x+t} \times \bar{a}_{x+t} - \left\{ \left[-e^{-\delta s} {}_s p_{x+t} \right]_0^\infty - \delta \int_0^\infty e^{-\delta s} {}_s p_{x+t} ds \right\} $$

$$ = \mu_{x+t} \times \bar{a}_{x+t} - 1 + \delta \times \bar{a}_{x+t} $$

$$= \mu_{x+t} \times {}_t\bar{V}_x - 1 + \delta \times {}_t\bar{V}_x$$

(ii) Consider a short time interval $(t, t + dt)$ then equation implies:

$${}_{t+dt}\bar{V} - {}_t\bar{V} = \mu_{x+t} \times {}_t\bar{V}_x \times dt - 1 \times dt + \delta \times {}_t\bar{V}_x \times dt + o(dt)$$

where

$\mu_{x+t} \times {}_t\bar{V}_x \times dt$ = reserve released as a result of deaths in time interval $(t, t + dt)$

$-1 \times dt$ = annuity payments made in time interval $(t, t + dt)$

$\delta \times {}_t\bar{V}_x \times dt$ = interest earned on reserve over time interval $(t, t + dt)$

In general very poorly answered on what was a standard bookwork question.

12 (i) Annual premium P for the term assurance policy is given by:

$$P = \frac{25{,}000\bar{A}^{1}_{[55]:\overline{10|}} + 25{,}000\bar{A}^{1}_{[55]:\overline{5|}}}{\ddot{a}_{[55]:\overline{10|}}}$$

where

$$25{,}000\bar{A}^{1}_{[55]:\overline{10|}} + 25{,}000\bar{A}^{1}_{[55]:\overline{5|}}$$

$$= 25{,}000 \times (1+i)^{1/2} \times \left((A_{[55]} - v^{10}\,{}_{10}p_{[55]}A_{65}) + (A_{[55]} - v^{5}\,{}_{5}p_{[55]}A_{60}) \right)$$

$$= 25{,}000 \times 1.019804 \times \begin{pmatrix} (0.38879 - 0.67556 \times \dfrac{8821.2612}{9545.9929} \times 0.52786) \\ +(0.38879 - 0.82193 \times \dfrac{9287.2164}{9545.9929} \times 0.4564) \end{pmatrix}$$

$$= 25{,}495.10 \times \left((0.38879 - 0.32953) + (0.38879 - 0.36496) \right) = 2118.39$$

Therefore

$$P = \frac{2118.39}{8.228} = 257.46$$

Net Premium Retrospective Reserves at the end of the fifth policy year is given by:

$$(1+i)^5 \times \frac{l_{[55]}}{l_{60}} \times \left[P\ddot{a}_{[55]:\overline{5}|} - 50,000\bar{A}^{1}_{[55]:\overline{5}|} \right]$$

$$= 1.21665 \times \frac{9545.9929}{9287.2164} \times \left[257.46 \times 4.59 - 50,000 \times 1.019804 \times (0.38879 - 0.36496) \right]$$

$$= -41.71$$

(ii) **Explanation** – more cover provided in the first 5 years than is paid for by the premiums in those years. Hence policyholder "in debt" at time 5, with size of debt equal to negative reserve.

Disadvantage – if policy lapsed during the first 5 years (and possibly longer), the company will suffer a loss which is not possible to recover from the policyholder.

Possible alterations to policy structure

Collect premiums more quickly by shortening premium payment term or make premiums larger in earlier years, smaller in later years

Change the pattern of benefits to reduce benefits in first 5 years and increase them in last 5 years.

(iii) Mortality Profit = EDS – ADS

Death strain at risk = 50,000 – (–42) = 50,042

$$EDS = (1000 - 20) \times q_{59} \times 50,042$$
$$= 980 \times 0.00714 \times 50,042 = 350,154$$

$$ADS = 8 \times 50,042 = 400,336$$

Total Mortality Profit = 350,154 – 400,336 = -£50,182 (i.e. a mortality loss)

Quite reasonably answered by the well prepared student.

In (i) it should be noted that in this case the retrospective and prospective reserves are equal. If the student recognised this, explicitly stated so and then did the easier prospective calculation full marks were given. No credit was given for a prospective calculation without explanation.

13

Annual premium	£4000.00	Allocation % (1st yr)	95.0%
Risk discount rate	7.0%	Allocation % (2nd yr)	100.0%
Interest on investments (1st yr)	5.5%	Allocation % (3rd yr)	105.0%
Interest on investments (2nd yr)	5.25%	B/O spread	5.0%
Interest on investments (3rd yr)	5.0%	Management charge	1.75%
Interest on non-unit funds	4.0%	Surrender penalty (1st yr)	£1000
Death benefit (% of bid value of units)	125%	Surrender penalty (2nd yr)	£500
		Policy Fee	£50

	£	*% prem*
Initial expense	200	15.0%
Renewal expense	50	2.0%
Expense inflation	2.0%	

(i) Multiple decrement table:

x	q_x^d	q_x^s
45	0.001201	0.12
46	0.001557	0.06
47	0.001802	0.00

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$
45	0.001201	0.11986	0.878943	1.000000
46	0.001557	0.05991	0.938536	0.878943
47	0.001802	0.00000	0.998198	0.824920

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
value of units at start of year	0.000	3690.074	7693.641
Alloc	3800.000	4000.000	4200.000
B/O	190.000	200.000	210.000
policy fee	50.000	50.000	50.000
Interest	195.800	390.604	581.682
management charge	65.727	137.037	213.768
value of units at year end	3690.074	7693.641	12001.554

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
unallocated premium + pol fee	250.000	50.000	–150.000
B/O spread	190.000	200.000	210.000
expenses	800.000	131.000	132.020
Interest	–14.400	4.760	–2.881
man charge	65.727	137.037	213.768
extra death benefit	1.108	2.995	5.407
surrender penalty	119.856	29.953	0.000
end of year cashflow	–189.926	287.755	133.461
probability in force	1	0.878943	0.824920
discount factor	0.934579	0.873439	0.816298
expected p.v. of profit	133.280		
premium signature	4000.000	3285.769	2882.069
expected p.v. of premiums	10167.837		
profit			
Margin	1.31%		

(ii) Revised profit vector (–309.781, 257.802, 133.461)
Revised profit signature (–309.781, 257.492, 133.093)

Revised PVFNP = –289.515 + 224.904+ 108.643 = 44.032

Again most well prepared students made a good attempt at this question. The most common error was to ignore dependent decrements.

Substantial credit was given to students who showed how they would tackle this question even if they did not complete all the arithmetical calculations involved.

14

(i) Let P be the quarterly premium. Then:

EPV of premiums:

$$4P\ddot{a}^{(4)}_{[35]:\overline{30|}} @ 6\% = 56.1408P$$

where

$$\ddot{a}^{(4)}_{[35]:\overline{30|}} = \ddot{a}_{[35]:\overline{30|}} - \frac{3}{8}\left(1 - {}_{30}p_{[35]}v^{30}\right)$$

$$= 14.352 - \frac{3}{8}\left(1 - \frac{8821.2612}{9892.9151} \times 0.17411\right)$$

$$= 14.0352$$

EPV of benefits:

$$100,000(q_{[35]}v^{0.5} + {}_{1|}q_{[35]}(1+b)v^{1.5} + \ldots + {}_{29|}q_{[35]}(1+b)^{29}v^{29.5})$$

$$+100,000 \times (1+b)^{30}v^{30}\ {}_{30}p_{[35]}$$

where $b = 0.0192308$

$$= \frac{100,000}{(1+b)^{0.5}} \times \frac{(1.06)^{0.5}}{(1+b)^{0.5}}\left(q_{[35]}(1+b)v\ + {}_{1|}q_{[35]}(1+b)^2v^2 + \ldots + {}_{29|}q_{[35]}(1+b)^{30}v^{30}\right)$$

$$+100,000(1+b)^{30}v^{30}\ {}_{30}p_{[35]}$$

$$= \frac{100,000}{(1+b)} \times (1.06)^{0.5} \times A^{1}_{[35]:\overline{30}|} @\, i' + 100,000v^{30}\ {}_{30}p_{[35]} @\, i'$$

$$= \frac{100,000 \times (1.06)^{0.5}}{(1+b)} \times \left(0.32187 - 0.30832 \times \frac{8821.2612}{9892.9151}\right)$$

$$+100,000 \times 0.30832 \times \frac{8821.2612}{9892.9151}$$

$$= 4,742.594 + 27,492.112 = 32,234.706$$

where

$$i' = \frac{1.06}{1+b} - 1 = 0.04$$

EPV of expenses (at 6%)

$$= P + 250 + 0.025 \times 4P\ddot{a}^{(4)}_{[35]:\overline{30}|} - 0.025 \times 4P\ddot{a}^{(4)}_{[35]:\overline{1}|} + 45\left[\ddot{a}_{[35]:\overline{30}|} - 1\right]$$

$$+500\bar{A}^{1}_{[35]:\overline{30}|} + 250v^{30}\ {}_{30}p_{[35]}$$

$$= P + 250 + 0.025 \times 56.1408P - 0.025 \times 4P \times 0.97857 + 45 \times 13.352$$

$$+500\times 1.06^{0.5}\left(0.18763-0.17411\times\frac{8821.2612}{9892.9151}\right)+250\times 0.17411\times\frac{8821.2612}{9892.9151}$$

$$=2.30566P+906.322$$

where

$$\ddot{a}^{(4)}_{[35]:\overline{1}|}=\ddot{a}_{[35]:\overline{1}|}-\frac{3}{8}\left(1-p_{[35]}v\right)$$

$$=1-\frac{3}{8}\left(1-\frac{9887.2069}{9892.9151}\times 0.9434\right)=0.97857$$

Equation of value gives:

$$56.1408P=32,234.706+2.30566P+906.322$$

$$\Rightarrow P=\frac{33,141.028}{53.8351}=£615.60$$

(ii) Gross prospective policy value (calculated at 4%) is given by:

$$V^{\text{prospective}}=\frac{245,000}{(1+b)}(1+i)^{1/2}A_{\substack{1\\60:\overline{5}|}}@i''+245,000\times v^5\,{}_5p_{60}@i''+0.025\times 4P\ddot{a}^{(4)}_{60:\overline{5}|}+90\ddot{a}_{60:\overline{5}|}-4P\ddot{a}^{(4)}_{60:\overline{5}|}$$
$$+1000\bar{A}^1_{60:\overline{5}|}+500v^5\,{}_5p_{60}$$

$$=\frac{245,000}{(1.04)^{0.5}}\times A^1_{60:\overline{5}|}@i''+245,000\times v^5\frac{l_{65}}{l_{60}}@i''+90\ddot{a}_{60:\overline{5}|}-0.975\times 4P\ddot{a}^{(4)}_{60:\overline{5}|}$$

$$+1000\times 1.04^{0.5}\left(A_{60:\overline{5}|}-v^5\frac{l_{65}}{l_{60}}\right)+500v^5\frac{l_{65}}{l_{60}}$$

where $\ddot{a}^{(4)}_{60:\overline{5}|}=\ddot{a}_{60:\overline{5}|}-\frac{3}{8}\left(1-v^5\times\frac{l_{65}}{l_{60}}\right)=4.55-\frac{3}{8}\left(1-0.82193\times\frac{8821.2612}{9287.2164}\right)=4.4678$

and $i''=\frac{1.04}{1.04}-1=0 \quad \Rightarrow \bar{A}^1_{60:\overline{5}|}@i''=\frac{\sum_0^4 d_{60+t}}{l_{60}}=\frac{465.9551}{9287.2164}=0.05017$

$$=\frac{245,000}{(1.04)^{0.5}}\times 0.05017+245,000\times 0.94983+90\times 4.55-0.975\times 4\times 615.60\times 4.4678$$

$$+1000\times 1.04^{0.5}\left(0.82499-0.78069\right)+500\times 0.78069$$

$$= 12,052.954 + 232,708.35 + 409.5 - 10,726.473 + 45.177 + 390.345 = 234,880$$

Part (i) answered reasonably well. Students had more problems with (ii)

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2010 examinations

Subject CT5 — Contingencies
Core Technical

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

T J Birse
Chairman of the Board of Examiners

December 2010

1 (a) $${}_{20|10}q_{[45]} = (l_{65} - l_{75}) / l_{[45]}$$
$$= (8,821.2612 - 6,879.1673) / 9,798.0837 = 0.198212$$

(b) $${}_{30}p_{[45]:[50]} = \frac{l_{75}}{l_{[45]}} \frac{l_{80}}{l_{[50]}} = \frac{6,879.1673}{9,798.0837} \frac{5,266.4604}{9,706.0977} = 0.380951$$

Question generally done well.

2 $${}_{.5}p_{45.75} = {}_{.25}p_{45.75} * {}_{.25}p_{46}$$

$${}_{.25}q_{45.75} = .25 * q_{45} / (1 - .75 * q_{45}) = .25 * .001465 / (1 - .75 * .001465)$$
$$= .000367 \text{ by UDD}$$
$${}_{.25}q_{46} = .25 * q_{46} = .25 * .001622 = .000406$$

Hence ${}_{.5}p_{45.75} = (1 - .000367) * (1 - .000406) = .999227$

In general question done well. However many students did not appreciate the split in line 1 above and attempted to apply formula directly.

3 Value of Single Premium is:

$$12 \times 1,000 \times \left(a^{(12)}_{55:\overline{20|}} - a^{(12)}_{50:55:\overline{20|}} \right)$$
$$= 12,000\left(\left[\left(\ddot{a}_{55} - \tfrac{13}{24}\right) - v^{20}\,{}_{20}p_{55}\left(\ddot{a}_{75} - \tfrac{13}{24}\right)\right] - \left[\left(\ddot{a}_{50:55} - \tfrac{13}{24}\right) - v^{20}\,{}_{20}p_{50:55}\left(\ddot{a}_{70:75} - \tfrac{13}{24}\right)\right]\right)$$
$$= 12,000\left(\left[\left(18.210 - \tfrac{13}{24}\right) - v^{20}\frac{8784.955}{9917.623}\left(10.933 - \tfrac{13}{24}\right)\right]\right.$$
$$\left. - \left[\left(16.909 - \tfrac{13}{24}\right) - v^{20}\frac{8784.955}{9917.623}\frac{9238.134}{9941.923}\left(8.792 - \tfrac{13}{24}\right)\right]\right)$$
$$= 12,000((17.668 - 4.201) - (16.367 - 3.099))$$
$$= 2,388$$

Many students struggled with how to break down the monthly annuity functions into those which could then utilise the Tables. However question generally done well by well prepared students.

4 The value of 1 per annum payable monthly for 1 year is

$$\ddot{a}^{(12)}_{x:\overline{1}|} = \ddot{a}^{(12)}_{x} - v.p_x \ddot{a}^{(12)}_{x+1} = \ddot{a}_{x:\overline{1}|} - 11/24(1 - v.p_x)$$

Where $\ddot{a}_{x:\overline{1}|} = 1$

Therefore

$$\ddot{a}^{(12)}_{x:\overline{1}|} = 1 - 11/24(1 - 0.99/1.06) = 0.96973$$

The probability of reaching the beginning of each year is :

Year $1 = 1$
Year 2 = 0.99*0.8 = 0.792
Year 3 = 0.792 * 0.792 = 0.6273

The value is therefore

$$120 \times 240 \times 0.96973 \times (1 + 0.792/1.06 + 0.6273/(1.06)^2)$$
$$= 64,388$$

This question was overall done very poorly with few students realising that the key element to the calculation involved a one year annuity due payable monthly.

5 The formula is:

$$25000\left[\sum_{t=15}^{19} \frac{12}{80} \frac{z_{45+t+0.5}}{s_{44}} \frac{r_{45+t}}{l_{45}} \frac{(rl)_{66+t}}{(rl)_{45+t+0.5}}\right] + 25000\left[\frac{12}{80} \frac{z_{65}}{s_{44}} \frac{r_{65}}{l_{45}} \frac{(rl)_{66}}{(rl)_{65}}\right]$$

Question done very poorly. Many students attempted to use annuity functions whereas the question sought was a pure cash flow one.

6 (a)

$$\bar{A}_{\overline{30:40}} = \int_0^\infty e^{-.04t}\{e^{-.01t}(1-e^{-.02t})*.01+e^{-.02t}(1-e^{-.01t})*.02\}dt$$
$$= \int_0^\infty \{.01*(e^{-.05t}-e^{-.07t})+.02*(e^{-.06t}-e^{-.07t})\}dt$$
$$= \int_0^\infty (.01*e^{-.05t}+.02*e^{-.06t}-.03*e^{-.07t})dt$$
$$= \left[-\frac{.01}{.05}*e^{-.05t}-\frac{.02}{.06}*e^{-.06t}+\frac{.03}{.07}*e^{-.07t}\right]_0^\infty$$
$$= (1/5+1/3-3/7) = .10476$$

(b)

$$\bar{a}_{30:40:\overline{20|}} = \int_0^{20} e^{-.04t}*e^{-.01t}*e^{-.02t}dt$$
$$= \int_0^{20} e^{-.07t}dt$$
$$= \left[-\frac{1}{.07}e^{-.07t}\right]_0^{20}$$
$$= (1/.07) - e^{-1.4}/.07) = 10.763$$

Question generally done well.

7 Let P be the monthly premium. Then equating expected present value of premiums and benefits gives:

$$12P\ddot{a}^{(12)}_{[55]:\overline{10|}} = 45000\bar{A}^{1}_{[55]:\overline{10|}} + 5000(I\bar{A})^{1}_{[55]:\overline{10|}}$$

where

$$\ddot{a}^{(12)}_{[55]:\overline{10|}} = \ddot{a}_{[55]:\overline{10|}} - \frac{11}{24}\left(1-v^{10}\times {}_{10}p_{[55]}\right) = 8.228-0.458\left(1-.67556\times\frac{8821.2612}{9545.9929}\right) = 8.056$$
$$\bar{A}^{1}_{[55]:\overline{10|}} = 1.04^{0.5}\left(A_{[55]:\overline{10|}} - v^{10}\times {}_{10}p_{[55]}\right) = 1.04^{0.5}\left(0.68354-0.62427\right) = 0.06044$$
$$(I\bar{A})^{1}_{[55]:\overline{10|}} = 1.04^{0.5}\left((IA)_{[55]} - v^{10}\times {}_{10}p_{[55]}\times(IA)_{65} - 10v^{10}\times {}_{10}p_{[55]}\times A_{65}\right)$$
$$= 1.04^{0.5}(8.58908-0.62427\times 7.89442-10\times 0.62427\times 0.52786) = 0.3728$$
$$\Rightarrow 12P = \frac{45000\times 0.06044+5000\times 0.3728}{8.056} = 568.99$$
$$\Rightarrow P = £47.42$$

In general question done well by well prepared students.

8 Occupation – either because of environmental or lifestyle factors mortality may be directly affected. Occupations may also have health barriers to entry, e.g. airline pilots

Nutrition – poor quality nutrition increases morbidity and hence mortality

Housing – standard of housing (reflecting poverty) increases morbidity

Climate – climate can influence morbidity and may also be linked to natural disaster

Education – linked to occupation but better education can reduce morbidity, e.g. by reducing smoking

Genetics – there is genetic evidence of a predisposition to contracting certain illnesses, even if this has no predictive capability

A straightforward bookwork question generally done well although not all students captured the full range. All valid examples not shown above were credited.

Students who misunderstood the question and tried to answer using Class, Time, Temporary Initial Selection were given no credit.

9 Use the formula

$$q_x^{\alpha} = \frac{(aq)_x^{\alpha}}{(1-0.5((aq)_x^{-\alpha}))}$$

to derive the independent probabilities:

$$q_x^{d} = \frac{(aq)_x^{d}}{(1-0.5((aq)_x^{-d}))} = \frac{(50/6548)}{(1-0.5*((219+516)/6548))} = 0.00809$$

$$q_x^{i} = \frac{(aq)_x^{i}}{(1-0.5((aq)_x^{-i}))} = \frac{(219/6548)}{(1-0.5*((50+516)/6548))} = 0.03496$$

$$q_x^{r} = \frac{(aq)_x^{r}}{(1-0.5((aq)_x^{-r}))} = \frac{(516/6548)}{(1-0.5*((50+219)/6548))} = 0.080455$$

Then the revised $q_x^d = 80\% * 0.00809 = 0.006472$
then use the formula

$$(aq)_x^{\alpha} = q_x^{\alpha}(1-\frac{1}{2}(q_x^{\beta}+...)+\frac{1}{3}(q_x^{\beta}.q_x^{\gamma}+...)-...)$$

to derive dependent probabilities:

$$(aq)_x^d = q_x^d(1-\frac{1}{2}(q_x^i + q_x^r) + \frac{1}{3}(q_x^i.q_x^r)) = 0.0061046$$

$$(aq)_x^i = q_x^i(1-\frac{1}{2}(q_x^d + q_x^r) + \frac{1}{3}(q_x^d.q_x^r)) = 0.0334465$$

$$(aq)_x^r = q_x^r(1-\frac{1}{2}(q_x^d + q_x^i) + \frac{1}{3}(q_x^d.q_x^i)) = 0.0787948$$

The resulting service table is:

l_x	d_x	i_x	r_x
6,548	40	219	516

This question was done poorly. Many students appeared not to remember the derivation process for multiple decrements etc. Some students wrote down the final table without showing intermediate working. This gained only a proportion of the marks.

10 (a) Crude mortality rate = actual deaths / total exposed to risk

$$= \frac{\sum_x E_{x,t}^c m_{x,t}}{\sum_x E_{x,t}^c}$$

where

$E_{x,t}^c$ is central exposed to risk in population between age x and $x+t$
$m_{x,t}$ is central rate of mortality in population between age x and $x+t$

(b) Indirectly standardised mortality rate

$$= \frac{\sum_x {}^sE_{x,t}^c\,{}^sm_{x,t}}{\sum_x {}^sE_{x,t}^c} \Bigg/ \frac{\sum_x E_{x,t}^c\,{}^sm_{x,t}}{\sum_x E_{x,t}^c m_{x,t}}$$

${}^{s}E^{c}_{x,t}$ is central exposed to risk in standard population between age x and $x+t$

${}^{s}m_{x,t}$ is central rate of mortality in standard population between age x and $x+t$

This question generally done well. Other symbol notation was accepted provided it was consistent and properly defined.

11

Year t	q_x	p_x	${}_{t-1}p_x$	$NUCF_t$	*Profit Signature*
1	0.01	0.99	1	–50.2	–50.2
2	0.01	0.99	0.99	–43.1	–42.7
3	0.01	0.99	0.9801	–32.1	–31.5
4	0.01	0.99	0.9703	145.5	141.2

(i) PV of profit @ 6%

$$= -50.2v - 42.7v^2 - 31.5v^3 + 141.2v^4$$
$$= -47.4 - 38.0 - 26.4 + 111.8$$
$$= 0.0 \quad \Rightarrow IRR = 6\%$$

(ii) $${}_2V = \frac{32.1}{1.025} = 31.3$$

$${}_1V \times 1.025 - p_x \times {}_2V = 43.1 \Rightarrow {}_1V = 72.3$$

revised cash flow in year 1 $= -50.2 - p_x \times {}_1V = -50.2 - 71.6 = -121.8$

and NPV of profit = –121.8/1.06 + 111.8 = -3.1

(iii) As expected, the *NPV* after zeroisation is smaller because the emergence of the non- unit cash flow losses have been accelerated and the risk discount rate is greater than the accumulation rate.

Parts (i) and (iii) done well generally. In Part (ii) many students failed to develop the formulae properly although they realised the effect in (iii).

12 (i) The gross future loss random variable is

$$50,000\left[1+b\left(K_{40}+1\right)\right]v^{T_{40}}+(I-e)+e\ddot{a}_{\overline{K_{40}+1}|}+fv^{T_{40}}-P\ddot{a}_{\overline{\min(K_{40}+1,25)}|}$$

Note: select functions also acceptable

where b is the annual rate of bonus
I is the initial expense
e is the annual renewal expense payable in the 2nd and subsequent years
f is the claim expense
P is the gross annual premium
$K_{40}(T_{40})$ is the curtate (complete) random future lifetime of a life currently aged 40

(ii) The annual premium P is given by

$$P\ddot{a}_{[40]:\overline{25}|}=50,250\bar{A}_{[40]}+1,250\left(I\bar{A}\right)_{[40]}+300+25(\ddot{a}_{[40]}-1)$$

$$\Rightarrow P\times 13.29=50,250\times 1.06^{0.5}\times 0.12296+1,250\times 1.06^{0.5}\times 3.85489$$
$$+300+25(15.494-1)$$

$$\Rightarrow 13.29P=6361.402+4961.065+300+362.35$$

$$\Rightarrow P=£901.79$$

(iii) The required reserve is

$$64,000\bar{A}_{50}+1,500\left(I\bar{A}\right)_{50}+35\ddot{a}_{50}-901.79\times\ddot{a}_{50:\overline{15}|}$$

$$=64,000\times 1.04^{0.5}\times 0.32907+1,500\times 1.04^{0.5}\times 8.55929$$
$$+35\times 17.444-901.79\times 11.253$$

$$=21,477.560+13,093.196+610.54-10,147.84$$

$$=£25,033.32$$

In general question done well by well prepared students. In (i) credit also given if the formulae included a limited term on the expense element although in reality this is unlikely.

13 (i) Let P be the annual premium. Then equating expected present value of premiums and benefits gives:

$$P\ddot{a}_{\overline{60^m:55^f}} = 100000\bar{A}_{\overline{60^m:55^f}}$$

where $\ddot{a}_{\overline{60^m:55^f}} = \ddot{a}_{60^m} + \ddot{a}_{55^f} - \ddot{a}_{60^m:55^f} = 15.632 + 18.210 - 14.756 = 19.086$

$$\bar{A}_{\overline{60^m:55^f}} = 1.04^{0.5} \times A_{\overline{60^m:55^f}} = 1.04^{0.5} \times (1 - d \times \ddot{a}_{\overline{60^m:55^f}})$$

$= 1.04^{0.5} \times (1 - 0.038462 \times 19.086) = 0.2711804$

$\therefore P \times 19.086 = 100000 * 0.2711804$

$\Rightarrow P = £1{,}420.83$.

(ii) Reserves at the end of the first policy year:

- Where both lives are alive:

$$100000 \times 1.04^{0.5} \times \left(1 - \frac{\ddot{a}_{\overline{61^m:56^f}}}{\ddot{a}_{\overline{60^m:55^f}}}\right)$$

$$= 100000 \times 1.04^{0.5} \times \left(1 - \frac{15.254 + 17.917 - 14.356}{15.632 + 18.210 - 14.756}\right) = 1448.01$$

- Where the male life is alive only:

$$100000\bar{A}_{61^m} - P\ddot{a}_{61^m}$$

$$100000 \times 1.04^{0.5} \times \left(1 - \frac{0.04}{1.04} \times 15.254\right) - 1420.83 \times 15.254 = 20475.94$$

- Where the female life is alive only:

$$100000\bar{A}_{56^f} - P\ddot{a}_{56^f}$$

$$100000 \times 1.04^{0.5} \times \left(1 - \frac{0.04}{1.04} \times 17.917\right) - 1420.83 \times 17.917 = 6247.12$$

Mortality Profit = Expected Death Strain – Actual Death Strain

(a) Both lives die during 2009 = 1 actual claim.

Mortality Profit

$$= \left(10,000 \times q_{60^m} \times q_{55^f} - 1\right) \times \left(100000 \times 1.04^{0.5} - 1448.01\right)$$
$$= \left(10,000 \times 0.002451 \times 0.001046 - 1\right) \times \left(100532.38\right) = -97954.99$$

(b) Males only die during 2009 = 20 actual deaths (and therefore we need to change reserve from joint life to female only surviving).

Mortality Profit

$$= \left(10,000 \times p_{55^f} \times q_{60^m} - 20\right) \times \left(6247.12 - 1448.01\right)$$
$$= \left(10,000 \times 0.998954 \times 0.002451 - 20\right) \times \left(4799.11\right) = 21520.95$$

(c) Females only die during 2009 = 10 actual deaths (and therefore we need to change reserve from joint life to male only surviving).

Mortality Profit

$$= \left(10,000 \times p_{60^m} \times q_{55^f} - 10\right) \times \left(20475.94 - 1448.01\right)$$
$$= \left(10,000 \times 0.997549 \times 0.001046 - 10\right) \times \left(19027.93\right) = 8265.02$$

Hence overall total mortality profit

$$= -97954.99 + 21520.95 + 8265.02 = -£68,169.01$$

i.e. a mortality loss

Part (i) generally done well. Part (ii) was challenging and few students realised the full implications of "reserve change" on 1st death. Only limited partial credit was given if students used only joint life situations.

14 Reserves required on the policy per unit sum assured are:

$$ {}_0V_{56:\overline{4|}} = 1 - \frac{\ddot{a}_{56:\overline{4|}}}{\ddot{a}_{56:\overline{4|}}} = 0 $$

$$ {}_1V_{56:\overline{4|}} = 1 - \frac{\ddot{a}_{57:\overline{3|}}}{\ddot{a}_{56:\overline{4|}}} = 1 - \frac{2.870}{3.745} = 0.23364 $$

$$ {}_2V_{56:\overline{4|}} = 1 - \frac{\ddot{a}_{58:\overline{2|}}}{\ddot{a}_{56:\overline{4|}}} = 1 - \frac{1.955}{3.745} = 0.47797 $$

$$ {}_3V_{56:\overline{4|}} = 1 - \frac{\ddot{a}_{59:\overline{1|}}}{\ddot{a}_{56:\overline{4|}}} = 1 - \frac{1.0}{3.745} = 0.73298 $$

Multiple decrement table:

T	$q^d_{[56]+t-1}$	$q^s_{[56]+t-1}$	$(aq)^d_{[56]+t-1}$	$(aq)^s_{[56]+t-1}$	$(ap)_{[56]+t-1}$	${}_{t-1}(ap)_{[56]}$
1	0.003742	0.1	0.003742	0.09963	0.896632	1.000000
2	0.005507	0.1	0.005507	0.09945	0.895044	0.896632
3	0.006352	0.1	0.006352	0.09936	0.894283	0.802525
4	0.007140	0.0	0.007140	0.0	0.992860	0.717685

Probability in force $(ap)_{[56]+t-1} = (1 - q^d_{[56]+t-1}) \times (1 - q^s_{[56]+t-1})$

The calculations of the profit vector, profit signature and NPV are set out in the table below:

Policy year	*Premium*	*Expenses*	*Interest*	*Death claim*	*Maturity claim*	*Surrender claim*	*In force cash flow*
1	5000	600.00	176.00	80.45	0.00	350.31	4145.23
2	5000	45.00	198.20	118.40	0.00	715.38	4319.42
3	5000	45.00	198.20	136.57	0.00	1096.1613	3920.475
4	5000	45.00	198.20	153.51	21346.49	0.00	–16346.80

Policy year	*Increase in reserves*	*Interest on reserves*	*Profit vector*	*Cum probability of survival*	*Discount factor*	*NPV profit*
1	4504.02	0.00	–358.78	1.00000	0.943396	–338.47
2	4174.53	200.93	345.82	0.89663	0.890000	275.96
3	3816.72	411.05	514.84	0.80253	0.839619	346.91
4	–15759.07	630.36	42.63	0.71768	0.792094	24.24

Total *NPV* = 308.63

The calculations of the premium signature and profit margin are set out in the table below:

Policy year	*1*	*2*	*3*	*4*
Premium	5000.00	5000.00	5000.00	5000.00
probability in force	1.00000	0.89663	0.80253	0.71768
discount factor	1.00000	0.943396	0.890000	0.839619
p.v. of premium signature	5000.000	4229.40	3571.22	3012.91
=> expected p.v. of premiums	**15813.53**			
profit margin =	**2.0%**			

Many well prepared students were able to outline the process required without being totally accurate on the calculation. Significant credit was awarded in such situation.

Many students failed to appreciate the multiple decrement element.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2011 examinations

Subject CT5 — Contingencies
Core Technical

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

T J Birse
Chairman of the Board of Examiners

July 2011

1 (a) Time selection – because it is based on a period of three calendar years
(b) Class selection – applies only to male pensioners
(c) Temporary initial selection – as there are select rates

Other valid answers acceptable

This question was generally done well. However some students did not supply <u>different</u> selection types for each part and this was penalised.

2 (a) $${}_{23}p_{65} = \frac{l_{88}}{l_{65}} = \frac{3534.054}{9647.797} = 0.366307$$

(b) $${}_{10|5}q_{60} = \frac{(l_{70} - l_{75})}{l_{60}} = \frac{(9238.134 - 8405.160)}{9826.131} = 0.084771$$

(c)

$$\ddot{s}_{65:\overline{10}|} = \frac{(1+i)^{10}\ddot{a}_{65:\overline{10}|}}{{}_{10}p_{65}} = \frac{(1+i)^{10}(\ddot{a}_{65} - v^{10}\,{}_{10}p_{65}\ddot{a}_{75})}{{}_{10}p_{65}}$$

$$= \frac{(1.04)^{10}(13.666 - (1.04^{-10}) \times \left(8,405.160\big/9,647.797\right) \times 9.456)}{\left(8,405.160\big/9,647.797\right)}$$

$$= 1.48024 \times (13.666 - 0.67556 \times 0.87120 \times 9.456) / 0.87120$$

$$= 13.764$$

This question was generally done well for parts (a) and (b) but students struggled more with part (c).

3

$$(I\bar{a})_x = \int_0^1 v^t\,{}_tp_x dt + 2\int_1^2 v^t\,{}_tp_x dt + 3\int_2^3 v^t\,{}_tp_x dt + ...$$

Now $vp_x = e^{-.04} * e^{-.02} = e^{-.06}$ throughout.

Hence

$(I\bar{a})_x = (1 + 2e^{-0.06} + 3(e^{-.06})^2 + 4(e^{-.06})^3 + ...)\bar{a}_{\overline{1}|}$ at force of interest 6%

$= (1/(1 - e^{-.06}))^2 \times ((1 - e^{-.06}) / .06)$

$= 294.8662 \times 0.970591$

$= 286.19$

This question was not done well. The majority of students failed to realise that the increasing function I was not continuous, although the payment ā is continuous. Instead most attempted to compute $(\bar{I}\bar{a})_x = \int_0^\infty tv^t\,{}_tp_x dt$ *. Only minimal credit was given for this.*

4 Schemes usually allow members to retire on grounds of ill-health and receive a pension benefit after a minimum length of scheme service.

Benefits are usually related to salary at the date of ill-health retirement in similar ways to age retirement benefits.

However, pensionable service is usually more generous than under age retirement with years beyond those served in the scheme being credited to the member e.g. actual pensionable service subject to a minimum of 20 years, or pensionable service that would have been completed by normal retirement age.

A lump sum may be payable on retirement and a spouse pension on death after retirement.

Other valid points were credited. Generally this bookwork question was done well.

5 The Kolmogorov equations in this case are:

$$\frac{\delta}{\delta t}\,{}_t(aq)_x^r = \rho e^{-(\mu+\rho)t}$$
$$\frac{\delta}{\delta t}\,{}_t(aq)_x^d = \mu e^{-(\mu+\rho)t}$$

For the case where t = 1 the solution for the dependent probability of retirement is:

$$(aq)_x^r = \frac{\rho}{\rho+\mu}(1-e^{-(\mu+\rho)})$$

Hence the dependent probability of retirement is

$$(aq)_x^r = \frac{0.08}{0.08+0.05}(1-e^{-(0.05+0.08)})$$
$$= 0.07502$$

The formula for the independent probability of death is

$$q_x^d = 1-e^{-\mu}$$

Hence the independent probability of death is:

$$q_x^d = 1-e^{-0.05} = 0.04877$$

Generally this question was completed satisfactorily by well prepared students.

6 (i) The definition of the uniform distribution of deaths (UDD) is ${}_sq_x = s.q_x$ (alternatively ${}_tp_x\mu_{x+t}$ is constant).

(ii) We have

$${}_{1.25}p_{65.5} = {}_{0.5}p_{65.5} \times {}_{0.75}p_{66}$$

$${}_{0.5}p_{65.5} = (1 - {}_{0.5}q_{65.5}) = (1-(0.5q_{65}/(1-0.5q_{65}))) \text{ by UDD}$$
$$= (1-((0.5\times 0.02447)/(1-0.5\times 0.02447)))$$
$$= 0.98761$$

$${}_{0.75}p_{66} = 1 - {}_{0.75}q_{66} = 1-0.75\times q_{66} = 1-0.75\times 0.02711$$
$$= 0.97967$$

Hence

$${}_{1.25}p_{65.5} = 0.98761\times 0.97967 = 0.96753$$
$$\Rightarrow {}_{1.25}q_{65.5} = 1 - {}_{1.25}p_{65.5} = 1 - 0.96753$$
$$= 0.03247$$

A straightforward question that was generally done well.

7 Education influences the awareness of a healthy lifestyle, which reduces morbidity.

Education includes formal and informal processes, such as public health awareness campaigns.

Shows in:

- Increased income
- Better diet
- Increased exercise
- Better health care
- Reduced alcohol and tobacco consumption
- Lower levels of illicit drug use
- Safer sexual practices

Some effects are direct (e.g. drug use); some are indirect (e.g. exercise)

Students generally scored on a range of points but in most cases did not write enough of them to gain all the marks.

Students who mentioned over indulgence risks for the better educated were given credit.

8 Let b be the simple bonus rate (expressed as a percentage of the sum assured). Then the equation of value at 4% p.a. interest is (where P = 3,212):

$$P(.975\ddot{a}_{[40]:\overline{25}|}+0.025)=(100{,}000+350)\bar{A}_{[40]}+1{,}000b(I\bar{A})_{[40]}+300+0.5P$$

$$P(.975\times 15.887+0.025)=$$

$$(100{,}000+350)\times(1.04)^{0.5}\times 0.23041+1{,}000b\times(1.04)^{0.5}\times 7.95835+300+0.5P$$

$$\Rightarrow 49{,}833.6179=23{,}579.5423+8{,}115.9564b+1{,}906$$

$$\Rightarrow b=\frac{24{,}348.0756}{8{,}115.9564}=3.00$$

i.e. a simple bonus rate of 3% per annum

Generally done well although some students treated b as not vesting in the first year.

9 Value of benefits using premium conversion

$$100{,}000\bar{A}_{52:50}=100{,}000\times(1.04)^{1/2}\times A_{52:50}$$

$$=100{,}000\times(1.04)^{1/2}\times(1-(0.04/1.04)\times\ddot{a}_{52:50})$$

$$=101{,}980.4\times(1-0.0384615\times 17.295)$$

$$=34{,}143.89$$

Value of monthly premium of P

$$12P\ddot{a}^{(12)}_{52:50:\overline{5}|}=12P\left(\ddot{a}^{(12)}_{52:50}-\left(v^5\times {}^{l_{57:55}}\!/\!_{l_{52:50}}\right)\times\ddot{a}^{(12)}_{57:55}\right)$$

$$\ddot{a}^{(12)}_{52:50}=\ddot{a}_{52:50}-11/24=17.295-0.458=16.837$$

$$\ddot{a}^{(12)}_{57:55}=\ddot{a}_{57:55}-11/24=15.558-0.458=15.100$$

$$\left(v^5\times {}^{l_{57:55}}\!/\!_{l_{52:50}}\right)=(0.82193\times 9{,}880.196\times 9{,}917.623)/(9{,}930.244\times 9{,}952.697)$$

$$=0.81491$$

Hence $12P\ddot{a}^{(12)}_{52:50:\overline{5}|}=12P(16.837-0.81491\times 15.100)=54.3823P$

Therefore:

$$P=34{,}143.89/54.3823=627.85$$

There was an anomaly in this question in that it was not fully clear that the premium paying period ceased on 1st death within the 5 year period. Even though the vast majority of students who completed this question used the above solution a small minority used $12P\ddot{a}^{(12)}_{\overline{5}|}$ *i.e. ignoring the joint life contingency. This was credited.*

None the less many students struggled with this question

10 Expected present value is $A_{[40]:\overline{15}|}$ where

$$A_{[40]:\overline{15}|} = A^{1}_{[40]:\overline{15}|} + A_{[40]:\overset{1}{\overline{15}|}}$$

$$= A_{[40]} - v^{15}\,{}_{15}p_{[40]}A_{55} + v^{15}\,{}_{15}p_{[40]}$$

$$= 0.23041 - \left(0.55526 \times \frac{9,557.8179}{9,854.3036} \times 0.38950\right) + 0.55526 \times \frac{9,557.8179}{9,854.3036}$$

$$= 0.23041 - 0.20977 + 0.53855$$

$$= 0.55919$$

Variance

$$= {}^{2}A_{[40]:\overline{15}|} - (A_{[40]:\overline{15}|})^2$$

$${}^{2}A_{[40]:\overline{15}|} = {}^{2}A^{1}_{[40]:\overline{15}|} + {}^{2}A_{[40]:\overset{1}{\overline{15}|}}$$

$$= {}^{2}A_{[40]} - (v^2)^{15}\,{}_{15}p_{[40]}\,{}^{2}A_{55} + (v^2)^{15}\,{}_{15}p_{[40]}$$

$$= 0.06775 - \left(0.30832 \times \frac{9,557.8179}{9,854.3036} \times 0.17785\right) + 0.30832 \times \frac{9,557.8179}{9,854.3036}$$

$$= 0.06775 - 0.05318 + 0.29904$$

$$= 0.31361$$

So variance $= 0.31361 - 0.55919^2 = 0.000917$

Note answers are sensitive to number of decimal places used.

Question done well by well prepared students. Many students failed to realise that the endowment function needed to be split into the term and pure endowment portions.

11 Summary of assumptions:

Annual premium	2,500.00	Allocation % (1st yr)	40%
Risk discount rate	5.5%	Allocation % (2nd yr +)	110%
Interest on investments	4.25%	Man charge	0.5%
Interest on sterling provisions	3.5%	B/O spread	5.0%
Minimum death benefit	10,000.00	Maturity benefit	105%

	£	% prm	Total
Initial expense	325	10.0%	575
Renewal expense	74	2.5%	136.5

(i) Multiple decrement table:

x	q_x^d	q_x^s
61	0.006433	0.06
62	0.009696	0.06
63	0.011344	0.06
64	0.012716	0.06

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	${}_{t-1}(ap)$
61	0.006240	0.05981	0.933953	1.000000
62	0.009405	0.05971	0.930886	0.933953
63	0.011004	0.05966	0.929337	0.869404
64	0.012335	0.05962	0.928047	0.807969

(ii) Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
value of units at start of year	0.00	985.42	3,732.08	6,581.15
alloc	1,000.00	2,750.00	2,750.00	2,750.00
B/O	50.00	137.50	137.50	137.50
interest	40.37	152.91	269.65	390.73
management charge	4.95	18.75	33.07	47.92
value of units at year end	985.42	3,732.08	6,581.15	9,536.46

Non-unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
unallocated premium	1,500.00	–250.00	–250.00	–250.00
B/O spread	50.00	137.50	137.50	137.50
expenses/commission	575.00	136.50	136.50	136.50
interest	34.12	–8.72	–8.72	–8.72
man charge	4.95	18.75	33.07	47.92
extra death benefit	56.25	58.95	37.62	5.72
extra surrender benefit	–58.94	–148.20	–168.91	–121.41
extra maturity benefit	0.00	0.00	0.00	442.51
end of year cashflow	1,016.76	–149.71	–93.36	–536.62

(iii)

probability in force	1	0.933953	0.869404	0.807969
discount factor	0.947867	0.898452	0.851614	0.807217
expected p.v. of profit	**419.03**			
premium signature	2,500.00	2,213.16	1,952.79	1,720.19
expected p.v. of premiums	**8,386.15**			
profit margin	**5.00%**			

Credit was given to students who showed good understanding of the processes involved even if the calculations were not correct. Generally well prepared students did this question quite well.

12 (i) Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25}|} @ 4\% = 186.996P$$

where

$$\ddot{a}^{(12)}_{[40]:\overline{25}|} = \ddot{a}_{[40]:\overline{25}|} - \frac{11}{24}\left(1 - {}_{25}p_{[40]}v^{25}\right)$$

$$= 15.887 - \frac{11}{24}\left(1 - \frac{8821.2612}{9854.3036} \times 0.37512\right)$$

$$= 15.583$$

EPV of benefits:

$$75,000(q_{[40]}v^{0.5} + {}_{1|}q_{[40]}(1+b)v^{1.5} + \ldots + {}_{24|}q_{[40]}(1+b)^{24}v^{24.5})$$

where b = 0.04

$$= \frac{75,000 \times (1+i)^{0.5}}{(1+b)} \times A^{1}_{[40]:\overline{25|}} @ i' = \frac{75,000 \times (1+i)^{0.5}}{(1+b)} \left[A_{[40]} - {}_{25}p_{[40]} v^{25} A_{65} \right] @ i'$$

$$= \frac{75,000}{(1.04)^{0.5}} \times \left(1 - \frac{8821.2612}{9854.3036} \times 1 \times 1 \right)$$

$$= 7709.6880$$

where

$$i' = \frac{1.04}{1+b} - 1 = 0.00 \text{ i.e. } i' = 0\%$$

EPV of expenses (at 4% unless otherwise stated

$$= 0.5 \times 12P + 400 + 0.025 \times 12P\ddot{a}^{(12)}_{[40]:\overline{25|}} - 0.025 \times 12P\ddot{a}^{(12)}_{[40]:\overline{1|}} + 75\left[\ddot{a}^{@0\%}_{[40]:\overline{25|}} - 1 \right]$$
$$+300\bar{A}^{1}_{[40]:\overline{25|}}$$
$$= 6P + 400 + 0.025 \times 12P \times 15.583 - 0.025 \times 12P \times 0.982025 + 75 \times 23.27542$$
$$+300 \times 0.05422$$
$$= 6P + 400 + 4.6749P - 0.2946P + 1745.6558 + 16.266$$
$$= 10.3803P + 2161.9218$$

where

$$\ddot{a}^{(12)}_{[40]:\overline{1|}} = \ddot{a}_{[40]:\overline{1|}} - \frac{11}{24}\left(1 - p_{[40]}v \right)$$
$$= 1 - \frac{11}{24}\left(1 - \frac{9846.5384}{9854.3036} \times 0.96154 \right) = 0.982025$$

$$\ddot{a}^{@0\%}_{[40]:\overline{25|}} - 1 = \frac{1}{l_{[40]}}\left(l_{[40]+1} + \ldots. + l_{64} \right) = e_{[40]} - \frac{l_{64}}{l_{[40]}} e_{64}$$
$$= 39.071 - \frac{8934.8771}{9854.3036} \times 17.421 = 23.27541$$

$$\bar{A}^{1}_{[40]:\overline{25}|} = 1.04^{0.5} A^{1}_{[40]:\overline{25}|} = 1.04^{0.5}\left[A_{[40]:\overline{25}|} - v^{25}\frac{l_{65}}{l_{[40]}}\right]$$
$$= 1.04^{0.5}\left[0.38896 - 0.37512 \times \frac{8821.2612}{9854.3036}\right] = 0.05422$$

Equation of value gives:

$$186.996P = 7709.6880 + 10.3803P + 2161.9218$$
$$\Rightarrow P = \frac{9871.6098}{176.6157} = £55.89$$

(ii) Gross prospective policy value at $t = 23$ (calculated at 4%) is given by:

$$V^{\text{prospective}} = 75{,}000 \times (1.04)^{23} \times v^{0.5}\left[q_{63} + (1.04) \times p_{63}q_{64} \times v\right] + 300v^{0.5}\left[q_{63} + p_{63}q_{64}v\right] + 0.025 \times 12P\ddot{a}^{(12)}_{63:\overline{2}|}$$
$$+75 \times (1.04)^{23}\left[1 + (1.04)p_{63}v\right] - 12P\ddot{a}^{(12)}_{63:\overline{2}|}$$

$$= 184{,}853.66 \times 0.98058 \times \left[0.011344 + (1.04) \times 0.988656 \times 0.012716 \times 0.96154\right]$$
$$+300 \times 0.98058 \times \left[0.011344 + 0.988656 \times 0.012716 \times 0.96154\right] + 0.025 \times 12 \times 55.89 \times 1.90629$$
$$+184.854\left[1 + (1.04) \times 0.988656 \times 0.96154\right] - 12 \times 55.89 \times 1.90629$$

where $\ddot{a}^{(12)}_{63:\overline{2}|} = \ddot{a}_{63:\overline{2}|} - \frac{11}{24}\left(1 - v^2 \times \frac{l_{65}}{l_{63}}\right) = 1.951 - \frac{11}{24}\left(1 - 0.92456 \times \frac{8821.2612}{9037.3973}\right) = 1.90629$

$$= 4{,}335.0628 + 6.8932 + 31.9628 + 367.6104 - 1{,}278.5106$$
$$= £3{,}463.02$$

This question was generally not done well especially part (ii). In part (i) although it was commonly recognised that a resultant rate of interest of 0% emerged students did not often seem to know how to progress from there.

13 (i) The death strain at risk for a policy for year $t + 1$ ($t = 0, 1, 2\ldots$) is the excess of the sum assured (i.e. the present value at time $t + 1$ of all benefits payable on death during the year $t + 1$) over the end of year provision.

i.e. DSAR for year $t + 1 = S - {}_{t+1}V$

The "expected death strain" for year $t + 1$ ($t = 0, 1, 2\ldots$) is the amount that the life insurance company expects to pay extra to the end of year provision for the policy.

i.e. EDS for year $t + 1 = q(S - {}_{t+1}V)$

The "actual death strain" for year t + 1 (t = 0, 1, 2…) is the observed value at t+1 of the death strain random variable

i.e. ADS for year $t + 1 = (S - {}_{t+1}V)$ if the life died in the year t to t+1
= 0 if the life survived to t + 1

Note: Full credit given if definition of death strain is given for a block of policies rather than for a single policy as per above.

(ii) (a) Annual premium for endowment assurance with £100,000 sum assured given by:

$$P^{EA} = \frac{100,000}{\ddot{a}_{35:\overline{25|}}} \times A_{35:\overline{25|}} = \frac{100,000}{16.027} \times 0.38359 = 2,393.40$$

Annual premium for term assurance with £200,000 sum assured given by:

$$P^{TA} = \frac{200,000 A^{1}_{40:\overline{25|}}}{\ddot{a}_{40:\overline{25|}}}$$

where $A^{1}_{40:\overline{25|}} = A_{40:\overline{25|}} - v^{25}\,{}_{25}p_{40}$

$$= 0.38907 - 0.37512 \times \frac{8,821.2612}{9,856.2863} = 0.38907 - 0.33573 = 0.05334$$

$$P^{TA} = \frac{200,000 \times 0.05334}{15.884} = 671.62$$

Reserves at the end of the 11th year:

– for endowment assurance with £100,000 sum assured given by:

$${}_{11}V^{EA} = 100,000 \times A_{46:\overline{14|}} - P^{EA}\ddot{a}_{46:\overline{14|}}$$

$$= 100,000 \times 0.58393 - 2,393.40 \times 10.818$$

$$= 58,393.0 - 25,891.8 = 32,501.2$$

– for term assurance with £200,000 sum assured given by:

$$ {}_{11}V^{TA} = 200,000 A^{1}_{51:\overline{14}|} - P^{TA}\ddot{a}_{51:\overline{14}|} $$

where $A^{1}_{51:\overline{14}|} = A_{51:\overline{14}|} - v^{14}\,{}_{14}p_{51}$

$$ = 0.58884 - 0.57748 \times \frac{8,821.2612}{9,687.7149} = 0.58884 - 0.52583 = 0.06301 $$

$$ {}_{11}V^{TA} = 200,000 \times 0.06301 - 671.62 \times 10.69 $$

$$ = 12,602.0 - 7,179.6 = 5,422.4 $$

Therefore, sums at risk are:

Endowment assurance: DSAR = 100,000 – 32,501.2 = 67,498.8

Term assurance: DSAR = 200,000 – 5,422.4 = 194,577.6

(b) Mortality profit = EDS – ADS

For endowment assurance

$$ EDS = 19768 \times q_{45} \times 67,498.8 = 19768 \times 0.001465 \times 67,498.8 = 1,954,773.3 $$

$$ ADS = 36 \times 67,498.8 = 2,429,956.8 $$

mortality profit = –475,183.5 (i.e. a loss)

For term assurance

$$ EDS = 9,855 \times q_{50} \times 194,577.6 = 9,855 \times .002508 \times 194,577.6 = 4,809,246.1 $$

$$ ADS = 22 \times 194,577.6 = 4,280,707.2 $$

mortality profit = 528,538.9
Hence, total mortality profit = 528,538.9 – 475,183.5 = £53,355.4

(c) Although there is an overall mortality profit in 2010, the actual number of deaths for the endowment assurances is approximately 25% higher than expected, which is a concern. Further investigation would be required to determine reasons for poor mortality experience for the endowment assurances, e.g. there may have been limited underwriting requirements applied to this type of contract when they were written.

Generally (a) was done well. The most common error in (b) was to assume reserves at 10 years rather than 11. On the whole well prepared students coped with (b) well. Many students did not attempt (c) or at best gave a somewhat sketchy answer.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2011 examinations

Subject CT5 — Contingencies
Core Technical

Purpose of Examiners' Reports

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and who are using past papers as a revision aid, and also those who have previously failed the subject. The Examiners are charged by Council with examining the published syllabus. Although Examiners have access to the Core Reading, which is designed to interpret the syllabus, the Examiners are not required to examine the content of Core Reading. Notwithstanding that, the questions set, and the following comments, will generally be based on Core Reading.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report. Other valid approaches are always given appropriate credit; where there is a commonly used alternative approach, this is also noted in the report. For essay-style questions, and particularly the open-ended questions in the later subjects, this report contains all the points for which the Examiners awarded marks. This is much more than a model solution – it would be impossible to write down all the points in the report in the time allowed for the question.

T J Birse
Chairman of the Board of Examiners

December 2011

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the September 2011 paper

The general performance was slightly worse than in April 2011 but well-prepared candidates scored well across the whole paper. Questions that were done less well were 7, 9, 10, 11 and 14(iii) and here more commentary is given to students to assist with further revision.

Most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions a reasonable level of credit is given if they can describe the right procedures although to score well reasonable accurate numerical calculation is necessary.

1 (a) $_{10|1}q_{[50]} = d_{60} / l_{[50]} = 74.5020 / 9,706.0977 = 0.00768$

(b) $_{10}p_{[60]+1} = l_{71} / l_{[60]+1} = 7,854.4508 / 9,209.6568 = 0.85285$

(c) $\ddot{a}^{(12)}_{[40];\overline{20}|} = (\ddot{a}_{[40]:\overline{20}|} - 11/24 \times (1 - \frac{v^{20} l_{60}}{l_{[40]}}))$

$= 12.000 - 11/24 \times (1 - 0.3118 \times 9,287.2164 / 9,854.3036)$

$= 11.676$

Straightforward question generally done well.

2 We have:

$_1p_{75} = 6,589.9258 / 6,879.1673 = 0.95795$

$= e^{-\mu}$ where μ is the constant force

Hence $\mu = -\ln(0.95795) = 0.04296$

Hence $_{0.5}p_{75.25} = e^{-\int_{75.25}^{75.75} 0.04296 dt}$

$= e^{-0.02148} = 0.97875$

Hence $_{0.5}q_{75.25} = 1 - {}_{0.5}p_{75.25} = 0.02125$

Again done well. Credit was given to those students who jumped straight to the solution ***of*** $(1 - ({}_1p_{75})^{0.5})$.

3 $p_{[x]} = {}_{0.5}p_{[x]} * {}_{0.5}p_{[x]+0.5} = (1 - {}_{0.5}q_{[x]}) * (1 - {}_{0.5}q_{[x]+0.5})$

$= (1 - 0.25q_x) * (1 - 0.45q_x) = (1 - 0.25(1 - p_x)) * (1 - 0.45(1 - p_x))$

$= (0.75 + .25p_x) * (.55 + .45p_x) = 0.4125 + 0.475p_x + 0.1125p_x^2$

This question was done reasonably well but many students failed to make the connection in line 1.

4 The expected value is $\bar{A}^{1}_{30:\overline{20}|}$

This equals

$$(A_{30} - (v^{20} \times (l_{50} / l_{30}) \times A_{50})) \times (1.04)^{1/2}$$
$$= (0.16023 - (0.45639 \times (9,712.0728 / 9,925.2094) \times 0.32907)) \times 1.019804$$
$$= 0.01353$$

The variance equals

$${}^2\bar{A}^{1}_{30:\overline{20}|} - (\bar{A}^{1}_{30:\overline{20}|})^2$$

$${}^2\bar{A}^{1}_{30:\overline{20}|} = (({}^2A_{30} - (v^{20})^2 \times (l_{50} / l_{30}) \times {}^2A_{50})) \times (1.04)$$
$$= (0.03528 - (0.20829 \times (9,712.0728 / 9,925.2094) \times 0.13065)) \times 1.04$$
$$= 0.008997$$

Variance $= 0.008997 - 0.01353^2 = 0.008814$

This question was done reasonably well. The most common error was to forget to use continuous functions which was penalised as one of the key attributes being tested was to see if students could work out the 1.04 factor for the variance.

5 (a)
$$\bar{Z} = \begin{cases} v_i^{T_x} & \text{if} \quad T_x \le T_y \\ 0 & \text{if} \quad T_x > T_y \end{cases}$$

where i is the valuation rate of interest.

(b)
$$\bar{A}^{\;1}_{x:y} = \int_0^\infty e^{-.04t} \cdot {}_tp_{xy} \cdot \mu_{x+t}\,dt$$

$$= \int_0^\infty e^{-.04t} \cdot e^{-.02t} \cdot e^{-.03t} \cdot (0.02)dt = 0.02\int_0^\infty e^{-.09t}dt$$

$$= 0.02 / 0.09$$

$$= 0.22222$$

In part(a)many students did not appreciate what a random variable form was. Part (b) was generally well done.

Part (a) comes directly from Core Reading but there is some debate about the situation where $T_x = T_y$ *i.e .a simultaneous death where it could be argued either that* $\bar{Z}$ *=0 or is undefined. The examiners decided to accept all these alternative situations.*

6

- When a life table is constructed it is assumed to reflect the mortality experience of a homogeneous group of lives i.e. all the lives to whom the table applies follow the same stochastic model of mortality represented by the rates in the table. This means that the table can be used to model the mortality experience of a homogeneous group of lives which is suspected to have a similar experience.

- If a life table is constructed for a heterogeneous group then the mortality experience will depend on the exact mixture of lives with different experiences that has been used to construct the table. Such a table could only be used to model mortality in a group with the same mixture. It would have very restricted uses.

- For this reason separate mortality tables are usually constructed for groups which are expected to be heterogeneous. This can manifest itself as class selection e.g. separate tables for males and females, whole life and term assurance policyholders, annuitants and pensioners, or as time selection e.g. separate tables for males in England and Wales in 1980–82 (ELT14) and 1990–92 (ELT15).

- Sometimes only parts of the mortality experience are heterogeneous e.g. the experience during the initial select period for life assurance policyholders, and the remainder are homogeneous e.g. the experience after the end of the select period for life assurance policyholders. In such cases the tables are separate (different) during the select period, but combined after the end of the select period. In fact there are separate (homogeneous) mortality tables for each age at selection, but they are tabulated in an efficient (space saving) way.

Well prepared students answered this question well. However many did not get to the heart of the homogeneity discussion and went off on tangents regarding various forms of selection.

7 EPV is

$$10,000(\bar{a}_{\overline{60:60}} - \bar{a}_{60:60}) + 10,000 \times \bar{a}_{\overline{5}|} \times \bar{A}_{\overline{60:60}}$$

$$\bar{a}_{\overline{60:60}} = \bar{a}^m_{60} + \bar{a}^f_{60} - \bar{a}_{60:60} = (15.632 - 0.5) + (16.652 - 0.5) - (14.090 - 0.5) = 17.694$$

$$\bar{A}_{\overline{x:y}} = (1 - \delta\bar{a}_{\overline{x:y}}) = 1 - \ln(1.04) \times 17.694 = 0.30603$$

Therefore

$$\begin{aligned} \text{EPV} &= 10,000 \times (17.694 - (14.090 - 0.5)) + 10,000 \times \frac{(1 - v^5)}{\delta} \times 0.30603 \\ &= 41,040 + 13,894 \\ &= 54,934 \end{aligned}$$

Many students struggled here with the second term in the equation in the 2nd line and did not appreciate how to mix a continuous assurance factor with an annuity.

8

	All professions		*Profession A*		
Age	*Population*	*Deaths*	*Population*	*Deaths*	*Expected deaths*
20–29	120,000	256	12,500	30	26.667
30–39	178,000	458	15,000	40	38.595
40–49	156,000	502	16,000	50	51.487
50–64	123,000	600	14,000	60	68.293
Total	577,000	1,816	57,500	180	185.042

(a) Total Expected deaths 185.042

Area comparability factor = $\dfrac{1,816}{577,000} \Big/ \dfrac{185.042}{57,500} = 0.978$

(b) Standardised mortality ratio = 180/185.042 = 0.973

Indirectly standardised mortality rate = $\dfrac{1,816}{577,000} \Big/ \dfrac{185.042}{180} = 0.003062$

Straightforward with no issues and generally well done.

9 Age retirement can be ignored in constructing the dependent decrements.

The following rates are required:

Age	q_x^d	q_x^i
59	0.01243	0.055
60	0.01392	0.06
61	0.01560	0.065

The dependent decrements are calculated as:

$$(aq)_x^{\alpha} = q_x^{\alpha}(1-0.5q_x^{\beta})$$

Age	$(aq)_x^d$	$(aq)_x^i$
59	0.012088	0.054658
60	0.013502	0.059582
61	0.015093	0.064493

Probability of reaching 60 = (1 – 0.012088 – 0.054658) = 0.933254
Probability of retiring at age 60 = 0.2 * 0.933254 = 0.186651
Probability of reaching 61 = 0.8 * 0.933254 * (1 – 0.013502-0.059582) = 0.692038
Probability of retiring at age 61 = 0.2 * 0.692038 = 0.138408
Probability of reaching 62 = 0.8 * 0.692038 * (1-0.015093 – 0.064493) = 0.509569
Probability of retiring at age 62 = 0.2 * 0.509569 = 0.101914
Overall required probability thus 10.19%

This question was not done well overall. Students struggled to follow through the logical sequences. In fact this question can be solved with the same answer without using multiple decrements and the few students who realised this were given credit.

10 (a) At the end of the 5th policy year, we have:

Year	*SA*	b_1	b_2	$\sum b$
0	150,000			
1	150,000	3,750	–	3,750.00
2	150,000	3,750	187.50	7,687.50
3	150,000	3,750	384.38	11,821.88
4	150,000	3,750	591.09	16,162.97
5	150,000	3,750	808.15	20,721.12

If net premium denoted by P then

$$P = \frac{150,000 A_{[30]}}{\ddot{a}_{[30]}} = \frac{150,000 \times 0.16011}{21.837} = 1099.81$$

Therefore, net premium reserve at end of 5th policy year is given by:

$$\begin{aligned} {}_5V &= (150,000 + 20,721.12) A_{35} - P\ddot{a}_{35} \\ &= 170,721.12 \times 0.19219 - 1,099.81 \times 21.003 \\ &= 32,810.89 - 23,099.31 = £9,711.58 \end{aligned}$$

(b) The sum assured and bonuses increase more slowly than under other methods for the same ultimate benefit, enabling the office to retain surplus for longer.

This method rewards longer standing policyholders and discourages surrenders, relative to other methods.

This question was also done poorly overall. A very large number of students attempted to construct a complex "net premium" from the existing bonus flow where the question was only seeking the normal net premium method. Part (b) was done better.

11 Retirement other than ill-health:

$$0.01 \times 20,000 \times \left(\sum_{t=0}^{65-30-1} t z_{30+t+0.5} r_{30+t} v^{t+0.5} \bar{a}^{*}_{30+t+0.5} + (65-30) z_{65} r_{65} v^{35} \bar{a}^{*}_{65} \right) / s_{30} l_{30}$$

Retirement due to ill-health:

$$0.01 \times 20,000 \times (65-30) \times \sum_{t=0}^{65-30-1} z_{30+t+0.5} i_{30+t} v^{t+0.5} \bar{a}^{*}_{30+t+0.5} / s_{30} l_{30}$$

Where

a_x^*	is the annuity value at age x including any contingent spouse pensions
i_x, r_x, l_x	are values from a multiple decrement table at age x
s_x	is the salary index for age x where s_{x+1} / s_x is the ratio of salary in the year beginning age $x+1$ to salary in the year beginning age x
z_x	$(s_{x-3} + s_{x-2} + s_{x-1})/3$

Other schemes were accepted but overall very few students managed to derive a full answer in this question.

12 (i)

- Allocated premiums are invested in a fund(s) chosen by the policyholder which purchases a number of units within that fund(s)
- Each investment fund is divided into units, which are priced regularly (usually daily)
- Policyholder receives the value of the units allocated to their own policy
- Benefits are directly linked to the value of the underlying investments
- Unallocated premiums are directed to the company's non-unit fund
- Bid/offer spread is used to help cover expenses and contribute towards profit
- Charges are made from the unit account periodically to cover expenses and benefits (i.e. fund management charge) and may be varied after notice of change given.
- Unit-linked contracts may offer guaranteed benefits (e.g. minimum death benefit)
- Unit-linked contracts are generally endowment assurance and whole of life contracts

(ii) To calculate the expected reserves at the end of each year we have (utilising the end of year cashflow figures):

$p_{58} = 0.99365 \quad p_{57} = 0.99435 \quad p_{56} = 0.99497$

$${}_3V = \frac{933.82}{1.045} = 893.61$$

$${}_2V \times 1.045 - p_{58} \times {}_3V = 292.05 \Rightarrow {}_2V = 1{,}129.17$$

$${}_1V \times 1.045 - p_{57} \times {}_2V = 334.08 \Rightarrow {}_1V = 1{,}394.13$$

The revised cash flow for year 1 will become:

$$1{,}525.89 - p_{56} \times 1{,}394.13 = 138.77$$

Revised profit vector becomes (138.77, 0, 0, 0) and
Net present value of profits = 138.77/(1.075) = 129.09

This question was generally done well.

13 (i) Reserves required on the policy per unit sum assured are:

$${}_0V_{57:\overline{3}|} = 1 - \frac{\ddot{a}_{57:\overline{3}|}}{\ddot{a}_{57:\overline{3}|}} = 0$$

$${}_1V_{57:\overline{3}|} = 1 - \frac{\ddot{a}_{58:\overline{2}|}}{\ddot{a}_{57:\overline{3}|}} = 1 - \frac{1.955}{2.870} = 0.318815$$

$${}_2V_{57:\overline{3}|} = 1 - \frac{\ddot{a}_{59:\overline{1}|}}{\ddot{a}_{57:\overline{3}|}} = 1 - \frac{1.0}{2.870} = 0.651568$$

Multiple decrement table:

T	$q^d_{[57]+t-1}$	$q^s_{[57]+t-1}$	$(aq)^d_{[57]+t-1}$	$(aq)^s_{[57]+t-1}$	$(ap)_{[57]+t-1}$	${}_{t-1}(ap)_{[57]}$
1	0.004171	0.10	0.004171	0.099583	0.896246	1.000000
2	0.006180	0.05	0.006180	0.049691	0.944129	0.896246
3	0.007140	0.00	0.007140	0.000000	0.992860	0.846172

Probability in force $(ap)_{[56]+t-1} = (1 - q^d_{[56]+t-1}) \times (1 - q^s_{[56]+t-1})$

The calculations of the profit vector, profit signature and NPV are set out in the table below:

Policy Year	*Premium*	*Expenses*	*Interest*	*Death claim*	*Maturity claim*	*Surrender claim*	*In force cash flow*
1	4700	470.00	211.50	62.57	0.00	351.03	4027.91
2	4700	65.00	231.75	92.70	0.00	350.02	4423.73
3	4700	65.00	231.75	107.10	14892.90	0.00	−10133.25

Policy year	*Increase in reserves*	*Interest on reserves*	*Profit vector*	*Cum probability of survival*	*Discount factor*	*NPV Profit*
1	4286.05	0.00	−258.15	1.00000	0.93458	−241.26
2	4445.24	239.11	217.60	0.89625	0.87344	170.34
3	−9773.52	488.68	128.95	0.84617	0.81630	89.07

Total NPV profit = 18.15

(ii) IRR is determined by solving the following equation for i:

$$-258.15(1+i)^{-1}+195.02(1+i)^{-2}+109.11(1+i)^{-3}=0$$

If i = 0.12 then LHS of equation = 2.64
If i = 0.13 then LHS of equation = –0.10
If i = 0.14 then LHS of equation = –2.74

Therefore IRR is 13%

(iii) The revised reserves required on the policy per unit sum assured are:

$$_0V_{57:\overline{3}|}=1-\frac{\ddot{a}_{57:\overline{3}|}}{\ddot{a}_{57:\overline{3}|}}=0$$

$$_1V_{57:\overline{3}|}=1-\frac{\ddot{a}_{58:\overline{2}|}}{\ddot{a}_{57:\overline{3}|}}=1-\frac{1.937}{2.817}=0.312389$$

$$_2V_{57:\overline{3}|}=1-\frac{\ddot{a}_{59:\overline{1}|}}{\ddot{a}_{57:\overline{3}|}}=1-\frac{1.0}{2.817}=0.645012$$

And the revised cashflows become:

Policy year	*Increase in reserves*	*Interest on reserves*	*Revised Profit vector*	*Cum probability of survival*	*Discount factor*	*NPV profit*
1	4199.66	0.00	–171.76	1.00000	0.93458	–160.52
2	4448.78	234.29	209.24	0.89625	0.87344	163.79
3	–9675.18	483.76	25.69	0.84617	0.81630	17.74

Total NPV profit = 21.02

The NPV of profit increases slightly if the reserving basis is weakened, as a result of the surplus emerging being brought forward and the fact that the risk discount rate is greater than the interest rate being earned on reserves.

In general well prepared students made a reasonable attempt with this question. Credit was given to students who showed they understood the processes even if not all the arithmetical calculations were correct.

Note that it is possible to solve (ii) using a quadratic equation process.

14 (i)

Formula is

$$({}_tV + P - e)\times(1+i) = q_{x+t}\times S + p_{x+t}\times {}_{t+1}V$$

Definitions:

${}_tV$ = gross premium reserve at time t

q_{x+t} / p_{x+t} = probability that a life aged $x+t$ dies within /survives one year on premium/valuation basis

P = office premium

e = initial/renewal expense incurred at start of policy year

i = rate of interest in premium/valuation basis

S = sum assured payable at end of year of death

(ii)

Let P be the annual premium. Then equation of value is:

$$P\ddot{a}_{[35]:\overline{30|}} = 50,000A^{1}_{[35]:\overline{30|}} + 100,000v^{30}\,{}_{30}p_{[35]} + 300 + 0.5P + 0.025P\left(\ddot{a}_{[35]:\overline{30|}} - 1\right)$$

where

$$A^{1}_{[35]:\overline{30|}} = A_{[35]:\overline{30|}} - v^{30}\,{}_{30}p_{[35]} = 0.32187 - 0.30832 \times \frac{8,821.2612}{9,892.9151} = 0.32187 - 0.27492 = 0.04695$$

$$\ddot{a}_{[35]:\overline{30|}} = 17.6313$$

$$\Rightarrow \quad 17.6313P = 50,000 \times 0.04695 + 100,000 \times 0.27492 + 300 + 0.5P + 0.025P \times 16.6313$$

$$P = \frac{30,139.5}{16.71552} = 1,803.08$$

(iii)

The gross premium prospective reserve per policy at the end of 2009 is given by:

$${}_{9}V^{PRO} = 50,000A^{1}_{44:\overline{21|}} + 100,000v^{21}\,{}_{21}p_{44} - 0.975P\ddot{a}_{44:\overline{21|}}$$

where

$$A^{1}_{44:\overline{21|}} = A_{44:\overline{21|}} - v^{21}\,{}_{21}p_{44} = 0.45258 - 0.43883 \times \frac{8,821.2612}{9,814.3359} = 0.45258 - 0.39443 = 0.05815$$

$$\ddot{a}_{44:\overline{21|}} = 14.2329$$

$$\Rightarrow {}_{9}V^{PRO} = 50,000 \times 0.05815 + 100,000 \times 0.39443 - 0.975P \times 14.2329$$

$$= 2,907.50 + 39,443.00 - 25,021.48 = 17,329.02$$

The gross premium prospective reserve per policy at the end of 2010 is given by:

$${}_{10}V^{PRO} = 50,000A^{1}_{45:\overline{20|}} + 100,000v^{20}\,{}_{20}p_{45} - 0.975P\ddot{a}_{45:\overline{20|}}$$

where

$$A^{1}_{45:\overline{20|}} = A_{45:\overline{20|}} - v^{20}\,{}_{20}p_{45} = 0.46998 - 0.45639 \times \frac{8,821.2612}{9,801.3123} = 0.46998 - 0.41075 = 0.05923$$

$\ddot{a}_{45:\overline{20|}} = 13.7805$

$\Rightarrow {}_{10}V^{PRO} = 50,000 \times 0.05923 + 100,000 \times 0.41075 - 0.975P \times 13.7805$

$= 2,961.50 + 41,075.00 - 24,226.16 = 19,810.34$

Combined mortality and interest profit =

$$385 \times \left(17,329.02 + 0.975 \times 1,803.08\right) \times 1.05 - 3 \times 50,000 - (385-3) \times 19,810.34$$
$$= 7,715,929.05 - 150,000 - 7,567,549.88$$
$$= -1,620.83$$

i.e. a combined mortality and interest loss of £1,620.83 which can be split between mortality profit and interest profit separately as follows:

$$DSAR = 50,000 - 19,810.34 = 30,189.66$$
$$EDS = 385 \times q_{44} \times DSAR = 385 \times 0.001327 \times 30,189.66 = 15,423.75$$
$$ADS = 3 \times DSAR = 3 \times 30,189.66 = 90,568.98$$

Therefore

Mortality profit = EDS – ADS = 15,423.75 – 90,568.98 = –75,145.23 (i.e. a mortality loss)

Interest profit = 385 × (17,329.02 + 0.975 × 1,803.08) × (0.05 – 0.04) = 73,489.04

Alternatively: Interest profit = 75,145.23 – 1,620.83 = 73,524.40 (the small discrepancy with the figure for interest profit above is due to figures being used from the Actuarial Tables with only a limited number of decimal places)

Part (i) and (ii) were done well. In part (iii) most well prepared students were able to derive the mortality profit but most struggled with the interest portion.

If a student got the combined total correct but then did not split up the content it was decided to give full credit.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2012 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and who are using past papers as a revision aid, and also those who have previously failed the subject. The Examiners are charged by Council with examining the published syllabus. Although Examiners have access to the Core Reading, which is designed to interpret the syllabus, the Examiners are not required to examine the content of Core Reading. Notwithstanding that, the questions set, and the following comments, will generally be based on Core Reading.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report. Other valid approaches are always given appropriate credit; where there is a commonly used alternative approach, this is also noted in the report. For essay-style questions, and particularly the open-ended questions in the later subjects, this report contains all the points for which the Examiners awarded marks. This is much more than a model solution – it would be impossible to write down all the points in the report in the time allowed for the question.

T J Birse
Chairman of the Board of Examiners

July 2012

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the April 2012 paper

The general performance was better this session than in recent diets and many students scored well with a very pleasing increase in the number passing. Questions that were done less well were 2, 10, 12, 13 and 15(i) and (iii) and here more commentary is given to students to assist with further revision.

Most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions a reasonable level of credit is given if they can describe the right procedures although to score well reasonable accurate numerical calculation is necessary.

1 (a) $_{4|5}q_{[60]+1}$ is the probability that a life now aged 61 exact who entered the selection period 1 year ago will die between the ages of 65 and 70 both exact

(b) $_{4|5}q_{[60]+1} = \left(l_{65} - l_{70}\right) / l_{[60]+1} = (8821.2612 - 8054.0544) / 9209.6568$
$= 0.0833$

A gentle starter generally done well

2 The death benefit in year 12 is £36,000

Profit emerging per policy in force at the start of the year is:

$$({}_{11}V + P - e) \times (1+i) - (q_{x+t} \times S) - p_{x+t} \times {}_{12}V$$

$$= (25{,}130 + 3{,}000 - 90) \times 1.04 - 36{,}000 \times 0.03 - (1 - 0.03) \times 28{,}950 = £0.10$$

This question overall caused problems and students sometimes had only a vague recall of the iterative formula in line 3. The most common error was to forget expenses and the survival factor before the closing reserve. It was also not on many occasions appreciated that the accumulation minus the benefit costs gave the profit.

3 (a)
$$a_{50:\overline{15}|} = a_{50} - v^{15}\left(\frac{l_{65}}{l_{50}}\right)(a_{65})$$
$$= (14.044 - 1) - 0.41727 \times \frac{8{,}821.2612}{9{,}712.0728} \times (10.569 - 1)$$
$$= 9.417$$

(b)
$$(IA)^{1}_{50:\overline{15}|} = (IA)_{50} - v^{15}\left(\frac{l_{65}}{l_{50}}\right)((IA)_{65} + 15A_{65})$$
$$= 4.84555 - 0.41727 \times \frac{8{,}821.2612}{9{,}712.0728}(5.50985 + 15 \times 0.40177)$$
$$= 0.47329$$

In (a) a surprising number of students thought that the required function could be derived from the a due function for the same term minus 1 which is, of course, wholly incorrect.

Otherwise the question was generally well done.

4 The value is $100000\bar{A}_{\overline{60:55}}$ where 60 relates to the male life and 55 the female life.

$$\begin{aligned}100000\bar{A}_{\overline{60:55}} &= 100000 \times (\bar{A}_{60} + \bar{A}_{55} - \bar{A}_{60:55}) \\ &= 100000 \times (1 - \ln(1.04)(\bar{a}_{60} + \bar{a}_{55} - \bar{a}_{60:55})) \\ &= 100000 \times (1 - \ln(1.04)([\ddot{a}_{60} - 1/2] + [\ddot{a}_{55} - 1/2] - [\ddot{a}_{60:55} - 1/2])) \\ &= 100000 \times (1 - 0.039221 \times (15.632 + 18.210 - 14.756 - 0.5)) \\ &= £27104\end{aligned}$$

Generally well done. Other methods such as multiplying non continuous functions by $(1.04)^{1/2}$ to obtain the continuous one were quite acceptable.

5 The reserves required at the beginning of policy years 6, 4, 3 and 2 are:

$$\begin{aligned}{}_5V &= \frac{4}{1.025} = 3.902 \\ {}_3V &= \frac{1}{1.025} = 0.976 \\ {}_2V &= \frac{1}{1.025}(6 + .995 \times {}_3V) = 6.801 \\ {}_1V &= \frac{1}{1.025}(12 + .995 \times {}_2V) = 18.309\end{aligned}$$

Revised cash flow in policy year 5 $= 5 - 0.995 \times {}_5V = 1.118$

Revised cash flow in policy year 1 $= -40 - 0.995 \times {}_1V = -58.218$

=> revised profit vector: (–58.22, 0, 0, 0, 1.12, 0, 8, 20, 25, 30)

Generally well done by well prepared students who were able to recall the techniques involved.

6 (a) $p_{67} = \exp(-\int_{67}^{68} \mu \; dx)$ where μ is the constant force.

$\Rightarrow \mu = -\ln(p_{67}) = -\ln(1-q_{67}) = -\ln(0.982176)$

$\Rightarrow \mu = 0.017985$

(b) Using the constant force assumption:

$${}_{0.5}q_{67.25} = 1 - {}_{0.5}p_{67.25} = 1 - \exp(-\int_{67.25}^{67.75} \mu \; dx)$$

$$= 1 - \exp(-0.5 \times \mu) = 1 - \exp(-0.5 \times 0.017985)$$

$$= 0.008952$$

Generally well done. However some students ignored the instruction to use (a) to get (b) choosing the more direct route. The examiners penalised this approach, although some credit was given.

7 Pension schemes usually have a fixed Normal Pension Age (NPA).

Age retirement benefits may be provided on early or late retirement.

Pension usually depends on pensionable service at retirement, as defined in the scheme rules, e.g. complete years of membership.

Pension for each year of service is usually related to pensionable salary, for example 1/80ths of pensionable salary for each year of service. 1/80 is described as the accrual rate.

Pensionable salary can be defined as:

1. Salary at retirement (final salary)
2. Annual salary averaged over a few years before retirement (final average salary)
3. Annual salary averaged the whole of service (career average salary)

Pensions are commonly increased in payment to offset the effect of inflation.

Some benefit may be in the form of cash, sometimes by converting pension to cash.

There can be a spouse's pension for married pensioners which is often a percentage of the main pension on the member's prior death.

Pensions may also be paid for an initial guarantee period like 5 years.

Other relevant comments were credited. No credit was given for any discussion on ill-health retirement as this was not required from the question.

Many students scored reasonable marks.

8 Occupation can have several direct and indirect effects on mortality and morbidity.

Occupation determines a person's environment for 40 or more hours each week. The environment may be rural or urban, the occupation may involve exposure to harmful substances e.g. chemicals, or to potentially dangerous situations e.g. working at heights. Much of this is moderated by health and safety at work regulations.

Some occupations are more healthy by their very nature e.g. bus drivers have a sedentary and stressful occupation while bus conductors are more active and less stressed. Some work environments e.g. publicans, give exposure to a less healthy lifestyle.

Some occupations by their very nature attract more healthy or unhealthy workers. This may be accentuated by health checks made on appointment or by the need to pass regular health checks e.g. airline pilots. However, this effect can be produced without formal checks, e.g. former miners who have left the mining industry as a result of ill-health and then chosen to sell newspapers. This will inflate the mortality rates of newspaper sellers.
A person's occupation largely determines their income, and this permits them to adopt a particular lifestyle e.g. content and pattern of diet, quality of housing. This effect can be positive and negative e.g. over indulgence.

Generally well done and credit was given for any other relevant points.

9 (i) Initial Expense
Renewal Expense
Claim Expense
Overhead Expense

(ii) Initial Expense

Underwriting (allowed for on a per policy basis although medical expenses might be sum assured related) **or**;
Processing proposal and issuing policy (allowed for on a per policy basis) **or**;
Commission (allowed for directly and usually premium related) **or**;
Marketing (allowed for on a per policy basis on estimated volumes)

Renewal Expense

Administration (allowed for on a per policy per annum basis with allowance for inflation) **or**;
Commission (allowed for directly and usually premium related) **or**;
Investment Expense (charged as a deduction from investment funds).

Claim Expense

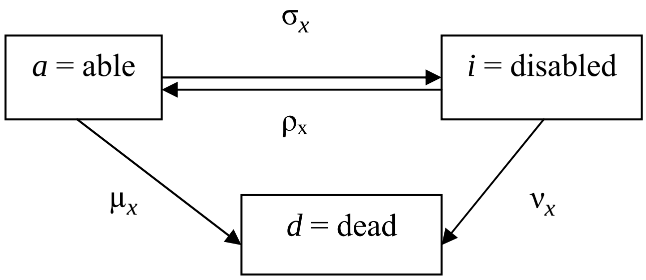
Calculation and payment of benefit (allowed for on a per policy per annum basis with allowance for inflation)

Overhead Expense

Central services e.g. premises, IT, legal (allowed for on a per policy per annum basis with allowance for inflation)]

Many students did not give a full answer referring only to direct and indirect expenses for which only partial credit was given, Also many did not give the full number of distinctly different examples. Other relevant examples were credited however.

10 The multiple state transition model is:



Define the force of interest δ

$$\text{Value of death benefit } = 10{,}000\int_0^{\infty} e^{-\delta t}\left({}_t p_x^{aa}.\mu_{x+t} + {}_t p_x^{ai}.\nu_{x+t}\right)dt$$

$$\text{Value of disablement benefit } = 1{,}000\int_0^{\infty} e^{-\delta t}\left({}_t p_x^{aa}.\sigma_{x+t}\right)dt$$

Generally the diagram was completed satisfactorily. Many students took the view that returning to the able state from the disabled one was impossible and thus omitted the return arrow. This was accepted so long as the assumptions were stated.

The resultant formulae were, however, on the whole poorly done.

11 (i) *Crude Mortality Rate*

Advantage – do not need population and deaths split by age
Disadvantage – differences in age structure between populations will be confounded

Directly Standardised Mortality Rate

Advantage – only reflects differences in mortality rates

Disadvantage – requires age specific mortality rates for the observed population

(ii) SMR = actual deaths / expected deaths

Expected deaths = 100000*0.00464+95000*0.00797+80000*0.01392 = 2335

SMR = 1250 / 2335 = 0.535

A straightforward question generally done well by well prepared students. Some students struggled to find the distinctive advantages and disadvantage given above.

12 The expected value is $100000\bar{A}^{1}_{x:\overline{n}|} + 50000A_{x:\overline{n}|}^{\ \ 1}$

$$\bar{A}^{1}_{x:\overline{n}|} = \int_0^{10} \exp((-\ln(1.05) - .03)t) \times .03dt$$

$$= 0.03\int_0^{10} \exp(-0.07879t)dt = 0.03\left[-\frac{\exp(-0.07879t)}{0.07879}\right]_0^{10}$$

$$= \frac{0.03}{0.07879}\left[1 - \exp(-0.7879)\right] \quad =0.20759$$

$$A_{x:\overline{n}|}^{\ \ 1} = \exp(-0.7879) = 0.45480$$

The expected value is thus $100000 \times 0.20759 + 50000 \times 0.45480 = £43499$

For the variance the rate used is $(1.05)^2 - 1 = 10.25\%$ and $\ln(1.1025) = 0.09758$.

Hence:

$${}^2\bar{A}^{1}_{x:\overline{n}|} = 0.03\int_0^{10} \exp(-0.12758t)dt = 0.03\left[-\frac{\exp(-0.12758t)}{0.12758}\right]_0^{10}$$

$$= \frac{0.03}{0.12758}\left[1 - \exp(-1.2758)\right] \quad =0.16949$$

$${}^2A_{x:\overline{n}|}^{\ \ 1} = \exp(-1.2758) = 0.27921$$

The variance is thus $\left(100000\right)^2 \times 0.16949 + \left(50000\right)^2 \times 0.27921 - (43499)^2 = (22378)^2$

The part relating to the expected value was generally done well. However by contrast the part relating to the variance was done poorly. Many students failed to realise that the integration process was the same as for the expected value with the exception of building in the 10.25% interest rate.

13 Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[20]:\overline{40}|} @ 6\% = 184.6092P$$

where

$$\ddot{a}^{(12)}_{[20]:\overline{40}|} = \ddot{a}_{[20]:\overline{40}|} - \frac{11}{24}\left(1 - v^{40}\ {}_{40}p_{[20]}\right)$$

$$= 15.801 - \frac{11}{24}\left(1 - 0.09722 \times \frac{9287.2164}{9980.2432}\right) = 15.801 - 0.4169 = 15.3841$$

EPV of benefits:

$$85{,}000\left(q_{[20]}v^{0.5} + {}_{1|}q_{[20]}(1+b)v^{1.5} + \ldots + {}_{39|}q_{[20]}(1+b)^{39}v^{39.5} + (1+b)^{40}\ {}_{40}p_{[20]}v^{40}\right)$$

where $b = 0.0192308$

$$= 85{,}000 \times \frac{(1.06)^{0.5}}{(1+b)}\left(q_{[20]}(1+b)v + {}_{1|}q_{[20]}(1+b)^2v^2 + \ldots + {}_{39|}q_{[20]}(1+b)^{40}v^{40}\right)$$
$$+85{,}000(1+b)^{40}v^{40}\ {}_{40}p_{[20]}$$

$$= \frac{85{,}000}{(1+b)} \times (1.06)^{0.5} \times A^{1}_{[20]:\overline{40}|} @ i' + 85{,}000v^{40}\ {}_{40}p_{[20]} @ i'$$

where

$$i' = \frac{1.06}{1+b} - 1 = 0.04$$

and $A^{1}_{[20]:\overline{40}|} @ i' = A_{[20]:\overline{40}|} - v^{40}\ {}_{40}p_{[20]}$

$$= 0.21746 - 0.20829 \times \frac{9287.2164}{9980.2432} = 0.21746 - 0.19383 = 0.02363$$

EPV of benefits

$$= \frac{85{,}000 \times (1.06)^{0.5}}{(1+b)} \times 0.02363 + 85{,}000 \times 0.19383$$

$$= 2{,}028.911 + 16{,}475.550 = 18{,}504.461$$

EPV of expenses

$$= 4.8P + 325 + 0.025 \times 12 \times P\ddot{a}^{(12)}_{[20]:\overline{40|}}$$

$$-0.025P + 65 \times \left[\ddot{a}_{[20]:\overline{40|}} - 1 \right] + 5 \times \left[(I\ddot{a})_{[20]:\overline{40|}} - 1 \right]$$

$$= 9.3902P + 325 + 65 \times 14.801 + 5 \times 208.366 = 9.3902P + 2{,}328.895$$

where

$$(I\ddot{a})_{[20]:\overline{40|}} = (I\ddot{a})_{[20]} - v^{40}\ {}_{40}p_{[20]} \left[40\ddot{a}_{60} + (I\ddot{a})_{60} \right]$$

$$= 262.666 - 0.09722 \times \frac{9287.2164}{9980.2432} \times \left[40 \times 11.891 + 113.516 \right]$$

$$= 262.666 - 53.300 = 209.366$$

Equation of value gives

$$184.6092P = 18{,}504.461 + 9.3902P + 2{,}328.895$$

$$\Rightarrow P = \frac{20{,}833.356}{175.219} = £118.90$$

The difficult part of this question was related to the EPV of Expenses and most students failed to complete this complex part. The rest of the question was however generally reasonably done by well prepared students.

14 (i) Annual net premium for the decreasing term assurance is given by:

$$P = \frac{210{,}000 A^{1}_{40:\overline{20|}} - 10{,}000 (IA)^{1}_{40:\overline{20|}}}{\ddot{a}_{40:\overline{20|}}}$$

where $A^{1}_{40:\overline{20|}} = A_{40:\overline{20|}} - v^{20}\ {}_{20}p_{40}$

$$= 0.46433 - 0.45639 \times \frac{9287.2164}{9856.2863} = 0.46433 - 0.43004 = 0.03429$$

and $(IA)^{1}_{40:\overline{20|}} = (IA)_{40} - v^{20}\ {}_{20}p_{40} \left[20A_{60} + (IA)_{60} \right]$

$$= 7.95699 - 0.43004 \times \left[20 \times 0.45640 + 8.36234 \right] = 0.43544$$

$$P = \frac{210,000 \times 0.03429 - 10,000 \times 0.43544}{13.927} = 204.39$$

(ii) Reserve at the <u>end</u> of the 10th policy year given by:

$$_{10}V = 110,000 A^{1}_{50:\overline{10|}} - 10,000 (IA)^{1}_{50:\overline{10|}} - P\ \ddot{a}_{50:\overline{10|}}$$

where

$$A^{1}_{50:\overline{10|}} = A_{50:\overline{10|}} - v^{10}\,{}_{10}p_{50}$$

$$= 0.68024 - 0.67556 \times \frac{9287.2164}{9712.0728} = 0.68024 - 0.64601 = 0.03423$$

and

$$(IA)^{1}_{50:\overline{10|}} = (IA)_{50} - v^{10}\,{}_{10}p_{50}\left[10A_{60} + (IA)_{60}\right]$$

$$= 8.55929 - 0.64601 \times \left[10 \times 0.45640 + 8.36234\right] = 0.20875$$

$$_{10}V = 110,000 \times 0.03423 - 10,000 \times 0.20875 - 204.39 \times 8.314$$

$$= 3,765.30 - 2,087.50 - 1,699.30 = -21.50$$

Therefore, sum at risk per policy in the 10th policy year is:

DSAR = 110,000 – (–21.50) = 110,021.50

Mortality profit = EDS – ADS

$$EDS = 625 \times q_{49} \times 110,021.50 = 625 \times 0.002241 \times 110,021.50 = 154,098.86$$

$$ADS = 3 \times 110,021.50 = 330,064.50$$

i.e. mortality profit = – 175,965.36 (i.e. a loss)

(iii) The death strain at risk per policy in the 10th policy year for this decreasing term assurance is very large (approximately equal to the sum assured payable in the event of death).

The actual number of deaths during the 10th policy year (at 3) is approximately double that expected (at 1.4) which accounts for the mortality loss.

However, a mortality experience investigation would need to consider a longer time period and ideally, a larger number of policies to determine whether actual mortality experience is heavier than expected.

Question generally done well by well prepared student. This was a straightforward question of its type.

15 (i) Gross future loss random variable =

$$150{,}000v^{K_{[57]}+1} + 350 + 50a_{\overline{K_{[57]}}|} - P(0.975\ddot{a}_{\overline{K_{[57]}+1)}|} - 0.125) \quad \text{if } K_{[57]} < 3$$

$$350 + 50a_{\overline{2}|} - P(0.975\ddot{a}_{\overline{3)}|} - 0.125) \quad if\ K_{[57]} \geq 3$$

(ii) E (Gross future loss random variable) = 0

$$\Rightarrow 150{,}000A^{1}_{[57]:\overline{3}|} + 350 + 50\left[\ddot{a}_{[57]:\overline{3}|} - 1\right] = P\left[0.975\ddot{a}_{[57]:\overline{3}|} - 0.125\right]$$

where $\mathrm{A}^{1}_{[57]:\overline{3}|} = \mathrm{A}_{[57]:\overline{3}|} - v^3\,{}_3p_{[57]} = 0.84036 - 0.83962 \times \dfrac{9287.2164}{9451.5938} = 0.84036 - 0.82502 = 0.01534$

and $\ddot{a}_{[57]:\overline{3}|} = 2.820$

$$\Rightarrow 150{,}000 \times 0.01534 + 350 + 50 \times 1.820 = 2.6245P$$

$$\Rightarrow P = \frac{2{,}301.0 + 350 + 91.0}{2.6245} = 1{,}044.77$$

(iii) Mortality table:

x	t	$q_{[x]+t-1}$	$p_{[x]+t-1}$	${}_{t-1}p_{[x]}$
57	1	0.004171	0.995829	1.000000
58	2	0.006180	0.993820	0.995829
59	3	0.007140	0.992860	0.989675

Cash flows (per policy at start of year) assuming annual premium is denoted by P:

Year	*1*	*2*	*3*
Premium	P	P	P
Expenses	$0.15P + 350$	$0.025P + 50$	$0.025P + 50$
Interest	$0.051P - 21$	$0.0585P - 3$	$0.0585P - 3$
Claim	625.65	927.00	1071.00
Profit vector	$0.901P - 996.65$	$1.0335P - 980.00$	$1.0335P - 1124.00$
Cumulative probability of survival	1.000000	0.995829	0.989675
Profit signature	$0.9010P - 996.650$	$1.0292P - 975.912$	$1.0228P - 1112.395$
Discount factor	0.94340	0.890000	0.83962
NPV of profit	$0.85P - 940.240$	$0.9160P - 868.562$	$0.8588P - 933.989$

Therefore:

$$\sum_{1}^{3} NPV = 0 = 2.6248P - 2742.791 \Rightarrow P = \frac{2742.791}{2.6248} = 1{,}044.95$$

which is consistent with the premium calculated in (ii) above (allowing for rounding)

(iv) (a) profit is deferred but as the earned interest rate is equal to the risk discount rate, there is no change to the *NPV* or premium

(b) profit is deferred and because the risk discount rate is greater than the earned interest rate, *NPV* falls. Therefore, the premium would need to be increased to satisfy the same profit criterion.

Most students struggled with part (i) but well prepared ones completed parts (ii) and (iv) satisfactorily. Part (iii) caused students great difficulties as often occurs with this approach and many even failed to realise that the answers to (ii) and (iii) should numerically be the same within rounding.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2012 examinations

Subject CT5 – Contingencies Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

D C Bowie
Chairman of the Board of Examiners

December 2012

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given to alternative valid points which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the September 2012 paper

The general performance was lower than average this session. Questions that were done less well were 6, 11, 12, and 15(ii) and (iii); more commentary on these questions is given in this report to assist candidates with further revision.

However most of the short questions were very straightforward and this is where many successful candidates scored particularly well. Students should note that for long questions a reasonable level of credit is given if they can describe the right procedures; however, to score well reasonable, accurate numerical calculation is necessary.

1 (a) $${}_{12}p_{43} = \frac{l_{55}}{l_{43}} = \frac{9557.8179}{9826.2060} = 0.97269$$

(b) $${}_{10|5}q_{55} = \frac{l_{65} - l_{70}}{l_{55}} = \frac{8821.2612 - 8054.0544}{9557.8179} = 0.08027$$

(c) $$\ddot{a}_{45:\overline{10|}} = \ddot{a}_{45} - \left(v^{10} \times \left(\frac{l_{55}}{l_{45}} \right) \times \ddot{a}_{55} \right) \text{ at } 6\%$$

$$= 14.850 - \left(0.55839 \times \frac{9557.8179}{9801.3123} \times 13.057 \right)$$

$$= 7.740$$

Generally well done

2

- Class selection – e.g. males and females have different mortality/longevity characteristics.

- Time Selection – mortality rates vary over time (in annuities generally improves).

- Adverse Selection – lives in better health than average may be more likely to purchase an annuity

Generally well done-other plausible types and examples were credited.

3 Nutrition has an important influence on morbidity and in the longer term on mortality.

Poor quality nutrition can increase the risk of contracting many diseases and hinder recovery from sickness.

Excessive or inappropriate (e.g. too much fat) eating can lead to obesity and an increased risk of associated diseases (e.g. heart disease, hypertension) leading to increased morbidity and mortality.

Inappropriate nutrition may be the result of economic factors e.g. lack of income to buy appropriate foods or the result of a lack of health and personal education resulting in poor nutritional choices.

Also, social and cultural factors encourage or discourage the eating of certain foods e.g. alcohol consumption.

Many candidates gave a reasonable answer but there was a tendency to overlook the obesity risk in the second paragraph.

4 $_3p_{55.75} = {}_{0.25}p_{55.75} \times p_{56} \times p_{57} \times {}_{0.75}\, p_{58}$

$$= (1 - {}_{0.25}\, q_{55.75}) \times (1 - q_{56}) \times (1 - q_{57}) \times (1 - {}_{0.75}\, q_{58})$$

$$= \left(1 - \left(\frac{0.25 \times 0.00475}{1 - 0.75 \times 0.00475}\right)\right) \times 0.99469 \times 0.99408 \times (1 - 0.75 \times 0.00660)$$

$$= 0.99881 \times 0.99469 \times 0.99408 \times 0.99505$$

$$= 0.98274$$

Generally well done.

5 (a) Definitions of three terms are:

$E^c_{x,t}$ Central exposed to risk in population being studied between ages x and $x + t$.

$m_{x,t}$ Central rate of mortality either observed or from a life table in population being studied for ages x to $x + t$.

${}^sE^c_{x,t}$ Central exposed to risk for a standard population between x and $x + t$.

${}^sm_{x,t}$ Central rate of mortality either observed or from a life table in standard population for ages x to $x + t$.

(b) The area comparability factor (F) is the ratio of the mortality rates in the standard population weighted by the age structure distribution of the standard population to the mortality rates in the standard population weighted by the age structure distribution of the observed population.

F is therefore is a measure of variation between population age structures.

Many candidates gave formulae that were not required. Also many did not give a complete answer.

6 The net future loss random variable is given by:

$$S(1+b)^{K_{40}+1}v^{T_{40}} - P\ddot{a}_{\overline{\min(K_{40}+1,45)}|}$$

b = annual rate of future bonus
P = annual net premium
K_{40} = curtate future lifetime of a life aged 40 exact
T_{40} = complete future lifetime of a life aged 40 exact

Generally not done well. It is often the case that candidates have difficulties in setting out the random variable expressions.

7 Reserve at the end of the 5th policy year is given by:

$$ {}_5V = 125{,}000v^{25}\, {}_{25}p_{40} - P\ddot{a}_{40:\overline{25}|} = 125{,}000 \times 0.37512 \times \frac{8821.2612}{9856.2863} - P \times 15.884 $$

where annual net premium for the policy is given by

$$ P = \frac{125{,}000v^{30}\, {}_{30}p_{[35]}}{\ddot{a}_{[35]:\overline{30}|}} = \frac{125{,}000 \times 0.30832 \times \dfrac{8821.2612}{9892.9151}}{17.631} = 1949.13 $$

$$ \Rightarrow {}_5V = 41{,}966.00 - 30{,}959.98 = 11{,}006.02 $$

$DSAR = 0 - {}_5V = -11{,}006.02$

$EDS = 3521q_{39} \times DSAR = 3521 \times .00087 \times DSAR = -33{,}714.41$

$ADS = -8 \times 11{,}006.02 = -88{,}048.16$

Profit = $EDS - ADS$ = 54,333.75

Generally done fine by well-prepared candidates

8

- Mortality just after birth ("infant mortality") is very high.

- Mortality then falls dramatically during the first few years of life and is at lowest around ages 8–10.

- There is a distinct "hump" in the deaths at ages around 18–25. This is often attributed to a rise in accidental deaths during young adulthood, and is called the "accident hump".

- From middle age onwards there is a steep increase in mortality, reaching a peak at about age 80.

- The number of deaths at higher ages falls again (even though the mortality rate q_x continues to increase) since the probabilities of surviving to these ages are small.

Generally well done but many candidates did not score all available marks.

9 (i) (a) The gross premium prospective policy reserve is the expected present value of future benefits (including declared bonus and an allowance for future bonus if applicable) and future expenses less the expected present value of future gross premiums.

(b) The gross premium retrospective policy reserve is the expected accumulation of past gross premiums received, less expected expenses and benefits including bonuses included in past claims.

(ii) Gross premium retrospective and prospective reserves will be equal if:

- the mortality and interest rate basis used is the same as used to determine the gross premium at the date of issue of the policy; and

- the expenses valued are the same as those used to determine the original gross premium; and

- the gross premium is that determined on the original basis (mortality, interest, expenses) using the equivalence principle

Generally done well but many answers were incomplete on a standard bookwork question.

10 Value of benefit:

$$\frac{r_{65}}{l_{35}} v^{(65-35)} 20{,}000\left(\frac{1}{12}\right)(65-35)\frac{\frac{s_{63}+s_{64}}{2}}{s_{35}}$$

$$= \frac{3757}{18866} v^{(65-35)} 20{,}000\left(\frac{1}{12}\right)(65-35)\frac{\frac{11.151+11.328}{2}}{6.655}$$

$= 5185$

Assume value of contributions is *K%* of salary

Value of contributions of *K%* of salary

$$20{,}000.K\%.\frac{{}^{s}\bar{N}_{35}}{{}^{s}D_{35}} = 20{,}000.K\%.\frac{502{,}836}{31{,}816} = 316{,}090.K\%$$

Therefore $K = 1.64$

This question was generally poorly answered despite being a relatively straightforward question. The main issue was in understanding how the benefit value arose.

11 Because the annuity is payable weekly this can reasonably be represented by continuous annuity functions.

Working initially for a unit annualised payment:

$$\text{PV} = \bar{a}_{\overline{5}|} + v^5 \frac{l_{70}^m}{l_{65}^m} \times \frac{l_{67}^f}{l_{62}^f} \times \left(\frac{2}{3} \times \bar{a}_{\overline{70(m):67(f)}} + \frac{1}{3} \times \bar{a}_{70(m):67(f)} \right)$$

$$+ v^5 \frac{l_{70}^m}{l_{65}^m} \times \left(1 - \frac{l_{67}^f}{l_{62}^f} \right) \times \left(\frac{2}{3} \bar{a}_{70(m)} \right) + v^5 \left(1 - \frac{l_{70}^m}{l_{65}^m} \right) \times \frac{l_{67}^f}{l_{62}^f} \times \left(\frac{2}{3} \bar{a}_{67(f)} \right) \text{ at } 4\%$$

$$\bar{a}_{\overline{5}|} = (i/\delta) \times a_{\overline{5}|} = 1.019869 \times 4.4518 = 4.5403$$

$$\frac{l_{70}^m}{l_{65}^m} = \frac{9238.134}{9647.797} = 0.95754, \quad \frac{l_{67}^f}{l_{62}^f} = \frac{9605.483}{9804.173} = 0.97973$$

$$\bar{a}_{70(m)} = \ddot{a}_{70(m)} - 0.5 = 11.062, \ \bar{a}_{67(f)} = \ddot{a}_{67(f)} - 0.5 = 13.611,$$

$$\bar{a}_{70(m):67(f)} = \ddot{a}_{70(m):67(f)} - 0.5 = 9.733$$

$$\bar{a}_{\overline{70(m):67(f)}} = 11.062 + 13.611 - 9.733 = 14.940$$

$$PV = 4.5403 + \left(0.82193 \times 0.95754 \times 0.97973 \times \left(\frac{2}{3} \times 14.940 + \frac{1}{3} \times 9.733 \right) \right)$$

$$+ 0.82193 \times 0.95754 \times 0.02027 \times \frac{2}{3} \times 11.062$$

$$+ 0.82193 \times 0.04246 \times 0.97973 \times \frac{2}{3} \times 13.611$$

$$= 4.5403 + 10.1816 + 0.1176 + 0.3103$$

$$= 15.1498$$

The annualised benefit is 500 × 52.18 = 26090 p.a. (NB 52 acceptable)

So PV = 26090 ×15.1498 = £395,258

The key to this question is to break down carefully the component parts of the annuity. Once this is done the question is then a relatively simple calculation of annuity functions. The question was generally done poorly and many candidates failed to realise that a weekly annuity could be closely approximated by a continuous one.

12 The value of the death benefit is:

$$100000\int_0^{25} e^{-.05t}\{e^{-.02t}(1-e^{-.03t})\times .02+e^{-.03t}(1-e^{-.02t})\times .03\}dt$$

$$=100000\int_0^{25}(.02e^{-.07t}+.03e^{-.08t}-.05e^{-.1t})dt$$

$$=100000\left\{\left[-\frac{0.02e^{-.07t}}{.07}\right]_0^{25}+\left[-\frac{0.03e^{-.08t}}{.08}\right]_0^{25}-\left[-\frac{.05e^{-.1t}}{.1}\right]_0^{25}\right\}$$

$$=100000\left\{\left(\frac{2}{7}+\frac{3}{8}-\frac{1}{2}\right)-\left(\frac{2e^{-1.75}}{7}+\frac{3e^{-2}}{8}-\frac{e^{-2.5}}{2}\right)\right\}$$

$$=100000\{(0.28571+0.375-0.5)-(.04965+.05075-.04104)\}$$

$$=10135$$

The value of the survival benefits are:

$$e^{-1.25}\times(100000e^{-.5}\times e^{-.75}+50000e^{-.5}(1-e^{-.75})+50000e^{-.75}(1-e^{-.5}))$$

$$=50000e^{-1.75}+50000e^{-2}$$

$$=8688.7+6766.8=15456 \text{ say}$$

The value of annualised premium P is:

$$P\int_0^{25} e^{-.05t}\{e^{-.02t}(1-e^{-.03t})+e^{-.03t}(1-e^{-.02t})+e^{-.05t}\}dt$$

$$=P\int_0^{25}(e^{-.07t}+e^{-.08t}-e^{-.1t})dt$$

$$=P\left\{\left[\frac{-e^{-.07t}}{0.07}\right]_0^{25}+\left[\frac{-e^{-.08t}}{0.08}\right]_0^{25}-\left[\frac{-e^{-.1t}}{0.1}\right]_0^{25}\right\}$$

$$=P\left\{\left(\frac{1}{.07}+\frac{1}{.08}-\frac{1}{.1}\right)-\left(\frac{e^{-1.75}}{.07}+\frac{e^{-2}}{.08}-\frac{e^{-2.5}}{.1}\right)\right\}$$

$$=P\{(14.2857+12.5-10)-(2.4825+1.6917-0.8208)\}$$

$$=13.432P$$

So $P = \dfrac{10135 + 15456}{13.432} = £1905.23$

Many well prepared candidates made a very good attempt at this difficult question but in general terms it was done quite poorly. As in Question 11 the key is to organise the component parts logically.

13 (i) If the monthly premium and sum assured are denoted by P and S respectively then:

$$0.975 \times 12 P \ddot{a}^{(12)}_{[55]:\overline{30}|} + 0.025P$$

$$= (0.98S + 200)\bar{A}_{[55]} + 0.02S(I\bar{A})_{[55]} + 275 + 65(\ddot{a}_{[55]} - 1) + 0.75 \times 12P$$

$$\Rightarrow 0.975 \times 12 P \ddot{a}^{(12)}_{[55]:\overline{30}|} + 0.025P$$

$$= (1.04)^{0.5}\left[(0.98 \times 75{,}000 + 200)A_{[55]} + 0.02 \times 75{,}000(IA)_{[55]}\right] + 275 + 65(\ddot{a}_{[55]} - 1) + 9P$$

where

$$\ddot{a}^{(12)}_{[55]:\overline{30}|} = \ddot{a}^{(12)}_{[55]} - v^{30}\,{}_{30}p_{[55]}\ddot{a}^{(12)}_{85}$$

$$= \left(\ddot{a}_{[55]} - \frac{11}{24}\right) - v^{30}\,{}_{30}p_{[55]}\left(\ddot{a}_{85} - \frac{11}{24}\right)$$

$$= \left(15.891 - \frac{11}{24}\right) - .30832 \times \frac{3385.2479}{9545.9929}\left(5.333 - \frac{11}{24}\right)$$

$$= 15.433 - 0.533 = 14.900$$

$$\Rightarrow (0.975 \times 12 \times 14.9 + 0.025)P$$

$$= (1.04)^{0.5}\left[73{,}700 \times 0.38879 + 1{,}500 \times 8.58908\right] + 275 + 65 \times 14.891 + 9P$$

$$\Rightarrow 174.355P = (1.04)^{0.5}[28{,}653.823 + 12{,}883.62] + 275 + 967.915 + 9P$$

$$\Rightarrow 165.355P = 42{,}360.046 + 275 + 967.915 \Rightarrow P = £263.69$$

(ii) Gross prospective policy value (calculated at 4%) is given by:

$$V^{\text{prospective}} = \left(0.975S + B + 250\right)\bar{A}_{85} + 0.025S(I\bar{A})_{85} + 80\ddot{a}_{85}$$

where $B = 30 \times 0.02 \times 75{,}000 = 45{,}000$

$\Rightarrow$

$$V^{\text{prospective}} = (1.04)^{0.5}\left\{\left(0.975 \times 75{,}000 + 45{,}000 + 250\right)A_{85} + 0.025 \times 75{,}000(IA)_{85}\right\} + 80\ddot{a}_{85}$$

$$= (1.04)^{0.5}(118{,}375 \times 0.7949 + 1{,}875 \times 4.40856) + 80 \times 5.333$$

$$= 104{,}389.51 + 426.64 = £104{,}816.15$$

Generally part (i) was done well. Very few candidates successfully completed part (ii) as is often the case with prospective reserve calculations.

14

We have the following multiple decrement table:

Year t	$(aq)^d_{[30]+t-1}$	$(aq)^s_{30+t-1}$	$(aq)^r_{30+t-1}$	$(ap)_{[x]+t-1}$	${}_{t-1}(ap)_{[30]}$
1	.000447	.098727	.023744	.877082	1.000000
2	.000548	.049361	.024368	.925723	.877082
3	.000602	.024680	.024680	.950038	.811935
4	.000636	0	0	.999364	.771370

Cash flows:

Year t	*Premium P*	*Expenses E*	*Interest on P-E*	*Death Claim*	*Surrender Claim*	*Redundancy Claim*	*Maturity Claim*	*Profit Vector*
1	14000.00	700.00	399.00	27.22	701.38	337.37	0	12633.04
2	14000.00	700.00	399.00	33.37	701.34	692.46	0	12271.82
3	14000.00	700.00	399.00	36.66	526.00	1051.99	0	12084.35
4	14000.00	700.00	399.00	38.73	0	0	59961.84	-46301.57

Note: allowance for ½ year interest roll up is included in death, surrender and redundancy costs

Year t	*Profit Vector*	*Cum probability of survival*	*Profit signature*	*Discount factor*	*NPV of Profit signature*
1	12633.04	1.0	12633.04	.952381	12031.46
2	12271.82	.877082	10763.40	.907029	9762.72
3	12084.35	.811935	9811.71	.863838	8475.72
4	-46301.57	.771370	-35715.60	.822702	-29383.31

=> Total NPV of profit = **886.59**

NPV of premium = 14,000 × (1 + .877082 × .952381 + .811935 × .907029 + .771370 × .863838)
= 45,333.44

Therefore, profit margin = 886.61/45,333.44 = 1.96%

Credit was given for correct data items and many well prepared candidates scored a reasonable proportion of the marks available. Very few got to the final answer however.

15

Annual premium	£3000.00	Allocation % (1st yr)	75.0%
Risk discount rate	6.5%	Allocation % (2nd yr)	100.0%
Interest on investments (1st yr)	5.0%	Allocation % (3rd yr)	105.0%
Interest on investments (2nd yr)	4.5%	B/O spread	5.0%
Interest on investments (3rd yr)	4.0%	Management charge	1.5%
Interest on non-unit funds	3.0%	Policy Fee	£35
Death benefit (% of bid value of units)	150%		

	£	*% premium*
Initial expense	275	20.0%
Renewal expense	80	2.5%
Expense inflation	2.0%	

Mortality table:

X	$q_{[x]+t-1}$	$p_{[x]+t-1}$	${}_{t-1}p_{[x]}$
45	0.001201	0.998799	1.000000
46	0.001557	0.998443	0.998799
47	0.001802	0.998198	0.997244

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
value of units at start of year	0.000	2174.511	5135.828
Alloc	2250.000	3000.000	3150.000
B/O	112.50	150.000	157.500
policy fee	35.000	35.000	35.000
interest	105.125	224.528	323.733
management charge	33.114	78.211	126.256
value of units at year end	2174.511	5135.828	8290.805

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
unallocated premium + pol fee	785.000	35.000	–115.000
b/o spread	112.500	150.000	157.500
expenses	875.000	156.600	158.232
interest	0.675	0.852	–3.472
man charge	33.114	78.211	126.256
extra death benefit	1.306	3.998	7.470
profit vector	54.984	103.464	–0.418

(i) If policyholder dies in the 3rd year of contract, non unit cash flows at end of each year are:

$$yr\ 1 = (785 + 112.5 - 875 + 0.675 + 33.114) = 56.289$$
$$yr\ 2 = (35 + 150 - 156.6 + 0.852 + 78.211) = 107.463$$
$$yr\ 3 = (-115 + 157.5 - 158.232 - 3.472 + 126.256 - 0.5 \times 8290.805) = -4138.351$$

=> expected present value of these cash flows is given by:

$$\left[56.289 \times v + 107.463 \times v^2 - 4138.351 \times v^3\right] \times p_{[45]} \times p_{[45]+1} \times q_{47}$$
$$= [52.854 + 94.746 - 3425.930] \times 0.998799 \times 0.998443 \times 0.001802 = -5.891$$

(ii) (a) If policyholder dies in the 1st year of contract, non unit cash flow at end of 1st year is:

$$yr\ 1 = (785 + 112.5 - 875 + 0.675 + 33.114 - 0.5 \times 2174.511) = -1030.967$$

=> expected present value of this cash flow is given by:

$$-1030.967 \times v \times q_{[45]} = -968.967 \times 0.001201 = -1.163$$

(b) If policyholder dies in the 2nd year of contract, non unit cash flows at end of each year are:

$$yr\ 1 == 56.289 \text{ (derived above)}$$
$$yr\ 2 = (35 + 150 - 156.6 + 0.852 + 78.211 - 0.5 \times 5135.828) = -2460.451$$

=> expected present value of these cash flows is given by:

$$\left[56.289 \times v - 2460.451 \times v^2\right] \times p_{[45]} \times q_{[45]+1}$$
$$= [52.854 - 2169.279] \times 0.998799 \times 0.001557 = -3.291$$

(iii) If policyholder survives until end of contract, non unit cash flows at end of each year are:

$yr\,1 == 56.289$ (derived above)
$yr\,2 = 107.463$ (derived above)
$yr\,3 = \left(-115 + 157.5 - 158.232 - 3.472 + 126.256\right) = 7.052$

=> expected present value of these cash flows is given by:

$$\left[56.289 \times v + 107.463 \times v^2 + 7.052 \times v^3\right] \times {}_3p_{[45]}$$
$$= \left[52.854 + 94.746 + 5.838\right] \times 0.998799 \times 0.998443 \times 0.998198 = 152.737$$

Expected present value of policy is therefore = $-1.163 - 3.291 - 5.891 + 152.737$
= **142.39**

Candidates generally found this question difficult particularly parts (ii) and (ii). Part credit was given in (i) for correctly calculating the data items and well prepared candidates scored a fair proportion of the marks here.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2013 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

D C Bowie
Chairman of the Board of Examiners

July 2013

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the April 2013 paper

The general performance was similar this session to previous ones although it was felt that this paper was a little easier than some previous ones. Questions that were done less well were 9, 10 (variance), 11, 12(b) and 14(ii). The examiners hope that the detailed solutions given below will assist students with further revision.

However most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions some credit is given if they can describe the right procedures although to score well reasonably accurate numerical calculation is necessary.

1 (a) $_{10|5}q_{40} = (l_{50} - l_{55}) / l_{40} = (9712.0728 - 9557.8179) / 9856.2863$
$= 0.01565$

(b) $\bar{a}_{65} = \ddot{a}_{65} - 1/2 = 11.776$

(c) $_{15}p_{[46]} = l_{61} / l_{[46]} = 9212.7143 / 9783.3371 = 0.94167$

Generally question done well.

2 The constant force of decrement is consistent with the Kolmogorov equations where the transition intensities are constant.

Thus:

$$(aq)_x^{\alpha} = \frac{\mu_\alpha}{(\mu_\alpha + \mu_\beta)} \times (1 - e^{-(\mu_\alpha + \mu_\beta)})$$

$$= \frac{0.1}{0.3} \times (1 - e^{-0.3}) = 0.086394$$

Generally question done well. Other approaches given credit.

3 Climate and geographical location are closely linked. Levels and patterns of rainfall and temperature lead to an environment which is amicable to certain kinds of diseases e.g. those associated with tropical regions.

Effects can also be observed within these broad categories e.g. the differences between rural and urban areas in a geographical region. Some effects may be accentuated or mitigated depending upon the development of an area e.g. industry leading to better roads and communications.

Natural disasters (such as tidal waves and famines) will also affect mortality and morbidity rates, and may be correlated to particular climates and geographical locations.

Generally question done well. Other valid points given credit.

4

- Terminal bonuses are allocated when a policy matures or becomes a claim as a result of the death of the life assured.

- Terminal bonuses are usually allocated as a percentage of the basic sum assured and the bonuses allocated prior to a claim.

- The terminal bonus percentage rate will vary with the term of the policy at the date of payment.

- Because the policy is being terminated, the terminal bonus rate is usually chosen so as to distribute all the surplus available to the policy based on asset share.

- Distributing available surplus as a terminal bonus delays the distribution of surplus and may allow the insurer to choose investments that are more volatile in the short term but are expected to be more profitable in the long term.

Generally question done well. Other valid points given credit. In particular comments about effects on lapse rates were an important extra point.

5 Past Service:

Value is $\dfrac{10}{60} * 40000 * \dfrac{({}^zM_{30}^{ra} + {}^zM_{30}^{ia})}{s_{29}D_{30}} = \dfrac{1}{6} * 40000 * \dfrac{128026 + 64061}{4.991 * 7874} = 32585.5$

Future Service:

Value is $\dfrac{1}{60} * 40000 * \dfrac{({}^z\bar{R}_{30}^{ra} + {}^z\bar{R}_{30}^{ia})}{s_{29}D_{30}} = \dfrac{1}{60} * 40000 * \dfrac{4164521 + 1502811}{4.991 * 7874} = 96140.1$

Total Value is 32585.5+96140.1 = £128,726 rounded

Generally question done well. It was not necessary to give the total in the last line for full credit.

6 The death benefit in policy year 10 is £65,000 which increases by £1,500 each year and the maturity value is £80,000. Therefore:

(a) Net premium P for the policy is given by

$$P = \frac{\left(50{,}000A^{1}_{30:\overline{20|}} + 1{,}500(IA)^{1}_{30:\overline{20|}} + 80{,}000 \times v^{20} \times \dfrac{l_{50}}{l_{30}}\right)}{\ddot{a}_{30:\overline{20|}}}$$

(b) Net premium prospective policy reserve at duration t = 9 is given by:

$${}_9V^{\text{Pro}} = 63{,}500A^{1}_{39:\overline{11|}} + 1{,}500(IA)^{1}_{39:\overline{11|}} + 80{,}000 \times v^{11} \times \frac{l_{50}}{l_{39}} - P\ddot{a}_{39:\overline{11|}}$$

Well prepared students scored good marks but many made elementary mistakes the most common of which was 48500 as the 1st factor in the numerator of the first formula above.

The alternative solution for the numerator in (a) is:

$$50000A_{30:\overline{20|}} + 1500(IA)_{30:\overline{20|}}$$

And for (b) overall:

$${}_9V = 63500A_{39:\overline{11|}} + 1500(IA)_{39:\overline{11|}} - P\ddot{a}_{39:\overline{11|}}$$

7 When a life table is constructed it is assumed to reflect the mortality experience of a homogeneous group of lives i.e. all the lives to whom the table applies follow the same stochastic model of mortality represented by the rates in the table. This means that the table can be used to model the mortality experience of a homogeneous group of lives which is suspected to have a similar experience.

If a life table is constructed for a heterogeneous group then the mortality experience will depend on the exact mixture of lives with different experiences that has been used to construct the table. Such a table could only be used to model mortality in a group with the same mixture. It would have very restricted uses.

For this reason separate mortality tables are usually constructed for groups which are expected to be heterogeneous. This can manifest itself as class selection e.g. separate tables for males and females, whole life and term assurance policyholders, annuitants and pensioners, or as time selection e.g. separate tables for males in England and Wales in 1980–82 (ELT14) and 1990–92 (ELT15).

Sometimes only parts of the mortality experience are heterogeneous e.g. the experience during the initial select period for life assurance policyholders, and the remainder are homogeneous e.g. the experience after the end of the select period for life assurance policyholders. In such cases the tables are separate (different) during the select period, but combined after the end of the select period. In fact there are separate (homogeneous) mortality tables for each age at selection, but they are tabulated in an efficient (space saving) way.

Generally question done well. Other valid points given credit.

8 (i) The crude mortality rate is defined as

$$\frac{\sum_x E^c_{x,t}\, m_{x,t}}{\sum_x E^c_{x,t}} = \frac{\text{actual deaths}}{\text{total exposed to risk}}$$

It is a weighted average of $m_{x,t}$ using $E^c_{x,t}$ as weights where:

$E^c_{x,t}$ Central exposed to risk in population being studied between ages x and $x + t$.

$m_{x,t}$ Central rate of mortality either observed or from a life table in population being studied for ages x to $x + t$.

The directly standardised mortality rate is defined as

$$\frac{\sum_x {}^sE^c_{x,t}\, m_{x,t}}{\sum_x {}^sE^c_{x,t}}$$

It is a weighted average of $m_{x,t}$ using ${}^sE^c_{x,t}$ as weights where

${}^sE^c_{x,t}$ Central exposed to risk for a standard population between x and $x + t$.

(ii)

Age	*Lives*	*Deaths*	*ELT15 M rate*	*Expected Deaths*
65	125000	2937	0.02447	3058.8
66	130000	3301	0.02711	3524.3
67	140000	3756	0.02997	4195.8
		9994		10778.9

Standardised mortality ratio is 9994/10778.9 = 0.927

Generally question done well. It was not necessary to make the weighted average remarks in line 3 and 7 above to obtain full marks.

9 $\text{PV}=10000\ddot{a}^{(12)}_{\overline{10}|}+5000*(\ddot{a}^{(12)}_{\overline{65:62}}+\ddot{a}^{(12)}_{65})+10000*(\bar{A}_{65}+\bar{A}_{62})$

where 65 relates to the male life and 62 the female.

$$10000\ddot{a}^{(12)}_{\overline{10}|}=10000*(i/d^{(12)})*a_{\overline{10}|}=1021537*8.1109=82855.85$$

$$\ddot{a}^{(12)}_{\overline{65:62}}=\left(\ddot{a}_{65}-\frac{11}{24}\right)+\left(\ddot{a}_{62}-\frac{11}{24}\right)-\left(\ddot{a}_{65:62}-\frac{11}{24}\right)=13.666+15.963-12.427-\frac{11}{24}$$

$$=16.744$$

$$\ddot{a}^{(12)}_{65}=\left(\ddot{a}_{65}-\frac{11}{24}\right)=13.666-\frac{11}{24}=13.208$$

$$10000\bar{A}_{65}=10000*(1-\ln(1.04)*(13.666-.5))=4836.20$$

$$10000\bar{A}_{62}=10000*(1-\ln(1.04)*(15.963-.5))=3935.30$$

$$\text{Total Value} = 82855.85 + 5000*(16.744 + 13.208) + 4836.20 + 3935.30$$

$$= £241387 \text{ rounded}$$

Well prepared students completed this question satisfactory but others had problems with the joint life portion. A very few students concluded that the question wording could be taken to mean that for the joint part the annuity ceases altogether on the female life death and examiners agreed that this was a potential ambiguity and the alternative approach was allowable. This alternative approach gave an answer of £228,994.

10 The expected value is:

$$1000(A_{40} + v^{20}\frac{l_{60}}{l_{40}}A_{60} + v^{40}\frac{l_{80}}{l_{40}}A_{80})$$

$$= 1000*(0.23056 + (0.45639*\frac{9287.2164}{9856.2863}*0.45640) + (0.20829*\frac{5266.4604}{9856.2863}*0.73775))$$

$$= 1000*(0.23056 + 0.19627 + 0.08211)$$

$$= £509 \text{ to nearer } £$$

To get the variance we calculate the second moment by defining the benefit as three temporary assurances, two of which are deferred, thus:
Benefit from age 40–60

$$(1000)^2*[{}^2A_{40} - v^{20}\frac{l_{60}}{l_{40}}\,{}^2A_{60}] \ (v \text{ at } 8.16\%)$$

$$= (1000)^2*(0.06792 - 0.20829*\frac{9287.2164}{9856.2863}*0.23723)$$

$$= 1,000,000*.021361 = 21,361$$

Benefit from age 60–80

$$(2000)^2*v^{20}\frac{l_{60}}{l_{40}}*[{}^2A_{60} - v^{20}\frac{l_{80}}{l_{60}}\,{}^2A_{80}] \ (v \text{ at } 8.16\%)$$

$$= (2000)^2*0.20829*\frac{9287.2164}{9856.2863}(0.23723 - 0.20829*\frac{5266.4604}{9287.2164}*0.56432)$$

$$= 4,000,000*.033478 = 133,911$$

Benefit from age 80

$(3000)^2 * v^{40} \frac{l_{80}}{l_{40}} * {}^2A_{80}$ (v at 8.16%)

$= (3000)^2 * 0.04338 * \frac{5266.4604}{9856.2863} * 0.56432$

$= 9,000,000 * .013082 = 117,735$

Second moment:

= 21,361 + 133,911 + 117,735

= 273,007

Variance = 273,007 – $(509)^2$

= 13,926

= $(£118)^2$

The calculation for the mean was generally well done but the calculation for the variance was poorly done overall.

11 (i) The probability is $(e^{(-20*.05)})^2 = e^{-2} = 0.13534$

(ii) The value is

$$1000*(1+(1.03/1.04)(e^{-.05})^2 + (1.03/1.04)^2 (e^{-.1})^2 + (1.03/1.04)^3 (e^{-.15})^2 + ...)$$
$$= 1000 * (1/(1-(1.03/1.04)e^{-.1})) = 1000/0.10386 = £9628$$

(iii) The value is

$$\int_0^{20} (100000 * (1.04)^{-t} * (2e^{-.05t}(1-e^{-.05t}) * .05) dt$$

$$= 10000 \int_0^{20} (e^{-t(\ln(1.04)+.05)} - e^{-t(\ln(1.04)+.1)}) dt$$

$$= 10000 \int_0^{20} (e^{-.089221t} - e^{-.139221t}) dt = 10000 \left[-\frac{e^{-.089221t}}{.089221} + \frac{e^{-.139221t}}{.139221} \right]_0^{20}$$

$$= 10000 * \left(-\frac{e^{-1.78442}}{.089221} + \frac{e^{-2.78442}}{.139221} + \frac{1}{.089221} - \frac{1}{.139221} \right)$$

$$=10000*(-1.88180+0.44365+11.20812-7.18282)$$

$$=£25872$$

Parts (ii) and (iii) were poorly done. In (ii) many students failed to realise that the expression needed was a geometric series rather than an integral.

12 (a) Let P be the annual premium for the policy. Then (functions at 6%):

EPV of premiums:

$$P\ddot{a}_{[50]}=14.051P$$

EPV of benefits:

$$75,000A_{[50]}$$

EPV of expenses:

$$P+325+\left(75+0.025P\right)a_{[50]}$$

Equation of value gives:

$$P\ddot{a}_{[50]}=75,000A_{[50]}+325+P+\left(0.025P+75\right)a_{[50]}$$

$$P\times 14.051=75,000\times 0.20463+325+P+\left(0.025P+75\right)\times 13.051$$

$$\Rightarrow P=\frac{16,651.075}{12.724725}=1,308.56$$

(b) The insurer's loss random variable for this policy is given by (where K and T denote the curtate and complete future lifetime of a policyholder):

$$L=75,000v^{K_{[50]}+1}+325+P'+\left(0.025P'+75\right)a_{\overline{K_{[50]}}|}-P'\ddot{a}_{\overline{K_{[50]}+1}|}$$

We need to find a value of t such that

$$P\left(L>0\right)=P\left(T<t\right)=0.1\Rightarrow P\left(T\geq t\right)=0.9$$

Using AM92 Select, we require:

$$\frac{l_{[50]+t}}{l_{[50]}}\geq 0.9\Rightarrow l_{[50]+t}\geq 0.9l_{[50]}=0.9\times 9706.0977=8735.488$$

As $l_{65} = 8821.2612$ *and* $l_{66} = 8695.6199$ then t lies between 15 and 16 so $K_{[50]}$ = 15.

We therefore need the minimum premium such that

$$L = 0 = 75{,}000v^{16} + 325 + P^{/} + \left(0.025P^{/} + 75\right)a_{\overline{15|}} - P^{/}\ddot{a}_{\overline{16|}}$$

$$\Rightarrow 0 = 75{,}000 \times 0.39365 + 325 + P^{/} + \left(0.025P^{/} + 75\right) \times 9.712254 - 10.712254P^{/}$$

$$\Rightarrow P^{/} = \frac{30{,}577.169}{9.46944765} = 3{,}229.03$$

Part (a) was done well. However very few students completed part (b).

13 (i) If P is the initial premium payable, then

EPV of premiums

$$= P\left[1 \times \frac{l_{56}}{l_{56}} + 0.75 \times \frac{l_{57}}{l_{56}} \times v + 0.5 \times \frac{l_{58}}{l_{56}} \times v^2 + 0.25 \times \frac{l_{59}}{l_{56}} \times v^3\right]$$

$$= \frac{P}{9515.104}\left[9515.104 + 0.75 \times 9467.2906 \times 0.9434 + 0.5 \times 9413.8004 \times .89 + 0.25 \times 9354.004 \times .83962\right]$$

$$= 2.350603P$$

EPV of benefits

$$= 100{,}000\left[q_{56} \times v + 0.75 \times p_{56} \times q_{57} \times v^2 + 0.5 \times {}_2p_{56} \times q_{58} \times v^3 + 0.25 \times {}_3p_{56} \times q_{59} \times v^4\right]$$

$$= 100{,}000\left[\begin{array}{l} 0.005025 \times 0.9434 + 0.75 \times 0.994975 \times 0.00565 \times 0.89 \\ +0.5 \times 0.989353 \times 0.006352 \times 0.83962 + 0.25 \times 0.983069 \times 0.00714 \times 0.79209 \end{array}\right]$$

$$= 1252.116$$

EPV of renewal expenses =

$$= 35\left[\ddot{a}^{@i^{/}}_{56:\overline{4|}} - 1\right] = 35 \times 2.745 = 96.075$$

where $i^{/} = \dfrac{1.06}{1.0192308} - 1 = 0.04$

EPV of other expenses =
$125 + 0.25P + 0.03 \times (EPV\ of\ \ premiums - 1)$
$= 125 + 0.25P + 0.040518P$

Equation of value gives:

2.350603P = 1252.116 + 96.075 + 125 + 0.25P + 0.040518P

P = 715.11

(ii) Prospective gross premium policy reserve at the end of the 1st policy year given by:

$${}_1V = EPV(\,future\,benefits + \exp enses - premiums) \text{ where:}$$

EPV of premiums

$$= P\left[0.75 \times \frac{l_{57}}{l_{57}} + 0.5 \times \frac{l_{58}}{l_{57}} \times v \ \ + 0.25 \times \frac{l_{59}}{l_{57}} \times v^2\right]$$

$$= \frac{715.11}{9467.2906}\left[0.75 \times 9467.2906 + 0.5 \times 9413.8004 \times .9434 + 0.25 \times 9354.004 \times .89\right] = 1028.952$$

EPV of benefits

$$= 100,000\left[0.75 \times q_{57} \times v \ \ + 0.5 \times \ \ p_{57} \times q_{58} \times v^2 + 0.25 \times {}_2p_{57} \times q_{59} \times v^3\right]$$

$$= 100,000\begin{bmatrix} 0.75 \times 0.00565 \times 0.9434 + 0.5 \times 0.99435 \times 0.006352 \times 0.89 \\ +0.25 \times 0.9880339 \times 0.00714 \times 0.83962 \end{bmatrix}$$

$$= 828.911$$

EPV of renewal expenses

$$= 35 \times 1.0192308 \times \ddot{a}^{@i'}_{57:\overline{3}|} = 35.673 \times 2.870 = 102.382$$

EPV of renewal commission
= 0.03 × EPV of premiums = 30.867

Therefore ${}_1V = 828.911 + 102.382 + 30.867 - 1028.952 = -66.79$

(iii) Therefore, sum at risk per policy in the 1st policy year is:

DSAR = 100,000 – (– 66.79) = 100,066.79
Mortality profit = EDS – ADS

$$EDS = 5000 \times q_{56} \times 100,066.79 = 5000 \times 0.005025 \times 100,066.79 = 2,514,178.1$$

$$ADS = 27 \times 100,066.79 = 2,701,803.3$$

i.e. mortality profit = –187,625.2 (i.e. a loss)

Mortality profit =

$$= 5000 \times ({}_0V + P - E) \times (1+i) - S \times actual\ deaths - {}_1V \times number\ of\ policies\text{ in } force$$
$$= 5000 \times (0 + 715.11 - 0.25 \times 715.11 - 125) \times 1.06 - 100,000 \times 27 - (-66.79) \times 4973$$
$$= -187,791.1$$

i.e. approximately the same figure as derived in (c) above

Reasonably well done by well prepared students. Partial credit was given in (b) for showing understanding of the processes involved.

14 (i) Let P be the annual premium required to meet the company's profit criteria.

Multiple decrement table – although deaths can be assumed to be uniformly distributed over the year, surrenders occur only at the year end. Therefore:

$$(aq)_x^d = q_x^d \text{ and } (aq)_x^w = q_x^w(1 - q_x^d)$$

x	q_x^d	q_x^w	$(aq)_x^d$	$(aq)_x^w$	$(ap)_x$	${}_{t-1}(ap)_x$
67	0.016042	0.08	0.016042	0.07872	0.905242	1
68	0.017922	0.04	0.017922	0.03928	0.942795	0.905242
69	0.020003	0.00	0.020003	0.0	0.979997	0.853458

Unit fund cashflows (per policy at start of year)

	Year 1	*Year 2*	*Year 3*
Value of units at start of year	0	0.490295P	1.584731P
Allocation	0.5P	1.1P	1.1P
Bid/offer	0.025P	0.055P	0.055P
Interest	0.019P	0.061412P	0.105189P
Management charge	0.003705P	0.011975P	0.020512P
Value of units at end of year	0.490295P	1.584731P	2.714408P

Non-unit fund cashflows

	Year 1	*Year 2*	*Year 3*
Unallocated premium	0.5*P*	–0.1P	–0.1*P*
Bid/offer	0.025*P*	0.055*P*	0.055*P*
Expenses	0.125*P*+235	0.025*P*+45	0.025*P*+45
Interest	0.012*P*–7.05	–0.0021*P*–1.35	–0.0021*P*–1.35
Management charge	0.003705*P*	0.011975*P*	0.020512*P*
Claim expense	7.10715	4.29015	1.500225
End of year cashflows	0.415705*P*–249.15715	–0.060125*P*–50.64015	–0.051588*P*–47.850225

Probability in force	1	0.905242	0.853458
Discount factor	0.943396	0.889996	0.839619
Expected present value of profit	0.392174*P*–235.0539	–0.048440*P*–40.7987	–0.036967*P*–34.2886

NPV of profit = .10P = 0.306767P – 310.1412 => *P* = £1500.0
(i.e. NPV of profit = £150.0)

(ii) The profit vector for the policy is (374.401, –140.827, –125.232)

In order to set up reserves in order to zeroise future expected negative cash flows, we require:

$$ {}_2V = \frac{125.232}{1.03} = 121.584 $$

$$ {}_1V \times 1.03 - (ap)_{68} \times {}_2V = 140.827 \Rightarrow {}_1V = 248.016 $$

revised cash flow in year 1 $= 374.401 - (ap)_{67} \times {}_1V = 149.887$

and NPV of profit = 149.887/1.06 = 141.402

Again reasonably well done by well prepared students for part (i). Part (ii) caused more difficulties however. As before partial credit was given for showing understanding of the processes involved.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2013 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

D C Bowie
Chairman of the Board of Examiners

December 2013

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the September 2013 paper

The general performance was similar this session to previous ones. Well prepared students generally scored well. Questions that were done less well were 15, 18, 11, 21, 22(iii) and 23(iii). The examiners hope that the detailed solutions given below will assist students with further revision.

As in past examinations most of the short questions were very straightforward and this is where many successful candidates scored particularly well. Students should note that for long questions some credit is given if they can describe the right procedures although to score well reasonably accurate numerical calculation is necessary.

11 (a) $${}_{10}q_{63} = \frac{l_{63} - l_{73}}{l_{63}} = \frac{9775.888 - 9073.650}{9775.888} = 0.07183$$

(b) $$\ddot{a}_{63}^{(2)} = \ddot{a}_{63} - \frac{1}{4} = 15.606 - 0.25 = 15.356$$

(c) $$s_{55:\overline{10}|} = \frac{(1.04)^{10} * a_{55:\overline{10}|}}{{}_{10}p_{55}} = \frac{(1.04)^{10} * ((\ddot{a}_{55} - 1) - (1.04)^{-10} * (l_{65} / l_{55}) * (\ddot{a}_{65} - 1))}{(l_{65} / l_{55})}$$

$$= \frac{(1.04)^{10} * 17.210 - (0.97843 * 13.871)}{0.97843}$$

$$= 12.166$$

This question was generally well done.

12 Temporary Initial Selection describes the modelling of rates by sub-dividing a population by duration since entry to that class. The rates modelled are dependent on duration up to a duration of s (the length of the select period) and after s they are independent of duration, so the effect is "temporary".

An example is a life purchasing a life assurance policy who has been medically selected and thus initially would be expected to enjoy better mortality. This advantage however wears off over time.

This question was generally well done. Credit was given for all relevant comments. To earn full marks it was important to stress in the answer the fact that the effect of selection wears off.

13 (a) For the first policy year

$$[{}_0V + P - \frac{a}{100}P - B] \times (1+i) = (1 + \frac{e}{100}) \times S \times q_x + {}_1V \times p_x$$

(b) For subsequent policy years

$$({}_tV + P - \frac{c}{100}P - D) \times (1+i) = (1 + \frac{e}{100}) \times S \times q_{x+t} + {}_{t+1}V \times p_{x+t}$$

Students had in many cases difficulties in setting out these standard formulae which are fundamental in CT5. In (a) expressing ${}_0V$ as zero was fine so long as this definition was stated. Also using t-1and t instead of t and t+1 respectively was acceptable.

14 $\quad {}_{2.25}p_{90.25} = {}_{0.75}p_{90.25} * p_{91} * {}_{.5}p_{92}$

$$= (1 - {}_{0.75}q_{90.25})(1 - q_{91})(1 - {}_{.5}q_{92})$$

$$= \left(1 - \frac{.75q_{90}}{(1 - .25q_{90})}\right)(1 - q_{91})(1 - .5q_{92})$$

$$= \left(1 - \frac{.75 * .170247}{(1 - .25 * .170247)}\right) * .815286 * .89996$$

$$= 0.63587$$

Generally well done. An alternative correct method is to use straight line interpolation on l factors. This is fine so long as it produces an accurate answer.

15 (a) If $q^d_{[40]}$ and q^s_{40} represent the independent rates of mortality and surrender respectively in the 1st policy year, then the dependent rate of surrender at the end of the 1st policy year is:

$$(aq)^s_{40} = \left[1 - q^d_{[40]}\right] \times q^s_{40} = (1 - 0.000788) \times 0.15 = 0.14988$$

The cash flows are now modified to include a surrender charge at the end of the 1st policy year

$$= 500 \times (aq)^s_{40} = 500 \times 0.14988 = 74.94$$

The revised profit vector = revised profit signature
= –209.80 + 74.94 = –134.86

(b) Although the profit vector for this policy will remain the same for policy years 2 and 3, the profit signature for each year will reduce as the probability of the policy being in force at the start of each year will reduce.

This question was done poorly overall with few students being able to derive the correct answer.

16

$$PV = 1100\ddot{a}_{75:\overline{10|}} + 100(I\ddot{a})_{75:\overline{10|}}$$

$$= 1100(\ddot{a}_{75} - v^{10} * {}_{10}p_{75}\ddot{a}_{85}) + 100((I\ddot{a})_{75} - v^{10} * {}_{10}p_{75}(10\ddot{a}_{85} + (I\ddot{a})_{85}))$$

$$= 1100\left(7.679 - 0.55839 * \frac{3385.2479}{6879.1673} * 4.998\right) + 100(48.128 - 0.55839 * \frac{3385.2479}{6879.1673} * (49.98 + 21.503))$$

$$= 6936.2 + 2848.6$$

= £9785 rounded

This was a very straightforward question that was generally well done. The most common error was for the first function above to be multiplied by 1000 rather than the correct 1100.

17 $EPV =$

$$\left(.03 * \int_0^{20} te^{-.08t}dt\right) + \left(.04 * e^{-1.6} * e^{1.8}\int_{20}^{\infty} te^{-.09t}dt\right) = \left(.03 * \int_0^{20} te^{-.08t}dt\right) + \left(.04 * e^{0.2}\int_{20}^{\infty} te^{-.09t}dt\right)$$

$$\int_0^{20} te^{-.08t}dt = \left[-\frac{te^{-.08t}}{.08}\right]_0^{20} + \frac{1}{.08}\int_0^{20} e^{-.08t}dt = \left[-\frac{te^{-.08t}}{.08} - \frac{e^{-.08t}}{(.08)^2}\right]_0^{20}$$

$$= -\frac{20e^{-1.6}}{.08} + 0 - \frac{e^{-1.6}}{(.08)^2} + \frac{1}{(.08)^2} = -50.474 - 31.546 + 156.25$$

$$= 74.230$$

$$\int_{20}^{\infty} te^{-.09t}dt = \left[-\frac{te^{-.09t}}{.09}\right]_{20}^{\infty} + \frac{1}{.09}\int_{20}^{\infty} e^{-.09t}dt = \left[-\frac{te^{-.09t}}{.09} - \frac{e^{-.09t}}{(.09)^2}\right]_{20}^{\infty}$$

$$= -0 + \frac{20e^{-1.8}}{.09} - 0 + \frac{e^{-1.8}}{(.09)^2} = 36.733 + 20.407 = 57.140$$

$$EPV = .03 * 74.230 + .04 * 1.2214 * 57.140$$

$$= 5.019$$

A challenging question. Well prepared students coped well but many failed at the basic level in constructing the integral.

18 Define the random variable $\mathbf{K}_x$ for the curtate duration of life aged x.

The expected present value is:

$$\Sigma_{k=0}^{n} 0 \times P[\mathbf{K}_x = k] + \Sigma_{k=n+1}^{\infty} {}_{n|}a_{\overline{k-n}|} \times P[\mathbf{K}_x = k]$$

$$= (\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n]) - (\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n])$$
$$+\Sigma_{k=n+1}^{\infty} {}_{n|}a_{\overline{k-n}|} \times P[\mathbf{K}_x = k]$$

$$= (\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n] + \Sigma_{k=n+1}^{\infty} {}_{n|}a_{\overline{k-n}|} \times P[\mathbf{K}_x = k])$$
$$-(\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n])$$

$$= (\Sigma_{k=0}^{\infty} a_{\overline{k}|} \times P[\mathbf{K}_x = k]) - (\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n])$$

$$= a_x - a_{x:\overline{n}|}$$

This is a straight bookwork question taken straight from Core Reading. Most students struggled to reproduce it and the primary error was that students did not appreciate the random variable aspect often trying to solve it in a non random variable manner. This gained no credit.

19 (i) (a)

	Region A			*Country*		
Age	*Population exposed*	*Number of Deaths*	*Mortality*	*Population exposed*	*Number of Deaths*	*Mortality*
18–35	25000	25	0.00100	500000	1000	0.00200
36–50	50000	80	0.00160	125000	375	0.00300
51–70	70000	170	0.00243	110000	500	0.00455
	145000	275		735000	1875	

The mortality rates are shown in Columns 4 and 7 above.

(b) Crude Mortality Rate (Region A) = 275/145000 = 0.00190
Crude Mortality Rate (Country) = 1875/735000 = 0.00255

(c) The directly standardised mortality rate for Region A is:

((500000 * .00100) + (125000 * .00160) + (110000 * .00243))/735000
= 0.00132

(d) The standardised mortality ratio for Region A is:

Actual deaths in Region A/Expected Deaths in Region A based on Country mortality rates i.e.

275/((25000 * .00200) + (50000 * .00300) + (70000 * .00455))
= 275/518.5 = 0.53

(ii)

- Crude mortality rate in Region A suggests Region A has only 75% of the mortality rate of Country as a whole.

- However the directly standardised mortality rate for Region A is significantly lighter than the appropriate crude rate.

- This difference is explained by the fact that Region A has a much higher proportion of older lives than the Country as a whole thus inflating the crude rate.

- The standardised mortality ratio shows the true difference i.e. the mortality rates for Region A are on average 53% of those for the Country as a whole.

Generally this was another straightforward question on which students did well. The most common error was that not all points were covered in (ii).

20

$$(aq)^d_{85} = \frac{1400}{10000} = 0.14; (aq)^d_{86} = \frac{1000}{6300} = 0.15873$$

$$(aq)^w_{85} = \frac{2300}{10000} = 0.23; (aq)^w_{86} = \frac{1100}{6300} = 0.17460$$

$$q^d_{85} = \frac{(aq)^d_{85}}{\left(1 - \frac{1}{2}(aq)^w_{85}\right)} = \frac{0.14}{0.885} = 0.158192$$

Similarly $q^d_{86} = \dfrac{0.15873}{0.9127} = 0.173913$

$$q^w_{85} = \frac{(aq)^w_{85}}{\left(1 - \frac{1}{2}(aq)^d_{85}\right)} = \frac{0.23}{0.93} = 0.247312$$

Similarly $q^w_{86} = \dfrac{0.17460}{0.9206} = 0.189659$

But q_{85}^{w} and q_{86}^{w} are now reduced by 50% so their new values are:

$$q_{85}^{w} = 0.123656 \text{ and } q_{86}^{w} = .0948295$$

$$\text{Hence } (aq)_{85}^{d} = 0.158192 * \left(1 - \frac{1}{2} * 0.123656\right) = 0.14841; (aq)_{86}^{d} = 0.173913 * \left(1 - \frac{1}{2} * .0948295\right) = 0.16567$$

$$\text{Hence } (aq)_{85}^{w} = 0.123656 * \left(1 - \frac{1}{2} * 0.158192\right) = 0.113875; (aq)_{86}^{w} = 0.0948295 * \left(1 - \frac{1}{2} * 0.173913\right) = 0.086583$$

Using the above the new table is:

Age x	$(al)_x$	$(ad)_x^d$	$(ad)_x^w$
85	10000	1484	1139
86	7377	1222	638
87	5518		

Note that values are sensitive to rounding-other close values accepted.

This was another relatively straightforward question generally well done by well prepared students. Most marks were awarded on knowing the principles of calculation rather than the precision of the calculations themselves.

21 (i) Assume that decrements on average occur at time x+ ½.

$$3 \times 25,000 \times \left\{ \left(\frac{d_{40}}{l_{35}} \frac{s_{39.5}}{s_{34}} \right) \right\}$$

$$= 3 \times 25,000 \times \left\{ \left(\frac{14}{18866} \frac{(7.623 + 7.814)/2}{6.389} \right) \right\}$$

$$= 67.24$$

(ii) Expected present value

$$\sum_{t=0}^{t=64-x} 3 \times 25,000 \times \frac{s_{34+t+1/2}}{s_{34}} \frac{d_{35+t}}{l_{35}} \frac{v^{35+t+½}}{v^{35}}$$

Define:

$$^{s}D_{35} \;=\; s_{34} l_{35} v^{35}$$

$$^{s}C^{d}_{x+t} \;=\; s_{34+t+½} d_{35+t} v^{35+t+½}$$

$$^{s}M^{d}_{35} \;=\; \sum_{t=0}^{t=64-x} {}^{s}C^{d}_{35+t}$$

Then the expected value is:

$$3 \times 25,000 \times \frac{^{s}M^{d}_{35}}{^{s}D_{35}}$$

This question was very poorly done. Students seem to struggle continually with questions involving pension commutation functions and this was felt to be a reasonably straightforward derivation from 1st principles.

22

(i) Let P be the annual premium for the policy. Then (functions at 4%):

EPV of premiums:

$$P\ddot{a}_{[40]:\overline{20}|} = 13.930P$$

EPV of benefits:

$$75,000 A^{1}_{[40]:\overline{20}|} + 150,000 v^{20}\,{}_{20}p_{[40]}$$

where:

$$v^{20}\,{}_{20}p_{[40]} \;=\; 0.45639 \times \frac{9287.2164}{9854.3036} = 0.43013$$

$$A^{1}_{[40]:\overline{20}|} \;=\; A_{[40]:\overline{20}|} - v^{20}\,{}_{20}p_{[40]} = 0.46423 - 0.43013 = 0.0341$$

$$= 75,000 \times 0.0341 + 150,000 \times 0.43013$$

$$= 67,077.0$$

EPV of expenses:

$$0.25P + 400 + 45(\ddot{a}_{[40]:\overline{20}|} - 1) \;=\; 0.25P + 400 + 45 \times 12.93$$

$$= 0.25P + 981.85$$

Equation of value gives:

$$13.93P = 67,077.0 + 0.25P + 981.85$$

$$\Rightarrow P = \frac{68,058.85}{13.68} = 4,975.06$$

(ii) The gross prospective policy reserve at the end of the 8th policy year is given by:

$${}_8V = 75,000A^{1}_{48:\overline{12}|} + 150,000v^{12}\,{}_{12}p_{48} + (45 - P)\ddot{a}_{48:\overline{12}|}$$

where:

$$v^{12}\,{}_{12}p_{48} = 0.62460 \times 0.95220 = 0.59474$$

$$\Rightarrow {}_8V = 75,000 \times (0.63025 - 0.59474) + 150,000 \times 0.59474 + (45 - 4975.06) \times 9.613$$

$$= 44,481.58$$

The gross prospective policy reserve at the end of the 9th policy year is given by:

$${}_9V = 75,000A^{1}_{49:\overline{11}|} + 150,000v^{11}\,{}_{11}p_{49} + (45 - P)\ddot{a}_{49:\overline{11}|}$$

where:

$$v^{11}\,{}_{11}p_{49} = 0.64958 \times 0.95411 = 0.61977$$

$$\Rightarrow {}_9V = 75,000 \times (0.65477 - 0.61977) + 150,000 \times 0.61977 + (45 - 4975.06) \times 8.976$$

$$= 51338.28$$

Note: students can alternatively calculate these reserves on a retrospective basis i.e.

$${}_8V = \frac{D_{[40]}}{D_{48}}\left[P\ddot{a}_{[40]:\overline{8}|} - 75,000A^{1}_{[40]:\overline{8}|} - 400 - 45(\ddot{a}_{[40]:\overline{8}|} - 1) - 0.25P\right]$$

where:

$$A^1_{[40]:\overline{8}|} = A_{[40]} - v^8 {}_8p_{[40]} \times A_{48} = 0.23041 - 0.73069 \times 0.98977 \times 0.30695 = 0.008419$$

and:

$$\ddot{a}_{[40]:\overline{8}|} = \ddot{a}_{[40]} - v^8 {}_8p_{[40]} \times \ddot{a}_{48} = 20.009 - 0.73069 \times 0.98977 \times 18.019 = 6.9774$$

$$\Rightarrow {}_8V = 1.382713[4975.06 \times 6.9774 - 75,000 \times 0.008419 - 400 - 45 \times 5.9774 - 0.25 \times 4975.06]$$

$${}_9V = \frac{D_{[40]}}{D_{49}}\left[P\ddot{a}_{[40]:\overline{9}|} - 75,000A^1_{[40]:\overline{9}|} - 400 - 45(\ddot{a}_{[40]:\overline{9}|} - 1) - 0.25P\right]$$

where:

$$A^1_{[40]:\overline{9}|} = A_{[40]} - v^9 {}_9p_{[40]} \times A_{49} = 0.23041 - 0.70259 \times 0.98778 \times 0.31786 = 0.009814$$

and:

$$\ddot{a}_{[40]:\overline{9}|} = \ddot{a}_{[40]} - v^9 {}_9p_{[40]} \times \ddot{a}_{49} = 20.009 - 0.70259 \times 0.98778 \times 17.736 = 7.7001$$

$$\Rightarrow {}_9V = 1.440915\left[4975.06 \times 7.7001 - 75,000 \times 0.009814 - 400 - 45 \times 6.7001 - 0.25 \times 4975.06\right]$$

$= 51,335.68$

(iii) Using the gross prospective policy reserve calculated in b) above then:

Sum at risk per policy in the 9th policy year is:

DSAR = 75,000 – 51,338.28 = 23,661.72

Mortality profit = EDS – ADS

EDS = $625 \times q_{48} \times 23,661.72 = 625 \times 0.002008 \times 23,661.72 = 29,695.46$

ADS = $3 \times 23,661.72 = 70,985.16$

i.e. mortality profit = –41,289.7 (i.e. a loss)

total profit/loss in 2012 =

$$= 625 \times ({}_8V + P - E) \times (1+i) - S \times \text{actual deaths} - {}_9V \times \text{number of policies inforce}$$

$= 625 \times (44,481.58 + 4,975.06 - 45) \times 1.045 - 75,000 \times 3 - 51,338.28 \times 622$

$= 114,567.22$

i.e. total profit from mortality, interest and expense combined = 114,567.22

As expenses incurred per policy during 2012 were the same as assumed in the premium basis, then expense surplus = 0

= $44,480.23$

Therefore interest surplus = 114,567.22 – (–41,289.7) = 155,856.92

Most well prepared students did parts (i) and (ii) well. Part (iii) was less well done as few students realised expense surplus was zero and many attempted only the mortality surplus.

23

(i) Let P be the monthly premium payable for this policy. Then:

EPV of premiums (at 6% p.a.)

$12P\ddot{a}^{(12)}_{[50]:\overline{15|}} = 117.114P$

where:

$$\ddot{a}^{(12)}_{[50]:\overline{15|}} = \ddot{a}_{[50]:\overline{15|}} - \frac{11}{24}(1 - v^{15}{}_{15}p_{[50]}) = 10.044 - \frac{11}{24}(1 - 0.379230) = 9.7595$$

EPV of benefits: (at 6% p.a.)

$$= 50,000\bar{A}^{\;1}_{[50]:\overline{15|}} + 10,000(I\bar{A})^{\;1}_{[50]:\overline{15|}}$$

$$= 50,000\{\bar{A}_{[50]} - v^{15}{}_{15}p_{[50]}\bar{A}_{65}\} + 10,000\{(I\bar{A})_{[50]} - v^{15}{}_{15}p_{[50]}(15\bar{A}_{65} + (I\bar{A})_{65})\}$$

$$= 1.06^{0.5} \times [50,000A_{[50]} + 10,000(IA)_{[50]} - v^{15}{}_{15}p_{[50]}(200,000A_{65} + 10,000(IA)_{65})]$$

$$= 1.02956\begin{bmatrix} 50,000 \times 0.20463 + 10,000 \times 4.84789 \\ -0.41727 \times \dfrac{8821.2612}{9706.0977}\left(200,000 \times 0.40177 + 10,000 \times 5.50985\right) \end{bmatrix}$$

$$= 1.02956[10,231.5 + 48,478.9 - 0.37923 \times (80,354.0 + 55,098.5)]$$

$= 7,559.80$

EPV of expenses (functions @6% p.a. unless otherwise stated):

$$225 + 0.3 \times 12P + 0.04 \times 12P\left(\ddot{a}^{(12)}_{[50]:\overline{15|}} - \frac{1}{12}\right) + 65\left(\ddot{a}^{4\%}_{[50]:\overline{15|}} - 1\right) + 275\bar{A}^{1\ \ 4\%}_{[50]:\overline{15|}}$$

$$= 225 + 3.6P + 0.48P \times 9.6762 + 65 \times 10.259 + 275\left(\bar{A}^{@4\%}_{[50]} - v^{15}_{(4\%)}\ {}_{15}p_{[50]}\bar{A}^{4\%}_{65}\right)$$

$$= 225 + 3.6P + 4.6446P + 666.835 + 275 \times 1.04^{0.5}[0.32868 - 0.55526 \times 0.90884 \times 0.52786]$$

$$= 909.307 + 8.2446P$$

Equation of value gives:

$117.114P = 7559.80 + 909.307 + 8.2446P \Rightarrow P = 77.79$

(ii) Gross prospective reserve at the end of the 14th policy year is given by (functions @6% p.a. unless otherwise stated):

$${}_{14}V = 200,000q_{64}v^{0.5} + 275(1.0192308)^{14}q_{64}v^{0.5}_{0.04}$$

$$+65(1.0192308)^{14} - 0.96 \times 12P\ddot{a}^{(12)}_{64:\overline{1|}}$$

$$= 200,000 \times 0.012716 \times 0.97129 + 359.044 \times 0.012716 \times 0.98058 + 84.865 - 867.967$$

$$= 2,470.185 + 4.4769 + 84.865 - 867.967 = 1691.60$$

where:

$$\ddot{a}^{(12)}_{64:\overline{1|}} = \ddot{a}_{64:\overline{1|}} - \frac{11}{24}(1 - v \ \times p_{64}) = 1 - \frac{11}{24}(1 - 0.9434 \times 0.98728) = 0.96856$$

(iii) If $K_{64} \geq 1$

GFLRV = $65(1.0192308)^{14} - 0.96 \times 12 \times 77.79 \times \ddot{a}^{(12)}_{\overline{1|}.06}$

If $K_{64} < 1$

$$\text{GFLRV} = 200,000v^{T_{64}}_{.06} + 275(1.0192308)^{14}v^{T_{64}}_{.04}$$

$$+65(1.0192308)^{14} - 0.96 \times 12 \times 77.79 \times \ddot{a}^{(12)}_{\overline{\frac{1}{12}\left(1+[12T_{64}]\right)|}} \ @6\%$$

where $[12T_{64}]$ represents the integer part of $12T_{64}$

Again well prepared students did part (i) well although part (ii) was done less well. Very few students made a serious attempt at part (iii) which was set to test higher skills.

24

(i)

Let P be the annual premium payable. Then equation of value gives (functions at 6% unless otherwise stated):

$$P\ddot{a}_{[56]:\overline{4}|} = 25,000A^{@i'}_{[56]:\overline{4}|} + 0.25P + 100 + (0.025P + 40)\left[\ddot{a}_{[56]:\overline{4}|} - 1\right]$$

where $i' = \left(\dfrac{1.06}{1.0192308}\right) - 1 = 0.04$

$$\Rightarrow 3.648P = 25,000 \times 0.8558 + 0.25P + 100 + (0.025P + 40) \times 2.648$$

$$\Rightarrow P = \frac{21,600.92}{3.3318} = 6,483.26$$

(ii) Decrement table

x	q_x	$q'_x = 0.8q_x$	p'_x	${}_{t-1}p'_x$
56	0.003742	0.002994	0.997006	1
57	0.005507	0.004406	0.995594	0.997006
58	0.006352	0.005082	0.994918	0.992614
59	0.007140	0.005712	0.994288	0.987570

Accrued bonus at start of policy year t for each in force policy is given by:

t	*Accrued bonus*
1	480.77
2	970.79
3	1470.22
4	1979.27

Reserves required on the policy at 4% interest are:

$${}_1V_{56:\overline{4}|} = 25,480.77A_{57:\overline{3}|} - NP\ddot{a}_{57:\overline{3}|}$$

$$= 25,000\left(1 - \frac{\ddot{a}_{57:\overline{3}|}}{\ddot{a}_{56:\overline{4}|}}\right) + 480.77A_{57:\overline{3}|} = 25,000\left(1 - \frac{2.87}{3.745}\right) + 480.77 \times 0.88963 = 6268.83$$

$$_2V_{56:\overline{4}|} = 25,000\left(1-\frac{\ddot{a}_{58:\overline{2}|}}{\ddot{a}_{56:\overline{4}|}}\right)+970.79A_{58:\overline{2}|} = 25,000\left(1-\frac{1.955}{3.745}\right)+970.79\times 0.92479 = 12847.04$$

$$_3V_{56:\overline{4}|} = 25,000\left(1-\frac{\ddot{a}_{59:\overline{1}|}}{\ddot{a}_{56:\overline{4}|}}\right)+1470.22A_{59:\overline{1}|} = 25,000\left(1-\frac{1.0}{3.745}\right)+1470.22\times 0.96154 = 19738.11$$

Cash flows for the policy under the profit test are given by:

Year T	*Opening reserve*	*Premium*	*Expense*	*Interest*	*Death Claim*	*Maturity Claim*	*Closing reserve*
1	0	6483.26	1720.82	357.18	76.28	0	6250.06
2	6268.83	6483.26	202.08	941.25	114.42	0	12790.44
3	12847.04	6483.26	202.08	1434.62	134.51	0	19637.81
4	19738.11	6483.26	202.08	1951.45	154.11	26825.16	0

Year t	*Profit vector*	$_{t-1}p$	*Profit signature*	*Discount factor*	*NPV of profit signature*
1	–1206.71	1.0	–1206.71	.913242	–1102.02
2	586.40	0.997006	584.64	.834011	487.60
3	790.51	0.992614	784.67	.761654	597.65
4	991.47	0.987570	979.15	.695574	681.07

NPV of profit signature = £664.30

Year t	*Premium*	$_{t-1}p$	*Discount factor*	*NPV of premium*
1	6483.26	1.0	1	6483.26
2	6483.26	0.997006	.913242	5903.06
3	6483.26	0.992614	.834011	5367.17
4	6483.26	0.987570	.761654	4876.62

NPV of premiums = £22,630.11

Profit margin = $\dfrac{664.30}{22,630.11} = 0.0294$ i.e. 2.94%

A relatively straightforward if detailed question where well prepared students scored well. In these types of question credit is given for understanding of the method and how to approach the calculations even if the calculation part contains numerical errors.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2014 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

D C Bowie
Chairman of the Board of Examiners

June 2014

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the April 2014 paper

The general performance was similar this session to previous ones. Questions that were done less well were Q3, Q5, Q8, Q11 part (iii) and Q13 part (ii). The examiners hope that the detailed solutions given below will assist students with further revision.

However most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions some credit is given if they can describe the right procedures although to score well reasonably accurate numerical calculation is necessary.

1 Each group is specified by a category or class of a particular characteristic of the population. The stochastic models (life tables) are different for each class. There are no common features to the models, they are different for all ages. This is termed class selection.

Examples are:

- Gender differences
- Distinction of Smoker and Non-Smoker status
- Occupation

Other examples credited.

Generally well done with no significant issues.

2 (a)

$$\ddot{a}^{(4)}_{25:\overline{20}|} = \ddot{a}^{(4)}_{25} - \frac{l_{45}}{l_{25}} \times v^{20} \times \ddot{a}^{(4)}_{45}$$

$$= \left(\ddot{a}_{25} - \frac{3}{8}\right) - \left(\frac{l_{45}}{l_{25}} \times v^{20} \times \left(\ddot{a}_{45} - \frac{3}{8}\right)\right)$$

$$= (22.520 - 0.375) - \left(\frac{9801.3123 \times 0.45639}{9953.6144} \times (18.823 - 0.375)\right)$$

$$= 22.145 - (0.4494 \times 18.448)$$

$$= 13.854$$

(b)

$$(I\bar{A})^{1}_{25:\overline{20}|} = (1.04)^{1/2} \times \left((IA)_{25} - \frac{l_{45}}{l_{25}} \times v^{20} \times ((IA)_{45} + 20 \times A_{45})\right)$$

$$= 1.0198 \times (6.33195 - 0.4494 \times (8.33628 + 20 \times 0.27605))$$

$$= 0.10656$$

Generally well done with no significant issues. The main error involved the accuracy of the formula in line 1 of part (b) above.

3

$${}_t p_x = \frac{l_0 e^{-0.15(x+t)}}{l_0 e^{-0.15x}} = e^{-0.15t}$$

Therefore:

$$\ddot{a}_{3:\overline{5}|} = \sum_{0}^{4} (1.05)^{-t} \times e^{-0.15t} = \frac{1 - ((1.05)^{-1} e^{-0.15})^5}{1 - ((1.05)^{-1} e^{-0.15})} = \frac{1 - 0.37011}{1 - 0.81972}$$

$$= 3.4940$$

Hence:

$$A_{3:\overline{5}|} = 1 - \left(\frac{.05}{1.05} \times 3.4940 \right)$$
$$= 0.83362$$

This question was poorly done. From the solution above it will be seen that the answer is very straightforward if premium conversion is used. Most students failed to realise this and attempted the question the longer direct way which involves a much more arduous calculation (full credit was given if this produced the correct answer).

4 Cash flow techniques promote understanding and clarity of thought

Cash flow techniques are more easily presented to non-actuaries

Cash flow techniques can be helpful when an office wishes to design an appropriate investment strategy to cope with expected future cash flows.

Cash flow techniques allow much more flexibility e.g.

- Premium basis with varying or stochastic interest rates
- Complex policy designs e.g. varying benefits or options
- Sensitivity analysis can be easy to do on a computer once the model has been set up
- Multiple state models (e.g. PHI) can be dealt with, which is not possible using commutation functions
- Allowance can be made for negative values.

Generally done reasonably well. Each distinct point mentioned above gained a mark up to the maximum for the question. Other valid points not contained above were also credited.

5 At 65 the member would have completed 20 + 16 = 36 years' service so the maximum of 2/3 applies in this case.

$$\text{Value } = \frac{2}{3} \times 40,000 \times \frac{{}^{z}C_{65}^{ra}}{s_{44} D_{45}}$$
$$= \frac{2}{3} \times 40,000 \times \left(\frac{45,467}{8.375 \times 2329} \right)$$
$$= 62,160$$

This very simple question was very poorly done. The question makes it clear that age retirement takes place at age 65 only. A large proportion of students tried to apply Past and Future Service values directly from the Tables which includes all other age retirement possibilities. This is not only arduous given the service limit but is invalid in this case.

6 (a) The method of approximation based on the assumption of a constant force of mortality assumes that for integer x and $0 \le t < 1$, we have:

$\mu_{x+t} = \mu =$ constant.

Then the appropriate relationship is:

$$_t p_x = \exp\left\{-\int_0^t \mu_{x+s}\, ds\right\} = e^{-t\mu}$$

From this μ can be derived.

(b) From the relationship in (a) we can derive:

$$_t p_x = ({}_1 p_x)^t$$

Therefore:

$$\begin{aligned}
{}_{2.75}p_{85.5} &= {}_{0.5}p_{85.5} \times p_{86} \times p_{87} \times {}_{0.25}p_{88} \\
&= (p_{85})^{0.5} \times p_{86} \times p_{87} \times p_{88}{}^{0.25} \\
&= (1-0.14372)^{0.5} \times (1-0.15585) \times (1-0.16848) \times (1-0.18061)^{0.25} \\
&= 0.92534 \times 0.84415 \times 0.83152 \times 0.95142 \\
&= 0.61797 \text{ so } {}_{2.75}q_{85.5} = (1-0.61797) \\
&= 0.38203
\end{aligned}$$

Generally well done. Part (a) above is a detailed explanation and lesser detail gained full credit.

7 The annuity is equivalent to:

£6,000 p.a. whilst at least one life survives

An additional £1,500 p.a. if the female is surviving

A further amount of £2,500 p.a. if both lives survive.

The expected present value is:

$$= 2500a_{65:62}^{(4)} + 1500a_{62}^{(4)} + 6000a_{\overline{65:62}}^{(4)}$$
$$= 2500(\ddot{a}_{65:62} - 0.625) + 1500(\ddot{a}_{62} - 0.625) + 6000(\ddot{a}_{65} + \ddot{a}_{62} - \ddot{a}_{65:62} - 0.625)$$
$$= 6000\ddot{a}_{65} + 7500\ddot{a}_{62} - 3500\ddot{a}_{65:62} - 6250$$
$$= (6000 \times 13.666) + (7500 \times 15.963) - (3500 \times 12.427) - 6250$$
$$= £151974$$

Other approaches were acceptable. Well prepared students coped well with this question but others found difficulties in analysing the contingencies.

8 (i) $l_{x+t}^{\beta} = l_x^{\beta} - t^5 d_x^{\beta} \Rightarrow {}_t p_x^{\beta} = 1 - t^5 q_x^{\beta}$ for $0 \le t \le 1$

Thus $\dfrac{\partial({}_t p_x^{\beta})}{\partial t} = -5t^4 q_x^{\beta}$

Also $\dfrac{\partial({}_t p_x^{\beta})}{\partial t} = -{}_t p_x^{\beta} \mu_{x+t}^{\beta}$ for $0 \le t \le 1$ (from ${}_t p_x^{\beta} = e^{-\int_0^t \mu_{x+r}^{\beta} dr}$)

Therefore ${}_t p_x^{\beta} \mu_{x+t}^{\beta} = 5t^4 q_x^{\beta}$ as required

(ii)
$$(aq)_x^{\beta} = \int_0^1 {}_t p_x^{\alpha}\, {}_t p_x^{\beta} \mu_{x+t}^{\beta} dt$$
$$= \int_0^1 {}_t p_x^{\alpha} (5t^4 q_x^{\beta}) dt \text{ from (i) above}$$
$$= \int_0^1 (1 - t^3 q_x^{\alpha})(5t^4 q_x^{\beta}) dt$$
$$= 5q_x^{\beta} \int_0^1 t^4 (1 - t^3 q_x^{\alpha}) dt$$
$$= 5q_x^{\beta} \left[\frac{t^5}{5} - \frac{t^8 q_x^{\alpha}}{8} \right]_0^1$$
$$= q_x^{\beta} \left(1 - \frac{5}{8} q_x^{\alpha} \right) \text{ as required}$$

This question was poorly done. Part (i) was essentially just the combining together of two bookwork formulae. Part (ii) could have been easily attempted just using the result of part (i) but a majority of students did not seem to really understand how to start this question.

9 $$EPV = \bar{A}^{1}_{x:\overline{20|}} = 0.03\int_0^{20} e^{-.05t} \times e^{-.03t} dt$$
$$= \frac{0.03}{0.08}\left[-e^{-.08t}\right]_0^{20}$$
$$= 0.375 \times (1 - e^{-1.6})$$
$$= 0.375 \times 0.79810$$
$$= 0.29929$$

For the Variance:

$$^2\bar{A}^{1}_{x:\overline{20|}} = 0.03\int_0^{20} e^{-.1t} \times e^{-.03t} dt$$
$$= \frac{0.03}{0.13}\left[-e^{-.13t}\right]_0^{20}$$
$$= \frac{0.03}{0.13}(1 - e^{-2.6})$$
$$= 0.23077 \times 0.92573$$
$$= 0.21363$$

Hence

$$\text{Variance } = {}^2\bar{A}^{1}_{x:\overline{20|}} - (\bar{A}^{1}_{x:\overline{20|}})^2$$
$$= 0.21363 - (0.29929)^2$$
$$= 0.12406$$
$$= (0.35221)^2$$

Generally well done. The question in essence should have been technically posed in random variable form as the $\bar{A}^{1}_{x:\overline{20|}}$ *function is already the expected value and strictly in those circumstances the variance could be argued as zero. However virtually all students produced the solution above and were not concerned with this point so no difficulties emerged. Anybody pointing out the anomaly gained full credit.*

10 (i) (a) Crude mortality rate is the ratio of the total number of deaths in a category to the total exposed to risk in the same category.

(b) Directly standardised mortality rate is the mortality rate of a category weighted according to a standard population.

(c) Indirectly standardised mortality rate is an approximation to the directly standardised mortality rate being the crude rate for the standard population multiplied by the ratio of actual to expected deaths for the region.

(d) The Area comparability factor is a measure of the crude mortality rate for the standard population divided by what the crude mortality rate is for the region being studied, assuming the mortality rates are the same as for the standard population..

(ii) Crude death rate: Occupational Group = 284/46000 = 0.006174

The Directly Standardised Mortality Rate is:

$$\left(\frac{(1000000\times\frac{67}{20000})+(1500000\times\frac{92}{15000})+(700000\times\frac{125}{11000})}{3200000} \right)$$

$$=\left(\frac{3350+9200+7954.55}{3200000}\right)$$

$$=0.00641$$

The Indirectly Standardised Mortality Rate can be calculated as follow:

Expected Deaths for Occupation:

$$\left(\frac{20000\times3500}{1000000}+\frac{15000\times7800}{1500000}+\frac{11000\times8000}{700000}\right)$$

$$=70+78+125.71=273.71$$

So the Indirectly Standardised Mortality Rate is:

$$\frac{0.006031\times284}{273.71}=0.00626$$

Generally well done. In part (i) students who put the formulae into words were given full credit.

11 (i) Let P be the annual premium for the policy.

Then (functions at 4%) equation of value gives:

$$P\ddot{a}_{[40]:\overline{20|}}=1{,}000A_{[40]:\overline{20|}}+0.54P+0.06P\ddot{a}_{[40]:\overline{20|}}$$

$$\Rightarrow P=\frac{1{,}000\times0.46423}{0.94\times13.930-0.54}=36.98$$

(ii) On 31 December 2012, the gross premium prospective reserve per £1,000 sum assured is given by:

$$_5V = 1{,}000A_{45:\overline{15}|} - 0.94\times 36.98\times \ddot{a}_{45:\overline{15}|}$$
$$\Rightarrow {}_5V = 1{,}000\times 0.56206 - 0.94\times 36.98\times 11.386$$
$$= 562.06 - 395.79 = 166.27$$

(iii) If we consider the total portfolio of non-profit endowment policies during 2013, we have:

Reserve on 31 December 2012 = 15,500 ×166.27 = 2,577,185

Premiums (***P***) paid on 1 January 2013 = 15,500 × 36.98 = 573,190

Expenses (***E***) incurred on 1 January 2013 = 76,500

Interest (***I***) earned during 2013 = 0.035 × (2,577,185 + 573,190 – 76,500) = 107,585.6

Death claims (***D***) during 2013 = 295,000

On 31 December 2013, the gross premium prospective reserve per £1,000 sum assured is given by:

$$_6V = 1{,}000A_{46:\overline{14}|} - 0.94\times 36.98\times \ddot{a}_{46:\overline{14}|}$$
$$\Rightarrow {}_6V = 1{,}000\times 0.58393 - 0.94\times 36.98\times 10.818$$
$$= 583.93 - 376.05 = 207.88$$

Total surrender values paid (***S***) during 2013
= 625 × 0.85 × 207.88 = 110,436.3

Total sum assured in force at 31 December 2013
= 15,500,000 – 295,000 – 625,000 = 14,580,000

Reserve on policies in force at 31 December 2013
= 14,580 × 207.88 = 3,030,890.4
Total Profit for 2013 =

$$= \sum {}_5V + P - E + I - D - S - \sum {}_6V$$
$$= 2{,}577{,}185 + 573{,}190 - 76{,}500 + 107{,}585.6 - 295{,}000 - 110{,}436.3 - 3{,}030{,}890.4$$
$$= -254{,}866.1$$

i.e. an experience loss of £254,866

Parts (i) and (ii) were generally well done. Part (iii) was less well done. Many students successfully obtained the mortality profit but were unable to quantify others as shown above. In particular identifying surrenders caused difficulties.

12 (i) Decrement table

age	q_x	p_x	${}_{t-1}p_x$
65	0.006032	0.993968	1.000000
66	0.007147	0.992853	0.993968
67	0.008439	0.991561	0.986864

(ii) Cash flows for policy:

(a) With reserves

Year	*Opening reserve*	*Premium*	*Initial expense*	*Interest*	*Annuity claim*	*Annuity expense*	*Closing reserve*	*Profit vector*
1	0.00	42000.00	770.00	2061.50	14909.52	56.31	29819.04	–1493.37
2	30000.00	0.00	0.00	1500.00	14892.80	57.93	14892.80	1656.47
3	15000.00	0.00	0.00	750.00	14873.42	59.59	0.00	816.99

Year	*Profit vector*	*Profit signature*	*Discount factor*	*PVFNP*
1	–1493.37	–1493.37	0.934579	–1395.67
2	1656.47	1646.48	0.873439	1438.10
3	816.99	806.26	0.816298	658.15

Total PVFNP = 700.58

(b) Without reserves

Year	*Opening reserve*	*Premium*	*Initial expense*	*Interest*	*Annuity claim*	*Annuity expense*	*Closing reserve*	*Profit vector*
1	0.00	42000.00	770.00	2061.50	14909.52	56.31	0.00	28325.67
2	0.00	0.00	0.00	0.00	14892.80	57.93	0.00	–14950.73
3	0.00	0.00	0.00	0.00	14873.42	59.59	0.00	–14933.01

Year	*Profit vector*	*Profit signature*	*Discount factor*	*PVFNP*
1	28325.67	28325.67	0.934579	26472.59
2	–14950.73	–14860.54	0.873439	–12979.78
3	–14933.01	–14736.85	0.816298	–12029.66

Total PVFNP = 1463.15

(ii) The net present value is smaller when reserves are set up because we are tying up money in the reserves which are subject to a lower rate of interest (5%) than the risk discount rate (7%) c.f. 700.58 compared to 1463.15.

(iii) With reserves, the net present value of the expected profit will increase if the risk discount rate is reduced from 7% per annum to 4% per annum because the positive profit signature in year 2 and 3 become more significant (note: NPV increases from 700.58 to 803.10).

Without reserves, the net present value of the expected profit will fall if the risk discount rate is reduced from 7% per annum to 4% per annum because the negative profit signature in year 2 and 3 become more significant (note: NPV decreases from 1463.16 to 395.81).

The net present value of expected profits without reserves would now be less than the net present value of expected profits with reserves. This is because the reserves are now subject to a higher rate of interest (5%) than the risk discount rate (4%) (note: 803.10 compared to 395.81).

Many well prepared students answered this question well. Other than accuracy of the numbers the main omission was the detail expected in part (iii). Note that in part (i) a general understanding of the methods needed to solve the problem earned proportionate credit even if the numerical accuracy was not always apparent.

13 (i) Let P be the annual premium for the contract. Then:

EPV of premiums is:

$$P\ddot{a}^{6\%}_{30:\overline{35|}} = 15.150P$$

EPV of benefits:

$$60,000\left[\frac{1}{1.0192308}\times(1.06)^{0.5}\times A^{1}_{30:\overline{35|}} + v^{35}\,{}_{35}p_{30}\right]@\,4\%$$

$$= 60,000[0.04176 + 0.22523] = 16,019.40$$

where

$$A_{30:\overline{35|}} = 0.26657$$

$$v^{35}\,{}_{35}p_{30} = 0.25342\times\frac{8821.2612}{9925.2094} = 0.22523$$

$$A^{1}_{30:\overline{35|}} = A_{30:\overline{35|}} - v^{35}\,{}_{35}p_{30} = 0.04134$$

EPV of expenses:

$$250 + 0.575P + 0.025P\ddot{a}^{6\%}_{30:\overline{35|}} = 250 + 0.95375P$$

Equation of value gives

$$15.15P = 16,019.40 + 250 + 0.95375P$$
$$\Rightarrow P = £1146.04$$

(ii) Gross future loss random variable

= *PV* future benefit payment + *PV* future expenses – *PV* of future premiums

$$= G(K_{30+t}) + 0.025 \times 1146.04\ddot{a}_{\overline{\min[K_{30+t}+1,35-t]}|} - 1146.04\ddot{a}_{\overline{\min[K_{30+t}+1,35-t]}|}$$

where

$$G(K_{30+t}) = 60,000 \times (1.0192308)^{t+K_{30+t}} \times v_{.06}^{T_{30+t}} \text{ if } K_{30+t} < 35 - t$$

or

$$G(K_{30+t}) = 60,000 \times (1.0192308)^{35} \times v_{.06}^{35-t} \text{ if } K_{30+t} \geq 35 - t$$

(iii) Sum assured and attaching bonuses at 31 December 2012

$$= 60,000(1.0192308)^{10} = 72,589.97$$

gross prospective reserve at the end of the 10th policy year is given by:

$${}_{10}V = 72,589.97\left[\frac{1}{1.0192308} \times (1.06)^{0.5} \times A^{1}_{40:\overline{25}|} + v^{25}\,{}_{25}p_{40}\right] @\,4\% - 0.975 \times 1146.04\ddot{a}^{6\%}_{40:\overline{25}|}$$

where

$$A_{40:\overline{25}|} = 0.38907$$

$$v^{25}\,{}_{25}p_{40} = 0.37512 \times \frac{8821.2612}{9856.2863} = 0.33573$$

$$A^{1}_{40:\overline{25}|} = A_{40:\overline{25}|} - v^{25}\,{}_{25}p_{40} = 0.05334$$

$$\ddot{a}^{6\%}_{40:\overline{25}|} = 13.288$$

$$\Rightarrow {}_{10}V = 72{,}589.97\left[0.05388 + 0.33573\right] - 0.975 \times 1146.04 \times 13.288$$
$$= 28{,}281.78 - 14{,}847.87 = £13{,}433.91$$

Part (i) was generally well done. Part (ii) was poorly done which is often the case for these types of question. Part (iii) gave more difficulties but was generally completed successfully by well prepared students.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2014 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

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F Layton
Chairman of the Board of Examiners

November 2014

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions' actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the September 2014 paper

The general performance was similar this session to previous ones although it was felt that this paper was possibly a little harder than some previous ones. Questions that were done less well were 4, 5(ii), 7 (variance), 11, 12(ii) and 14(iii). The examiners hope that the detailed solutions given below will assist students with further revision.

However most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonable accurate numerical calculation is necessary.

1 Within a population mortality (or morbidity) varies with calendar time. This effect is usually observed at all ages. The usual pattern is for mortality rates to become lighter (improve) over time, although there can be exceptions, due, for example, to the increasing effect of AIDS in some countries.

For example a separate model or table will be produced for different calendar periods e.g. English Life Table No 14 1980–82 and English Life Table No 15 1990–92. The difference between the tables is termed time selection. [2]

This question was generally well done. Other valid examples were credited.

2

- Withdrawal often acts as a selective decrement in respect of mortality. Those withdrawing tend to have lighter mortality than those who keep their policies in force.
- This selective effect results in mortality rates which increase markedly with policy duration and resembles temporary initial selection.

[3]

Generally well done. The main omission was mentioning the worsening mortality of those who did not lapse.

3

$$ {}_{2.5}q_{75.75} = (1 - {}_{2.5}p_{75.75}) = 1 - ({}_{0.25}p_{75.75}) \times p_{76} \times p_{77} \times {}_{0.25}p_{78} $$

$$ p_{76} = 1 - q_{76} = 0.967821 $$

$$ p_{77} = 1 - q_{77} = 0.963304 $$

$$ {}_{0.25}p_{78} = 1 - 0.25 \times q_{78} = 0.989575 $$

$$ {}_{0.25}p_{75.75} = 1 - \frac{0.25q_{75}}{1 - 0.75q_{75}} = 0.992818 $$

$$ {}_{2.5}q_{75.75} = 1 - (0.992818 \times 0.967821 \times 0.963304 \times 0.989575) $$

$$ = 0.08404 $$

[4]

Generally well done.

4

$$ {}_{5|3}q^{1}_{40:40} = \frac{l_{45}}{l_{40}} \times \frac{l_{45}}{l_{40}} \times \frac{1}{2}\, {}_3q_{45:45} $$

$$ = 0.5 \times \left(\frac{l_{45}}{l_{40}}\right)^2 \times (1 - {}_3p_{45:45}) $$

$$ = 0.5 \times \left(\frac{l_{45}}{l_{40}}\right)^2 \times \left(1 - \frac{l_{48}}{l_{45}}\frac{l_{48}}{l_{45}}\right) $$

$$ = 0.5 \times \left(\frac{9801.3123}{9856.2863}\right)^2 \times \left(1 - \frac{9753.4714}{9801.3123} \times \frac{9753.4714}{9801.3123}\right) $$

$$ = 0.00482 $$ [4]

This question caused many students problems. The main issue missed was the relationship between the first of 2 equal ages to die and the joint mortality function.

5 (i)

$$ F = \frac{\sum_x {}^sE^c_{x,t}\ {}^sm_{x,t}}{\sum_x {}^sE^c_{x,t}} \Bigg/ \frac{\sum_x E^c_{x,t}\ {}^sm_{x,t}}{\sum_x E^c_{x,t}} $$

F is called the area comparability factor and is a measure of the crude mortality rate for the standard population divided by what the crude mortality rate is for the region being studied, assuming the mortality rates are the same as for the standard population. [2]

(ii) If its age/sex profile is such that if it experienced the same age/sex specific mortality rates as the country, then its crude death rate would be 2/3 of that of the country, i.e. the region has either a younger age structure or a higher female proportion (or both) than the country. [2]

[Total 4]

The first part was straight bookwork. Part (ii) was generally poorly explained and the two-thirds relationship was not appreciated.

6 Past Service

$$30000 \times \frac{10}{80} \times \frac{{}^{z}M_{30}^{ia}}{{}^{s}D_{30}} = 30000 \times \frac{10}{80} \times \frac{64061}{41558} = £5781$$

For future service note that maximum ill-health pension will accrue by age 60

$$30000 \times \frac{1}{80} \times \frac{{}^{z}\overline{R}_{30}^{ia} - {}^{z}\overline{R}_{60}^{ia}}{{}^{s}D_{30}} = 30000 \times \frac{1}{80} \times \frac{1502811 - 35570}{41558} = £13240$$ [4]

Well prepared students did this question well. Many others did not allow for the age limitation properly just setting out the standard formula which was not credited.

7 EPV $= 10000\left(A_{[40]:\overline{20|}} + v^{20}\,\frac{l_{60}}{l_{[40]}} \right)$

$$= 10000\left(0.46423 + \left(0.45639 \times \frac{9287.2164}{9854.3036} \right) \right)$$

$$= 8943.6$$

For the variance we need the second moment which can be found as:

$$(10000)^2 \left({}^{2}A_{[40]} - (v^{20})^2\,\frac{l_{60}}{l_{[40]}}\,{}^{2}A_{60} \right) + (20000)^2 (v^{20})^2\,\frac{l_{60}}{l_{[40]}}$$

$$= (10000)^2 \left(0.06775 - 0.20829 \times \frac{9287.2164}{9854.3036} \times 0.23723 + 4 \times 0.20829 \times \frac{9287.2164}{9854.3036} \right)$$

$$= (10000)^2 (0.06775 - 0.04657 + 0.78521)$$

$$= (10000)^2 \times 0.80639$$

Hence Variance is:

$$(10000)^2 \times 0.80639 - (10000)^2 \times (0.89436)^2$$

$$= (10000)^2 \times 0.00651$$

$$= (807)^2$$ [6]

The mean was generally easily calculated but many students struggled with the variance not coping properly with the double payment on survival.

8 (i) T_x is the total future lifetime of an ultimate life aged x [2]

K_x is the curtate future lifetime of an ultimate life aged x

(ii) (a) v^{T_x}

(b) $a_{\overline{K_x}|}$

(c) $v^{\min[K_{[x]}+1,n]}$

(d) $v^5 \ddot{a}_{\overline{K_x-4}|}$ if $K_x \geq 5$

0 otherwise

[5]

[Total 7]

Very straightforward quick question which well prepared students did well. Main omission was inaccuracies in (ii)(d).

9 The annual premium is found from

$$P\ddot{a}_{60:\overline{20}|} = P(IA)^1_{60:\overline{5}|} + 100,000 \times v^5 \times \frac{l_{65}}{l_{60}} \times A^1_{65:\overline{15}|}$$

$$\ddot{a}_{60:\overline{20}|} = \ddot{a}_{60} - v^{20} \times \frac{l_{80}}{l_{60}} \times \ddot{a}_{80} = 14.134 - 0.45639 \times \frac{5266.4604}{9287.2164} \times 6.818 = 12.369$$

$$(IA)^1_{60:\overline{5}|} = (IA)_{60} - v^5 \times \frac{l_{65}}{l_{60}} \times ((IA)_{65} + 5A_{65}) = 8.36234 - 0.82193 \times \frac{8821.2612}{9287.2164} \times (7.89442 + 5 \times 0.52786)$$

$$= 0.13874$$

$$A^1_{65:\overline{15}|} = A_{65} - v^{15} \times \frac{l_{80}}{l_{65}} \times A_{80} = 0.52786 - 0.55526 \times \frac{5266.4604}{8821.2612} \times 0.73775 = 0.28330$$

Hence:

$$12.369P = 0.13874P + 100000 \times 0.82193 \times \frac{8821.2612}{9287.2164} \times 0.28330$$

$$12.230P = 22117.02$$

$P = £1808$ to nearer £ [7]

Generally done well by well prepared students. Main error related to the treatment of the return of premiums in the first 5 years.

10 (i) The accumulated net cash flow at end of t^{th} policy year per policy in force at the start of that year is given by:

$$(CF)_t = (P - E_t)\times(1+i_t) - (aq)^d_{x+t-1}\times D_t - (aq)^w_{x+t-1}\times B_t$$
$$-\left(1-(aq)^d_{x+t-1}-(aq)^w_{x+t-1}\right)\times S_t$$

[2]

(ii) We need to set the expected present value of the profit signature of the policy equal to zero using a risk discount rate of j% per annum. Hence, if

$$(ap)_{x+k} = \left(1-(aq)^d_{x+k}-(aq)^w_{x+k}\right) \Rightarrow {}_t(ap)_x = \prod_{k=0}^{t-1}(ap)_{x+k}$$

then the level annual premium P is derived from the following equation:

$$\sum_{t=1}^{n} (CF)_t \times {}_{t-1}(ap)_x \times v^t_{j\%} = 0$$ [3]

(iii) Expected profit at the end of the t^{th} policy year for each policy in force at the start of that year

$$= {}_{t-1}V \times (1+i_t) + (CF)_t - (ap)_{x+t-1} \times {}_tV$$ [2]

[Total 7]

Generally done well although students struggled to explain part (ii). Credit was given for reasonable alternative explanations.

11 (i) The reserve for the death claim at 31 December 2013 was

$${}_{14}V = 15{,}000\left(1-\frac{\ddot{a}_{70}}{\ddot{a}_{56}}\right) = 15{,}000\left(1-\frac{10.375}{15.537}\right) = 4{,}983.59$$

Total death strain at risk ($DSAR$) at 31 December 2013:

$$DSAR = 740{,}000 - (371{,}000 + 4{,}983.59) = 364{,}016.41$$

Expected death strain (EDS) =
$q_{69} \times DSAR = 0.022226 \times 364{,}016.41 = 8{,}090.63$

Actual death strain (ADS) = $(15{,}000 - 4{,}983.59) = 10{,}016.41$

Mortality profit = EDS – ADS = 8,090.63 – 10,016.41 = –1,925.78 i.e. a loss

[5]

(ii) Expected claims = $q_{69} \times 740,000 = 16,447.24$ [1]

(iii) Actual claims = 15,000

Actual claims were lower than expected although the company made a mortality loss. This was due to the DSAR (expressed as a % of the sum assured) on the one death claim policy being significantly higher than for the group of policies on average. [2]

[Total 8]

This question was not done well overall. Many students failed to understand how to derive the reserve using premium conversion techniques and basically ignored it. This led to totally the wrong conclusions.

12 (i) The probability is:

$${}_{25}p_{30} - {}_{35}p_{30} = \frac{l_{55}}{l_{30}} - \frac{l_{65}}{l_{30}}$$

$$= \frac{91217 - 79293}{97645} = 0.12212$$ [2]

(ii) $${}_{25}p_{30} = \exp\left(-\int_0^{25} \mu_{30+t}\,dt\right)$$

$$= \exp\left(-\int_0^{25} 0.005e^{0.09(30+t-20)}\,dt\right)$$

$$= \exp\left(-0.005 \times e^{0.9} \int_0^{25} e^{0.09t}\,dt\right)$$

$$= \exp\left(-0.005 \times e^{0.9} \times \left[\frac{e^{0.09t}}{.09}\right]_0^{25}\right)$$

$$= \exp\left(-0.005 \times e^{0.9} \times \left[\frac{e^{2.25} - 1}{0.09}\right]\right)$$

$$= \exp(-1.159803)$$

$$= 0.313548$$

Similarly:

$$ {}_{35}p_{30} = \exp\left(-\int_0^{35} \mu_{30+t}\,dt\right)$$

$$= \exp\left(-\int_0^{35} 0.005e^{0.09(30+t-20)}\,dt\right)$$

$$= \exp\left(-0.005\times e^{0.9}\int_0^{35} e^{0.09t}\,dt\right)$$

$$= \exp\left(-0.005\times e^{0.9}\times\left[\frac{e^{0.09t}}{.09}\right]_0^{35}\right)$$

$$= \exp\left(-0.005\times e^{0.9}\times\left[\frac{e^{3.15}-1}{0.09}\right]\right)$$

$$= \exp(-3.052103)$$

$$= 0.047259$$

Hence required probability

$${}_{25}p_{30} - {}_{35}p_{30} = 0.313548 - 0.047259 = 0.266289$$ [7]

[Total 9]

Part (i) was straightforward and well done. Part (ii)was generally poorly done although in essence it was a simple subtraction of 2 similar integrals.

13 (i) Multiple decrement table

	q_x^d	q_x^w	q_x^i	$(aq)_x^d$	$(aq)_x^w$	$(aq)_x^i$	$(ap)_x$	${}_{t-1}(ap)_x$
55	0.004916	0.100	0.040	0.00458	0.09776	0.03791	0.85975	1.00000
56	0.005528	0.080	0.050	0.00518	0.07779	0.04787	0.86917	0.85975
57	0.006215	0.060	0.060	0.00585	0.05802	0.05802	0.87811	0.74727

[3]

(ii) Cash flows for the policy

Let P be the level annual premium for the policy, then

Yr	*Prm*	*Exp*	*Interest*	*Death claim*	*Surrender claim*	*Ill-health claim*	*Mat claim*	*Profit vector*
1	P	150.00	$0.05P$–7.50	916.00	$0.09972P$	3791.00	0.00	$0.95028P$–4864.50
2	P	25.00	$0.05P$–1.25	1036.00	$0.16028P$	4787.00	0.00	$0.88972P$–5849.25
3	P	25.00	$0.05P$–1.25	1170.00	$0.18112P$	5802.00	8781.10	$0.86888P$–15779.25

Yr	*Profit vector*	${}_{t-1}(ap_x)$	*Profit signature*	*Discount factor*	*PVFNP*
1	$0.95028P$–4864.50	1.00000	$0.95028P$–4864.50	0.952381	$0.90503P$–4632.86
2	$0.88972P$–5849.25	0.85975	$0.76494P$–5028.89	0.907029	$0.69382P$–4561.35
3	$0.86888P$–15779.25	0.74727	$0.64929P$–11791.36	0.863838	$0.56088P$–10185.82

Total PVFNP = $2.15973P - 19380.03 = 0.05P$

$$\Rightarrow P = \frac{19380.03}{2.10973} = 9186.02$$ [9]

(iii) The cash flows show that for this policy, the expected profit vector is positive for policy years 1 and 2 but negative (significantly) for the last policy year (which is expected due to the survival amount being paid at the end of the term of the policy). Unless the company builds up reserves over the period of the policy, it may not have sufficient funds available to pay claims in policy year 3. Therefore, it would be prudent for the company to hold reserves at the beginning and end of each policy year. Indeed, regulations may force the company to do so. [2]

(iv) As the discount rate and the interest rate earned on cash flows items (including reserves) is the same at 5% per annum, holding reserves will not change the premium required for this policy. [2]

[Total 16]

This question was generally well done by students who had prepared well. Other approaches were credited especially where a non tabular approach was adopted.

Credit was also given if the student could demonstrate how the problem might be approached without getting all the arithmetic entirely accurate.

14 (i) Multiple decrement table:

x	q_x^d	q_x^s
61	0.009009	0.075
62	0.010112	0.025
63	0.011344	0.000

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$
61	0.008108	0.07439	0.917500	1.000000
62	0.009101	0.02477	0.966127	0.917500
63	0.010210	0.00000	0.989790	0.886421

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
value of units at start of year	0.00	729.36	3052.03
allocation	750.00	2362.50	3450.00
B/O spread	45.00	141.75	207.00
interest	31.73	132.75	283.28
management charge	7.37	30.83	65.78
value of units at year end	729.36	3052.03	6512.53

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
unallocated premium	750.00	–112.50	–450.00
B/O spread	45.00	141.75	207.00
expenses	275.00	111.25	130.00
interest	13.00	–2.05	–9.32
man charge	7.37	30.83	65.78
extra death benefit	48.82	33.65	2.42
claim expense	6.19	2.54	0.77
profit vector	485.36	–89.41	–319.73
probability in force	1	0.917500	0.886421
profit signature	485.36	–82.03	–283.42
discount factor	0.938967	0.881659	0.827849
PVFNP	455.74	–72.33	–234.63

Total PVFNP = 148.78

	yr 1	*yr 2*	*yr 3*
premium signature	1500.000	1938.38	2344.57

Total PV of premiums = 7197.448

Total PV of premiums = 5782.95

Profit margin = $\dfrac{148.78}{5782.95} = 2.57\%$ [13]

(ii) Reserves might be required to eliminate/zeroise expected negative cash flows in the future so that the company does not expect to have to input further capital in the future. [2]

(iii) The profit vector for the policy is (485.36, –89.41, –319.73)

In order to set up reserves to zeroise future expected negative cash flows, we require:

$${}_2V = \frac{319.73}{1.025} = 311.93$$

$${}_1V \times 1.025 - (ap)_{62} \times {}_2V = 89.41 \Rightarrow {}_1V = 381.24$$

revised cash flow in year 1 $= 485.36 - (ap)_{61} \times {}_1V = 135.57$

and PVFNP = 135.57/1.065 = 127.30

$\Rightarrow$Profit margin = $\dfrac{127.30}{5782.95} = 2.20\%$ [4]

[Total 19]

Again well prepared students scored good marks on this question and credit was given if a good understanding of the process was demonstrated even if the result was not entirely arithmetically accurate.

The main difficulty here was the interpretation of zeroising the result in part (iii).

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INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2015 examinations

Subject CT5 – Contingencies
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F Layton
Chairman of the Board of Examiners

June 2015

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the April 2015 paper

The general performance was higher than usual this session compared to previous ones although it was felt that this paper was roughly of the same standard as previous ones. Questions that were done less well were Q1, Q10 part (ii), Q11, Q13 part (ii) and Q13 part (iv) and Q14 part (ii). The examiners hope that the detailed solutions given below will assist students with further revision.

However most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonable accurate numerical calculation is necessary.

1 $\ddot{a}_{50:\overline{4}|} = 1 + \dfrac{(1-.05)}{1.06} + \dfrac{(1-.05)(1-.06)}{(1.06)^2} + \dfrac{(1-.05)(1-.06)(1-.06(1.1))}{(1.06)^3}$

$= 1 + 0.89623 + 0.79477 + 0.70029 = 3.39129$

$A_{50:\overline{4}|} = 1 - d(6\%)\ddot{a}_{50:\overline{4}|} = 1 - \dfrac{.06}{1.06}(3.39129) = 0.80804$

This question gave many students difficulties. The answer was most easily obtained quickly using premium conversion formulae as above. The alternative method of direct computation is, of course, possible but is more involved.

2 The standard of housing encompasses not only all aspects of the physical quality of housing (e.g. state of repair, type of construction, heating, sanitation) but also the way in which the housing is used e.g. overcrowding and shared cooking.

These factors have an important influence on morbidity, particularly that related to infectious diseases (e.g. from tuberculosis and cholera to colds and coughs) and thus on mortality in the longer term.

The effect of poor housing is often confounded with the general effects of poverty.

A straightforward bookwork question generally well answered. The main omission by students was the comment in the 3rd paragraph.

3 $(aq)_x^{\alpha} = \dfrac{\mu_x^{\alpha}}{\mu_x^{\alpha} + \mu_x^{\beta}}\left(1 - e^{-(\mu_x^{\alpha} + \mu_x^{\beta})}\right)$ and

$(aq)_x^{\beta} = \dfrac{\mu_x^{\beta}}{\mu_x^{\alpha} + \mu_x^{\beta}}\left(1 - e^{-(\mu_x^{\alpha} + \mu_x^{\beta})}\right)$

Thus $(aq)_x = (aq)_x^{\alpha} + (aq)_x^{\beta} = \left(1 - e^{-5\mu_x^{\beta}}\right)$

Question was generally well done. Students who left the final answer in integral form also received full credit.

4 (a) $${}_{10|15}q_{60} = \frac{l_{70} - l_{85}}{l_{60}} = \frac{8054.0544 - 3385.2479}{9287.2164} = 0.50271$$

(b) $${}_{12}p_{[50]+1} = \frac{l_{63}}{l_{[50]+1}} = \frac{9037.3973}{9686.9669} = 0.93294$$

(c) $$a^{(4)}_{40:\overline{10|}} = a^{(4)}_{40} - \frac{v^{10} l_{50}}{l_{40}} a^{(4)}_{50} \text{ at } 6\%$$

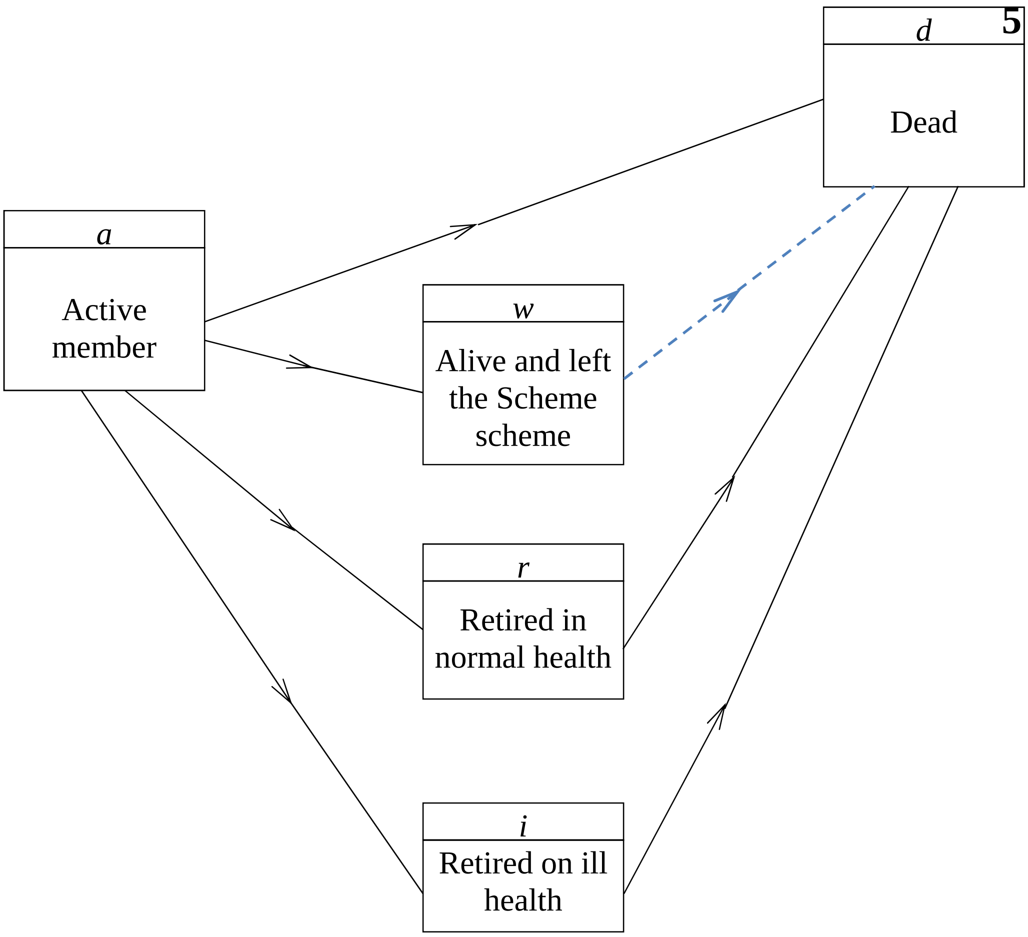
$$= \left(\ddot{a}_{40} - \frac{5}{8}\right) - \frac{v^{10} l_{50}}{l_{40}} \left(\ddot{a}_{50} - \frac{5}{8}\right)$$

$$= (15.491 - 0.625) - \frac{0.55839 \times 9712.0728}{9856.2863}(14.044 - 0.625)$$

$$= 14.866 - 7.383 = 7.483$$

Parts (a) and (b) were straightforward and well done. Many students in (c) did not obtain the correct relationship for the adue function in line 2 of the formulae above.

5



Straightforward question generally well done. Note there is in reality no connection from Withdrawn to Dead as this is not a feature of the PEN tables and lives have left the scheme experience altogether. Also there are no probabilities shown for the PEN tables for states w, r and i to d so students who did not include these in the diagram were given full credit.

6

	$q_{[x]}$	$q_{[x-1]+1}$	$q_{[x-2]+2}$
55	**0.003358**		
56		**0.004903**	
57			**0.005650**

EPV premiums $= 900\{1 + v^*p_{[55]} + v^{2*}\ p_{[55]}{}^*p_{[55]+1}\}$

$= 900\{1 + v^*(1 - 0.003358) + v^{2*}(1 - 0.003358)^*(1 - 0.004903)\}$

$= 900(1 + 0.96761 + 0.93482) = 2612.19$

EPV benefits $= 150{,}000\{v^*q_{[55]} + v^{2*}p_{[55]}{}^*q_{[55]+1} + v^{3*}p_{[55]}{}^*p_{[55]+1}{}^*q_{57}\}$

$= 150{,}000\ \{0.0032602 + 0.0046060 + 0.0051279\} = 1949.12$

EPV expenses $= 260 + 70\{v^*p_{[55]} + v^{2*}\ p_{[55]}{}^*p_{[55]+1}\}$

$= 260 + 70\{v^*(1 - 0.003358) + v^{2*}(1 - 0.003358)^*(1 - 0.004903)\}$

$= 260 + 70(0.96761 + 0.93482) = 393.17$

EPV profit $= 2612.19 - 1949.12 - 393.17 = 269.90$

Alternatively, using cash flow approach:

Yr	*premium*	*expense*	*interest*	*claim*	*profit vector*	*cumulative probability of survival*	*discount factor*	*net present value*
1	900.00	260.00	19.20	503.70	155.50	1.000000	.97087	150.97
2	900.00	70.00	24.90	735.45	119.45	0.996642	.94260	112.22
3	900.00	70.00	24.90	847.50	7.40	0.991755	.91514	6.72

Total net present value of profit = 269.91

This question was generally well done by well prepared students.

7 (a)

$$ {}_{1.75}p_{82.75} = {}_{0.25}p_{82.75} \times p_{83} \times {}_{0.5}p_{84} $$

$$ = (1 - {}_{0.25}q_{82.75})(1 - q_{83})(1 - {}_{0.5}q_{84}) $$

$$ = \left(1 - \frac{0.25q_{82}}{(1 - 0.75q_{82})}\right)(1 - q_{83})(1 - 0.5q_{84}) $$

$$ = \left(1 - \frac{0.25 \times 0.11279}{(1 - 0.75 \times 0.11279)}\right) \times (1 - 0.12235) \times (1 - 0.5 \times 0.13270) $$

$$ = 0.79418 $$

(b)

$$ {}_{1.75}p_{82.75} = {}_{0.25}p_{82.75} \times p_{83} \times {}_{0.5}p_{84} $$

$$ = (p_{82})^{0.25} \times p_{83} \times (p_{84})^{0.5} $$

$$ = (1 - 0.11279)^{0.25} \times (1 - 0.12235) \times (1 - 0.13270)^{0.5} $$

$$ = 0.79325 $$

This question was generally well done.

8 (a) When a life table is constructed it is assumed to reflect the mortality experience of a homogeneous group of lives. This table can then be used to model the experience of a homogeneous group of lives which is suspected to have a similar experience.

If a table is constructed for heterogeneous group then the mortality experience will depend on the exact mixture of lives with different experiences used to construct the table. Such a table could only be used to model mortality in a group with the same mixture.

For this reason separate mortality tables are usually constructed for groups which are expected to be heterogeneous.

(b) Choose from:

- Full choice available here from
- Temporary Initial Selection
- Class Selection
- Adverse Selection
- Time Selection
- Spurious Selection

A straight bookwork question generally well done.

9 (i) Let P be the annual premium for the contract. Then:

EPV of premiums is:

$$P\ddot{a}_{[45]:\overline{20|}} = 11.888P$$

EPV of benefits and claim expense:

$$125,325A_{[45]} = 125,325 \times 0.15918 = 19,949.23$$

EPV of other expenses:

$$0.75P + 0.05P\left[\ddot{a}_{[45]:\overline{20|}} - 1\right] = 1.2944P$$

Equation of value gives

$$11.888P = 19,949.23 + 1.2944P$$

$$\Rightarrow P = \frac{19,949.23}{10.5936} = £1,883.14$$

(ii) gross prospective reserve

$$= 125,000A_{60} - 1883.14\ddot{a}_{60:\overline{5|}} = 125,000 \times 0.32692 - 1883.14 \times 4.39$$
$$= 40,865.0 - 8,266.98 = 32,598.02$$

Generally well done. The main omission that some students counted the claim expense within gross prospective reserve.

10 (i) $$\bar{A}^{1}_{40:50} = \int_0^\infty v^t\, {}_tp_{40:50}\mu_{40+t}dt = .04\int_0^\infty e^{-(.04+.06+\ln 1.05)t}dt = .04\int_0^\infty e^{-0.14879t}dt$$

$$= .04\left[-\frac{e^{-.14879t}}{.14879}\right]_0^\infty = \frac{.04}{.14879} = 0.26884$$

(ii) $$\bar{a}_{40:50:\overline{20|}} = \int_0^{20} v^t\, {}_tp_{40:50}dt = \int_0^{20} e^{-.14879t}dt$$

$$= \left[-\frac{e^{-0.14879t}}{0.14879}\right]_0^{20} = \frac{1}{0.14879}(1 - e^{-2.976}) = 6.378$$

$$\bar{a}_{40:50:\overline{30}|} = \int_0^{30} v^t\, {}_t p_{40:50} dt = \int_0^{30} e^{-.14879t} dt$$

$$= \left[-\frac{e^{-0.14879t}}{0.14879} \right]_0^{30} = \frac{1}{0.14879}(1 - e^{-4.464}) = 6.643$$

Let Premium = P, then

$$P(0.75 \times 6.643 + .25 \times 6.378) = 75,000 \times 0.26884$$

$$P = \frac{20163}{6.577} \Rightarrow P = 3065.7$$

Generally part (i) was done well but part (ii) was poorly done. A large proportion of students did not appreciate how to derived the premium relationship described in the question. Another common error was to take the force of interest as 5% rather than ln(1.05)%.

11 The annuity can be written as (with 65 denoting the male life and 62 the female):

$$50000 a^{(12)}_{\overline{65:62}} + 25000 a^{(12)}_{65:62} + 25000 a^{(12)}_{65} + 20000(v^{10}\, {}_{10}p_{\overline{65:62}} + v^{20}\, {}_{20}p_{\overline{65:62}})$$

$$a^{(12)}_{65} = \ddot{a}_{65} - \frac{13}{24} = 13.666 - \frac{13}{24} = 13.124$$

$$a^{(12)}_{65:62} = \ddot{a}_{65:62} - \frac{13}{24} = 12.427 - \frac{13}{24} = 11.885$$

$$a^{(12)}_{\overline{65:62}} = \ddot{a}_{65} + \ddot{a}_{62} - \ddot{a}_{65:62} - \frac{13}{24} = 13.666 + 15.963 - 12.427 - \frac{13}{24} = 16.660$$

$$v^{10}\, {}_{10}p_{\overline{65:62}} = \frac{1 - (1 - l_{75}/l_{65})(1 - l_{72}/l_{62})}{(1.04)^{10}}$$

$$= \frac{1 - (1 - 8405.16/9647.797)(1 - 9193.86/9804.173)}{1.48024}$$

$$= 0.67015$$

$$v^{20}\ {}_{20}p_{\overline{65:62}} = \frac{1-(1-l_{85}/l_{65})(1-l_{82}/l_{62})}{(1.04)^{20}}$$

$$= \frac{1-(1-4892.878/9647.797)(1-7147.965/9804.173)}{2.19112}$$

$$= 0.39545$$

So value is:

(50000*16.660)+(25000*11.885)+(25000*13.124)+20000*(.67015+.39545)

= 1479537

Other formulae approaches credited. Also the final answer is very sensitive to rounding and full credit was given to +/–00 to the answer.

Many students found difficulty in reproducing the correct annuities to make up the total value.

12 Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25}|} @ 6\% = 155.1272P$$

where

$$\ddot{a}^{(12)}_{[40]:\overline{25}|} = \ddot{a}_{[40]:\overline{25}|} - \frac{11}{24}\left(1 - {}_{25}p_{[40]}v^{25}\right)$$

$$= 13.290 - \frac{11}{24}\left(1 - \frac{8821.2612}{9854.3036} \times 0.233\right) = 12.9273$$

EPV of benefits:

$$72,750\bar{A}^{\ \ 1}_{[40]:\overline{25}|} + 2250\left(I\bar{A}\right)^{\ \ 1}_{[40]:\overline{25}|} + 131,250A_{[40]:\overset{\ \ 1}{\overline{25}|}} \quad @ 6\%$$

$$= 72,750 \times 0.04032 + 2250 \times 0.62876 + 131,250 \times 0.208574$$

$$= 2,933.561 + 1,414.71 + 27,375.3375 = 31,723.609$$

where

$$\bar{A}^{\ 1}_{[40]:\overline{25}|} = 1.06^{0.5} A^{\ 1}_{[40]:\overline{25}|} = 1.06^{0.5}\left[A_{[40]:\overline{25}|} - v^{25}\ {}_{25}p_{[40]}\right]$$

$$= 1.06^{0.5}\left[0.24774 - 0.233\times\frac{8821.2612}{9854.3036}\right] = 0.04032$$

$$\left(I\bar{A}\right)^{\ 1}_{[40]:\overline{25}|} = 1.06^{0.5}\left(IA\right)^{\ 1}_{[40]:\overline{25}|} = 1.06^{0.5}\left[\left(IA\right)_{[40]} - v^{25}\ {}_{25}p_{\ [40]}\left(25A_{65} + \left(IA\right)_{65}\right)\right]$$

$$= 1.06^{0.5}\left[3.85489 - 0.208574\times\left(25\times 0.40177 + 5.50985\right)\right] = 0.62876$$

EPV of expenses:

$$\begin{aligned}
&= 1.15P + 210 + 0.025\times 12\times P\ddot{a}^{(12)}_{[40]:\overline{25}|} - 0.025P + 85\left[\ddot{a}^{@i'}_{[40]:\overline{25}|} - 1\right] \\
&= 1.15P + 210 + 0.025\times 12\times P\times 12.9273 - 0.025P + 85\times\left[15.887 - 1\right] \\
&= 5.00319P + 1,475.395
\end{aligned}$$

where

$$i' = \frac{1.06}{1+b} - 1 = 0.04$$

Equation of value gives:

$$155.1272P = 31,723.609 + 5.00319P + 1475.395$$

$$\Rightarrow P = \frac{33,199.00}{150.1240} = £221.14$$

Well prepared students completed this question satisfactorily. Others found difficulty in deriving in particular the expense values. Credit was given in part to the correct approach even if the final arithmetic proved to be inaccurate.

13 (i) Let P be the net annual premium. Then:

EPV of premiums:

$$P\ddot{a}_{45:\overline{15}|} = 11.386P$$

EPV of benefits:

$$60000\bar{A}^{1}_{45:\overline{10}|} + 40000\bar{A}^{1}_{45:\overline{15}|}$$

$$= 60000\frac{i}{\delta}\left(A_{45} - v^{10}\,{}_{10}p_{45}A_{55}\right) + 40000\frac{i}{\delta}\left(A_{45} - v^{15}\,{}_{15}p_{45}A_{60}\right)$$

$$= 60000\frac{0.04}{0.039221}\left(0.27605 - 1.04^{-10}\frac{9557.8179}{9801.3123}0.38950\right)$$

$$+\ 40000\frac{0.04}{0.039221}\left(0.27605 - 1.04^{-15}\frac{9287.2164}{9801.3123}0.45640\right)$$

$$= 1190.567 + 1465.406 = £2655.973$$

Equation of value gives:

$$P = \frac{2655.973}{11.386} = £233.27$$

(ii) The net premium reserve at 31.12.13 is given by:

$${}_{10}V_{45:\overline{15}|} = 40000\bar{A}^{1}_{55:\overline{5}|} - 233.27\ddot{a}_{55:\overline{5}|} = 40000\frac{i}{\delta}\left(A_{55} - v^{5}\,{}_{5}p_{55}A_{60}\right) - 233.27\times 4.585$$

$$= 40000\frac{0.04}{0.039221}\left(0.38950 - 1.04^{-5}\frac{9287.2164}{9557.8179}0.4564\right) - 233.27\times 4.585$$

$$= 1019.53 - 1069.54 = -£50.01$$

(iii) Explanation:

Policyholder "in debt" at time 10 (with size of debt equal to the negative reserve) as more life cover provided in the first 10 years than is paid for by the level premiums in those years.

Disadvantages:

If policy is lapsed during first ten years (possibly longer) the company will suffer a loss.

Not possible to recover this loss from policyholder.

Possible alterations:

Collect the premiums more quickly e.g. shorten premium paying term, make premiums larger in earlier years, smaller in later years.

Change the pattern of benefits to reduce benefits in first ten years and increase them in last five years.

(iv) During 2013, we have:

Death strain at risk = 100,000 $(1.04)^{1/2}$ + 50.01 = 102,030.40

EDS = $2878q_{54} \times 102,030.40 = 2878 \times 0.003976 \times 102,030.40 = 1,167,526.52$

ADS = $12 \times 102,030.40 = 1,224,364.80$

Mortality profit = $1,167,526.52 - 1,224,364.80$ = –£56,838.28 (i.e. a loss)

Question done well for students who had prepared. Common errors were in (ii) where immediate payment on death not computed and not getting the profit correct in (iv). Students were given reasonable credit if they showed understanding of the problem even if all arithmetical calculations not correct.

14 (i) Multiple decrement table:

x	q_x^d	q_x^s
58	0.004649	0.1
59	0.006929	0.1
60	0.008022	0.1

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$
58	0.004649	0.09954	0.895816	1.000000
59	0.006929	0.09931	0.893764	0.895816
60	0.008022	0.09920	0.892780	0.800648

Unit fund (per policy at start of year)

	Yr 1	*Yr 2*	*Yr 3*
value of units at start of year	0.00	2206.33	5072.05
allocation	2250.00	2850.00	3450.00
B/O spread	112.50	142.50	172.50
interest	85.50	196.55	333.98
management charge	16.67	38.33	65.12
value of units at end of year	2206.33	5072.05	8618.41

Cash flows (per policy at start of year)

	Yr 1	*Yr 2*	*Yr 3*
unallocated premium	750.00	150.00	–450.00
B/O spread	112.50	142.50	172.50
expenses	425.00	130.00	130.00
interest	8.75	3.25	–8.15
management charge	16.67	38.33	65.12
extra death benefit	31.58	27.22	3.06
profit vector	431.34	176.86	–353.59

profit vector	431.34	176.86	–353.59
probability in force	1.0	0.895816	0.800648
profit signature	431.34	158.43	–283.10
discount factor	0.943396	0.889996	0.839619
expected p.v. of profit	406.92	141.01	–237.69

Total NPV of expected profit = 310.24

	Yr 1	*Yr 2*	*Yr 3*
premium signature	3000.00	2687.45	2401.94
discount factor	1.0	0.943396	0.889996
expected p.v. of premiums	3000.00	2535.33	2137.72

Total PV of premiums = 7673.05

Profit margin = $\dfrac{310.24}{7673.05} = 4.04\%$

(ii) To calculate the expected provisions at the end of each year we have (utilising the end of year cash flow figures and decrement tables in (i) above):

$$_2V = \frac{353.59}{1.02} = 346.66$$

$$_1V \times 1.02 - (ap)_{59} \times {}_2V = -176.86 \Rightarrow {}_1V = 130.36$$

The revised cash flow for year 1 will become:

$$431.34 - (ap)_{58} \times {}_1V = 314.56$$

Hence the table below can now be completed for the revised net present value of expected profit.

	Yr 1	*Yr 2*	*Yr 3*
revised end of year cash flow	314.56	0	0
probability in force	1	0.895816	0.800648
discount factor	0.943396	0.889996	0.839619
expected p.v. of profit	296.76		

Profit margin = $\frac{296.76}{7673.05} = 3.87\%$

Question again done well by students properly prepared. Part (ii) gave more issues as many students could not seem to remember the zeroisation procedure.

Again reasonable credit given for understanding the process where computational errors had occurred.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2015

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chairman of the Board of Examiners
December 2015

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

3. Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate valid points are made which do not appear in the solutions below.

4. In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

5. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonable accurate numerical calculation is necessary.

B. General comments on *student performance in this diet of the examination*

1. The general performance was lower in this session than the exceptionally high result of the April 2015 examination although reasonably in line with earlier sessions.

2. Well prepared students on the whole did very well in this paper in most questions. In general the questions that were done less well were 2, 8 and 12. The examiners hope that the detailed solutions given below will assist students with further revision.

3. Most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well.

4. It is worth repeating that reasonable credit was given if a student could demonstrate on the longer questions that they understood the processes required even if not all computations were accurate.

C. Comparative pass rates for the past 3 years for this diet of examination

Year	*%*
September 2015	51
April 2015	59
September 2014	52
April 2014	52
September 2013	56
April 2013	53

Reasons for any significant change in pass rates in current diet to those in the past:

See B. above.

Generally this paper was deemed to be a similar standard as those in the past except for April 2015 which students found more straightforward than anticipated. Otherwise there is reasonable consistency.

September 2015 was a little lower because of very poor experience in certain overseas centres (others performed to high standard).

Solutions

Q1 (a) $${}_{25}p_{40} = l_{65} / l_{40} = 8821.2612 / 9856.2863 = 0.894988$$

(b) $${}_{10|}q_{[53]} = d_{63} / l_{[53]} = 102.5202 / 9621.1006 = 0.010656$$

(c) $$\bar{a}_{55:\overline{10}|} = (\ddot{a}_{55} - 0.5 - v^{10}(l_{65} / l_{55})(\ddot{a}_{65} - 0.5))$$

$$= 15.873 - 0.5 - 0.67556 \times \left(\frac{8821.2612}{9557.8179}\right) \times (12.276 - 0.5)$$

$$= 8.031$$

(a) and (b) were done well.

There was a surprising poor showing on (c) where the most common error was to assume the answer was to deduct 0.5 from a 10 year life annuity due

Q2 $\Pr\left(T_x \le n\right) = 0.5$

Therefore $e^{-.01*10} \times e^{-.02*10} \times e^{-.03*(n-20)} = 0.5$

$\therefore e^{0.3-0.03n} = .5$

$\Rightarrow -0.3 + 0.03n = -\ln(0.5) = 0.69315$

$\Rightarrow n = \dfrac{0.99315}{.03} = 33.11$

So the total median future lifetime is 33 to nearest whole year

> A very simple question which was poorly done overall. Many students did not appear to know how to start the question.

Q3 (i) An overhead expense is an expense that <u>does not</u> vary with the amount of business written

A direct expense is an expense that <u>does</u> vary with the amount of business written

(ii) (a) <u>Overhead Expense</u>

Central services e.g. premises, IT, legal (allowed for on a per policy per annum basis with allowance for inflation)

<u>Direct Expense</u>

- Underwriting (allowed for on a per policy basis although medical expenses might be sum assured related)
- Processing proposal and issuing policy (allowed for on a per policy basis)
- Initial Commission (allowed for directly and usually premium related)
- Renewal Administration (allowed for on a per policy per annum basis with allowance for inflation)
- Renewal Commission (allowed for directly and usually premium related)

(b) See (a) for how expenses are allowed for (shown in brackets).

> Generally well done. All reasonable descriptions were credited

Q4 (i) Education influences awareness of healthy lifestyle that reduces morbidity and hence mortality

Education includes formal education and public health campaigns

(ii) This manifests itself through many proximate determinants:

- Increased income
- Better diet choices
- Exercise
- Health care
- Moderation in alcohol consumption or smoking
- Awareness of dangers of drug abuse
- Awareness of safe sexual lifestyle

> Generally well done. All reasonable descriptions were credited.

Q5 $\text{EPV} = 5000\ddot{a}_{\overline{5}|}^{(12)} + 6000\,{}_{5|}\ddot{a}_{60}^{(12)} + 1000\,{}_{10|}\ddot{a}_{60}^{(12)}$

$$= 5000 \times \left(\frac{1-v^5}{d^{(12)}}\right) + 6000 \times v^5\,{}_5p_{60}\left(\ddot{a}_{65} - 11/24\right) + 1000 \times v^{10}\,{}_{10}p_{60}\left(\ddot{a}_{70} - 11/24\right)$$

$$= 5000 \times \left(\frac{1-v^5}{0.039157}\right) + 6000v^5 \times \frac{9647.797}{9826.131} \times \left(13.666 - \frac{11}{24}\right)$$

$$+ 1000v^{10} \times \frac{9238.134}{9826.131} \times \left(11.562 - \frac{11}{24}\right)$$

$$= 22738.32 + 63952.31 + 7052.36$$

$$= 93743 \text{ rounded}$$

> Generally well done. However many students gave themselves considerable extra work by valuing a deferred 5 year annuity for 6000 for a 5 year term at 60 and then a deferred 10 year annuity for 7000 at 60. The above approach which relies only on whole life annuities is much easier.

Q6 Using:

$$(aq)_{50}^{d} = \frac{\mu_{50}^{d}}{\mu_{50}^{d}+\mu_{50}^{w}}\left(1-e^{-(\mu_{50}^{d}+\mu_{50}^{w})}\right) = \frac{0.001}{0.151}\left(1-e^{-0.151}\right) = 0.0009282$$

$$(aq)_{50}^{w} = \frac{\mu_{50}^{w}}{\mu_{50}^{d}+\mu_{50}^{w}}\left(1-e^{-(\mu_{50}^{d}+\mu_{50}^{w})}\right) = \frac{0.15}{0.151}\left(1-e^{-0.151}\right) = 0.1392241$$

Construct a multiple decrement table assuming the radix of the table is 100,000 lives.

At age 50:

Number of deaths over year = $100,000 \times (aq)_{50}^{d} = 92.82$

Number of withdrawals over year = $100,000 \times (aq)_{50}^{w} = 13,922.41$

Age	*No of lives*	*No of deaths over year*	*No of withdrawals over year*
50	100,000.00	92.82	13,922.41
51	85,984.77		

At age 51:

$$(aq)_{51}^{d} = \frac{\mu_{51}^{d}}{\mu_{51}^{d}+\mu_{51}^{w}}\left(1-e^{-(\mu_{51}^{d}+\mu_{51}^{w})}\right) = \frac{0.0015}{0.1015}\left(1-e^{-0.1015}\right) = 0.0014264$$

Number of deaths over year = $85,984.77 \times (aq)_{51}^{d} = 122.65$

Probability that a new employee aged 50 exact will die as an employee at age 51 last birthday = 122.65 / 100,000 = 0.00123 i.e. **0.123%**

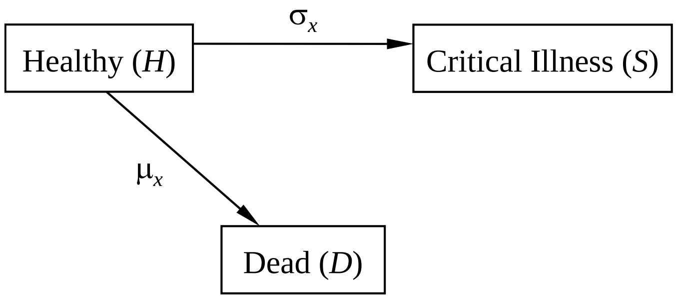
Assumption: The independent forces of mortality and withdrawal are constant over each year of age.

The above solution is a complete analysis. Note the numerical part could also be simply solved directly as follows:

$$(ap)_{50}.(aq)_{51}^{d} = e^{-(0.001+0.15)}.\frac{0.0015}{(0.1+0.0015)}\left(1-e^{-(0.1+0.0015)}\right) = 0.0012265$$

Reasonably done but many students failed to organise the probabilities properly. Full credit was given for the shorter direct approach as long as the assumptions were also stated.

Q7 Firstly we need the transition model:



The appropriate expression is:

$$EPV = \int_0^\infty 100,000\, e^{-\delta t}\, {}_t p_x^{HH} \left(\sigma_{x+t} + \mu_{x+t}\right) dt$$

Or

$$EPV = \int_0^\infty 100,000\, e^{-\delta t}\, {}_t(ap)_x \left((a\mu)_{x+t}^S + (a\mu)_{x+t}^D\right) dt$$

> A very easy question generally well done. Students who drew a transition line from (*S)* to (*D)* were penalised as that did not form part of the benefit structure.

Q8 $a_{73.25}^{(4)} = 0.25\,{}_{0.25}p_{73.25}v^{0.25} + 0.25\,{}_{0.5}p_{73.25}v^{0.5} + \ddot{a}_{74}^{(4)}\,{}_{0.75}p_{73.25}v^{0.75}$

Assuming a constant force of mortality between ages 73 and 74 we are required to solve for the constant μ not using μ_{73}

$$p_{73} = 1 - q_{73} = 1 - 0.014973 = 0.985027 = e^{-\mu} \text{ hence } \mu = \ln(.985027) = 0.015086$$

$${}_{0.25}p_{73.25} = e^{-0.25\times 0.015086} = 0.996236$$

$${}_{0.5}p_{73.25} = e^{-0.5\times 0.015086} = 0.992485$$

$${}_{0.75}p_{73.25} = e^{-0.75\times 0.015086} = 0.988749$$

$$\ddot{a}_{74}^{(4)} = \ddot{a}_{74} - \frac{3}{8} = 11.333 - 0.375 = 10.958$$

Hence

$$a^{(4)}_{73.25} = 0.25 \times 0.996236 v^{0.25} + 0.25 \times 0.992485 v^{0.5} + 10.958 \times 0.988749 v^{0.75}$$

$$= 11.011$$

> A question which combined a non integer age annuity using a constant force of mortality with a whole life constituent also. The question was very poorly done.
>
> It is also possible to start with $a^{(4)}_{73}$ and deduct off the first quarter.

Q9 (i)

$$\frac{25,000}{60}\left[\frac{10\left({}^{z}M^{ia}_{45} + {}^{z}M^{ra}_{45}\right) + \left({}^{z}\overline{R}^{ia}_{45} + {}^{z}\overline{R}^{ra}_{45}\right)}{s_{44}D_{45}}\right]$$

$$= \frac{25,000}{60}\left[\frac{10(52554 + 128026) + (609826 + 2244130)}{8.375 \times 2329}\right]$$

= 99,540

(ii)

$$5\% \times 25,000 \times \left[\frac{{}^{s}\overline{N}_{45}}{s_{44}D_{45}}\right]$$

$$= 5\% \times 25,000 \times \left[\frac{253080}{8.375 \times 2329}\right]$$

= 16,219

> An easy question generally very well done if students had prepared.

Q10 (i) The area comparability factor is the ratio of the crude mortality rate in a standard population to the crude mortality rate of a sub-population, if that sub-population exhibited standard mortality.

(ii)

	Country A			*Area N*		
Age	*Population*	*Number of deaths*	*Mortality rate*	*Population*	*Actual deaths*	*Expected deaths*
60	100,235	566	0.005647	25,366	125	143
61	95,666	621	0.006491	22,159	121	144
62	92,386	635	0.006873	21,864	135	150
Total	288,287	1,822		69,389	381	437

The area comparability factor = ((1,822/288,287) / (437/69,389)) = 1.0027 (after rounding deaths to 1 decimal place)

(iii) The directly standardised mortality rate for Area N is

(100,235 * 125/25,366 + 95,666 * 121 / 22,159
+ 92,386 * 135 / 21,864) / (100,235 + 95,666 + 92,386) = 0.0055

Note that this question is sensitive to rounding.

Generally straightforward and well done.

Q11 (i) Using the premium conversion relationship:

$$\text{Value} = 10,000 \times \bar{A}_{\overline{55:50}}$$

$$= 10,000 \times (1.04)^{1/2} \times \left(1 - \frac{.04}{1.04} \times \ddot{a}_{\overline{55:50}}\right)$$

$$= 10,000 \times (1.04)^{1/2} \times \left(1 - \frac{.04}{1.04} \times \left(\ddot{a}_{55} + \ddot{a}_{50} - \ddot{a}_{55:50}\right)\right)$$

$$= 10,000 \times (1.04)^{1/2} \times \left(1 - \frac{.04}{1.04} \times (17.364 + 19.539 - 16.602)\right)$$

$$= 2,235$$

(ii) Let the status $u = \overline{x:y}$

Then $\bar{A}_u = \int_0^\infty v^t \mu_{u+t} dt$

The second moment is $\int_0^\infty \left(v^t\right)^2 \mu_{u+t} dt = {}^2\bar{A}_u$

Assuming the two lives are independent then the variance is

$$(10{,}000)^2 \left({}^2\bar{A}_u\right) - (10{,}000\bar{A}_u)^2$$

Alternatively:

$$\operatorname{var}\left[10{,}000\, v^{T_{\overline{55:50}}}\right] = 10{,}000^2 \left\{ E\left[\left(v^{T_{\overline{55:50}}}\right)^2\right] - \left(E\left[v^{T_{\overline{55:50}}}\right]\right)^2 \right\}$$

$$= 10{,}000^2 \left\{ E\left[\left(v^2\right)^{T_{\overline{55:50}}}\right] - \left(\bar{A}_{\overline{55:50}}\right)^2 \right\}$$

$$= 10{,}000^2 \left\{ {}^2\bar{A}_{\overline{55:50}} - \left(\bar{A}_{\overline{55:50}}\right)^2 \right\}$$

Part (i) was done well but part (ii) gave difficulties.

It should be noted that there was a small omission in the question wording. The basis should also, of course, have included the male single mortality table. Most students gave the correct answer in any event but any student using female mortality throughout for the single life function was given full credit.

Q12

Annual premium	6000.00	Allocation % (1st yr)	98.0%
Risk discount rate	6.0%	Allocation % (2nd yr)	98.0%
Interest on investments (1st yr)	5.0%	B/O spread	6.0%
Interest on investments (2nd yr)	4.5%	Management charge	1.25%
Interest on non-unit funds (1st and 2nd yrs)	3.0%	Policy Fee	£50
Death benefit (% of bid value of units)	200%		

		% premium
Initial expense/commission	225	7.5%
Renewal expense/commission	80	2.5%
Death claim expense	90	
Maturity claim expense	55	

Mortality table:

x	t	$q^{d}_{[x]+t-1}$	$q^{s}_{[x]+t-1}$	$(aq)^{d}_{[x]+t-1}$	$(aq)^{s}_{[x]+t-1}$	$(ap)_{[x]+t-1}$	${}_{t-1}(ap)_{[x]}$
45	1	0.001201	0.02500	0.001201	0.02497	0.973829	1.000000
46	2	0.001557	0.00000	0.001557	0.00000	0.998443	0.973829

Unit fund (per policy at start of year)

	yr 1	*yr 2*
value of units at start of year	0.000	5679.172
alloc	5880.000	5880.000
B/O	–352.80	–352.800
policy fee	–50.000	–50.000
interest	273.860	502.037
management charge	–71.888	–145.730
value of units at year end	5679.172	11512.678

Non Unit fund cash flows (per policy at start of year)

	yr 1	*yr 2*
unallocated premium + policy fee	170.000	170.000
b/o spread	352.800	352.800
expenses	–675.000	–230.000
interest	–4.566	8.784
man charge	71.888	145.730
extra death benefit	-6.821	–17.925
surrender penalty	12.485	0.000
claim expense (death/maturity)	–0.108	–55.054
end of year cash flow	–79.322	374.335

(i) (a) if p/h dies in the 1st year of contract, non unit cash flows at end of the year are:

$yr1 = (170 + 352.80 - 675 - 4.566 + 71.888 - 5679.172 - 90) = -5854.050$

(b) if p/h surrenders in the 1st year of contract, non unit cash flows at end of the year are:

$yr1 = (170 + 352.80 - 675 - 4.566 + 71.888 + 500) = 415.122$

(c) if p/h dies in the 2nd year of contract, non unit cash flows at end of each year are:

$$yr1 = (170 + 352.80 - 675 - 4.566 + 71.888) = -84.878$$

$$yr\ 2 = (170 + 352.8 - 230 + 8.784 + 145.73 - 11512.678 - 90) = -11155.364$$

(d) if p/h survives to the end of the contract, non unit cash flows at end of each year are:

$yr\ 1 = -84.878$ (derived above)

$$yr\ 2 = (170 + 352.8 - 230 + 8.784 + 145.73 - 55) = 392.314$$

(ii) (a) if p/h dies in the 1st year of contract, expected present value of profit is given by:

$$-5854.050 \times v \times (aq)^d_{[45]} = -5522.689 \times 0.001201 = -6.633$$

(b) if p/h surrenders in the 1st year of contract, expected present value of profit is given by:

$$415.122 \times v \times (aq)^s_{[45]} = 391.624 \times 0.02497 = 9.779$$

(c) if p/h dies in the 2nd year of contract, expected present value of profit is given by:

$$\left[-84.878 \times v - 11155.364 \times v^2\right] \times (ap)_{[45]} \times (aq)^d_{[45]+1}$$

$$= [-80.074 - 9928.234] \times 0.973829 \times 0.001557 = -15.175$$

(d) if p/h survives to the end of the contract, expected present value of profit is given by:

$$\left[-84.878 \times v + 392.314 \times v^2\right] \times {}_2(ap)_{[45]}$$

$$= [-80.074 + 349.158] \times 0.973829 \times 0.998443 = 261.634$$

(iii) Expected present value of the profit of the policy is therefore

= –6.633 + 9.779 – 15.175 + 261.634 = **249.605**

> This question proved to be the most difficult on the paper and was in general poorly done. In essence the question was about breaking the final Present Value of Future Profits down into constituent parts which would need to be carried out in any event and each part in itself is relatively straightforward.
>
> Reasonable partial credit was given if a good understanding was shown without the calculations being fully accurate.

Q13 (i) Let P be the monthly premium for the contract. Then:

EPV of premiums is:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25}|} = 12P\left[\ddot{a}_{[40]:\overline{25}|} - \frac{11}{24}\left(1 - v^{25}\,{}_{25}p_{[40]}\right)\right]$$

$$= 12P\left[15.887 - \frac{11}{24}\left(1 - 0.37512 \times \frac{8821.2612}{9854.3036}\right)\right]$$

$$= 186.9909P$$

EPV of death benefits:

$$260,000\bar{A}^{1}_{[40]:\overline{25}|} - 10,000\left(I\bar{A}\right)^{1}_{[40]:\overline{25}|} = 10,000 \times (1.04)^{0.5}\left[26A^{1}_{[40]:\overline{25}|} - (IA)^{1}_{[40]:\overline{25}|}\right]$$

$$= 10198.04\left[26 \times 0.05316 - 0.87602\right] = 5161.64$$

where

$$A^{1}_{[40]:\overline{25}|} = A_{[40]:\overline{25}|} - v^{25}\,{}_{25}p_{[40]} = 0.38896 - 0.33580 = 0.05316$$

and

$$(IA)^{1}_{[40]:\overline{25}|} = (IA)_{[40]} - v^{25}\,{}_{25}p_{[40]}\left[25A_{65} + (IA)_{65}\right]$$

$$= 7.95835 - 0.33580\left[25 \times 0.52786 + 7.89442\right] = 0.87602$$

EPV of annuity:

$$v^{25}\,{}_{25}p_{[40]}\left[28500\ddot{a}_{65}+1500\left(I\ddot{a}\right)_{65}\right]$$

$$=0.33580\left[28500\times 12.276+1500\times 113.911\right]=174{,}861.97$$

EPV of expenses:

(a) Death claim

$$275\left[1.04^{0.5}q_{[40]}v^{0.5}+1.04^{1.5}\,p_{[40]}q_{[40]+1}v^{1.5}+.....+1.04^{24.5}\,{}_{24}p_{[40]}q_{64}v^{24.5}\right]$$

$$=275\times {}_{25}q_{[40]}=275\left(1-0.895168\right)=28.83$$

(b) Annuity

0.025 × *EPV* of annuity = 4,371.55

(c) Premium related

$$0.35\times 12P+0.05\times 12P\left[\ddot{a}^{(12)}_{[40]:\overline{25}|}-\frac{1}{12}\right]=4.2P+0.6P\times\left(15.5826-0.08333\right)$$

$$=13.49956P$$

(d) Other

$$225+55\left(\ddot{a}^{@0\%}_{[40]:\overline{25}|}-1\right)=225+55\left(e_{[40]}-\frac{l_{65}}{l_{[40]}}\left(1+e_{65}\right)\right)$$

$$=225+55\left(39.071-\frac{8821.2612}{9854.3036}\times 17.645\right)=1505.17$$

Equation of value gives:

$$186.9909P=5161.64+174861.97+28.83+4371.55$$
$$+13.49956P+1505.17$$

$$\Rightarrow 173.49134P=185929.16$$

$$\Rightarrow P=£1071.69$$

A typical CT5 question, well done by prepared students.

The only real uncertainty was treatment of the death claim expenses.

Again reasonable partial credit was given for understanding without full computational accuracy.

Q14 (i) The death strain at risk for a policy for year $t + 1$ ($t = 0, 1, 2\ldots$) is the excess of the sum assured (i.e. the present value at time $t + 1$ of all benefits payable on death during the year $t + 1$) over the end of year reserve and any benefit payable if the life survives to the end of year $t + 1$.

i.e. DSAR for year $t + 1 = S - \left({}_{t+1}V + R\right)$

(ii) Annual premium for pure endowment with £75,000 sum assured given by:

$$P^{PE} = \frac{75,000}{\ddot{a}_{55:\overline{5}|}} \times v^5 \times {}_5p_{55} = \frac{75,000}{4.585} \times 0.82193 \times \frac{9287.2164}{9557.8179} = 13,064.223$$

Annual premium for term assurance with £75,000 sum assured given by:

$$P^{TA} = P^{EA} - P^{PE} = \frac{75,000 A_{55:\overline{5}|}}{\ddot{a}_{55:\overline{5}|}} - P^{PE}$$

$$= \frac{75,000 \times 0.82365}{4.585} - 13,064.223 = 408.786$$

Reserves at the end of the fourth policy year:

for pure endowment with £75,000 sum assured given by:

$${}_4V^{PE} = 75,000 \times v \times {}_1p_{59} - P^{PE}\ddot{a}_{59:\overline{1}|}$$

$$= 75,000 \times 0.96154 \times \frac{9287.2164}{9354.0040} - 13,064.223 = 58,536.372$$

for term assurance with £75,000 sum assured given by:

$${}_4V^{TA} = {}_4V^{EA} - {}_4V^{PE}$$

$$= 75,000 A_{59:\overline{1}|} - (13,064.223 + 408.786)\ddot{a}_{59:\overline{1}|} - 58,536.372$$

$$= 75,000 \times 0.96154 - (13,064.223 + 408.786) \times 1 - 58,536.372 = 106.119$$

for temporary immediate annuity paying an annual benefit of £15,000 given by:

$$ {}_4V^{IA} = 15,000 a_{\overline{59:1}|} $$

$$ = 15,000 \times v \times {}_1p_{59} = 15,000 \times 0.96154 \times \frac{9826.131}{9846.908} $$

$$ = 14,392.644 $$

Death strain at risk per policy:

Pure endowment: DSAR = 0 − 58,536.372 = −58,536.372

Term assurance: DSAR = 75,000 – 106.119 = 74,893.881

Immediate annuity: DSAR = 0 − (14,392.644 + 15,000) = −29,392.644

(iii) Mortality profit = EDS – ADS

For pure endowment

$$ EDS = 984 \times q_{58} \times -58,536.372 = 984 \times .006352 \times -58,536.372 = -365,873.866 $$

$$ ADS = 5 \times -58,536.372 = -292,681.86 $$

mortality profit = −73,192.00

For term assurance

$$ EDS = 3950 \times q_{58} \times 74,893.881 = 3950 \times .006352 \times 74,893.881 = 1,879,117.432 $$

$$ ADS = 22 \times 74,893.881 = 1,647,665.382 $$

mortality profit = 231,452.05

For temporary immediate annuity

$$ EDS = 495 \times q_{58} \times -29,392.644 = 495 \times .001814 \times -29,392.644 = -26,391.746 $$

$$ ADS = 2 \times -29,392.644 = -58,785.288 $$

mortality profit = 32,393.54

Hence, total mortality profit = −73,192.00 + 231,452.05 + 32,393.54

= £190,653.59

Another typical CT5 question which was well done by prepared students. Again reasonable partial credit was given for understanding without full computational accuracy.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2016 (with mark allocations)

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chair of the Board of Examiners
June 2016

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

3. Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate valid points are made which do not appear in the solutions below.

4. In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by the Examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

5. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonably accurate numerical calculation is necessary.

B. General comments on *student performance in this diet of the examination*

1. The general performance was slightly lower than usual this session compared to previous ones although it was felt that this paper was roughly of the same standard..

2. Many well prepared students gained very high marks but there were some concerns that some students had just not prepared for the examination satisfactorily and scored very minimal marks overall

3. Questions that were done less well were 4, 9, 11 and 12 part (ii). The Examiners hope that the detailed solutions given below will assist students with further revision.

4. However most of the short questions 1–8 and 10 were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well.

C. Pass Mark

The Pass Mark for this exam was 60%.

Solutions

Q1 $${}_{0.5}p_{90.25} = 1 - \frac{0.5q_{90}}{1-0.25q_{90}}$$

$$= 1 - \frac{0.5 \times 0.20465}{1-(0.25 \times 0.20465)}$$

$$= 0.892158$$

[2 for formula, 1 for result]
[TOTAL 3]

A very straightforward question which was generally done very well.

Q2 (i) The net premium retrospective reserve will be equal to the net premium prospective reserve if:

The retrospective and prospective reserves are calculated on the same basis [1]
and,
This basis is the same as the basis used to calculate the premiums used in the reserve calculation. [1]

(ii) In practice these conditions rarely hold since:

The assumptions which are appropriate for the retrospective calculation (based on the experienced conditions over the duration of the contract up to the valuation date) are not generally appropriate for the prospective calculation (based on assumptions considered suitable for the remainder of the term) [1]
and,
The assumptions considered appropriate at the time the premium was calculated may not be appropriate for the retrospective or prospective reserves some years later [1]
[TOTAL 4]

A bookwork question answered well by students who had prepared satisfactorily for the examination.

Q3 (a) $${}_{25}p_{30} = \frac{l_{55}}{l_{30}} = \frac{9557.8179}{9925.2094} = 0.962984$$ [½]

(b) $$\ddot{a}^{(4)}_{[40]:\overline{15|}} = \ddot{a}^{(4)}_{[40]} - v^{15} \times \frac{l_{55}}{l_{[40]}} \times \ddot{a}^{(4)}_{55} = \ddot{a}_{[40]} - 0.375 - v^{15} \times \frac{l_{55}}{l_{[40]}} \times (\ddot{a}_{55} - 0.375)$$

$$= 19.634 - 0.55526 \times \frac{9557.8179}{9854.3036} \times 15.498 = 11.287$$ [2]

(c) $$A^{1}_{50:\overline{20|}} = A_{50} - v^{20} \times \frac{l_{70}}{l_{50}} \times A_{70}$$

$$= 0.32907 - 0.45639 \times \frac{8054.0544}{9712.0728} \times 0.60097 = 0.10162$$ [1½]

[TOTAL 4]

Generally well done. The main issue was with part (ii) where often students did not perform the quarterly adjustment properly.

Q4 For those currently paying contributions the decrements of interest are death, withdrawal and retirement. For those receiving benefit or entitled to a deferred benefit the only decrement of interest is death.

The mortality of those who retired early (but in good health) or at normal retirement age is likely to be lower than that of ill-health retirement pensioners. This is an example of class selection. [1]

The mortality of ill-health retirement pensioners is likely to depend on duration since retirement for a few years following the date of retirement, and subsequently only on age attained. This is an example of temporary initial selection. [1]

Underwriting at the date of joining a scheme tends to be very limited, e.g. actively at work, and so there tends to be only very slight temporary initial selection. [1]

Different sections of a large scheme, e.g. works and staff, may exhibit different levels of mortality. This is an example of class selection. [1]

Among the active members of the scheme ill-health retirement acts as a selective decrement, resulting in lighter mortality among the remaining active members. This is sometimes termed the "healthy worker" or the "active lives mortality" effect. [1]

Withdrawal from a scheme is associated with voluntary or compulsory termination of employment (changing jobs or redundancy). If voluntary resignation is the cause this tends to select those with lighter mortality (and ill-health retirement) rates. If redundancy is the cause withdrawal rates tend to vary markedly over time as economic conditions vary. This is an example of time selection. [1]

Marks were awarded for additional examples, e.g. class selection between males and females, or time selection of the mortality rates.

[MAX 6]

All other reasonable comments were credited. In general this question was poorly answered. Many students described the selection processes in the abstract without properly relating them to a pension scheme environment. To score well there needed to be a strong linkage demonstrated between the various types of selection and the operation of such schemes.

Q5 (i)

Age	*Population*	*Number of deaths*	*Standard q_x*	*Expected deaths*
60	9,950	52	0.01392	138.50
61	8,020	68	0.01560	125.11
62	6,997	73	0.01749	122.38
Total	24,967	193		385.99

[2 for whole table]

$$\text{The SMR} = \frac{\sum_x E^c_{x,t}\, m_{x,t}}{\sum_x E^c_{x,t}\; {}^s m_{x,t}} = \frac{\text{Actual deaths in population}}{\text{Expected deaths in population}}$$

$= 193/385.99 = 0.500$ [2]

[Total 4]

(ii) An SMR less than 1 indicates a population with mortality lighter than that in the standard population, allowing for the distribution by age and sex in the observed population. [1]

A value of 0.5 indicates that population has half the number of expected deaths. [1]

[Total 2]

[TOTAL 6]

Part (i) was very straightforward and generally done well.

Often however students gave only one of the statements in part (ii), usually the former.

Q6 (i)

$$\bar{A}_{x:\overline{n}|} = \int_0^n \mu e^{-(\mu+\partial)t}dt + e^{-(\mu+\partial)n} = \mu\bar{a}_{x:\overline{n}|} + e^{-(\mu+\partial)n}$$

But $\bar{a}_{x:\overline{n}|} = \int_0^n e^{-(\mu+\partial)t}dt = \dfrac{1-e^{-(\mu+\partial)n}}{\mu+\partial}$ so $e^{-(\mu+\partial)n} = 1-(\mu+\partial)\bar{a}_{x:\overline{n}|}$

Thus $\bar{A}_{x:\overline{n}|} = \mu\bar{a}_{x:\overline{n}|} + 1-(\mu+\partial)\bar{a}_{x:\overline{n}|} = 1-\partial\bar{a}_{x:\overline{n}|}$ as required

[1 mark for each line]
[Max 3]

(ii) In this case we need to calculate $10000\bar{P}_{40:\overline{20}|}$.

From the formula given in (i) by dividing throughout by $\bar{a}_{x:\overline{n}|}$ we can deduce for age and term that:

$$\bar{P}_{40:\overline{20}|} = \frac{1}{\bar{a}_{40:\overline{20}|}} - \partial$$

In this case $\mu = .01$ and $\partial = \ln(1.05) = .048790$.

Thus $10000\bar{P}_{40:\overline{20}|} = 10000\times\left[\dfrac{.01+.048790}{1-e^{-20(.01+.048790)}} - .048790\right]$

$$= 10000\times\left[\frac{.058790}{.691428} - .048790\right]$$

$= £362.4$

[1 for line 3, ½ for line 4, 2 for line 5 and ½ for result]
[Max 4]
[TOTAL 7]

Part (i) overall was poorly done. The most common error was to omit the $e^{-(\mu+\partial)n}$ term which of course leaves a term assurance function and not an endowment.

Although the question asked for a proof using a constant force of mortality some students offered a generalised proof involving random variables. If done correctly this method was credited.

Part (ii) was better answered.

Q7 (i) As this unit linked policy produces negative cash flows after the initial funding, these can be zeroised by establishing reserves from earlier cash flows. [1]

(ii) It is prudent that once sold and funded at outset, a policy should be self-supporting financially. This implies that the profit signature has a single negative value (funds are provided by the insurance company) at policy duration zero. [1]

(iii) To calculate the expected reserves at the end of each year we have (utilising the end of year cash flow figures):

$$p_{63} = 0.988656 \quad p_{62} = 0.989888 \quad p_{61} = 0.990991 \quad p_{60} = 0.991978$$
$$\Rightarrow {}_4p_{60} = 0.962062$$

$${}_3V = \frac{192.05}{1.035} = 185.556$$
$${}_2V \times 1.035 - p_{62} \times {}_3V = 267.57 \Rightarrow {}_2V = 435.990$$
$${}_1V \times 1.035 - p_{61} \times {}_2V = 321.06 \Rightarrow {}_1V = 727.654$$

[3]

The revised cash flow for year 1 will become:

$$751.25 - p_{60} \times 727.654 = 29.433$$

[1]

Revised profit vector becomes (29.43, 0, 0, 0, 201.75) and

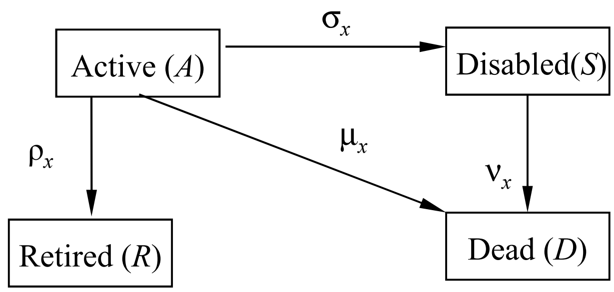
Net present value of profits $= \dfrac{29.433}{1.06} + {}_4p_{60} \times \dfrac{201.75}{1.06^5} = 27.77 + 145.04 = 172.81$

[1]

[TOTAL 7]

This straightforward bookwork and application question was generally well done.

Q8 (i)



[Max 4]

(ii) The expected present value is given by the following expression:

$$20{,}000 \times 0.5 \times \int_0^{65-x} \frac{s_{x+t}}{s_x} e^{-\delta t}\, {}_tp_x^{AA}\, \sigma_{x+t}\, a_{x+t}^*\, dt \qquad [1]$$

with s_{x+t} being proportionate to the current annual rate of salary at exact age $x+t$,

$${}_tp_x^{AA} = \exp\left[-\int_{r=0}^{t} \left(\rho_{x+r} + \mu_{x+r} + \sigma_{x+r}\right) dr \right]$$

Where the integral is limited to the normal retirement age of 65. [½]

${}_tp_x^{AS} = e^{-\int_0^t {}_sp_x^{AA} \sigma_{x+s} ds}$ is the probability that an Active life age x is Disabled at age $x + t$ [1]

${}_tp_x^{AA}$ is the probability that an Active life age x is still Active at age $x + t$ [½]

a_{x+t}^* is the expected present value at age $x + t$ of an annuity to a disabled life [½]

$\frac{s_{x+t}}{s_x}$ denotes the increase in salary for the period from age x to age $x + t$ [½]

Alternatively

$$\frac{20{,}000 \times 0.5}{s_{x-½}\, l_x} \times \left[s_x\, v^{½}\, \bar{a}^i_{x+½}\, i_x + s_{x+1}\, v^{1½}\, \bar{a}^i_{x+1½}\, i_{x+1} + \cdots + s_{64}\, v^{64½-x}\, \bar{a}^i_{64½}\, i_{64} \right]$$

is acceptable in pension summation form

[Max 3]
[TOTAL 7]

> Most students offered satisfactory diagrams and had some idea about the correct formulae. Most marks were lost for not specifying the bases.

Q9 Firstly calculate μ^d for each age from q_x as it is the constant force required.

For age 63-64 $u^d = -\ln p_{63} = -\ln(1-q_{63}) = 0.01985$

For age 64-65 $u^d = -\ln p_{64} = -\ln(1-q_{64}) = 0.02224$

$$(aq)^s_x = \frac{\sigma}{\mu+\sigma}\left(1-e^{-(\mu+\sigma)}\right)$$

$$(aq)^d_x = \frac{\mu}{\mu+\sigma}\left(1-e^{-(\mu+\sigma)}\right)$$

[2]

Age	*t*	μ	σ	$(aq)^d_x$	$(aq)^s_x$	$_{t-1}(ap)_x$
63	1	0.01985	0.03	0.019363	0.029265	1
64	2	0.02224	0.03	0.021669	0.029230	0.951372

[2 for multiple decrements, ½ for $(ap)_x$]

Assume benefits payable uniformly through year of age so discount factors = $v^{t+½}$ [1]

Age	$_{t-1}(ap)_x$	*Salary*	*Benefit*	*Discount factors*	*Present value*
63	1	50,000	250,000	0.975900	7,140
64	0.951372	51,500	257,500	0.929429	6,655
					13,795

[2 for complete table, ½ for result]
[TOTAL 8]

> This rather challenging question was poorly done overall. Most students did not realise that an equivalent constant force of mortality needed to be first calculated and used the varying force instead .

> Some students used the old formula approach $(aq)^d = q^d \times (1 - \frac{1}{2}q^s)$ etc. which is no longer used in this course. The Examiners decided to allow this approach.

Q10 Value of lump sum death benefit

$$50000\bar{A}^1_{55:\overline{10}|} = 50000 \times (1.04)^{0.5} \times A^1_{55:\overline{10}|}$$

$$A^1_{55:\overline{10}|} = A_{55} - v^{10} \times \frac{l_{65}}{l_{55}} \times A_{65}$$

As

$$A_x = 1 - d\ddot{a}_x$$

then

$$A_{65} = 1 - \frac{.04}{1.04} \times 13.666 = 0.474385$$

Hence value of death benefits $= 50000 \times (1.04)^{0.5} \times \left(0.332154 - v^{10} \times \frac{9647.797}{9904.805} \times 0.474385\right)$

$= 1019.4$ [3]

Value of survival benefit

Let P be the annual premium, then value is:

$$0.25 \times 5 \times P \times v^{10} \times {}_{10}p_{55}$$

$$= 1.25 \times P \times v^{10} \times \frac{9647.797}{9904.805}$$

$= .82254P$ [1½]

Value of reversionary annuity

$$5000\ddot{a}^{(12)}_{50|55} = 5000(\ddot{a}^{f}_{50} - \ddot{a}^{f\ m}_{50:55})$$

$$= 5000\times(19.539-16.602)$$

$$= 14685 \qquad [2]$$

Value of premiums

$$P\ddot{a}_{55:\overline{5}|} = P\left(\ddot{a}_{55} - v^5 \times {}_5p_{55} \times \ddot{a}_{60}\right)$$

$$= P\left(17.364 - v^5 \times \frac{9826.131}{9904.805} \times 15.632\right)$$

$$= 4.61769P \qquad [2]$$

Equation of value is

$$4.61769P = 1019.4 + 0.82254P + 14685$$

$$\Rightarrow P = 4138 \qquad [½ \text{ for result}]$$

[TOTAL 9]

> This fairly standard style premium valuation question was well done by fully prepared students.

Q11 Reserves at the end of the 3rd policy year:

- Where both lives are alive:

$$75000\times1.04^{0.5}\times\left(1-\frac{\ddot{a}_{\overline{68^m:63^f}}}{\ddot{a}_{\overline{65^m:60^f}}}\right)$$

$$= 75000\times1.04^{0.5}\times\left(1-\frac{12.412+15.606-11.372}{13.666+16.652-12.682}\right) = 4293.52 \qquad [1½]$$

- Where the male life is alive only:

$$75000\bar{A}_{68^m} - P\ddot{a}_{68^m}$$

$$75000\times1.04^{0.5}\times\left(1-\frac{0.04}{1.04}\times12.412\right) - 1395.11\times12.412 = 22656.29 \qquad [1]$$

- Where the female life is alive only:

$$75000\bar{A}_{63^f} - P\ddot{a}_{63^f}$$

$$75000 \times 1.04^{0.5} \times \left(1 - \frac{0.04}{1.04} \times 15.606\right) - 1395.11 \times 15.606 = 8804.38$$ [1]

Mortality Profit = Expected Death Strain – Actual Death Strain

(a) Both lives die during 2014 = 2 actual claims.

Mortality Profit

$$= \left(5997 \times q_{67^m} \times q_{62^f} - 2\right) \times \left(75000 \times 1.04^{0.5} - 4293.52\right)$$
$$= (5997 \times 0.008439 \times 0.002885 - 2) \times (72191.77) = -133843.10$$ [2]

(b) Males only die during 2014 = 12 actual deaths (and therefore we need to change reserve from joint life to female only surviving).

Mortality Profit

$$= \left(5997 \times p_{62^f} \times q_{67^m} - 12\right) \times (8804.38 - 4293.52)$$
$$= (5997 \times 0.997115 \times 0.008439 - 12) \times (4510.86) = 173499.75$$ [2]

(c) Females only die during 2014 = 8 actual deaths (and therefore we need to change reserve from joint life to male only surviving).

Mortality Profit

$$= \left(5997 \times p_{67^m} \times q_{62^f} - 8\right) \times (22656.29 - 4293.52)$$
$$= (5997 \times 0.991561 \times 0.002885 - 8) \times (18362.77) = 168117.38$$ [2]

Hence overall total mortality profit

$$= -133843.10 + 173499.75 + 168117.38 = £207774.03$$ [½]

[TOTAL 10]

This question proved to be the most challenging on the paper and was generally not answered well.

The most common error was to use only the situation where both lives were alive thus ignoring the other two states.

Also many students failed to calculate the reserves correctly.

Credit was given for the correct method where calculations were inaccurate.

Q12 (i) Let P be the monthly premium for the contract. Then:

EPV of premiums (valued at 6%) is:

$$12P\ddot{a}^{(12)}_{[45]:\overline{40|}} = 170.928P$$

where

$$\ddot{a}^{(12)}_{[45]:\overline{40|}} = \ddot{a}^{(12)}_{[45]} - v^{40}\,{}_{40}p_{[45]}\ddot{a}^{(12)}_{85} = \left(\ddot{a}_{[45]} - \frac{11}{24}\right) - v^{40}\,{}_{40}p_{[45]}\left(\ddot{a}_{85} - \frac{11}{24}\right)$$

$$= \left[(14.855 - 0.458) - 0.09722 \times \frac{3385.2479}{9798.0837} \times (4.998 - 0.458)\right] = [14.397 - 0.153] = 14.244$$

[2]

EPV of death benefits:

$$150,000\bar{A}_{[45]} @ i' = 42,597.65$$

where

$$\bar{A}_{[45]} = (1.06)^{0.5} \times A^{@i'\%}_{[45]} = 1.029563 \times 0.27583$$ [2]

EPV of expenses:

$$0.65 \times 12P + 0.05 \times 12P\ddot{a}^{6\%}_{[45]:\overline{40|}} = 7.8P + 0.6P \times 14.687 = 16.6122P$$

where:

$$\ddot{a}_{[45]:\overline{40|}} = \ddot{a}_{[45]} - v^{40}\,{}_{40}p_{[45]}\ddot{a}_{85}$$

$$= \left[14.855 - 0.09722 \times \frac{3385.2479}{9798.0837} \times 4.998\right] = 14.687$$ [2]

Equation of value gives:

$$170.928P = 42,597.65 + 16.6122P$$

$$\Rightarrow P = £276.04$$ [1]

[Total 7]

(ii) Sum assured and attaching bonuses at 1 March 2015
$= 150{,}000(1.02)^{18} = 214{,}236.94$ [½]

Gross prospective policy reserve immediately before alteration is given by (valued at 6%):

$$214{,}236.94\bar{A}_{63} - 276.04 \times 12\ddot{a}^{(12)}_{63:\overline{22|}} = 48{,}078.29$$

where

$$\bar{A}_{63} = (1.06)^{0.5} \times A_{63} = 1.029563 \times 0.37091 = 0.38188$$

$$\ddot{a}^{(12)}_{63:\overline{22|}} = \ddot{a}^{(12)}_{63} - v^{22}\,{}_{22}p_{63}\ddot{a}^{(12)}_{85} = \left(\ddot{a}_{63} - \frac{11}{24}\right) - v^{22}\,{}_{22}p_{63}\left(\ddot{a}_{85} - \frac{11}{24}\right)$$

$$= \left[(11.114 - 0.458) - 0.27751 \times \frac{3385.2479}{9037.3973} \times 4.998 - 0.458\right]$$

$$= [10.656 - 0.4719]$$

$$= 10.1841$$ [3]

Let S be the revised sum assured after alteration. Value of gross prospective policy reserve immediately after alteration (valued at 6%) is given by:

$$S\bar{A}_{63} = S \times 0.38188$$ [1]

Allowing for cost of alteration, and equating reserves before and after alteration, we have:

$$0.38188S + 175 = 48{,}078.29$$

$$\Rightarrow S = 125{,}440.69$$

i.e. the sum assured is reduced to £125,441 [1½]

[Total 6]

[TOTAL 13]

Part (i) was generally done well.

Part (ii) caused greater problems. In many cases students did not understand the process of equating before and after reserves so were not able to develop a satisfactory solution.

Q13 (i) If P is the annual office premium, the gross future loss random variable ($GFLRV$)

$$= v^{K_{[56]}+1}(140,000 + 20,000 \times K_{[56]}) + 275 + 55a_{\overline{K_{[56]}}|} - P\left(.975\ddot{a}_{\overline{K_{[56]}+1}|} - 0.275\right)$$

for $K_{[56]} < 4$

or

$$= 275 + 55a_{\overline{3}|} + 0.5 \times 4Pv^4 - P\left(.975\ddot{a}_{\overline{4}|} - 0.275\right)$$

for $K_{[56]} \geq 4$

[Total 3]

(ii) If $E(GFLRV) = 0$ then we have:

$$P(0.975\ddot{a}_{[56]:\overline{4}|} - 0.275) = 120,000A^{1}_{[56]:\overline{4}|} + 20,000(IA)^{1}_{[56]:\overline{4}|} + 0.5 \times 4Pv^4\,{}_4p_{[56]} + 275 + 55[\ddot{a}_{[56]:\overline{4}|} - 1]$$

$$\Rightarrow P(0.975 \times 3.648 - 0.275) = 120,000 \times 0.01927 + 20,000 \times 0.051424 + 2P \times 0.774228 + 275 + 55 \times 2.648$$

$$\Rightarrow P = \frac{3761.52}{1.733344} = 2170.09$$ [3]

where

$$A^{1}_{[56]:\overline{4}|} = A_{[56]:\overline{4}|} - v^4\,{}_4p_{[56]} = 0.79350 - 0.79209 \times \frac{9287.2164}{9501.4839}$$

$$= 0.01927$$

and

$$(IA)^{1}_{[56]:\overline{4}|} = (IA)_{[56]} - v^4\,{}_4p_{[56]}[4A_{60} + (IA)_{60}]$$

$$= 5.29558 - 0.774228[4 \times 0.32692 + 5.46572]$$

$$= 0.051424$$ [1]

[Total 4]

(iii) Decrement table

x	t	q_x^d	$p_{[x]+t-1}$	$_{t-1}p_{[x]}$
56	1	0.003742	0.996258	1.0000000
57	2	0.005507	0.994493	0.9962580
58	3	0.006352	0.993648	0.9907716
59	4	0.007140	0.992860	0.9844782

[½]

Cash flows for the policy:

Yr	*Prm*	*Exp*	*Interest*	*Death claim*	*Mat claim*	*Profit vector*
1	P	$0.3P$+275	$0.042P$–16.50	523.88	0.00	$0.742P$–815.38
2	P	$0.025P$+55	$0.0585P$–3.3	881.12	0.00	$1.0335P$–939.42
3	P	$0.025P$+55	$0.0585P$–3.3	1143.36	0.00	$1.0335P$–1201.66
4	P	$0.025P$+55	$0.0585P$–3.3	1428.00	$1.98572P$	$-0.95222P$–1486.30
	[½]	[½]	[½]	[½]	[½]	[½]

Yr	*Profit vector*	$_{t-1}p_{[x]}$	*Profit signature*	*Discount factor*	*PVFNP*
1	$0.742P$–815.38	1.0000000	$0.742P$–815.38	0.943396	$0.7P$–769.23
2	$1.0335P$–939.42	0.9962580	$1.02963P$–935.90	0.889996	$0.91637P$–832.95
3	$1.0335P$–1201.66	0.9907716	$1.02396P$–1190.57	0.839619	$0.85974P$–999.63
4	$-0.95222P$– 1486.30	0.9844782	$-0.93744P$–1463.23	0.792094	$-0.74254P$–1159.02
			[½]	[½]	[½]

Total PVFNP = $1.73359P - 3760.83 = 0$

$$\Rightarrow P = \frac{3760.80}{1.73359} = 2169.40$$ [1]

[Total 6]

(iv) (a) If reserves are established, profit is deferred but the PVFNP will be the same as the earned interest rate on reserves is equal to the discount rate. [1]

(b) Again, if reserves are established, profit is deferred but now the PVFNP will be lower as the earned interest rate on reserves is lower than the discount rate. The premium would therefore have to be increased to achieve the same profit criteria. [2]

[Total 3]

[TOTAL 16]

Well prepared students made very good progress with this question.

The main area of problems were in part (i). Part (ii) was done well.

Credit was given in part (iii) for the correct method even if the calculations were inaccurate.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2016

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
December 2016

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

3. Credit is given to students who produce alternative correct numerical solutions. In the case of descriptive answers credit is also given where appropriate valid points are made which do not appear in the solutions below.

4. In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by the Examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

5. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonably accurate numerical calculation is necessary.

B. General comments on *student performance in this diet of the examination*

1. In general this paper was very well done by students who had prepared fully for the exam. Most questions were straightforward and capable of being answered in the allotted time. The questions that gave most difficulty were 3, 5, 11 and 12.

2. Detailed solutions are given below together with commentary from the examiners which we hope will be of assistance.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1 (a) $EPV = 100000 \times (1.04)^{1/2} \times A_{45} = 101980.4 \times 0.27605 = 28151.7$ [1]

(b) $$\text{Variance} = (100000)^2 \times {}^2A_{45} \times (1.04) - (28151.7)^2$$
$$= 10^{10} \times 0.09458 \times 1.04 - 7.925 \times 10^8$$
$$= 10^8 \times 1.9113$$
$$= (13825)^2$$

[3]
[Total 4]

Generally well done. The most common error was in (b) where students did not square the 100000 figure.

Q2 The reserves required at the beginning of policy years 7, 3 and 2 are:

$${}_6V = \frac{3}{1.015} = 2.956$$ [½]

Revised cash flow in policy year 6 $= 5 - 0.9975 \times {}_6V = 2.052$ [1]

$${}_2V = \frac{10}{1.015} = 9.852$$ [½]

$${}_1V = \frac{1}{1.015}(10 + .9975 \times {}_2V) = 19.535$$ [½]

Revised cash flow in policy year 1 $= -50 - 0.9975 \times {}_1V = -69.486$ [1]

=> revised profit vector: (–69.49, 0, 0, 5, 5, 2.05, 0, 15, 40, 60) [½]
[Total 4]

Again well done in general.

Q3 (i) Value is given by:

$$100{,}000 \times \frac{2}{3} \times \frac{{}^zM^{ia}_{42}}{s_{41}D_{42}} = 100{,}000 \times \frac{2}{3} \times \frac{56093}{7.980 \times 2799} = 167{,}422$$ [2]

(ii) Value of this benefit given by:

$$100{,}000 \times 4 \times \frac{{}^{s}M_{42}^{d}}{s_{41} D_{42}}$$ [2]

[Total 4]

This question was poorly done overall. The most common errors were using 42 instead of 41 for the salary index, using *R* functions instead of *M* and valuing age retirement instead of ill health. In (ii) all that was required for the limited marks was the final formula shown whilst students often spent a long time preparing detailed formulae

Q4

(a) $${}_{10|5}q_{65} = \frac{l_{75} - l_{80}}{l_{65}} = \frac{6879.1673 - 5266.4604}{8821.2612} = 0.18282$$ [1½]

(b)

$$\ddot{a}^{(12)}_{[30]:\overline{15}|} = \ddot{a}^{(12)}_{[30]} - v^{15} \times \frac{l_{45}}{l_{[30]}} \times \ddot{a}^{(12)}_{45}$$

$$= \left(\ddot{a}_{[30]} - \frac{11}{24}\right) - v^{15} \times \frac{l_{45}}{l_{[30]}} \times \left(\ddot{a}_{45} - \frac{11}{24}\right)$$

$$= \left(21.837 - \frac{11}{24}\right) - 0.555265 \times \frac{9801.3123}{9923.7497} \times \left(18.823 - \frac{11}{24}\right)$$

$$= 21.378 - 10.071$$

$$= 11.307$$

[2½]

A straightforward typical CT5 short question which was generally well done.

Q5

- Life insurance companies provide a service of pooling independent homogeneous risks. If a company is able to do this then as a result of the Central Limit Theorem the profit per policy will be a random variable that follows the normal distribution with a known mean and standard deviation. [1]

- The company can use this result to set premium rates which ensure that the probability of a loss of a portfolio of policies is at an acceptable level. [1]

- The independence of risks usually follows naturally from the way in which life insurance policies are sold. Rarely does the death of one policyholder influence the mortality of another policyholder. [1]

- Careful underwriting is the mechanism by which the company ensures that its risk groups are homogeneous. The risk groups are defined by the use of rating factors, e.g. age, sex, medical history, height, weight, lifestyle. [1]

- In theory, a company should continue to add rating factors to its underwriting system until the differences in mortality between the different categories of the next rating factor are indistinguishable from the random variation between lives that remains after using the current list of rating factors. [1]

- In reality the ability of prospective policyholders to provide accurate responses to questions and the cost of collecting information also limit the extent to which rating factors can be used. [1]

- For example, a proposal form should not ask for information which requires a specialist knowledge. Height is acceptable, but blood pressure is not. For example, the cost of undertaking extensive blood tests has to be weighed against the expected cost of claims that will be "saved" as a result of having this information. [1]

- From a marketing point of view proposers are anxious that the process of underwriting should be straightforward and speedy. [1]

- In practice, rating factors will be included if they avoid any possibility of selection against the company, and satisfy the time and cost constraints of marketing. This decision is often driven by competitive pressures. If several companies introduce a new rating factor, which in fact influences mortality levels significantly, then other companies will need to follow this lead or risk adverse selection against them. [1]

[Max 6]

Students did not score well on this question often only covering a limited number of points. The above solution is a full one. Any other valid points not shown were credited.

Q6 (i) $_{2.5}q_{80.75} = 1 - {}_{2.5}p_{80.75}$

$$_{2.5}p_{80.75} = {}_{0.25}p_{80.75} \times p_{81} \times p_{82} \times {}_{0.25}p_{83}$$

$$= \frac{l_{81}}{l_{80.75}} \times \frac{l_{82}}{l_{81}} \times \frac{l_{83}}{l_{82}} \times \frac{l_{83.25}}{l_{83}}$$

$$= \frac{l_{83.25}}{l_{80.75}} = \frac{110-83.25}{110-80.75} = \frac{26.75}{29.25} = \frac{107}{117} \text{ using UDD}$$

$$_{2.5}q_{80.75} = 1 - \frac{107}{117} = \frac{10}{117} \text{ as required}$$

[3]

(ii) $$\ddot{a}_{80:\overline{4|}} = 1 + \frac{_1p_{80}}{1.05} + \frac{_2p_{80}}{1.05^2} + \frac{_3p_{80}}{1.05^3}$$

$$= 1 + \frac{29}{30} \times \frac{1}{1.05} + \frac{28}{30} \times \frac{1}{1.05^2} + \frac{27}{30} \times \frac{1}{1.05^3}$$

$$= 1 + 0.9206 + 0.8466 + 0.7775$$

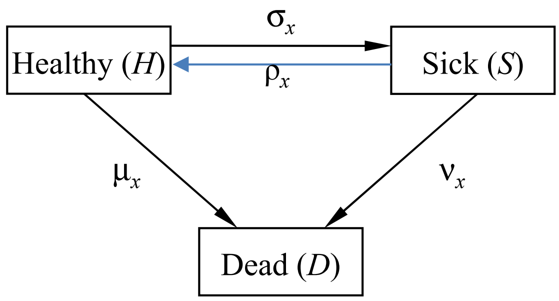
$$= 3.545 \text{ to 3 decimal places}$$

[3]
[Total 6]

Generally well done. Students who obtained decimal answers in (i) were given full credit if equivalent although this made the work more time consuming.

Students answered part (ii) very well.

Q7 (i) Transition model:



[5 marks for complete diagram]

(ii) Temporary Initial Selection – assuming that there is initial underwriting to exclude unhealthy lives we would expect to see lower levels of disability in the initial period.

Class Selection – we would expect to see different rates of sickness, recovery and mortality for different classes of policyholder, e.g. Male/Female, age.

[1 each example, max 2]
[Total 7]

> A very straightforward question, generally well done. Candidates lost marks in part (i) if labelling was unclear or incomplete. Note it was not necessary to define symbols in this case.
>
> In (ii) other valid examples were credited. Marks were lost if the reason for the selection chosen was not explained.

Q8 (i) The main advantage of the use of single figure indices is their simplicity for summary and comparison compared to the use of a set of age specific rates. Some indices are particularly designed for comparison with the mortality in a standard population, e.g. mortality rates used for premium calculation. This makes their use particularly relevant in an actuarial context.

The main disadvantage of the use of single figure indices is the loss of information as a result of summarising the set of age specific rates, and any distortions that may be introduced by the choice of weights for the averaging process. [2]

(ii)

	Country			*Sub-population*			
Age	*Population*	*Number of deaths*	*Mortality rate*	*Population*		*Actual deaths*	*Expected deaths*
40–44	834,561	3,510	0.0042	123,978	0.0029	360	521
45–49	779,862	3,153	0.0040	116,853	0.0033	386	467
50–54	750,234	3,620	0.0048	102,800	0.0051	524	493
Total						1,270	1,481

[3 for complete table]

The standardised mortality ratio = 1,270 / 1,481 = 0.858 [1]

(iii) The value less than 1 indicates that the mortality of the sub-population is lighter than for the country as a whole, after allowing for changes in the structure of the population [2]

[Total 8]

A straightforward question generally well done. Despite asking for words only some students still gave formulae in (i).

In (ii) the final answer is very sensitive to rounding. Retaining decimals for Expected Deaths refines the answer to 0.852.

Q9 Let b be the simple bonus rate (expressed as a percentage of the sum assured). Then the equation of value at 4% p.a. interest is (where P = 3,090):

$$P(.975\ddot{a}_{[30]:\overline{35|}}+0.025)=(125{,}000\times(1-b)+375)A_{[30]:\overline{35|}}+125{,}000bv^{35}\,{}_{35}p_{[30]}$$

$$+125{,}000b(IA)_{[30]:\overline{35|}}+325+0.75P$$

[4½]

where

$$(IA)_{[30]:\overline{35|}}=(IA)_{[30]}-v^{35}\,{}_{35}p_{[30]}(35A_{65}+(IA)_{65})+35v^{35}\,{}_{35}p_{[30]}$$

$$=6.91644-0.25342\times\frac{8821.2612}{9923.7497}(35\times0.52786+7.89442)+35\times0.25342\times\frac{8821.2612}{9923.7497}$$

$$=6.91644-0.22527\times26.36952+7.88431=8.86049$$

[3]

Equation of value becomes:

$$P(.975\times 19.072+0.025)=(125,375-125,000b)\times 0.26647+125,000b\left(0.25342\times\frac{8821.2612}{9923.7497}+8.86049\right)$$
$$+325+0.75P$$

$$\Rightarrow 57,536.42=33,408.68+1,102,411b+2,642.5$$

$$\Rightarrow b=\frac{21,485.24}{1,102,411}=0.0195$$

[1½]
[Total 9]

> Generally well done. The main difficulty was the correct valuation of the *IA* factor in the above equation

Q10 (i) Let *P* be the net premium for the policy payable annually in advance. Then, equation of value becomes:

$$P\ddot{a}_{[40]:\overline{25}|}=25,000(A_{[40]:\overline{25}|}+v^{25}{}_{25}p_{[40]})$$

$$15.887P=25,000\left(0.38896+0.37512\times\frac{8821.2612}{9854.3036}\right)=25,000(0.38896+0.33580)$$

$$\Rightarrow P=£1,140.49$$ [2]

Net premium reserve at the end of the 17th policy year is

$${}_{17}V=25,000(A_{57:\overline{8}|}+v^{8}{}_{8}p_{57})-P\ddot{a}_{57:\overline{8}|}$$

$$=25,000\left(0.73701+0.73069\times\frac{8821.2612}{9467.2906}\right)-1,140.49\times 6.838$$

$$=35,445.98-7,798.67=27,647.31$$ [2]

Death strain at risk per policy = 25,000 – 27,647.31 = −2,647.31 [1]

$$EDS=5,374q_{56}\times -2,647.31=5,374\times 0.005025\times -2,647.31=-71,488.85$$
$$ADS=24\times -2,647.31=-63,535.44$$ [1½]

mortality profit = −71,488.85 + 63,535.44 = -£7,953.41 i.e. a loss [½]

(ii) The death strain at risk is negative. Hence, the life insurance company makes money on deaths. [1]

Less people die than expected during the year considered so the company makes a mortality loss. [1]

[Total 9]

> Generally done well by fully prepared students. The main error was in trying to use premium conversion to fix the premium in (i) which is not correct. Students also used ${}_{18}V$ instead of ${}_{17}V$ in the net premium reserve.

Q11

$$\text{EPV} = 50000\times\left(a_{\overline{10}|}^{(12)} + v^{10}\times\frac{l_{70}}{l_{60}}\times a_{70}^{(12)}\right)$$

$$+20000\times\left[v^{10}\times\left(1-\frac{l_{70}}{l_{60}}\right)\times\frac{l_{72}}{l_{62}}\times a_{72}^{(12)} + v^{10}\times\frac{l_{70}}{l_{60}}\times\frac{l_{72}}{l_{62}}\times a_{70|72}^{(12)}\right] \quad [4]$$

$$a_{\overline{10}|}^{(12)} = \frac{i}{i^{(12)}}\times a_{\overline{10}|} = 1.018204\times 8.1109 = 8.259 \quad [½]$$

$$v^{10} = 0.67556 \qquad \frac{l_{70}}{l_{60}} = \frac{9238.134}{9826.131} = 0.94016 \qquad \frac{l_{72}}{l_{62}} = \frac{9193.860}{9804.173} = 0.93775 \quad [1½]$$

$$a_{70}^{(12)} = \ddot{a}_{70} - \frac{13}{24} = 11.020 \quad [½]$$

$$a_{72}^{(12)} = \ddot{a}_{72} - \frac{13}{24} = 11.593 \quad [½]$$

$$a_{70|72}^{(12)} = a_{72}^{(12)} - a_{70:72}^{(12)} = 11.593 - (9.404 - \frac{13}{24}) = 2.731 \quad [1½]$$

$$EPV = 50000\times(8.259 + 0.67556\times 0.94016\times 11.020)$$
$$+20000\times 0.67556\times 0.93775\times(0.05984\times 11.593 + 0.94016\times 2.731)$$

$$= 762909.1 + 41321.1$$

$$= 804230 \text{ rounded} \quad [1½]$$

[Total 10]

> This question was often poorly done because students did not distinguish the four payment situations properly.

Q12 (i)

$$\text{GFLRV} = 275 + 0.3P + 0.05P \times a^{4\%}_{\overline{\min(K_{[50]},14)}|} + 68 \times a^{0\%}_{\overline{\min(K_{[50]},14)}|} - P \times \ddot{a}^{4\%}_{\overline{\min(K_{[50]}+1,15)}|}$$

$$+ v^{T_{[50]}}(450{,}000 - 30{,}000 \times K_{[50]}) + 315 \qquad (\text{if } K_{[50]} < 15) \qquad [4]$$

(ii) $$P\ddot{a}^{4\%}_{[50]:\overline{15}|} = 207 + 0.25P + 0.05P\ddot{a}^{4\%}_{[50]:\overline{15}|} + 68\ddot{a}^{0\%}_{[50]:\overline{15}|}$$

$$+480{,}000\bar{A}^{1}_{[50]:\overline{15}|} - 30{,}000(I\bar{A})^{1}_{[50]:\overline{15}|} + 315 \times {}_{15}q_{[50]} \qquad [4]$$

$$\ddot{a}^{4\%}_{[50]:\overline{15}|} = 11.259$$

$$\ddot{a}^{0\%}_{[50]:\overline{15}|} = (1 + e_{[50]}) - {}_{15}p_{[50]}(1 + e_{65})$$

$$= 30.583 - \left(\frac{8{,}821.2612}{9{,}706.0977}\right)17.645$$

$$= 30.583 - (0.90884)17.645$$

$$= 14.547 \qquad [1]$$

$$\bar{A}^{1}_{[50]:\overline{15}|} = (1.04)^{0.5}(A_{[50]\overline{15}|} - v^{15}\,{}_{15}p_{[50]})$$

$$= (1.04)^{0.5}(0.56695 - 0.55526 \times 0.90884)$$

$$= 0.06354 \qquad [1]$$

$$(I\bar{A})^{1}_{[50]:\overline{15}|} = (1.04)^{0.5}[(IA)_{[50]} - v^{15}\,{}_{15}p_{[50]}(15A_{65} + (IA)_{65}]$$

$$= (1.04)^{0.5}[8.56390 - 0.55526 \times 0.90884(15 \times 0.52786 + 7.89442)]$$

$$= 0.59590 \qquad [1½]$$

$$11.259P = 207 + 0.25P + 0.05P \times 11.259 + 68 \times 14.547 + 480{,}000 \times 0.06354$$

$$-30{,}000 \times 0.59590 + 315 \times (1 - 0.90884)$$

$$\Rightarrow 10.4461P = 207 + 989.196 + 30{,}499.2 - 17{,}877.0 + 28.715$$

$$\Rightarrow P = 13{,}847.11 / 10.4461 = 1{,}325.58 \qquad [½]$$

(iii) $$ {}_{14}V = q_{64} v^{0.5}[30{,}000 + 315 \times (1.04)^{14.5}] + 68 \times (1.04)^{14} - 0.95P $$

$$ = (0.012716)(0.980581)[30{,}000 + 556.28] + 117.75 - 0.95 \times 1{,}325.58 $$

$$ = 381.01 + 117.75 - 1{,}259.30 = -760.54 $$ [2]

[Total 14]

> This proved to be the hardest question on the paper. Many struggled to get the Random Variable solution in part (i) (although correct alternatives were credited).
>
> In part (ii) the main issue was the Increasing Assurance functions.
>
> In part (iii) it is very common for students to have difficulties with the calculation of retrospective reserves.

Q13 (i) The dependent rates of decrement are calculated for policy years 1 and 2 using:

$$ (aq)_x^j = \frac{\mu^j}{\mu^d + \mu^m + \mu^s}\left[1 - e^{-(\mu^d + \mu^m + \mu^s)}\right] $$

where d denotes mortality, m marriage and s surrenders

$\Rightarrow$

$$ (aq)_x^d = \frac{\mu^d}{\mu^d + \mu^m + \mu^s}\left[1 - e^{-(\mu^d + \mu^m + \mu^s)}\right] = \frac{0.01}{0.235}\left[1 - e^{-(0.235)}\right] = 0.008912 $$

$$ (aq)_x^m = \frac{\mu^m}{\mu^d + \mu^m + \mu^s}\left[1 - e^{-(\mu^d + \mu^m + \mu^s)}\right] = \frac{0.15}{0.235}\left[1 - e^{-(0.235)}\right] = 0.133678 $$

$$ (aq)_x^s = \frac{\mu^s}{\mu^d + \mu^m + \mu^s}\left[1 - e^{-(\mu^d + \mu^m + \mu^s)}\right] = \frac{0.075}{0.235}\left[1 - e^{-(0.235)}\right] = 0.066839 $$

The dependent rates of decrement are calculated for policy year 3 using:

$$ (aq)_x^j = \frac{\mu^j}{\mu^d + \mu^m}\left[1 - e^{-(\mu^d + \mu^m)}\right] $$

where d denotes mortality and m marriage

$\Rightarrow$

$$(aq)_x^d = \frac{\mu^d}{\mu^d + \mu^m}\left[1 - e^{-(\mu^d + \mu^m)}\right] = \frac{0.01}{0.16}\left[1 - e^{-(0.16)}\right] = 0.009241$$

$$(aq)_x^m = \frac{\mu^m}{\mu^d + \mu^m}\left[1 - e^{-(\mu^d + \mu^m)}\right] = \frac{0.15}{0.16}\left[1 - e^{-(0.16)}\right] = 0.138615$$ [4]

Multiple decrement table:

t	$(aq)_{x+t}^d$	$(aq)_{x+t}^m$	$(aq)_{x+t}^s$	$(ap)_{x+t}$	$_{t-1}(ap)_x$
1	0.008912	0.133678	0.066839	0.790571	1.000000
2	0.008912	0.133678	0.066839	0.790571	0.790571
3	0.009241	0.138615	0.00	0.852144	0.625002

(ii) The expected cash flows for the policy are:

Yr	*Opening reserve*	*Premium*	*Expense*	*Interest*	*Death claim*	*Surrender claim*	*Marriage claim*	*Maturity claim*	*Closing reserve*
1	0.00	9516.00	142.74	328.06	89.12	318.02	1367.49	0.00	0.00
2	0.00	9516.00	142.74	328.06	178.24	636.04	2734.98	0.00	0.00
3	0.00	9516.00	142.74	328.06	277.23	0.00	4253.98	25564.31	0.00
		[½]	[½]	[½]	[1½]	[1½]	[1½]	[1]	

(iii)

Yr	*Profit vector*	$_{t-1}(ap)_x$	*Profit signature*	*Discount factor*	*Present value of profit*
1	7926.70	1.000000	7926.70	0.96154	7621.84
2	6152.07	0.790571	4863.65	0.92456	4496.74
3	–20394.20	0.625002	–12746.42	0.88900	–11331.57
	[½]		[½]		

Total present value of profit = 787.01 [1]

(iv) The cash flows show that for this policy, the expected profit vector is positive for policy years 1 and 2 but negative (significantly) for the last policy year (which is expected due to the maturity value being paid at the end of the term of the policy). The company may not have sufficient funds available to pay claims in policy year 3 and therefore, it would be prudent for the company to hold reserves at the beginning and end of each policy year. [2]

[Total 15]

Generally the question was answered well by fully prepared students. Credit was given to students who could outline the processes even if the final calculations were not always correct.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2017

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
July 2017

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

3. Credit is given to students who produce alternative correct numerical solutions. In the case of descriptive answers credit is also given where appropriate valid points are made which do not appear in the solutions below.

4. In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by the Examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

5. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonably accurate numerical calculation is necessary.

B. General comments on *student performance in this diet of the examination*

1. In general this paper was done less well than previous recent papers although well prepared students managed to score very reasonable marks. Most questions were straightforward and capable of being answered in the allotted time. The questions that gave most difficulty were 4, 6, 7 and 8.

2. Detailed solutions are given below together with commentary from the examiners which we hope will be of assistance.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1

$${}_{2.75}p_{77.4} = {}_{0.6}p_{77.4} \times {}_{2}p_{78} \times {}_{0.15}p_{80}$$

$$= \frac{p_{77}}{{}_{0.4}p_{77}} \times {}_{2}p_{78} \times {}_{0.15}p_{80}$$

$$= \frac{p_{77}}{(1-0.4q_{77})} \times {}_{2}p_{78} \times (1-0.15q_{80})$$

$$= \frac{(1-0.036696)}{(1-0.4\times 0.036696)} \times \frac{6953.536}{7615.818} \times (1-0.15\times 0.053303)$$

$$= 0.88550$$

[3]

> Straightforward and generally done well.

Q2

The death benefit in year 19 is 631 × 19 = 11,989. [½]

Profit emerging per policy in force at the start of the year is:

$$([{}_{18}V + P]\times 1.045) - (11{,}989 \times 0.015) - ([1-0.015]\times {}_{19}V)$$ [2]

$$= ([17{,}095 + 631]\times 1.045) - (11{,}989\times 0.015) - (0.985\times 18{,}510) = 111.49$$ [½]

[Total 3]

> This question was done less well. The main issue was the correct use of the traditional reserve movement formula which many students failed to recall.

Q3 (a) $${}_{12}p_{73} = \frac{l_{85}}{l_{73}} = \frac{3385.2479}{7403.0084} = 0.45728$$ [½]

(b) $${}_{10|}a_{56} = v^{10} \times \frac{l_{66}}{l_{56}} \times (\ddot{a}_{66} - 1)$$ [1]

$$= 0.67556 \times \frac{8695.6199}{9515.104} \times 10.896$$

$$= 6.727$$

(c)

$$A_{64:\overline{10|}} = A_{64} - v^{10} \times \frac{l_{74}}{l_{64}} \times A_{74} + v^{10} \times \frac{l_{74}}{l_{64}}$$

$$= 0.51333 - \left(0.67556 \times \frac{7150.2401}{8934.8771} \times (0.65824 - 1) \right)$$

$$= 0.69809$$

[1½]
[Total 3]

> Straightforward and generally done well. Quite a few students valued a Term Assurance in (c) by omitting the final survival factor.

Q4 Under a non-unitised accumulating with-profits (AWP) contract, the basic benefit takes the form of an accumulating fund of premiums with a discretionary annual bonus interest determined by the insurance company each year. [1]

If the accumulating fund at time t is denoted by F_t, the simplest form of an AWP contract follows the following recursive formula: $F_t = (F_{t-1} + P)(1+g)(1+b_t)$ where P is the annual premium, g is the guaranteed annual interest and b_t is the discretionary annual bonus interest declared for year t. [1]

The discretionary bonus interest will reflect both the returns achieved on the underlying assets over the period plus any additional profits made on the contract in this time. [1]

It is unusual for any guaranteed rate to be applied to AWP in modern conditions (other than the degenerate case where g = 0). [1]

The regular bonus interest under AWP can be reduced so as to retain profit for subsequent deferred payment as a terminal bonus payable on death or survival. [1]

With the simple AWP contract, the bonus interest would distribute profits net of all expenses and other costs incurred. [1]

[Max 4]

> This question was answered very poorly. Most students failed to understand the features of an AWP and either left the question unanswered or gave an answer of a traditional with – profit contract which was not asked for and thus failed to score marks.

Q5

Age Band	***City A*** *Actual Deaths*	***City B*** *Actual Deaths*	*Total Deaths*	*Total Exposed*	*(i)**	*(ii)***
20–29	230.0	312.5	542.5	450000	562.5	301.4
30–39	654.5	555.8	1210.3	675000	1154.3	582.7
40–49	1608.0	1342.5	2950.5	775000	2774.5	1427.7
50–59	3465.0	3030.0	6495.0	450000	6817.5	2886.7
60–69	4271.0	5460.0	9731.0	225000	9828.0	5406.1
TOTAL	10228.5	10700.8	20929.3	2575000	21136.8	10604.5

* City B expected deaths using Total Exposed and City B mortality rate

** City B expected deaths using Total mortality rate and City B exposure

[4 for table values]

Directly Standardised mortality rate = $\dfrac{21136.8}{2575000} = 0.008208$ [½]

Indirectly Standardised mortality rate = $\dfrac{20929.3}{2575000} \times \dfrac{10700.8}{10604.5} = 0.008202$ [1½]

[Total 6]

> Generally well answered.

Q6 The present value is:

$$= 0.01 \times 10 \times \left[\begin{array}{l} \left[\sum_{t=0}^{10} \left(30000 \times \frac{z_{54.5+t}}{s_{53}} - 1000 \right) \times \frac{r_{54+t}}{l_{54}} \times v^{t+0.5} \times \bar{a}^{r}_{54.5+t} \right] \\ + \left(30000 \times \frac{z_{65}}{s_{53}} - 1000 \right) \times \frac{r_{65}}{l_{54}} \times v^{11} \times \bar{a}^{r}_{65} \end{array} \right]$$

[5]

Ignoring suffixes and the compound interest factor:

s = salary index, *z*-average salary factor over 3 years prior to retirement
r = age retirement factor
l = lives factor

and

$\bar{a}^r$ = the continuous annuity factor payable for life for the retiree

[2]
[Total 7]

This question was done quite poorly. Despite being asked not to use commutation factors many students did rather than develop the formula model above.

It was noted that some students interpreted the second paragraph of the question to mean – "age retirement is not permitted before age 60". Their summation expressions therefore started with $t = 6$, rather than $t = 0$. This approach was given full credit.

Q7 Underwriting is the process by which life insurance companies divide lives into homogeneous risk groups by using the values of certain factors (rating factors) recorded for each life. [1]

(a) Adverse selection is characterised by the way in which the select groups are formed rather than by the characteristics of those groups. Any form of selection may also exhibit adverse selection. Adverse selection usually involves an element of self-selection, which acts to disrupt (act against) a controlled selection process which is being imposed on the lives. This adverse selection tends to reduce the effectiveness of the controlled selection. [1]

If prospective policyholders know that a company does not use a particular rating factor, e.g. smoking status, then lives who smoke will opt to buy a policy from this company rather than a company that uses smoking status as a rating factor. [1]

The outcome will be to lessen the effect of the controlled selection being used by the company as part of the underwriting process. The effect of self-selection by smokers is adverse to the company's selection process. [1]

(b) When homogeneous groups are formed we usually tacitly infer that the factors used to define each group are the cause of the differences in mortality observed between the groups. However, there may be other differences in composition between the groups, and it is these differences rather than the differences in the factors used to form the groups that are the true causes of the observed mortality differences. [1]

Ascribing mortality differences to groups formed by factors which are not the true causes of these differences is termed spurious selection. [1]

For example, when the population of England and Wales is divided by region of residence, some striking mortality differences are observed. However, a large part of these differences can be explained by the different mix of occupations in each region. The class selection ascribed to regions is spurious and is in part the effect of compositional differences in occupation between the regions. [1]

[Total 7]

> This question was generally poorly answered. The main issue was that many students just gave bookwork definitions without linking their answer to the underwriting process.

Q8 (i) $(I\bar{A})_{x:\overline{n}|}$ is the expected present value of an increasing endowment assurance on a life aged x for n years whereby the sum assured is 1 in the first year increasing by 1 every full year to n at the end of the term. The death claim is paid immediately on death. The maturity value is paid at the end of the term.

$(\bar{I}\bar{A})_{x:\overline{n}|}$ is similar except that the increase in sum assured from 1 to n is continuous from inception. [2]

(ii)

$$(I\bar{A})_{x:\overline{15}|} = 0.02 \times \int_0^1 e^{-.05t} dt + 0.02 \times 2e^{-.05} \times \int_0^1 e^{-.05t} dt + 0.02 \times 3e^{-.05\times 2} \times \int_0^1 e^{-.05t} dt$$

$$+ ... + 0.02 \times 15 \times e^{-.05\times 14} \times \int_0^1 e^{-.05t} dt + 15 \times e^{-.05\times 15}$$

$$= 0.02 \times \int_0^1 e^{-.05t} dt \times \left(1 + 2e^{-.05} + 3e^{-.10} + ... + 15e^{-.70}\right) + 15e^{-.75}$$

$$= 0.02 \times \left(\frac{1-e^{-.05}}{.05}\right) \times \left(\frac{1-e^{-.75}}{(1-e^{-.05})^2} - \frac{15e^{-.75}}{(1-e^{-.05})}\right) + 15e^{-.75}$$

$$= 0.4 \times 0.04877 \times \left(\frac{0.52763}{0.00238} - \frac{7.08550}{0.04877}\right) + 7.08550$$

$$= 1.4906 + 7.0855$$

$$= 8.576 \quad \text{(to 3dp)}$$

[3½ marks lines 1 to 4, 1 mark line 5, ½ mark for result]

In Part (i) some students thought that there was no difference in the symbols depicted but there was clearly an extension of the bar across the top of the symbol in the second case compared to the first.

Part (ii) proved to be the most challenging question on the paper and was consequently very poorly done with few students making progress on this.

Some students actually wrongly calculated $(\overline{IA})_{x:\overline{n}|}$ which has a simpler derivation.

Q9 (i)

$$EPV = \bar{A}_{\overline{55:50}} = \bar{A}_{55} + \bar{A}_{50} - \bar{A}_{55:50}$$

$$\bar{A}_{55} = 0.03 \times \int_0^\infty e^{-0.07t} dt = \frac{3}{7}$$

$$\bar{A}_{50} = 0.02 \times \int_0^\infty e^{-0.06t} dt = \frac{1}{3}$$

$$\bar{A}_{55:50} = \int_0^\infty e^{-0.09t}(0.03+0.02)dt = \frac{5}{9}$$

$$\Rightarrow \bar{A}_{\overline{55:50}} = \frac{3}{7} + \frac{1}{3} - \frac{5}{9} = \frac{13}{63} = 0.20635$$

[½ mark for lines 1 and 5, 1 mark each other line – Total 4]

(ii)

$$\text{Variance} = {}^2\bar{A}_{55} + {}^2\bar{A}_{50} - {}^2\bar{A}_{55:50} - (EPV)^2$$

$${}^2\bar{A}_{55} = 0.03 \times \int_0^\infty e^{-0.11t} dt = \frac{3}{11}$$

$${}^2\bar{A}_{50} = 0.02 \times \int_0^\infty e^{-0.10t} dt = \frac{1}{5}$$

$${}^2\bar{A}_{55:50} = \int_0^\infty e^{-0.13t}(0.03+0.02)dt = \frac{5}{13}$$

$${}^2\bar{A}_{55} + {}^2\bar{A}_{50} - {}^2\bar{A}_{55:50} = \frac{3}{11} + \frac{1}{5} - \frac{5}{13} = 0.08811$$

$$\text{Variance} = 0.08811 - (0.20635)^2 = 0.04553$$

[2 marks for knowing to square interest – 2 marks for rest]
[Total 8]

A straightforward question of its type – well done by well prepared students

Q10 Value of lump sum death benefit:

$$EPV = 20000 \times \left(\bar{A}_{55} + \bar{A}_{50} \right)$$

$$\bar{A}_{55} = (1.04)^{0.5} \times (1 - d\ddot{a}_{55}) = 1.019804 \times (1 - 0.038462 \times 17.364) = 0.338724$$

$$\bar{A}_{50} = (1.04)^{0.5} \times (1 - d\ddot{a}_{50}) = 1.019804 \times (1 - 0.038462 \times 19.539) = 0.253412$$

$$EPV = 20000 \times (0.338724 + 0.253412) = 11842.7$$

[3]

Value of deferred annuity:

$$EPV = 5000 \times ({}_{10|}\ddot{a}^{(12)}_{55} + {}_{10|}\ddot{a}^{(12)}_{50})$$

$$= v^{10} \times 5000 \times \left(\frac{9647.797}{9904.805} \times \left(13.666 - \frac{11}{24} \right) + \frac{9848.431}{9952.697} \times \left(16.652 - \frac{11}{24} \right) \right)$$

$$= 0.67556 \times 5000 \times (12.865 + 16.024)$$

$$= 97581.3$$

[3]

Value of premiums:

Let P be the monthly premium

$$EPV = 12P\ddot{a}^{(12)}_{55:50:\overline{10}|}$$

$$= 12P \times (\ddot{a}^{(12)}_{55:50} - v^{10}\, {}_{10}p_{55}\, {}_{10}p_{50} \ddot{a}^{(12)}_{65:60})$$

$$= 12P \times \left(16.602 - \frac{11}{24} - v^{10} \times \frac{9647.797}{9904.805} \times \frac{9848.431}{9952.697} \times \left(12.682 - \frac{11}{24} \right) \right)$$

$$= 12P \times (16.144 - (0.67556 \times 0.97405 \times 0.98952 \times 12.224))$$

$$= 98.215P$$

[2½]

Hence monthly premium = (11842.7 + 97581.3) / 98.215 = 1114 nearer whole no.

[½]

[Total 9]

> A straightforward question of its type – well done by well prepared students. The only real issue was in valuing the deferred annuity portion.

Q11 (i) $P\ddot{a}_{[40]:\overline{20|}} = 60,000A^{\ \ 1}_{[40]:\overline{20|}} = 60,000v^{20}\ {}_{20}p_{[40]}$

$\Rightarrow P(13.930) = (60,000)(0.45639)(0.94245)$

$\Rightarrow P = 25,807.49 / 13.93 = 1,852.66$

[2]

Mortality profit = Expected Death Strain – Actual Death Strain

$DSAR = 0 - {}_{17}V = -(60,000A^{\ \ 1}_{57:\overline{3|}} - P\ddot{a}_{57:\overline{3|}})$

$= -(60,000v^3\ {}_3p_{57} - P\ddot{a}_{57:\overline{3|}})$

$= -\{(60,000)(0.88900)(0.98098) - (1,852.66)(2.870)\} = -47,008.34$

[2]

EDS = (q_{56})(18230)(–47,008.34) = (0.005025)(18230)(–47,008.34)

= –4,306,234.24 [1½]

ADS = (86)(–47,008.34) = –4,042,717.24 [1]

Mortality Profit = –4,306,234.24 – (–4,042,717.24) = –263,517
i.e. a loss. [½]

(ii) We expected 18230q_{56} = 91.6 deaths. Actual deaths were 86. With pure endowments, the death strain is negative because no death claim is paid and there is a release of reserves to the company on death. In this case, less deaths than expected means this release of reserves is less than required by the equation of equilibrium and the company therefore makes a loss. [2]

[Total 9]

Well prepared students completed this question very satisfactorily.

The main errors were students valuing an endowment assurance rate than a pure endowment or using ${}_{16}V$ instead of ${}_{17}V$.

The conclusion was also not clearly stated in many cases

Q12 (i) If the monthly premium and sum assured are denoted by P and S respectively then:

$$0.975\times 12P\ddot{a}^{(12)}_{[35]:\overline{30|}}+0.025P$$

$$=(0.975S+275)\bar{A}^{\;1}_{[35]:\overline{30|}}+Sv^{30}\,{}_{30}p_{[35]}+0.025S(I\bar{A})_{[35]:\overline{30|}}$$

$$+325+70(\ddot{a}_{[35]:\overline{30|}}-1)+0.7\times 12P \qquad [6]$$

where $(I\bar{A})_{[35]:\overline{30|}}=(I\bar{A})^{\;1}_{[35]:\overline{30|}}+30v^{30}\,{}_{30}p_{[35]}$ [½]

$$\Rightarrow 0.975\times 12P\ddot{a}^{(12)}_{[35]:\overline{30|}}+0.025P$$

$$=(1.04)^{0.5}\left[(0.975\times 100{,}000+275)A^{\;1}_{[35]:\overline{30|}}+0.025\times 100{,}000(IA)^{\;1}_{[35]:\overline{30|}}\right]$$

$$+(1+30\times 0.025)\times 100{,}000v^{30}\,{}_{30}p_{[35]}+325+70(\ddot{a}_{[35]:\overline{30|}}-1)+8.4P$$

where

$$\ddot{a}^{(12)}_{[35]:\overline{30|}}=\ddot{a}^{(12)}_{[35]}-v^{30}\,{}_{30}p_{[35]}\ddot{a}^{(12)}_{65}$$

$$=\left(\ddot{a}_{[35]}-\frac{11}{24}\right)-v^{30}\,{}_{30}p_{[35]}\left(\ddot{a}_{65}-\frac{11}{24}\right)$$

$$=\left(21.006-\frac{11}{24}\right)-.30832\times\frac{8821.2612}{9892.9151}\left(12.276-\frac{11}{24}\right)$$

$$=20.548-3.249=17.299 \qquad [½]$$

$$A^{\ 1}_{[35]:\overline{30}|} = A_{[35]:\overline{30}|} - v^{30}\ {}_{30}p_{[35]} == 0.32187 - 0.27492 = 0.04695$$ [½]

$$(IA)^{\ 1}_{[35]:\overline{30}|} = (IA)_{[35]} - v^{30}\ {}_{30}p_{[35]}\left(30A_{65} + (IA)_{65}\right)$$

$$= 7.47005 - 0.30832 \times \frac{8821.2612}{9892.9151}(30 \times 0.52786 + 7.89442)$$

$$= 7.47005 - 6.52394 = 0.94611$$

[1]

$$\Rightarrow (0.975 \times 12 \times 17.299 + 0.025)P$$

$$= (1.04)^{0.5}\left[97,775 \times 0.04695 + 2,500 \times 0.94611\right]$$

$$+175,000 \times 0.27492 + 325 + 70 \times 16.631 + 8.4P$$

$$\Rightarrow 202.423P = (1.04)^{0.5}[4,590.536 + 2,365.275]$$

$$+48,111.0 + 325 + 1,164.17 + 8.4P$$

$$\Rightarrow 194.023P = 7,093.563 + 48,111.0 + 325 + 1,164.17 \Rightarrow P = 292.20$$ [½]

(ii) Gross prospective policy value (calculated at 4%) is given by:

$$V^{\text{prospective}} = 170,000\bar{A}_{63:\overline{2}|} + (0 + 300)q_{63}v^{0.5} + (2750 + 300)\,p_{63}q_{64}v^{1.5}$$

$$+5500\ {}_{2}p_{63}v^{2} + 85\ddot{a}_{63:\overline{2}|} - 0.975 \times 12P\ddot{a}^{(12)}_{63:\overline{2}|}$$ [4]

where $B = 28 \times 0.025 \times 100,000 = 70,000$ [½]

and

$$\ddot{a}^{(12)}_{63:\overline{2}|} = \ddot{a}^{(12)}_{63} - v^{2}\ {}_{2}p_{63}\ddot{a}^{(12)}_{65}$$

$$= \left(\ddot{a}_{63} - \frac{11}{24}\right) - v^{2}\ {}_{2}p_{63}\left(\ddot{a}_{65} - \frac{11}{24}\right)$$

$$= \left(13.029 - \frac{11}{24}\right) - .92456 \times \frac{8821.2612}{9037.3973}\left(12.276 - \frac{11}{24}\right)$$

$$= 12.57067 - 0.90245 \times 11.81767 = 1.90582$$ [½]

$$\bar{A}_{63:\overline{2}|} = (1.04)^{0.5} A^{\ 1}_{63:\overline{2}|} + v^2 \ {}_2p_{63} = (1.04)^{0.5}(0.92498 - 0.90245) + 0.90245$$

$$= 0.92543$$

[½]

$$V^{\text{prospective}} = 170,000 \times 0.92543 + 300 \times 0.011344 \times 0.98058$$

$$+3050 \times 0.98866 \times 0.012716 \times 0.94287$$

$$+5500 \times 0.90245 + 85 \times 1.951 - 0.975 \times 12 \times 292.20 \times 1.90582$$

$$= 157323.1 + 3.3371 + 36.1534 + 4963.475 + 165.835 - 6515.50$$

$$= 155,976.39$$

[½]
[Total 15]

Part (i) was generally done well. The main issue was the development of the bonus part of the formula. Part (ii) was generally less well done.

Reasonable credit was given if students showed overall understanding of the processes even if some of the calculations lacked accuracy.

Q13

Annual premium	£9000.00	Allocation % (1st yr)	80.0%
Risk discount rate	6.5%	Allocation % (2nd yr)	100.0%
Interest on investments (1st yr)	4.5%	Allocation % (3rd yr)	100.0%
Interest on investments (2nd yr)	4.0%	B/O spread	5.0%
Interest on investments (3rd yr)	3.5%	Management charge	1.5%
Interest on non-unit funds	2.0%	Surrender penalty (1st yr)	£600
Death benefit (% of bid value of units)	125%	Surrender penalty (2nd yr)	£300
		Policy Fee	£25

	£	*% prem*
Initial expense	220	30.0%
Renewal expense	75	1.5%
Expense inflation	2.0%	

(i) Using $\mu^d_{[x]+t} = -\ln\left(1 - q^d_{[x]+t}\right)$ we have:

x	t	$q^d_{[x]+t}$	μ^s_{x+t}	$\mu^d_{[x]+t}$
60	0	0.005774	0.10	0.005791
61	1	0.008680	0.05	0.008718
62	2	0.010112	0.00	0.010163

The dependent rates of decrement are calculated for each policy year using:

$$(aq)^j_x = \frac{\mu^j}{\mu^d + \mu^s}\left[1 - e^{-(\mu^d + \mu^s)}\right]$$

where d denotes mortality and s surrender

$\Rightarrow$

$$(aq)^d_{60} = \frac{\mu^d}{\mu^d + \mu^s}\left[1 - e^{-(\mu^d + \mu^s)}\right] = \frac{0.005791}{0.105791}\left[1 - e^{-(0.105791)}\right] = 0.005495$$

$$(aq)^s_{60} = \frac{\mu^s}{\mu^d + \mu^s}\left[1 - e^{-(\mu^d + \mu^s)}\right] = \frac{0.1}{0.105791}\left[1 - e^{-(0.105791)}\right] = 0.094892$$

$$(aq)^d_{61} = \frac{\mu^d}{\mu^d + \mu^s}\left[1 - e^{-(\mu^d + \mu^s)}\right] = \frac{0.008718}{0.058718}\left[1 - e^{-(0.058718)}\right] = 0.008467$$

$$(aq)^s_{61} = \frac{\mu^s}{\mu^d + \mu^s}\left[1 - e^{-(\mu^d + \mu^s)}\right] = \frac{0.05}{0.058718}\left[1 - e^{-(0.058718)}\right] = 0.048560 \qquad [4]$$

$$(aq)^d_{62} = \frac{\mu^d}{\mu^d + \mu^s}\left[1 - e^{-(\mu^d + \mu^s)}\right] = \left[1 - e^{-(0.010163)}\right] = 0.010112$$

Multiple decrement table:

x	$(aq)^d_x$	$(aq)^s_x$	(ap)	${}_{t-1}(ap)$
60	0.005495	0.094892	0.899613	1.000000
61	0.008467	0.048560	0.942973	0.899613
62	0.010112	0.000000	0.989888	0.848310

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
value of units at start of year	0.000	7021.026	15926.629
alloc	7180.000	8975.000	8975.000
B/O	359.000	448.750	448.750
interest	306.945	621.891	855.851
management charge	106.919	242.538	379.631
value of units at year end	7021.026	15926.629	24929.099

[3]

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
unallocated premium + pol fee	1820.000	25.000	25.000
B/O spread	359.000	448.750	448.750
expenses	2920.000	211.500	213.030
interest	–14.820	5.245	5.214
man charge	106.919	242.538	379.631
extra death benefit	9.645	33.712	63.021
surrender penalty	56.935	14.568	0.000
end of year cashflow / profit vector	–601.611	490.888	582.545

[4]

probability in force	1	0.899613	0.848310	
discount factor	0.938967	0.881659	0.827849	
expected p.v. of profit	233.56			[1½]
premium signature	9000.000	7602.362	6731.287	
expected p.v. of premiums	23333.649			[1½]
profit margin	1.00%			[1]

(ii) The non unit fund cash flows that change are:

	yr1	*yr2*	*yr3*
extra death benefit	10.135	34.561	-
surrender penalty	0	0	-

Multiple decrement table becomes:

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	${}_{t-1}(ap)$
60	0.005774	0	0.994226	1.000000
61	0.008680	0	0.991320	0.994226
62	0.010112	0	0.989888	0.985596

[2]

Revised profit vector (–659.036, 475.472, 582.545) [1]

Revised profit signature (–659.036, 472.727, 574.154) [½]

Revised PVFNP = –618.813 + 416.784 + 475.313 = 273.284 [½]

[Total 19]

This question was done well by those students who had prepared.

Again partial credit was given to students who understood the processes where calculations were not always accurate.

Students should note that the use of the approximation $(aq)_x^d = q_x^d(1 - \frac{1}{2}q_x^s)$ is no longer in the core reading and has been replaced by the force of mortality version.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2017

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
December 2017

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

B. General comments on *student performance in this diet of the examination*

Well prepared students did very well in this relatively straightforward exam where the main questions of challenge were Q7, Q9(ii), Q12 and Q13(iii). Q1 was in addition done very poorly.

There was evidence that may people attempted this examination without robust preparation and in these cases many of the simpler questions were not answered satisfactorily.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1

- Salaries and salary related expenses [½]
- Buildings and other property costs [½]
- Computing and associated costs [½]
- Costs involved with the investment of funds [½]

[Total 2]

> This question was generally poorly answered. Many students simply ignored the reference to inflation and gave a generalised answer on costs which was not what was required.

Q2

When homogeneous groups are formed we usually tacitly infer that the factors used to define each group are the cause of the differences in mortality observed between the groups. However, there may be other differences in composition between the groups, and it is these differences rather than the differences in the factors used to form the groups that are the true causes of the observed mortality differences.

Ascribing mortality differences to groups formed by factors which are not the true causes of these differences is termed spurious selection. [1½]

For example, when the population of England and Wales is divided by region of residence, some striking mortality differences are observed. However, a large part of these differences can be explained by the different mix of occupations in each region. The class selection ascribed to regions is spurious and is in part the effect of compositional differences in occupation between the regions.

In statistical terminology the occupational differences in mortality are confounded (mixed up) with the regional differences. [1½]

[Total 3]

> Generally well done. Other valid examples were given credit, particularly with reference to changing underwriting standards as explained in the Core Reading

Q3

$$_{2.25}p_{85.5} = {}_{0.5}p_{85.5}\,p_{86}\,{}_{0.75}p_{87}$$

$$= \frac{l_{86}}{l_{85.5}} \times \frac{l_{87}}{l_{86}} \times \frac{l_{87.75}}{l_{87}}$$

$$= \frac{l_{87.75}}{l_{85.5}}$$

$$= \frac{0.75 l_{88} + 0.25 l_{87}}{0.5 l_{86} + 0.5 l_{85}}$$

$$= \frac{0.75 \times 11,874 + 0.25 \times 14,280}{0.5 \times 16,917 + 0.5 \times 19,756}$$

$$= \frac{12,475.5}{18,336.5}$$

$$= 0.68036$$

$$_{2.25}q_{85.5} = 1 - {}_{2.25}p_{85.5}$$

$$= 0.31964$$

[2 marks for first 4 lines plus 2 for calculations]
[Total 4]

Generally done well.

Q4

The expected cost of paying benefits usually increases as the life ages and the probability of a claim by death increases. [1]

Level premiums received in the early years of a contract are more than enough to pay the benefits that fall due in those early years, but in the later years the premiums are too small to pay for the benefits. It is therefore prudent for the premiums that are not required in the early years of the contract to be set aside, or reserved, to fund the shortfall in the later years of the contract. [2]

If premiums received that were not required to pay benefits were spent by the company, perhaps by distributing to shareholders, then later in the contract the company may not be able to find the money to pay for the excess of the cost of benefits over the premiums received. [1]
[Total 4]

Many students did not clearly describe the fundamental timing issue given in the second paragraph above. A small credit was given if students mentioned legislative requirements.

Q5

(i)

$$\bar{A}_{47:\overline{11}|} = (1.04)^{1/2} \times A^{1}_{47:\overline{11}|} + (1.04)^{-11} \times \frac{l_{58}}{l_{47}}$$

$$= 1.0198 \times (A_{47} - (1.04)^{-11} \times \frac{l_{58}}{l_{47}} \times A_{58}) + (1.04)^{-11} \times \frac{l_{58}}{l_{47}}$$

$$= 1.0198 \times (0.29635 - 0.64958 \times \frac{9413.8004}{9771.0789} \times 0.42896) + 0.64958 \times \frac{9413.8004}{9771.0789}$$

$$= 0.02845 + 0.62583$$

$$= 0.65428$$

[2 marks lines 1 and 2; 1 mark for calculation]

(ii)

$$\ddot{a}^{[4]}_{[53]:\overline{13}|} = \ddot{a}^{[4]}_{[53]} - (1.04)^{-13} \times \frac{l_{66}}{l_{[53]}} \times \ddot{a}^{[4]}_{66}$$

$$= (\ddot{a}_{[53]} - 0.375) - (1.04)^{-13} \times \frac{l_{66}}{l_{[53]}} \times (\ddot{a}_{66} - 0.375)$$

$$= 16.163 - 0.60057 \times \frac{8695.6199}{9621.1006} \times 11.521$$

$$= 9.909$$

[1 mark lines 1 and 2; 1 mark for calculation]
[Total 5]

Generally done well. Some students lost marks in (i) by mistakenly calculating the temporary assurance function only. Others forgot the $(1.04)^{1/2}$ adjustment for immediate claim payment.

Q6

(i)

$$\ddot{a}_{40:\overline{4}|} = 1 + \frac{0.992}{1.05} + \frac{0.981}{(1.05)^2} + \frac{0.967}{(1.05)^3}$$

$$= 1 + 0.94476 + 0.88980 + 0.83533$$

$$= 3.6699$$

[2]

(ii)

$$A_{40:\overline{4}|} = 1 - d(5\%) \times \ddot{a}_{40:\overline{4}|} = 1 - \frac{0.05}{1.05} \times 3.6699$$

$$= 0.82524$$

$$A^{1}_{40:\overline{4}|} = A_{40:\overline{4}|} - (1.05)^{-4} \times \frac{l_{44}}{l_{40}} = 0.82524 - 0.82270 \times 0.947 = 0.04614$$

[2 marks 1st 2 lines in and 2 marks for 3rd line]
[Total 6]

Generally well done. Despite the question wording requirement in (ii), some students attempted to calculate $A^{1}_{40:\overline{4|}}$ in the long method way rather than the simple premium conversion route. This received full credit if the answer was correct.

Q7

$$(aq)^{\beta}_{40} = \int_0^1 ({}_t p^{\alpha}_{40} \times {}_t p^{\beta}_{40} \times \mu^{\beta}_{40+t})dt$$

$${}_t p^{\alpha}_{40} = \exp(-\int_0^t \mu^{\alpha}_{40+s} ds) = \exp(-\int_0^t (\frac{1}{110-(40+s)})ds) = \exp(-\int_0^t (\frac{1}{70-s})ds$$

$$= \exp(\ln[70-s]_0^t) = \frac{70-t}{70}$$

$${}_t p^{\beta}_{40} = \exp(-\int_0^t \mu^{\beta}_{40+s} ds) = \exp(-0.03t)$$

$$\mu^{\beta}_{40+t} = 0.03$$

$$\Rightarrow (aq)^{\beta}_{40} = \int_0^1 ((\frac{70-t}{70}) \times e^{-0.03t} \times .03)dt = 0.03\int_0^1 (e^{-0.03t} - \frac{t}{70} \times e^{-0.03t})dt$$

$$= 0.03 \times \left[-\frac{e^{-0.03t}}{0.03}\right]_0^1 - \frac{0.03}{70}\int_0^1 t \times e^{-0.03t} dt$$

$$= (1-e^{-.03}) - \frac{0.03}{70} \times 0.490112$$

$$= 0.029554 - 0.000210$$

$$= 0.029344$$

[Total 7]

This was a very challenging question which many students did not attempt fully. Many that did assumed μ^{α}_{40} was a constant 1/70 but this was not correct although the numerical answer was nearly the same (reasonable credit was given for this approach however), The question did not give the assumption that μ^{α}_{x} was constant through each year of age *x* and to score high marks this non constant factor needed to be taken into account.

Q8

(i)

Past benefits:

$$= 35000 \times \frac{15}{80} \times \left(\frac{{}^{z}M_{45}^{ra} + {}^{z}M_{45}^{ia}}{s_{44}D_{45}} \right) = 6562.5 \times \left(\frac{128026 + 52554}{8.375 \times 2329} \right) = 60755.4$$

Future benefits:

$$= 35000 \times \frac{1}{80} \times \left(\frac{{}^{z}\bar{R}_{45}^{ra} + {}^{z}\bar{R}_{45}^{ia}}{s_{44}D_{45}} \right) = 437.5 \times \left(\frac{2244130 + 609826}{8.375 \times 2329} \right) = 64013.4$$

Total Value $= 60755.4 + 64013.4 = 124769$ rounded

[2 marks for each formula plus 1 for result]

(ii)

Let the contribution rate be *k*

Then $\frac{k}{100} \times 35000 \times \frac{{}^{s}\bar{N}_{45}}{s_{44}D_{45}} = 124769$

$$\Rightarrow k = \frac{124769 \times 100 \times 8.375 \times 2329}{253080 \times 35000} = 27.47$$

i.e. 27.47% of salary

[2 marks line 2 plus 1 for result]
[Total 8]

Generally done well. The most common error was the definition of *s* and forgetting to calculate both forms of retirement benefit in (i).

Q9

(i)

$$\begin{aligned}
\text{EPV} &= A^{1}_{50:\overline{10}|} + 0.75 \times v^{10} \times \frac{l_{60}}{l_{50}} \times A_{60} \\
&= (A_{50} - v^{10} \times \frac{l_{60}}{l_{50}} \times A_{60}) + 0.75 \times v^{10} \times \frac{l_{60}}{l_{50}} \times A_{60} \\
&= A_{50} - 0.25 \times v^{10} \times \frac{l_{60}}{l_{50}} \times A_{60} \\
&= 0.32907 - 0.25 \times 0.67556 \times \frac{9287.2164}{9712.0728} \times 0.45640 \\
&= 0.25536
\end{aligned}$$

[2 marks first 3 lines; 1 mark for result]

(ii)

First calculate 2nd moment

$$\begin{aligned}
\text{Value} \quad &= ({}^{2}A_{50} - v^{20} \times \frac{l_{60}}{l_{50}} \times {}^{2}A_{60}) + (0.75)^{2} \times v^{20} \times \frac{l_{60}}{l_{50}} \times {}^{2}A_{60} \\
&= {}^{2}A_{50} - 0.4375 \times v^{20} \times \frac{l_{60}}{l_{50}} \times {}^{2}A_{60} \\
&= 0.13065 - 0.4375 \times 0.45639 \times \frac{9287.2164}{9712.0728} \times 0.23723 \\
&= 0.08535
\end{aligned}$$

$$\text{Variance} \quad = 0.08535 - (0.25536)^{2} = 0.02014 = (0.14192)^{2}$$

[3 marks for the first 3 lines; 2 marks for remainder]
[Total 8]

Part (i) was generally answered well'

Part (ii) was poorly answered where the main issue was developing correctly the 2nd moment value above especially the treatment of the 0.75 multiplier.

Q10

(i) (a) Crude mortality rate is the ratio of the total number of deaths in a category to the total exposed to risk in the same category. [1]

(b) Directly standardised mortality rate is the mortality rate of a category weighted according to a standard population. [1]

(c) Indirectly standardised mortality rate is an approximation to the directly standardised mortality rate being the crude rate for the standard population multiplied by the ratio of actual to expected deaths for the region. [1]

(d) Standardised Mortality Ratio is the ratio of the actual deaths in the category to the expected deaths in the same category using the mortality rates from the standard population. [1]

(ii) The crude death rate is $\dfrac{287}{150000} = 0.001913$ [½]

The Directly Standardised Mortality Rate is:

$$\left(\frac{(1000000 \times \frac{42}{40000}) + (1600000 \times \frac{135}{75000}) + (900000 \times \frac{110}{35000})}{3500000}\right)$$

$$= \frac{1050 + 2880 + 2828.6}{3500000} = 0.001931$$ [2]

The Indirectly Standardised Mortality Rate can be calculated as follows:

Expected deaths for regional group:

$$\left(\frac{40000 \times 1300}{1000000} + \frac{75000 \times 3200}{1600000} + \frac{35000 \times 2500}{900000}\right)$$

$$= 52 + 150 + 97.22 = 299.22$$

So the Indirectly Standardised Mortality rate is:

$$\frac{0.002 \times 287}{299.22} = 0.001918$$ [2]

The Standardised Mortality Ratio is $\dfrac{287}{299.22} = 0.9592$ [½]

[Total 9]

Generally straightforward and done well.

Q11

(i)

t	*Profit vector*	${}_{t-1}p_{65}$	*Profit signature*
1	185.21	1	185.21
2	–121.52	0.985757	–119.79
3	–5.28	0.970044	–5.12
4	12.95	0.952754	12.34

[1] [½]

Let X be the reserve required at t =1 in order to zeroise negative cash flows at t = 2 and t = 3.

Then:

$X = 119.79v + 5.12v^2$ at 5% $= 118.73$ [1]

Revised cash flow at t = 1 is 185.21 – 118.73 = 66.48

Hence profit signature is: (66.48, 0, 0, 12.34) [½]

(ii) Multiple decrement table – although deaths can be assumed to be uniformly distributed over the year, surrenders occur only at the year end. Therefore:

$(aq)_x^d = q_x^d$ and $(aq)_x^s = q_x^s(1 - q_x^d)$

x	q_x^d	q_x^s	$(aq)_x^d$	$(aq)_x^s$	$(ap)_x$	${}_{t-1}(ap)_x$
65	0.014243	0.03	0.014243	0.029573	0.956184	1
66	0.015940	0.03	0.015940	0.029522	0.954538	0.956184
67	0.017824	0.00	0.017824	0.00	0.982176	0.912714
68	0.019913	0.00	0.019913	0.00	0.980087	0.896446

[2½]

t	*Revised profit vector*	${}_{t-1}(ap)_{65}$	*Revised profit signature*
1	$185.21+50(aq)_{65}^s = 186.69$	1	186.69

2	$-121.52+50\left(aq\right)^{s}_{66}=-120.04$	0.956184	–114.78
3	–5.28	0.912714	–4.82
4	12.95	0.896446	11.61

[2]

Let Y be the reserve required at t =1 in order to zeroise negative cash flows at t = 2 and t = 3.

Then:

$Y = 114.78v + 4.82v^2$ at $5\% = 113.69$ [1]

Revised cash flow at t = 1 is 186.69 – 113.69 = 73.00

Hence revised profit signature is: (73.00, 0, 0, 11.61) [½]

(iii) NPV of revised profit signature $= 73.00v + 11.61v^4$ at $8\% = 76.13$ [1]

[Total 10]

> Straightforward and done well by students who had prepared thoroughly. Reasonable credit was given for method even if the values were not always accurate.

Q12

(i)

The sum assured and attaching bonus at the beginning of each policy year is:

Policy year	*Sum assured + Bonus*
1	78,000
2	81,000
3	84,000

Let NP be the net premium for the policy. Then

$$NP = \frac{75000A_{62:\overline{3}|} + 3000(IA)_{62:\overline{3}|}}{\ddot{a}_{62:\overline{3}|}}$$

$$= \frac{75000 \times 0.89013 + 3000 \times 2.64053}{2.857} = 26139.78$$

where

$$(IA)_{62:\overline{3}|} = (IA)_{62} - v^3 \frac{l_{65}}{l_{62}}\left(3A_{65} + (IA)_{65}\right) + 3v^3 \frac{l_{65}}{l_{62}}$$

$$= 8.20491 - 0.889 \times \frac{8821.2612}{9129.7170}\left(3 \times 0.52786 + 7.89442\right) + 3 \times 0.889 \times \frac{8821.2612}{9129.7170}$$

$$= 8.20491 - 8.14126 + 2.57688 = 2.64053$$

[4]

Alternatively

$$NP = \frac{78000 q_{62} v + 81000 p_{62} q_{63} v^2 + 84000 p_{62} p_{63} v^3}{\ddot{a}_{62:\overline{3}|}}$$

$$= (758.40 + 840.953 + 73082.462) / 2.857 = 26139.94$$

The net premium reserve ${}_tV$ at duration t for the policy is given by:

$${}_1V = 81,000 q_{63} v + 84,000 p_{63} v^2 - P\ddot{a}_{63:\overline{2}|}$$

$$= 81,000 \times 0.011344 \times 0.96154 + 84,000 \times 0.988656 \times 0.92456 - 26,139.78 \times 1.951$$

$$= 883.52 + 76781.72 - 50998.71 = 26,666.53$$

$${}_2V = 84,000 v \ - P\ddot{a}_{64:\overline{1}|}$$

$$= 84,000 \times 0.96154 - 26,139.78 \times 1.0$$

$$= 80,769.23 - 26,139.78 = 54,629.45$$

[4]

(ii)

Let P be the annual office premium for the policy. Then the expected cash flows for the policy are:

Yr	*Opening reserve*	*Premium*	*Expense*	*Interest*	*Death claim*	*Maturity claim*	*Closing reserve*
1	0.00	P	0.15P	0.0425P	788.74	0.00	26396.88
2	26666.53	P	0.05P	1333.33+0.0475P	918.86	0.00	54009.73
3	54629.45	P	0.05P	2731.47+0.0475P	1068.14	82931.86	0.00
	[½]	[½]	[½]	[1]	[1]	[½]	[1]

Yr	*Profit vector*	$_{t-1}(ap)_x$	*Profit signature*	*Discount factor*	*Present value of profit*
1	0.8925*P*–27185.62	1.000000	0.89250*P*–27185.62	0.94340	0.84198*P*–25646.91
2	0.9975*P*–26928.73	0.989888	0.98741*P*–26656.43	0.89000	0.87880*P*–23724.22
3	0.9975*P*–26639.08	0.978659	0.97621*P*–26070.58	0.83962	0.81965*P*–21889.38

[½] [½] [½] [½] [½]

Total present value of profit = 2.54043*P* – 71260.51 [½]

However we require:

Total present value of profit = 0 to achieve an internal rate of return of 6% p.a. [½]

Therefore *P* = 71260.51/2.54043 = 28050.57 [½]

[Total 17]

Many students in part (i) overlooked the fact that the bonus was guaranteed throughout from outset i.e. it was effectively a product with an increasing sum assured. Therefore the net premium calculation needed to reflect this guarantee rather than use the traditional net premium non-profit basis.

Part (ii) was quite challenging and credit was given for method employed even when not all the arithmetic calculations were correct.

Q13

(i)

Let P be the annual net premium for the increasing term assurance policy. Then the equation of value is given by:

$$P \;=\frac{\dfrac{50,000}{1.0192308}A^{1}_{[40]:\overline{25}|}@4\%+25PA_{[40]:\overline{25}|}^{\;\;1}@6\%}{\ddot{a}^{@6\%}_{[40]:\overline{25}|}}$$ [2½]

where at 4%

$$A^{1}_{[40]:\overline{25}|}=A_{[40]:\overline{25}|}-v^{25}\,{}_{25}p_{[40]}$$

$$=0.38896-0.37512\times\frac{8821.2612}{9854.3036}=0.38896-0.33580=0.05316$$

[½]

and at 6%

$$A_{[40]:\overline{25}|}^{\;\;\;\;\;\;\;1} = v^{25}\,{}_{25}p_{[40]}$$

$$= 0.233 \times \frac{8821.2612}{9854.3036} = 0.20857$$

[½]

$$\Rightarrow P \;= \frac{\frac{50,000}{1.0192308} \times 0.05316 + 25P \times 0.20857}{13.29} \Rightarrow P = \frac{2607.849}{8.07575} = 322.95$$ [½]

(ii)

Reserve at the <u>end</u> of the 17th policy year given by:

$${}_{17}V \;= 50,000\left(1.0192308\right)^{16} A^{1}_{57:\overline{8}|} @\, 4\% + 25PA_{57:\overline{8}|}^{\;\;\;\;\;\;1} @\, 6\% - P\;\ddot{a}_{57:\overline{8}|} @\, 6\%$$ [2½]

where at 4%

$$A^{1}_{57:\overline{8}|} = A_{57:\overline{8}|} - v^{8}\,{}_{8}p_{57}$$

$$= 0.73701 - 0.73069 \times \frac{8821.2612}{9467.2906} = 0.73701 - 0.68083 = 0.05618$$

[½]

and at 6%

$$A_{57:\overline{8}|}^{\;\;\;\;\;\;1} = v^{8}\,{}_{8}p_{57}$$

$$= 0.62741 \times \frac{8821.2612}{9467.2906} = 0.58460$$

[½]

$$\Rightarrow {}_{17}V \;= 50,000\left(1.0192308\right)^{16} \times 0.05618 + 25 \times 322.95 \times 0.58460 - 322.95 \times 6.433$$

$$= 3,809.957 + 4,719.914 - 2,077.537 = 6,452.334$$

[½]

Therefore, death strain at risk (*DSAR*) per policy in the 17th policy year is:

$$DSAR = 50,000\left(1.0192308\right)^{16} - 6,452.334 = 61,363.425$$ [1]

Mortality profit = expected death strain (*EDS*) – actual death strain (*ADS*) [½]

$$EDS \;= 1425 \times q_{56} \times 61,363.425 = 1425 \times 0.005025 \times 61,363.425 = 439,400.475$$

$ADS = 10 \times 61{,}363.425 = 613{,}634.25$ [1]

i.e. mortality profit = –174,233.77 (i.e. a loss) [½]

(iii)

Reserve at the <u>end</u> of the 16th policy year given by:

$${}_{16}V = 50{,}000(1.0192308)^{15}\ A^{1}_{56:\overline{9}|}\ @\ 4\% + 25PA_{56:\overline{9}|}^{\ \ 1}\ @\ 6\% - P\ \ddot{a}_{56:\overline{9}|}\ @\ 6\%$$ [2½]

where at 4%

$$A^{1}_{56:\overline{9}|} = A_{56:\overline{9}|} - v^9\ {}_9p_{56}$$

$$= 0.70993 - 0.70259 \times \frac{8821.2612}{9515.1040} = 0.70993 - 0.65136 = 0.05857$$

[½]

and at 6%

$$A_{56:\overline{9}|}^{\ \ 1} = v^9\ {}_9p_{56}$$

$$= 0.5919 \times \frac{8821.2612}{9515.1040} = 0.54874$$

[½]

$$\Rightarrow {}_{16}V = 50{,}000(1.0192308)^{15} \times 0.05857 + 25 \times 322.95 \times 0.54874 - 322.95 \times 7.038$$

$$= 3{,}897.026 + 4{,}430.390 - 2{,}272.922 = 6{,}054.494$$

[½]

Let *i'* be the actual interest rate earned. Then the interest profit is given by:

$$[i' - 0.06] \times 1425 \times ({}_{16}V + P) = 174{,}233.77$$

$$[i' - 0.06] \times 9{,}087{,}857.7 = 174{,}233.77$$

$$[i' - 0.06] = 0.019172 \Rightarrow i' = 7.92\%$$ [2]

[Total 17]

Parts (i) and (ii) generally well done by students who had thoroughly prepared. The main error in (ii) was having the wrong duration for the reserve.

Part (iii) was more challenging and done less well.

Again credit was given for understanding the method even if calculations were not always accurate.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2018

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
December 2018

A. General comments on the aims *of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

B. General comments on *student performance in this diet of the examination*

This exam was done well by those students who had prepared thoroughly. There were a large number of students however who were unprepared.

Most questions were very straightforward. Those of particular challenge were Questions 5, 7, 10 (b) and 12 (b).

The Examiners hope the comments below will assist in future studies.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1

(a) $_{12}q_{[54]} = \dfrac{\left(l_{[54]} - l_{66}\right)}{l_{[54]}} = \dfrac{\left(9585.6916 - 8695.6199\right)}{9585.6916} = 0.09285$ [1]

(b) $\ddot{a}_{65}^{(6)} = \ddot{a}_{65} - \dfrac{5}{12} = 12.276 - \dfrac{5}{12} = 11.859$ [1]

(c)

$$\bar{s}_{43:\overline{10}|} = (1.04)^{10} \times \frac{l_{43}}{l_{53}} \times \bar{a}_{43:\overline{10}|} = (1.04)^{10} \times \frac{l_{43}}{l_{53}} \times \bar{a}_{43} - \bar{a}_{53}$$

$$= (1.04)^{10} \times \frac{9826.2060}{9630.0522} \times \left(19.319 - 0.5\right) - \left(16.524 - 0.5\right) = 12.400$$

[2]

[Total 4]

Generally well done. The main issue was remembering the definition of $\bar{s}_{43:\overline{10}|}$ in (c)

Q2

(a)

$$\bar{Z} = \begin{cases} \bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{T_x}|} & \text{if } T_y > T_x \\ 0 & \text{otherwise} \end{cases}$$

Alternatives

(1) $\bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{\min\left(T_x, T_y\right)}|}$

(2) $\bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{T_{x:y}}|}$

(3) $v^{T_x}\, \bar{a}_{\overline{T_y - T_x}|}$

[2]

(b) $E(\bar{Z}) = \bar{a}_y - \bar{a}_{xy} = \frac{1-\bar{A}_y}{\delta} - \frac{1-\bar{A}_{xy}}{\delta} = \frac{\bar{A}_{xy}-\bar{A}_y}{\delta}$ [2]

[Total 4]

Surprisingly many students struggled with this and tried to answer with non-random variable functions. This gained no marks. A mark was also lost if the 0 otherwise" was missing from a random variable answer.

Q3

Value of benefits:

$$= 30000 \times (\ddot{a}_{\overline{5}|}^{(12)} + v^5 \times \frac{l_{70}}{l_{65}} \times \ddot{a}_{70}^{(12)}) \qquad [1]$$

$$= 30000 \times (\frac{1-v^5}{d^{(12)}} + v^5 \times \frac{l_{70}}{l_{65}} \times (\ddot{a}_{70} - \frac{11}{24})) \qquad [2]$$

$$= 30000 \times (\frac{(1-0.82193)}{0.039157} + v^5 \times \frac{9392.621}{9703.708}(12.934 - \frac{11}{24})) \qquad [1]$$

$$= 434{,}189$$

A very straightforward question of its type. Generally answered well. Main error was incorrect application of the monthly adjustment.

Q4

(i) A group of lives is selected from a larger group in a non-random way with regard to their mortality. [1]

For example, in the underwriting of applicants for life insurance policies, the lives selected for standard premium rates have lower mortality rates than the population from which they were selected. [1]

Some of this effect on mortality will be temporary, i.e. the longer the time since the selection date, the less will be the difference in mortality of the select group compared to the equivalent non-selected group at the same age. [1]

Thus, the effect of the initial selection tends to 'wear off' over time. This is temporary initial selection. [1]

[Maximum 3 marks]

[Credit any sensible example]

(ii) The initial underwriting process is based on a proposal form and may sometimes include a medical questionnaire and a medical examination. The absence of medical questions will allow more impaired lives to be included within the initial group of lives. [1]

The impact of temporary selection will be reduced in two ways:

- The period of selection will be shorter [1/2]
- The select mortality rates will be closer to the ultimate rates [1/2]

[Total 5]

Most students got some of the valid points above, but many had omissions. Credit was given by examiners for any acceptable points or examples not contained in the solution above.

Q5

In the first instance the following relationship is used:

$p_x = e^{-\int_0^1 \mu_{x+t} dt}$ which is equivalent to $e^{-\mu}$ for constant μ

$\Rightarrow \mu = -\ln(p_x)$

Thus the following values can be obtained:

$\mu_x = -\ln(p_x) = -\ln(0.99) = 0.01005$ and $\mu_{x+1} = -\ln(p_{x+1}) = -\ln(0.975) = 0.02532$

[2]

EPV of benefits:

$$100,000 \times \int_0^1 (0.01005 \times e^{-(0.01005+.05)t}) dt + 150,000 \times e^{-0.06005} \times \int_0^1 (0.02532 \times e^{-(0.02532+.05)t}) dt$$

$$= 1005 \times \left(\frac{1 - e^{-0.06005}}{0.06005} \right) + 3798 \times e^{-0.06005} \times \left(\frac{1 - e^{-0.07532}}{0.07532} \right)$$

$$= 1005 \times 0.970567 + 3798 \times 0.941717 \times 0.963268$$

$$= 975.42 + 3445.26$$

$= 4421$ rounded

[2 marks each of first 2 lines; 1 mark for result]

Overall this question was poorly answered. Many students failed to appreciate the simple link $\mu = -\ln(p_x)$ and tried all sorts of approximations. A small amount of credit was given for close approximations but to score well required the above solution.

Q6

(i)

(a) The directly standardised mortality rate is calculated as

$$\frac{\sum_x {}^sE^c_{x,t}\, m_{x,t}}{\sum_x {}^sE^c_{x,t}}$$

[2]

(b)

The indirectly standardised mortality rate is calculated as

$$\frac{\sum_x {}^sE^c_{x,t}\ {}^sm_{x,t}}{\sum_x {}^sE^c_{x,t}} \Bigg/ \frac{\sum_x E^c_{x,t}\ {}^sm_{x,t}}{\sum_x E^c_{x,t}\ m_{x,t}}$$

$$= \begin{array}{c}\text{Crude mortality rate}\\ \text{for standard population}\end{array} \Bigg/ \frac{\text{Expected deaths in population}}{\text{Actual deaths in population}}$$

[2]

(ii)

Directly standardised

(140,000 x 125 / 10,236 + 156,000 x 156 / 11,256 + 168,000 x 166 / 10,633)
464,000

=0.013997

[2]

Indirectly standardised

Crude mortality rate for standard population
(140,000 x 0.00169 + 156,000 x 0.00220 + 168,000 x 0.00277) / 464,000 = 0.002253
Expected deaths in population
(10,236 x 0.00169 + 11,256 x 0.00220 + 10,633 x 0.00277) = 71.5
Actual deaths in population = 447
Indirectly standardised rate = 0.002253 / (71.5 / 447) = 0.014079 [2]

[Total 8]

Part (i) was knowledge based and part (ii) a straightforward application. The question was generally well done by fully prepared students.

Q7

Derive the constant force of mortality for 70 to 71 using

$$\mu = -\log(p_{70}) = -\log\left(\frac{9112.449}{9238.134}\right) = 0.013698 \qquad [1]$$

$$_{0.25}p_{70.75} = e^{-0.25\times 0.013698} = 0.996581 \qquad [1/2]$$

$$\bar{A}_{70.75} = \int_0^{0.25} v^t \times {}_t p_{70.75} \times \mu dt + v^{0.25} \times {}_{0.25}p_{70.75} \times \bar{A}_{71} \qquad [2]$$

$$\bar{A}_{71} \approx 1.04^{0.5} \times A_{71} = 1.04^{0.5} \times (1 - d\ddot{a}_{71}) = 1.04^{0.5} \times \left(1 - \frac{0.04}{1.04} \times 11.136\right) = 0.583014 \qquad [2]$$

$$\int_0^{0.25} v^t \times {}_t p_{70.75} \times \mu dt = 0.013698 \int_0^{0.25} e^{-\log(1.04)t} \times e^{-0.013698t} dt$$

$$= 0.013698 \times \left[\frac{-e^{-(\log(1.04)+0.013698)t}}{\log(1.04)+0.013698}\right]_0^{0.25} \qquad [2]$$

$$= 0.003402$$

Therefore $\bar{A}_{70.75} = 0.003402 + v^{0.25} \times 0.996581 \times 0.583014 = 0.578754$ [1/2]

[Total 8]

Students generally struggled with this question and failed to spot the same link as in Q5. Because of the low impact on the final answer from the first quarter many students successfully got an answer close to the solution using other approximations. In this case this was given a fair proportionate credit.

Q8

(i)

- Allocated premiums are invested in the fund(s) chosen by the policyholder which purchases a number of units within the fund(s)
- Each investment fund is divided into units, which are priced regularly (usually daily)
- Policyholder receives the value of the units allocated to their own policy
- Benefits are directly linked to the value of the underlying investments
- Unallocated premiums are directed to the company's non-unit fund
- Bid/offer spread is used to help cover expenses and contribute towards profit
- Charges are made from the unit account periodically to cover expenses and benefits (i.e. fund management charge) and may be varied after notice of change given.
- Unit-linked contracts may offer guaranteed benefits (e.g. minimum death benefit)

- Unit-linked contracts are generally endowment assurance and whole of life contracts

0.5 mark for each feature [max 4]

(ii) To calculate the expected reserves at the end of each year we have (utilising the end of year cashflow figures):

$p_{53} = 0.996461 \quad p_{52} = 0.996848 \quad p_{51} = 0.997191$

$${}_3V = \frac{1075.23}{1.025} = 1{,}049.00$$

$${}_2V \times 1.025 - p_{53} \times {}_3V = 355.10 \Rightarrow {}_2V = 1{,}366.23$$

$${}_1V \times 1.025 - p_{52} \times {}_2V = 401.56 \Rightarrow {}_1V = 1{,}720.47$$

[3]

The revised cash flow for year 1 will become:

$1{,}798.01 - p_{51} \times 1{,}720.47 = 82.37$ [1]

Revised profit vector becomes (82.37, 0, 0, 0) and
Net present value of profits = 82.37/(1.045) = 78.82 [1]

[Total 9]

In (i) very few students got the maximum number of points requested. There were no issues with (ii) except for many students got arithmetical errors whilst appreciating the techniques required.

Q9

(i) The dependent rates of decrement are calculated for each policy year using:-

$$(aq)_x^j = \frac{\mu^j}{\mu^d + \mu^r + \mu^s}\left[1 - e^{-(\mu^d + \mu^r + \mu^s)}\right]$$

where d denotes mortality, r retirement and s surrender

$\Rightarrow$ for policy years 1 and 2

$$(aq)^d = \frac{\mu^d}{\mu^d + \mu^r + \mu^s}\left[1 - e^{-(\mu^d + \mu^r + \mu^s)}\right] = \frac{0.015}{0.085}\left[1 - e^{-(0.085)}\right] = 0.01438$$

$$(aq)^r = \frac{\mu^r}{\mu^d + \mu^r + \mu^s}\left[1 - e^{-(\mu^d + \mu^r + \mu^s)}\right] = \frac{0.02}{0.085}\left[1 - e^{-(0.085)}\right] = 0.019174$$

$$(aq)^s = \frac{\mu^s}{\mu^d + \mu^r + \mu^s}\left[1 - e^{-(\mu^d + \mu^r + \mu^s)}\right] = \frac{0.05}{0.085}\left[1 - e^{-(0.085)}\right] = 0.047934$$

$\Rightarrow$ for policy year 3

$$(aq)^d = \frac{\mu^d}{\mu^d + \mu^r}\left[1 - e^{-(\mu^d + \mu^r)}\right] = \frac{0.015}{0.035}\left[1 - e^{-(0.035)}\right] = 0.014741$$

$$(aq)^r = \frac{\mu^r}{\mu^d + \mu^r}\left[1 - e^{-(\mu^d + \mu^r)}\right] = \frac{0.02}{0.035}\left[1 - e^{-(0.035)}\right] = 0.019654$$

which gives the following multiple decrement table:

Year t	μ^d	μ^s	μ^r	$(aq)^d$	$(aq)^s$	$(aq)^r$	(ap)	${}_{t-1}(ap)$
1	.015	.05	.02	.014380	.047934	.019174	.918512	1.0
2	.015	.05	.02	.014380	.047934	.019174	.918512	.918512
3	.015	0	.02	.014741	0	.019654	.956505	.843665

[3]

(ii) Cash flows:

Year t	Premium P	Expenses E	Interest on P-E	Death Claim	Surrender Claim	Redundancy Claim	Maturity Claim	Profit Vector
1	12500.00	312.50	304.69	545.96	200.18	254.78	0	11491.27
2	12500.00	312.50	304.69	545.96	400.37	509.56	0	11036.30
3	12500.00	312.50	304.69	559.64	0	783.49	37658.61	-26509.55

[4]

Note: allowance for ½ year interest roll up is included in death, surrender and redundancy costs

Year t	Profit Vector	Cum probability of survival	Profit signature	Discount factor	NPV of Profit signature
1	11491.27	1	11491.27	.961538	11049.30
2	11036.30	.918512	10136.98	.924556	9372.21
3	-26509.55	.843665	-22365.18	.888996	-19882.56

=> Total NPV of profit = 538.94 [1½]

NPV of premium = 12,500 x (1 + .918512x .961538 + .843665 x .924556)
= 33,290.01 [1]

Therefore, profit margin = 538.94/33,290.01 = 1.62% [½]

Generally well attempted although often with arithmetical errors. Credit was given for understanding the process even though accuracy was not always attained.

Q10

(a) Let P be the annual premium for the policy. Then (functions at 6%):-

EPV of premiums:

$P\ddot{a}_{[35]} = 15.993P$

[0.5]

EPV of benefits:

$85,000A_{[35]}$

[1]

EPV of expenses:

$0.75P + 350 + (85 + 0.025P)a_{[35]}$

[2]

Equation of value gives:-

$$P\ddot{a}_{[35]} = 85,000A_{[35]} + 350 + 0.75P + (0.025P + 85)a_{[35]}$$

$$P \times 15.993 = 85,000 \times 0.09475 + 350 + 0.75P + (0.025P + 85) \times 14.993$$

$$\Rightarrow P = \frac{9,678.155}{14.868175} = 650.93$$

[0.5]

(b) Let $P^{/}$ be the required minimum office premium. Then the insurer's loss random variable for this policy is given by (where K and T denote the curtate and complete future lifetime of a policyholder):-

$$L = 85,000v^{K_{[35]}+1} + 350 + 0.75P^{/} + (0.025P^{/} + 85)a_{\overline{K_{[35]}|}} - P^{/}\ddot{a}_{\overline{K_{[35]}+1|}}$$ [2]

We need to find a value of t such that

$P(L > 0) = P(T < t) = 0.05 \Rightarrow P(T \geq t) = 0.95$ [0.5]

Using AM92 Select, we require:-

$\frac{l_{[35]+t}}{l_{[35]}} \geq 0.95 \Rightarrow l_{[35]+t} \geq 0.95l_{[35]} = 0.95 \times 9892.9151 = 9398.269$ [1]

As $l_{58} = 9413.8004$ *and* $l_{59} = 9354.004$ then t lies between 23 and 24 so $K_{[35]} = 23$.

[0.5]

We therefore need the minimum premium such that

$$L = 0 = 85,000v^{24} + 350 + 0.75P^{/} + (0.025P^{/} + 85)a_{\overline{23|}} - P^{/}\ddot{a}_{\overline{24|}}$$

$$\Rightarrow 0 = 85,000 \times 0.24698 + 350 + 0.75P^{/} + (0.025P^{/} + 85) \times 12.3034 - 13.3034P^{/}$$

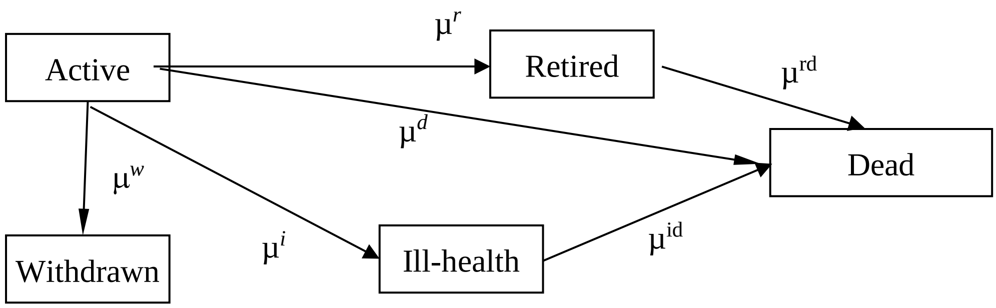
$$\Rightarrow P^{/} = \frac{22,389.089}{12.2458} = 1828.31$$

[2]

No issues with part (i). Most students struggled with part (ii) generally failing to appreciate how to approach the problem.

Q11

(i)



Ill health means ill health retirement

[1/2 for each box, 1/2 for each labelled arrow, extra ½ if complete, total 6]

(ii)

$$\text{Value} = 30{,}000 \times \frac{s_{50}}{s_{49}} \times \frac{1}{80} \times \frac{\left[\left(10({}^{z}M_{50}^{ra} + {}^{z}M_{50}^{ia}\right) + \left({}^{z}\bar{R}_{50}^{ra} + {}^{z}\bar{R}_{50}^{ia}\right)\right]}{{}^{s}D_{50}}$$

$$= 30{,}000 \times \frac{9.165}{9.031} \times \frac{1}{80} \times \frac{\left[10\left(128026 + 45392\right) + (1604000 + 363963)\right]}{16460}$$

$$= 85{,}596$$

[3 marks for formula, 1 for calculation]

Very straightforward question done well. Main omissions were parts of the diagrams and the salary factor in (ii).

Q12

(a) Let P be the monthly premium for version A of the contract. Then equation of value (at 4% p.a. interest) is: -

$$12P\ddot{a}^{(12)}_{[35]:\overline{30}|} = \frac{100,000}{1.04} \times 1.04^{0.5} \times A^{@0\%}_{[35]} + 225 \times 1.04^{0.5} \times A^{@4\%}_{[35]} + 275 + 0.4 \times 12P$$

[½] [1] [½] [½] [½]

$$+0.025 \times 12P(\ddot{a}^{(12)}_{[35]:\overline{30}|} - \frac{1}{12}) + 0.025 \times 12P(\ddot{a}^{(12)}_{[35]:\overline{30}|} - \frac{1}{12})$$

[½]

$$12P(.95\ddot{a}^{(12)}_{[35]:\overline{30}|}) - 4.75P = \frac{100,000}{1.04} \times 1.04^{0.5} \times A^{@0\%}_{[35]} + 225 \times 1.04^{0.5} \times A^{@4\%}_{[35]} + 275$$

where $\ddot{a}^{(12)}_{[35]:\overline{30}|} = \ddot{a}_{[35]:\overline{30}|} - \frac{11}{24}\left(1 - v^{30}\ {}_{30}p_{[35]}\right) = 17.631 - \frac{11}{24}\left(1 - 0.30832 \times \frac{8821.2612}{9892.9151}\right)$

$= 17.631 - \frac{11}{24}\left(1 - 0.27492\right) = 17.2987$ [½]

and $A^{@0\%}_{[35]} = 1$ [½]

$$\Rightarrow 12P\ (.95 \times 17.2987) - 4.75P = \frac{100,000}{(1.04)^{0.5}} + 225 \times 1.04^{0.5} \times 0.19207 + 275$$

$$192.4552P = 98,058.0676 + 44.0716 + 275$$

$\Rightarrow P' = \frac{98,377.1392}{192.4552} = £511.17$ [½]

[Total 5]

(b) Let b be the simple bonus per annum for version B of the contract that can be supported by a monthly premium P = 511.17. Then equation of value (at 4% p.a. interest) is:

$$12P(.95\ddot{a}^{(12)}_{[35]:\overline{30}|}) - 4.75P = (100,000(1-b) + 225)\bar{A}_{[35]} + 100,000b(I\bar{A})_{[35]} + 275$$

[$1\frac{1}{2}$] [1] [$\frac{1}{2}$] [1] [$\frac{1}{2}$]

$$\Rightarrow 192.4552P = (100,000(1-b) + 225) \times 1.04^{0.5} \times 0.19207 + 100,000b \times 1.04^{0.5} \times 7.47005 + 275$$

$$98,377.32 = 19,587.374(1-b) + 44.0716 + 761,798.61b + 275$$

$\Rightarrow b = \frac{78,470.88}{742,211.24} = .1057 \Rightarrow 10.6\%$ [$\frac{1}{2}$]

[Total 5]

> *Part (a) was generally done well although some students struggled to include all the required parameters in the equation of value.*
>
> *Part (b) proved challenging for many despite the fact that it was in reality a quite simple formula.*

Q13

(a) Annual net premium for the decreasing term assurance is given by:

$$P = \frac{520,000A^{1}_{40:\overline{25}|} - 20,000(IA)^{1}_{40:\overline{25}|}}{\ddot{a}_{40:\overline{25}|}}$$

$$\textit{where } A^{1}_{40:\overline{25}|} = A_{40:\overline{25}|} - v^{25}\,{}_{25}p_{40}$$

$$= 0.38907 - 0.37512 \times \frac{8821.2612}{9856.2863} = 0.38907 - 0.33573 = 0.05334$$

$$\textit{and } (IA)^{1}_{40:\overline{25}|} = (IA)_{40} - v^{25}\,{}_{25}p_{40}\left[25A_{65} + (IA)_{65}\right]$$

$$= 7.95699 - 0.33573 \times \left[25 \times 0.52786 + 7.89442\right] = 0.87612$$

$$P = \frac{520,000 \times 0.05334 - 20,000 \times 0.87612}{15.884} = 643.06$$

2½ marks for premium formula, ½ mark for evaluating the first assurance function, 1 mark for evaluating the second assurance function and ½ mark for correct answer for *P* [4.5]

(b) Reserve at the <u>end</u> of the 17th policy year given by:

$${}_{17}V = 180,000A^{1}_{57:\overline{8}|} - 20,000(IA)^{1}_{57:\overline{8}|} - P\ \ddot{a}_{57:\overline{8}|}$$

$$\textit{where } A^{1}_{57:\overline{8}|} = A_{57:\overline{8}|} - v^{8}\,{}_{8}p_{57}$$

$$= 0.73701 - 0.73069 \times \frac{8821.2612}{9467.2906} = 0.73701 - 0.68083 = 0.05618$$

$$\textit{and } (IA)^{1}_{57:\overline{8}|} = (IA)_{57} - v^{8}\,{}_{8}p_{57}\left[8A_{65} + (IA)_{65}\right]$$

$$= 8.52268 - 0.68083 \times \left[8 \times 0.52786 + 7.89442\right] = 0.27286$$

$${}_{17}V = 180,000 \times 0.05618 - 20,000 \times 0.27286 - 643.06 \times 6.838$$

$$= 10,112.40 - 5,457.20 - 4,397.24 = 257.96$$

2½ marks for reserve formula, ½ mark for evaluating the first each assurance function, 1 mark for evaluating the second assurance function and ½ mark for correct answer for *V* [4]

Therefore, sum at risk per policy in the 17th policy year is:

DSAR = 180,000 – 257.96 = 179,742.04

[1]

Mortality profit = EDS – ADS

$$EDS = 1527 \times q_{56} \times 179,742.04 = 1527 \times 0.005025 \times 179,742.04 = 1,379,192.13$$

$$ADS = 9 \times 179,742.04 = 1,617,678.36$$

[1]

i.e. mortality profit = - 238,486.23 (i.e. a loss) [0.5]

Generally well done by students who had prepared. The main error was establishing the correct duration and sum assured for the reserve in (b).

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2018

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
June 2018

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

B. **General comments on *student performance in this diet of the*** examination

In general, students who had prepared properly for this examination did well and passed relatively easily. However, it was clear that a large number of students were sitting CT5 in advance of the transition to Curriculum 19 (there was a virtually double normal intake this session). There was considerable evidence that many such attempts were poorly prepared for, resulting in very low marks.

Most questions were easily answered. The ones which gave most difficulty were Q7, 8(ii), 9(ii), 10 and 12(i) and it is hoped the detailed solutions will assist students in future preparation.

C. Pass Mark

The Pass Mark for this exam was 57.

Solutions

Q1 $\quad {}_{2.75}p_{84.5} = {}_{0.5}p_{84.5} \times p_{85} \times p_{86} \times {}_{0.25}p_{87}$

$$= (p_{84})^{0.5} \times p_{85} \times p_{86} \times (p_{87})^{0.25}$$

$$= (1-q_{84})^{0.5} \times (1-q_{85}) \times (1-q_{86}) \times (1-q_{87})^{0.25}$$

$$= (1-0.101007)^{0.5} \times (1-0.110600) \times (1-0.120929) \times (1-0.132028)^{0.25}$$

$$= 0.94815 \times 0.88940 \times 0.87907 \times 0.96522$$

$$= 0.71553$$

$${}_{2.75}q_{84.5} = 1 - {}_{2.75}p_{84.5}$$

$$= 0.28447$$

[2 marks for first 2 lines; 1 for result]
[Total 3]

Straightforward and generally well answered. The most common error was not using the method asked for and using the uniform distribution of deaths method instead. This was given minor credit if done correctly.

Q2 The following are three types of reversionary bonuses. The bonuses are usually allocated annually in arrears, following a valuation.

Simple – the rate of bonus each year is a percentage of the initial (basic) sum assured under the policy. The effect is that the sum assured increases linearly over the term of the policy. [1]

Compound – the rate of bonus each year is a percentage of the initial (basic) sum assured and the bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy. [1]

Super compound – two compound bonus rates are declared each year. The first rate (usually the lower) is applied to the initial (basic) sum assured. The second rate is applied to bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy. The sum assured usually increases more slowly than under a compound allocation in the earlier years and faster in the later years. [2]
[Total 4]

(Note: credit to be given if special reversionary bonus mentioned.)

Relatively well answered. To score full marks the exponential growth aspect in Super Compound needed to be mentioned.

Q3 Net retrospective reserve = accumulated value of single premium less accumulated value of benefits provided [½]

$$= \frac{l_{[50]} \times (1+i)}{l_{[50]+1}} \left[7500 a_{[50]:\overline{10|}} - 7500 a_{[50]:\overline{1|}} \right]$$ [1½]

$$= 7500 \times \frac{9706.0977}{9686.9669} \times 1.04 \times 7.0048 = 54,745.03$$ [½]

where $a_{[50]:\overline{10|}} = \ddot{a}_{[50]:\overline{10|}} - 1 + v^{10} \frac{l_{60}}{l_{[50]}} = 8.318 - 1 + 0.67556 \times \frac{9287.2164}{9706.0977} = 7.9644$ [1]

and $a_{[50]:\overline{1|}} = v \frac{l_{[50]+1}}{l_{[50]}} = 0.96154 \times \frac{9686.9669}{9706.0977} = 0.9596$ [½]

[Total 4]

Reasonable marks scored but many had errors in the first formula above, particularly using just 7500 without the 1 year annuity factor in accumulated value of benefits provided.

Q4

- Occupation determines a person's environment for 40 or more hours each week.
    - The environment may be rural or urban.
    - The occupation may involve exposure to harmful substances e.g. chemicals, or to potentially dangerous situations e.g. working at heights.
    - Much of this is moderated by health and safety at work regulations.

- Some occupations are more healthy by their very nature e.g. bus drivers have a sedentary and stressful occupation while bus conductors are more active and less stressed.

- Some work environments e.g. publicans, give exposure to a less healthy lifestyle.

- Some occupations by their very nature attract more healthy or unhealthy workers. This may be accentuated by health checks made on appointment or by the need to pass regular health checks e.g. airline pilots.

- This effect can be produced without formal checks, e.g. former miners who have left the mining industry as a result of ill-health and then chosen to sell newspapers. This will inflate the mortality rates of newspaper sellers.

- A person's occupation largely determines their income, and this permits them to adopt a particular lifestyle e.g. content and pattern of diet, quality of housing. This effect can be positive and negative e.g. over indulgence.

[1 for each bullet; maximum 5]

Most students covered some of the points but generally omitted some as well. Other valid points were credited especially the relationship between level of education and health.

Q5 (i) The death strain at risk (DSAR) per annuity is given by:

$$\left[0-(\text{payment due 31.12.17}+\text{reserve at 31.12.17})\right]$$
$$=[0-30,000-30,000a_{73}]=-30,000\left(1+9.288\right)=-308,640$$ [1]

Expected death strain (EDS) =
$$-q_{72}\times 5,650\times 308,640=-(0.01838)\times 5,650\times 308,640=-32,051,338$$ [1]

Actual death strain (ADS) $=-80\times 308,640=-24,691,200$ [1]

Profit = EDS–ADS = –32,051,338 + 24,691,200 = – 7,360,138 i.e. a mortality loss [1]

(ii) The company expected 103.8 deaths during 2017 and experienced fewer than this (i.e. 80). There is no death benefit for the annuity. However, there is a release of reserves on death, so fewer actual deaths than expected leads to a mortality loss. [2]

[Total 6]

Part (i) was done well by many well-prepared students. The explanation in (ii) was generally not well stated, especially the release of reserves point.

Q6 (i) (a) $E\left[\dfrac{1-v^{\min(K_x+1,n)}}{d}\right]$ [1]

(b) $E\left[v^{\min(T_x,T_y)}\right]$ [1]

where:

x = age first life, y=age second life
n = duration of temporary annuity
K_x = random variable for curtate duration of life
T_x , T_y = random variables for complete duration of life

[½ for each definition]
[Total 4]

(ii) (a) $\bar{Z} = \bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{T_x}|}$ if $T_y > T_x$

$= 0$ otherwise [2]

(b) $E(\bar{Z}) = \int_0^{\infty} (v^t\ {}_t p_{xy} \mu_{x+t} \bar{a}_{y+t}) dt$ [2]

[Total 8]

Generally well answered. There were many different correct ways to express the answers all of which were given credit.

Q7 (i) $$\text{EPV} = \frac{500 \times n \times (3 \times M_x^r + 2 \times M_x^i) + 500 \times (3 \times \bar{R}_x^r + 2 \times \bar{R}_x^i)}{D_x}$$ [3]

(ii) $$\text{EPV} = \frac{500 \times 15 \times (3 \times M_{45}^r + 2 \times M_{45}^i) + 500 \times (3 \times \bar{R}_{45}^r + 2 \times \bar{R}_{45}^i)}{D_{45}}$$

$$= \frac{500 \times 15 \times (3 \times 782 + 2 \times 334) + 500 \times (3 \times 13773 + 2 \times 3916)}{2329}$$

$$= \frac{22605000 + 24575500}{2329}$$

$$= 20257.8$$

[1 for line 1; 2 for calculation]

(iii) If C is the contribution then:

$$C \times \frac{\bar{N}_{45}}{D_{45}} = 20257.8$$

$$\Rightarrow C \times \frac{26693}{2329} = 20257.8$$

$$\text{Hence } C = \frac{20257.8 \times 2329}{26693} = 1767.52$$

[1 for line 2; 1 for calculation]
[Total 8]

> *Poorly answered for a straightforward question. . A very large percentage of students used pension annuity and salary scaled functions which was totally wrong given the question was about a lump sum benefit without scaling.*

Q8 (i) EPV $= 20000\int_0^{20}(0.03 \times e^{-0.08t})dt - 10000\int_0^{10}(0.03 \times e^{-0.08t})dt$

$$= 600 \times \left[-\frac{e^{-0.08t}}{0.08}\right]_0^{20} - 300 \times \left[-\frac{e^{-0.08t}}{0.08}\right]_0^{10}$$

$$= 7500 \times \left(1 - e^{-1.6}\right) - 3750 \times \left[1 - e^{-0.8}\right]$$

$$= 3750 + (3750 \times 0.44933) - (7500 \times 0.20190)$$

$$= 3920.8 \text{ or say } 3921.$$

[1 mark lines 1 and 2, ½ mark line 3 and ½ mark for answer]

(ii) First calculate 2nd moment at interest rate ∂ of 10%

Value $= (10000)^2 \int_0^{10} 0.03 \times e^{-0.13t}dt + (20000)^2 \times \int_{10}^{20} 0.03 \times e^{-0.13t}dt$

$$= 0.03 \times (10)^8 \times \left(\frac{1 - e^{-1.3}}{0.13}\right) + 0.03 \times 4 \times (10)^8 \times \left(\frac{e^{-1.3} - e^{-2.6}}{0.13}\right)$$

$$= 16787727.9 + 18300758.3$$

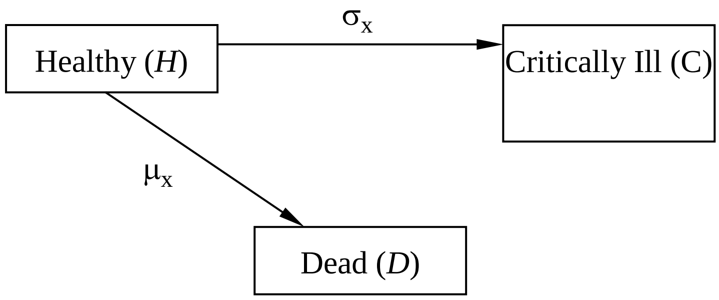
$$= 35088486.2$$

Variance $= 35088486.2 - (3920.8)^2 = 19715813.6 = (4440)^2$

[2 mark lines 1 and 2, 1 mark for answer]
[Total 8]

> *Part (i) was generally done well. Part (ii) was done less well as the approach was different to (i) with the necessity to return to first principles to ensure the squaring of the benefit was properly allowed for. The answer in (ii) was very susceptible to rounding and other close values were accepted provided the method was correct.*

Q9 (i)



[2½ mark plus ½ for not including the irrelevant mortality of C]

(ii) The expected present value of premiums is:

$$(750/12)\times\sum_{1}^{240} e^{-.046(t-1)/12} = (750/12)\times\left(\frac{1-e^{-0.92}}{1-e^{-(.046/12)}}\right)$$

$$= 62.5\times\left(\frac{1-0.398519}{1-0.996174}\right) = 9825.56$$

[3]

The expected present value of benefits is:

$$\text{EPV} = 100000\times\int_0^{20} e^{-0.04t}\, p_{45,t}^{HH}\, m_{45+t}\,dt + 50000\times\int_0^{20} e^{-0.04t}\, p_{45,t}^{HH}\, s_{45+t}\,dt$$

$$= (100000\times 0.004 + (50000\times 0.002))\times\int_0^{20} e^{-0.046t}\,dt$$

$$= 500\times 13.07567$$

$$= 6537.8$$ [2]

So value to company = 9,825.6 – 6,537.8 = 3,288 rounded [1]
[Total 9]

Part (i) was very straightforward. Part (ii) was more complex and many students struggled to develop the correct formulae.

Q10 Value of benefits:

$$\text{EPV} = v^{4.25} \times \frac{l_{65}}{l_{60.75}} \times \left[12,000 \times \ddot{a}_{\overline{5}|}^{(2)} + 12,000 \times v^5 \times \frac{l_{70}}{l_{65}} \times \ddot{a}_{70:\overline{5}|}^{(2)} + 10,000 \times v^{10} \times \frac{l_{75}}{l_{65}} \times (1 + \ddot{a}_{75}^{(2)}) \right]$$

$$= v^{4.25} \times \frac{9703.708}{9833.230} \times \left[12,000 \times 4.58483 + 12,000 \times 0.82193 \times \frac{9392.621}{9703.708} \times 4.471 + 10,000 \times 0.67556 \times \frac{8784.955}{9703.708} (1 + 10.683) \right]$$

$$= 141,303$$ [6]

where:

$$l_{60.75} = \left(0.75 l_{61} + 0.25 l_{60}\right) = \left(0.75 \times 9828.163 + 0.25 \times 9848.431\right) = 9833.230$$

$$\ddot{a}_{\overline{5}|}^{(2)} = \frac{\left(1 - v^5\right)}{d^{(2)}} = \frac{(1 - 0.82193)}{0.038839} = 4.58483$$

$$\ddot{a}_{70:\overline{5}|}^{(2)} = (\ddot{a}_{70} - \frac{1}{4}) - v^5 \times \frac{l_{75}}{l_{70}} \times (\ddot{a}_{75} - \frac{1}{4})$$

$$= (12.934 - 0.25) - 0.82193 \times \frac{8784.955}{9392.621} \times (10.933 - 0.25)$$

$$= 4.471$$

$$\ddot{a}_{75}^{(2)} = 10.933 - 0.25 = 10.683$$

Value of premiums:

Let quarterly premium = P

Value of premiums:

$$= P \times (1 + v^{1/4} \times \frac{l_{61}}{l_{60.75}} \times 4 \times \ddot{a}^{(4)}_{61:\overline{4|}})$$

$$= P \times (1 + 0.99025 \times \frac{9828.163}{9833.230} \times 4 \times ((16.311 - 0.375)$$

$$- 0.85480 \times \frac{9703.708}{9828.163} \times (14.871 - 0.375)))$$

$$= P \times (1 + 3.95896 \times (15.936 - 12.234))$$

$$= 15.656P$$

Hence:

Quarterly premium = 141,303/ 15.656 = 9026 or 9030 approximately [3]

[Total 9]

This higher skills question was generally poorly done. Credit was given for correct formulae and approach even if all the arithmetic was not accurate.

Q11 (i) Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[35]:\overline{30|}} \ @\ 4\% = 207.5844P$$

where:

$$\ddot{a}^{(12)}_{[35]:\overline{30|}} = \ddot{a}_{[35]:\overline{30|}} - \frac{11}{24}\left(1 - {}_{30}p_{[35]}v^{30}\right)$$

$$= 17.631 - \frac{11}{24}\left(1 - \frac{8821.2612}{9892.9151} \times 0.30832\right) = 17.2987$$ [2]

EPV of benefits and claim expense:

$$= 90,295 \times \bar{A}^{1}_{[35]:\overline{30|}} + 45,000 \times v^{15} \times {}_{15}p_{[35]}\bar{A}^{1}_{50:\overline{15|}}$$

$$= 90,295 \times (1.04)^{0.5}\left[A_{[35]} - v^{30} \times {}_{30}p_{[35]}A_{65}\right]$$

$$+45,000\times(1.04)^{0.5}\times v^{15}\times {}_{15}p_{[35]}\left[A_{50}-v^{15}\times {}_{15}p_{50}A_{65}\right]$$

$$=90,295\times(1.04)^{0.5}\left[0.19207-0.30832\times\frac{8821.2612}{9892.9151}\times0.52786\right]$$

$$+45,000\times(1.04)^{0.5}\times0.55526$$
$$\times\frac{9712.0728}{9892.9151}\left[0.32907-0.55526\times\frac{8821.2612}{9712.0728}\times0.52786\right]$$

$$=90,295\times(1.04)^{0.5}\times0.04695+45,000\times(1.04)^{0.5}\times0.54511\times0.06285$$

$$=4323.306+1572.239=5895.545$$ [3]

EPV of expenses (at 4% unless otherwise stated)

$$=0.5\times12P+375+0.025\times12P\ddot{a}^{(12)}_{[35]:\overline{30}|}-0.025\times12P\ddot{a}^{(12)}_{[35]:\overline{1}|}+72\left[\ddot{a}^{@0\%}_{[35]:\overline{30}|}-1\right]$$

$$=6P+375+0.025\times12P\times17.2987-0.025\times12P\times0.98212+72\times28.1751$$

$$=6P+375+5.18961P-0.29464P+2028.607$$

$$=10.89497P+2403.607$$

where:

$$\ddot{a}^{(12)}_{[35]:\overline{1}|}=\ddot{a}_{[35]:\overline{1}|}-\frac{11}{24}\left(1-v\times p_{[35]}\right)$$

$$=1-\frac{11}{24}\left(1-\frac{9887.2069}{9892.9151}\times0.96154\right)=0.98212$$

$$\ddot{a}^{@0\%}_{[35]:\overline{30}|}-1=\frac{1}{l_{[35]}}\left(l_{[35]+1}+....+l_{64}\right)=e_{[35]}-\frac{l_{64}}{l_{[35]}}e_{64}$$

$$=43.909-\frac{8934.8771}{9892.9151}\times17.421=28.1751$$ [4½]

Equation of value gives:

$$207.584P=5895.545+10.89497P+2403.607$$

$$\Rightarrow P=\frac{8299.152}{196.689}=42.19$$ [½]

[Total 10]

Well done by fully prepared students. Credit was given for developing the correct methods.

Q12 (i)

$$150,000\left[1+0.04\left(K_{[25]}+1\right)\right]v^{T_{[25]}}+315v^{T_{[25]}}+265+0.05P\left(\ddot{a}_{\overline{K_{[25]}+1}|}-1\right)-P\ddot{a}_{\overline{K_{[25]}+1}|}$$

[1] [½] [½] [½] [½]

where P is the annual premium.

(ii) The annual gross premium P is given by:

$$P\ddot{a}_{[25]}=150,315\bar{A}_{[25]}+6,000\left(I\bar{A}\right)_{[25]}+265+0.05Pa_{[25]}$$ [3]

$$\Rightarrow P\times 16.662=150,315\times 1.06^{0.5}\times 0.05686+6,000\times 1.06^{0.5}\times 2.40151$$
$$+265+0.05P\times(16.662-1)$$

$$\Rightarrow 15.8789P=8799.583+14835.035+265$$

$$\Rightarrow P=1505.12$$ [1]

(iii) After 5 years, the total bonus on the policy is given by:

$$150,000\times\left[0.04\times 2+0.0375+0.035+0.03\right]=27,375$$

Therefore, the gross premium prospective reserve is:

$$177,700\bar{A}_{30}+4,500\left(I\bar{A}\right)_{30}-0.95\times 1,505.12\times\ddot{a}_{30}$$ [3]

$$=177,700\times 1.04^{0.5}\times 0.16023+4,500\times 1.04^{0.5}\times 6.91559$$
$$-0.95\times 1,505.12\times 21.834$$

$$=29,036.745+31,736.456-31,219.651$$

$$=29,553.55$$ [1]

[Total 11]

Part (i) was poorly done-students traditionally have problems in developing random variable expressions. The other parts were done well by properly prepared students.

Q13 Multiple decrement table

x	q_x^d	q_x^s
62	0.010112	0.100
63	0.011344	0.050
64	0.012716	0.000

x	$(aq)_x^d$	$(aq)_x^s$	$(ap)_x$	${}_{t-1}(ap)_{62}$
62	0.010112	0.09899	0.890899	1.000000
63	0.011344	0.04943	0.939223	0.890899
64	0.012716	0.00000	0.987284	0.836753

[2]

Unit fund

Year	*1*	*2*	*3*
Fund at the start of the year	0	5332.635	10875.909
Premium allocation	5400.000	5400.000	5400.000
B/O spread	270.00	270.00	270.00
Interest	256.500	523.132	800.295
Management charge	53.865	109.858	168.062
Fund at the end of the year	5332.635	10875.909	16638.142

[½ mark for each line]
[Total 3]

Non-unit fund before reserves

Year	*1*	*2*	*3*
Unallocated premium + B/O spread	870.00	870.00	870.00
Expenses	525.00	215.00	215.00
Interest	10.350	19.650	19.650
Extra death cost	67.420	12.752	0
Extra maturity cost	0	0	1642.657
Management charge	53.865	109.858	168.062
End of year cash flow	341.795	771.756	–799.945

[½ mark for each line; ½ extra for death cos]
[Total 4]

Reserve required at the start of year 3 = 799.945 / 1.03 =776.646 [1]

Reduced profit at the end of year 2 = 771.756 – 776.646 × $(ap)_{63}$ = 42.312 [1]

Revised profit vector: $(341.795, 42.312, 0)$ [½]

Net present value = $\frac{341.795}{1.07} + \frac{42.312 \times (ap)_{62}}{1.07^2} = 352.359$ [1½]

Present value of premiums = $6000 \times \left(1 + \frac{(ap)_{62}}{1.07} + \frac{{}_2(ap)_{62}}{1.07^2}\right) = 15,380.812$

[1½]

Profit margin = $\frac{352.359}{15380.812} = 2.29\%$ [½]

[Total 6]
[Total 15]

This question was generally done reasonably well by prepared students. The main errors other than arithmetical were omitting calculating dependant decrements and the revision of the profit vector. Again the correct approach earned substantial credit even where there were arithmetic errors.

END OF EXAMINERS' REPORT