(4) interest as $((2)+(3))i$ (*ie* interest earned on the profit on allocation less the expenses incurred)

(5) expected death cost as $q_{50+t-1} \times \max(5{,}000-(8)\text{ in unit fund table}, 0)$

(6) management charge from (7) in unit fund table

(7) expected profit as the sum of all the above.

Be careful about signs here. From the point of view of the company (*ie* the non-unit fund's point of view), the management charge is a positive item because it is something deducted from the unit fund (the 'policyholder's fund') – although it was a negative when we were considering it in the context of the unit fund.

One way of thinking about all of these items is 'if it's good for profits, it must be positive' – so a management charge must be positive, because if the company increases the management charge then profits go up. Similarly the death cost must be negative: if there are more deaths, company profits go down.

1.4 Example 4: Single premium unitised with-profits contract

Unitised with-profits contracts operate in a similar way to unit-linked contracts, in that the benefits are expressed as an accumulating (unit) fund of premiums and explicit charges are deducted to cover expenses and other costs. Non-unit cashflows can therefore be projected in the same way as for unit-linked contracts, and used in profit testing as described in Section 2 below.

For this example we assume the contract has a five-year term, paid for by a single premium. The allocation rate is 100% and there is no bid-offer spread. The only charges under the contract are a policy fee equal to 0.5% of the bid value of the unit fund, deducted at the end of each year by cancelling units, and a penalty on surrender calculated as a percentage of the unit fund value depending on policy duration as follows:

After 1 year: 2.4%

After 2 years: 1.8%

After 3 years: 1.2%

After 4 years: 0.6%

The unit price increases each year according to the insurer's declared bonus interest rate. The full value of the unit fund, plus any discretionary terminal bonus, is paid out on maturity or at the end of the year of earlier death. Surrender is permitted only at the end of each year, when the fund value less the above surrender penalty is payable.

We will assume that, over the long term, all profits earned from investment returns are fully distributed to policyholders through the bonus payments (we explain the significance of this assumption at the end of this section, below).

We shall calculate the projected profit for each policy year based on the following assumptions:

Age at entry:	45 exact
Single premium:	£20,000
Mortality:	$q_x = 0.002$ for $x = 45, 46, \ldots, 49$
Surrender rate:	5% of all policies in force at the end of each of Years 1-4
Bonus interest rate:	3.5% *pa*
Terminal bonus rate:	Nil
Non-unit interest:	2% *pa*
Initial expenses:	£300
Renewal expenses:	£25 at the start of Year 2, and thereafter at the start of each subsequent year, inflating at 2.5% *pa*
Claim expenses:	£100 per death, surrender, or maturity
Non-unit reserves:	£50 per policy in force at the start of Year 5

The first thing we need to do is to project the unit fund values at the end of each year allowing for the deduction of the policy fee. At the end of the first year, the fund value (before deduction of the policy fee) is the single premium plus one year's bonus:

$$20{,}000 \times 1.035 = 20{,}700$$

The policy fee at the end of the year is:

$$-20{,}700 \times 0.005 = -103.50$$

This means that the fund remaining at the end of the year is:

$$20{,}700 - 103.50 = 20{,}596.50$$

Working through the other years in a similar way, we obtain the following table of projected values:

Year *t*	Fund at end of year before deduction of policy fee (A)	Policy fee (B)	Fund at end of year after deduction of policy fee (C)
1	20,700	–103.50	20,596.50
2	21,317.38	–106.59	21,210.79
3	21,953.17	–109.77	21,843.40
4	22,607.92	–113.04	22,494.88
5	23,282.20	–116.41	23,165.79

where, for years $t = 2, 3, 4, 5$:

$$(A)_t = (C)_{t-1} \times 1.035$$

$$(B) = (A) \times 0.005$$

$$(C) = (A) + (B)$$

Next we need to work out the surrender penalties. These are equal to the end-year fund values (after charges) multiplied by the appropriate percentage rates:

Year t	Fund at end of year	Surrender penalty
1	20,596.50	494.32
2	21,210.79	381.79
3	21,843.40	262.12
4	22,494.88	134.97
5	23,165.79	0

We will also need the dependent probabilities of decrement by death and surrender. Because surrenders take place at the end of each year, the dependent probability of dying is the same as the independent probability, *ie* equal to 0.002 each year. The dependent surrender probability is:

$$(1 - 0.002) \times 0.05 = 0.0499$$

as we are told that 5% of the *end* of year in-force policies surrender each year.

The dependent probability of a policy staying in force over any particular year is then:

$$1 - 0.002 - 0.0499 = 0.9481$$

(or alternatively this can be calculated as $(1 - 0.002) \times (1 - 0.05) = 0.9481$).

The non-unit cashflows can now be calculated. For the first four years of the contract, these are as follows:

Year t	Initial and renewal expenses (1)	Interest (2)	Policy fee (3)	Expected surrender profit (4)	Expected claim expenses (5)	Expected non-unit cashflow (6)
1	–300	–6	103.5	24.67	–5.19	–183.02
2	–25	–0.5	106.59	19.05	–5.19	94.95
3	–25.62	–0.51	109.77	13.08	–5.19	91.53
4	–26.27	–0.53	113.04	6.73	–5.19	87.78
5						

where:

$$(1)_t = (1)_{t-1} \times 1.025 \quad (3 \le t \le 5)$$

$$(2) = (1) \times 0.02$$

$$(3) = \text{from table of fund calculations}$$

$$(4) = \{\text{surrender penalty}\} \times 0.0499$$

$$(5) = -100 \times (0.002 + 0.0499) \quad (1 \le t \le 4)$$

$$(6) = (1) + (2) + (3) + (4) + (5)$$

Question

Calculate the table entries for Year 5.

Solution

The expenses in Year 5 are the Year 4 expenses increased by inflation at 2.5%. The amount is therefore:

$$-26.27 \times 1.025 = -26.93$$

The interest is equal to the interest lost over the year due to the expenses incurred at the start of the year. Its amount is:

$$-26.93 \times 0.02 = -0.54$$

The policy fee income is read from the fund calculation table, and for Year 5 this is 116.41.

As there is no surrender penalty (and no-one surrenders!) then the expected surrender profit from the surrender penalty is zero.

The claim expenses of 100 are paid out on all policies that become claims during the year. All policies in force at the start of Year 5 will ultimately claim at the end of the year – either by surviving and receiving the maturity benefit, or by dying during the year and receiving the death benefit. So the expected amount of claim expenses is –100.

The expected cashflow for Year 5 is then calculated by summing across all of the above items:

$$= -26.93 - 0.54 + 116.41 - 100 = -11.06$$

The completed cashflow table now reads as follows:

Year t	Initial and renewal expenses (1)	Interest (2)	Policy fee (3)	Expected surrender profit (4)	Expected claim expenses (5)	Expected non-unit cashflow (6)
1	−300	−6	103.5	24.67	−5.19	−183.02
2	−25	−0.5	106.59	19.05	−5.19	94.95
3	−25.62	−0.51	109.77	13.08	−5.19	91.53
4	−26.27	−0.53	113.04	6.73	−5.19	87.78
5	−26.93	−0.54	116.41	0	−100	−11.06

Having worked out the expected cashflows, we now need to calculate the expected profit for each year, taking into account the effect of the non-unit reserves.

For this policy, we are told that a non-unit reserve of 50 is required per policy in force at the start of Year 5 (and also therefore per policy in force at the *end* of Year 4); at all other times no non-unit reserve is to be held.

This will affect the expected profit in Year 4. For a policy in force at the start of Year 4, the insurer expects a cashflow of 87.78 at the end of the year. However, the insurer now has to set aside reserves of 50 for each policy that's remained in force through to the end of the year. The probability of doing this is 0.9481 (from above).

So the expected profit for Year 4 (per policy in force at the start of that year) reduces to:

$$87.78 - 50 \times 0.9481 = 40.38$$

Question

Calculate the expected profit at the end of Year 5, per policy in force at the start of Year 5, allowing for the reserve of 50 held at the start of that year.

Solution

The expected profit for Year 5 will equal:

$$\{\text{expected cashflow}\} + \{\text{reserve at start of year}\} + \{\text{interest on reserve}\}$$
$$- \{\text{expected cost of reserve at end of year}\}$$

Putting in the values (and noting that the reserve at the end of the year is zero) we obtain:

$$-11.06 + 50 + 0.02 \times 50 - 0 = 39.94$$

As there are no reserves held at any time in Years 1-3, the expected profits for these years are all equal to the expected cashflows shown in the table above.

Non-unit reserves, such as in this example, are required for unit-linked and unitised with-profits policies wherever a future negative cashflow is expected.

Question

Given that there is an expected negative cashflow of 11.06, it might seem logical to hold the smallest reserve possible to cover this expected cost, *ie* 11.06 rather than the 50 actually held. Suggest a reason why the insurer might use the higher figure.

Solution

All reserves need to be prudent, in order for there to be a high probability that the liabilities (future outgo) will be covered. The insurer will therefore project its cashflows on more cautious assumptions leading to a somewhat higher negative cashflow than 11.06 in Year 5, and as a result may decide that a reserve of 50 is necessary to cover this.

The reason for profit-testing unitised with-profits contracts

As explained in an earlier chapter, the idea of a with-profits contract is to return to the policyholder the profits earned by the insurer on the policy over the policy term. One approach for unitised with-profits is to earmark the investment profits for policyholders (via the unit fund), and deduct explicit charges to cover the insurer's non-unit expenses and other costs, as in the example we have just looked at. By projecting the future non-unit profits, the insurer can check that its charges are on track to cover these outgoes.

This was why in the above example we assumed that all the investment profits were distributed to policyholders. An exam question may or may not state this assumption explicitly, but it generally would be implied, as the main point of profit testing these contracts in this way is to 'test' the adequacy of the charges in covering the non-unit liabilities.

2 Profit tests for annual premium contracts

Having now considered how to project the revenue accounts for a policy, we see how to use that projection.

The first step in the profit testing of a contract is the construction of the projected revenue accounts for the non-unit (cash) fund for each policy year.

For some contracts, other funds (*eg* unit fund for unit-linked assurances, reserves for traditional assurances) provide cashflows to the non-unit fund.

For conventional (or 'traditional') products, the whole profit test is really a 'non-unit fund projection', where one of the elements is the interest on reserves.

In such cases these funds will need to be projected so that the expected cashflows to the non-unit fund can be determined. These calculations will require data items about the contract, *eg* proportion of premium allocated to purchase of units, bid-offer spread in unit prices; and assumptions which form a basis for the calculations, *eg* growth rate of unit fund, mortality and interest rate basis used to calculate required reserves.

These expected cashflows, together with the direct expected cashflows into and out of the non-unit fund, are the components of the projected revenue account. The calculation of the direct expected cashflows will also require data items about the contract, *eg* initial and renewal expenses; and assumptions to form a basis, *eg* mortality of policyholders, rate of return earned on non-unit fund.

Profit vector

The vector of balancing items in the projected revenue accounts for each policy year is called the profit vector; $(PRO)_t$, $t = 1, 2, 3, \ldots$. The profit vector gives the expected profit at the end of each policy year per policy in force at the beginning of that policy year.

For example, the conventional endowment assurance policy studied in Section 1.2 gave the following profit vector:

$$(-803.99, 186.97, 178.12, 206.47, 215.23)$$

Profit signature

The vector of expected profits per policy issued is called the profit signature. This is obtained by multiplying the profit vector by the probability of a policy remaining in force from policy duration 0 to policy duration $t-1$, *ie*:

$$(PS)_t = {}_{t-1}(ap)_x \, (PRO)_t$$

where $(PS)_t$ denotes the tth entry in the profit signature and $(PRO)_t$ denotes the tth entry in the profit vector.

Summarising the above, we have the very important distinction:

- profit vector = vector whose entries represent the expected profit at the end of each year per policy in force at the start of that year
- profit signature = vector whose entries represent the expected profit at the end of each year per policy in force at inception.

For example, for the conventional endowment assurance policy studied in Section 1.2, we found the second element of the profit vector to be £186.97.

To obtain the corresponding entry in the profit signature, we multiply the entry in the profit vector by the probability that the policy is still in force at the start of Year 2. This is:

$$186.97 \times (ap)_{55}$$

Question

Use the relevant table of dependent decrement probabilities from Section 1.2 (reproduced below) to calculate the required probability, and hence calculate the value of the second entry in the profit signature.

Age x	$(aq)_x^d$	$(aq)_x^w$
55	0.005	0.1
56	0.006	0.05
57	0.007	0.05
58	0.008	0.01
59	0.009	0

Solution

The probability that a policy is still in force at the start of Year 2 is:

$${}_1(ap)_{55} = 1 - (aq)_{55}^d - (aq)_{55}^w = 1 - 0.005 - 0.1 = 0.895$$

So the second entry in the profit signature is:

$$186.97 \times {}_1(ap)_{55} = 186.97 \times 0.895 = 167.34$$

The other elements of the profit signature can be obtained in the same way: by multiplying the profit vector element for a given year by the *probability of a policy remaining in force from policy inception to the start of that year*.

Question

Calculate the other elements of the profit signature for the conventional endowment insurance policy studied in Section 1.2.

Solution

The first element of the profit signature is unchanged from the profit vector, because the amount (-803.99) is already the amount per policy in force at the beginning of year 1, *ie* at issue.

For Year 3 we need:

$$(PS)_3 = (PRO)_3 \times {}_2(ap)_{55}$$

We calculate $(ap)_{55}$, $(ap)_{56}$, *etc* from:

$$(ap)_x = 1-(aq)_x^d-(aq)_x^w$$

(using the dependent probabilities given in Section 1.2); and ${}_2(ap)_{55}$ (for example) from the cumulative probability:

$${}_2(ap)_{55} = (ap)_{55}\,(ap)_{56}$$

The required probabilities are:

Year t	$(ap)_{54+t}$	${}_{t-1}(ap)_{55}$
1	0.895	1
2	0.944	0.895
3	0.943	0.8449
4	0.982	0.7967
5	not required	0.7824

(For example, $0.7967 = 0.895 \times 0.944 \times 0.943$.)

The tth entry of the profit signature is obtained by multiplying the tth entry of the profit vector by ${}_{t-1}(ap)_{55}$:

Year t	Profit in year t	${}_{t-1}(ap)_{55}$	Profit signature
1	–803.99	1	–803.99
2	186.97	0.895	167.34
3	178.12	0.8449	150.49
4	206.47	0.7967	164.49
5	215.23	0.7824	168.40

So the profit signature is (–803.99, 167.34, 150.49, 164.49, 168.40).

The vector representing the profit signature $(PS)_t$; $t = 1, 2, 3, \dots$ can be displayed graphically to illustrate the way in which profits are expected to emerge over the lifetime of the policy. However, it is difficult to compare this information for different policies when there is a need to evaluate alternative designs for a product (policy) or to decide which of several different possible policies is the most profitable. Decisions like this are usually made easier by summarising each profit signature as a single figure. We describe three such summary measures below.

2.1 Summary measures of profit

Summary measures usually involve determining the present values of the expected cashflows. In some cases, this requires an assumption about the discount rate. This rate is chosen to equal the cost of capital plus a risk premium, and is called the risk discount rate, i_d.

The cost of capital is the rate at which funds can be borrowed, or the rate of return such funds would earn if invested elsewhere, (*ie* the 'opportunity cost').

The risk premium reflects the risks and uncertainties surrounding the cashflows to and from the policy.

Writing a policy can be thought of as an investment by the shareholders of the company, because they supply the capital to make good the shortfall between the premium income and the outgo of expenses and the setting up of reserves. If the shareholders are providing capital, then they expect a return on that capital appropriate to the riskiness of their investment. It is much riskier investing in life insurance business than buying government bonds, because more things could go wrong.

To allow for the extra risk, we add a margin to the investment returns on relatively risk-free assets such as government bonds, and then price the product using the resulting risk discount rate. This will then give us premiums that contain an adequate allowance for the risk.

We can now find the price of the product by projecting cashflows, and then varying the premium amount until we meet the profit criterion (as described below) which has been calculated using our risk discount rate.

Net present value (NPV)

This is the present value of the profit signature determined using the risk discount rate.

$$\text{NPV} = \sum_{t=1}^{t=\infty} (1+i_d)^{-t} (PS)_t$$

The NPV can be interpreted as the EPV of the future profits from the policy, for a single policy as at the start date of the contract.

Question

Calculate the net present value of the conventional endowment assurance policy studied in Section 1.2, using a risk discount rate of 7%.

Solution

Discounting the profit signature at 7%, we obtain:

Year t	Profit signature	Discount factor v^t	Discounted profit
1	–803.99	0.9346	–751.41
2	167.34	0.8734	146.16
3	150.49	0.8163	122.84
4	164.49	0.7629	125.49
5	168.40	0.7130	120.07
Total			–236.85

So the net present value of profits is –£236.85.

The expected profits for each year include all interest earned (on cashflows and reserves) during the year. So the profits are essentially already accumulated with interest to the end of each year, and all we have to do is to discount *from* the end of each year.

Question

Give two possible reasons why this net present value is negative.

Solution

One reason is that the pricing of the contract did not allow for withdrawals, and on early withdrawal the company will lose out as it has not recouped the initial expenses of the policy.

A second reason (which makes the answer more negative than it already is) is that we have valued profits using a risk discount rate that is much higher than the 4% interest rate used in calculating the reserves for the product. The effect of such differences is discussed in more detail in the next chapter.

Profit margin

This is the NPV expressed as a percentage of the EPV of the premium income. If the premium paid at the beginning of the t th policy year is P_t, this is:

$$\frac{\sum_{t=1}^{t=\infty} (1+i_d)^{-t} \, (PS)_t}{\sum_{t=1}^{t=\infty} (1+i_d)^{-(t-1)} \; {}_{t-1}(ap)_x \; P_t}$$

In other words, the profit margin is:

$$\frac{\text{NPV}}{\text{EPV premiums}}$$

where the risk discount rate is used to do the discounting in both the numerator and the denominator.

Question

Calculate the profit margin of the conventional endowment assurance policy studied in Section 1.2 , using a risk discount rate of 7%.

Solution

The discounted present value of the premiums is £8,059.11, calculated as in the table below. Note that, because premiums are payable in advance, the discount factors are based on the duration at the *start* of each year.

Year t	Premium	In force probability ${}_{t-1}(ap)_{55}$	Discount factor v^{t-1}	Discounted premium
1	2,108.81	1	1	2,108.81
2	2,108.81	0.895	0.93458	1,763.91
3	2,108.81	0.84488	0.87344	1,556.20
4	2,108.81	0.79672	0.81630	1,371.49
5	2,108.81	0.78238	0.76290	1,258.70
Total				8,059.11

So the profit margin is $\dfrac{-236.85}{8,059.11} = -2.9\%$.

The internal rate of return (IRR)

This has been described earlier in this subject in relation to non-contingent cashflows, but it can also be calculated for contingent cashflows, as here. It is defined as the discount rate that would make the NPV of the contract equal to zero.

The internal rate of return does not always exist. It does exist where the profit signature has a single financing phase, *ie* where the first profit flow is negative and the profit flows in all subsequent years are positive. This situation commonly occurs, however, and so the IRR is often a useful measure in practice.

Suppose that the profit signature on a five year life insurance policy is:

$$(-300,\ 100,\ 100,\ 100,\ 100)$$

The internal rate of return is the discount rate (r) such that:

$$\frac{-300}{1+r}+\frac{100}{(1+r)^2}+\frac{100}{(1+r)^3}+\frac{100}{(1+r)^4}+\frac{100}{(1+r)^5}=0$$

$$\Rightarrow -300+100a_{\overline{4}|}=0$$

$$\Rightarrow a_{\overline{4}|}=\frac{300}{100}$$

$$\Rightarrow \frac{1-(1+r)^{-4}}{r}=3$$

By trial and error, the value of r that satisfies this equation is 12.6%, so this is the IRR in this case.

This is the kind of calculation that is easy to do using a spreadsheet or other suitable computer software, and also using Table mode on many calculators.

Question

A policy with a 3-year term has a profit vector of $(-428,\ 72,\ 560)$. The policyholder is aged 55 exact at entry. Calculate the internal rate of return for this policy, assuming AM92 Select mortality.

Solution

First of all, we need to calculate the profit signature, by multiplying each element of the profit vector by the probability of the policy being in force at the start of each year. The vector of probabilities is:

$$\left(1,\ {}_1p_{[55]},\ {}_2p_{[55]}\right)$$

$$=\left(1,\ \frac{l_{[55]+1}}{l_{[55]}},\ \frac{l_{57}}{l_{[55]}}\right)$$

$$=\left(1,\ \frac{9{,}513.9375}{9{,}545.9929},\ \frac{9{,}467.2906}{9{,}545.9929}\right)$$

$$=\left(1,\ 0.996642,\ 0.991755\right)$$

Multiplying the profit vector by the probabilities we get the profit signature to be:

$$\left(-428\times 1,\ 72\times 0.996642,\ 560\times 0.991755\right)=\left(-428,\ 71.76,\ 555.38\right)$$

The IRR is then the rate of discount, r, that satisfies:

$$\frac{-428}{1+r}+\frac{71.76}{(1+r)^2}+\frac{555.38}{(1+r)^3}=0 \quad\Rightarrow\quad -428\times(1+r)^2+71.76\times(1+r)+555.38=0$$

Using the quadratic formula, we can solve for $1+r$:

$$1+r=\frac{-71.76\pm\sqrt{71.76^2-4\times(-428)\times 555.38}}{2\times(-428)}$$

for which the positive root is:

$$1+r=1.2260 \quad\Rightarrow\quad r=0.2260$$

That is, the IRR is 22.60% *pa*.

3 Profit testing using the present value random variable

3.1 Introduction

In this section we describe an alternative approach to calculating the expected present value of the future profits when profit testing. It is not the method usually used in practice but could come up in exam questions.

So far we have calculated the expected values of each year's future profit, and then discounted them to obtain the expected *present* value of the profits. These values sum to give the net present value (NPV), as we saw in Section 2.1 above.

As explained in Section 2.1, the NPV is the same as the expected present value (EPV) of the future profits. So, to arrive at the NPV, we could alternatively do our discounting first, and then take the expectation afterwards. We have seen this approach used for calculating EPVs almost everywhere else in this course.

3.2 Example

Let's see how this works for the 5-year conventional endowment assurance example first shown in Section 1.2. The first thing to do is to write down the present value as a random variable. This is not a trivial task.

To do this we consider what the present value would be on the occurrence of each possible event (contingency) that could happen to the policy during the potential 5 years of the policy. For example, the policyholder could die in the first policy year, in which case the policy would terminate after just one year with the payment of a death claim at the end of the year. If this occurred, then the profit for the policy would be calculated as:

Premium*	Expenses*	Interest*	Claim cost	Profit
2,108.81	− 1,054.41	42.18	− 10,000	− 8,903.42

* *as shown in the example in Section 1.2.*

We here deduct 10,000 for the death claim. This is because we are calculating the profit *if* the policyholder dies in Year 1, in which case the full sum assured is paid out at the end of the year.

The present value of the profit (discounting at the risk discount rate of 7% *pa*), *given* (*ie* if) the policy terminates by death in the first year, is then:

$$\left\{PV \middle| \text{Dies in first year}\right\} = \frac{-8{,}903.42}{1.07} = -8{,}320.95$$

Question

Calculate the present value of the profit assuming the policy is surrendered in Year 1.

Solution

The elements of the profit are the same as before, except instead of the death claim at the end of the year, the surrender value is paid out. The Year 1 surrender value is 2,108.81 (equal to the total premium paid to date), and so the profit is:

Premium	Expenses	Interest	Claim cost	Profit
2,108.81	− 1,054.41	42.18	− 2,108.81	− 1,012.23

So the present value of the profit, if the policy surrenders in Year 1, is:

$$\left\{\text{PV}\middle|\text{Surrenders in first year}\right\} = \frac{-1{,}012.23}{1.07} = -946.01$$

Next we calculate the present value of the profit if the policyholder dies in the second year. So we calculate:

Yr	Premium	Expense	Int	Reserve at start of year	Interest on reserve	Claim cost	Reserve at end of year
1	2,108.81	−1,054.41	42.18	0	0	0	−1,832.06
2	2,108.81	−105.44	80.13	1,832.06	73.28	−10,000	0

In the first year, we have to set up the reserve of 1,832.06 out of cashflows, so this is deducted from the Year 1 profit.

In Year 2, assets of 1,832.06 are available and, along with interest, contribute to the profit made in Year 2.

Summing each row gives the profit for each year. We then discount the two profit flows at the risk discount rate, and sum to give the total present value. The Year 1 figure is divided by 1.07 as it emerges at the end of Year 1, and the Year 2 value is divided by 1.07^2 as it emerges at the end of Year 2. So we obtain:

Yr	Profit	Present value
1	−735.48	−687.36
2	−6,011.16	−5,250.38
		−5,937.74

Continuing in this way for all possible future events, we arrive at the following table, which defines the probability distribution of the present value random variable for a single policy at outset.

Event	Present value of profit | Event occurs
Dies in Year 1	–8,320.95
Surrenders in Year 1	–946.01
Dies in Year 2	–5,937.74
Surrenders in Year 2	–887.18
Dies in Year 3	–3,757.17
Surrenders in Year 3	–758.44
Dies in Year 4	–1,764.93
Surrenders in Year 4	–571.18
Dies in Year 5	52.43
Stays in force to end of term	52.43

Question

Verify the present value of the profit assuming:

(a) the policy is surrendered in Year 3

(b) the policy stays in force to the end of the term.

Solution

(a) ***Present value if the policy is surrendered in year 3***

The present values of the profits for each of the three years, assuming the policy is surrendered in the third year, are shown in the following table:

Yr	Premium – Expense + Interest	Reserve at start of year	Interest on reserve	Claim cost	Reserve at end of year	Profit	Present value
1	1,096.58	0	0	0	–1,832.06	–735.48	–687.36
2	2,083.50	1,832.06	73.28	0	–3,740.46	248.39	216.95
3	2,083.50	3,740.46	149.62	–6,326.43	0	–352.85	–288.03
						Total	–758.44

In this case, the surrender value (equal to three times the annual amount of premium) is paid out at the end of Year 3, at which point the policy terminates.

The present values in the last column are equal to the profits for each year discounted at 7% *pa* to the start of the policy.

(b) ***Present value if the policy stays in force until the end of the term***

The present values of the profits for each of the five years, assuming the policy stays in force until the end of the term, are shown in the following table:

Yr	Premium – Expense + Interest	Reserve at start of year	Interest on reserve	Claim cost	Reserve at end of year	Profit	Present value
1	1,096.58	0	0	0	–1,832.06	–735.48	–687.36
2	2,083.50	1,832.06	73.28	0	–3,740.46	248.39	216.95
3	2,083.50	3,740.46	149.62	0	–5,736.10	237.48	193.86
4	2,083.50	5,736.10	229.44	0	–7.818.97	230.08	175.53
5	2,083.50	7,818.97	312.76	–10,000	0	215.23	153.46
						Total	52.43

In this case the policy pays out the maturity benefit of 10,000 at the end of the five-year period.

We now calculate the EPV as:

$$EPV = \sum_{\text{All events}} \left\{ \text{Present value of profit} \middle| \text{Event occurs} \right\} \times P\left(\text{Event occurs}\right) \quad (*)$$

So we need to calculate the probability that each event occurs. For this we need deferred dependent probabilities of the form:

$${}_{n|}(aq)^k_{55} = {}_n(ap)_{55}\,(aq)^k_{55+n} \qquad \text{for } k = d, w$$

The required calculations are shown in the following table:

n	${}_n(ap)_{55}$	$(aq)^d_{55+n}$	$(aq)^w_{55+n}$	${}_{n\|}(aq)^d_{55}$	${}_{n\|}(aq)^w_{55}$
0	1	0.005	0.1	0.005	0.1
1	0.895	0.006	0.05	0.00537	0.04475
2	0.84488	0.007	0.05	0.005914	0.042244
3	0.796722	0.008	0.01	0.006374	0.007967
4	0.782381	0.009	0	0.007041	0
5	0.775339				

Question

(i) Verify the probability that a policy issued at age 55 will terminate by death in the third policy year.

(ii) Calculate the expected present value of the profit using (*) above.

Solution

(i) ***Probability that policy will terminate by death in third year***

This probability is ${}_{2|}(aq)^d_{55} = {}_2(ap)_{55}\,(aq)^d_{57}$, where:

$${}_2(ap)_{55} = \left[1-(aq)^d_{55}-(aq)^w_{55}\right]\times\left[1-(aq)^d_{56}-(aq)^w_{56}\right]$$

$$=(1-0.005-0.1)\times(1-0.006-0.05)=0.84488$$

So:

$${}_{2|}(aq)^d_{55} = 0.84488\times 0.007 = 0.005914$$

(ii) ***Expected present value of profit***

The required calculations are shown in the following table:

Event	Present value of profit | Event occurs	Probability	EPV
D Yr 1	–8,320.95	0.005	–41.60
S Yr 1	–946.01	0.1	–94.60
D Yr 2	–5,937.74	0.00537	–31.89
S Yr 2	–887.18	0.04475	–39.70
D Yr 3	–3,757.17	0.005914	–22.22
S Yr 3	–758.44	0.042244	–32.04
D Yr 4	–1,764.93	0.006374	–11.25
S Yr 4	–571.18	0.007967	–4.55
D Yr 5	52.43	0.007041	0.37
Matures	52.43	0.775339	40.65
		Total	–236.83

This is the same (apart from rounding differences) as the value obtained for the net present value using the standard profit-testing approach shown in Section 2.1.

4 Pricing (*ie* determining premiums) using a profit test

The actual profitability is not known until each respective contract terminates and we know the actual experience. However, we can calculate the expected profitability using criteria set out in Section 4.1 below. To do this we need details of the premium, any charges and other relevant data items relating to the contract. A projection basis is also needed.

When products are designed, the expected level of profit will usually be specified as an objective and the features of the product can be set to achieve this objective. For conventional contracts, the benefits and attaching terms and conditions are usually specified in advance, therefore the key variable to determine when setting the profit objective is the level and pattern of premium payments. For unit-linked and unitised with-profits contracts the key variable is usually the charges, such as premium allocation rate and the management charge.

4.1 Profit criterion

The objective specified for the expected level of profit is termed the 'profit criterion'.

Careful choice of a profit criterion is central to the actuarial management of the company selling the products. It is common for those marketing and selling the products to receive part of their salary in the form of a 'productivity' bonus, *eg* commission which is a percentage of the total premiums for the policies sold. If the profit criterion chosen is directly related to this 'productivity' bonus, the company's profits will be maximised if the salesforce maximises its income. Such considerations are important in choosing the profit criterion to be used.

Examples of the profit criterion are:

NPV = 40% of Initial Sales Commission

Profit Margin = 3% of EPV of premium income

For conventional products, the profit test is completed using a spreadsheet or similar software, and the premiums are varied until the required 'target' *ie* NPV, level of profit margin, IRR, is achieved. The premium or price of the product has been determined using a profit test.

For unit-linked and unitised with-profits products, the management charges are varied to try to achieve an acceptable charging structure (in comparison with other products in the market) which satisfies the profit criterion.

It is common practice to investigate the sensitivity of this profit to variation in a number of factors such as the key features of the product design and/or the assumptions made in the projection basis. This is done by performing a 'sensitivity test' whereby each proposed premium or charging structure will be input into the model and the assumptions varied to determine the impact on the expected profit. The objective is to design a product which is robust and exhibits minimal variation in the expected profit to feasible changes in the data and the assumptions used. Sensitivity tests are covered in the next chapter.

So the steps in profit testing can be summarised as follows:

- decide on the structure of the product, *eg* single premium unit-linked deferred annuity
- build a model to project cashflows for the product
- choose some specimen policies – what age, sex, level of cover do we expect?
- decide on a risk discount rate and profit criterion
- choose a basis – probably our best estimate – of all important parameters required for the profit test, *eg* unit growth, levels of withdrawals, *etc*
- decide on some 'first draft' premiums (conventional product) / charges (unit-linked product)
- profit test our specimen policies using these premiums
- vary the premiums / charges until our profit criterion is met, remembering that premiums need to be acceptable in the market
- sensitivity test by varying key parameters (*eg* investment return, mortality, expense inflation) in the cashflow projection, to check that our product design is sufficiently resilient to adverse changes in future experience
- keep varying premiums and, if necessary, features of the product design until we have a product that:
  - meets the profitability criterion,
  - is marketable, and
  - is resilient to adverse future experience.

The chapter summary starts on the next page so that you can keep all the chapter summaries together for revision purposes.

Chapter 26 Summary

Projecting profit flows – conventional products

Per policy in force at start of each year, project expected amounts of:

(+) premiums
(–) expenses
(+) investment income
(–) benefit payouts (death, maturity, surrender)
(–) increase in reserves*
= profit vector

* Increase in reserves =

$$\{\text{probability of staying in force over the year}\} \times \{\text{reserve per policy at end of year}\}$$
$$-\{\text{reserve at start of year}\} - \{\text{interest on start year reserve}\}$$

Projecting cashflows – unit-linked and unitised with-profits

Project unit fund at end of each year as:

(+) Premium $\times$ allocation rate $\times(1-\text{bid-offer spread})$ *
(+) unit fund from end of previous year
(+) expected unit fund growth
(–) management charge
= fund at end of year after charges

* 'Cost of allocation'

Per policy in force at start of each year, project expected amounts of:

(+) premium less cost of allocation
(–) expenses
(+) interest on non-unit fund
(–) non-unit benefit costs
(+) non-unit surrender profit
(+) management charge transferred from unit fund
(+) other charges from unit fund, if applicable
= non-unit cashflow vector (= profit vector in absence of any non-unit reserves)

The effect of non-unit reserves on the profit vector is covered in the next chapter.

Summary measures of profit

Profit signature = vector whose entries represent the expected profit emerging at the end of year per policy in force at inception.

Net present value (NPV) = total present value of profit signature, discounted at risk discount rate.

Profit margin = $\dfrac{NPV}{EPV\ premiums}$

Internal rate of return (IRR) = discount rate that makes the NPV equal to zero.

We determine the risk discount rate as:

risk discount rate = cost of capital + risk premium

where the cost of capital is the return that can be obtained by investing money without risk, and the risk premium reflects the degree of riskiness associated with the product.

Using present value random variable approach

Alternatively, the NPV can be calculated using:

$$NPV = \sum_{\text{All events}} \{\text{Present value of profit} \mid \text{Event occurs}\} \times P(\text{Event occurs})$$

Profit testing

We can set the premiums for a product to give a desired level of profitability by projecting cashflows under a certain set of assumptions, deciding on a risk discount rate and profit criterion, and then varying the premium amount until the profit criterion is satisfied.

Chapter 26 Practice Questions

26.1 An actuary is profit testing a 15-year endowment assurance policy. The sum assured is £25,000 payable on survival or at the end of the year of earlier death. On surrender, a return of premiums is paid without interest at the end of the year of surrender.

A level premium of £1,500 *pa* is payable annually in advance.

For a policy in force at the start of the eighth year the remaining details are as follows:

	(£)
Renewal expenses	35
Claim expenses on death or surrender	75
Reserve at the start of year, ${}_7V$	8,000
Reserve at end of year per survivor, ${}_8V$	9,300
Rate of interest	8% *pa*
Dependent probability of death during 8th year	0.02
Dependent probability of surrender during 8th year	0.05

Calculate the profit expected to emerge at the end of the eighth year, per policy in force at the start of that year.

26.2 A unit-linked policy issued to lives aged 50 has a minimum death benefit of £3,000 (payable at the end of the year). Write down an expression for the expected death cost in the non-unit fund for Year 2 for a policy in force at the start of the year, expressed in terms of F_i, the size of the unit fund at the end of year i.

26.3 Amit, aged 60, invests £100 at the beginning of each month in an account earning interest at the rate of 1% per month. Amit requires a guaranteed amount of £3,000 at the end of the month of his death. To provide this guarantee, he buys a decreasing term assurance with a sum assured payable at the end of the month following death equal to the difference between the balance in the account and £3,000. The gross premium for the assurance is £10 per month. The insurance company incurs initial expenses of £25 and renewal expenses of £5 per month. The mortality basis for premium calculations is AM92 Ultimate and a uniform distribution of deaths over each year of age is assumed.

Determine the expected net outgo for the 18th month of the assurance contract as at policy outset. (Ignore interest earned by the insurance company.)

26.4 A life insurance company sells five-year-term, single-premium, unit-linked policies each for a premium of £10,000. There is no bid/offer spread and the allocation percentage is 100%. The only charge is a 2% annual management charge. The maturity, death and surrender benefits are equal to the value of the units at maturity, or at the end of the year of death or surrender, as appropriate, after deduction of the annual management charge in each case.

(i) Assuming unit growth of 9% *pa*, calculate the value of the units at the start and end of each year after deduction of the management charge, and the amount of management charge each year.

(ii) Calculate the net present value of the contract assuming:

- Commission of 5% of the premium
- Initial expenses of £50
- Annual renewal expenses of £20 in the 1st year, inflating at 5% *pa*
- Independent probability of mortality is 0.5% at each age
- Independent probability of surrender is 5% at each age
- Non-unit fund interest rate is 9% *pa*
- Risk discount rate 12% *pa*

The company holds unit reserves equal to the full value of the units (after deduction of annual management charge) and zero non-unit reserves.

You may assume that expenses are incurred at the start of the year and that death and surrender payments are made at the end of the year.

26.5 A life insurance company issues five-year without profit endowment assurances for an annual premium of £3,600 and a sum assured of £20,000 payable on maturity or at the end of the year of death if earlier.

The company uses the following assumptions for profit testing:

Year	Mortality probability	Surrender probability	Expenses at start year per policy	Reserves at end of year per policy	Surrender value at end of year per policy
1	0.01	0.05	£750	£3,100	£2,800
2	0.01	0.05	£15	£6,800	£6,250
3	0.01	0.05	£15	£10,900	£10,000
4	0.01	0.05	£15	£15,300	£14,500
5	0.01	0	£15	–	–

Surrenders occur only at the end of a year immediately before a premium is paid. The surrender probabilities shown in the table above are applied to the number of policies in force at each year-end. The company assumes it will earn 6% *pa* on its investments.

(i) Set out the column headings and the formulae you would use to calculate the profit arising each year per policy in force at the beginning of the year.

(ii) Calculate (to the nearest 1%) the internal rate of return obtained by the company.

26.6 Exam style

A life insurance company is planning to issue a new series of three-year unit linked endowment policies. Two designs of policy are under consideration, both having level annual premiums of £1,000.

Type A: 85% of the first year's premium and 101% of each subsequent premium is invested in units. On surrender the bid value of the units allocated is paid.

Type B: 95% of each premium is invested in units. On surrender the bid value of the units allocated is paid less a penalty of 10% of the total premiums outstanding under the policy.

There is a bid/offer spread in unit values, the bid price being 95% of the offer price. A fund management charge of 1% of the value of the policyholder's fund is deducted at the end of each policy year.

The death benefit, which is payable at the end of the year of death, and the maturity value are equal to the bid value of the units allocated. Surrenders are assumed to take place at the end of the year.

The insurance company's expenses in respect of the policy are £100 at the start of the first year and £30 at the start of the second and third years.

The insurance company holds unit reserves equal to the bid value of the units and zero non-unit reserves.

The dependent probability of mortality at each age is assumed to be 1% and the dependent probability of surrender at each duration is 5%.

The non-unit fund is assumed to grow at the rate of 7½% *pa*.

(i) Calculate the unit fund values at the end of each year assuming that the growth in unit value is 7½% *pa* and hence calculate the estimated maturity proceeds for each policy type. [6]

(ii) Calculate the net present value of the profit that is expected to arise under each policy type, using a discount rate of 10% *pa*. [8]

[Total 14]

26.7 A two-year insurance policy has the following particulars:

- annual premium of 520
- death benefit of 15,000, paid at the end of the year of death
- surrender value in Year 1 of 420, paid at the end of the year
- age at entry 50
- reserve at the end of Year 1 of 510 per policy then in force

The policy is to be profit tested using the following assumptions:

- mortality AM92 Ultimate
- 15% of policies surviving to the end of Year 1 surrender at that time
- expenses of 250 and 25 at the start of Years 1 and 2 respectively
- return on assets of 2% *pa*
- risk discount rate 5% *pa*

(i) Calculate the present value of the profit for this policy at outset, using the risk discount rate, assuming the policyholder:

(a) dies in Year 1

(b) surrenders in Year 1

(c) dies in Year 2.

(ii) The present value of the profit from the policy, assuming the policyholder neither dies in the first two years nor surrenders in the first year, is 250.

Calculate the net present value of the policy.

Chapter 26 Solutions

26.1 The expected profit is:

$$(8{,}000+1{,}500-35)\times 1.08$$
$$-(8\times 1{,}500+75)\times 0.05-(25{,}000+75)\times 0.02$$
$$-9{,}300\times 0.93$$
$$=£467.95$$

26.2 For a policy in force at the start of Year 2, the probability of a death claim at the end of Year 2 is q_{51} (as policyholders were aged 50 at issue).

If the unit fund value at the end of Year 2 is greater than 3,000, then the death benefit is the bid value of the units and there is no cashflow from the non-unit fund. However, if the unit fund value at the end of Year 2 is less than 3,000, then a benefit of 3,000 is payable if the policyholder dies in Year 2, and the shortfall must come from the non-unit fund. This shortfall is $3{,}000-F_2$.

So the expected death cost in the non-unit fund is:

$$\max\{0,\ 3{,}000-F_2\}\times q_{51}$$

26.3 The cashflows occurring at the end of the 18th month are:

(1) a premium of £10 paid if Amit is alive at the beginning of the month

(2) renewal expenses of £5 paid if Amit is alive at the beginning of the month

(3) a sum assured of $3{,}000-100\ddot{s}_{\overline{18}|}^{@1\%}$ paid if Amit dies in the 18th month.

The probability that Amit survives to the beginning of the 18th month is:

$$\frac{l_{61\frac{5}{12}}}{l_{60}}$$

Assuming deaths are uniformly distributed over the year of age we can calculate the value of l_x at the non-integer age using linear interpolation between the values at adjacent integer ages. So the required probability is equal to:

$$\frac{\left(\frac{7}{12}l_{61}+\frac{5}{12}l_{62}\right)}{l_{60}}=\frac{\left(\frac{7}{12}\times 9{,}212.7143+\frac{5}{12}\times 9{,}129.7170\right)}{9{,}287.2164}=0.98825$$

The probability that Amit dies in the 18th month is:

$$p_{60}\ {}_{\frac{1}{12}}q_{61}=\frac{1}{12}\times\frac{d_{61}}{l_{60}}=\frac{82.9973}{12\times 9{,}287.2164}=0.0007447$$

The sum assured required on death in the 18th month is 3,000 less the accumulated fund value at that point. The accumulated fund value is:

$100\ddot{s}_{\overline{18|}}$ calculated at $i = 1\%$

$$= 100 \times \left(\frac{1.01^{18} - 1}{0.01} \right) \times 1.01 = 1{,}981.09$$

So the expected net outgo is:

$$(3{,}000 - 1{,}981.09) \times 0.0007447 - (10 - 5) \times 0.98825 = -£4.18$$

26.4 (i) ***Unit reserves***

The following table shows the figures required:

Year	Value of units start of year	Value of units at end of year before mgmt charge	Mgmt charge	Value of units at end of year after mgmt charge
1	10,000.00	10,900.00	–218.00	10,682.00
2	10,682.00	11,643.38	–232.87	11,410.51
3	11,410.51	12,437.46	–248.75	12,188.71
4	12,188.71	13,285.69	–265.71	13,019.98
5	13,019.98	14,191.78	–283.84	13,907.94

(ii) ***Net present value***

The following table shows the calculation of the profit signature.

Year	Expenses	Interest	Mgmt charge	Profit vector	Prob in force	Profit signature
1	–570.00	–51.30	218.00	–403.30	1.000	–403.30
2	–21.00	–1.89	232.87	209.98	0.9453	198.48
3	–22.05	–1.98	248.75	224.71	0.8935	200.78
4	–23.15	–2.08	265.71	240.48	0.8446	203.10
5	–24.31	–2.19	283.84	257.34	0.7983	205.44

where probability in force at start of year $t = (0.95 \times 0.995)^{t-1}$.

There are no expected costs from deaths, surrenders or at maturity as the benefit level is equal to the unit fund value in all cases.

The net present value of the contract at a risk discount rate of 12% is:

$$-\frac{403.30}{1.12}+\frac{198.48}{1.12^2}+\frac{200.78}{1.12^3}+\frac{203.10}{1.12^4}+\frac{205.44}{1.12^5}=186.69$$

26.5 (i) ***Column headings***

The column headings and formulae required are:

(t)	Year	$t=1,2,\ldots,5$
(1)	Premium received	£3,600
(2)	Expenses	£750 (Year 1)
		£15 (Years 2-5)
(3)	Interest earned	$0.06\times[(1)-(2)]$
(4)	Expected death cost	$0.01\times 20{,}000$
(5)	Expected maturity cost	$0.99\times 20{,}000$ (Year 5 only)
(6)	Expected surrender cost	$0.05\times 0.99\times SV$ (Years 1-4 only)
	(The surrender value SV is a data item.)	
(7)	Expected in force cashflow	$(1)-(2)+(3)-(4)-(5)-(6)$
(8)	Reserve at start of year	${}_{t-1}V$
	(The reserve at the start of year 1 is zero.)	
(9)	Interest on reserves	$0.06\times {}_{t-1}V$
(10)	Cost of end year reserve	$0.99\times 0.95\times {}_tV$
	(The reserve at the end of year 5 is zero.)	
(11)	Profit vector	$(7)+(8)+(9)-(10)$

We have adopted the convention here that the figures in the expenses and claims columns are shown as positive entries. If you have shown these as negative numbers and adjusted the signs in the other column definitions accordingly, that is an equally valid approach.

(ii) ***Internal rate of return***

The calculations required to determine the profit signature are set out in the tables below, which include two extra columns:

(12)	Probability in force	$0.95^{t-1}\times 0.99^{t-1}$
(13)	Profit signature	$(11)\times(12)$

We then have:

Year	Premiums	Expenses	Interest	Expected death cost	Expected maturity cost	Expected surrender cost
t	(1)	(2)	(3)	(4)	(5)	(6)
1	3,600	750	171	200	–	138.60
2	3,600	15	215.10	200	–	309.38
3	3,600	15	215.10	200	–	495.00
4	3,600	15	215.10	200	–	717.75
5	3,600	15	215.10	200	19,800	–

Year	Expected in force cashflow	Reserve at start of year	Interest on reserves	Cost of end year reserve	Profit vector	Prob in force
t	(7)	(8)	(9)	(10)	(11)	(12)
1	2,682.40	0	0.00	2,915.55	−233.15	1.0000
2	3,290.73	3,100	186.00	6,395.40	181.33	0.9405
3	3,105.10	6,800	408.00	10,251.45	61.65	0.8845
4	2,882.35	10,900	654.00	14,389.65	46.70	0.8319
5	−16,199.90	15,300	918.00	0	18.10	0.7824

Year	Profit signature
t	(13)
1	−233.15
2	170.54
3	54.53
4	38.85
5	14.16

The internal rate of return is the interest rate that satisfies the equation:

$$-233.15v+170.54v^2+54.53v^3+38.85v^4+14.16v^5=0$$

By trial and error:

$i = 11\%\ \ LHS = 2.24$

$i = 12\%\ \ LHS = -0.68$

So the internal rate of return is approximately 11.8%, or 12% to the nearest percent.

26.6 (i) ***Unit fund values and maturity proceeds***

The calculations of the build up of each fund are set out in the tables below:

TYPE A							
Policy year	Prem	Prem all'd	Cost of all'n	Fund after all'n	Fund before charge	Annual charge	Fund at year end
1	1,000.00	850.00	807.50	0.00	868.06	–8.68	859.38
2	1,000.00	1,010.00	959.50	859.38	1,955.30	–19.55	1,935.75
3	1,000.00	1,010.00	959.50	1,935.75	3,112.39	–31.12	3,081.27

So the estimated maturity value for policy Type A is £3,081.27. [3]

TYPE B							
Policy year	Prem	Prem all'd	Cost of all'n	Fund after all'n	Fund before charge	Annual charge	Fund at year end
1	1,000.00	950.00	902.50	0.00	970.19	–9.70	960.49
2	1,000.00	950.00	902.50	960.49	2,002.71	–20.03	1,982.68
3	1,000.00	950.00	902.50	1,982.68	3,101.57	–31.02	3,070.55

So the estimated maturity value for policy Type B is £3,070.55. [3]

(ii) ***Net present value***

The calculations of the net present values of the profits are set out in the tables below:

TYPE A					
Policy year	Profit on allocation	Expenses	Non-unit interest	Annual charge	Profit vector
1	192.50	–100.00	6.94	8.68	108.12
2	40.50	–30.00	0.79	19.55	30.84
3	40.50	–30.00	0.79	31.12	42.41

[2]

Since the dependent probabilities of mortality and withdrawal are 1% and 5%, the probability of remaining in force until the end of the year is $1-0.01-0.05=0.94$. [1]

So the net present value for policy Type A is:

$$NPV_A = 108.12v + 30.84v^2 \times 0.94 + 42.41v^3 \times 0.94^2 = £150.40$$ [1]

TYPE B						
Policy year	Profit on allocation	Expenses	Non-unit interest	Expected surrender profit	Annual charge	Profit vector
1	97.50	−100.00	−0.19	10.00	9.70	17.01
2	97.50	−30.00	5.06	5.00	20.03	97.59
3	97.50	−30.00	5.06	0.00	31.02	103.58

[3]

The expected surrender profit is equal to the amount of surrender penalty recovered on a surrender, multiplied by the dependent probability of surrender occurring during the year. So, for example, at the end of Year 1 the total premiums outstanding is £2,000, so the surrender penalty is 10% of this, which is £200. The dependent probability of surrender in Year 1 is 5%, and so the expected surrender profit at the end of Year 1 is:

$$0.05 \times 200 = £10$$

The values in other years are calculated similarly.

So the net present value for policy Type B is:

$$NPV_B = 17.01v + 97.59v^2 \times 0.94 + 103.58v^3 \times 0.94^2 = £160.03$$ [1]

26.7 (i) ***Present values of profits***

(a) If the policyholder dies in Year 1, the present value of the profit (*PV*) is:

$$PV_a = \frac{(P-e_1)(1+i)-S}{1+r}$$
$$= \frac{(520-250)\times 1.02 - 15{,}000}{1.05} = \frac{275.4-15{,}000}{1.05} = -14{,}023.43$$

(b) If the policyholder surrenders at the end of Year 1, the *PV* is:

$$PV_b = \frac{(P-e)(1+i)-SV}{1+r} = \frac{275.4-420}{1.05} = -137.71$$

(c) If the policyholder dies in Year 2, the *PV* is:

$$PV_c = \frac{(P-e_1)(1+i) - {}_1V}{1+r} + \frac{({}_1V + P - e_2)(1+i) - S}{(1+r)^2}$$

$$= \frac{275.4 - 510}{1.05} + \frac{(510+520-25)\times 1.02 - 15{,}000}{1.05^2}$$

$$= -12{,}899.07$$

(ii) ***Net present value***

For this we will need the probabilities of the four possible outcomes.

The probabilities of events (a), (b), and (c), denoted by P_a, P_b, and P_c, are:

$$P_a = q_{50} = 0.002508$$

$$P_b = (1-q_{50})\times 0.15 = 0.997492 \times 0.15 = 0.149624$$

$$P_c = (1-q_{50})\times(1-0.15)\times q_{51} = 0.997492 \times 0.85 \times 0.002809 = 0.002382$$

The probability of none of these events occurring (which is the probability that the policy matures at time 2) is:

$$P' = 1 - P_a - P_b - P_c = 1 - 0.002508 - 0.149624 - 0.002382 = 0.845487$$

So the net present value is:

$$NPV = -14{,}023.43 \times 0.002508 - 137.71 \times 0.149624 - 12{,}899.07 \times 0.002382 + 250 \times 0.845487$$

$$= 124.87$$

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Reserving aspects of profit testing

Syllabus objectives

6.4 Project expected future cashflows for whole life, endowment and term assurances, annuities, unit-linked contracts, and conventional/unitised with-profits contracts, incorporating multiple decrement models as appropriate.

 6.4.3 Show how gross premium reserves can be computed using the above cashflow projection model and included as part of profit testing.

6.5 Show how, for unit-linked contracts, non-unit reserves can be established to eliminate ('zeroise') future negative cashflows, using a profit test model.

0 Introduction

In the previous chapter, we saw how to project the cashflows for policies. In this chapter, we describe how such techniques can be used to set reserves for both unit-linked and conventional contracts, and how changes in the reserving and pricing assumptions affect profit.

1 Pricing and reserving bases

In this section we consider the different bases that may be used in the financial management of a life insurance contract.

The profit test, as described in the previous chapter, needs many assumptions to be made in order to compute the expected future profits of a contract for comparison with a stated profit criterion.

The assumptions will, in the first instance, be the insurer's best estimate of expected future experience. This can be termed the *experience basis*.

Many life insurance companies see this as an acceptable basis for pricing because of the implicit allowance for risk in the pricing basis, through the risk discount rate. In fact, competition may force a company into marketing a 'loss leader'.

Assuming that the chosen risk discount rate reflects fully the uncertainties in the assumptions, no further margins would be taken.

Thus, we arrive at the *pricing basis* for the contract, *ie* the insurer's realistic expected outlook that it chooses to use in setting premiums and/or charges.

In practice, the actual profits arising in the future will depend on the actual future experience values of the items for which assumptions have been made. The insurer must consider two implications of this:

(i) How will profits be affected by the actual values for assumptions turning out to be different from the pricing basis?; and

(ii) Might the actual experience give rise to a need for additional finance?

The answer to both questions lies in re-running the profit test with a different set of assumptions from the pricing basis.

To answer question (i), typically the insurer will choose a variety of different assumptions in order to determine how quickly the expected future profit changes on varying any particular assumption. Such alternative bases represent *sensitivity test assumptions*. Such 'sensitivity tests' can give the insurer an understanding of how profits might be increased as well as how they might decrease, or even become losses. The results of these tests may indicate ways in which a product might be re-designed to minimise the volatility of expected profits. Any redesign would need to be profit tested itself, so this process can be iterative.

A similar approach could be taken to answer question (ii), but in this case the alternative assumptions would concentrate just on situations where profits would be reduced to the extent that external finance, or capital, would be required. These external capital requirements must then be supported by *reserves* held by the insurer (see Sections 2 and 3 below). The profit test can be re-run for a range of scenarios which give rise to varying levels of reserves. The more the assumptions diverge from the pricing basis, the greater the required reserves will become, and it becomes necessary to choose a single set of assumptions for which the reserves would provide adequate protection to policyholders without being beyond the means of an insurer's finances.

This is one of the most important, and difficult, aspects of life insurance company management.

Consider the following two extremes:

- A company whose reserving basis is too optimistic runs a significant risk of paying out too much profit to its capital providers. The result could be insufficient assets in future to meet liabilities, and the company would become insolvent.
- A company whose reserving basis is too pessimistic will be holding extremely large reserves. Large reserves require large amounts of capital from the shareholders (or with-profits policyholders). The profits from the business then have to be higher in order to provide the required rate of return on that capital, which means greater cost to the customer through having to pay higher premiums or charges.

Policyholders will be prepared to pay higher premiums up to a point, in return for the increased security of the fund (and therefore obtaining greater certainty that the company will meet its future obligations to them). On the other hand, if the cost of capital is too great, the company will lose customers and could go out of business.

The actuary therefore has to determine the level of reserve that leaves the company with an acceptably low probability (or risk) of insolvency occurring in the future, whilst at the same time imposing a cost of capital on the company that the policyholders are willing to pay for. The result is that the reserving basis will be prudent. A significantly more pessimistic basis than 'best estimate' will be assumed, but it will not be beyond the realms of reasonable possibility (for example, we would not assume that all the company's life assurance policyholders will die on the day after the valuation date!).

This single *valuation (or reserving) basis* is set by an insurer's actuary to ensure that an adequate assessment of the reserves is made. In practice, the valuation basis chosen will have to satisfy any local legislation and professional guidance which exists to protect the interests of policyholders. Detail is beyond the scope of this course.

Once the actuary has decided on an appropriate level of reserves to be held by the insurer, the profit test would be finally re-run, still on the original pricing basis, but with the cashflows paid into reserves modelled explicitly. This may mean a reassessment of premiums, benefits, and charges with consequential reassessment of valuation bases. The approach of pricing a contract can therefore be iterative.

2 Calculating reserves for unit-linked contracts

In this section we see how cashflow projections can be used to set reserves. This is particularly important for unit-linked products, where it is the only way of determining appropriate reserves, but we shall also see that we can apply the same methodology to conventional products. We first take another look at reserves in order to put things in perspective.

2.1 Reserves revisited

We have already come across the idea of a reserve as being a sum of money that a life insurance company puts aside to meet future liabilities. Now that we have discussed cashflow projections, we can look at reserving in a different light.

Let's consider what might happen if we project the cashflows for a policy, where no reserves are established, *ie* if we just look at the cash income (premiums and investment return) less cash outgo (expenses and claims) for each year. For instance, for a regular premium endowment assurance of term n years we would expect something like the following pattern of cashflows:

- in Year 1 — small and positive, or possibly negative, due to initial expenses
- in Years 2 to $n-1$ — positive due to premiums exceeding outgo
- in Year n — very negative due to maturity payout.

For an example consider the endowment assurance we looked at in the previous chapter. The policy is a 5-year regular-premium endowment assurance, with sum assured £10,000 payable on survival or at the end of the year of earlier death. Premiums of £2,108.81 are payable annually in advance, and on surrender a return of premiums is paid, without interest, at end of the year of surrender.

Using appropriate assumptions for interest, mortality, surrender and expenses, we obtain the following yearly cashflows:

Year	Premium	Expense	Interest	Expected claim cost	Expected surrender cost	Expected cashflow per policy in force at start of year
1	2,108.81	−1,054.41	42.18	−50	−210.88	835.70
2	2,108.81	−105.44	80.13	−60	−210.88	1,812.62
3	2,108.81	−105.44	80.13	−70	−316.32	1,697.18
4	2,108.81	−105.44	80.13	−80	−84.35	1,979.15
5	2,108.81	−105.44	80.13	−10,000	0	−7,916.50

where the expected cashflow values are calculated by summing over all the columns for each year. So, the insurer needs to put aside money early on to provide for the large negative cashflow in the last year.

We have already seen how to calculate the required amount of reserve prospectively, by valuing the stream of future benefit payments and future expenses and deducting future premiums. In other words we take each item of the expected cashflow and sum (with discounting and allowing for the probabilities of remaining in force) over all future years. We would get the same answer by summing each year's cashflow (again with discounting and allowing for the probabilities of remaining in force) and multiplying by –1, as we now explain below.

The calculation of a reserve involves summing all the elements in the following box, with suitable discounting for interest and allowing for staying in force:

Year	**Discounted premium (positive)**	**...**	**Discounted expenses (negative)**	**Discounted claims (negative)**	***etc* ...**	**Sum = discounted cashflow**
1	$+P_1$		$-e_1$	$-C_1$		EPV at time 0 of premiums, expenses, *etc* in year 1
...						
t	$+P_t$		$-e_t$	$-C_t$		EPV at time 0 of premiums, expenses, *etc* in year t
...						
Sum	Sum of discounted premiums		Sum of discounted expenses			Sum of sums × –1 = reserve

We can either sum the rows first and then sum the row totals, or sum the columns first and then sum the column totals. We then multiply by -1 so as to convert from:

'EPV of future income less outgo'

to:

'EPV of future outgo less income'

which is what we require in order to produce a (prospective) reserve value. So we can take our cashflow projections – these are our row totals above – and use these to calculate suitable reserves.

We illustrate how this can be done for conventional contracts in Section 3. In the rest of this section we look at the situation for unit-linked contracts. This is slightly different from the simple summation implied above.

2.2 Calculating reserves for unit-linked contracts

Unit-linked contracts require a unit reserve, which is equal to the unit fund value at any particular time, and a non-unit reserve. The calculation of the non-unit reserve follows the procedure described below, which is sometimes referred to as *zeroising negative casflows*.

We have already come across the idea that the reserve for a unit-linked contract should be the bid value of the units. However, this will clearly not be adequate if we expect future negative non-unit cashflows. In that case we need to set up a cash, or non-unit, reserve to fund for those future negative cashflows. We do this using the projected cashflows.

It is a principle of prudent financial management that, once sold and funded at the outset, a product should be self-supporting. This implies that the profit signature has a single negative value (funds are provided by the insurance company) at policy duration zero. This is often termed 'a single financing phase at the outset'.

Many unit-linked products naturally produce profit signatures which have a single financing phase. However some products, particularly those with substantial expected outgo at later policy durations, can give profit signatures which have more than one financing phase. In such cases these later negative non-unit cashflows (financing phases) should be reduced to zero by establishing reserves in the non-unit fund at earlier durations. These reserves are funded by reducing earlier positive non-unit cashflows. Good financial management dictates that these reserves should be established as late as possible during the term of the contract.

So we find the latest negative cashflow, set up a reserve the year before to fund for that negative, and if necessary carry on working back until we have no negative cashflows.

Suppose we have a four-year unit-linked policy with expected non-unit cashflows of:

$$(10, 15, -5, 8)$$

per policy in force at the start of the year. The expected cashflow at the end of Year 4 is positive, so no reserve is required at the start of Year 4 (*ie* at time 3). However, the expected cashflow at the end of Year 3 is negative, so we need to set up a reserve at the start of Year 3 (*ie* at time 2) to zeroise this. For simplicity, we will ignore interest and mortality for now.

If we set up a reserve of 5 at the end of the second year, then the new cashflow in the third year will be $5 + (-5) = 0$. However, setting up the reserve of 5 in the second year costs 5 at the end of that second year. So the second year's cashflow is reduced by 5, from 15 to 10.

So our pattern of expected cashflows is now (10, 10, 0, 8).

Question

It might be argued that we don't need a reserve at the start of Year 3, because we are expecting more than enough money in Year 4 to make up the loss. That is:

$$\{\text{Reserve at time } 2\} = 5 - 8 = -3$$

and so we'd assume a reserve of zero for prudence. Explain the flaws in this argument.

Solution

One problem is that the positive cashflow comes after the negative cashflow, so if the company does not have any reserves available in Year 3, it might find it physically impossible to pay out all its claims and expenses at the required time.

A second and ultimately more serious problem is that the fourth policy year may never happen (because the policy may lapse at the start of Year 4). The insurer is therefore potentially permanently short of money (and, in the scenario we have just described, might become completely bankrupt).

So, when working out non-unit reserves for unit-linked contracts, it is necessary to *ignore all positive cashflows that occur after the last negative cashflow*.

General approach

A policy has a non-unit cashflow vector (profit vector without non-unit reserves) of $(\text{NUCF})_t$; $t = 1, 2, 3, \ldots$ determined using the methods described in the previous chapter. Non-unit reserves (reserves in the cash fund) are to be set up so that there is only a single financing phase. The reserves to be established at policy duration t are ${}_tV$.

The reserving basis interest rate is i_s, and the probability of a policy staying in force for one year at age x is $(ap)_x$. After establishing non-unit reserves the profit vector is $(\text{PRO})_t$, $t = 1, 2, 3, \ldots$.

So, in the example at the start of this section, (10, 15, –5, 8) is the cashflow vector, and (10, 10, 0, 8) is the profit vector.

The equation of value at the end of policy year t, for cashflows in policy year t, per policy in force at time $t-1$, is:

$$(\text{NUCF})_t + {}_{t-1}V(1+i_s) - (ap)_{x+t-1}\,{}_tV = (\text{PRO})_t$$

In this equation, the entry in the profit vector corresponding to the end of year t is equal to:

- the non-unit cashflow at time t
- plus the accumulated amount of the reserve that was set up at time $t-1$
- minus the cost of setting up the reserve at time t for each policy that's remained in force over the policy year.

The process of establishing reserves begins at the greatest duration t for which $(\text{NUCF})_t$ is negative. Let this be duration $t = m$. Non-unit reserves will not be required at durations $t \geq m$ because during these policy years the product is expected to be self-financing. Hence we know that ${}_tV = 0$ for $t \geq m$.

For policy year m we can write:

$$(\text{NUCF})_m + {}_{m-1}V(1+i_s) - (ap)_{x+m-1} \times 0 = (\text{PRO})_m$$

where $(\text{NUCF})_m < 0$ and we wish to choose ${}_{m-1}V$ so that $(\text{PRO})_m = 0$.

Thus ${}_{m-1}V$ should be chosen to be:

$$ {}_{m-1}V = -\frac{(\text{NUCF})_m}{(1+i_s)} \qquad (*)$$

$(\text{NUCF})_m < 0$ has been 'turned into' $(\text{PRO})_m = 0$; the expected cashflows have been zeroised.

So we set up a reserve at time $m-1$ (the beginning of the year m). This reserve, when accumulated at the rate of interest i_s assumed in the basis, is exactly enough to cancel out the negative cashflow at the end of year m. So the profit at the end of year m is now 0.

However, setting up a reserve at time $m-1$ will now have an impact on the non-unit profit at time $m-1$.

The reserve ${}_{m-1}V$ will be established at policy duration $m-1$ out of the funds available at duration $m-1$. A reserve of ${}_{m-1}V$ is required for every policyholder alive at the start of the m th policy year. The non-unit cashflow $(\text{NUCF})_{m-1}$ at time $m-1$ is $(\text{NUCF})_{m-1}$ for every policyholder alive at the start of the $(m-1)$ th policy year. If we take the required reserve ${}_{m-1}V$ out of this cashflow, then the adjusted cashflow $(NUCF)'_{m-1}$ is given by:

$$(\text{NUCF})'_{m-1} = (\text{NUCF})_{m-1} - (ap)_{x+m-2}\ {}_{m-1}V$$

We use the probability $(ap)_{x+m-2}$ because we are interested in the probability of the policy staying in force to age $x+m-1$ from a year before that age, *ie* from age $x+m-2$ to age $x+m-1$.

If $(\text{NUCF})_{m-1} < 0$, then $(\text{NUCF})'_{m-1}$ will be negative. However, if $(\text{NUCF})_{m-1} > 0$, then $(\text{NUCF})'_{m-1}$ may be positive or negative. If $(\text{NUCF})'_{m-1} > 0$ then:

$$(\text{PRO})_{m-1} = (\text{NUCF})'_{m-1}$$

So if the non-unit cashflow is positive after setting up the reserve, then the entry in the profit vector is equal to this cashflow. However, if the non-unit cashflow is now negative, we have to zeroise this as well.

If $(\text{NUCF})'_{m-1} < 0$, then we repeat the process establishing non-unit reserves ${}_{m-2}V$ at policy duration $m-2$.

So we have:

$$(\mathrm{NUCF})'_{m-1} + {}_{m-2}V(1+i_s) = (\mathrm{PRO})_{m-1}$$

and choose ${}_{m-2}V$ so that $(PRO)_{m-1} = 0$, *ie*:

$${}_{m-2}V = -\frac{(\mathrm{NUCF})'_{m-1}}{(1+i_s)}$$

The process then repeats as from (*) above, and continues in this way until all the necessary reserves at durations $t = 1, 2, 3, \ldots, m-1$ have been obtained.

Let's look at an example. Suppose we have a five-year unit-linked policy. Assuming no non-unit reserves are held, the expected non-unit cashflows emerging at the end of each year, per policy in force at the start of that year, are:

$$(-60.20,\ -20.50,\ -17.00,\ 50.13,\ 85.75)$$

We shall calculate the reserves required if negative cashflows other than in Year 1 are to be eliminated, and hence obtain the revised profit vector allowing for the reserves. We'll assume that reserves earn interest at a rate of 5% per annum, and we'll ignore mortality.

Here we have negative cashflows in Years 2 and 3 that need to be dealt with. If the company sets up reserves at the end of Years 1 and 2 that can release:

- £20.50 at the end of Year 2
- £17.00 at the end of Year 3

then the negative cashflows at the end of Years 2 and 3 will be matched exactly by a positive cashflow from reserves, and the profit vector will show a zero entry for these two years.

No reserves are required after Year 3 since there are no losses after Year 3.

So, we require a reserve at the start of Year 3, ${}_2V$, such that:

$${}_2V = \frac{17.00}{1.05} = 16.19$$

Setting up this reserve turns the cashflow of –17.00 at the end of Year 3 into a profit of 0, and (since we are ignoring mortality) it turns the cashflow at the end of Year 2 into an 'adjusted cashflow' of:

$$-20.50 - 16.19 = -36.69$$

We now need to set up a reserve at time 1 to zeroise this negative. The reserve at time 1 is assumed to earn interest at the rate of 5% *pa*, so its accumulated value at time 2 is ${}_1V \times 1.05$. In order to eliminate the adjusted cashflow of –36.69 at time 2, we require:

$${}_1V = \frac{36.69}{1.05} = 34.94$$

This corresponds to the approach set out in the above Core Reading because we are ignoring mortality (and other decrements), *ie* setting $(ap)_{x+m-2}=1$.

No other reserves are required, since negative cashflows except in Year 1 have been eliminated. So, the reserves required are:

$${}_1V=34.94,\ {}_2V=16.19$$

and $\quad {}_0V={}_3V={}_4V={}_5V=0$

The net effect is that the loss in Year 1 is increased by the amount of the reserve set up, and the losses in Years 2 and 3 have been zeroised, so the profit vector is:

$$(-95.14,\ 0,\ 0,\ 50.13, 85.75)$$

Question

Under a five-year unit-linked policy, and assuming no non-unit reserves are held, the expected non-unit cashflows emerging at the end of each year, per policy in force at the start of that year, are:

$$\left(-10,\ -20,\ 5,\ -15,\ 40\right)$$

Calculate the non-unit reserves that should be set up to zeroise the negative cashflows, and give the profit vector. Assume 6% *pa* interest, and ignore mortality.

Solution

We do not need to set up a reserve at the start of Year 5 since the expected cashflow for the 5th policy year is positive.

We want to zeroise the –15 in Year 4. So we need a reserve in place at the start of that year, *ie* at time 3, equal to:

$${}_3V=\frac{15}{1.06}=14.15$$

The Year 3 adjusted cashflow is $5-14.15=-9.15$ and so this now needs to be eliminated.

We therefore need a reserve at the start of Year 3 (*ie* at time 2) of:

$${}_2V=\frac{9.15}{1.06}=8.63$$

This means the adjusted cashflow for Year 2 is $-20-8.63=-28.63$, so we need a reserve at the start of Year 2 (*ie* at time 1) of:

$${}_1V=\frac{28.63}{1.06}=27.01$$

This changes the year 1 cashflow to $-10-27.01=-37.01$.

So the profit vector is (–37.01, 0, 0, 0, 40).

In the above example and question we have ignored mortality. However, we normally need to take mortality into account in determining the reserves to be established.

To show this, we'll take the same example as above, which had in-force expected cashflows (before reserves) of $(-60.20, -20.50, -17.00, 50.13, 85.75)$. We'll again calculate the non-unit reserves and the profit vector, but this time we'll assume that the policy is issued to lives aged 55 with mortality assumed to be such that:

$$q_{55+t}=0.01+0.001t \quad \text{for } t=0,1,\dots,4$$

We will assume 5% *pa* interest, as before.

The reserve required at the start of Year 3 (time 2) does not change, *ie* we still have:

$${}_2V=16.19$$

since we still need to reduce the expected loss of 17.00 at the end of Year 3 to zero for each policy that was in force at the start of that year (*ie* at time 2), allowing for interest at 5%.

Now let's consider the reserve we need at time 1, *ie* ${}_1V$. This is the reserve that needs to be held *per policy in force at time 1*. This reserve has to cover:

(1) the expected cash outgo from this in-force policy, occurring at the end of Year 2

(2) the required reserves that need to be carried over to the third policy year, which will be needed *only for those policies that are still in force at the end of Year 2*, *ie* for the proportion surviving from time 1 to time 2.

To assess this, we work out the adjusted cashflow at time 2, per policy in force at time 1.

This is:

$$\begin{aligned}(NUCF)_2' &= -20.50 - {}_2V \times p_{56} \\ &= -20.50 - 16.19 \times \left(1-q_{56}\right) \\ &= 16.19 \times 0.989 \\ &= -36.51\end{aligned}$$

In other words, the total expected cash *outgo* at the end of Year 2, per policy in force at the *start* of Year 2, is $+36.51$, and it is this amount that the reserve for each policy that starts the year needs to cover. So, to eliminate this loss, we need:

$${}_1V=\frac{36.51}{1.05}=34.77$$

Holding this reserve (and then using it to cover the year's adjusted cashflow) will now result in a zero expected profit for Year 2.

The other reserves are zero as before.

Finally, we turn to Year 1. The ${}_1V$ reserves now become part of the cash outgo for this year. However, the profit vector element for this year is defined as the expected profit per policy in force at the start of *that* year, *ie* as at time 0. As the ${}_1V$ reserves are only required for the survivors of Year 1, the expected adjusted cashflow at the end of Year 1 is:

$$(NUCF)_1' = -60.20 - {}_1V \times p_{55} = -60.20 - 34.77 \times 0.99 = -94.62$$

which is also the profit for that year. So, the revised profit vector is:

$$(-94.62, 0, 0, 50.13, 85.75)$$

Question

The in-force expected non-unit cashflows for a five-year unit-linked contract taken out by a person aged exactly 50 are $(-10,\ -20,\ 5,\ -15,\ 40)$. Calculate the non-unit reserves required to zeroise any negative cashflows other than those occurring in the first policy year. Assume AM92 Ultimate mortality and 6% *pa* interest.

Solution

We do not need to set up a reserve at the start of Year 5 since the expected cashflow for the 5th policy year is positive.

We want to zeroise the cashflow of –15 in Year 4. So we need a reserve in place at the start of that year, *ie* at time 3, equal to:

$${}_3V = \frac{15}{1.06} = 14.15$$

Setting up this reserve at time 3 affects the expected non-unit cashflow at the end of the third policy year. The probability that a policy is still in force at the end of the third year given that it was in force at the start of the third year is p_{52}. So the (adjusted) expected cashflow at the end of Year 3, per policy in force at the start of Year 3, is now:

$$5 - {}_3V \times p_{52} = 5 - 14.15 \times 0.996848 = -9.11$$

Since this is negative, we need to set up a reserve at the start of Year 3 to zeroise it. We require:

$${}_2V \times 1.06 = 9.11$$

So ${}_2V = 8.59$.

We now repeat this process for Year 2. Setting up the reserve of 8.59 at time 2 affects the expected non-unit cashflow at the end of the second policy year. The probability that a policy is still in force at the end of the second year given that it was in force at the start of the second year is p_{51}. So the (adjusted) expected cashflow at the end of Year 2, per policy in force at the start of Year 2, is:

$$-20 - {}_2V \times p_{51} = -20 - 8.59 \times 0.997191 = -28.57$$

Since this is negative, we need to set up a reserve at the start of Year 2 to zeroise it. So we require:

$${}_1V \times 1.06 = 28.57 \;\Rightarrow\; {}_1V = 26.95$$

(We have not needed to use a survival probability for Year 4 in any of these calculations because there was no reserve required at the end of Year 4.)

In some cases, the vector of non-unit cashflows may show several runs of negative entries. In this case, the above method is repeated as many times as necessary until all the negative entries have been eliminated.

For example, suppose the in-force expected cashflows for a 6-year policy issued to lives aged x is:

$$(-131.53, -70.11, 25.00, -20.15, 55.74, 157.91)$$

We shall calculate the reserves required and the resulting profit vector if negative cashflows after Year 1 are to be zeroised. We shall assume non-unit reserves earn interest at 6% *pa* and that the probability of death during any year is 0.01.

No reserves will be required after Year 4 as the expected non-unit cashflows in Years 5 and 6 are positive. To zeroise the negative cashflow in Year 4, we require a reserve at the start of Year 4 such that:

$${}_3V \times 1.06 = 20.15 \;\Rightarrow\; {}_3V = 19.01$$

The expected cashflow in the previous year is +25.00. This is sufficient to set up the reserve. Allowing for the reserve, the expected profit emerging at the end of Year 3 per policy in force at the start of Year 3 is:

$$25.00 - 0.99 \times 19.01 = 6.18$$

We now need to zeroise the negative cashflow in Year 2. This will require a reserve at the start of Year 2 such that:

$${}_1V = \frac{70.11}{1.06} = 66.14$$

The expected profit for Year 1 then becomes:

$$-131.53 - 0.99 \times 66.14 = -197.01$$

So the profit vector is:

$$(-197.01\,,\ 0\,,\ 6.18\,,\ 0\,,\ 55.74\,,\ 157.91)$$

Question

Calculate the reserves required for the policy in the above example if the cashflow in Year 3 is 15.00 instead of 25.00.

Solution

As before, we have a reserve at the end of Year 3 of 19.01. So the adjusted cashflow in Year 3 becomes $15-0.99\times 19.01=-3.82$. We will need to set up a reserve at the start of Year 3 in respect of this negative result.

The start of Year 3 reserve is then:

$${}_2V=\frac{3.82}{1.06}=3.60\,.$$

Then the expected profit at the end of Year 2 per policy in force at the start of Year 2 becomes:

$$-70.11-{}_2V\times p_{x+1}=-70.11-3.60\times 0.99=-73.68$$

So we need a reserve at the start of Year 2 of:

$${}_1V=\frac{73.68}{1.06}=69.51$$

Summary of the method

To recap all of the above, here is a summary of the steps required to zeroise negative cashflows.

Step 1

Starting from the last negative entry in the vector of non-unit cashflows (in Year m say), calculate ${}_{m-1}V$ as:

$$ {}_{m-1}V = \frac{-\{\text{Cashflow in Year } m\}}{1+i} $$

Step 2

Calculate the adjusted expected non-unit cashflow in the previous year, as:

$$ \{\text{Adjusted cashflow in Year } m-1\} = \{\text{Cashflow in Year } m-1\} - {}_{m-1}V \times (ap)_{x+m-2} $$

using the value of ${}_{m-1}V$ found from Step 1, and where the (ap) probability is the probability of staying in force over Year $m-1$. Any surrenders, as well as mortality, will need to be taken into account in this probability.

Step 3

If the adjusted cashflow from Step 2 is negative, then find ${}_{m-2}V$ from:

$$ {}_{m-2}V = \frac{-\{\text{Adjusted cashflow in Year } m-1\}}{1+i} $$

using the adjusted cashflow value found from Step 2, and go to Step 4.

Step 4

Carry on working backwards through the profit vector by repeated application of Steps (3) and (4) until either Year 1 is reached, or a positive value for the adjusted cashflow is obtained.

If Year 1 is reached, the profit for Year 1 is equal to the adjusted cashflow for Year 1, *ie*:

$$ \{\text{Cashflow in Year } 1\} - {}_{1}V \times (ap)_x $$

If a positive adjusted cashflow is obtained, (in Year k say, $k > 1$), the process stops for this run of negative entries, the profit for Year k is equal to the adjusted cashflow for Year k, and the reserve at the start of the year (*ie* ${}_{k-1}V$) is equal to zero.

Step 5

Repeat Steps 1 to 4 for any other runs of negative entries.

2.3 Incorporating non-unit reserves into the profit test

The assumptions (of interest and mortality – and indeed expenses) that we use for calculating the non-unit reserves will be the insurer's valuation (or reserving) basis that was referred to towards the end of Section 1. In other words, we calculate the non-unit reserves that zeroise the expected negative cashflows that arise *when using the reserving basis*.

These reserves so calculated will then be put into the company's pricing profit-test model, as also described in that section.

Question

In an earlier example, we looked at a five-year unit-linked contract which had in-force expected cashflows (before reserves) of $(-60.20, -20.50, -17.00, 50.13, 85.75)$. Assuming 5% *pa* interest and mortality probabilities:

$$q_{55+t} = 0.01 + 0.001t \quad \text{for } t = 0, 1, \ldots, 4$$

it was found that this policy required non-unit reserves of ${}_1V = 34.77$ and ${}_2V = 16.19$. These reserves resulted in a profit vector of $(-94.62, 0, 0, 50.13, 85.75)$, when calculated on the same basis.

Assume that the assumptions that were used for these calculations were more prudent than the insurer's current best estimate basis.

Without doing any calculations, explain how the profit vector produced would be expected to change, when we run the company's pricing profit-test model using best estimate assumptions and incorporating the above reserves.

Solution

The profit test basis will be less cautious than the reserving basis. This means we will have some or all of the following:

- lower expected expenses and expense inflation
- higher investment returns
- lower expected claim costs

than before. These will in turn lead to having:

- higher expected profits from expenses and expense inflation
- higher investment profits
- higher mortality profits

than before. As this will apply to every year of the projection, all the profits will be larger (*ie* become more positive or less negative) than they were when calculated using the reserving basis.

This means the expected profits in Years 2 and 3 in this example will now be *greater* than zero, not equal to zero. So, while our non-unit reserves will zeroise profits in certain years on the *reserving* basis, our *best estimates* of profits in those years will be *greater* than zero.

When answering profit-testing questions where non-unit reserves are required, these reserves should be calculated on the basis specified for these in the question.

2.4 Full unit-linked example profit test

The following example is quite long and complex, as it involves calculations on two bases (the reserving basis, and the profit test basis). This level of computation is beyond what would be expected in the paper-based exam, but is possible in the computer-based assessment. It would be good practice to try to match the figures obtained below in Excel.

Policy details

A life insurance company issues a 4-year unit-linked endowment assurance policy to a person currently aged exactly 50, that offers the following benefits:

- on death, the higher of the unit fund value or 50,000 is paid
- on surrender, the unit fund value less a surrender penalty is paid. The surrender penalty is 225 if surrender occurs at the end of the first year, 150 if it occurs in the second year, and 75 if it is in the third year
- on maturity, the unit fund value is paid.

The death and surrender benefits are payable at the end of the policy year of claim, based on unit fund values at the end of the year, after deduction of all charges.

Premiums of 3,000 are payable annually in advance throughout the term of the policy, ceasing immediately on earlier death or on surrender of the policy.

The contract has the following charging structure:

- an allocation rate in the first year of 80% of the first premium
- allocation rate in all other years equal to 104% of each premium
- bid-offer spread: 5%
- fund management charge: 0.75% of the unit fund value deducted at the end of each year.

Profit test experience basis

The company currently uses the following basis to profit test this policy:

- AM92 Select mortality.
- Surrenders in the first year occur at a rate of 15% of all policies in force at the end of the year. In the second year, the rate is 8%, and in the third year the rate is 3%.
- Unit growth rate: 5% per annum effective.
- Interest earned on non-unit cashflows: 2% per annum effective.

- Initial expenses: 300.
- A renewal expense of 40 is incurred at the start of each year except the first.
- Non-unit reserves are set up at the start of each year that will exactly zeroise any negative cashflows that are expected according to the company's current reserving basis (see below). Negative non-unit reserves are not permitted.
- Risk discount rate: 7% per annum effective.

Reserving basis

- AM92 Select mortality.
- Surrenders are ignored.
- Unit growth rate: 3% per annum effective.
- Interest earned on non-unit cashflows 1% per annum effective.
- Initial expenses: 300.
- A renewal expense of 50 is incurred at the start of each year except the first.

Question

(i) Calculate the required non-unit reserves per policy in force at the start of each year.

(ii) Calculate the profit margin for this policy.

Solution

(i) ***Non-unit reserves***

We need to work out the non-unit reserves required according to the assumptions set out in the reserving basis. The first step is to project the unit fund using the reserving basis assumptions.

Unit fund (reserving basis)

The unit fund projection using the reserving basis is shown in the table below:

Policy year	Cost of allocation (1)	Total unit fund at start of year (2)	End of year unit fund before FMC (3)	Fund management charge (FMC) (4)	End of year unit fund after FMC (5)
1	2,280.00	2,280.00	2,348.40	17.61	2,330.79
2	2,964.00	5,294.79	5,453.63	40.90	5,412.73
3	2,964.00	8,376.73	8,628.03	64.71	8,563.32
4	2,964.00	11,527.32	11,873.14	89.05	11,784.09

where:

$$(1) = \{\text{premium}\} \times \{\text{allocation rate}\} \times \left(1 - \{\text{bid-offer spread}\}\right)$$

$$(2)_t = (5)_{t-1} + (1)_t \text{ for year } t = 2,3,4$$

$$(3) = (2) \times \left(1 + \{\text{unit growth rate}\}\right) = (2) \times 1.03$$

$$(4) = (3) \times \{\text{fund management charge rate}\} = (3) \times 0.0075$$

$$(5) = (3) - (4)$$

Next we need to calculate the expected non-unit cashflows at the end of each year per policy in force at the start of the year (the expected non-unit cashflows).

Non-unit cashflows (reserving basis)

Policy year	Premium minus cost of allocation (6)	Expenses (7)	Non-unit interest (8)	FMC (9)	Mortality rate (10)	Expected death cost (11)
1	720.00	300.00	4.20	17.61	0.001971	93.96
2	36.00	50.00	–0.14	40.90	0.002732	121.81
3	36.00	50.00	–0.14	64.71	0.003152	130.61
4	36.00	50.00	–0.14	89.05	0.003539	135.25

Policy year	Expected non-unit cashflow at end of year (*NUCF*) (12)
1	347.86
2	–95.05
3	–80.04
4	–60.34

where:

$$(6) = \{\text{premium}\} - (1)$$

$$(8) = [(6)-(7)] \times \{\text{non-unit interest rate}\} = [(6)-(7)] \times 0.01$$

$$(11) = \max\left[\{\text{minimum death benefit}\} - (5), 0\right] \times (10)$$

$$(12) = (6) - (7) + (8) + (9) - (11)$$

Non-unit reserves

The non-unit reserves are now calculated as the amount of money that we need to hold at the start of each year, that will lead to expected profits of (not less than) zero at the end of the year. For this we continue to use the reserving basis assumptions as necessary.

As the most distant negative cashflow is in year 4, we require the non-unit reserve at the start of year 4 to be:

$${}_3V = \frac{-NUCF_4}{1+\{\text{non-unit interest rate}\}} = \frac{60.34}{1.01} = 59.74$$

As the expected non-unit cashflows in years 2 and 3 are also negative, the non-unit reserves at the start of year t $(t = 2, 3)$ can be calculated recursively using:

$${}_{t-1}V = \frac{-NUCF_t + \{\text{probability of staying in force over } [t-1, t]\} \times {}_tV}{1+\{\text{non-unit interest rate}\}}$$

$$= \frac{-NUCF_t + \left(1 - \{\text{mortality rate}\}\right) \times {}_tV}{1.01}$$

The non-unit reserve at the start of year 1 is assumed to be zero. This leads to the following reserves:

Policy year	Expected non-unit cashflow at end of year (*NUCF*) (12)	Probability of staying in force over the year (13)	Non-unit reserve at start of year (${}_{t-1}V$) (14)
1	347.86	0.998029	0
2	–95.05	0.997268	230.57
3	–80.04	0.996848	138.21
4	–60.34	–	59.74

(ii) ***Profit margin***

We now need to calculate the expected non-unit profits using the assumptions set out in the profit test experience basis. This means first calculating the projected unit fund values and expected non-unit cashflows in a similar way to part (i), but using the profit test basis. The resulting values are shown in the following tables.

Unit fund (profit test basis)

Policy year	Cost of allocation (15)	Total unit fund at start of year (16)	End of year unit fund before FMC (17)	Fund management charge (FMC) (18)	End of year unit fund after FMC (19)
1	2,280.00	2,280.00	2,394.00	17.96	2,376.05
2	2,964.00	5,340.05	5,607.05	42.05	5,564.99
3	2,964.00	8,528.99	8,955.44	67.17	8,888.28
4	2,964.00	11,852.28	12,444.89	93.34	12,351.56

Non-unit cashflows (profit test basis)

Policy year	Premium minus cost of allocation (20)	Expenses (21)	Non-unit interest (22)	FMC (23)	Mortality rate (24)	Expected death cost (25)
1	720.00	300.00	8.40	17.96	0.001971	93.87
2	36.00	40.00	–0.08	42.05	0.002732	121.40
3	36.00	40.00	–0.08	67.17	0.003152	129.58
4	36.00	40.00	–0.08	93.34	0.003539	133.24

Policy year	Survival probability (26)	Probability of surrender (27)	Expected surrender profit (28)	Expected non-unit cashflow (29)
1	0.998029	0.149704	33.68	386.17
2	0.997268	0.079781	11.97	–71.46
3	0.996848	0.029905	2.24	–64.26
4	–	–	–	–43.98

where:

$$(26) = 1 - (24)$$

$$(27) = (26) \times \{\text{surrender rate}\}$$

$$(28) = (27) \times \{\text{surrender penalty}\}$$

$$(29) = (20) - (21) + (22) + (23) - (25) + (28)$$

Profit vector

The next step is to calculate the expected profits at the end of each year, per policy in force at the start of the year (the profit vector). This means allowing for the impact of the non-unit reserves that we calculated in part (i).

Policy year	Expected non-unit cashflow (29)	Non-unit reserve at start of year (14)	Interest on reserve (30)	Probability of staying in force (31)	Expected cost of end year reserve (32)	Profit vector (33)
1	386.17	0	0	0.848325	195.60	190.57
2	–71.46	230.57	4.61	0.917487	126.80	36.93
3	–64.26	138.21	2.76	0.966943	57.77	18.95
4	–43.98	59.74	1.19	–	–	16.95

where:

$$(30) = (14) \times \{\text{non-unit interest rate}\} = (14) \times 0.02$$

$$(31) = 1 - \{\text{mortality probability}\} - \{\text{surrender probability}\} = (26) - (27)$$

$$(32)_t = (31)_t \times (14)_{t+1} \quad \text{for year } t = 1,2,3$$

$$(33) = (29) + (14) + (30) - (32)$$

If the profit test assumptions had been identical to the reserving basis, the profits in years 2-4 would have been exactly equal to zero. We expect positive profits here because:

- *the non-unit reserves will cover the level of negative cashflows expected on the* ***reserving*** *basis*
- *the profit test assumptions are less cautious than the reserving basis: for example, the unit growth rate is higher (producing higher expected fund management charges) and we are including the expected profits from surrender, producing higher (less negative) expected cashflows than before*

so the reserves held will be ***more*** *than enough to cover the expected level of negative cashflows on the* ***profit test*** *basis, leading to surplus cashflow (ie a profit) being expected each year.*

Net present value

We now need the net present value of the policy, which is calculated by multiplying each year's profit by the probability of the policy remaining in force from policy outset to the start of the year, and then discounting the resulting profit signature at the risk discount rate: summing the resulting values gives the net present value.

The net present value (NPV) is obtained using the figures calculated in the following table:

Policy year	Profit vector (33)	Probability of staying in force to start of year (34)	Profit signature (35)	Discount (36)	Expected present value (37)
1	190.57	1	190.57	0.934579	178.10
2	36.93	0.848325	31.33	0.873439	27.36
3	18.95	0.778326	14.75	0.816298	12.04
4	16.95	0.752597	12.76	0.762895	9.73

where:

$$(34)_t = (34)_{t-1} \times (31)_{t-1} \quad \text{for year } t = 2,3,4$$

$$(35) = (33) \times (34)$$

$$(36)_t = \left(1 + \{\text{risk discount rate}\}\right)^{-t} = 1.07^{-t} \quad \text{for year } t = 1,2,3,4$$

$$(37) = (35) \times (36)$$

The NPV is then the sum of the values in Column 37, which is 227.24.

Profit margin

The profit margin is equal to the NPV divided by the expected present value of the premiums using the profit test basis discounted at the risk discount rate.

The expected present value of the premiums is:

$$3{,}000 \times \left(1 + \frac{0.848325}{1.07} + \frac{0.778326}{1.07^2} + \frac{0.752597}{1.07^3}\right) = 9{,}260.97$$

Hence the profit margin is:

$$\frac{227.24}{9{,}260.97} = 2.45\%$$

3 Calculating reserves for conventional contracts using a profit test

In the previous section we saw how to determine non-unit reserves for unit-linked products by working backwards from the last negative cashflow. We can apply exactly the same methodology to conventional contracts.

A profit test can also be used to determine the reserves for a conventional (*ie* non unit-linked) policy. We illustrate the procedures by using a without-profit endowment assurance with a term of n years, a sum assured of S payable at the end of the year of death or on survival to the end of the term, and a surrender value payable at the end of year t of U_t, which is secured by a level annual premium of P.

A basis is required for the projection of the cashflows and for calculating the required reserves. This will consist of an interest rate i, dependent probabilities of death and surrender $(aq)^d_{x+t}$ and $(aq)^s_{x+t}$, dependent probabilities of remaining in force ${}_t(ap)_x$, and expenses per policy in force at time t of e_t.

$(\text{CF})_t$, the expected cashflow at time t per policy in force at time $t-1$, ignoring reserves, is:

$$(\text{CF})_t = \left(P - e_{t-1}\right)(1+i) - S\,(aq)^d_{x+t-1} - U_t\,(aq)^s_{x+t-1} \qquad t = 1, 2, 3, \ldots, n-1$$

$$(\text{CF})_n = \left(P - e_{n-1}\right)(1+i) - S$$

assuming there is no surrender value paid at the end of the last policy year.

Technically it is not possible to surrender an annual premium endowment assurance in the last policy year, because all the premiums will have been paid by then and the policy will simply terminate as a maturity, receiving the full sum assured in benefit payment.

These cashflows will usually be positive for earlier years of a contract and negative during the later years. For example a five-year endowment assurance with a sum assured of 1,000 might have cashflows:

t	1	2	3	4	5
$(\text{CF})_t$	156.39	187.41	186.33	185.14	–803.17

If the contract is to be self-funding, then reserves must be established using the earlier positive cashflows. These reserves should be sufficient to pay the later expected negative cashflows. This requirement is exactly analogous to the need to establish reserves in the non-unit fund for a unit-linked contract. Reserves can be established for conventional assurances using the same procedures as those used to establish non-unit reserves.

Let $(\text{CF})_t$ denote the expected cashflow at the end of Year t, and let $(\text{PRO})_t$ denote the t th entry in the profit vector. Calculations begin at the longest policy duration, m, at which there is a negative cashflow.

For policy year m, we write:

$$(\mathrm{CF})_m + {}_{m-1}V(1+i) - (ap)_{x+m-1} \times 0 = (\mathrm{PRO})_m$$

where $(\mathrm{CF})_m < 0$ and we wish to choose ${}_{m-1}V$ so that $(\mathrm{PRO})_m = 0$.

This requires that ${}_{m-1}V$ is chosen to be:

$${}_{m-1}V = \frac{-(\mathrm{CF})_m}{(1+i_r)} \qquad (**)$$

We set up this reserve from $(\mathrm{CF})_{m-1}$ and determine the adjusted cashflow:

$$(\mathrm{CF})'_{m-1} = (\mathrm{CF})_{m-1} - (ap)_{x+m-2}\ {}_{m-1}V$$

If $(\mathrm{CF})'_{m-1} > 0$, then ${}_{m-2}V = 0$. If $(\mathrm{CF})'_{m-1} < 0$, then we calculate:

$${}_{m-2}V = \frac{-(\mathrm{CF})'_{m-1}}{(1+i)}$$

The process then repeats as from (**) above, and continues in this way until all the necessary reserves at durations $t = 1, 2, 3, \ldots, m-1$ have been obtained.

For conventional assurances it is usually the case that reserves are needed at all policy durations. So the calculation begins with $(\mathrm{CF})_n$ and concludes with $(\mathrm{CF})_1$.

For the example cashflows shown above, if the reserving basis is 3% *pa* interest, mortality follows the AM92 Ultimate for a life assumed to be aged 55 exact at entry, and surrenders are ignored, we obtain:

t	1	2	3	4	5
$(\mathrm{CF})_t$	156.39	187.41	186.33	185.14	–803.17
${}_tV$	177.20	371.79	572.51	779.78	0

These figures are calculated as follows. First we need a reserve at time 4 to cover the negative cashflow at time 5. We simply discount the negative cashflow at time 5 over one year, and multiply by -1. So:

$${}_4V = \frac{803.17}{1.03} = 779.78$$

Next we need to work out the expected cashflow at time 4 allowing for the reserve. This is:

$$(CF)'_4 = 185.14 - 779.78\,p_{58} = 185.14 - 779.78 \times (1 - 0.006352) = -589.68$$

We only need reserves at the end of Year 4 for those policyholders that have *survived* Year 4, so we multiply the required reserve by p_{58} (the probability of surviving Year 4). Remember that all of these end-year cashflows are expressed per policy in force at the *start* of each year.

As $(CF)'_4$ is negative, we need a reserve at the start of Year 4 of:

$$_3V = \frac{589.68}{1.03} = 572.51$$

The other figures are obtained by continuing this process until $(CF)'_t$ becomes positive or, as in this case, we get to the start date of the policy.

When a policy has a cashflow pattern like this one, where only the final cashflow is negative, calculating reserves using:

1. the EPV future outgo – EPV future income formula
2. discounted future cashflows
3. zeroisation of future negative cashflows

all give the same result.

This is also true when the cashflows become increasingly negative over time, as in the case of a term assurance. However, as we discussed in Section 2.2, when a policy produces some negative cashflows but the last cashflow is positive, zeroisation of future negative cashflows gives a different answer to the other two methods and is preferable on the grounds of prudence.

As before, these reserves are then put back into the profit testing model used to price the contract, as described in Section 1 above.

So we:

- calculate reserves on a prudent basis,
- calculate cashflows on a realistic basis, then allow for the prudent reserves so as to give the projected profits for the profit test,
- play around with premiums and product design until we achieve our profitability criterion using these 'realistic experience but prudent reserves' cashflows.

Clearly we cannot derive prudent reserves from realistic (*ie* best estimate) cashflows. We go into more detail on the interaction between a realistic experience basis and a prudent reserving basis in the next section of this chapter.

4 Effect of pricing and reserving bases on a profit test

The writing of each contract represents an investment of an insurer's capital. It has been seen from above that the expected profit from a contract is the present value of the projected profit test net cashflows at the risk discount rate.

By 'net' cashflows we mean the values of the profit vector, as described earlier.

The chosen pricing and valuation assumptions will each have an impact on the expected profit.

Remember by 'valuation assumptions' we mean the assumptions used to calculate the reserves.

For example, the assumed investment return in the pricing basis would normally, by definition of the risk discount rate, be less than that risk discount rate. A lower assumption will lead to lower present values of profits.

A valuation basis which gives rise to positive reserves will normally reduce the present value of profits from a contract. This is because the reserves reduce otherwise positive cashflows and are then invested in lower risk assets whose rate of investment return is expected to be less than the risk discount rate. Over the whole term of a contract, and assuming that experience turns out to be in line with the assumed pricing basis, stronger (*ie* larger) reserves will not reduce the overall aggregate size of the cashflows, but they will delay the cashflows so that they emerge later than they would have done using a weaker valuation basis, hence reducing the present value of profits.

In the above description of the profit test calculations, we have assumed that reserves are set up at the end of the policy year, and this is conventionally how profit tests are carried out. However, it is more prudent to assume that reserves are set up at the beginning of the policy year as soon as a premium is paid. In this case, the usual assumption that reserves will earn less than the risk discount rate gives rise to a slightly smaller profit. In practice, insurance companies calculate cashflows monthly rather than annually, and therefore the convention of end-of-period reserves does not lead to a material overstatement of profit.

A more general approach which can be used to discount cashflows, given sufficient computing power, would be to discount cashflows as soon as they arise, rather than first accumulating them to the end of policy years at an assumed rate of interest. This approach would be more prudent for all negative cashflows including reserve cashflows, such as expenses paid.

For example, if we had an initial expense of 100, accumulating this negative cashflow to the end of the year at 8% and then discounting back at 12% gives:

$$100\times\frac{1.08}{1.12}=96.43$$

So it is more prudent to account for the negative cashflow of 100 as soon as it arises rather than accumulate to the end of the policy year and discount.

We can illustrate the above concepts by considering a very simple single premium 5-year pure endowment in a zero-mortality zero-expense world. The sum assured is 1,000.

Suppose we have the following bases for the various rates of interest:

Pricing	6% *pa*	
Experience	7% *pa*	(on cash and reserves)
Reserving	5% *pa*	
Risk discount rate	9% *pa*	

So this means we use 6% *pa* interest when calculating the premium, 5% *pa* interest when calculating the amounts of the reserves, 7% *pa* interest when calculating the interest earned on the premium and reserves, and 9% *pa* interest to discount the projected profits.

Expenses, mortality and withdrawals are zero for all bases (pricing, reserving and expected experience).

So the premium is $1{,}000v^5$ at 6% giving 747.26.

The projected cashflows, showing them both before and after reserving, are shown in the tables below.

Year	Premium	Interest	Benefit outgo	Cashflow (before reserves)
1	747.26	52.31	0	799.57
2	–	0	0	0
3	–	0	0	0
4	–	0	0	0
5	–	0	–1,000	–1,000

Year	End of year reserve	Interest earned on reserves	Cost of increasing reserves (ignoring interest on reserves)	Profit vector
1	822.70	0	–822.70	–23.14
2	863.84	57.59	–41.14	16.45
3	907.03	60.47	–43.19	17.28
4	952.38	63.49	–45.35	18.14
5	0.00	66.67	952.38	19.05
Total			0.00	47.78

Since there is no mortality, the cost of increasing the reserve at the end of Year 1 is calculated as ${}_1V - {}_0V = {}_1V$. Since this is a cost to the company, it is shown as a negative entry in the table above. The other entries in this column are calculated in a similar way.

Question

(i) Verify the entries above for the second year.

(ii) Calculate the net present value of the profits for this policy.

Solution

(i) ***Verifying the Year 2 entries***

There is no premium, so no interest on the premium. There is no benefit either. So the only cashflows come from the reserves.

At the start of the year, the company holds a reserve of 822.70 per policy in force. Interest on reserves is 7%, which gives interest of 57.59.

The reserve required at the end of the year is $1{,}000v^3$. Valuing at 5%, this is 863.84. Since there is no mortality, the cost of increasing the reserve is:

$$863.84 - 822.70 = 41.14$$

So profit is $57.59 - 41.14 = 16.45$.

(ii) ***Net present value***

Net present value of profits is

$$-23.14v + 16.45v^2 + \cdots + 19.05v^5 = 31.19$$

discounting at 9%.

In this example, the value of the profit in the first policy year is negative. This means selling one of these policies will require an injection of extra money equal to this amount, to avoid the company becoming insolvent, and this required extra money is referred to as the *new business strain* of the contract.

Extra money such as this has to be provided from the company's capital resources.

Question

(i) Explain why the new business strain arises with this contract, given that there are no expenses.

(ii) State the one feature in the basis that tells us that the total non-discounted profits will be positive.

Solution

(i) ***Reason for new business strain***

New business strain has arisen because the reserving basis is stronger (*ie* more pessimistic) than the pricing basis. The premium contains 747.26 in respect of the value of future benefits, while the reserves we require to set up, after receiving the premium, have a present value of

$\frac{822.70}{1.05} = 783.53$ at the start of the year.

(ii) ***Reason for positive total profit***

It is because the experience investment income assumption is greater than the pricing assumption.

Now let's consider the effect of strengthening the reserving basis. So, if we were to calculate reserves using 3% *pa* interest, for example, the cashflows before reserves are unchanged from before, and the projected profits become:

Year	End of year reserve	Interest on reserves	Cost of increasing reserves	Profit vector
1	888.49	0	–888.49	–88.92
2	915.14	62.19	–26.65	35.54
3	942.60	64.06	–27.45	36.61
4	970.87	65.98	–28.28	37.70
5	0.00	67.96	970.87	38.83
Total			0.00	59.76

The net present value of these profits is now 28.55. The total amount of profits is slightly higher because we have set up bigger reserves and those reserves bring in interest – but that interest is lower than the risk discount rate, so the effect is a reduction in the *value* of profits.

We see that the profitability of the contract has gone down due to the increase in the gap between the investment return earned on cash and reserves, and the required rate of return.

We could also have increased that gap by increasing the risk discount rate – this would have reduced the net present value of profits.

We can see also that strengthening the reserving basis has increased the new business strain.

5 Setting out the calculations

We have used several different ways of showing our calculations in this and in the previous chapter, *eg* sometimes showing the interest on the reserves separately and sometimes including it within the 'increase in reserves' column; or sometimes showing every reserve-related item in separate columns (*ie* 'reserve at start of year' + 'interest on reserves' – 'expected cost of reserve at end of year'). There is no fixed rule as to which of these is 'best' to use, as they all achieve the same thing and result in the same answers. Different people find different approaches more intuitive and easier to remember or use, and all (correct) methods are equally acceptable in the exam.

The chapter summary starts on the next page so that you can keep all the chapter summaries together for revision purposes.

Chapter 27 Summary

Calculating reserves using cashflow projections

We can use cashflow projections to calculate reserves by:

- identifying negative cashflows (starting from the end of the projection) in year t,
- calculating the reserves needed at the end of the previous year $t-1$ to fund such negative cashflows,
- paying for such a reserve from the cashflow of year $t-1$,
- and if the cashflow of year $t-1$ is now negative, repeat the above process.

We need to do this with unit-linked contracts to ensure that policies in force will not require further financing by the life company.

We can use the same technique with conventional contracts, as an alternative to using a prospective gross premium formula approach.

Different bases

A basis is a set of assumptions about quantities such as future investment returns, mortality rates, surrender rates and expenses.

Assumptions that are our best estimates of the future give an experience basis.

The basis we use to set premiums is called the pricing basis and can be the same as the experience basis, or slightly more prudent, or more risky.

The basis we use to determine reserves – the valuation or reserving basis – is generally much more prudent than the experience basis. This will defer the emergence of profits from the contract. If the required rate of return is greater than the investment return on reserves, this will reduce the profitability of the contract.

The practice questions start on the next page so that you can keep the chapter summaries together for revision purposes.

Chapter 27 Practice Questions

27.1 Explain why a life insurance company might need to set up non-unit reserves in respect of a unit-linked life assurance contract.

27.2 The following table shows (in £'s) a profit testing calculation (with some of the entries missing) for three-year endowment assurance contracts issued to lives aged exactly 57 with a sum assured of £5,000 payable at the end of the year of death. Outgo terms are shown as negative entries.

Year	Premium	Expenses	Interest	Expected cost of death and maturity claims	Expected cost of increasing reserves (*)	Profit vector
1	1,530	–50	?	?	?	–51
2	1,530	?	?	?	?	21
3	1,530	?	?	?	?	45

The mortality probability at each age is 1%. The rate of interest earned on cashflows and reserves is 6%. Reserves are calculated using an interest rate of 4%. The reserves are zero at the start and end of the contract. The interest earned on the reserve in the third year is £195.

(i) Complete the table.

(ii) Calculate the internal rate of return.

(iii) Explain the effect that changing to a weaker reserving basis would have on the internal rate of return.

(iv) Calculate the net present value using a risk discount rate of 7%.

(v) Explain the effect that changing to a weaker reserving basis would have on the net present value.

(*) *Allowing for interest earned on reserves.*

27.3 A profit test for unit-linked policies issued to lives aged 60 has been carried out. The expected non-unit cashflows before setting up non-unit reserves are as follows:

Year	Expected non-unit cashflows per policy in force at start of year
1	–30
2	–12
3	–6
4	20
5	30

Write down an expression for the expected loss at the end of Year 1, after zeroisation of all negative cashflows.

27.4 A conventional 3-year endowment assurance is issued to a life aged exactly 56. The details are:

- sum assured 10,000 payable after 3 years or at the end of the year of death, if earlier
- surrender value equal to the return of premiums without interest, less 400, at the end of the year of surrender
- annual premium 3,250 paid at the start of each year.

The company calculates its reserves for this contract on the following basis:

Expenses: 30 at the start of Year 2

32 at the start of Year 3

Surrender probabilities: 4% of policies in force at the end of Year 2 only

Mortality: AM92 Ultimate

Interest: 2% *pa*

(i) Calculate the prospective gross premium reserves required per policy in force at the start of Years 2 and 3, according to the above basis, using a cashflow projection approach.

(ii) Calculate the expected profit arising in the second year per policy in force at the start of Year 2, assuming the following profit test experience basis:

Expenses: as reserving basis

Surrenders: as reserving basis

Reserves: as calculated in part (i)

Mortality: 75% of AM92 Select from policy outset

Interest: 4% *pa*

(iii) Explain why the expected profit calculated in part (ii) is not zero.

27.5 *Exam style*

A life insurance company issues a number of 3-year term assurance contracts to lives aged exactly 60. The sum assured under each contract is £200,000, payable immediately on death. Premiums are payable annually in advance for the term of the policy, ceasing on earlier death.

The company carries out profit tests for these contracts using the following assumptions:

Initial expenses:	£200 plus 35% of the first year's premium
Renewal expenses:	£25 plus 3% of the annual premium, incurred at the beginning of the second and subsequent years
Mortality:	AM92 Ultimate
Investment return:	7% per annum
Risk discount rate:	15% per annum
Reserves:	One year's office premium

(i) Show that the office premium, to the nearest pound, is £2,610, if the net present value of the profit is 25% of the office premium. [10]

(ii) Calculate the expected in-force cashflows if the company holds zero reserves throughout the contract, using a premium of exactly £2,610. [2]

(iii) Explain why the company might not hold reserves for this contract and the impact on profit if it did not hold any reserves. [3]

[Total 15]

27.6 *Exam style*

A 5-year unit-linked endowment assurance is issued to a male aged exactly 55. The expected year-end cashflows in the non-unit fund, $(t=1,\ldots,5)$ per policy in force at the start of Year t are:

Year t	1	2	3	4	5
$(NUCF)_t$	−200	+20	+45	−60	+480

You are given:

Independent probabilities of mortality:	AM92 Select
Independent probabilities of withdrawal:	0.1 for Years 1 and 2
	0.05 for Years 3 and 4
	0 for Year 5

Withdrawals can occur at any time over the policy year.

(i) Calculate the net present value of profit at a risk discount rate of 10% *pa* assuming that the company holds no non-unit reserves. [4]

The rate of interest earned on non-unit reserves is assumed to be 8% *pa*.

(ii) (a) Calculate the reserves that are required at times $t = 1, .., 4$ in order to zeroise future negative cashflows.

(b) Calculate the net present value of the policy assuming that the company holds the non-unit reserves calculated in (i)(a). [6]

(iii) Without carrying out any more calculations, explain the effect on the net present value if non-unit reserves earned interest at the rate of 10% *pa*. [2]

[Total 12]

27.7 A special endowment policy pays a sum assured of £20,000 to a life who is currently aged exactly 57 after three years or at the end of the year of earlier death.

Exam style

Annual reversionary bonuses are declared at the end of each policy year, and an additional terminal bonus is payable at maturity only.

Policies may be surrendered only at the end of each policy year. On surrender, the policyholder receives a return of premiums with interest calculated at the rate of 3% per annum.

A level premium of £8,000 is paid at the start of each year.

The premium basis is as follows:

Interest:		7% per annum on cashflows and reserves
Mortality:		AM92 Select
Surrender rates:		15% of all policies in force at the end of year 1
		5% of all policies in force at the end of year 2
Reversionary bonuses:		6% per annum compound
Terminal bonus:		10% of all other benefits payable at maturity
Expenses:	Initial	£500
	Renewal	£30 at start of year 2
		£35 at start of year 3
	Termination	£100 per termination (death, surrender or maturity)
Reserves:		7,500 at the start of year 2 and 15,000 at the start of year 3 for each life in force.

(i) Calculate the profit signature for this policy according to the premium basis. [13]

(ii) By accumulating the elements of the profit signature to the maturity date, explain briefly whether you think the company expects to declare the bonus rates it has assumed in its premium basis, assuming all the other assumptions in the basis are realistic. [2]

[Total 15]

27.8 Craig, aged 40, buys a four-year unit-linked endowment policy under which level annual premiums of £1,000 are payable. 75% of the first premium and 105% of each subsequent premium is invested in units. There is a bid/offer spread in unit values, the bid price being 95% of the offer price.

Exam style

A fund management charge of 0.75% of the value of the policyholder's fund is deducted at the end of each policy year.

The death benefit, which is payable at the end of the year of death, is £3,000 or the bid value of the units if greater. The maturity value is equal to the bid value of the units.

The insurance company incurs expenses of £150 at the start of the first year, £75 at the start of the second year, and £25 at the start of each of the third and fourth years.

The mortality probability (q_x) is assumed to be 0.01 at each age and withdrawals may be ignored.

(i) Assuming that the growth in the unit value is 5% *pa*, the non-unit interest rate is 5% *pa*, and the insurance company holds unit reserves equal to the value of units and zero non-unit reserves, calculate the expected profit emerging in each policy year. [10]

(ii) Calculate the revised profit emerging each year assuming that the office sets up non-unit reserves to ensure that the expected profit emerging in the second and subsequent policy years is non-negative. [5]

[Total 15]

The solutions start on the next page so that you can separate the questions and solutions.

Chapter 27 Solutions

27.1 A life office might set up reserves in the non-unit fund if the overall cashflow in any year other than Year 1 would otherwise be negative. It is desirable that the policy be self-funding after Year 1.

Reserves are set up early in the contract so that money can be released from the reserves as required to eliminate the negative cashflow. The regulatory authorities, *eg* the Financial Conduct Authority in the UK, may insist on this. Setting up reserves will result in larger losses (or smaller profits) in the early years of the contract, but the life office will not expect to have to find extra capital to support the policies later on.

27.2 (i) ***Completed table***

The completed table is:

Year	Premiums	Expenses	Interest	Expected cost of claims	Expected cost of increasing reserves	Profit vector
1	*1,530*	*–50*	88.80 (1)	–50 (2)	–1,569.80 (3)	*–51.00*
2	*1,530*	–13.30 (4)	91.00 (6)	–50 (2)	–1,536.70 (4)	*21.00*
3	*1,530*	–20.57 (5)	90.57 (6)	–5,000 (2)	3,445.00	*45.00*

(Pre-calculated figures are shown in italics.) The missing figures can be derived using the following steps (which are indicated in brackets beside the figures in the table).

(1) Interest in the first year is $(1{,}530-50)\times 0.06$.

(2) The expected cost of claims in Years 1 and 2 is $5{,}000\times 0.01$. The expected cost of claims in Year 3 is 5,000 since all policies in force at the start of Year 3 will receive a benefit of 5,000 at the end of Year 3. These figures are shown as negative entries in the table as they are outgo for the insurer.

(3) Let e_t denote the amount of the expense paid at time t and PRO_t be the profit vector term at time t. If the policyholder is aged x at time 0, then the recursive formula tells us that:

$$\left({}_tV+P-e_t\right)(1+i)={}_{t+1}V\,p_{x+t}+S\,q_{x+t}+PRO_{t+1} \quad \text{for } t=0,1 .$$

This can also be written as:

$$PRO_{t+1}=\left(P-e_t\right)(1+i)-\underbrace{\left({}_{t+1}V\,p_{x+t}-{}_tV(1+i)\right)}_{\substack{\text{expected cost of increasing}\\ \text{the reserve at time } t+1}}-S\,q_{x+t}$$

So for Year 1, we have:

$$-51=(1{,}530-50)\times 1.06-\underbrace{\left({}_1V\times 0.99-0\right)}_{\substack{\text{expected cost of}\\ \text{increasing the}\\ \text{reserve at time 1}}}-5{,}000\times 0.01$$

and hence the expected cost of increasing the reserve at the end of Year 1 is 1,569.80 (which is shown in the table as a negative since it represents a cost to the insurer) …

… and:

$${}_1V=\frac{1{,}569.80}{0.99}=1{,}585.66$$

(4) For Year 2, the recursive formula is:

$$21=(1{,}530-e_1)\times 1.06-\underbrace{\left({}_2V\times 0.99-1{,}585.66\times 1.06\right)}_{\substack{\text{expected cost of}\\ \text{increasing the}\\ \text{reserve at time 2}}}-5{,}000\times 0.01$$

But we know that the interest earned on the reserve in the third year is 195, so:

$${}_2V\times 0.06=195 \Rightarrow {}_2V=3{,}250$$

Hence the expected cost of increasing the reserve at the end of the second year is:

$$3{,}250\times 0.99-1{,}585.66\times 1.06=1{,}536.70$$

and:

$$e_1=1{,}530-\frac{21+50+1{,}536.70}{1.06}=13.30$$

(5) For Year 3, the recursive formula is:

$$45=(1{,}530-e_2)\times 1.06-\underbrace{\left({}_3V\times 0.99-{}_2V\times 1.06\right)}_{\substack{\text{expected cost of}\\ \text{increasing the}\\ \text{reserve at time 3}}}-5{,}000$$

But ${}_3V=0$, so the expected cost of increasing the reserve at time 3 is

$$-{}_2V\times 1.06=-3{,}250\times 1.06=-3{,}445$$

This is negative because the reserves that have been built up over the term of the policy are released at time 3 to cover the benefits.

We can summarise the expected cost of increasing reserves calculations in a table:

Year	Reserve at start of year	Interest	Expected reserve at end of year	Expected cost of increasing reserves
1	0	0	1,569.80	−1,569.80
2	1,585.66	95.14	3,217.50	− 1,536.70
3	3,250.00	195.00	0	3,445.00

So we have:

$$e_2 = 1{,}530 - \frac{45 + 5{,}000 - 3{,}445}{1.06} = 20.57$$

(6) So, finally, the interest earned on the fund in Years 2 and 3 is:

$$(1{,}530 - 13.30) \times 0.06 = 91.00 \quad \text{for Year 2}$$

$$(1{,}530 - 20.57) \times 0.06 = 90.57 \quad \text{for Year 3}$$

(ii) ***Internal rate of return***

The internal rate of return is the interest rate that satisfies the equation:

$$-51v + 21 \times 0.99v^2 + 45 \times 0.99^2 v^3 = 0$$

(where the 0.99 factors represent the probability of surviving each year).

Dividing through by v gives:

$$-51 + 20.79v + 44.10v^2 = 0$$

For the quadratic equation $ax^2 + bx + c = 0$, we know that the roots are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Applying this quadratic formula gives (ignoring the negative root):

$$v = 0.8652 \Rightarrow i = \frac{1}{0.8652} - 1 = 15.6\%$$

(iii) ***Effect on internal rate of return***

The internal rate of return will increase.

This is because the weaker reserving basis reduces the amounts of the reserves, causing profits to be released sooner, and this will increase their expected present value.

(iv) ***Net present value***

The net present value calculated using a risk discount rate of 7% is:

$$NPV = -51v + 20.79v^2 + 44.10v^3 = -47.66 + 18.16 + 36.00 = £6.50$$

(v) ***Effect on net present value***

The net present value will increase.

This is because the weaker reserving basis reduces the amounts of the reserves, causing profits to be released sooner, and this will increase their expected present value.

27.3 The latest negative cashflow is at the end of Year 3. So first we need a non-unit reserve at the start of Year 3 of:

$${}_2V = 6v$$

where v is calculated at the valuation (reserving) rate of interest.

Setting up the reserve at the start of Year 3 produces an adjusted expected non-unit cashflow at the end of Year 2 of:

$$-12 - {}_2V \times p_{61} = -\left(12 + 6vp_{61}\right)$$

The non-unit reserve at the start of Year 2 required to zeroise this is:

$${}_1V = -\left(12 + 6vp_{61}\right)v = 12v + 6v^2 p_{61}$$

So the expected loss at the end of Year 1 is:

$$L = 30 + {}_1V \times p_{60} = 30 + \left(12v + 6v^2 p_{61}\right)p_{60} = 30 + 12vp_{60} + 6v^2{}_2p_{60}$$

27.4 (i) ***Reserves at the start of Years 2 and 3***

As we only need reserves at the start of Years 2 and 3, we only need to project cashflows for those last two years. The calculations are shown in the following table:

Year	Premium	Expenses	Interest (1)	Expected claim cost	Expected surrender cost	Cashflow
2	3,250	–30	64.40	–56.50 (2)	–242.62 (3)	2,985.28
3	3,250	–32	64.36	–10,000	0	–6,717.64

where:

$$(1) = \left(\{\text{premium}\} - \{\text{expenses}\}\right) \times 0.02$$

$$(2) = -10{,}000 \times q_{57} = -10{,}000 \times 0.005650 = -56.50$$

$$(3) = -\left(3{,}250 \times 2 - 400\right) \times \left(1 - 0.005650\right) \times 0.04 = -242.62$$

The reserve at the start of Year 3 is:

$${}_2V = \frac{6,717.64}{1.02} = 6,585.92$$

The reserve at the start of Year 2 is:

$${}_1V = \frac{(1-0.005650)\times(1-0.04)\times 6,585.92 - 2,985.28}{1.02} = 3,236.75$$

(ii) ***Expected profit at end of Year 2***

The expected cashflow at the end of Year 2, per policy in force at the start of the year, is:

$$\{\text{premium}\} - \{\text{expenses}\} + \{\text{interest}\} - \{\text{expected death cost}\} - \{\text{expected surrender cost}\}$$

$$= 3,250 - 30 + (3,250-30)\times 0.04 - 0.75q_{[56]+1}\times 10,000$$

$$- \left(1-0.75q_{[56]+1}\right)\times 0.04\times(2\times 3,250 - 400)$$

$$= 3,220 + 128.80 - 0.75\times 0.005507\times 10,000 - (1-0.75\times 0.005507)\times 0.04\times 6,100$$

$$= 3,064.51$$

Allowing for the start and end of year reserves, the expected profit at the end of Year 2, per policy in force at the start of the year, is:

$$\{\text{expected cashflow}\} + \{\text{reserve at start of year}\} + \{\text{interest on reserve}\}$$

$$- \{\text{expected cost of end-year resserve}\}$$

$$= 3,064.51 + {}_1V + {}_1V\times 0.04 - (ap)_{[56]+1}\times {}_2V$$

$$= 3,064.51 + 3,236.75\times 1.04 - (1-0.75\times 0.005507)\times(1-0.04)\times 6,585.92$$

$$= 134.35$$

(iii) ***Why the profit is not zero***

If the projection assumptions had been the same as the reserving basis, then an expected profit of exactly zero would have been calculated for Year 2. However, the expected future experience has:

- lower mortality rates than the reserving basis, which reduces the expected death cost
- a higher interest rate on cashflows and reserves, which will increase investment income

both of which will increase the expected profit, which will therefore be greater than zero.

27.5 (i) ***Office premium***

The table for the profit test is as follows:

Year	Premium	Expenses	Interest	Expected death cost
1	P	$-200-0.35P$	$0.07(0.65P-200)$	−1,659.60
2	P	$-25-0.03P$	$0.07(0.97P-25)$	−1,863.80
3	P	$-25-0.03P$	$0.07(0.97P-25)$	−2,091.99
		[½]	[1]	[1]

Year	Expected end of year cashflow	Expected cost of increase in reserves	Profit vector
1	$0.6955P-1{,}873.60$	$-0.9920P$	$-0.2965P-1{,}873.60$
2	$1.0379P-1{,}890.55$	$0.0790P$	$1.1169P-1{,}890.60$
3	$1.0379P-2{,}118.74$	$1.07P$	$2.1079P-2{,}118.74$
	[1]	[1½]	[½]

Year	Probability in force	Profit signature
1	1	$-0.2965P-1{,}873.60$
2	0.991978	$1.1079P-1{,}875.38$
3	0.983041	$2.0722P-2{,}082.81$
	[1]	[1]

The expected death cost in Year t is calculated as:

$$200{,}000\times q_{60+t-1}\times 1.07^{½}$$

This assumes the immediate death payment occurs on average half way through the year, and so earns half a year's interest by the end of the year.

So the expected net present value of the profit is:

$$\frac{-0.2965P-1{,}873.60}{1.15}+\frac{1.1079P-1{,}875.38}{1.15^2}+\frac{2.0722P-2{,}082.81}{1.15^3}$$

$$=1.9424P-4{,}416.76 \qquad [1½]$$

Setting this equal to $0.25P$ and solving for P gives $P=£2{,}610$ to the nearest £1. [1]

[Total 10]

(ii) ***Cashflows, ignoring reserves***

The cashflows are given in the table below:

Year	Premium	Expenses	Interest	Expected death cost	Expected end of year cashflow
1	2,610	−1,113.5	104.76	−1,659.60	−58.34
2	2,610	−103.3	175.47	−1,863.80	818.37
3	2,610	−103.3	175.47	−2,091.99	590.18
		[½]	[½]		[1]

[Total 2]

(iii) ***Why reserves are not needed and impact of holding reserves***

As the table above shows, the policy is self-funding after the first year. [½]

So there is no need to hold reserves to cover future outgo. [½]

If the company didn't hold reserves, this would accelerate the emergence of profit. [1]

Since the risk discount rate is higher than the investment return, not holding reserves would increase the net present value of the profit. [1]

[Total 3]

27.6 (i) ***Net present value assuming no non-unit reserves***

The independent probabilities of mortality are:

$$q_{55}^d = q_{[55]} = 0.003358$$

$$q_{56}^d = q_{[55]+1} = 0.004903$$

$$q_{57}^d = q_{57} = 0.005650$$

$$q_{58}^d = q_{58} = 0.006352$$ [½]

So we have:

$$p_{55}^d = 0.996642 \qquad p_{55}^w = 0.9$$

$$p_{56}^d = 0.995097 \qquad p_{56}^w = 0.9$$

$$p_{57}^d = 0.994350 \qquad p_{57}^w = 0.95$$

$$p_{58}^d = 0.993648 \qquad p_{58}^w = 0.95$$ [1]

and:

$$(ap)_{55} = p^d_{55} \times p^w_{55} = 0.896978$$

$$(ap)_{56} = p^d_{56} \times p^w_{56} = 0.895587$$

$$(ap)_{57} = p^d_{57} \times p^w_{57} = 0.944633$$

$$(ap)_{58} = p^d_{58} \times p^w_{58} = 0.943966$$ [1]

It follows that:

$$_2(ap)_{55} = (ap)_{55} \times (ap)_{56} = 0.803322$$

$$_3(ap)_{55} = {}_2(ap)_{55} \times (ap)_{57} = 0.758844$$

$$_4(ap)_{55} = {}_3(ap)_{55} \times (ap)_{58} = 0.716323$$ [½]

So the net present value of the profit from the contract is:

$$-\frac{200}{1.1} + \frac{20 \times 0.896978}{1.1^2} + \frac{45 \times 0.803322}{1.1^3} - \frac{60 \times 0.758844}{1.1^4} + \frac{480 \times 0.716323}{1.1^5} = 42.56$$ [1]

[Total 4]

(ii)(a) ***Reserves required to zeroise negative cashflows***

We do not need a reserve at time 4 since the cashflow at time 5 is positive. So:

$$_4V = 0$$ [½]

We do need a reserve at time 3 since the cashflow at time 4 is negative. We require:

$$_3V(1+i) = 60 \Rightarrow {}_3V = \frac{60}{1.08} = 55.56$$ [½]

The non-unit cashflow at time 3 then becomes:

$$(NUCF)'_3 = 45 - {}_3V(ap)_{57} = 45 - 55.56 \times 0.944633 = -7.48$$ [1]

We now need a reserve at time 2 to zeroise this negative:

$$_2V(1+i) = 7.48 \Rightarrow {}_3V = \frac{7.48}{1.08} = 6.93$$ [½]

The non-unit cashflow at time 2 then becomes:

$$(NUCF)'_2 = 20 - {}_2V(ap)_{56} = 20 - 6.93 \times 0.895587 = 13.80$$ [1]

Since this is positive, we do not need a reserve at time 1, *ie*:

$$_1V = 0$$ [½]

(ii)(b) ***Net present value assuming non-unit reserves are set up***

The profit vector is the vector of non-unit cashflows after the reserves have been set up. So for this policy the profit vector is:

$$\left(-200,\ 13.80,\ 0,\ 0,\ 480\right)$$ [½]

The net present value is then:

$$-\frac{200}{1.1}+\frac{13.80\times 0.896978}{1.1^2}+\frac{480\times 0.716323}{1.1^5}=41.91$$ [1½]

[Total 6]

(iii) ***If non-unit reserves earned 10% pa interest***

Holding reserves delays the emergence of profit. [½]

However, if the rate of interest earned on the reserves is 10%, then we are accumulating and discounting at the same rate. In this case, delaying the emergence of profit will have no effect on the net present value for the contract. [1]

So the net present value would be 42.56 as in (i). [½]
[Total 2]

27.7 (i) ***Profit signature***

The calculations are shown in the following tables.

Year t	Premium (1)	Expenses (2)	Interest (3)	Mortality probability (4)	Death benefit + termination expenses (5)	Expected death cost (6)
1	8,000	–500	525.00	0.004171	20,100	–83.84
2	8,000	–30	557.90	0.006180	21,300	–131.63
3	8,000	–35	557.55	0.007140	22,572	–161.16

[3]

Year t	Dependent surrender probability (7)	Surrender value + terminat'n expenses (8)	Expected surrender cost (9)	Maturity value + terminat'n expenses (10)	Survival probability (11)	Expected maturity cost (12)
1	0.149374	8,340.00	–1,245.78	0	0.846455	0
2	0.049691	16,827.20	–836.16	0	0.944129	0
3	0	0	0	26,302.35	0.992860	–26,114.55

[6]

Year t	Cashflow (13)	Reserve at start of year (14)	Interest on reserve (15)	Expected cost of end yr reserve (16)	Profit vector (17)	Survival probability (18)
1	6,695.38	0	0	−6,348.41	346.97	1
2	7,560.11	7,500	525	−14,161.94	1,423.17	0.846455
3	−17,753.16	15,000	1,050	0	−1,703.17	0.799162

[3½]

Year t	Profit signature (19)
1	346.97
2	1,204.65
3	−1,361.11

[½]

Key to tables:

$(3)=\left[(1)+(2)\right]\times 0.07$ (as (2) is a deduction)

$(5)_t=20{,}000\left(1.06\right)^{t-1}+100$

$(6)=-(4)\times(5)$

$(7)=\left[1-(4)\right]\times\left[\text{surrender probability}\right]$

$(8)_1=8{,}000\times 1.03+100$

$(8)_2=8{,}000\left(1.03^2+1.03\right)+100$

$(9)=-(7)\times(8)$

$(10)_3=20{,}000\times 1.06^3\times 1.1+100$

$(11)=1-(4)-(7)$

$(12)=-(10)\times(11)$

$(13)=(1)+(2)+(3)+(6)+(9)+(12)$

Column (14) shows the reserve at time $t-1$.

$(15)=(14)\times 0.07$

$(16)_t=-(14)_{t+1}\times(11)_t$

$(17)=(13)+(14)+(15)+(16)$

$(18)_1=1$

$(18)_t=(18)_{t-1}\times(11)_{t-1},\ t>1$

$(19)=(17)\times(18)$ [Total 13]

(ii) ***Affordability of future bonuses***

Assuming 7% *pa* investment return, the first two years' cashflows accumulate to the following value at the end of Year 3:

$$346.97\times 1.07^2+1,204.65\times 1.07=£1,686.22$$ [1]

The outgo in the third year is £1,361.11. So the company can afford all the assumed bonuses during the policy with an additional profit of £325.11 at the end of the term. So, given these assumptions, it would be quite likely for the company to pay higher bonuses than these (though it depends upon how much profit is required for any shareholders). [1]

[Total 2]

27.8 (i) ***Emerging profit***

The tables required for the calculations are shown below:

Unit fund

Policy year	Premium allocated	Cost of allocation	Plus fund b/f	Fund before charge	Annual charge	Fund c/f
1	750.00	712.50	712.50	748.13	–5.61	742.52
2	1,050.00	997.50	1,740.02	1,827.02	–13.70	1,813.32
3	1,050.00	997.50	2,810.82	2,951.36	–22.14	2,929.22
4	1,050.00	997.50	3,926.72	4,123.06	–30.92	4,092.14

[5]

Non-unit fund

Policy year	Premium minus cost of allocation	Expenses	Non unit interest	Annual charge	Expected death cost	Expected in-force cashflow
1	287.50	–150.00	6.88	5.61	–22.57	127.42
2	2.50	–75.00	–3.63	13.70	–11.87	–74.30
3	2.50	–25.00	–1.13	22.14	–0.71	–2.20
4	2.50	–25.00	–1.13	30.92	0.00	7.29

[5]

The expected death cost is calculated as:

$$-\max\left[3{,}000-\{\text{unit fund at end of year}\},0\right]\times\{\text{probability of dying}\}$$

Eg in Year 1 we calculate:

$$-\max\left[3{,}000-742.52,0\right]\times 0.01=-2{,}257.48\times 0.01=-22.57$$

[Total 10]

(ii) ***Revised profit***

First of all we calculate the non-unit reserves.

The expected cashflow at the end of Year 4 is positive, so we do not need to hold a reserve at the start of year 4, *ie* ${}_3V=0$. [½]

At the end of Year 3, there is an expected cashflow of –2.20 per policy in force at the start of that year. To zeroise this, we need to hold a reserve of:

$${}_2V = \frac{2.20}{1.05} = 2.10$$ [½]

Holding this reserve has an effect on the expected cashflow at the end of Year 2. The adjusted cashflow is:

$$-74.30 - {}_2V \times p_{41} = -74.30 - 2.10 \times 0.99 = -76.38$$ [1]

To zeroise this, we need a non-unit reserve at the start of Year 2 equal to:

$${}_1V = \frac{76.38}{1.05} = 72.74$$ [½]

Applying these reserves will give the following expected profits, per policy in force at the start of each year:

Year 1

The expected profit is equal to the expected cashflow less the expected cost of setting up the end-year reserve. This is:

$$127.42 - {}_1V \times p_{40} = 127.42 - 72.74 \times 0.99 = 55.41$$ [1]

Years 2 and 3

The negative cashflows in these two years have both been zeroised by setting up the reserves. So the expected profit for both of these years is zero. [½]

Year 4

There are no reserves held at the start or end of Year 4, and so the expected profit is equal to the expected cashflow for this year, *ie* 7.29. [½]

The profit vector is therefore:

$$(55.41, 0, 0, 7.29)$$ [½]

[Total 5]

End of Part 5

What next?

1. Briefly **review** the key areas of Part 5 and/or re-read the **summaries** at the end of Chapters 23 to 27.
2. Ensure you have attempted some of the **Practice Questions** at the end of each chapter in Part 5. If you don't have time to do them all, you could save the remainder for use as part of your revision.
3. Attempt **Assignment X5**.
4. Attempt the questions relating to Chapters 23 to 27 of the **Paper B Online Resources (PBOR)**.
5. Attempt **Assignment Y2**

And finally …

Good luck!

Subject CM1: Assignment X1

2022 Examinations

Time allowed: 2¾ hours

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Have you used the solutions? Yes [ ] No [ ]

Are you allowed extra time or other special conditions in the profession's exams (if you wish to share this information)?
Yes [ ] No [ ]

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Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q6	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Total	
_ / 1	_ / 2	_ / 3	_ / 3	_ / 3	_ / 4	_ / 4	_ / 4	_ / 5	_ / 5	_ / 5	_ / 6	_ / 7	_ / 7	_ / 7	_ / 14	_ / 80	= _____%

Grade: A B C D E **Marker's initials:** ________

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Prize draw: For your chance to win £150 of gift vouchers, please submit your feedback and enter our six-monthly prize draw.

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A = Excellent progress B = Good progress C = Average progress
D = Below average progress E = Well below average progress

X1.1 Calculate the present value of a perpetuity paying £50 *pa* in arrears at an annual effective rate of interest of 6%. [1]

X1.2 Calculate the length of time it will take £800 to accumulate to £1,000 at a simple rate of interest of 4% *pa*. [2]

X1.3 Explain the main differences between a deterministic model and a stochastic model. [3]

X1.4 A 91-day government bill is discounted at a simple rate of discount of 10% *pa*. Calculate the annual effective rate of interest earned on this investment. [3]

X1.5 Describe the cashflows experienced by a term assurance policyholder. [3]

X1.6 An annuity of $300 *pa* is paid annually in advance for seven years, followed by $100 *pa* paid quarterly in arrears for a further five years. The rate of interest is 6% *pa* convertible half-yearly.

Determine the accumulated amount at the end of twelve years. [4]

X1.7 An investor, who has a sum of £10,000 to invest, wishes to purchase an annuity certain which makes payments over a ten-year period. Calculate the amount of the payments that can be provided if the annuity takes each of the following forms (assuming interest of 8% *pa* effective):

(i) a level annuity payable monthly in arrears [2]

(ii) a level annuity due payable half-yearly, commencing in 2 years' time. [2]

[Total 4]

X1.8 Explain when you would use real and money rates of interest. Give an example of when each rate of interest would be used. [4]

X1.9 (i) Calculate the nominal annual rate of discount convertible quarterly equivalent to a nominal rate of interest of 10% *pa* convertible quarterly. [2]

(ii) A single investment of £500 is accumulated at a nominal rate of discount of 6% *pa* convertible half-yearly for 1 year, followed by a nominal rate of interest of 6% *pa* convertible every 4 months for 1 year. Calculate the accumulated amount of this investment after 2 years. [3]

[Total 5]

X1.10 List the factors that should be considered when assessing the suitability of an actuarial model for its purpose. [5]

X1.11 For each of the following calculate the equivalent effective annual rate of interest:

(i) an effective rate of interest of 12.7% paid every 2 years [1]

(ii) an effective rate of discount of 5.75% *pa* [1]

(iii) a force of interest of ½% per month [1]

(iv) a nominal rate of discount of 6% *pa* convertible quarterly [1]

(v) a nominal rate of interest of 14% *pa* convertible every 2 years. [1]

[Total 5]

X1.12 An investor receives payments half-yearly in arrears for 20 years. The first payment is £250, and each payment is 2% higher than the previous one.

The interest rate is 6% *pa* effective for the first 10 years and 4% *pa* effective for the final 10 years.

Calculate, showing all workings, the present value of the payments. [6]

X1.13 The force of interest, $\delta(t)$, is a function of time and at time t, measured in years, is given by:

$$\delta(t) = 0.03 - 0.005t + 0.001t^2 \qquad 0 \le t \le 10$$

(i) Calculate the equivalent constant force of interest per annum for the period $t = 0$ to $t = 10$. [3]

(ii) Calculate, showing all workings, the accumulated value at time $t = 7$ of an investment of £250 at time $t = 0$ plus a further investment of £150 at time $t = 5$. [4]

[Total 7]

X1.14 (i) Prove that:

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$ [3]

(ii) An annuity payable monthly in arrears has a first payment of £300, with subsequent payments decreasing by £10 each month, until a final payment of £70 is made in two years' time.

Calculate the present value of the payments from this annuity using an effective rate of interest of 6% *pa*. [4]

[Total 7]

X1.15 In return for a fixed initial deposit, an investor receives a continuously payable annuity for a term of 15 years. The annual rate of payment is 50 in the first year, and the rate of payment increases in each subsequent year.

The investor can select either:

- Option 1: the rate of payment increases by 2 at the end of each year,
- Option 2: the rate of payment increases by 3% *pa* compound at the end of each year.

Determine which option would provide the better deal for the investor at an annual effective interest rate of 7%. [7]

X1.16 The force of interest $\delta(t)$ is a function of time, and at any time t, measured in years, is given by the formula:

$$\delta(t)=\begin{cases} 0.04+0.005t & 0\leq t<6 \\ 0.16-0.015t & 6\leq t<8 \\ 0.04 & 8\leq t \end{cases}$$

(i) Derive expressions in terms of t for the accumulated amount at time t of an investment of 1 at time 0. [6]

(ii) (a) Calculate the value at time 0 of £100 due at time 9.

(b) Calculate the annual effective rate of discount implied by the transaction in (a). [3]

(iii) A continuous payment stream is received at a rate of $45e^{0.01t}$ units per annum between time 10 and time 15. Calculate, showing all workings, the present value at time 4 of this payment stream. [5]

[Total 14]

END OF PAPER

Subject CM1: Assignment X2

2022 Examinations

Time allowed: 2¾ hours

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2022 Examinations

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Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	**Total**	
$\frac{\quad}{2}$	$\frac{\quad}{4}$	$\frac{\quad}{5}$	$\frac{\quad}{7}$	$\frac{\quad}{9}$	$\frac{\quad}{9}$	$\frac{\quad}{12}$	$\frac{\quad}{13}$	$\frac{\quad}{19}$	$\frac{\quad}{80}$	= _____%

Grade: A B C D E **Marker's initials:** ________

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X2.1 At time $t = 0$, the 2-year spot rate is 4% *pa* effective, the 3-year spot rate is 5% *pa* effective and the 4-year spot rate is 6% *pa* effective.

Calculate the 2-year continuous-time forward rate from time $t = 2$. [2]

X2.2 A borrower has agreed to repay a loan of £20,000 with payments of £427.90 made monthly in arrears for 5 years.

Calculate, showing all workings, the APR charged on the loan. [4]

X2.3 An investor, who is liable to income tax at 20% but is not liable to capital gains tax, wishes to earn a net effective rate of return of at least 5% per annum. A bond that pays coupons half-yearly in arrear at a rate of 6.25% per annum has just been issued. The bond will be redeemed at par on a coupon date between 10 and 15 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

Determine the maximum price that the investor is willing to pay for £100 nominal of the bond. [5]

X2.4 An investor finances a project with an initial payment of £10,000. Income from the project is received half-yearly in arrears for the next 10 years. The income is £1,500 *pa* for the first 5 years, and £3,000 *pa* for the final 5 years.

Using an effective rate of interest of 8% *pa*, calculate:

(i) the net present value [2]

(ii) the discounted payback period. [5]

[Total 7]

X2.5 The liabilities of a fund consist of two lump sum payments due at known times in the future. The second lump sum is due for payment 5 years after the first and is twice the amount of the first.

The total present value and the discounted mean term of the liabilities, both calculated using an interest rate of 6% *pa* effective, are £75,000 and 8 years, respectively.

(i) Determine the timing and amounts of the liability payments. [6]

The assets of the fund consist of a single zero-coupon bond that will mature 8 years from now with a redemption payment of £119,540.

(ii) Explain what Redington's theory of immunisation tells you about this fund's portfolio. [3]

[Total 9]

X2.6 An index-linked bond was issued on 1 January 2010 with a term of 7 years. Coupons were payable annually and the redemption value before indexing was £105 per £100 nominal. Coupons and redemption payments were indexed by reference to the value of an inflation index with a time lag of 6 months. The annual nominal coupon rate on the bond was 4% and a coupon of £4.31 per £100 nominal was paid on 1 January 2014.

A tax-exempt investor purchased £100 nominal of the bond on 1 July 2014 and held it to redemption. The inflation index was as follows:

Date	***Index value***
1 July 2013	125.0
1 January 2014	127.1
1 July 2014	128.2
1 January 2015	130.9
1 July 2015	131.3
1 January 2016	132.6
1 July 2016	134.8
1 January 2017	136.0

(i) Show that the value of the inflation index on 1 July 2009 was 116.0. [1]

(ii) Calculate the amounts of the coupon payments and the redemption payment received by the investor who purchased the bond on 1 July 2014. [4]

(iii) Determine the price paid by the investor on 1 July 2014, given that he achieved a real return of 5% *pa* effective from owning the bond. [4]

[Total 9]

X2.7 A loan is to be repaid by a series of instalments payable annually in arrears for 20 years. The first instalment is £1,400 and payments increase thereafter by £300 per year. Repayments are calculated using an annual effective interest rate of 7%.

(i) Calculate the amount of the loan. [3]

(ii) Calculate the total amount of money owed by the borrower immediately after the third instalment has been paid. [2]

(iii) Explain your answer to (ii). [2]

Immediately after the third instalment has been paid, it is decided to restructure the loan, so that level payments are made quarterly in arrears for the remaining term of the loan. The interest rate on the restructured loan is 9% *pa* convertible half-yearly.

(iv) Calculate the amount of the quarterly payment. [3]

(v) Calculate the total interest paid over the whole term of the loan. [2]

[Total 12]

X2.8 One year ago, a bond was issued with a coupon rate of 14% *pa*, payable half-yearly in arrears. The bond will be redeemed at £110% in nine years' time.

The bond was issued at a price such that an investor subject to income tax at 35%, but not subject to capital gains tax, would obtain a net yield of 9.5% *pa*.

(i) Calculate, showing all workings, the issue price for £100 nominal. [3]

The investor has now decided to sell the bond and has found a potential buyer, who is subject to income tax at 25% and capital gains tax at 35%, and who is prepared to buy the bond provided that he obtains a net yield of at least 10% *pa*.

(ii) Calculate, showing all workings, the best price (per £100 nominal) the original investor can expect to obtain from the potential buyer. [5]

(iii) Calculate the net running yield obtained by the buyer. [1]

(iv) Determine the net yield that will be obtained by the original investor if the bond is sold to the buyer at the price determined in (ii). [4]

[Total 13]

X2.9 (i) Explain what is meant by the discounted payback period from an investment project. [2]

(ii) An insurance company is considering setting up a branch in a country in which it has not previously operated. The company is aware that access to capital may become difficult in twelve years' time. It therefore has two decision criteria. The cashflows from the project must provide an internal rate of return greater than 9% per annum effective and the discounted payback period at a rate of interest of 7% per annum effective must be less than twelve years.

The following cashflows are generated in the development and operation of the branch.

Cash Outflows

Between the present time and the opening of the branch in three years' time the insurance company will spend $£1.5m$ per annum on research, development and the marketing of products. This outlay is assumed to be a constant continuous payment stream. The rent on the branch building will be $£0.3m$ per annum paid quarterly in advance for twelve years starting in three years' time. Staff costs are assumed to be $£1m$ in the first year, $£1.05m$ in the second year, rising by 5% per annum each year thereafter. Staff costs are assumed to be incurred at the beginning of each year starting in three years' time and assumed to be incurred for 12 years.

Cash Inflows

The company expects the sale of products to produce a net income at a rate of $£1m$ per annum for the first three years after the branch opens rising to $£1.9m$ per annum in the next three years and to $£2.5m$ for the following six years. This net income is assumed to be received continuously throughout each year. The company expects to be able to sell the branch operation 15 years from the present time for $£8m$.

Determine which, if any, of the decision criteria the project fulfils. [17]

[Total 19]

END OF PAPER

Subject CM1: Assignment X3

2022 Examinations

Time allowed: 2¾ hours

Instructions to the candidate

1. Please:
   - attempt all of the questions
   - begin your answer to each question on a new page
   - leave at least 2cm margin on all borders
   - either type your answers in Word or write your answers using black ink and a medium-sized nib (because we will be unable to mark illegible scripts)
   - use a minimum font size of 11pt, if typing your answers
   - note that assignment marking is not included in the price of the course materials. Please purchase Series Marking or a Marking Voucher before submitting your script.
   - note that we only accept the current version of assignments for marking, *ie* you can only submit this assignment in the sessions leading to the 2022 exams.
2. Please **do not**:
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At the end of the assignment

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Cover sheet

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You can download the Word version of the cover sheet to include at the start of your script from our website **www.ActEd.co.uk** under Products, Marking, Coversheets. Failing to attach your completed cover sheet may delay the return of your marked script.

Submitting a script completed in Word

Ensure that you include the completed cover sheet at the start of your document. Convert your Word document to a PDF file.

Submitting a handwritten script

Your script should be scanned, together with the completed cover sheet (and Marking Voucher if applicable) and saved as a PDF file. Please ensure that all pages have been scanned the right way up (so that they can be read normally without rotation) and as a single document. We cannot accept individual files for each page. Before sending to ActEd, please check carefully that your scanned assignment includes all pages.

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Please name the PDF file '**CM1 Assignment X3 No. 12345**', inserting your ActEd Student Number for 12345.

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Please title your email '**CM1 Assignment X3 No. 12345**', inserting your ActEd Student Number for 12345.

Subject CM1: Assignment X3

2022 Examinations

Please complete the following information:

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Note: Your ActEd Student Number is printed on all personal correspondence from ActEd. Quoting it will help us to process your scripts quickly. If you do not know your ActEd Student Number, please email us at **ActEd@bpp.com**.

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Have you used the solutions? Yes ☐ No ☐

Are you allowed extra time or other special conditions in the profession's exams (if you wish to share this information)?
Yes ☐ No ☐

If Yes, you can provide further information on the extra time / other conditions if you wish:

____

____

Time to do assignment
(see Note below): _____ hrs _____ mins

Note: If you take more than 2¾ hours, you should indicate how much you completed within this exam time so that the marker can provide useful feedback on your progress.

Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Total	
$\frac{}{2}$	$\frac{}{3}$	$\frac{}{4}$	$\frac{}{5}$	$\frac{}{5}$	$\frac{}{5}$	$\frac{}{6}$	$\frac{}{6}$	$\frac{}{6}$	$\frac{}{7}$	$\frac{}{7}$	$\frac{}{11}$	$\frac{}{13}$	$\frac{}{80}$	=_____%

Grade: A B C D E **Marker's initials:** ________

Please tick the following checklist so that your script can be marked quickly. Have you:

[ ] Checked that you are using the latest version of the assignments, *ie* 2022 for the sessions leading to the 2022 exams?

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[ ] Completed your ActEd Student Number in the box above?

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How was your marking?

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Prize draw: For your chance to win £150 of gift vouchers, please submit your feedback and enter our six-monthly prize draw.

Notes on marker's section

The main objective of marking is to provide specific advice on how to improve your chances of success in the exam. The most useful aspect of the marking is the comments the marker makes throughout the script, however you will also be given a percentage score and the band into which that score falls. Each assignment tests only part of the course and hence does not give a complete indication of your likely overall success in the exam. However it provides a good indicator of your understanding of the material tested and the progress you are making with your studies:

A = Excellent progress B = Good progress C = Average progress
D = Below average progress E = Well below average progress

X3.1 A two-year term assurance policy is issued to a life aged 75 exact. The benefit amount is 100,000 if the life dies in the first year, and 200,000 if the life dies in the second year. Benefits are payable at the end of the year of death.

Calculate the expected present value of the benefits from this policy. [2]

Basis: Mortality: PFA92C20

Interest: 5% *pa* effective

X3.2 A life insurance company issues an annuity to a life aged 60 exact, under which the annual payment is 17,000. The annuity is payable monthly in advance and is guaranteed to be paid for a period of 10 years and for the whole of life thereafter.

Calculate the expected present value of the annuity payments. [3]

Basis: Mortality: AM92 Ultimate

Interest: 6% *pa* effective

X3.3 A life insurance company issues a deferred annuity contract to a life aged exactly 40. Premiums of £4,500 are payable annually in advance for 25 years or until earlier death.

The policy provides the following benefits:

- On survival to age 65, an annuity of £15,000 *pa* is payable continuously for the whole of the policyholder's life.
- In the event of the policyholder's death during the deferment period, a lump sum is payable at the end of the year of death equal to the total amount of premiums paid to date, without interest.

Calculate the total expected present value of benefits from this policy. [4]

Basis Mortality: AM92 Select before age 65

PMA92C20 after age 65

Interest: 4% *pa* effective

X3.4 Calculate the exact value of $\bar{A}^{\,1}_{70:\overline{1|}}$ assuming the force of mortality is constant between consecutive integer ages. [5]

Basis Mortality: ELT15 (Males)

Interest: 7.5% *pa* effective

X3.5 A man aged exactly 42 purchases a whole life assurance policy with a sum assured of 5,000 payable immediately on death.

(i) Write down an expression for the random variable representing the present value of the benefits from this policy. [1]

(ii) Show that the variance of this random variable is $5{,}000^2\left({}^2\bar{A}_{42}-(\bar{A}_{42})^2\right)$. [2]

(iii) Calculate the variance of the present value of the benefits from this assurance policy, assuming AM92 Ultimate mortality and 4% *pa* interest. [2]

[Total 5]

X3.6 A policyholder aged 65 exact buys a whole life annuity that provides payments at the end of each policy year. The first payment is £10,000, and subsequent payments increase by 3% *pa* compound.

Let X denote the present value random variable for this annuity.

Interest is assumed to be 3% *pa* effective and mortality is assumed to follow the AM92 Ultimate table.

(i) Derive an expression for X and simplify this as much as possible. [2]

(ii) Calculate $E(X)$. [1]

(iii) Calculate the probability that the present value of the benefit received by the policyholder is greater than £250,000. [2]

[Total 5]

X3.7 A population is subject to the force of mortality $\mu_x = 0.03e^{0.015x-1.8}$.

Calculate the probability that a life now aged 35 exact:

(i) survives to age 65 exact [3]

(ii) dies between ages 65 exact and 80 exact. [3]

[Total 6]

X3.8 Estimate ${}_2p_{63.25}$ assuming ELT15 (Males) mortality at integer ages and:

(a) a uniform distribution of deaths between integer ages

(b) a constant force of mortality between integer ages. [6]

X3.9 (i) Explain what is meant by the following notation, and calculate its value using AM92 mortality:

$${}_{3|}q_{[55]+1}$$ [2]

(ii) Calculate the value of the following symbols, using AM92 mortality and an interest rate of 6% *pa* effective:

(a) $\ddot{a}^{(4)}_{[40]:\overline{5}|}$

(b) $(I\bar{a})_{70:\overline{10}|}$ [4]

[Total 6]

X3.10 In 20 years' time a sum of 20,000 is to be divided equally amongst the survivors of two independent lives now aged 30 and 40 and a charitable trust.

Determine the expected present value and the variance of the present value of the amount due to the charitable trust. [7]

Basis: Mortality: AM92 Ultimate

Interest: 4% *pa* effective

X3.11 (i) Describe the four different methods of allocating bonuses to conventional with-profits contracts. [3]

A life insurance company issues a 15-year with-profits endowment assurance policy to a life aged 50. The sum assured of 50,000 plus declared reversionary bonuses is payable on survival to the end of the term or immediately on earlier death.

The company assumes that it will award compound bonuses at a rate of 1.92308% *pa* at the end of each policy year (*ie* the death benefit does not include any bonus relating to the policy year of death).

(ii) Calculate the expected present value of the benefits from this policy. [4]

Basis: Mortality: AM92 Ultimate

Interest: 6% *pa* effective

[Total 7]

X3.12 Let K denote the curtate future lifetime random variable of a life aged exactly x.

(i) Describe the benefit whose present value random variable is:

$$W = \begin{cases} 10{,}000\ddot{a}_{\overline{K+1}|} & \text{if } K < 10 \\ 10{,}000\ddot{a}_{\overline{10}|} & \text{if } K \geq 10 \end{cases}$$ [1]

(ii) Prove the premium conversion formula:

$$A_{x:\overline{n}|} = 1 - d\ddot{a}_{x:\overline{n}|}$$ [2]

(iii) Calculate the expected present value and the standard deviation of the present value of the benefit in (i), assuming:

- a force of interest of 0.04 *pa*
- the life is subject to a constant force of mortality of 0.02 *pa*. [8]

[Total 11]

X3.13 A pension fund provides its members with benefits on retirement at exact age 65. Members can choose to receive their benefits in one of three forms:

A A lump sum of £62,500 immediately on retirement, followed by a flat pension of £21,500, which is guaranteed for five years. Pension payments cease on the death of the member or at the end of the guarantee period, whichever is later.

B A lump sum of £100,000 paid on either the member's 80th birthday or on death, if earlier, plus a pension that starts at £20,000 and increases by 2.5% *pa* simple, payable until death.

C A pension that starts at £20,000 and increases in line with inflation, payable until the member dies, plus a lump sum payable on death of five times the most recent pension instalment.

Pensions are paid annually in advance. Increases are applied on the anniversary of the member's retirement (*ie* the first increase is applied at exact age 66). Lump sum death benefits are payable at the end of the year of death.

(i) Calculate the expected present values of the three benefit options. [9]

(ii) Discuss how a member might decide which benefit is best for them. You may assume that the member knows the expected present value of each option. [4]

Basis: Mortality: AM92 Select

Interest: 6% *pa* effective

Inflation: 1.92308% *pa* compound

[Total 13]

END OF PAPER

Subject CM1: Assignment X4

2022 Examinations

Time allowed: 3¼ hours

Instructions to the candidate

1. Please:
   - attempt all of the questions
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At the end of the assignment

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Submission for marking

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Subject CM1: Assignment X4

2022 Examinations

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Have you used the solutions? Yes [ ] No [ ]

Are you allowed extra time or other special conditions in the profession's exams (if you wish to share this information)?
Yes [ ] No [ ]

If Yes, you can provide further information on the extra time / other conditions if you wish:

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Time to do assignment
(see Note below): _____ hrs _____ mins

Note: If you take more than 3¼ hours, you should indicate how much you completed within this exam time so that the marker can provide useful feedback on your progress.

Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	**Total**	
$\frac{\ }{2}$	$\frac{\ }{4}$	$\frac{\ }{4}$	$\frac{\ }{5}$	$\frac{\ }{5}$	$\frac{\ }{5}$	$\frac{\ }{5}$	$\frac{\ }{6}$	$\frac{\ }{6}$	$\frac{\ }{9}$	$\frac{\ }{10}$	$\frac{\ }{10}$	$\frac{\ }{11}$	$\frac{\ }{18}$	$\frac{\ }{100}$	= _____%

Grade: A B C D E **Marker's initials:** ________

Please tick the following checklist so that your script can be marked quickly. Have you:

[ ] Checked that you are using the latest version of the assignments, *ie* 2022 for the sessions leading to the 2022 exams?
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D = Below average progress E = Well below average progress

X4.1 A 5-year endowment assurance is issued to a life aged 60. The policy pays a benefit of £20,000 on maturity or at the end of the year of earlier death. Level premiums are payable annually in advance while the policy is in force.

Calculate the net premium reserve at the end of the second policy year using AM92 Ultimate mortality and an effective annual interest rate of 6%. [2]

X4.2 A 65-year-old male buys a one-year term assurance policy with a sum assured of 200,000 paid immediately on death. Assuming that the force of mortality is constant between integer ages, calculate the single premium that he should pay for this policy.

Basis: Mortality: ELT15 (Males)

Interest: $\delta = 2.5\%$ *pa*

Expenses: none [4]

X4.3 Calculate ${}_{5|4}q^{\;1}_{[61]:[61]}$ where both lives are subject to AM92 mortality. [4]

X4.4 Define $\ddot{a}^{(12)}_{60:50:\overline{20|}}$ fully in words and calculate its value using an interest rate of 4% *pa* effective, and PMA92C20 and PFA92C20 mortality for the two lives respectively. [5]

X4.5 A joint life annuity of 1 *pa* is payable continuously to lives currently aged x and y while both lives are alive. The present value of the annuity payments is expressed as a random variable, in terms of the joint future lifetime of x and y.

Derive, and simplify as far as possible, expressions for the expected present value and the variance of the present value of the annuity, in terms of assurance functions. [5]

X4.6 A life insurance company issues whole life assurance policies with a sum assured of £100,000 payable at the end of the year of death to lives aged exactly 35. Level premiums are charged annually in advance while the policy is in force.

Calculate the minimum premium the office could charge in order that the probability of making a loss on any one policy would be 1% or less. [5]

Basis: Mortality: AM92 Select

Interest: 6% *pa* effective

Expenses: 5% of each premium

X4.7 A population is subject to the force of mortality at age x of $\mu_x = e^{0.0002x} - 1$.

Calculate the probability that a life now aged 20 exact:

(i) survives to age 70 exact [2]

(ii) dies between ages 60 exact and 70 exact. [3]

[Total 5]

X4.8 An n-year term assurance with a sum assured of 1 payable at the end of the year of death is issued to a life aged x. Level premiums are payable annually in advance throughout the term of the policy or until the policyholder's earlier death. The premium includes an initial expense loading of I, and a renewal expense loading of e at the start of each policy year, including the first.

(i) Write down expressions, in terms of standard actuarial functions, for:

(a) the gross premium

(b) the prospective gross premium reserve at the end of the t th policy year (where $t < n$)

(c) the retrospective gross premium reserve at the end of the t th policy year (where $t < n$). [3]

(ii) Hence show that, if all three of the expressions in (i) are calculated on the same basis, the prospective and retrospective gross premium reserves are equal. [3]

[Total 6]

X4.9 A policyholder, aged exactly 50, takes out a 15-year term assurance where the sum assured is £10,000 for the first 5 years and £15,000 thereafter. The sum assured is payable at the end of the year of death.

Level premiums are payable annually in advance for at most 10 years while the policy is in force.

Calculate the annual premium. [6]

Basis:	Mortality:	AM92 Select	
	Interest:	4% *pa* effective	
	Expenses:	Initial:	25% of the first premium
		Renewal:	5% of the second and subsequent premiums

X4.10 A life insurance company issues an annuity policy to two lives each aged 60 exact in return for a single premium. Under the policy, an annuity of £10,000 *pa* is payable annually in advance while at least one of the lives is alive.

(i) Write down an expression for the net future loss random variable at the outset for this policy. [2]

(ii) Calculate the single premium, using the equivalence principle.

Basis: Mortality: PMA92C20 for the first life, PFA92C20 for the second life

Interest: 4% *pa* effective

Expenses: Ignore [3]

(iii) Calculate the standard deviation of the net future loss random variable at the outset for this policy, using the basis in part (ii).

You are given that $\ddot{a}_{\overline{60:60}} = 11.957$ at a rate of interest 8.16% *pa*. [4]

[Total 9]

X4.11 An index-linked deferred annuity is issued to a life currently aged exactly 51. The first annuity payment is made at exact age 65 and continues at annual intervals thereafter until the death of the policyholder. The benefit level, which at the outset of the policy is 20,000 *pa*, increases annually in line with inflation, both while in payment and in deferment.

Level premiums are paid annually in advance until age 65 or the earlier death of the policyholder.

On death before age 65, all premiums paid to the date of death are returned without interest, paid at the end of the year of death.

Calculate the annual premium. [10]

Basis: Mortality: AM92 Select

Interest: 4% *pa* effective

Inflation: 4% *pa* compound

Expenses: Initial: 300

Renewal: 3% of each premium excluding the first

Claim: 150 on death before age 65 (payable at the same time as the death claim), inflating at 4% *pa* from policy outset
plus
0.25% of each annuity payment

X4.12 A life insurance company issues an annuity contract to a man aged 65 exact and his wife aged 62 exact. Under the contract, an annuity of £20,000 *pa* is guaranteed payable for a period of 5 years and thereafter during the lifetime of the man. On the man's death, an annuity of £10,000 *pa* is payable to his wife, if she is then alive. This annuity commences on the monthly payment date next following, or coincident with, the date of his death or from the 5th policy anniversary, if later, and is payable for the lifetime of his wife. Annuities are payable monthly in advance.

Calculate the single premium required for the contract. [10]

Basis: Mortality: PMA92C20 for the male and PFA92C20 for the female

Interest: 4% *pa* effective

Expenses: None

X4.13 A life insurance company issued a with-profits whole life policy to a life aged 20 exact, on 1 July 2015. Under the policy, the basic sum assured of £100,000 and attaching bonuses are payable immediately on death. The company declares simple reversionary bonuses at the start of each year. Level premiums are payable annually in advance under the policy.

(i) Give an expression for the gross future loss random variable under the policy at the outset. Define symbols where necessary. [3]

(ii) Calculate the annual premium, using the equivalence principle.

Basis: Mortality: AM92 Select

Interest: 6% *pa* effective

Bonus loading: 3% *pa* simple

Expenses: Initial: £200

Renewal: 5% of each premium payable in the second and subsequent years

Assume bonus entitlement is earned immediately on payment of premium. [4]

(iii) On 30 June 2018 the policy is still in force. A total of £10,000 has been declared as a simple bonus to date on the policy.

The company calculates reserves for the policy using a gross premium prospective basis, with the following assumptions:

Mortality: AM92 Ultimate

Interest: 4% *pa* effective

Bonus loading: 4% *pa* simple

Renewal expenses: 5% of each premium

Calculate the reserve for the policy as at 30 June 2018. [4]

[Total 11]

X4.14 A special term assurance policy is such that a sum of £20,000 is payable if a life (x) dies within a 20-year period. The sum assured is payable immediately on (x)'s death if another life (y) dies before (x). However, if (y) is alive at the time of (x)'s death, payment of the sum assured is deferred until the end of the 20-year period. A continuous level premium is payable.

(i) State, with reasons, the appropriate annuity factor that should be used to calculate the expected present value of the premiums, as at policy outset. [3]

(ii) Calculate the annual rate of premium payable, assuming a constant annual force of interest of 0.05 throughout, and that both lives are subject to the same constant annual force of mortality of 0.005 at all ages. Ignore expenses. [9]

(iii) Assuming the same interest and mortality basis as in (ii), and that no benefit has yet been paid out under the policy, calculate the prospective reserve on this policy after exactly 4 years under each of the following scenarios:

(a) only life (x) is alive at that time

(b) only life (y) is alive at that time

(c) both lives are dead by that time. [5]

(iv) Comment briefly on the differences between the answers you have obtained in (iii). [1]

[Total 18]

END OF PAPER

Subject CM1: Assignment X5

2022 Examinations

Time allowed: 3¼ hours

Instructions to the candidate

1. Please:
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   - note that we only accept the current version of assignments for marking, *ie* you can only submit this assignment in the sessions leading to the 2022 exams.
2. Please **do not**:
   - use headed paper
   - use highlighting in your script.

At the end of the assignment

If your script is being marked by ActEd, please follow the instructions on the reverse of this page.

Submission for marking

You should aim to submit this script for marking by the recommended submission date. The recommended and deadline dates for submission of this assignment are listed on our website at **www.ActEd.co.uk** (Products, Marking, Deadlines).

Scripts received after the deadline date will not be marked, unless you are using a Marking Voucher. **It is your responsibility to ensure that scripts reach ActEd in good time**. If you are using Marking Vouchers, then please make sure that your script reaches us by the Marking Voucher deadline date to give us enough time to mark and return the script before the exam.

Cover sheet

You **must** ensure that a completed cover sheet is part of the single PDF file that you submit for marking. You should complete all information, not forgetting the checklist, and include the 'Feedback from marker' sheet.

You can download the Word version of the cover sheet to include at the start of your script from our website **www.ActEd.co.uk** under Products, Marking, Coversheets. Failing to attach your completed cover sheet may delay the return of your marked script.

Submitting a script completed in Word

Ensure that you include the completed cover sheet at the start of your document. Convert your Word document to a PDF file.

Submitting a handwritten script

Your script should be scanned, together with the completed cover sheet (and Marking Voucher if applicable) and saved as a PDF file. Please ensure that all pages have been scanned the right way up (so that they can be read normally without rotation) and as a single document. We cannot accept individual files for each page. Before sending to ActEd, please check carefully that your scanned assignment includes all pages.

Requirements for the PDF file

Please name the PDF file '**CM1 Assignment X5 No. 12345**', inserting your ActEd Student Number for 12345.

It is important that the PDF file submitted:

- is legible (so check the resolution setting if necessary)
- is less than 10 MB in size
- is not protected in any way (otherwise your marker will not be able to return the script to ActEd, which causes delays).

Sending to ActEd

Email the PDF file to **ActEdMarking@bpp.com**. If you are submitting a Marking Voucher and this has not been included in the PDF, the voucher should also be attached to the email.

Please title your email '**CM1 Assignment X5 No. 12345**', inserting your ActEd Student Number for 12345.

Subject CM1: Assignment X5

2022 Examinations

Please complete the following information:

Name:

ActEd Student Number (see Note below):

Note: Your ActEd Student Number is printed on all personal correspondence from ActEd. Quoting it will help us to process your scripts quickly. If you do not know your ActEd Student Number, please email us at **ActEd@bpp.com**.

Your ActEd Student Number is not the same as your IFoA Actuarial Reference Number or ARN.

Have you used the solutions? Yes [ ] No [ ]

Are you allowed extra time or other special conditions in the profession's exams (if you wish to share this information)?
Yes [ ] No [ ]

If Yes, you can provide further information on the extra time / other conditions if you wish:

____

____

Time to do assignment
(see Note below): _____ hrs _____ mins

Note: If you take more than 3¼ hours, you should indicate how much you completed within this exam time so that the marker can provide useful feedback on your progress.

Score and grade for this assignment (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	**Total**	
$\frac{}{3}$	$\frac{}{4}$	$\frac{}{4}$	$\frac{}{6}$	$\frac{}{7}$	$\frac{}{7}$	$\frac{}{9}$	$\frac{}{9}$	$\frac{}{11}$	$\frac{}{12}$	$\frac{}{13}$	$\frac{}{15}$	$\frac{}{100}$	= _____%

Grade: A B C D E **Marker's initials:** ________

Please tick the following checklist so that your script can be marked quickly. Have you:

[ ] Checked that you are using the latest version of the assignments, *ie* 2022 for the sessions leading to the 2022 exams?

[ ] Completed your full name in the box above?

[ ] Completed your ActEd Student Number in the box above?

[ ] Recorded your time taken and whether you have used the solutions or not?

[ ] Checked that all pages have been included and in the right order, if you have scanned your script?

[ ] Included your Marking Voucher or ordered Series X Marking?

[ ] Rated your X4 marker at **www.ActEd.co.uk/marking**?

Please follow the instructions on the previous page when submitting your script for marking.

Feedback from marker

How was your marking?

You can provide feedback on the marking of this and earlier assignments at www.ActEd.co.uk/marking.

We want the standard of our marking to be as high as possible so we pay markers on the basis of student feedback. Your feedback is very important to us and your marker so please do leave a grade and comment.

Prize draw: For your chance to win £150 of gift vouchers, please submit your feedback and enter our six-monthly prize draw.

Notes on marker's section

The main objective of marking is to provide specific advice on how to improve your chances of success in the exam. The most useful aspect of the marking is the comments the marker makes throughout the script, however you will also be given a percentage score and the band into which that score falls. Each assignment tests only part of the course and hence does not give a complete indication of your likely overall success in the exam. However it provides a good indicator of your understanding of the material tested and the progress you are making with your studies:

A = Excellent progress B = Good progress C = Average progress
D = Below average progress E = Well below average progress

X5.1 A 10-year endowment assurance policy has a sum assured of £12,000 payable on survival or at the end of the year of earlier death. If the policy is surrendered, the policyholder will receive a return of premiums without interest. Surrenders can occur only at the end of a policy year.

A level premium of £1,100 *pa* is payable annually in advance.

For a policy in force at the start of the fifth year you are given the following details:

	(£)
Renewal expenses	40
Claim expenses on death or surrender	100
Reserve at the start of year	5,000
Reserve at end of year (per policy still in force)	6,500
Rate of interest	8% *pa* effective
Dependent probability of death	0.01
Dependent probability of surrender	0.07

Calculate the profit expected to emerge at the end of the fifth year, per policy in force at the start of that year. [3]

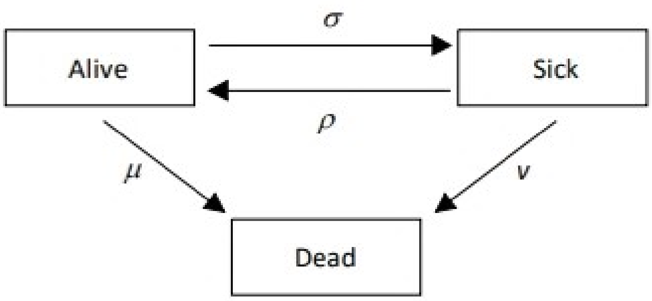
X5.2 A 4-year conventional endowment assurance policy issued to lives aged exactly 61 has a sum assured of £10,000. The profit signature, calculated assuming AM92 Ultimate mortality and making no allowance for surrenders, is (–100, –20, 80, 140). Reserves have been calculated on a net premium basis using 6% *pa* interest and AM92 Ultimate mortality.

The calculations are modified to allow for 10% of policies in force at the end of the first year to be surrendered with a surrender value of £1,500.

(i) Calculate the revised profit in the first year. [3]

(ii) Comment on the impact on the profit signature in years 2 to 4. [1]
[Total 4]

X5.3 A three-state transition model is shown in the following diagram:



Assume that the transition intensities are constant at all ages with $\mu = 0.02$ *pa* , $\nu = 0.04$ *pa* , $\rho = 0.01$ *pa* and $\sigma = 0.05$ *pa* .

Calculate the expected present value of a sickness benefit of £2,000 *pa* paid continuously to a sick life now aged 40 exact, for this period of sickness only, discounted using an interest rate of 4% *pa* effective and payable to a maximum age of 60 exact. [4]

X5.4 (i) Outline the main features of a (non-unitised) accumulating with-profits contract. [4]

(ii) You are given the following details about a unitised with-profits contract:

- fund value on 11th March: £65,292
- monthly premium (payable on first day of month): £600
- annual bonus interest rate for the calendar year: 4.25%
- monthly policy fee (payable on fifteenth day of month): £3

The bonus interest is credited to the policy on a daily basis, by increasing the unit price at the appropriate daily effective interest rate.

Assuming there are no other charges on the contract, calculate the fund value for this policy as at 6th April of the same year. You should assume that there are 365 days in this calendar year. [2]

[Total 6]

X5.5 A unit-linked policy has the following profit vector:

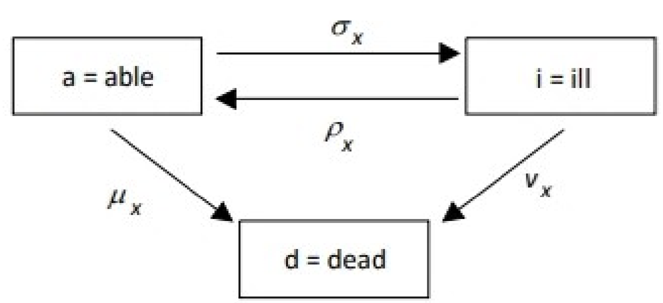
Year	*In-force profit*
1	–25
2	–12
3	–6
4	25
5	35

(i) Calculate the reserves required, in order to zeroise the losses occurring at the end of years 2 and 3. Assume a rate of accumulation of 8% *pa* effective, and that $q_x = 0.01$ at each age. [2]

(ii) If the risk discount rate used is 10% *pa* effective, determine the net present value of the profits before and after zeroisation and state with reasons which of these figures you would expect to be greater. [5]

[Total 7]

X5.6 Consider the following three-state illness-death model:



Let ${}_tp_x^{jk}$ denote the probability that a life in state j at age x will be in state k at age $x+t$, and let ${}_tp_x^{\overline{jj}}$ denote the probability that a life in state j at age x will remain in state j for at least t years. Given a constant force of interest of δ *pa*, write down integral expressions for the expected present value of each of the following benefits payable to a policyholder who is currently aged exactly 50 and in the able state:

(i) a benefit of £50,000 payable immediately on death, provided that death occurs within the next 10 years [2]

(ii) a benefit of £50,000 payable immediately on death, provided that death occurs within the next 10 years and the life has been sick for at least a year at the time of death [3]

(iii) a sickness benefit of £5,000 *pa* payable continuously throughout any period of sickness, ceasing at age 60. [2]

[Total 7]

X5.7 A life office issues 3-year term assurance policies to a group of lives aged 62 exact. Each policy has a sum assured of 150,000, payable at the end of the year of death. Premiums are payable annually in advance, ceasing on earlier death.

The life office calculates reserves on a net premium basis using an interest rate of 4% *pa* effective, and AM92 Ultimate mortality.

The life office makes the following additional assumptions when carrying out a profit test:

Mortality:	AM92 Select
Expenses:	Initial: 400
	Renewal: 50 at the start of the second and subsequent policy years
Interest rate:	6% *pa* effective on investments
Risk discount rate:	9% *pa* effective

Carry out a profit test to determine the premium that will produce a net present value of zero on the above basis. [9]

X5.8 On 1 January 2014 an insurer issued a block of 25-year annual premium endowment policies that pay £120,000 at maturity, or £60,000 at the end of the year of earlier death, to lives aged exactly 65.

The premium basis assumed 4% *pa* effective interest, AM92 Select mortality and allowed for an initial expense of £200 and renewal expenses of 1% of each subsequent premium. The annual premium, calculated on the premium basis, was £3,071.40.

(i) Calculate the reserve required per policy at 31 December 2018, assuming that reserves are calculated on the same basis as the premiums. [3]

(ii) There were 197 policies in force on 1 January 2018. During 2018 there were 9 deaths, interest was earned at twice the rate expected and expenses were incurred at twice the rate expected. By considering the total reserve required at the start and end of the year, and all the cashflows during the year, calculate the profit or loss made by the insurer from all sources (not just from mortality) in respect of these policies for the 2018 calendar year.
[6]
[Total 9]

X5.9 A special 3-year term assurance issued to a man aged exactly 62 pays 50,000 immediately on death within the policy term. On survival to the end of the term, or immediately on earlier surrender, half of the total premiums paid to date (without interest) will be returned to the policyholder. A level annual premium is paid at the start of each year.

The insurance company uses the following basis to calculate its premiums:

Mortality:	independent probabilities as defined by the AM92 Select table
Surrender:	forces of surrender of 5%, 2.5% and 1% in policy years 1, 2, and 3 respectively
Interest:	3% *pa* effective
Expenses:	none

(i) Assuming that forces of decrement are constant over each year of age, construct a multiple decrement table that would be suitable for valuing the cashflows for this policy, using a radix of $(al)_{62} = 100{,}000$. [5]

(ii) Using the entries in the multiple decrement table, or otherwise, calculate the annual premium for this policy. [6]

[Total 11]

X5.10 A life insurance company issues a 3-year unit-linked endowment assurance contract to a female life aged 60 exact under which level premiums of £5,000 per annum are payable in advance. In the first year, 85% of the premium is allocated to units and 104% in the second and third years. The units are subject to a bid-offer spread of 5%, and an annual management charge of 0.75% of the bid value of the units is deducted at the end of each year.

If the policyholder dies during the term of the policy, the death benefit of £20,000 or the bid value of the units after the deduction of the management charge, whichever is higher, is payable at the end of the year of death. On survival to the end of the term, the bid value of the units is payable.

The company holds unit reserves equal to the full bid value of the units but does not set up non-unit reserves. It uses the following assumptions in carrying out profit tests of this contract:

Mortality:		AM92 Ultimate
Surrenders:		None
Expenses:	Initial:	600
	Renewal:	100 at the start of each of the second and third policy years
Unit fund growth rate:		6% per annum
Non-unit fund interest rate:		4% per annum
Risk discount rate:		10% per annum

Calculate the profit margin on this contract. [12]

X5.11 A 3-year unitised with-profits endowment is to be issued to a man aged exactly 55. The policy includes the following features:

- Allocation rate of 85% in year 1 and 100% thereafter.
- Premium of 5,000 paid at start of each year.
- Death benefit, paid at the end of the year of death, equal to the end year unit fund value plus terminal bonus, or 15,000, if higher.
- Maturity benefit equal to the end-year unit fund value plus terminal bonus.
- Surrender is permitted at the end of the first and second years, equal to the unit fund value plus terminal bonus less a surrender penalty of 80 per surrender.
- A policy fee is deducted at the start of each year except the first, equal to 1.5% of the unit fund value immediately **after** the premium for that year has been paid.

Calculate the net present value for this policy on the following assumptions:

- Initial expenses: 500
- Renewal expenses: 30
- Termination expenses: 50 per termination (death, surrender or maturity)
- Initial commission: 5% of the first year's premium
- Renewal commission: 1% of the second and third year's premiums
- Investment and actuarial management expenses:
  0.25% of the end-year unit fund value each year
- Mortality: 80% of AM92 Select
- Surrender probability: 10% of all policies in force at the end of each year
- Regular bonus interest: 4% per annum
- Terminal bonus rates: 1% of the unit fund value after 1 year
  3% of the unit fund value after 2 years
  6.5% of the unit fund value after 3 years
- Non-unit interest: 2% per annum
- Risk discount rate: 8% per annum
- All expected investment returns are assumed to be distributed to the policyholder through the regular and terminal bonuses.

[13]

X5.12 (i) Consider a policy issued t years ago to a policyholder then aged x. The policy provides a benefit of S at the end of the policy year of death (and no benefit on survival).

In relation to the policy year $(t, t+1)$, write down in the form of symbols, and also explain in words, the expressions 'death strain', 'death strain at risk', 'expected death strain' and 'actual death strain'. [6]

(ii) A life insurance company issues the following policies:

- 15-year term assurances with a sum assured of £150,000 where the death benefit is payable at the end of the year of death
- 5-year single premium temporary immediate annuities with an annual benefit payable in arrear of £25,000.

On 1 January 2015, the company sold 5,000 term assurance policies to male lives aged 45 exact and 1,000 temporary immediate annuity policies to male lives aged 55 exact. For the term assurance policies, premiums are payable annually in advance. During the first two years, there were fifteen actual deaths from the term assurance policies written and five actual deaths from the immediate annuity policies written.

(a) Calculate the death strain at risk for each type of policy during 2017.

(b) During 2017, there were eight actual deaths from the term assurance policies written and one actual death from the immediate annuity policies written.

Calculate the total mortality profit or loss to the office in the year 2017.

Basis: Interest: 4% *pa* effective

Mortality: AM92 Ultimate for term assurances

PMA92C20 for annuities

[9]

[Total 15]

END OF PAPER

Contents

There are five parts to the Subject CM1 course. The parts cover related topics and are broken down into chapters. At the end of each part there are assignments testing the material from that part.

The following table shows how the parts and chapters relate to each other. The final three columns show how the chapters relate to the assignments and to the days of the regular tutorials. This table should help you plan your progress across the study session.

1.2 Subject CM1 – Syllabus and Core Reading

Syllabus

The Syllabus for Subject CM1 is given here. To the right of each objective are the chapter numbers in which the objective is covered in the ActEd course.

Aim

The aim of the Actuarial Mathematics subject is to provide a grounding in the principles of modelling as applied to actuarial work – focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival or other uncertain risks.

Competences

On successful completion of this subject, a student will be able to:

1. describe the basic principles of actuarial modelling
2. describe, interpret and discuss the theories on interest rates
3. describe, interpret and discuss mathematical techniques used to model and value cashflows which are contingent on mortality and morbidity risks.

Syllabus topics

1.	The basics of modelling	(10%)
2.	Theory of interest rates	(20%)
3.	Equation of value and its applications	(15%)
4.	Single decrement models	(10%)
5.	Multiple decrement and multiple life models	(10%)
6.	Pricing and reserving	(35%)

The weightings are indicative of the approximate balance of the assessment of this subject between the main syllabus topics, averaged over a number of examination sessions.

The weightings also have a correspondence with the amount of learning material underlying each syllabus topic. However, this will also reflect aspects such as:

- the relative complexity of each topic, and hence the amount of explanation and support required for it
- the need to provide thorough foundation understanding on which to build the other objectives
- the extent of prior knowledge that is expected
- the degree to which each topic area is more knowledge- or application-based.