1 Define in words ${}_{10|4}q_{[27]+2}$ [3]

2 Calculate ${}_{2.75}q_{84.5}$ using the method of uniform distribution of deaths.

Basis:
Mortality ELT15(Females)

[3]

3 Describe the main features of an endowment assurance contract. [3]

4 A life insurance company provides the following benefits:

- an annuity, on survival to age 65, of £15,000 per annum payable monthly in advance
- a spouse's annuity of £8,000 per annum payable monthly in advance on the death of the policyholder, provided that the policyholder survives to age 65.

No benefit is payable if the policyholder dies before age 65.

Calculate the single premium in respect of a female policyholder currently aged exactly 50 who has a male spouse currently aged exactly 53.

Basis:

Mortality	Female	PFA92C20
	Male	PMA92C20
Interest rate	4% per annum	
Expenses	Ignore	

[6]

5 Calculate, as a percentage to four decimal places, the nominal rate of interest per annum convertible half-yearly which is equivalent to:

(i) an effective rate of discount of 0.5% per month. [2]

(ii) a nominal rate of discount of 6% per annum convertible every two years. [2]

(iii) a nominal rate of interest of 6% per annum convertible quarterly. [2]

[Total 6]

6 A life insurance company issues a 20-year term assurance with additional permanent disability benefit. The benefits provided are:

- on death (whether the life was previously healthy or permanently disabled) a lump sum payment of £150,000 payable immediately
- on permanent disability a lump sum of £75,000 payable immediately.

(i) Draw a transition state model for this policy, labelling your diagram. [2]

(ii) Calculate the total expected present value of the benefits. [8]

Basis:

Force of mortality from healthy	0.03 for all ages
Force of mortality from permanent disability	0.08 for all ages
Force of permanent disability	0.001 for all ages
Force of interest	5% per annum

[Total 10]

7 An individual buys an annuity from an insurance company for a single lump sum premium. The annuity will pay £10,000 annually in arrears for 15 years. The insurance company invests the premium in a fixed-interest bond which pays coupons at the rate of 6% per annum annually in arrears and is redeemable at par in exactly nine years.

(i) (a) Calculate the duration of the annuity at an interest rate of 5% per annum effective.

(b) Calculate the duration of the bond at an interest rate of 5% per annum effective.

[5]

(ii) Explain whether the insurance company will make a profit or a loss if interest rates decrease slightly at all terms. [3]

[Total 8]

8 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \le t < 2 \\ 0.045 - 0.0025t & 2 \le t < 10 \\ 0.02 & t \ge 10 \end{cases}$$

(i) Calculate the accumulated amount at time $t = 9$ of an investment of £15,000 made at time $t = 1$. [4]

(ii) Calculate the present value at time $t = 0$ of a payment stream paid continuously from time $t = 10$ to time $t = 12$, under which the rate of payment at time t is $\rho(t) = 60e^{0.02t}$. [6]

[Total 10]

9 Describe four limitations of using models in actuarial work. [8]

10 (i) Define the term "prospective reserve" when used for a life insurance contract. [2]

(ii) State the conditions necessary for the prospective reserve to equal the retrospective reserve. [2]

A life insurance company issues a whole life assurance with sum assured S to a life aged exactly x. Annual premiums, payable annually in advance, are paid throughout the policy term. The benefit is payable immediately on death and there are no expenses.

(iii) Demonstrate that the prospective reserve is equal to the retrospective reserve at time t, assuming that the conditions referred to in part (ii) are met. [4]

[Total 8]

11 On 1 February 2017, an investor was considering purchasing ordinary shares in Actuaria PLC.

Dividends are payable annually, and a dividend of £0.40 per share had just been paid.

At the date of purchase, dividends were expected to grow each year on a compound basis. The rate of growth was expected to be 5% in the first year, 4% in the second year and 3% per annum thereafter.

The investor was not entitled to the dividend which had just been paid.

(i) Calculate the maximum price per share the investor would have been prepared to pay at this date to give a rate of return of 9% per annum effective, assuming the investor holds the share in perpetuity. [6]

The investor purchased a holding of shares on 1 February 2017 at a price of £7.00 per share and sold the holding at a price of £7.50 per share on 1 February 2019, immediately after receiving the dividend payment then due.

(ii) Calculate the effective annual real rate of return achieved by the investor between 1 February 2017 and 1 February 2019 using the following information:

Date	*Inflation index*	*Dividend per share*
1 February 2017	211.0	£0.400
1 February 2018	215.7	£0.428
1 February 2019	221.2	£0.449

[5]

[Total 11]

12 A loan of £80,000 was taken out on 1 January 2016. The loan was to be repaid over 10 years in level instalments payable monthly in arrears.

(i) Calculate the level monthly instalment using an effective rate of interest of 8% per annum. [2]

(ii) Calculate the amount of the loan outstanding on 1 November 2018 immediately after payment of the instalment then due. [3]

On 1 November 2018, immediately after payment of the instalment, the borrower asked that the monthly instalment be reduced to £900 and the remaining term extended as required to clear the outstanding loan amount. The final payment would be equal to the outstanding loan at the time, if less than £900.

The lender agreed to this change, subject to the following conditions:

- the interest rate applied in future is increased to 9% per annum convertible monthly; and
- an administration fee of £250 is added to the loan outstanding at 1 November 2018.

(iii) (a) Determine the new date on which the loan will be repaid.
(b) Calculate the final instalment paid.

[6]
[Total 11]

13 On 1 January 2002 a life insurance company issued the following policies:

- Identical 25-year without profit endowment assurances each with a sum assured of £200,000 payable at the end of the policy term or at the end of year of death if earlier. Premiums are payable annually in advance throughout the term or until earlier death. The policies were issued to lives aged 35 exact.
- Identical level whole life annuities, each payable annually in advance at a rate of £10,000 per annum, issued to lives aged 65 exact.

An extract from the company's records gives the following information for 2018 in respect of these policies:

Policy type	*Endowment assurance*	*Annuity*
Number of policies in force on 1 January 2018	15,203	12,352
Total annual premium for in-force policies as at 1 January 2018	82,774,000	–
Number of policyholder deaths during 2018	46	746

There were no other exits in 2018.

(i) Calculate the mortality profit for the year ended 31 December 2018 in respect of:

(a) endowment assurances
(b) annuities.

[8]

Basis:

Mortality	Endowment assurances	AM92
	Annuities	PMA92C20
Interest rate	4% per annum	
Expenses	Ignore	

(ii) Discuss your answers in part (i). [5]

[Total 13]

END OF PAPER

1 Describe the main features of an income protection health insurance contract. [4]

2 List five key preparatory steps in a data analysis process prior to performing an exploratory analysis of the data. [5]

3 (i) Explain what is meant by the expression ${}_{5|17}q^{1}_{40:40}$ [2]

Two lives, each aged exactly 40, are independent with respect to mortality and are each subject to a constant force of mortality of 0.01 per annum.

(ii) Calculate the value of the expression in part (i). [4]

[Total 6]

4 Describe the properties that can lead to data being classified as "big data". [5]

5 A life office issued a whole of life assurance contract to a life aged x exact with a sum assured of 1 payable at the end of the year of death.

Level premiums of P are payable annually in advance, ceasing on death. Ignore expenses.

(i) Using standard actuarial notation, write down:

(a) The equation of value at time 0.
(b) An expression for the prospective reserve at duration t years, denoted by ${}_tV^P_x$.
(c) An expression for the retrospective reserve at duration t years, denoted by ${}_tV^R_x$.

[3]

Assume that the basis used to calculate both the prospective and retrospective reserves is the same as that used to calculate the premium, P.

(ii) Show that the prospective and retrospective reserves are equal at time t. [4]

[Total 7]

6 The annual effective forward rate applicable over the period from t to $t + r$ is defined as $f_{t,r}$ where t and r are measured in years.

You are informed that $f_{0,1} = 4\%$, $f_{1,1} = 5\%$, $f_{2,1} = 6\%$ and $f_{3,1} = 7\%$.

(i) Determine the gross redemption yield at issue for a four-year bond, redeemable at par, with a 4% coupon payable annually in arrears. [7]

(ii) Explain why the gross redemption yield in part (ii) is lower than $f_{3,1}$. [3]

[Total 10]

7 On 1 January 2002, a life insurance company issued whole life increasing assurances to lives then aged 45 exact.

The initial sum assured was £20,000, which increased by £2,000 on each policy anniversary.

Benefits are payable at the end of the year of death. Premiums are payable annually in advance for a maximum of 20 years, ceasing on earlier death.

On 1 January 2018, there were 378 policies in force and, during 2018, 4 of these policyholders died.

(i) Calculate the mortality profit during 2018, assuming the insurance company uses the following basis for both premiums and reserves.

Mortality	AM92 ultimate
Interest	4% per annum
Expenses	none

[9]

(ii) Explain why the result in part (i) has arisen. [3]

[Total 12]

8 A loan of £1,000,000 nominal is issued with coupons payable half-yearly in arrears at a rate of 9% per annum. The loan is to be redeemed at £110 per £100 nominal on a single coupon date between 20 and 25 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 15% but not liable to capital gains tax wishes to purchase the loan at the date of issue.

(i) Calculate the price the investor should pay to ensure a net effective yield of at least 8% per annum. [5]

The investor purchases the loan for the price calculated in part (i). Exactly ten years later, immediately after the payment of the coupon then due, a second investor, who is liable to income tax at 25% and capital gains tax of 35%, purchases the loan for a price such that the first investor obtained a net effective yield of 8% per annum. The second investor holds the loan to maturity.

(ii) Calculate:

(a) the price paid by the second investor
(b) the minimum net redemption yield earned by the second investor, to the nearest 0.1% per annum.

[6]
[Total 11]

9 A man aged 60 exact purchases a whole life level annuity of £20,000 per annum payable monthly in arrears with payment guaranteed for the first five years.

In addition, a reversionary annuity of £10,000 per annum is payable to the man's wife, who is two years younger. This reversionary annuity commences on the monthly payment date following the man's death or on completion of the five-year guaranteed period, if later. The annuity is payable monthly in arrears until the wife's death.

Calculate the single premium payable using the following basis:

Interest:	4% per annum
Mortality:	PMA92C20 for the policyholder
	PFA92C20 for the spouse
Expenses:	Initial expenses of £250 plus £10 on each annuity payment date

[11]

10 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.01t & 0 \le t < 4 \\ 0.07 & 4 \le t < 6 \\ 0.09 & t \ge 6 \end{cases}$$

(i) Calculate the accumulated amount at time $t = 6$ of a lump sum of 10 units invested at time $t = 0$. [3]

(ii) Calculate the present value at time $t = 0$ of a deferred annuity certain of 5 units per year payable continuously from time $t = 4$ to $t = 10$. [6]

(iii) Determine, to the nearest 0.1%, the constant annual effective rate of interest earned by an investor who invests the present value calculated in part (ii) at time $t = 0$ to obtain the payment stream described in part (ii). [3]

[Total 12]

11 A life insurance company issues a with profit whole life policy. The benefit is payable at the end of the year of death and is equal to the basic sum assured plus any attaching bonus.

Level premiums are paid monthly in advance ceasing after 25 years or on the death of the policyholder if earlier.

Simple reversionary bonuses are added at the start of each policy year.

The company uses the following basis to calculate premiums:

Mortality	AM92 Select
Interest Rate	6% per annum
Commission	
Initial	25% of the total premium payable in the first policy year
Renewal	2.5% of the second and subsequent monthly premiums
Expenses	
Initial	£300
Renewal	£75 at the start of each year from the first year increasing at a rate of 1.92308% per annum. The first increase will take place at the start of the second year.
Claim	£50 inflating at 1.92308% per annum. The first increase will take place at the end of the first year.
Bonus	Simple bonus rate of 1.5% of basic sum assured

The company issues the policy to lives aged 50 exact. The basic sum assured is £150,000.

(i) Show that the monthly premium is approximately £303. [9]

For the first 24 years of the policy, the actual simple bonuses declared have been at an annual rate of 1% per annum.

(ii) Calculate the gross premium prospective reserve at the end of the 24th policy year.

Prospective Reserving Basis	
Mortality	AM92 Ultimate
Interest Rate	4% per annum
Commission	
Renewal	2.5% of the monthly premiums
Expenses	
Renewal	£125 per annum
Claim	£75
Bonus	Simple bonus rate of 0.75% of basic sum assured

[8]

[Total 17]

END OF PAPER

1 Describe the cashflows for an investor who purchases an index-linked bond. [3]

2 Calculate, using standard approximations where necessary:

(a) ${}_{10|4}q_{[36]}$

(b) $\overline{A}^{1}_{46:\overline{25|}}$

Basis
Mortality: AM92
Interest rate: 4% per annum

[5]

[**Note**: You should show your working, but intermediate steps can be shown using numerical values – no additional notation is required.]

3 Two independent lives aged x and y have constant annual forces of mortality of 0.04 and 0.03 respectively.

The probability ${}_{6}q^{2}_{xy}$ can be expressed as

$$\int_{t=a}^{b}\int_{s=c}^{d} Xe^{-ps}\times e^{-qt}ds\,dt$$

where time periods are measured in years.

(i) State the values of a, b, c, d, p, q and X. [3]

(ii) Calculate the probability ${}_{6}q^{2}_{xy}$ based on your answers to part (i). [4]

[Total 7]

4 An insurance company issues 20-year joint life term assurances. The sum assured of \$150,000 is payable at the end of the year of the first death, if it occurs within the policy term. The premium is payable monthly in advance throughout the term of the policy or until the first death, if earlier.

Calculate the monthly premium for a policy issued to a male life aged 55 exact and a female life aged 53 exact.

Basis

Mortality of male life:	PMA92C20
Mortality of female life:	PFA92C20
Interest rate:	4% per annum
Expenses:	Ignore

Assume that lives are independent with respect to mortality. [9]

5 A company invests \$50,000 now and receives the following income over the next 12 years:

During the first 4-year period: \$4,000 per annum paid quarterly in arrears.
During the second 4-year period: \$$X$ per annum paid half-yearly in arrears.
During the final 4-year period: \$12,000 per annum paid continuously.

There are no other payments under the investment.

Calculate X assuming the company achieves a nominal rate of return of 9% per annum convertible monthly. [11]

6 An investment bank borrows \$39.5 million at an effective rate of interest of 8% per annum. The bank uses the money to invest in a capital project that provides an income of \$5 million per annum payable quarterly in arrears for a term of 15 years. This income is used to repay the loan. Once the loan has been repaid, the bank can earn interest on the income at an effective rate of interest of 6% per annum.

(i) Calculate the discounted payback period for this project. [4]

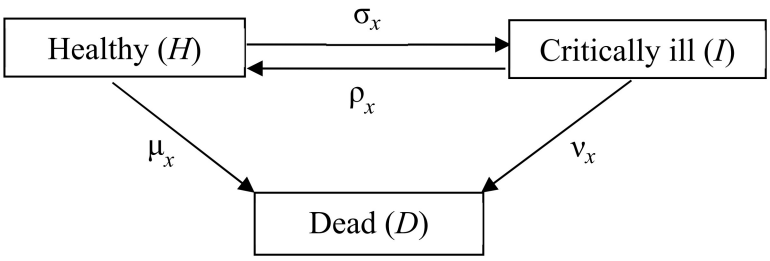
(ii) Calculate the accumulated profit the bank will have made at the end of the term. [6]

[Total 10]

7 A life insurance company sells a policy with a 20-year term that provides a benefit of \$100,000 payable immediately on death or on earlier diagnosis of a critical illness. No further benefit is paid in the event of death after a critical illness claim has been paid.

Premiums of P are paid annually in advance throughout the term or until a claim, if earlier.

The company prices the policy using the following multiple state model using the forces of transition σ_x, ρ_x, μ_x and ν_x for a life aged x.



Basis:

$\mu_x = 0.025$ for all ages
$\sigma_x = 0.015$ for all ages
Interest rate is 3% per annum effective

(i) Determine, with a reason, the values for ρ_x and ν_x that should be used in the pricing model for this policy. [1]

The present value of the benefits on this policy is given by the following formula

$$\boldsymbol{a} \times \int_{t=\boldsymbol{c}}^{\boldsymbol{d}} \boldsymbol{b} \times e^{\boldsymbol{z}t}\, dt.$$

(ii) State the values of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$ and $\boldsymbol{z}$. [3]

(iii) Calculate the present value of the benefits for this policy based on your answers to part (ii). [2]

The present value of the annual premiums for this policy is given by the following formula

$$P \sum_{t=\boldsymbol{f}}^{\boldsymbol{g}} e^{t\boldsymbol{h}}.$$

(iv) State the values of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$. [2]

(v) Calculate the annual premium for this policy based on your answers to parts (iii) and (iv). [1]

[Total 9]

8 A loan is to be repaid by a series of instalments made annually in arrears. The first instalment is \$200 per annum and thereafter instalments increase by \$15 each year. The instalments are paid for 16 years and are calculated using an effective rate of interest of 5% per annum.

(i) Calculate the amount of the loan. [3]

(ii) Construct a loan schedule that shows the capital and interest elements included in, and the amount of loan outstanding after, each of the 5th and 6th instalments. [5]

(iii) Calculate the capital and interest elements of the final instalment. [2]

[Total 10]

9 A fixed interest security of nominal amount \$1,000,000 is to be issued paying coupons quarterly in arrears at a rate of 6% per annum. The security is to be redeemed with a capital payment of \$105 per \$100 nominal on a coupon date between 20 and 25 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

An investor, who is liable to income tax at 20% and capital gains tax of 25%, wishes to purchase the entire security at the date of issue, at a price that ensures she achieves a net effective yield of at least 4.9% per annum.

(i) Determine whether the investor would make a capital gain if she holds the security until redemption. [3]

(ii) Explain how your answer to part (i) influences the assumptions made in calculating the price the investor should pay. [2]

(iii) Calculate the maximum price that the investor should pay per \$100 nominal. [5]

(iv) Explain, without carrying out any further calculations, how your answer to part (iii) would change if the coupons had been payable half-yearly in arrears. [2]

[Total 12]

10 A life insurance company is proposing to launch a 3-year with-profits endowment assurance policy. Compound reversionary bonuses are declared at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death). If the policyholder dies during the term, the basic sum assured plus any attaching reversionary bonus is payable at the end of the year of death. On survival to maturity, the basic sum assured plus any attaching reversionary bonus plus a terminal bonus is payable.

Premiums are paid yearly in advance throughout the term of the policy or until earlier death.

Profit test assumptions	
Rate of interest:	2% per annum
Mortality:	120% AM92 Ultimate
Initial expenses:	\$200
Renewal expenses:	\$30 per annum on the second and third premium dates
Initial commission:	15% of the first premium
Renewal commission:	1.5% of the second and third years' premiums
Claim expense:	\$50 (payable on death and maturity)
Reversionary bonus:	2% per annum compound
Terminal bonus:	10% of the sum of the basic sum assured and the attaching reversionary bonuses. This is payable on maturity only
Risk discount rate:	7% per annum
Reserves:	Ignore
Surrenders:	Ignore

(i) Calculate the premium, using a discounted cash flow projection, for a policy with a basic sum assured of \$15,000 issued to a life aged 62 exact. [14]

The life insurance company intends to set up the following reserves for the policy at duration t.

Duration t	${}_tV$
0	0
1	\$5,000
2	\$10,000

(ii) Calculate the premium for the policy in part (i) after allowing for the reserves. [6]

Assume that the life insurance company charges the premium calculated in part (ii) and sets up reserves as detailed above.

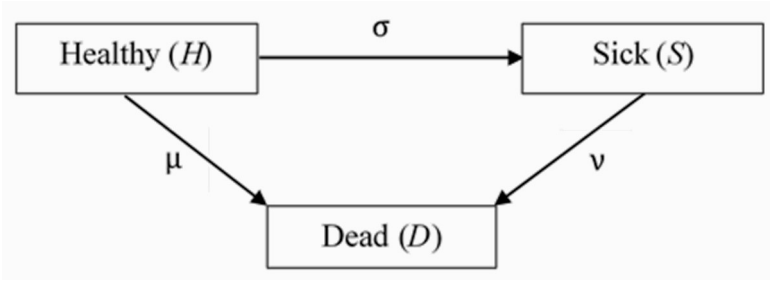
(iii) Discuss briefly whether the reserves set up by the life insurance company would be sufficient. [4]

[Total 24]

END OF PAPER

1 Calculate ${}_{3|5}q_{45:45}^{\ 1}$ assuming AM92 mortality for both individuals and that the individuals are independent with regards to mortality. [4]

2 A life insurance company uses the following three-state model, with constant forces of transition, to price its stand-alone critical illness policies.



Under these policies, a lump sum benefit is payable when a life becomes critically ill during the policy term. No other benefits are payable.

A 30-year policy with sum assured \$150,000 is issued to a healthy life aged 35 exact.

The expected present value of the benefit at outset is given by the following formula:

$$\boldsymbol{m} \times \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{n} \times e^{\boldsymbol{z}t}\ dt.$$

(i) State the numerical values of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{m}$, $\boldsymbol{n}$ and $\boldsymbol{z}$. [3]

(ii) Calculate the expected present value of the benefit for this policy based on your answer to part (i).

Basis: $\mu = 0.01$
$\sigma = 0.02$
$\nu = 0.04$

Interest: 3% p.a. effective

[3]
[Total 6]

3 A fixed interest security of nominal amount \$100,000 was issued on 1 March 2017 and was redeemed at par on 1 March 2020. Coupons were paid at the rate of 4% p.a. annually in arrears.

The value of the inflation index at various dates during the term of the security was as follows:

Date	*Inflation index*
1 March 2017	240.5
1 March 2018	256.0
1 March 2019	272.8
1 March 2020	286.6

(i) Demonstrate that the effective annual real rate of return achieved over the term of the security is approximately equal to –1.9% p.a. [5]

(ii) Comment on the result in part (i). [3]

[Total 8]

4 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \le t \le 6 \\ 0.1 - 0.01t & t > 6 \end{cases}$$

$A(0,t)$, the accumulation at time t of a unit of money invested at time 0, can be written as:

$$A(0,t) = \begin{cases} e^{\boldsymbol{a}+\boldsymbol{b}t+\boldsymbol{c}t^2} & 0 \le t \le 6 \\ e^{\boldsymbol{f}+\boldsymbol{g}t+\boldsymbol{h}t^2} & t > 6 \end{cases}$$

(i) Calculate the values of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$. [5]

A sum of \$5,000 is invested at $t = 2$ for 5 years.

(ii) Calculate the annual nominal rate of return convertible monthly on the investment. [3]

[Total 8]

5 The force of mortality, μ_x^*, experienced by a particular population at all ages x (where x is not necessarily an integer) is 20% higher than that under the PMA92C20 table.

Calculate the following, based on μ_x^*, assuming a rate of interest of 7% p.a.

(i) $\ddot{a}_{70:\overline{3}|}$ [5]

(ii) $A_{70:\overline{3}|}$ [2]

[Total 7]

6 A special whole life assurance policy is issued to a life aged 45 exact. The policy provides a benefit of \$40,000 on death within 15 years of inception, and \$50,000 on death thereafter. Benefits are payable at the end of the year of death.

(i) Calculate the expected present value of the benefit payments. [3]

(ii) Calculate the variance of the present value of the benefit payments.

Basis:
Mortality: AM92 Ultimate
Interest: 6% p.a.

[6]

[Total 9]

7 A life insurance company issues a 20-year with-profit endowment assurance policy to a life aged 45 exact for a sum assured of \$150,000. The sum assured, together with any attaching bonuses, is payable on survival to the end of the term or at the end of the year of death if earlier.

The company assumes that future reversionary bonuses will be declared on the policy annually at a rate of 1.92308% of the sum assured, compounded and vesting at the end of the policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

Calculate the level premium payable annually in advance throughout the policy term, and ceasing on earlier death.

Basis:

Mortality:	AM92 Select
Interest:	6% p.a.
Initial expenses:	\$200 plus 75% of the annual premium
Renewal expenses:	2.5% of each annual premium excluding the first
Claim expenses:	\$140 on death or maturity

[10]

8 On 1 January 2022, a student plans to take out a 10-year bank loan for \$15,000.

Under the repayment schedule, instalments will be paid monthly in arrears until the end of the term. The first instalment, at the end of January 2022, will be X, and the second instalment, at the end of February 2022, will be $2X$, and so on, until the instalment at the end of December 2026, which will be $60X$. The remaining instalments from the end of January 2027 will also be $60X$.

The bank charges a rate of interest of 12% p.a. effective.

(i) Write down an equation of value to calculate X. [2]

(ii) Calculate the value of X using the equation of value in part (i). [5]

(iii) Write down an equation to calculate the loan outstanding, after the instalment paid at the end of December 2026, using the retrospective method. [2]

(iv) Calculate the loan outstanding after the instalment at the end of December 2026 has been paid, using the equation in part (iii). [1]

(v) Comment on your answer to part (iv). [2]

(vi) Write down an equation to calculate the total interest paid during 2027. [2]

(vii) Calculate the total interest paid during 2027 using the equation in part (vi). [2]

The bank also offers the 10-year loan with the same interest rate but where the monthly instalments remain level throughout the term.

(viii) Comment on whether the total interest paid by the student under this revised offer would be greater or less than that paid under the original repayment schedule. You should not perform any further calculations. [2]

[Total 18]

9 The Green Investment Company has the opportunity to purchase a factory for $400,000. The factory is to be leased and two different companies, A and B, are interested in the lease. The two companies have made the following proposals.

Company A
The Green Investment Company will need to spend another $50,000 refurbishing the factory for Company A.

Company A will pay rent annually in advance for 20 years starting immediately. The rent will increase by 3% p.a. compound each year. At the end of 20 years, Company A will purchase the factory from the Green Investment Company for $450,000.

Company B
Company B will pay rent at an initial level amount of $44,600 p.a. payable monthly in advance starting immediately. The rent will increase by 50% at the end of the 10th year and remain at this level for the next 10 years. At the end of 20 years, ownership of the factory will pass to Company B at no further cost.

(i) Calculate the initial annual rent payable by Company A, to give the Green Investment Company an internal rate of return of 9% p.a. effective on the proposal. [3]

(ii) Demonstrate that the internal rate of return from Company B's proposal would be greater than 9% p.a. effective. [3]

The Green Investment Company does not have the capital available to purchase the factory but can take out a loan at an interest rate of 9.5% p.a. effective. The loan is to be repaid over 20 years in level instalments payable annually in arrears.

The Green Investment Company decides to accept the proposal from Company B, and takes out a loan in order to purchase the factory.

(iii) Calculate the accumulated profit of the investment after 20 years using an effective rate of interest of 9.5% p.a. [4]

[Total 10]

10 On 1 January 2011, a life insurance company planned to issue the following two policies to lives then aged 45 exact:

- a 15-year without-profit endowment assurance with a sum assured of S payable on maturity or immediately on earlier death, and with premiums of P payable annually in advance
- a 15-year temporary life annuity payable annually in advance purchased with a single premium of \$50,000.

The annual annuity payments were calculated to be exactly sufficient to pay the premiums for the endowment assurance as they fell due.

(i) Calculate P and S. [7]

The policies were actually issued with S = \$90,000 payable under each endowment assurance policy and an annual premium of P = \$4,450.

On 31 December 2020, there were 550 policies still in force. During 2020, there were six deaths with no other decrements taking place.

(ii) Calculate the mortality profit for the calendar year 2020. [10]

(iii) Comment on your numerical result obtained in part (ii).

Basis:
Mortality: AM92 Ultimate
Interest: 4% p.a.
Expenses: Ignore

[3]
[Total 20]

END OF PAPER

1 A 10-year unit linked contract has the following profit signature before any non-unit reserves are created:

Year	1	2	3	4	5	6	7	8	9	10
	+1	–1	+1	+1	+1	–1	0	–1	+1	+1

Non-unit reserves are set up to zeroise the negative cashflows.

Determine the revised profit signature, ignoring interest and mortality. [3]

2 Calculate, showing all working

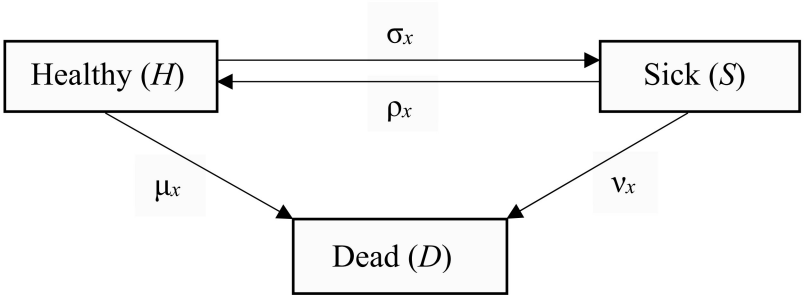
(a) $\mu_{55:60}$

(b) ${}_5p_{55:60}$

(c) ${}_2q^{1}_{60:60}$

Assume lives are independent with regards to mortality and that both lives are subject to the PFA92C20 mortality table. [4]

3 A life insurance company uses the following multiple state model to price its sickness policies.



Describe, in words, what each of the following integral expressions represents. You may assume the time periods are measured in years.

(a) $2{,}000 \times \int_0^{20} e^{-\delta t} \times {}_tp_{40}^{HS}\,dt$

(b) $1{,}000 \times \int_0^{20} e^{-\delta t} \times {}_tp_{40}^{\overline{HH}}\,dt$

(c) $20{,}000 \times \int_0^{20} e^{-\delta t} \times {}_tp_{40}^{HS} \times v_{40+t}\,dt$

[6]

4 A company has agreed to build and operate a ferry service for a regional government.

The company will invest \$10 million at the outset, and a further \$8 million after 1 year.

The ferry will then come into operation and the company will receive payments at the end of each year, the first payment occurring at the end of the second year of the project.

The amount of payment at the end of the second year will be \$4 million, increasing by \$0.5 million in each of the subsequent years until the annual payment is \$7 million, after which the payments will reduce by \$1 million each year. When the payments have reduced to zero, the company's involvement in the project will end.

Calculate the net present value of the project at a rate of interest of 6% p.a. effective.
Note: You should show your working and determine the present value of income using annuity functions.

[6]

5 An equity is expected to pay its first dividend in exactly 2 years' time. It is assumed that this dividend will be \$0.20 per share.

Subsequent annual dividends are assumed to grow at 6% p.a. compound for the following 10 years, and at 3% p.a. compound in perpetuity thereafter.

Calculate, showing all working, the price of the share to the nearest \$0.01, that would give an effective rate of return of 7% p.a. [7]

6 A bond is issued at time $t = 0$ at a price of \$107.60 per \$100 nominal. The bond pays coupons of 6% p.a., annually in arrears, and will be redeemed at par in 3 years' time.

The 2-year par yield at time $t = 0$ is 6.5% p.a. The 1-year forward rate of interest at time $t = 1$ year is 4.5% p.a. effective.

[7]

Calculate, showing all working and assuming no arbitrage, the implied 1-year, 2-year and 3-year annual effective spot rates.

7 A life insurance company issues a reversionary annuity policy to a male and female, both aged exactly 65.

The annuity of \$30,000 p.a., payable monthly in arrears, commences on the first death, and payments cease on the death of the second life, or on the 15th anniversary of the policy inception if earlier.

Calculate, showing all working, the single premium for the policy.

Basis:

Mortality:	PMA92C20 for the male life and PFA92C20 for the female life The lives are independent with respect to mortality
Interest:	4% p.a.
Expenses:	Initial: \$250 incurred at the outset Renewal: 3% of each annuity payment

[10]

8 The force of interest, $\delta(t)$, is a function of time, and at any time, t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 + 0.02t & 0 \le t \le 4 \\ 0.08 - 0.01t & t > 4 \end{cases}$$

$A(0,t)$ the accumulation at time t of a unit of money invested at time 0, can be written as:

$$A(0,t) = \begin{cases} e^{\boldsymbol{a}+\boldsymbol{b}t+\boldsymbol{c}t^2} & 0 \le t \le 4 \\ e^{\boldsymbol{f}+\boldsymbol{g}t+\boldsymbol{h}t^2} & t > 4 \end{cases}$$

(i) Determine the values of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$. [6]

A sum of \$600 is invested at $t = 3$ and a further sum of \$900 is invested at $t = 9$.

(ii) Calculate, showing all working, the accumulated amount at $t = 13$. [4]

(iii) Calculate, showing all working, the yield of the investment described in part (ii) expressed as an effective rate of interest per month to the nearest 0.1% [3]

(iv) Comment on your answer to part (iii). [1]

[Total 14]

9 A life insurance company issues 30-year pure endowment assurance policies to a group of lives aged exactly 30. Each policy provides a sum assured of \$50,000 payable on survival to the end of the term. Premiums on the policy are payable annually in advance for 30 years or until earlier death.

There were two deaths during the 25th policy year and the number of policies in force at the end of that year was 315. There were no exits other than death during the year.

(i) Calculate, showing all working, the mortality profit or loss arising in the 25th policy year. [5]

(ii) Comment on your result obtained in part (i).

Basis:

Mortality:	AM92 Ultimate
Interest:	4% p.a. effective
Expenses:	None

[3]

[Total 8]

10 The table below is an extract from a multiple decrement table that is currently used to model the deaths and withdrawals of employees working for a large company in the hospitality industry. No decrements occur other than by death or withdrawal.

Age (x)	*Number of employees* $(al)_x$	*Number of deaths* $(ad)_x^d$	*Number of withdrawals* $(ad)_x^w$
47	50,000	390	1,500

Recent experience has resulted in an estimate that, at all ages:

- the annual independent force of mortality for employees is now 60% of that implied by the q_x rates in the ELT15 (Females) table.
- the annual independent probability of withdrawal for employees is now 250% of that used to construct the above table.

(i) Calculate, showing all working, the revised independent forces of mortality and withdrawal, each to six significant figures, for age 47. You should state any assumptions that you make. [7]

(ii) Construct the revised multiple decrement table, showing your results to two decimal places. [5]

(iii) Identify any concerns with the use of this revised multiple decrement table to model the future deaths and withdrawals of employees of the company. [3]

[Total 15]

11 A life insurance company issues a 15-year with profit endowment assurance policy to a life aged 50 exact. Premiums are payable monthly in advance for 15 years or until earlier death. The sum assured is payable at the end of the year of death or at the end of the term if earlier.

(i) Demonstrate that the basic sum assured a policyholder can purchase for a premium of \$500 per month is approximately \$93,000 (to the nearest \$1,000). [8]

Pricing basis:

Mortality:	AM92 Ultimate
Interest:	6% p.a. effective
Reversionary bonus:	1.9231% p.a. compound, vesting at the end of each year (i.e. The death benefit does not include the bonus relating to the policy year of death).
Initial expenses:	60% of the annual premium, incurred at policy commencement
Renewal expenses:	4% of the annual premium, incurred annually from the start of the second year onwards

Assume that the policyholder purchased a basic sum assured of \$93,000 with the premium of \$500 per month.

(ii) Demonstrate that the annual effective rate of return that a policyholder will earn on this contract, if they survive to the end of the 15 years, is at least 0.434% p.a. [3]

(iii) Explain why the contract may be attractive to policyholders in spite of the low level of minimum rate of return given in part (ii). [3]

During each of the first 5 years of the contract, the office declared compound reversionary bonuses of 5% p.a.

(iv) Calculate, showing all working, the prospective gross premium reserve at the end of the fifth year of the contract, using the basis given in part (i). [6]

[Total 20]

END OF PAPER