INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2019 Examinations

Subject CM1A – Actuarial Mathematics

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
July 2019

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Actuarial Mathematics subject is to provide a grounding in the principles of modelling as applied to actuarial work – focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

2. Candidates may have concluded to different answers than what is shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may lose marks where excessive rounding has been used or where insufficient working is shown.

B. General comments on *student performance in this diet of the examination*

1. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well by most candidates. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.

2. This was a new large subject which was broadly a merging of the old CT1 and CT5 subjects. There appeared to be a large number of ill-prepared candidates who had underestimated the quantity of study required for the new larger subject. However, given this is a new subject, it is difficult to compare the performance of candidates in this diet with those in previous years.

C. Pass Mark

The Pass Mark for this exam in combination with CM1B was 58.

Solutions

Q1

${}_{10|4}q_{[27]+2}$ is the probability:

- that a life aged 29 [½]
- who joined the select population ... [1]
- .... at age 27 [½]
- will survive for 10 years [½]
- but then die in the subsequent 4 years. [½]

[Total 3]

This question was answered well. An answer defining the periods in terms of the relevant ages was given full credit. The most common error was to omit mention of select mortality.

Q2

$${}_{2.75}q_{84.5} = 1 - {}_{2.75}p_{84.5} = 1 - {}_{0.5}p_{84.5} \times {}_{2}p_{85} \times {}_{0.25}p_{87}$$

[1]

using UDD $\quad {}_{t-s}p_{x+s} = 1 - \dfrac{(t-s)q_x}{1-sq_x}$ for $0 \le s < t < 1$ and ${}_tq_x = tq_x$ for $0 \le t \le 1$

$${}_{0.5}p_{84.5} = 1 - \frac{0.5q_{84}}{1-0.5q_{84}} = 1 - \frac{0.5\times(0.08757)}{1-0.5\times(0.08757)} = 0.95421$$

[½]

$${}_{2}p_{85} = \frac{l_{87}}{l_{85}} = \frac{30,651}{38,081} = 0.80489$$ [½]

$${}_{0.25}p_{87} = 1 - 0.25q_{87} = 1 - 0.25\times(0.11859) = 0.97035$$

[½]

$${}_{2.75}p_{84.5} = 1 - 0.95421\times 0.80489 \times 0.97035 = 0.74526$$

$${}_{2.75}q_{84.5} = 1 - 0.74526 = 0.25474$$ [½]

[Total 3]

<u>Alternatively :</u>

$${}_{2.75}q_{84.5} = 1 - {}_{2.75}p_{84.5} = 1 - \frac{l_{87.25}}{l_{84.5}}$$

$$= 1 - \frac{\left(\frac{1}{4}l_{88} + \frac{3}{4}l_{87}\right)}{\left(\frac{1}{2}l_{84} + \frac{1}{2}l_{85}\right)} = 1 - \frac{\left(\frac{1}{4}\times 27,017 + \frac{3}{4}\times 30,651\right)}{\frac{1}{2}\left(41,736 + 38,081\right)}$$

$$= 1 - \frac{29,742.5}{39,908.5} = 0.25473$$

Generally well-answered although some candidates multiplied the q factors together in an attempt to calculate the overall probability of death rather than multiplying the p factors together to get an overall probability of survival.

Q3

An endowment assurance provides a survival benefit at the end of the term, but it also provides a lump sum benefit on death before the end of the term. [1½]

The benefits are provided in return for a series of regular premiums (or a single premium). [1]

The sum assured payable on death or survival need not be the same, although they often are. [½]

[Total 3]

A knowledge based question generally well-answered although weaker candidates tended to only make the first of the points above.

Q4

The value of the policyholders annuity benefit is given by:

$$v^{15} \times \frac{l_{65}^{f}}{l_{50}^{f}} \times \left(15,000\ddot{a}_{65}^{(12)}\right)$$

$$= v^{15} \times \frac{l_{65}^{f}}{l_{50}^{f}} \times \left(15,000\left(\ddot{a}_{65} - \frac{11}{24}\right)\right) = 0.555265 \times \left(\frac{9,703.708}{9,952.697}\right) \times 15,000 \times \left(14.871 - \frac{11}{24}\right)$$

$$= 15,000 \times 7.80363 = 117,039.50$$

[2½]

The value of the spouse's annuity benefit is given by:

$$v^{15} \times \frac{l_{65}^{f}}{l_{50}^{f}} \times \frac{l_{68}^{m}}{l_{53}^{m}} \times \left(8,000\left(\ddot{a}_{68}^{(12)} - \ddot{a}_{68:65}^{(12)}\right)\right)$$

$$= 0.555265 \times \left(\frac{9,703.708}{9,952.697} \right) \times \left(\frac{9,440.717}{9,922.995} \right) \times \left(8,000 \times \left(12.412 - 11.112 \right) \right)$$

$$= 0.555265 \times \left(0.974983 \right) \times \left(0.951398 \right) \times \left(8,000 \times 1.3 \right)$$

$$= 5,356.64$$

[3]

The total value of benefits is therefore $117,039.50 + 5,356.64 = 122,396.14$ [½]

[Total 6]

A more challenging question that distinguished well between stronger and weaker candidates. Many candidates did not identify the reversionary element of the benefit. Another common error was not to include the survival probability of the male life during the deferred period of the reversionary annuity.

Q5

(i) With $d = 0.005$ per month, equivalent nominal rate of interest per annum convertible half-yearly is $i^{(2)}$ given by:

$$1 = \left(1 + \frac{i^{(2)}}{2} \right)^2 \times (1 - d)^{12} = \left(1 + \frac{i^{(2)}}{2} \right)^2 \times (1 - 0.005)^{12} \Rightarrow i^{(2)} = 0.061064$$ [2]

(ii) With $d^{(0.5)} = 0.06$, equivalent nominal rate of interest per annum convertible half-yearly is $i^{(2)}$ given by:

$$1 = \left(1 + \frac{i^{(2)}}{2} \right)^4 \times \left(1 - \frac{d^{(0.5)}}{0.5} \right) = \left(1 + \frac{i^{(2)}}{2} \right)^4 \times (1 - 0.12) \Rightarrow i^{(2)} = 0.064949$$ [2]

(iii) With $i^{(4)} = 0.06$, equivalent nominal rate of interest per annum convertible half-yearly is $i^{(2)}$ given by:

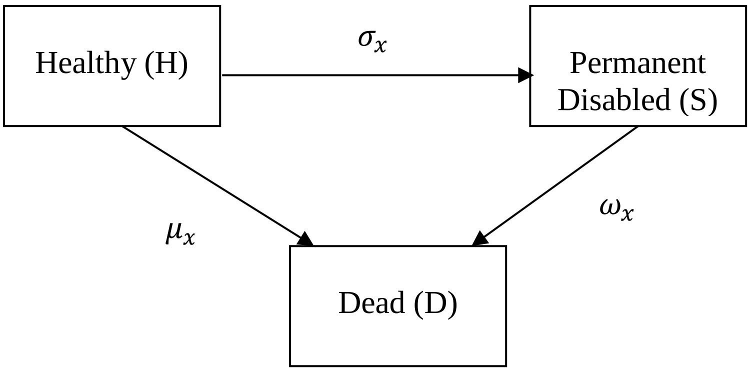
$$\left(1 + \frac{i^{(4)}}{4} \right)^4 = \left(1 + \frac{i^{(2)}}{2} \right)^2 \Rightarrow \left(1 + \frac{0.06}{4} \right)^4 = \left(1 + \frac{i^{(2)}}{2} \right)^2 \Rightarrow i^{(2)} = 0.060450$$ [2]

[Total 6]

Parts (ii) and (iii) were generally done well although many candidates failed to follow the rounding instructions.

Q6

(i) The transition state model is shown by:



[2]

(ii) The expected present value of death benefits arising from the state H is given by:

$$\int_{t=0}^{20} 150,000\mu e^{-(\delta+\mu+\sigma)t}dt = 150,000\times(0.03)\int_{t=0}^{20} e^{-0.081t}dt = 4,500\times\left[\frac{-e^{-0.081t}}{0.081}\right]_{t=0}^{20}$$

$$= 55,555.56\times(1-0.197899) = 44,561.17$$

[2½]

The expected present value of the permanent disability benefit given by:

$$\int_{t=0}^{20} 75,000\sigma e^{-(\delta+\mu+\sigma)t}dt$$

$$= 75,000\times(0.001)\int_{t=0}^{20} e^{-(\delta+\mu+\sigma)t}dt = 75\int_{t=0}^{20} e^{-0.081t}dt = 75\times\left[\frac{-e^{-0.081t}}{0.081}\right]_{t=0}^{20}$$

$$= 925.93\times(1-0.197899) = 742.69$$

[2]

Therefore, EPV of death benefit from sick state is:

$$\int_{t=0}^{20} 150000\omega_t\, p_x^{HS} e^{-\delta t}dt$$

where

$$ {}_t p_x^{HS} = \int_0^t {}_s p_x^{\overline{HH}} \times \sigma_{x+s} \times {}_{t-s} p_{x+s}^{\overline{SS}} ds = \int_0^t e^{-0.031s} \times 0.001 \times e^{-0.08(t-s)} ds $$

$$ = \int_0^t 0.001 \times e^{-0.031s+0.08s-0.08t} ds = \int_0^t 0.001 \times e^{+0.049s-0.08t} ds $$

$$ = 0.001 \times e^{-0.08t} \times \left[\frac{e^{0.049s}}{0.049} \right]_0^t = \frac{0.001}{0.049} \times \left(e^{-0.031t} - e^{-0.08t} \right) $$

Thus EPV

$$ = \int_{t=0}^{20} 150000 \times 0.08 \times \left[\frac{0.001}{0.049} \left(e^{-0.031t} - e^{-0.08t} \right) \right] e^{-0.05t} dt $$ [2]

$$ = 244.90 \int_{t=0}^{20} \left(e^{-0.081t} - e^{-0.13t} \right) dt $$

$$ = 244.90 \times \left[\frac{e^{-0.081t}}{-0.081} - \frac{e^{-0.13t}}{-0.13} \right]_0^{20} = 681.19 $$ [1]

Therefore the total value of the expected benefits is:

$44{,}561.17 + 742.69 + 681.19 = 45{,}985.05$

i.e. Approximately £45,985 [½]

[Total 10]

Part (i) was done well. In part (ii), candidates found the calculation of the various probabilities to be challenging. The determination of ${}_t p_x^{HH}$ that was needed to calculate the EPV of death benefits arising from the healthy state should have been straightforward but most marginal candidates struggled with this. The ${}_t p_x^{HS}$ probability that was needed to calculate the EPV of death benefits arising from the sick state was considerably more difficult and candidates were given credit for any reasonable approach.

Q7

(i) (a) Duration of the annuity is $\dfrac{10{,}000 (Ia)_{\overline{15|}}}{10{,}000\, a_{\overline{15|}}}$ at 5%

$$ = \frac{(Ia)_{\overline{15|}}}{a_{\overline{15|}}} = \frac{73.6677}{10.3797} = 7.0973 \text{ years} $$

[2]

(b) Duration of bond is $\dfrac{6(Ia)_{\overline{9}|}+900V^9}{6a_{\overline{9}|}+100V^9}$ at 5%

$$=\frac{6\times 33.2347+900\times 0.64461}{6\times 7.1078+100\times 0.64461}$$

$$= =\frac{779.5572}{107.1078}=7.2782 \text{ years}$$ [3]

(ii) The duration of the assets (the bond) is greater than the duration of the liabilities (the annuity). [1]

Therefore, if there is a small decrease in interest rates then the present value of the assets increases by more than the present value of the liabilities. [1½]

Therefore, the insurance company would make a profit. [½]

[Total 8]

Part (i) was done well. It is much more straightforward to calculate the duration/DMT of the bond directly than via the calculation of the volatility which involves some relatively complex differentiation. Part (ii) was very poorly done with many candidates assuming that because the company was not immunised, it must follow that it would make a loss.

Q8

(i) We have the accumulated amount

$$\begin{aligned}
&= 15{,}000\times\exp\left(\int_1^9 \delta(t)\,dt\right)\\
&= 15{,}000\times\exp\left(\int_1^2 (0.03+0.005t)\,dt+\int_2^9 (0.045-0.0025t)\,dt\right)\\
&= 15{,}000\times\exp\left(\left[0.03t+0.0025t^2\right]_{t=1}^{t=2}+\left[0.045t-0.00125t^2\right]_{t=2}^{t=9}\right)\\
&= 15{,}000\times\exp(0.0375+0.21875)\\
&= 19{,}381.14
\end{aligned}$$

[1 for formula + 3 for solution]

(ii) The PV of the payment stream is $PV=\int_{10}^{12}\rho(t)v(t)\,dt$

where $v(t)=\exp\left(-\int_0^t \delta(s)\,ds\right)$. [1]

Then, for $t\geq 10$, we have:

$$
\begin{aligned}
v(t) &= \exp\left[-\left(\int_0^2 (0.03+0.005s)\,ds + \int_2^{10}(0.045-0.0025s)\,ds + \int_{10}^t 0.02ds\right)\right] \\
&= \exp\left[-\left(\left[0.03s+0.0025s^2\right]_{s=0}^{s=2} + \left[0.045s-0.00125s^2\right]_{s=2}^{s=10} + \left[0.02s\right]_{s=10}^{s=t}\right)\right] \\
&= \exp\left[-\left(0.07+0.24+\left[0.02t-0.20\right]\right)\right] \\
&= \exp\left[-\left(0.02t+0.11\right)\right]
\end{aligned}
$$

[3]

Thus, the PV of the payment stream is:

$$
\begin{aligned}
&= \int_{10}^{12} 60e^{0.02t} \times e^{-(0.02t+0.11)}dt \\
&= 60e^{-0.11} \times \int_{10}^{12} dt \\
&= 120e^{-0.11} \\
&= 107.50
\end{aligned}
$$

[2]

[Total 10]

<u>Alternatively:</u>

The value of the payment stream at $t = 10$

$$= \int_{10}^{12} \rho(t)\exp\left(-\int_{10}^{t} \delta(s)\,ds\right)dt$$

with

$$\exp\left(-\int_{10}^{t}\delta(s)\,ds\right) = \exp\left(-\int_{10}^{t} 0.02ds\right)$$

$$= \exp\left(-\left[0.02s\right]_{10}^{t}\right) = \exp\left(0.2-0.02t\right)$$

So value at $t = 10$

$$= \int_{10}^{12} 60e^{0.02t}e^{0.2-0.02t}dt = 60e^{0.2}\int_{10}^{12} dt = 60e^{0.2}\left[t\right]_{t=10}^{t=12} = 120e^{0.2}$$

PV $= 60e^{0.2}v(10)$ where

$$
\begin{aligned}
v(10) &= \exp\left[-\left(\int_0^2 (0.03+0.005s)\,ds + \int_2^{10}(0.045-0.0025s)\,ds\right)\right] \\
&= \exp\left[-\left(\left[0.03s+0.0025s^2\right]_{s=0}^{s=2} + \left[0.045s-0.00125s^2\right]_{s=2}^{s=10}\right)\right] \\
&= \exp\left[-(0.07+0.24)\right] \\
&= \exp\left[-0.31\right]
\end{aligned}
$$

and so PV $= 120e^{-0.11} = 107.50$

Generally well done although some candidates in part (ii), discounted using a fixed discount factor, e.g. $v(10)$, rather than a time-dependent discount factor.

Q9

Costs [½]

- Model development requires a considerable investment of time, and expertise.
- An example i.e. financial costs of development can be quite large given the need to check the validity of the model's assumptions, the computer code, the reasonableness of results and the way in which results can be interpreted in plain language by the target audience. [1½]

Multiple Runs [½]

- In a stochastic model, for any given set of inputs each run gives only estimates of a model's outputs.
- So, to study the outputs for any given set of inputs, several independent runs of the model are needed. [1½]

Input vs Output [½]

- As a rule, models are more useful for comparing the results of input variations than for optimising outputs. [1½]

Real world relevance [½]

- Models can look impressive when run on a computer so that there is a danger that one gets lulled into a false sense of confidence.
- If a model has not passed the tests of validity and verification its impressive output is a poor substitute for its ability to imitate its corresponding real-world system [1½]

Quality of data [½]

- Models rely heavily on the data input.
- if the data quality is poor or lacks credibility, then the output from the model is likely to be flawed
- parameter error [1½]

Black Box effect [½]

- users of the model must understand the model and the uses to which it can be safely put
- danger of using a model as a 'black box' from which it is assumed that all results are valid without considering the appropriateness of using that model for the data input and the output expected. [1½]

Predictability [½]

- It is not possible to include all future events in a model.
- For example, a change in legislation could invalidate the results of a model, but may be impossible to predict when the model is constructed. [1½]

Interpretation of results [½]

- It may be difficult to interpret some of the outputs of the model.
- They may only be valid in relative rather than absolute terms, as when, for example, comparing the level of risk of the outputs associated with different inputs. [1½]

[max 8]
[Total 8]

It was pleasing to see many candidates make a reasonable attempt at this question which was taken from part of the syllabus not previously included in CT1 or CT5.

Q10

(i) The prospective reserve is the expected present value of the future outgo less the expected present value of the future income. [2]

(ii) If

- the retrospective and prospective reserves are calculated on the same basis; and
- this basis is the same as the basis used to calculate the premiums used in the reserve calculation, using the equivalence principle

then the retrospective reserve will be equal to the prospective reserve [2]

(iii) The premium is given by $P = \dfrac{S\bar{A}_x}{\ddot{a}_x}$ [½]

The prospective reserve at time *t* is

$${}_tV_x^{\,prosp} = S\bar{A}_{x+t} - P\ddot{a}_{x+t}$$ [½]

The retrospective reserve at time *t* is

$${}_tV_x^{\,retro} = \frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_{x:\overline{t|}} - S\bar{A}^{1}_{x:\overline{t|}}\right)$$ [½]

From above

$$P\ddot{a}_x - S\bar{A}_x = 0$$

Multiplying by $\frac{l_x}{l_{x+t}}(1+i)^t$ gives

$$\frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_x - S\bar{A}_x\right) = 0$$

Adding this to ${}_tV_x^{\,prosp}$

$${}_tV_x^{\,prosp} = S\bar{A}_{x+t} - P\ddot{a}_{x+t} + \frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_x - S\bar{A}_x\right)$$

$$= S\left(\bar{A}_{x+t} - \frac{l_x}{l_{x+t}}(1+i)^t\,\bar{A}_x\right) - P\left(\ddot{a}_{x+t} - \frac{l_x}{l_{x+t}}(1+i)\,\ddot{a}_x\right)$$

$$= -S\,\frac{l_x}{l_{x+t}}(1+i)^t\,\bar{A}_{x:\overline{t|}} + P\,\frac{l_x}{l_{x+t}}(1+i)\,\ddot{a}_{x:\overline{t|}}$$

$$= \frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_{x:\overline{t|}} - S\bar{A}_{x:\overline{t|}}\right)$$

${}_tV_x^{\,retro}$ [2½]

[Total 8]

Alternatively:

$$P\ddot{a}_x = S\bar{A}_x$$

$$P\left(\ddot{a}_{x:\overline{t|}} + {}_tp_x \times v^t \times \ddot{a}_{x+t}\right) = S\left(\bar{A}^1_{x:\overline{t|}} + {}_tp_x \times v^t \times \bar{A}_{x+t}\right)$$

$$P\ddot{a}_{x:\overline{t|}} - S\bar{A}^1_{x:\overline{t|}} = \left({}_tp_x \times v^t\right)S\bar{A}_{x+t} - \left({}_tp_x \times v^t\right)P\ddot{a}_{x+t}$$

$$P\ddot{a}_{x:\overline{t|}} - S\bar{A}^1_{x:\overline{t|}} = \left({}_tp_x \times v^t\right)\left(S\bar{A}_{x+t} - P\ddot{a}_{x+t}\right)$$

$$\frac{1}{{}_tp_x \times v^t}\left(P\ddot{a}_{x:\overline{t|}} - S\bar{A}^1_{x:\overline{t|}}\right) = S\bar{A}_{x+t} - P\ddot{a}_{x+t}$$

$$\frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_{x:\overline{t|}} - S\bar{A}^1_{x:\overline{t|}}\right) = S\bar{A}_{x+t} - P\ddot{a}_{x+t}$$

$${}_tV_x^{\,retro} = {}_tV_x^{\,prosp}$$

In part (i), weaker candidates tended to miss out 'expected' and/or present value' from their definitions. *Part (iii) was answered poorly with many candidates unable to state the equation for the retrospective reserve.*

Q11

(i) At 1/2/17, PV of future dividends

$$= 0.40 \times \left(1.05v + 1.04 \times 1.05v^2 + \sum_{k=1}^{\infty} 1.05 \times 1.04 \times 1.03^k v^{k+2} \right)$$

[2]

$$= 0.40 \times \left(\frac{1.05}{1.09} + \frac{1.05 \times 1.04}{(1.09)^2} + \frac{1.05 \times 1.04}{(1.09)^2} \times \sum_{k=1}^{\infty} \left(\frac{1.03}{1.09} \right)^k \right)$$

$$= 0.40 \times \left(\frac{1.05}{1.09} + \frac{1.05 \times 1.04}{(1.09)^2} + \frac{1.05 \times 1.04}{(1.09)^2} \times a_{\overline{\infty}|}^{i'\%} \right)$$

where $\frac{1}{1+i'} = \frac{1.03}{1.09} \Rightarrow i' = 0.058252427$

$$\Rightarrow a_{\overline{\infty}|}^{i'} = \frac{1}{i'} = 17.1\dot{6}$$

Hence $PV = 0.40 \times \left(0.9633028 + 0.9191146 + 0.9191146 \times 17.1\dot{6} \right)$

$$= 7.0642 \quad (ie\ £7.06)$$

[4]

(ii) Let i denote real return achieved:

$$7.00 \times \frac{221.2}{211.0} \times (1+i)^2 = 0.428 \times \frac{221.2}{215.7} \times (1+i) + (0.449 + 7.50)$$

[2]

$$\Rightarrow 7.33839(1+i)^2 - 0.43891(1+i) - 7.949 = 0$$

$$\Rightarrow 1+i=\frac{0.43891\pm\sqrt{\left(0.43891^2+4\times 7.33839\times 7.949\right)}}{2\times 7.33839}$$

=1.0711 (+ ve root)

$$\Rightarrow i\ = 7.11\%\,pa$$

[3]
[Total 11]

Alternatively

$$7.00 = 0.428\times\frac{211}{215.7}\times v+\left(0.449+7.50\right)\times\frac{211}{221.2}\times v^2$$

$$0 = 7.5825\times v^2+0.41867\times v-7.00$$

$$\Rightarrow v=\frac{-0.41867\pm\sqrt{\left(\left(-0.41867\right)^2+4\times 7.5825\times 7\right)}}{2\times 7.5825}$$

$v = 0.933613058$ (+'ve root)

$$\Rightarrow i = 7.11\%\,pa$$

Generally well-done. Common errors were:

- *not to include the 5%/4% growth rates in the valuation of the dividends from year 3 onwards*
- *to include the dividend that had just been paid*

Many candidates used a trial and error/interpolation approach to find the yield in part (ii). Full credit was given for this approach.

Q12

(i) Let R denote the level monthly instalment.

Then, we have:

$$\begin{aligned} & 12R\times a^{(12)}_{\overline{10}|8\%} &=& \ 80{,}000 \\ \Rightarrow\ & 12R\times 1.036157\times 6.7101 &=& \ 80{,}000 \\ \Rightarrow\ & R &=& \ 958.86 \end{aligned}$$

[2]

(ii) On 1st November 2018, remaining term is 7 years and 2 months (i.e. $7\frac{2}{12}$ years). [½]

Then, outstanding loan is:

$$L = 12R\times a^{(12)}_{\overline{7\frac{2}{12}}|8\%} = 12\times 958.86\times\frac{1-v_{8\%}^{7\frac{2}{12}}}{0.077208} = 63{,}180.76$$ [2½]

Or alternatively, working in months and using effective interest rate of 0.6434% per month, we have:

$$L = 958.86 \times a_{\overline{86}|0.6434\%} = 958.86 \times \frac{1 - v_{0.6434\%}^{86}}{0.006434} = 63{,}180.54$$

(iii) (a) Work in months, where 9% per annum convertible monthly $\Rightarrow$ 0.75% per month.

Let n denote remaining number of months, given by:

$$\begin{aligned}
& 900 \times a_{\overline{n}|0.75\%} && \geq && 63180.76 + 250 \\
\Rightarrow\ & \frac{1 - v_{0.75\%}^{n}}{0.0075} && \geq && 70.47862 \\
\Rightarrow\ & v_{0.75\%}^{n} && \leq && 0.471410 \\
\Rightarrow\ & n \times \ln\left(v_{0.75\%}\right) && \leq && \ln(0.471410) \\
\Rightarrow\ & n && \geq && 100.646
\end{aligned}$$

[3]

Thus, loan will be repaid in 101 months (or 8 years and 5 months) from 1st November 2018 $\Rightarrow$ final payment will now be made on 1st April 2027. [½]

(b) Let X denote amount of final instalment. Then, we have:

$$63180.76 + 250 = 900 \times a_{\overline{100}|0.75\%} + Xv_{0.75\%}^{101} \Rightarrow X = \frac{63430.76 - 900 \times 70.1746}{0.470164} = 581.97$$

[2½]

[Total 11]

This question was done well apart from part (iii)(b). A common error for that part was to not discount the final payment by the correct number of months (or indeed to discount it at all).

Q13

(i) (a) Total Reserve for the Endowment Assurance portfolio at 1st January 2019, ${}_{17}V^{EA}$ is:

$${}_{17}V^{EA} = 15{,}203 \times 200{,}000 A_{52:\overline{8}|} - 82{,}774{,}000 \times \ddot{a}_{52:\overline{8}|}$$

$$= 15{,}203 \times 200{,}000(0.73424) - 82{,}774{,}000 \times 6.910 = 1{,}660{,}561{,}804$$

(Reserve is 109,226 per policy in force on 1 Jan 2018) [2]

Death Strain at Risk :

$DSAR^{EA} = 15,203 \times 200,000 - 1,660,561,804 = 1,380,038,196$ (or 90,774 per policy in force on 1 Jan 2018)

[1]

Mortality Profit is given by:

$$MP^{EA} = q_{51} \times 1{,}380{,}038{,}196 - \left(\frac{46}{15{,}203}\right) \times 1{,}380{,}038{,}196$$

$$= 0.002809 \times 1,380,038,196 - \left(\frac{46}{15,203}\right) 1,380,038,196 = 3,876,527 - 4,175,607$$

= -299,080

[1½]

(b) Total Reserve for the Annuity portfolio as at 1st January 2019, ${}_{17}V^{ann}$,

$${}_{17}V^{ann} = 12,352 \times 10,000 \ddot{a}_{82}$$

$= 12,352 \times 10,000(6.801) = 840,059,520$ (or 68,010 per policy in force on 1 Jan 2018) [1]

Death Strain at Risk

$DSAR^{ann} = 0 - (840,059,520 + 0) = -840,059,520$ (or -68,010 per policy in force on 1 Jan 2018) [1]

Mortality Profit is given by:

$$MP^{ann} = q_{81} \times -840,059,520 - \left(\frac{746}{12,352}\right)(-840,059,520)$$

$$= 0.059952 \times -840,059,520 - \left(\frac{746}{12,352}\right)(-840,059,520) = -50,363,248 - (-50,735,460)$$

=372,212

[1½]

(ii) Endowment Assurance Policies

- With endowment assurances earlier than expected deaths lead to an earlier payment of the benefit - the benefit is paid as a death benefit rather than as a maturity benefit. This implies earlier than expected deaths leads to a mortality loss [1½]

- The company expected approximately 42.7 deaths, whereas 46 deaths actually occurred. So actual mortality was heavier than expected. [½]
- Here more deaths occurred than was expected and so the company suffers a mortality loss of £299,080. [½]

Annuity Policies

- With annuities there is no death benefit, however when a death occurs it leads to the release of the reserve being held to cover the future annuity payments. [1½]
- The company expected approximately 740.5 deaths, whereas 746 deaths actually occurred. So actual mortality was heavier than expected. [½]
- Here more deaths occurred than expected and so the company has a greater release of reserves than expected. Hence the company sees a mortality profit of £372,212 for these annuities. [½]

Mortality profit/loss from the 2 products cancel each other out to an extent. [1]

Mortality profit relatively small compared to book of business, especially for annuity business. [1]

[max 5]

The intention of the question was that the endowment assurance reserve should be calculated using the premium data given in the question. Instead some candidates calculated a net premium reserve.
Part (ii) was poorly answered despite the points required being comparatively straightforward.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORTS

September 2019

Subject CM1A - Actuarial Mathematics

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
September 2019

A. General comments on the *aims of this subject and how it is marked*

1. CM1 provides a grounding in the principles of modelling as applied to actuarial work - focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

2. Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may lose marks where excessive rounding has been used or where insufficient working is shown.

B. Comments on *student performance in this diet of the examination.*

1. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well by most candidates. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.
2. This was a new large subject which was broadly a merging of the old CT1 and CT5 subjects. To an even greater extent than in April, there appeared to be a large number of ill-prepared candidates who had underestimated the quantity of study required for the new larger subject. However, given this is a new subject, it is difficult to compare the performance of candidates in this diet with those in previous years.

C. Pass Mark

The Pass Mark for this exam in combination with CM1B was 55

Q1

An income protection insurance contract pays an income to the policyholder [½]
while that policyholder is deemed as being 'sick'. [½]
The definition of sickness will be carefully specified in the policy conditions. [½]

If the policyholder recovers, the cover under the policy usually continues, so that subsequent bouts of qualifying sickness would merit further benefit payments. [1]

Such policies are usually subject to a deferred period (e.g. three months) of continuous sickness that has to have elapsed before any benefits start to be paid, and during which no benefit is payable. [1]

Premiums for these policies would normally be regular (e.g. monthly) or single premium [½]

Premiums would typically be waived during periods of qualifying sickness. This means that premiums would not be paid at the same time as benefits are payable. [1]

Off-period [½]
Benefit linked to salary [½]
Benefit can be converted to lump-sum on permanent disability [½]
Term of policy (e.g. ceases at retirement) [½]
[½ each]
[Marks available 7, maximum 4]

This question was generally poorly answered. Many candidates did not seem able to distinguish between income protection contracts and other health-based contracts e.g. critical illness.
Even candidates who identified the correct policy type appeared to struggle to make enough points (e.g. deferred periods, waiver of premiums were mentioned by few candidates)

Q2

Develop a well-defined set of objectives which need to be met by the results of the data analysis. [1]
Identify the data items required for the analysis. [1]
Collection of the data from appropriate sources. [1]
Processing and formatting data for analysis, e.g. inputting into a spreadsheet, database or other model. [1]
Cleaning data, e.g. addressing unusual, missing or inconsistent values. [1]
(Also ensure that legal requirements / professional guidance requirements are satisfied) [1]
[Marks available 6, maximum 5]

Many candidates scored well on this question although a significant minority of candidates failed to score any marks at all. Some candidates wasted time in describing the whole data analysis process.

Q3

(i)

The probability that, of two lives aged 40, one particular life dies first and the death occurs between 5 and 22 years from now (i.e. between age 45 and 62). [2]

(ii)

$$\begin{aligned} {}_{5|17}q^{1}_{40:40} &= \tfrac{1}{2} \times {}_{5|17}q_{40:40} \\ &= \tfrac{1}{2} \times \left[{}_{5}P_{40:40}(1 - {}_{17}P_{45:45}) \right] \\ &= \tfrac{1}{2}\left({}_{5}p_{40}^{2} - {}_{22}p_{40}^{2} \right) \end{aligned}$$

[1½]

$$\text{and } {}_{t}p_{x} = e^{-\int_0^t \mu_{x+s} ds} = e^{-\int_0^t 0.01 ds} = e^{-0.01t}$$

[1]

$$\begin{aligned} \Rightarrow {}_{5|17}q^{1}_{40:40} &= \tfrac{1}{2}\left[(e^{-0.05})^2 - (e^{-0.22})^2 \right] \\ &= \tfrac{1}{2}[0.9048374 - 0.6440364] \\ &= 0.13040 \end{aligned}$$

[1½]

Alternative solution:

$$\int_5^{22} {}_{t}p_{40}\, {}_{t}p_{40}\, \mu_{40+t} dt$$

[2]

$$= \int_5^{22} e^{-0.02t} 0.01 dt$$

$$= \frac{1}{2}\left[e^{-0.02t} \right]_{22}^{5}$$

[1]

$$= \frac{1}{2}\left(e^{-0.1} - e^{-0.44} \right) = 0.13040$$

[1]

[Marks available, maximum 6]

Part (i) was answered well. Part (ii) was answered well by the strongest candidates but others struggled to make much headway. In questions such as these, it is helpful to remember the integral form of joint lives probabilities as this can be the a quicker way to solve the problem without the need to remember specific formulas

A common error, using the first methodology above, was to exclude the '½' factor.

Q4

Properties include:

- Size/volume, not only does big data include a [½]
    - very large number of individual cases, but each [½]
    - might include very many variables, [½]
    - a high proportion of which might have empty (or null) values - leading to sparse data; [½]

[Maximum 1½]

- Speed/velocity, the data to be analysed [½]
    - might be arriving in real time at a very fast rate - [½]
    - for example, from an array of sensors taking measurements thousands of times every second; [½]

- variety, big data is [½]
    - often composed of elements from many different sources [½]
    - which could have very different structures - [½]
    - or is often largely unstructured [½]

[Maximum 1½]

- reliability/veracity, given the above three characteristics [½]
    - we can see that the reliability of individual data elements might be difficult to ascertain [½]
    - and could vary over time (for example, an internet connected sensor could go offline for a period) [½]

[Marks available 7, maximum 5]

Generally poorly answered except by the strongest candidates.

Q5

(i)

(a)

Equation of value at time 0 is [1]

$$P \times \ddot{a}_x = A_x$$

(b)

Prospective reserve at time is given by [1]

$$_{t}V_x^P = A_{x+t} - P \times \ddot{a}_{x+t}$$

(c)

Retrospective reserve at time t is given by [1]

$$\frac{(1+i)^t}{{}_t p_x}\left(P \times \ddot{a}_{x:\overline{t}|} - A^{\,1}_{x:\overline{t}|}\right)$$

(ii)

$$\begin{aligned}
{}_tV_x^P &= A_{x+t} - P \times \ddot{a}_{x+t} \\
&= A_{x+t} - \underbrace{P \times \frac{l_x}{l_{x+t}} \times (1+i)^t \times {}_{t|}\ddot{a}_x}_{\left[\frac{1}{2}\right]} - \underbrace{P \times \frac{l_x}{l_{x+t}} \times (1+i)^t \times \ddot{a}_{x:\overline{t}|} + P \times \frac{l_x}{l_{x+t}} \times (1+i)^t \times \ddot{a}_{x:\overline{t}|}}_{\left[\frac{1}{2}\right]} \\
&= P \times \frac{l_x}{l_{x+t}} \times (1+i)^t \times \ddot{a}_{x:\overline{t}|} - P \times \frac{l_x}{l_{x+t}} \times (1+i)^t \times \underbrace{\left(\ddot{a}_{x:\overline{t}|} + {}_{t|}\ddot{a}_x\right)}_{\left[\frac{1}{2}\right]} + A_{x+t} \\
&= P \times \frac{l_x}{l_{x+t}} \times (1+i)^t \times \ddot{a}_{x:\overline{t}|} - \frac{l_x}{l_{x+t}} \times (1+i)^t \times P \times \underbrace{\ddot{a}_{x+t}}_{\left[\frac{1}{2}\right]} + A_{x+t} \\
&= P \times \frac{l_x}{l_{x+t}} \times (1+i)^t \times \ddot{a}_{x:\overline{t}|} - \underbrace{\frac{l_x}{l_{x+t}} \times (1+i)^t \times \left(\underbrace{A_x}_{[0.5]} - \frac{l_{x+t}}{l_x} \times v^t \times A_{x+t}\right)}_{\left[\frac{1}{2}\right]} \\
&= \underbrace{\frac{l_x}{l_{x+t}} \times (1+i)^t \times \left(P \times \ddot{a}_{x:\overline{t}|} - \underbrace{A^{\,1}_{x:\overline{t}|}}_{[0.5]}\right)}_{\left[\frac{1}{2}\right]} = {}_tV_x^R
\end{aligned}$$

[4]

[Total 7]

Part (a) was well answered although many candidates included an endowment assurance expression within the formula for the retrospective reserve rather than a term assurance. This caused problems when trying to manipulate part (b).

A valid alternative approach to part (b) was to start directly from the premium equation.

Q6
(i)

$$v(1) = \frac{1}{1.04} = 0.961538$$ [½]

$$v(2) = \frac{1}{1.04 \times 1.05} = 0.915751$$ [1]

$$v(3) = \frac{1}{1.04 \times 1.05 \times 1.06} = 0.863916$$ [1]

$$v(4) = \frac{1}{1.04 \times 1.05 \times 1.06 \times 1.07} = 0.807398$$ [½]

$$\text{Then price} = 4\left(v(1) + v(2) + v(3) + v(4)\right) + 100v(4) = 94.9342$$ [½]

Find i such that: [1]

$$94.9342 = 4a_{\overline{4|}} + 100v^4$$
$$Try\ i = 5\% \qquad RHS = 96.4540$$
$$Try\ i = 5.5\% \qquad RHS = 94.7423$$
$$i = 0.05 + \left(\frac{96.4540 - 94.9342}{96.4540 - 94.7423}\right) \times 0.005$$
$$= 5.444\%\ \text{pa}$$ [1½]

(ii)
The forward rates are increasing with term. [1]
The gross redemption yield is a weighted average of those increasing rates. [1]
It is therefore lower than the 1-year yield forward rate $f_{3,1}$. [1]
[Total 10]

Part (i) was answered well although some candidates did not appreciate the need to calculate the price of the bond in order to calculate the gross redemption yield.
Part (ii) was poorly answered by marginal candidates as this calls for a good understanding of how gross redemption yield relates to forward rates. Note that marks are available for apparently obvious observations e.g. 'forward rates are increasing with term'.

Q7
(i)
Let P denote the level annual premium.
Then, we have:

$$P \times \ddot{a}^{4\%}_{45:\overline{20|}} = (20{,}000 - 2{,}000) \times A^{4\%}_{45} + 2{,}000 \times (IA)^{4\%}_{45}$$

$$\Rightarrow \quad P = \frac{18{,}000 \times 0.27605 + 2{,}000 \times 8.33628}{13.780}$$

$$= 1{,}570.50$$

[2]

We need the reserve at the end of 2018 (i.e. at time 17), when the lives are age 62. [1]
The benefit on death in 2019 (i.e. year 18) is (20,000−2,000) + 18×2,000=54,000. [1]
Thus, the reserve at time 17 is given by:

$${}_{17}V = 52{,}000 \times A_{62} + 2{,}000 \times (IA)_{62} - P \times \ddot{a}_{62:\overline{3|}} = 37{,}121.06$$

[2]

Benefit on death during 2018 is 52,000, so that we have $DSAR = SA - {}_{17}V = 14{,}878.94$. [1]

Then, we have:

$EDS = 378 \times q_{61} \times 14{,}878.94 = 50{,}668.77$ where $q_{61} = 0.009009$ [1]

and $ADS = 4 \times 14{,}878.94 = 59{,}515.76$ [½]

Hence, mortality profit for 2018 is $EDS - ADS = -8{,}846.99$. [½]

(ii)
For a whole life assurance policy early deaths lead to losses for the company. [1]
A loss has arisen here as there were more deaths than expected.
Actual deaths = 4 compared with expected deaths of 0.009009 x 378 =3.4054 [2]

[Total 12]

Generally answered well. Common errors in part (i) included calculating the reserve for the wrong time period and/or calculating the wrong death benefit for that period.

Q8
(i)
Is there a Capital gain?

$$i^{(2)} = 2\left(1.08^{1/2} - 1\right) = 7.846\%$$ [1]

$$\frac{D}{R}(1-t) = \frac{9}{1.1} \times 0.85 = 6.955\%$$ [1]

$$i^{(2)} \geq \frac{D}{R}(1-t) \Rightarrow \text{Capital gain}$$

And so loan will be assumed to be redeemed as late as possible since that is worst case scenario for the investor. [1]

Working per £100 nominal

$$\text{Price} = 0.85 \times 9a_{\overline{25}|}^{(2)} + 110v^{25} \quad \text{at } 8\%$$

$$= 83.264 + 16.062$$ [1]

$$= £99.326 \text{ per £100 nominal} = £993,260 \text{ for whole loan}$$ [1]

(ii)
(a)
Price paid is P per £100 nominal where:

$$99.326 = 0.85 \times 9a_{\overline{10}|}^{(2)} + Pv^{10} \quad \text{at } 8\%$$ [1½]

$$\Rightarrow P = (99.326 - 52.339) \times 1.08^{10} = £101.441 \text{ per £100 nominal}$$

$$= £1,014,410 \text{ for whole loan}$$ [1½]

Alternative solution

$$\text{Price} = 0.85 \times 9a_{\overline{15}|}^{(2)} + 110v^{15} = 101.44 \quad \text{at } 8\%$$

(b)
Net redemption yield is i where:
R > P→ CGT → worst case scenario redeem as late as possible [½]

$$101.441 = 0.75 \times 9a_{\overline{15}|}^{(2)} + 110v^{15} - 0.35 \times (110 - 101.441)v^{15} \quad \text{at } i$$

$$i = 7\% \Rightarrow RHS = 101.319$$

$$i = 6\% \Rightarrow RHS = 111.176$$

Interpolating gives $i = 7.0\%$ [2½]

[3]

[Marks available 14, maximum 11]

Part (i) was answered well, although some candidates neglected to test whether there would be a capital gain.

Some candidates did not appreciate that the bond should be assumed to be redeemed as late as possible as that is the worst case scenario for the investor. As the redemption choice is outside the investor's control the investor must assume that it is redeemed at the worst time possible.

In part (b) many candidates struggled to recognise what cashflows were paid/received by each of the two investors and hence failed to formulate the equations of value.

Q9 Let P denote the single premium payable at age 60.

Then, equation of value is given by:

$$P = \underbrace{250+(20{,}000+12\times 10)\times\left(a_{\overline{5}|}^{(12)}+\underbrace{v^5\times {}_5p_{60}\times a_{65}^{(12)}}_{\text{PMA92C20}}\right)}_{\text{EPV of benefit payable to husband (including annuity expenses)}}$$

$$+(10{,}000+12\times 10)\times\left[\underbrace{v^5\times {}_5p_{60:58}\times a_{65|63}^{(12)}}_{\substack{\text{EPV of reversionary annuity}\\ \text{assuming both lives survive}\\ \text{to end of guaranteed period}}}+\underbrace{v^5\times(1-{}_5p_{60})\times {}_5p_{58}\times a_{63}^{(12)}}_{\substack{\text{but, if husband dies during guaranteed period}\\ \text{then EPV of benefit after end of guaranteed}\\ \text{period is based on single-life annuity for wife}}}\right]$$

[1 for expenses + 2 for benefit to husband + 3 for benefit to wife]

Then, using 4% per annum interest, we have:

$$a_{\overline{5}|}^{(12)}=\frac{i}{i^{(12)}}\times a_{\overline{5}|}=1.018204\times 4.4518=4.532841$$ [½]

using PMA92C20, we have:

$$v^5\times {}_5p_{60}=0.82193\times\frac{9{,}647.797}{9{,}826.131}=0.807013$$ [½]

using PMA92C20, we have:

$$a_{65}^{(12)}=\ddot{a}_{65}-\frac{12+1}{24}=13.666-\frac{13}{24}=13.124333$$ [½]

using PMA92C20/PFA92C20, we have:

$${}_5p_{60:58}={}_5p_{60}\times {}_5p_{58}=\frac{9{,}647.797}{9{,}826.131}\times\frac{9{,}775.888}{9{,}881.764}=0.971331$$ [½]

using PMA92C20/PFA92C20, we have:

$$a_{65|63}^{(12)}=a_{63}^{(12)}-a_{65:63}^{(12)}=\left(\ddot{a}_{63}-\frac{13}{24}\right)-\left(\ddot{a}_{65:63}-\frac{13}{24}\right)=15.606-12.282=3.324$$ [1½]

using PFA92C20, we have:

$$a_{63}^{(12)} = \ddot{a}_{63} - \frac{13}{24} = 15.606 - \frac{13}{24} = 15.064333$$ [½]

Thus, we have:

$$\begin{aligned} P &= 250 + 20{,}120 \times (4.532841 + 0.807013 \times 13.124333) \\ &\quad + 10{,}120 \times \begin{bmatrix} 0.82193 \times 0.971331 \times 3.324 \\ +0.82193 \times \left(1 - \frac{9{,}647.797}{9{,}826.131}\right) \times \frac{9{,}775.888}{9{,}881.764} \times 15.064333 \end{bmatrix} \\ &= 250 + 304{,}301.89 + 29{,}105.91 \\ &= £333{,}657.80 \end{aligned}$$

[1]
[Total 11]

This was a challenging questions and few candidates correctly incorporated all the components required. Marginal candidates would have benefited from breaking down each component of the benefit and the expenses and from showing more detailed working. This would have added greater clarity to their attempts and may well have led to greater marks for some partially correct solutions.

Q10

(i)

$$10A(0,6) = 10 \times \exp\left(\int_0^6 \delta(t)\,dt\right)$$ [½]

$$= 10 \times \exp\left(\int_0^4 (0.03 + 0.01t)\,dt + \int_4^6 0.07\,dt\right)$$ [1]

$$= 10 \times \exp\left[\left[0.03t + \frac{0.01}{2}t^2\right]_{t=0}^{t=4} + (0.07 \times 6 - 0.07 \times 4)\right]$$ [1]

$$10\exp(0.20 + 0.14) = 14.0495$$ [½]

(ii)

Present value is given by:

$\int_4^{10} 5v(t)\,dt$ where $v(t) = \exp\left(-\int_0^t \delta(s)\,ds\right)$ [0.5 for integral + 0.5 for {4 to10}]

Then, for $4 \le t < 6$, we have:

$$\begin{aligned} v(t) &= \exp\left[-\left(\int_0^4 (0.03+0.01s)\,ds + \int_4^t 0.07\,ds\right)\right] \\ &= \exp\left[-\left(\left[0.03s + \frac{0.01}{2}s^2\right]_{s=0}^{s=4} + \left[0.07s\right]_{s=4}^{s=t}\right)\right] \\ &= \exp\left[-\left(0.20 + \left[0.07t - 0.28\right]\right)\right] \\ &= \exp\left(-0.07t + 0.08\right) \end{aligned}$$

[1½]

And, for $t \ge 6$, we have:

$$\begin{aligned} v(t) &= \exp\left[-\left(\int_0^4 (0.03+0.01s)\,ds + \int_4^6 0.07\,ds + \int_6^t 0.09\,ds\right)\right] \\ &= \exp\left[-\left(0.20 + \left[0.07\times 6 - 0.07\times 4\right]\right) + \left[0.09s\right]_{s=6}^{s=t}\right] \\ &= \exp\left[-\left(0.34 + \left[0.09t - 0.54\right]\right)\right] \\ &= \exp\left(-0.09t + 0.20\right) \end{aligned}$$

[1½]

Thus, present value is given by:

$$\begin{aligned} & 5\times\int_4^6 e^{-0.07t+0.08}\,dt + 5\times\int_6^{10} e^{-0.09t+0.20}\,dt \\ =\; & 5e^{0.08}\times\left[\frac{e^{-0.07t}}{-0.07}\right]_{t=4}^{t=6} + 5e^{0.20}\times\left[\frac{e^{-0.09t}}{-0.09}\right]_{t=6}^{t=10} \\ =\; & \frac{5e^{0.08}}{0.07}\times\left(e^{-0.28} - e^{-0.42}\right) + \frac{5e^{0.20}}{0.09}\times\left(e^{-0.54} - e^{-0.90}\right) \\ =\; & 7.6400 + 11.9547 \\ =\; & 19.5947 \end{aligned}$$

[2]

(iii)
We need to find i such that:

$19.5947 = 5\left(\bar{a}_{\overline{10}|} - \bar{a}_{\overline{4}|}\right)$

[1]

Thus, we have:

$$\left.\begin{array}{lll} \text{try } i = 6\% & \Rightarrow & RHS = 20.0536 \\ \text{try } i = 7\% & \Rightarrow & RHS = 18.8112 \end{array}\right\} \Rightarrow i \approx 0.06 + (0.07-0.06)\times\frac{20.0536 - 19.5947}{20.0536 - 18.8112} = 0.0637$$

Hence, to nearest 0.1%, equivalent effective rate of interest is 6.4% per annum. [2]

[Total 12]

Part (i) was answered well. Common errors in part (ii) included
- *assuming that a single payment was being valued rather than a continuous payment stream.*
- *the force of interest was 0.07 throughout the entire annuity payment.*

Candidates who made an error in part (ii) often used the correct methodology in part (iii) and they were awarded full credit.

Q11

(i)

All functions are valued at 6% per annum unless otherwise noted.

Value of Premiums

Let P be the monthly premium, then the value of future premiums is given by

$$=12P\ddot{a}^{(12)}_{[50]:\overline{25}|}=12P\left(\ddot{a}_{[50]}-\frac{11}{24}-\frac{v^{25}l_{75}}{l_{[50]}}\left(\ddot{a}_{75}-\frac{11}{24}\right)\right)$$

$$=12P\left(14.051-\frac{11}{24}-\frac{(0.232999)6879.1673}{9706.0977}\left(7.679-\frac{11}{24}\right)\right)$$

$$=12P\left(14.051-\frac{11}{24}-0.165137\left(7.679-\frac{11}{24}\right)\right)$$

$$=12P(12.400)=148.803P$$

[2]

Value of benefits

$$150000\left(A_{[50]}+0.015(IA)_{[50]}\right)$$

$$=150000(0.20463+0.015\times 4.84789)$$

$$=30,695+10,908=41,603$$

[1½]

Value of Commission

$$12P\left(0.25+0.025\left(\ddot{a}^{(12)}_{[50]:\overline{25}|}-{}^{1}\!/_{12}\right)\right)$$

$$=12P\left(0.25+0.025\left(12.400-{}^{1}\!/_{12}\right)\right)=6.6951P$$

[1]

Value of Expenses

Renewal expenses:

Calculating at an interest rate given by

$$1+i'=\frac{1.06}{1.0192308}=1.04$$

i.e. at $i'\% = 4\%$ [1]

$\Rightarrow$ EPV of renewal expenses = $75\ddot{a}^{4\%}_{[50]}$ [½]

Claim expenses:
If we assume first inflation increase occurs at same time as claim is paid (and so claim expense at end of year 1 would be £50 x 1.0192308) then

$EPV \quad = 50A_{[50]} \qquad @4\%$ [1]

So total expenses:

$300 + 75\ddot{a}^{4\%}_{[50]} + 50A^{4\%}_{[50]}$

$= 300 + 75 \times 17.454 + 50 \times 0.32868$

$= 1625.19$ [1½]

Thus, monthly premium is given by

$148.803P = 41,603 + 6.6951P + 1,625.19$

$$\Rightarrow P = \frac{43228}{142.108}$$

$\Rightarrow P = 304.19$

Thus, the monthly premium is approximately £304 (not £303 as shown in question). [½]

(ii)
All functions valued at 4%.
At the end of the 24th policy year the sum assured payable is equal to:
150,000(1+24×0.01) = 186,000 [1]

<u>EPV of future benefits</u>:

$186,000A_{74} + 150,000 \times 0.0075(IA)_{74}$

$= 186,000(0.65824) + 150,000 \times 0.0075(6.50913)$

$= 122,432.64 + 7,322.77 = 129,755.41$ [2]

<u>EPV of Future Premiums</u>:

$$12P\left(\ddot{a}_{74:\overline{1}|} - \left(\frac{11}{24}\right) \times \left(1 - \frac{v\, l_{75}}{l_{74}}\right)\right)$$

$$= 12P\left(1 - \left(\frac{11}{24}\right) \times \left(1 - \frac{6879.1673}{1.04(7150.2401)}\right)\right)$$

$$= 12P\left(1 - \left(\frac{11}{24}\right) \times (1 - 0.92509)\right) = 11.588P = 11.588 \times 304.19 = 3,525$$ [2]

EPV of Commission:

$$12P\left(0.025\ddot{a}^{(12)}_{\overline{74:1}|}\right)$$

$$=12P\left(0.025\ddot{a}^{(12)}_{\overline{74:1}|}\right)=11.588P\times 0.025=0.290P=88.22$$ [1]

EPV of Expenses:

$$125\ddot{a}_{74}+75A_{74}$$

$$=125\times 8.886+75\times 0.65824=1{,}160.12$$ [1]

Therefore, the gross prospective reserve at the end of the 24th policy year is given by:
129,755+88+1,160-3,525=127,478
i.e. £127,478 [1]

[Total 17]

Using the standard approximations, the premium to the nearest £ was £304 rather than the £303 as stated in the question. Allowance was made for candidates who appeared to have an answer of £304 but then spent time trying to adjust this answer. However, few marginal candidates appeared to have been affected. Candidates who used £303 in part (b) were given full credit. Full credit was also given to candidates who assumed the first inflation increase occurred after the first year so that the claim expense at the end of the first year would be £50.
In general part (i) of this question generated clearer attempts by candidates (e.g. when compared to Q9). Attempts to part (ii) were less clear possibly due to time pressure.

END OF MARKING SCHEDULE

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS’ REPORTS

September 2020

Subject CM1A - Actuarial Mathematics

Introduction

The Examiners’ Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

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Mike Hammer
Chair of the Board of Examiners
September 2020

A. General comments on the *aims of this subject and how it is marked*

1. CM1 provides a grounding in the principles of modelling as applied to actuarial work - focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

2. Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may lose marks where excessive rounding has been used or where insufficient working is shown.

3. These solutions use full actuarial notation although candidates who used notation based on standard keystrokes were given full credit.

B. Comments on *candidates' performance in this diet of the examination.*

1. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well by most candidates. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.
2. The move to an online exam meant that there were less knowledge based questions on the paper. The examiners included new types of questions (Qs 3 and 7) which reduced the need for typing of long formulae although these questions proved to be among the more challenging on the paper. In addition, the longest question (Q10) was designed so that it should not take longer to answer in a typed format rather than a handwritten format. Other questions offered more marks than would have been offered in previous years to allow for the greater length of time to type the answers. Despite this, there was some limited evidence that the typing requirement led to greater time pressure for the marginal candidate and the pass mark was slightly lowered to allow for this.
3. Where candidates made numerical errors, examiners would award marks for the correct method used and also for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.
4. The Examiners felt that the "open book" nature of the online exam led some candidates to rely on their notes much more than if the exam had been "closed book". The Examiners strongly recommend that candidates prepare for online exams just as thoroughly as they would do if the exam were of the traditional "closed book" format. Candidates should treat it as a bonus that they can refer to their notes but they should not be relying on being able to do so.

C. Pass Mark

The Pass Mark for this exam in combination with CM1B was 58.
1717 presented themselves and 797 passed.

Q1

The investor has an initial negative cashflow. [½]
In return the investor receives…
… a series of regular interest payments… [½]
…which are linked to an 'index'… [½]
…which reflects the effects of inflation… [½]
…and a final capital repayment… [½]
…that is also linked to the index. [½]
The indexation might be subject to a time lag [½]

[Marks available 3 ½, maximum 3]

This Questions was generally well-answered although some candidates chose to include non-cashflow related characteristics in their answer.

Q2

(a)

$$ {}_{10|4}q_{[36]} = \frac{l_{46} - l_{50}}{l_{[36]}} $$ [1 formula]

$$ = \frac{9,786.9534 - 9,712.0728}{9,886.0395} $$

$$ = 0.007574 $$ [1]

(b)

$$ \bar{A}^{\,1}_{46:\overline{25}|} = (1+i)^{0.5}\left(A_{46} - v^{25}\frac{l_{71}}{l_{46}}A_{71}\right) $$ [1.5 formula]

$$ = \sqrt{1.04}\left(0.28605 - 0.375117\frac{7,854.4508}{9,786.9534}0.61548\right) $$

$$ = \sqrt{1.04}\,(0.10076) = 0.10276 $$ [1.5 calculation]

[Total 5]

This question was well-answered by the candidates.

Q3

(i) $a = 0, b = 6, c = 0, d = t, p = 0.03, q = 0.04, X = 0.03 \times 0.04 = 0.0012$
[or $a = 0, b = 6, c = t, d = 6, p = 0.04, q = 0.03, X = 0.03 \times 0.04 = 0.0012$ – see end of part (ii) for solution with these numbers]
[3, deduct 0.5 mark for each error, min(0)]

(ii) Answer = $\int_{t=0}^{6} 0.03\times 0.04 e^{-0.04t}\left(\int_{s=0}^{t} e^{-0.03s}ds\right)dt$

$$= 0.04\int_{t=0}^{6} e^{-0.04t}\left(1-e^{-0.03t}\right)dt = 0.04\int_{t=0}^{6}\left(e^{-0.04t}-e^{-0.07t}\right)dt \qquad [1]$$

$$= 0.04\left[\frac{e^{-0.04t}}{-0.04}\right]_{t=0}^{6} - 0.04\left[\frac{e^{-0.07t}}{-0.07}\right]_{t=0}^{6} = (1-0.786628) - \frac{0.04}{0.07}(1-0.657047) \qquad [2]$$

$$= 0.017399 \qquad [1]$$

[or Answer = $\int_{t=0}^{6} 0.03\times 0.04 e^{-0.03t}\left(\int_{s=t}^{6} e^{-0.04s}ds\right)dt$

$$= 0.03\int_{t=0}^{6} e^{-0.03t}\left(e^{-0.04t}-e^{-0.24}\right)dt = 0.03\int_{t=0}^{6} e^{-0.07t} - e^{-0.03t-0.24}dt$$

$$= \frac{0.03}{0.07}\left[1-e^{-0.42}\right] - \left(e^{-0.24}-e^{-0.42}\right) = 0.017399]$$

[Total 7]

Part (i) was a new style of question which aimed to reduce the amount of typing required from candidates. Most candidates scored some marks on part (i) and a significant number of candidates were then able to use the answers from part (i) to make an attempt at part (ii).

Q4 Let P denote the the monthly premium, then the present value of future premiums is given by

$$12P\ddot{a}^{(12)}_{55:53:\overline{20|}} = 12P\left(\ddot{a}_{55:53:\overline{20|}} - \left(\frac{11}{24}\right)\times\left(1-\frac{v^{20}\times l_{75:73}}{l_{55:53}}\right)\right) \qquad [2]$$

$$= 12P\times\left(13.463 - \frac{11}{24}\times(1-0.353725)\right)$$

$$= 12\times P\times 13.16683$$

$$= 158.002P \qquad [1]$$

where $\ddot{a}_{55:53:\overline{20|}} = \ddot{a}_{55:53} - \left(\dfrac{v^{20} \times l_{75:73}}{l_{55:53}}\right) \times \ddot{a}_{75:73} = 16.284 - 0.353725 \times 7.975 = 13.463$

and

$$\left(\frac{v^{20} \times l_{75:73}}{l_{55:53}}\right) = v^{20} \times \left(\frac{l_{75}^{m} \times l_{73}^{f}}{l_{55}^{m} \times l_{53}^{f}}\right) = 0.456387 \times \left(\frac{8,405.160 \times 9,073.650}{9,904.805 \times 9,934.574}\right) = 0.353725$$

[2]

The present value of death benefits is given by

$$150000 \times \left(1 - d \times \ddot{a}_{55:53:\overline{20|}} - \left(\frac{v^{20} \times l_{75:73}}{l_{55:53}}\right)\right)$$

[2]

$$= 150,000 \times \left(1 - \left(\frac{0.04}{1.04}\right) \times 13.463 - 0.353725\right)$$

$$= 150,000 \times (0.482191 - 0.353725) = 150,000 \times 0.128465 = 19269.82$$ [1]

<u>Alternativly present value of death benefit is given by :</u>

$$150000 \times \left(A_{55:53} - \left(\frac{v^{20} \times l_{75:73}}{l_{55:53}}\right) \times A_{75:73}\right)$$

$$= 150000 \times \left(\left(1 - d \times \ddot{a}_{55:53}\right) - \left(\frac{v^{20} \times l_{75:73}}{l_{55:53}}\right) \times \left(1 - d \times \ddot{a}_{75:73}\right)\right)$$

$$= 150000 \times \left(\left(1 - \left(\frac{0.04}{1.04}\right) \times 16.284\right) - 0.353725 \times \left(1 - \left(\frac{0.04}{1.04}\right) \times 7.975\right)\right)$$

Using the principle of equivalence the monthly premium is found by equating the present values of premiums and benefits.

$$\Rightarrow 158.002P = 19269.82$$

$$\Rightarrow P = \$121.96$$

[1]

[Total 9]

Common errors here were

- *to incorrectly apply the monthly premium adjustment (or to ignore the adjustment completely)*
- *to calculate the benefit as an endowment assurance*

Q5

Equation of value is given by:

$$50,000 = 4,000a_{\overline{4}|}^{(4)} + Xa_{\overline{4}|}^{(2)} \times v^4 + 12,000\bar{a}_{\overline{4}|} \times v^8$$ [2]

where the effective rate of interest per annum is i such that $i^{(12)} = 9\%$.

Thus, we have: $1+i = \left(1+\frac{i^{(12)}}{12}\right)^{12} = \left(1+\frac{0.09}{12}\right)^{12} \Rightarrow i = 9.38069\%$ per annum [1]

Thus, we have:

$$a_{\overline{4}|}^{(4)} = \frac{1-v^4}{i^{(4)}} = \frac{1-0.6986141}{0.0906767} = 3.323742, \text{ where}$$

$$v^4 = \left(\frac{1}{1+i}\right)^4 = \left(\frac{1}{1.0938069}\right)^4 = 0.6986141$$

$$\left(1+\frac{i^{(4)}}{4}\right)^4 = 1+i = 1.0938069 \Rightarrow i^{(4)} = 0.0906767$$

[2½]

$$a_{\overline{4}|}^{(2)} = \frac{1-v^4}{i^{(2)}} = \frac{1-0.6986141}{0.0917045} = 3.286491, \text{ where}$$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1+i = 1.0938069 \Rightarrow i^{(2)} = 0.0917045$$ [2]

$$\bar{a}_{\overline{4}|} = \frac{1-v^4}{\delta} = \frac{1-0.6986141}{0.0896642} = 3.361274, \text{ where}$$

$$e^{\delta} = 1+i = 1.0938069 \Rightarrow \delta = 0.0896642$$

[1½]

Thus, we have:

$$50,000 = 4,000 \times 3.323742 + 0.6986141 \times 3.286491X$$

$$+12,000 \times \left(\frac{1}{1.0938069}\right)^8 \times 3.361274$$

$$= 13,294.968 + 2.2959890X + 19,686.109$$

$$\Rightarrow X = \$7,412.46$$

[2]

[Total 11]

Candidates generally answered this question well Common errors were

> - *to assume that the effective interest rate was 9% per annum which simplified the calculations significantly and so was penalised accordingly*
> - *to omit the deferral factors for the second and third elements of the annuity*

Q6

(i) Let DPP be t. Working is \$000's, we want

$39{,}500 = 5{,}000 \; a_{\overline{t}|}^{(4)}$ at 8% pa [1]

$$= 5{,}000 \frac{i}{i^{(4)}} a_{\overline{t}|}$$

$$\Rightarrow a_{\overline{t}|} = \frac{39{,}500}{5000 \times 1.029519} = 7.67349$$ [1]

$$\Rightarrow v^t = 1 - 7.67349 \times 0.08$$

$$\Rightarrow t = \frac{\ln 0.38612}{\ln (1/1.08)} = 12.365$$ [1]

So DPP is 12 years 6 months [1]

[Total 4]

(ii) Profit at the end of 15 years is:

$$-39500 \times (1.08)^{12.5} \times (1.06)^{2.5} + 5000 s_{\overline{12.5}|@8\%}^{(4)} (1.06)^{2.5} + 5000 s_{\overline{2.5}|@6\%}^{(4)}$$ [2]

where:

$$s_{\overline{12.5}|8\%}^{(4)} = \frac{(1.08)^{12.5} - 1}{i^{(4)}} = \frac{(1.08)^{12.5} - 1}{0.077706} = 20.80868$$ [1½]

$$s_{\overline{2.5}|@6\%}^{(4)} = \frac{(1.06)^{2.5} - 1}{i^{(4)}} = \frac{(1.06)^{2.5} - 1}{0.058695} = 2.67173$$ [1½]

Profit = -119,580 + 120,359 + 13,359 =14,138 (=\$14,138,000) [1]

[Total 6]

> *Part (i) was answered well although many candidates did not recognise that the end of the DPP would be measured in a whole number of quarters i.e. the investment would only become profitable at the time of an income payment.*
> *Part (ii) was answered less well with many candidates incorrectly assuming that the accumulated profit would be zero at the end of the DPP.*

Q7

(i) Since the policy will terminate on a critical illness claim, we do not need to model ρ_x and ν_x and these can be assumed to be zero. [1]

(ii)

$a = 100{,}000$ [½]
$b = 0.04$ [½]
$c = 0$ [½]
$d = 20$ [½]
$z = -ln(1.03) - 0.04 = -0.069559$ [1]

(iii) $\frac{4{,}000}{-0.069559}\left[e^{-0.069559t}\right]_0^{20}$ [1]

$= \$43{,}199$ [1]

(iv) $f = 0$
$g = 19$ [1 for *f* and *g*]

$h = -ln(1.03) - 0.04 = -0.069559$ [1]

(v) $P\left(\frac{1-e^{-0.069559\times 20}}{1-e^{-0.069559}}\right) = 11.1797P$ [½]

43,199 = 11.1797P
P = $3,864 [½]

<u>Alternatively:</u>

$\ddot{a}_{\overline{20|}}$ with $\delta = 0.069559$

[Total 9]

Some of the points highlighted in Q3 also apply here with many candidates struggling to use the equations developed in parts (ii) and (iv) to obtain answers to parts (iii) and (v). In general this question was not answered well with many candidates making errors without showing much intermediate working, especially in part (iii).

Q8

(i) Loan = 185 $a_{\overline{16|}} + 15(Ia)_{\overline{16|}}$ at 5% [1½]

$= 185 \times 10.8378 + 15 \times 80.9975$
$= \$3{,}219.96$ [1½]

(ii) Capital o/s after 4 payments comes from:

5th payment =$260 [½]

$\Rightarrow$ Capital o/s $= 245a_{\overline{12}|} + 15(Ia)_{\overline{12}|}$ at 5%

$= 245 \times 8.8633 + 15 \times 52.4873 = 2958.82$ [2]

Year	Capital o/s at start	Instalment	Interest	Capital repaid	Capital o/s at end
5	2958.82	260.00	147.94	112.06	2846.76
6	2846.76	275.00	142.34	132.66	2714.10
7	2714.10	290.00			

[2½]

(iii) Final instalment =$425

$\Rightarrow$ Loan o/s at start of final year = 425v = $404.76 [1]

= capital paid in final payment

$\Rightarrow$ Interest = 425 – 404.76 = $20.24 [1]

[Total 10]

This question was well answered by candidates. Many candidates seemed to use Excel to help with their calculations although their final answers needed to be typed into their script in order to receive credit.

Q9

(i)

$$i^{(4)} = 4\left(1.049^{1/4} - 1\right) = 4.812\%$$

$$\frac{D}{R}(1 - t_1) = \frac{6}{1.05} \times 0.8 = 4.571\%$$ [3]

$$i^{(4)} > \frac{D}{R}(1 - t_1) \Rightarrow \text{Capital gain}$$

(ii) Since there is a capital gain, the security is least valuable to the investor if the repayment is made by the borrower at the latest possible date. [1]
Since that <u>decision is beyond the control of the investor</u>, we must assume that the redemption occurs after 25 years to find the minimum yield obtained. [1]

(iii) If P is the price per $100 of the security:

$$P = 100 \times 0.06 \times 0.8a_{\overline{25}|}^{(4)} + \left(105 - 0.25(105 - P)\right)v^{25} \text{ at } 4.9\%$$ [2]

where:

$$a_{\overline{25}|}^{(4)} = \frac{1-v^{25}}{0.04812} = 14.4496 \quad [1]$$

$$\Rightarrow P = 4.8 \times 14.4966 + \left(105 - 0.25\left(105 - P\right)\right) \times 0.30242$$

$$\Rightarrow P = \frac{69.5837 + 23.8156}{1 - 0.25 \times 0.30242} = \$101.03157 \quad [2]$$

(iv) If the coupons were paid less frequently (i.e. half-yearly not quarterly) then the investor would have to wait longer, on average, for the coupon payments to be made. This will make the investment less valuable, and therefore the price would be lower than in (iii). [2]

[Total 12]

The calculation elements of the question were answered well but the explanations given as part of answers to parts (ii) and (iv) were often unclear.

Q10

(i) Mortality Table

	[½]	[1]	[½]	[1]
x	q_x^{base}	120% q_x^{base}	p_x	${}_{t-1}(ap)$
62	0.010112	0.012134	0.987866	1
63	0.011344	0.013613	0.986387	0.987866
64	0.012716	0.015259	0.984741	0.974418

Year	1	2	3	
Death Benefit	15,000	15,300	15,606	[1½]
Maturity Benefit			17,509.93	[1½]

Profit Test

Year	1	2	3	
Premium	P	P	P	[½]
Commission	0.15P	0.015P	0.015P	[½]
Expenses	200	30	30	
Interest	0.017P-4	0.0197P-0.6	0.0197P-0.6	[1]
Death Outgo	182.02	208.28	238.14	[1]
Maturity Outgo			17,242.74	[½]
Claim Expense	0.61	0.68	50	[½]

Profit Vector	0.867P-386.62	1.0047P-239.56	1.0047P-17,561.48

[1]

Probability in force	1	0.987866	0.974418
Profit Signature	0.867P-386.62	0.993P-236.65	0.979P-17,112.22
Discount Factor	0.934579	0.873439	0.816298
Present Value of Future Profits	0.810P-361.33	0.867P-206.70	0.799P-13,968.67

[2]

Total Present Value of Future Profits is 2.47633P-14,536.70

[½]

Thus the annual premium is found by:

$$P = \frac{14,536.70}{2.47633} = 5,870.26$$

i.e. the annual premium is approximately $5,871 [½]

(ii) Taking the profit vector from above and allowing for the transfer to reserves.

Year	1	2	3	
Profit Vector	0.867P-386.62	1.0047P-239.56	1.0047P-17,561.48	[½]
Reserve at start	0	5,000	10,000	[½]
Interest on reserve	0	100	200	[½]
Reserve at end	4,939.33	9,863.87	0	[1]
Transfer to Reserves	-4,939.33	-4,763.87	10,200	[1]
Revised Profit Vector	0.867P-5,325.95	1.0047P-5,003.43	1.0047P-7,361.48	[½]
Probability in force	1	0.987866	0.974418	
Revised Profit Signature	0.867P-5,325.95	0.993P-4,4942.71	0.979P-7,173.16	[½]
Discount Factor	0.934579	0.873439	0.816298	
Present Value of Future Profits	0.810P-4,977.52	0.867P-4,317.16	0.799P-5,855.43	[½]

Total Present Value of Future Profits is 2.47633P - 15,150.11

$$P = \frac{15,150.11}{2.47633} = \$6,117.87$$ [1]

(iii) With the premium calculated in (ii) we can calculate the expected cashflows arising in each policy year.

Year	1	2	3
Profit Vector	-$21	$1,144	-$1,214

[1]

Here we see negative cashflows arising in policy years 1 and 3 and a positive cashflow for policy year 2. [1]

Despite having set up reserves the life insurance company is facing a negative cashflow in policy year 3. The company may find itself with insufficient funds to meet all the claims that fall due in policy year 3. [1]
This implies that the reserves set up are not sufficient. It would be prudent to hold larger reserves. [1]

[Total 24]

This question seemed to discriminate well between the stronger and weaker candidates although some candidates seemed to be under time pressure. Despite the instruction to use a discounted cash-flow approach, many candidates used an equation of value approach in part (i) and were given partial credit for this.
In parts (i) and (ii), a zero profit- criterion was implicitly assumed although candidates who used other criteria were given full credit.
Common errors in part (i) were

- *to incorrectly calculate the maturity benefit (although full credit was given for candidates who did not include the final reversionary bonus)*
- *to ignore claims expenses*

There was a significant amount of calculation work and it was not surprising that even the strongest candidates made the occasional numerical error.
Many candidates failed to score any marks at all on parts (ii) and (iii) although it was not clear whether this was due to time pressure.

END OF MARKING SCHEDULE

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2021

Subject CM1 - Actuarial Mathematics
Core Principles
Paper A

Introduction

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Paul Nicholas
Chair of the Board of Examiners
July 2021

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2. Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations, but candidates are not penalised for this. However, candidates may not be awarded marks where excessive rounding has been used or where insufficient working is shown.

3. These solutions use full actuarial notation although candidates who used notation based on standard keystrokes were given full credit.

B. Comments on *candidate performance in this diet of the examination.*

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2. There appeared to be a large number of ill-prepared candidates who had underestimated the quantity of study required for the subject.
3. The nature of the online exam format meant that there was little on the paper that could be answered via knowledge based alone.
4. Where candidates made numerical errors, examiners awarded marks for the correct method used and also for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.
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1,856 presented themselves and 941 passed.

Solutions for CM1A – April 2021

Q1 $\quad {}_{3|5}q^{\ 1}_{45:45} = \dfrac{l_{48}\times l_{48}}{l_{45}\times l_{45}}\times\dfrac{1}{2}\times{}_5q_{48:48} = \dfrac{l_{48}\times l_{48}}{l_{45}\times l_{45}}\times\dfrac{1}{2}\times\left(1-\dfrac{l_{53}\times l_{53}}{l_{48}\times l_{48}}\right)$ [2]

$= \dfrac{(9,753.4714)^2}{(9,801.3123)^2}\times\dfrac{1}{2}\times\left(1-\dfrac{(9,630.0522)^2}{(9,753.4714)^2}\right)$ [1]

$= 0.990261683\times\dfrac{1}{2}\times\left(0.02514763\right)$ [1]

$= 0.012451366$

This question was generally well-answered.

Q2

(i)

a=0,

b=30 [a and b together ½]

m= 150,000 [½]

n=0.02 [1]

$z = -ln\left(1.03\right)-0.01-0.02 = -0.059559$ [1]

(ii)

$= -\dfrac{150000\times 0.02}{0.059559}\left[e^{-0.059559t}\right]_0^{30}$ [1½]

= 50,370.22 [1 – 0.167500] [1]

= 41, 933.21 (without rounding 41,933.28) [½]

[Total 6]

Most candidates scored some marks in part (i) and a significant number of candidates were then able to use the answers to part (i) to make an attempt at part (ii). However, a surprising number of candidates struggled to perform the straightforward integration in part (ii)

A common error in part (i) was to confuse the effective rate of interest, i, with the force of interest, δ .

Q3

(i)

Interest paid per year is $0.04\times 100,000 = 4,000$ [½]

Hence, by expressing cash flows in 1 March 2017 purchasing power, the effective annual real rate of return achieved, i, is found by solving:

$$100{,}000 = 4{,}000 \times \frac{240.5}{256.0} \times v_{i\%} + 4{,}000 \times \frac{240.5}{272.8} \times v_{i\%}^2 + (4{,}000 + 100{,}000) \times \frac{240.5}{286.6} \times v_{i\%}^3$$

$$\Rightarrow \quad 100{,}000 = 3{,}757.81 v_{i\%} + 3{,}526.39 v_{i\%}^2 + 87{,}271.46 v_{i\%}^3$$

[3]

Then, we have:

$$\left.\begin{array}{lll} i = -2\% & \Rightarrow & RHS = 100{,}230.69 \\ i = -1.5\% & \Rightarrow & RHS = 98{,}769.15 \end{array}\right\}$$

$$\Rightarrow i \approx -0.02 + [-0.015 - (-0.02)] \times \frac{100{,}230.69 - 100{,}000}{100{,}230.69 - 98{,}769.15} \approx -1.9\%$$

[1½]

(ii)
High actual inflation over the term of the loan has eroded the real return achieved. [1]
The nominal rate of return achieved is 4% per annum. However, as average inflation over the term of the loan (i.e. 6.02% pa) has exceeded 4% per annum, the real rate of return achieved by the lender is negative. [2]

[Total 8]

Part (i) was generally well answered. Common errors included: -
not discounting the individual cashflows back to time t=0,
ignoring the amounts of the cashflows and using an average inflation over the entire period,
getting the ratios of the inflation indices the wrong way round.

The question omitted to say that the bond was issued at par. In practice, nearly all the candidates assumed this to be the case; where an alternative assumption was made candidates were not penalised.

Where candidates made comments in part (ii), they were often of insufficient quality to demonstrate understanding. Candidates often failed to distinguish between nominal return and real return.

Q4
(i)
For $0 \le t \le 6$:

$$A(0,t) = \exp\left(\int_0^t \delta(s)\,ds\right) = \exp\left(\int_0^t 0.03 + 0.005s\,ds\right)$$

$$= \exp\left[0.03s + 0.0025s^2\right]_0^t = e^{0.03t + 0.0025t^2}$$

and $A(0,6) = e^{0.03\times6+0.0025\times36} = e^{0.27}$

For $t > 6$:

$$A(0,t) = \exp\left(\int_0^6 \delta(s)\,ds + \int_6^t \delta(s)\,ds\right) = A(0,6) \times \exp\left(\int_6^t 0.1 - 0.01s\,ds\right)$$

$$= e^{0.27}\exp\left[0.1s - 0.005s^2\right]_6^t = e^{0.27}\exp\left[0.1t - 0.005t^2 - (0.6 - 0.18)\right]$$
$$= e^{0.1t - 0.005t^2 - 0.15}$$

$a = 0$ [½]
$b = 0.03$ [½]
$c = 0.0025$ [1]
$f = -0.15$ [1]
$g = 0.1$ [1]
$h = -0.005$ [1]

(ii)

Nominal rate of return is $i^{(12)}$ where $\left(1 + \frac{i^{(12)}}{12}\right)^{60} = \frac{A(7)}{A(2)}$ [1]

$$= \frac{e^{0.1\times 7 - 0.005\times 49 - 0.15}}{e^{0.03\times 2 + 0.0025\times 4}} = \frac{e^{0.305}}{e^{0.07}} = e^{0.235}$$ [1]

Therefore $i^{(12)} = 12\left(e^{0.235/60} - 1\right) = 4.709\%$ [1]

[Total 8]

Part (i) was generally well answered. A common error was to evaluate A(0,6) incorrectly or omit it entirely in the derivation of f, g and h. It was not necessary to show the derivation of the numerical results in order to gain full marks. The derivation is included here to aid candidates' understanding.

In part (ii) some candidates did not appreciate that the accumulation factors derived in part (i) were only applicable if accumulated from time t=0. Many candidates attempted to derive new accumulation factors from time t=2, which is a perfectly valid approach but takes more time. Common errors included:
Integrating over an incorrect time period;
Using a formula for δ(t) over a time period for which it was not relevant.

Q5
(i)

$\mu_x^* = 1.2\mu_x$ where $t \le 1$

${}_t p_x^* = e^{-\int_0^t \mu_{x+s}^* ds}$ where $t \le 1$

$\Rightarrow {}_t p_x^* = e^{-\int_0^t 1.2\mu_{x+s} ds}$ where $t \le 1$ [1]

$\Rightarrow p_x^* = (p_x)^{1.2}$ [1]

$\ddot{a}^*_{70:\overline{3|}} = 1 + v \times p^*_{70} + v^2 \times {}_2p^*_{70}$ at 7% pa [1]

$$\Rightarrow \ddot{a}^*_{70:\overline{3|}} = 1 + v \times \left(\frac{9112.449}{9238.134}\right)^{1.2} + v^2 \times \left(\frac{8968.099}{9238.134}\right)^{1.2}$$ [1]

$= 2.762234$ [1]

(ii) $A_{70:\overline{3}|} = 1 - d\ddot{a}^*_{70:\overline{3}|} = 1 - \left(\frac{0.07}{1.07}\right) \times (2.762234)$ [1½]

$= 0.819293$ [½]

[Total 7]

This question was poorly answered. Many candidates did not appreciate that the values of μ_x shown in the tables are values at exact age x and do not apply over the whole period of x to x+1. It is therefore necessary to derive an average value of μ_x for age x from the tabulated value of p_x.

In part (i) many candidates forgot that the first payment in a life annuity in advance is 1 at time t=0, and that to receive the 3rd payment the life only needs to survive 2 years.
Another common error was to use only a one-year survival probability, rather than the survival probability from outset.

In part (ii) where candidates used the valid (but unnecessarily time consuming) approach of deriving the endowment assurance factor from first principles, a common error was to miss out the pure endowment benefit.

Q6

(i)

$$EPV = 40,000 \times A^1_{45:\overline{15}|} + 50,000 \times v^{15}_{6\%} \times \frac{l_{60}}{l_{45}} \times A_{60}$$ [1]

Where

$$A^1_{45:\overline{15}|} = A_{45:\overline{15}|} - v^{15}_{6\%} \times \frac{l_{60}}{l_{45}} = 0.42556 - 0.417265 \times \frac{9287.2164}{9801.3123}$$

[1]

$$= 0.030181218$$

$$EPV = 40,000 \times 0.030181218 + 50,000 \times 0.417265 \times \frac{9287.2164}{9801.3123} \times 0.32692$$

$= \$7,670.11$ [1]

(ii)

To get the variance, we calculate the 2nd moment by defining the benefit as a combination of a temporary assurance and a deferred whole life assurance.

Therefore:

Benefit from age 45 to 60:

$40,000^2 \times {}^2A^1_{45:\overline{15}|}$ with i at 6% [1]

$$= 40,000^2 \times \left[{}^2A_{45} - v^{15}_{12.36\%} \times \frac{l_{60}}{l_{45}} \times {}^2A_{60} \right]$$

$$= 40{,}000^2 \times \left[0.04172 - 0.174110 \times \frac{9287.2164}{9801.3123} \times 0.14098\right]$$ [1½]

$=1{,}600{,}000{,}000 \times 0.018461 = \$29{,}537{,}600.$ [½]

Benefit from age 60:

$$50{,}000^2 \times v^{15}_{12.36\%} \times \frac{l_{60}}{l_{45}}\, {}^2A_{60} \text{ with } i \text{ at } 6\%$$ [1]

$$= 50{,}000^2 \times 0.174110 \times \frac{9287.2164}{9801.3123} \times 0.14098$$

$$= 50{,}000^2 \times 0.023259 = \$58{,}147{,}500$$ [½]

Then total 2nd moment = 29,537,600 + 58,147,500

= 87,685,100 (with no rounding 87,684,707) [½]

Variance = 87,685,100 – $(7{,}670.11)^2$ = $\$^2$28,854,513 = $(\$5{,}372)^2$ [1]

[Total 9]

Part (i) was generally well answered. Common errors included: -
Using an endowment assurance factor rather than a term assurance factor for the first benefit.
Missing out the survival probability to age 60 in the second benefit.

Part (ii) was poorly answered. The simplest approach is to treat the benefit payments as a term assurance and deferred whole life, which are independent and therefore the covariance between them is zero.

Candidates who treated the benefits as a whole life benefit plus a deferred whole life failed to appreciate that these are not independent and hence failed to address the covariance between them.

Other common errors when calculating the second moment in part (ii) included: -
Omitting the square of the sums assured,
Squaring the survival probability for the deferred whole life benefit.

Q7

EPV Premiums $P\ddot{a}^{6\%}_{[45]:\overline{20}|} = 11.888P$ [1]

The interest rate for valuing the benefits is 4% p.a. $\dfrac{1.0192308}{1.06} = \dfrac{1}{1.04}$ [1]

EPV Benefit

$$= \$150{,}000\left(\frac{1}{1.0192308} A^1_{[45]:\overline{20}|} + A_{[45]:\overset{\;1}{\overline{20}|}}\right) \text{ at } 4\%$$ [2]

$$= \$150,000\left(\frac{1}{1.0192308}\left(A_{[45]:\overline{20}|} - v_{4\%}^{20}\frac{l_{65}}{l_{[45]}}\right) + v^{20}\frac{l_{65}}{l_{[45]}}\right)$$

$$= \$150,000\left(\begin{array}{l}\frac{1}{1.0192308}\left(0.46982 - (1.04)^{-20}\frac{8,821.2612}{9,798.0837}\right) \\ \\ +(1.04)^{-20}\frac{8,821.2612}{9,798.0837}\end{array}\right) \quad [1½]$$

$$= \$150,000(0.057821 + 0.41089) = \$70,306 \quad [½]$$

EPV Expenses

$$= \$200 + 0.75P + 0.025P\left(\ddot{a}_{[45]:\overline{20}|}^{6\%} - 1\right) + 140A_{[45]:\overline{20}|}^{6\%}$$

$$= \$200 + 0.75P + 0.025 \times 10.888P + 140 \times 0.32711 = \$245.80 + 1.0222P \quad [3]$$

Thus

$$10.866P = \$70,552 \Rightarrow P = \$6493.03 \quad [1]$$

[Total 10]

This question was generally well answered. Common errors included: -
Not splitting the death and survival benefits to correctly allow for bonuses vesting at the end of the policy year if a policyholder survives to the end of that policy year.
Using annuity and assurance factors at the adjusted interest rate to value cashflows that do not attract bonuses, in particular the claim expense

Q8

(i)

$$\$15,000 = X\,(Ia)_{\overline{60}|@j} + 60Xv_j^{60}a_{\overline{60}|@j} \quad [2]$$

where $j = \frac{i^{(12)}}{12}$

<u>Alternative</u>

$$\$15,000 = X\,(Ia)_{\overline{60}|@j} + 12 \times 60Xv_i^5 a_{\overline{5}|@i}^{(12)}$$

(ii)

$$(Ia)_{\overline{60}|@j} = \frac{\ddot{a}_{\overline{60}|j} - 60v_j^{60}}{j} = \frac{\frac{1-1.0094888^{-60}}{0.0094888/1.0094888} - 60 \times 1.0094888^{-60}}{0.0094888} = 1261.989$$

[2]

$$a_{\overline{5}|@i}^{(12)} = 3.6048 \times \frac{0.12}{0.11387} = 3.7990 \qquad \text{OR} \qquad a_{\overline{60}|@j} = 45.58779473 \quad [1]$$

where $i = 12\%$ and $j = \frac{i^{(12)}}{12} = 1.12^{1/12} - 1 = 0.0094888$ [1]

So $\$15,000 = 1261.989 \times X + 60 \times 45.58779473 \times (1.0094888)^{-60} \times X$

$\Rightarrow X = \frac{15,000}{2,814.053} = \$5.33$ [1]

(iii)
Loan outstanding at the end of December 2026

$15,000(1.12)^5 - 5.33 \times (Ia)_{\overline{60}|@j}(1.12)^5$ [2]

(iv)

$\$26,435.13 - \$11,855.09 = \$14,580$ [1]

(v)
As interest exceeds repayments at early durations the loan outstanding will increase [1]
Repayments will only start to reduce loan outstanding once the repayments increase beyond a certain amount [½]
Therefore by halfway through the term the loan outstanding has barely reduced - thus very little of the initial loan has been paid off. [½]

(vi)
Interest repaid during 2027 = Total repayments less capital repaid

$12 \times 60X -$ (loan o/s Dec 2026 $-$ loan o/s Dec 2027) [1]

$= 12 \times 60X - \left(\left[15,000(1.12)^5 - 5.33 \times (Ia)_{\overline{60}|@j}(1.12)^5\right] - 60Xa_{\overline{48}|@j}\right)$

$= 12 \times 60X - \left(\$14,580 - 60Xa_{\overline{48}|@j}\right)$ [1]

<u>Alternative</u>

$= 12 \times 60X - \left(\left[15,000(1.12)^5 - 5.33 \times (Ia)_{\overline{60}|@j}(1.12)^5\right] - 12 \times 60Xa^{(12)}_{\overline{4}|@i}\right)$

$= 12 \times 60X - \left(\$14,580 - 12 \times 60Xa^{(12)}_{\overline{4}|@i}\right)$ [2]

(vii)

$60Xa_{\overline{48}|@j} = 60 \times 5.33 \times 38.411832159 = \$12,284$ [1]

Total repayments during 2027 $= 12 \times 60X = \$3,837.88$ [½]

So interest repaid during 2027 = Total repayments less capital repaid

$= \$3,837.88 - (\$14,580 - \$12,284) = \$1,542$ [½]

<u>Alternative</u>

$12 \times 60Xa^{(12)}_{\overline{4}|@i} = 720 \times 5.33 \times 3.0373 \times \frac{0.12}{0.11387} = \$12,285$

So interest repaid during 2027 = Total repayments less capital repaid

$= \$3,837.88 - (\$14,580 - \$12,285) = \$1,543$

(viii)
Under the revised repayment schedule, more of the loan will be repaid earlier and so the loan outstanding at any one time will be less than under the original schedule [1]
and so less interest will be paid in total. [1]

[Total 18]

The early parts of this question were generally well answered.

In parts (i) and (ii) common errors included: -
Using $(Ia)_{\overline{5|}}^{(12)}$ *to value the first 60 payments. This is incorrect as it allows for annual increases and not the monthly increases required. (This compound interest function is not currently covered by the CM1 syllabus.)*
Where $a_{\overline{5|}}^{(12)}$ *was used in the alternative solution many candidates missed that the payment needed to be multiplied by 12 to reflect the total annual payment was now 12 × 60X.*
The commentary given by candidates for part (v) was often unclear.

For parts (vi) and (vii) many candidates attempted to calculate the interest by calculating (loan outstanding × interest rate) as you would do for a single payment.

The question asked for equations in parts (i), (iii) and (vi) to be used to calculate parts (ii), (iv) and (vii) respectively. Where this was not done, limited credit was given for (ii), (iv) and (vii).

Q9

(i)

$$\$450,000 = X\left[1+(1.03)v_{9\%}^{1}+(1.03)^{2}v_{9\%}^{2}+\ldots+(1.03)^{19}v_{9\%}^{19}\right]+\$450,000v_{9\%}^{20}$$ [1]

$$\text{With } \frac{1.03}{1.09}=\frac{1}{1+j}\Rightarrow j=0.058252427$$ [½]

$$\$450,000 = X\ddot{a}_{\overline{20|}j\%}+\$450,000v_{9\%}^{20}$$ [½]

$$\$450,000 = X\times 12.31216704+\$80,293.9004\Rightarrow X=\$30,027.70 \text{ per annum}$$ [1]

(ii)

$$i=9\%\Rightarrow d^{(12)}=8.58689942\%$$

$$\$44,600\times\ddot{a}_{\overline{10|}9\%}^{(12)}+\$44,600\times 1.5\times\ddot{a}_{\overline{10|}9\%}^{(12)}\times v_{9\%}^{10}$$ [1½]

$$=\$44,600\times 6.72639989+\$44,600\times 1.5\times 6.72639989\times 0.42241081$$

$$=\$299,997.4351+\$190,083.2379=\$490,080.67>\$400,000 \textit{ Purchase price}$$ [1]

$\Rightarrow$ IRR > 9% per annum [½]

(iii)

Profit $=\left[\$44,600\times \ddot{a}^{(12)}_{\overline{10|}9.5\%}+\$44,600\times 1.5\times \ddot{a}^{(12)}_{\overline{10|}9.5\%}\times(1.095)^{-10}-400,000\right]\times(1.095)^{20}$ [2½]

$\ddot{a}^{(12)}_{\overline{10|}9.5\%}=6.5974$

$=\left[472,342.60-400,000\right]\times(1.095)^{20}=72,342.60\times(1.095)^{20}=444,300.21$ [1½]

[Total 10]

Parts (i) and (ii) were generally well answered.

In part (iii) candidates who used a discounted payback period approach were given credit, but most candidates over-simplified the calculation and so did not score full marks.

Q10

(i)

$\$50,000=P\ddot{a}_{45:\overline{15|}}=P\left(11.386\right)\Rightarrow P=\$4,391.36$ [2]

$\$4,391.36\ddot{a}_{45:\overline{15|}}=S\bar{A}_{45:\overline{15|}}$ OR $\$50,000=S\bar{A}_{45:\overline{15|}}$ [1½]

$50,000=S\left[(1.04)^{0.5}\times A^{1}_{45:\overline{15|}}+A_{45:\overset{1}{\overline{15|}}}\right]$ [1]

where

$(1.04)^{0.5}A^{1}_{45:\overline{15|}}+A_{45:\overset{1}{\overline{15|}}}=(1.04)^{0.5}\times 0.035920087+0.526139912=0.562771357$ [2]

$50,000=S\left[0.562771357\right]$

$S=\$88,846.03$ [½]

(ii)

<u>Endowment</u>:

${}_{10}V=\$90,000\bar{A}_{55:\overline{5|}}-\$4,450\ddot{a}_{55:\overline{5|}}=90,000\times 0.824144965-4,450\times 4.585$
$=\$53,769.80$ [1]

Where $(1.04)^{0.5}A^{1}_{55:\overline{5|}}+A_{55:\overset{1}{\overline{5|}}}=(1.04)^{0.5}\times 0.024993342+0.798656658=0.824144965$

[1]

DSAR = $\$90,000(1.04)^{0.5}-\$53,769.80=\$38,012.55$ [2]

E(deaths)= $q_{54}\times\left(550+6\right)=0.003976\times 556=2.210656$ [1]

EDS $=2.210656\times\$38,012.55=\$84,032.67$ [½]

ADS $=6\times\$38,012.55=\$228,075.30$ [½]

EDS-ADS = $-\$144,042.63$ [½]

<u>Annuity:</u>

${}_{10}V=\$4,450\times\ddot{a}_{55:\overline{5|}}=\$20,403.25$ [1]

DSAR = $-\$20,403.25$ [½]

E(deaths)= $q_{54} \times (550+6) = 0.003976 \times 556 = 2.210656$

EDS $= 2.210656 \times -\$20,403.25 = -\$45,104.57$ [½]

ADS $= 6 \times -\$20,403.25 = -\$122,419.5$ [½]

EDS-ADS = $\$77,314.93$ [½]

Total mortality profit: $-\$144,042.64 + \$77,314.93 = -\$66,727.70$ [½]

(iii)
The insurance company expected approximately 2.21 deaths, whereas 6 deaths actually occurred. So actual mortality was heavier than expected. [1]
With endowment assurances, earlier-than-expected deaths lead to an earlier payment of the benefit - the benefit is paid as a death benefit rather than as a maturity benefit. This implies earlier than expected deaths lead to a mortality loss. Here, as actual mortality was heavier than expected, there is a mortality loss on the endowments. [1]

With an annuity, early deaths imply no future benefits are paid. Thus earlier-than-expected deaths lead to a mortality profit. Here, as actual mortality was heavier than expected, there is a mortality profit on the annuities. [1]

The mortality loss on the endowments > the mortality profit on the annuities, thus overall there is a total mortality loss. [1]

[Marks available 4, maximum 3]

Part (i) was generally well answered. A common error was applying the claim acceleration adjustment to both the death benefit and the survival benefit when calculating $\bar{A}_{45:\overline{15|}}$. *(This also applied in part (ii)).*

In part (ii) many candidates only calculated the mortality profit arising from the endowment assurance and ignored the mortality profit of the annuity policy. Other common errors included: -
Making no adjustment to the sum assured to allow for the immediate payment on death when calculating the DSAR.
Using the number of policies in force at end of the year (550) rather than at the start of the year (550+6=556);
Using the mortality rate for the age at the end of year instead of the age at the start of year when calculating the expected number of deaths.

In part (iii) many candidates lost marks as they only commented on the results for the endowment assurance and not the annuity policy.

[Paper Total 100]

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS’ REPORT

September 2021

Subject CM1 – Actuarial Mathematics
Core Principles
Paper A

Introduction

The Examiners’ Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners’ approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
December 2021

A. General comments on the *aims of this subject and how it is marked*

CM1 provides a grounding in the principles of modelling as applied to actuarial work - focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may lose marks where excessive rounding has been used or where insufficient working is shown.

Although the solutions show full actuarial notation, candidates were generally expected to use standard keystrokes in their solutions.

Candidates should pay attention to any instructions included in questions; failure to do so can lead to fewer marks being awarded.

In particular, where the instruction, "*showing all working*" is included and the candidate shows little or no working, then the candidate will be awarded very few marks.

Where a question specifies a method to use (e.g. *determine the present value of income using annuity functions*) then when a candidate uses a different method the candidate will not be awarded full marks.

Candidates are advised to familiarise themselves with the meaning of the command verbs (e.g. state, determine, calculate). These identify what needs to be included in answers in order to be awarded full marks.

B. Comments on *candidate performance in this diet of the examination.*

The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those scripts.

A large number of candidates appeared to be inadequately prepared, in terms of not having sufficiently covered the entire breadth of the subject. We would advise candidates not to underestimate the quantity of study required for this subject.

Candidates should be aware that the questions cannot be answered using knowledge alone and well prepared candidates will demonstrate application of their knowledge to the questions asked.

Where candidates made numerical errors, the examiners awarded marks for the correct method used and also for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.

The Examiners felt that the "open book" nature of the online exam led some candidates to rely on their notes much more than if the exam had been "closed book". The Examiners strongly recommend that candidates prepare for online exams just as thoroughly as they would do if the exam were of the traditional "closed book" format. Candidates are recommended to use their notes only as a tool to check or confirm answers where necessary, rather than as a source for looking up the answers.

C. Pass Mark

The Pass Mark for this exam was 53.
1344 presented themselves and 533 passed.

Whilst the paper was of a similar standard to previous sittings there were questions within this paper that covered areas of the syllabus which hadn't been examined recently. It was apparent that many candidates were not adequately prepared for this and therefore the examiners took this into consideration when setting the pass mark.

Candidates need to be aware that examiners can ask questions from across the breadth of the syllabus and that they will be asked to apply their knowledge in different situations. Candidates are advised to access as much learning material as available to them and not rely on past papers alone when preparing to sit the examination.

Solutions for Subject CM1 Paper A– September 2021

Q1

t	1	2	3	4	5	6	7	8	9	10
	+1	-1	+1	+1	+1	-1	0	-1	+1	+1

t=10 and 9: adjusted cashflows = +1 and +1

t=8: zeroise cashflow, adjusted cashflow = 0, need reserves @ t=7 of +1

t=7: adjusted cashflow =0 -1 = -1 (after reserve),
Zeroise cashflow, adjusted cashflow = 0, need reserves @ t=6 of +1

t=6: adjusted cashflow = -1-1= -2 (after reserve),
Zeroise cashflow, adjusted cashflow = 0, need reserves @ t=5 of +2

t=5: adjusted cashflow = 1-2=-1 (after reserve),
Zeroise cashflow, adjusted cashflow = 0, need reserves @ t=4 of +1

t=4: adjusted cashflow = 1-1 = 0 (after reserve),
STOP THIS RUN OF ZEROISING

t=3: cashflow = +1

t=2: zeroise cashflow, adjusted cashflow = 0, need reserves @ t=1 of +1 t=1: adjusted cashflow = +1-1=0 (after reserve)

Reserve

$_tV$	1	2	3	4	5	6	7	8	9	10
	+1	0	0	+1	+2	+1	+1	0	0	0

[1]

Revised profit signature is

t	1	2	3	4	5	6	7	8	9	10
	0	0	+1	0	0	0	0	0	+1	+1

[2]

Generally well answered.

Candidates needed to include some explanation on the derivation of the revised profit signature to gain full marks since the command word was "determine".

Q2

(a)

$$\mu_{55:60} = \mu_{55} + \mu_{60} = 0.000976 + 0.001918 = 0.002894$$ [1]

(b)

$${}_5p_{55:60} = \frac{l_{60}}{l_{55}} \times \frac{l_{65}}{l_{60}} = \frac{l_{65}}{l_{55}} = \frac{9703.708}{9917.623} = 0.97843082$$ [1]

(c)

$${}_2q^1_{60:60} = \frac{1}{2} \times \left({}_2q_{60:60} \right) = \frac{1}{2} \times \left(1 - {}_2p_{60:60} \right) = \frac{1}{2} \times \left(1 - {}_2p_{60} \times {}_2p_{60} \right)$$ [1]

$$= \frac{1}{2} \times \left(1 - \left(\frac{l_{62}}{l_{60}} \right)^2 \right) = \frac{1}{2} \times \left(1 - \left(\frac{9804.173}{9848.431} \right)^2 \right) = 0.004483816$$ [1]

[Total 4]

Well answered.

Q3

In all parts we are calculating expected present values [½]
and the life is healthy at t = 0 (x = 40) [½]

(a)

2,000 per annum payable continuously [½]
when the life is in the sick state [½]
for the first and any subsequent bouts of sickness [½]

All payments cease at time 20 (age 60) [½]

(b)
1,000 per annum payable continuously [½]
while the life remains in the healthy state [½]
The contract ceases when the life leaves the healthy state for the first time or at time 20 (or when life attains age 60) [½]

(c)
20,000 lump sum payable immediately [½]
on death from the sick state [½]
if death occurs before time 20 (age 60) [½]

[Total 6]

Few candidates scored full marks. Most candidates omitted important elements in their descriptions such as the initial states of the life (healthy), annuity payments being made continuously, lump sums being paid immediately, whether return to the healthy state was permitted.

A number of candidates were unable to demonstrate they understood the difference between an annuity type benefit and a lump sum benefit payable immediately when a transition occurs.

Q4

Working in \$m

(a) PV of outgo $= 10 + 8v = 17.5472$ [1]

(b) PV of income

$$= v\left(0.5\left(Ia\right)_{\overline{6}|} + 3.5a_{\overline{6}|}\right) + v^7\left(8a_{\overline{7}|} - \left(Ia\right)_{\overline{7}|}\right)$$

[3]

$$= v\left(0.5\times 16.3767 + 3.5\times 4.9173\right) + v^7\left(8\times 5.5824 - 21.0321\right)$$

$$= 23.9612 + 15.7134 = 39.6746$$

[1½]

NPV = 39.6746 – 17.5472 =\$22.1274m [½]

Generally well answered.

This question specified that candidates should use annuity functions; those that did not were heavily penalised.

Some candidates were unable to determine as to when annuity cashflows started. This led them to include erroneous discount factors.

Q5

The price is given by the present value of the expected future dividend payments:

$$P = v^2 D_0 \left(1 + 1.06v + 1.06^2 v^2 + ... + 1.06^{10} v^{10} \left(1 + 1.03v + 1.03^2 v^2 + ...\right)\right) \ @ \ 7\% \ p.a.$$

$$= v^2 D_0 \left(\ddot{a}_{\overline{10}|}^{j\%} + 1.06^{10} v^{10} \ddot{a}_{\overline{\infty}|}^{i^*\%} \right)$$

[3]

where $\frac{1.06}{1.07} = \frac{1}{1+j} \Rightarrow j = 0.94340\%$ and $\frac{1.03}{1.07} = \frac{1}{1+i^*} \Rightarrow i^* = 3.88350\%$ [1]

and

$D_0 = \$0.20$ and $\ddot{a}_{\overline{10}|}^{i'} = 9.58975$ and $\ddot{a}_{\overline{\infty}|}^{i*} = 26.75$ [2]

$$P = \frac{0.20}{1.1449}\left(9.58975 + 0.910376 \times 26.75\right) = 5.929$$

[½]

Therefore, the required price is \$5.93 [½]

Many candidates failed to demonstrate how to translate a description of cashflows into an actuarial equation that accurately valued those cashflows.

A common error was leaving out the second (6%) growth rate.

A number of candidates missed out on marks by not giving their final answer to 2 decimal places.

Q6

Let $i_t =$ annual effective spot rate for t-year period

$f_{t,1}$ = annual effective one-year forward rate at time t

107.60 = $6(v_{i_1} + v_{i_2}^2 + v_{i_3}^3) + 100v_{i_3}^3$ (1) [1]

100 = $6.5(v_{i_1} + v_{i_2}^2) + 100v_{i_2}^2$ (2) [1]

$(1+i_2)^2 = (1+i_1) \times (1+f_{1,1}) = (1+i_1) \times 1.045$ (3) [1]

From (3), $v_{i_2}^2 = v_{i_1} \times \frac{1}{1.045}$

$\Rightarrow$ from (2), 100 = $6.5 \times v_{i_1} + (6.5 + 100) \times v_{i_1} \times \frac{1}{1.045}$

$\Rightarrow$ 100 = $v_{i_1} \times (6.5 + \frac{6.5}{1.045} + \frac{100}{1.045})$

$\Rightarrow i_1 = 8.41388\%$ p.a. [2]

And from (3), $i_2 = 6.43895\%$ p.a. [1]

And from (1), 107.60 = $6 \times (v_{i_1} + v_{i_2}^2) + 106 \times v_{i_3}^3 \Rightarrow i_3 = 3.08345\%$ p.a. [1]

Poorly answered.

Many candidates appeared to not understand the relationship between spot rates and forward rates, nor the meaning of par yield. This led many candidates to setting up incorrect formulae.

Many candidates also calculated the gross redemption yield for the 3-year fixed interest bond. This provided no additional information to solve the question and therefore no marks for awarded for this.

Q7 $EPV\ Expenses = 30,000 \times 0.03 \times Z + 250$ [1]

Where Z= EPV of \$1 pa of the reversionary annuities

$= a^{(12)m}_{65:\overline{15}|} + a^{(12)f}_{65:\overline{15}|} - 2a^{(12)mf}_{65:65:\overline{15}|}$ [2]

$$a^{(12)m}_{65:\overline{15}|} = a^{(12)m}_{65} - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times a^{(12)m}_{80} = \left(\ddot{a}^m_{65} - \frac{13}{24}\right) - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times \left(\ddot{a}^m_{80} - \frac{13}{24}\right)$$

$$= \left(13.666 - \frac{13}{24}\right) - v^{15} \times \frac{6,953.536}{9,647.797} \times \left(7.506 - \frac{13}{24}\right) = 10.3372$$

$$a^{(12)f}_{65:\overline{15}|} = a^{(12)f}_{65} - v^{15} \times \frac{l^f_{80}}{l^f_{65}} \times a^{(12)f}_{80} = \left(\ddot{a}^f_{65} - \frac{13}{24}\right) - v^{15} \times \frac{l^f_{80}}{l^f_{65}} \times \left(\ddot{a}^f_{80} - \frac{13}{24}\right)$$

$$= \left(14.871 - \frac{13}{24}\right) - v^{15} \times \frac{7,724.737}{9,703.708} \times \left(8.989 - \frac{13}{24}\right) = 10.5954$$

$$a^{(12)mf}_{65:65:\overline{15}|} = a^{(12)mf}_{65:65} - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times \frac{l^f_{80}}{l^f_{65}} \times a^{(12)mf}_{80:80} = \left(\ddot{a}^{mf}_{65:65} - \frac{13}{24}\right) - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times \frac{l^f_{80}}{l^f_{65}} \times \left(\ddot{a}^{mf}_{80:80} - \frac{13}{24}\right)$$

$$= = \left(11.958 - \frac{13}{24}\right) - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times \frac{l^f_{80}}{l^f_{65}} \times \left(5.857 - \frac{13}{24}\right) = 9.7230$$ [5½]

$\Rightarrow\ Z = 10.3372 + 10.5954 - 2 \times 9.7230 = 1.4867$ [½]

Single Premium = EPV Expenses + EPV annuity

$30,000 \times 1.03 \times 1.4867 + 250$ = 46,189.13

Single Premium is \$46,189 [1]

Poorly answered.

Many candidates appeared to not understand how to value a reversionary annuity.

Several candidates missed part of the annuity payment details when valuing the benefit. For example, assuming the benefit was payable in advance, or annually, or assuming the benefit was payable for the whole of the policyholder's life.

One common error was using an incorrect adjustment to the tabulated annuity rates to arrive at the monthly in arrears annuity rate.

Q8

(i)

If $0 < t \le 4$ then accumulation to time t is

$$e^{\int_0^t \delta(t)dt} = e^{\int_0^t (0.06+0.02t)dt} = e^{(0.06t+0.01t^2)}$$ [1]

If $4 < t$ then accumulation to time t is

$$e^{\left[\int_0^4 (0.06+0.02t)dt+\int_4^t (0.08-0.01t)dt\right]}$$ [1]

$$= e^{\left(0.06\times4+0.01\times4^2\right)}.e^{\left[0.08t-0.005t^2\right]_4^t} = e^{0.4}.e^{\left[0.08t-0.005t^2-(0.32-0.08)\right]} = e^{\left(0.16+0.08t-0.005t^2\right)}$$ [1]

So:

a = 0, [½]

b = 0.06 [½]

c = 0.01 [½]

f = 0.16 [½]

g = 0.08 [½]

h= -0.005 [½]

(ii)

Accumulated amount at t=13 is:

$$600\times\frac{A(0,13)}{A(0,3)}+900\times\frac{A(0,13)}{A(0,9)}$$ [2]

$$=600\times\left(\frac{e^{0.355}}{e^{0.270}}\right)+900\times\left(\frac{e^{0.355}}{e^{0.475}}\right)$$ [1]

= 653.23 + 798.23 = £1,451.46 [1]

(iii)

Let j% per monthly yield

Then $600(1+j)^{120}+900(1+j)^{48}=1,451.46$ [1]

It can be seen that j = 0% gives a left-hand side value of 1,500 which implies that j must be slightly negative [½]

Try j = -0.1% and left-hand side value is 1,389.92 (which is further from 1451.46 than 1500) [1]

Hence, answer is 0.0 % per month [½]

(iv)
The force of interest is negative for all times after time 8 [½]
So the 900 decreases in value from the time of investment onwards. Whereas the 600 increases for a period and then decreases [½]
The overall effect is that the accumulated value of the investments returns roughly to the original amounts invested, hence the overall yield is approximately 0.0% per annum [½]
[Marks available 1½, maximum 1]

Part (i) Generally well answered. Candidates should remember that the command verb "Determine" means that candidates will only be awarded full marks if they made clear their reasoning. Listing the numeric answers is not sufficient to gain full marks.

Part (ii) Many candidates appeared to be unfamiliar with how the accumulation factors derived in part (i) could be used to answer part (ii), and so used a valid alternative but more time-consuming approach.

A common error was to use the accumulation factors derive in part (i) but not to apply it from time zero.

Part (iii) Poorly answered. A common error was to calculate the effective interest rate per annum instead of per month.

Part (iv) Poorly answered.

Q9
(i)

EPV Premiums $P\ddot{a}^{4\%}_{30:\overline{30|}} = 50,000 \times v^{30}\dfrac{l_{60}}{l_{30}}$

$$P \times 17.756 = 50,000 \times 0.30832 \times \frac{9,287.2164}{9,925.2094} = 14,425.00 \Rightarrow P = 812.40$$

[½]

Reserve: ${}_{25}V = 50,000 \times v^5 \times \dfrac{l_{60}}{l_{55}} - 812.40 \times \ddot{a}_{55:\overline{5|}}$ [½]

$$= 50,000 \times 0.821927 \times \frac{9,287.2164}{9,557.8179} - 812.40 \times 4.585 = 36,207.97$$

[½]

Mortality profit

$DSAR = 0 - 36,207.97 = -36,207.97$ [1]

$E(death) = (315+2) \times q_{54} = 317 \times 0.003976 = 1.2604$ [1]

$EDS = 1.2604 \times (-36,207.97) = -45,636.24$ [½]

$ADS = 2\times\left(-36,207.97\right) = -72,415.95$ [½]

$EDS - ADS = 26,779.71$

Mortality profit is $26,780. [½]

(ii)

A death leads to a release of reserve which contributes to profit [1]
With a Pure Endowment no death benefit is paid [1]
So higher than expected deaths leads to higher than expected profit [½]
The company expected approximately 1.3 deaths whereas 2 deaths actually occurred
So, mortality was heavier than expected [1]
Thus higher mortality led to a mortality profit [½]

[Marks available 4, maximum 3]

Well answered.

Common errors included using a mortality rate when calculating the number of expected deaths for an age that did not correspond to the reserve calculated and including a sum assured when calculating the death sum at risk.

In part (ii) well prepared candidates covered the importance of the release of the reserve

Q10

(i)

(Denoting the revised rates with a ' suffix)
If we assume forces of decrement are constant over individual years of age [½]
and that independent and dependent forces of decrement are equal, [½]
Then we can use: -

$$\mu_{47}^{w} = \frac{(ad)_{47}^{w}}{(ad)_{47}^{d}+(ad)_{47}^{w}}\times\left(-\ln\left(ap\right)_{47}\right) = \frac{1,500}{390+1,500}\times\left(-\ln\left(0.9622\right)\right) = 0.0305817$$

[2]

$$q_{47}^{w} = \left(1-e^{-\mu_{47}^{w}}\right) = \left(1-e^{-0.0305817}\right) = 0.0301188$$

[1]

The revised independent probability of withdrawal, q'^{w}_{47}, is therefore
$2.5\times 0.0301188 = 0.0752970$ [½]

And

$$\mu_{47}^{'w} = -\ln\left(1-q_{47}^{'w}\right) = 0.0782827$$

[1]

The force of mortality at age 47 implied by the ELT15 Females rates will be:

$$-\ln\left(1-q_{47}^{ELT15(F)}\right) = -\ln\left(1-0.00219\right) = 0.00219240$$

[1]

and the revised independent force of mortality,

$$\mu_{47}^{'d} = 0.6 \times 0.00219240 = 0.00131544$$ [½]

(ii)
We have

$$(aq)_{47}^{'d} = \frac{\mu_{47}^{'d}}{\mu_{47}^{'d} + \mu_{47}^{'w}}\left(1 - e^{-\left(\mu_{47}^{'d} + \mu_{47}^{'w}\right)}\right)$$ [1]

$$= \frac{0.00131544}{0.00131544 + 0.0782827}\left(1 - e^{-(0.00131544 + 0.0782827)}\right) = 0.00126445$$ [1]

$$(aq)_{47}^{'w} = \frac{\mu_{47}^{'w}}{\mu_{47}^{'d} + \mu_{47}^{'w}}\left(1 - e^{-\left(\mu_{47}^{'d} + \mu_{47}^{'w}\right)}\right)$$

$$= \frac{0.078287}{0.00131544 + 0.0782827}\left(1 - e^{-(0.00131544 + 0.0782827)}\right) = 0.0752482$$ [1]

Thus

Age (x)	Number of employees $(al)_x$	Number of deaths $(ad)_x^d$	Number of withdrawals $(ad)_x^w$
47	50,000.00	63.22	3,762.41

[2]

(iii)
Concerns with the use of the revised multiple decrement table:
The population in the future may be different to the past population on which the analysis was conducted [1]
No allowance is made for future improvements in mortality or company initiatives that may reduce withdrawals [1]
There may be group events (such as workplace fire, outbreak of Covid) which may occur in the future but did not occur in the investigation period [1]
The investigation estimates may not be accurate [½]
The assumptions underlying the calculations in part (i) may not be valid [½]
The new table is significantly different from the original table [½]
Other valid points on why using the past to model the future may not be valid [½]
[Marks available 5, maximum 3]
[Total 15]

Part (i) Very poorly answered.

Most candidates did not appear to appreciate the difference between dependent and independent decrement rates, nor the difference between forces and rates of decrement. Most candidates missed out steps when deriving the required results. Many candidates lost out on marks as they did not quote the results to 6 significant figures.

Part (ii) Many candidates made a good attempt, even if they had struggled in part (i).

Part (iii) Poorly answered.

Q11

(i)

$$6,000\ddot{a}^{(12)}_{50:\overline{15}|6\%} = 0.6\times(6,000)+0.04\times(6,000)\times\left(\ddot{a}_{50:\overline{15}|6\%}-1\right)$$

$$+\frac{1}{1.019231}\times S\times A^{1}_{50:\overline{15}|j\%}+S\times\frac{l_{65}}{l_{50}}\times v^{15}_{j\%}\text{with }\frac{1.019231}{1.06}=\frac{1}{1+j}\Rightarrow j=0.04$$

$$\text{with }\frac{1.019231}{1.06}=\frac{1}{1+j}\Rightarrow j=0.04$$

[5]

$$\ddot{a}_{50:\overline{15}|6\%}=10.038;\quad \frac{l_{65}}{l_{50}}=\frac{8,821.2612}{9,712.0728};\quad A_{50:\overline{15}|4\%}=0.56719;$$

$$A^{1}_{50:\overline{15}|4\%}=A_{50:\overline{15}|4\%}-\frac{l_{65}}{l_{50}}v^{15}_{4\%}=0.06286$$ [2½]

$$\ddot{a}^{(12)}_{50:\overline{15}|6\%}=\ddot{a}_{50:\overline{15}|6\%}-\frac{11}{24}\left(1-\frac{l_{65}}{l_{50}}v^{15}_{6\%}\right)=9.753$$

$$\Rightarrow 6000\times 9.753=3,600+0.04\times 6,000\times 9.038+S\left(0.06167+0.50433\right)$$

$$\Rightarrow S=93,199$$

\$93,199 rounded to the nearest \$1,000 is \$93,000. [½]

(ii)

Require $93,000=6,000\ddot{s}^{(12)}_{\overline{15}|@i}$ [1]

Now

$$\ddot{s}^{(12)}_{\overline{15}|@0.434\%} = \frac{(1.00434)^{15}-1}{d^{(12)}}$$

$$\text{where }\left(1-\frac{d^{(12)}}{12}\right)^{-12}=1.00434\ \Rightarrow d^{(12)}=0.004329828$$

$$\ddot{s}^{(12)}_{\overline{15}|@0.434\%} = 15.5007$$

[1]

$$6,000\ddot{s}^{(12)}_{\overline{15}|@0.434\%}=6,000\times 15.50071$$

$$=93,004.2\approx 93,000$$

[½]

Therefore rate of return at least 0.434% per annum [½]

(iii)

Policyholder would expect a higher maturity benefit than basic sum assured of \$93,000 [½]
as they would expect bonuses to be declared every year [1]
A higher maturity benefit will give a higher return [½]

In addition to the survival benefit the policyholder will receive death cover during the term of the contact which is not measured by the return calculated in part (ii) but has value to the policyholder [1]

The policy may be obligatory, for example to back a mortgage [½]

This policy may be cheaper than others on the market [½]

Although 0.434% is low, it is guaranteed over 15 years and this guarantee might be important, for example for those planning to retire at 65 [½]

[Marks available 4½, maximum 3]

(iv)

$$SA = 93,000(1.05)^5 = 118,694$$ [1]

$$_5V = \frac{118,694}{1.019231} A^1_{55:\overline{10}|4\%} + 118,694 v^{10}_{4\%} \frac{l_{65}}{l_{55}} + 0.04 \times 6,000 \ddot{a}_{55:\overline{10}|6\%} - 6,000 \ddot{a}^{(12)}_{55:\overline{10}|6\%}$$

[3]

$$\ddot{a}_{55:\overline{10}|6\%} = 7.610; \quad \frac{l_{65}}{l_{55}} = \frac{8,821.2612}{9,557.8179}; \quad A_{55:\overline{10}|4\%} = 0.68388;$$

$$A^1_{55:\overline{10}|4\%} = A_{55:\overline{10}|4\%} - \frac{l_{65}}{l_{55}} v^{10}_{4\%} = 0.06037$$

$$\ddot{a}^{(12)}_{55:\overline{10}|6\%} = \ddot{a}_{55:\overline{10}|6\%} - \frac{11}{24}\left(1 - \frac{l_{65}}{l_{55}} v^{10}_{6\%}\right) = 7.388$$

[1]

$$_5V = 7,031.18 + 74,006.18 + 1826.40 - 44,327.25 = 38,536.51$$

Reserve is \$38,537 [1]

[Total 20]

Part (i) Generally well answered.

Common errors centred on how the benefits were valued, with many candidates not splitting the benefits into those payable on death and those on survival, in order to be able to apply the bonus adjustment only to the benefit on death. Another common error was valuing the renewal expenses using a monthly annuity rate.

Part (ii) Poorly answered.

Many candidates attempted to include mortality in their calculations. Candidates did not seem able to switch between the mathematics of finance and life contingencies within one question.

Part (iii) Many candidates failed to consider the nature of the benefits provided by a with profit endowment assurance and instead concentrated on the size of the premium.

Part (iv) Common errors included using a sum assured that ignored historic bonuses, using expense elements taken from the initial equation of value (e.g., including a deduction from the renewal expense annuity, or including an initial expense).

[Paper Total 100]

END OF EXAMINERS' REPORT

