

EXAMINATION

6 April 2005 (pm)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 Explain the difference between a profit vector and a profit signature. [2]

2 A 20-year temporary annuity-due of 1 per annum is issued to a life aged 50 exact.

(a) Express the expected present value of the annuity in terms of an assurance function.

(b) Hence calculate the value using the mortality table AM92 Ultimate with 4% interest.

[3]

3 A life insurance company sells an annual premium whole life assurance policy where the sum assured is payable at the end of the year of death. Expenses are incurred at the start of each policy year, and claim expenses are nil.

(a) Write down a recursive relationship between the gross premium provisions at successive durations, with provisions calculated on the premium basis. Define all the symbols that you use.

(b) Explain in words the meaning of the relationship.

[4]

4 A life insurance company issues an annuity to a life aged 60 exact. The purchase price is £200,000. The annuity is payable monthly in advance and is guaranteed to be paid for a period of 10 years and for the whole of life thereafter.

Calculate the annual annuity payment.

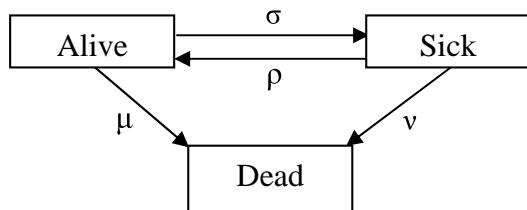
Basis:

Mortality AM92 Ultimate

Interest 6% per annum

[4]

- 5** A three-state transition model is shown in the following diagram:



Assume that the transition probabilities are constant at all ages with $\mu = 2\%$, $\nu = 4\%$, $\rho = 1\%$ and $\sigma = 5\%$.

Calculate the present value of a sickness benefit of £2,000 p.a. paid continuously to a life now aged 40 exact and sick, during this period of sickness, discounted at 4% p.a. and payable to a maximum age of 60 exact. [4]

- 6** Calculate the probability of survival to age 60 exact using ELT15 (Males) for a life aged 45½ exact using two approximate methods. State any assumptions you make. [5]
- 7** A joint life annuity of 1 per annum is payable continuously to lives currently aged x and y while both lives are alive. The present value of the annuity payments is expressed as a random variable, in terms of the joint future lifetime of x and y . Derive and simplify as far as possible expressions for the expected present value and the variance of the present value of the annuity. [5]
- 8** A pension scheme provides a pension on ill-health retirement of $1/80^{\text{th}}$ of Final Pensionable Salary for each year of pensionable service subject to a minimum pension of $20/80^{\text{ths}}$ of Final Pensionable Salary. Final Pensionable Salary is defined as the average salary earned in the three years before retirement. Normal retirement age is 65 exact. Derive a formula for the present value of the ill-health retirement benefit for a member currently aged 35 exact with exactly 10 years past service and salary for the year before the calculation date of £20,000. [5]
- 9** Explain how an insurance company uses risk classification to control the profitability of its life insurance business. [5]

- 10** You are given the following statistics in respect of the population of Urbania:

<i>Age band</i>	<i>Males</i>		<i>Females</i>	
	<i>Exposed to risk</i>	<i>Observed Mortality rate</i>	<i>Exposed to risk</i>	<i>Observed Mortality rate</i>
20–29	125,000	0.00356	100,000	0.00125
30–39	200,000	0.00689	250,000	0.00265
40–49	100,000	0.00989	200,000	0.00465
50–59	90,000	0.01233	150,000	0.00685

Calculate the directly and indirectly standardised mortality rates for the female lives, using the combined population as the standard population. [6]

- 11** A life insurance company issues a 25-year with profits endowment assurance policy to a male life aged 40 exact. The sum assured of £100,000 plus declared reversionary bonuses are payable on survival to the end of the term or immediately on death, if earlier.

Calculate the monthly premium payable in advance throughout the term of the policy if the company assumes that future reversionary bonuses will be declared at a rate of 1.92308% of the sum assured, compounded and vesting at the end of each policy year.

Basis:

Interest	6% per annum
Mortality	AM92 Select
Initial commission	87.5% of the total annual premium
Initial expenses	£175 paid at policy commencement date
Renewal commission	2.5% of each monthly premium from the start of the second policy year
Renewal expenses	£65 at the start of the second and subsequent policy years
Claim expense	2.5% of the claim amount

[10]

- 12** (i) By considering a term assurance policy as a series of one year deferred term assurance policies, show that:

$$\bar{A}_{x:n}^1 = \frac{i}{\delta} A_{x:n}^1 \quad [5]$$

- (ii) Calculate the expected present value and variance of the present value of a term assurance of 1 payable immediately on death for a life aged 40 exact, if death occurs within 30 years.

Basis:

Interest 4% per annum

Mortality AM92 Select

Expenses: None

[6]

[Total 11]

- 13** A life insurance company issues a 4-year unit-linked endowment assurance contract to a male life aged 40 exact under which level premiums of £1,000 per annum are payable in advance. In the first year, 50% of the premium is allocated to units and 102.5% in the second and subsequent years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.5% of the bid value of the units is deducted at the end of each year.

If the policyholder dies during the term of the policy, the death benefit of £4,000 or the bid value of the units after the deduction of the management charge, whichever is higher, is payable at the end of the year of death. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the year of exit.

The company uses the following assumptions in its profit test of this contract:

Rate of growth on assets in the unit fund	6% per annum
Rate of interest on non-unit fund cashflows	4% per annum
Independent rates of mortality	AM92 Select
Independent rate of withdrawal	10% per annum in the first policy year; 5% per annum in the second and subsequent policy years.
Initial expenses	£150 plus 100% of the amount of initial commission
Renewal expenses	£50 per annum on the second and subsequent premium dates
Initial commission	10% of first premium
Renewal commission	2.5% of the second and subsequent years' premiums
Risk discount rate	8% per annum

- (i) Calculate the profit margin on the assumption that the office does not zeroise future negative cashflows and that decrements are uniformly distributed over the year. [13]
 - (ii) Suppose the office does zeroise future negative cashflows.
 - (a) Calculate the expected provisions that must be set up at the end of each year, per policy in force at the start of each year.
 - (b) Calculate the profit margin allowing for the cost of setting up these provisions. [4]
- [Total 17]

14 (i) Write down in the form of symbols, and also explain in words, the expressions “death strain at risk”, “expected death strain” and “actual death strain”. [6]

(ii) A life insurance company issues the following policies:

- 15-year term assurances with a sum assured of £150,000 where the death benefit is payable at the end of the year of death
- 15-year pure endowment assurances with a sum assured of £75,000
- 5-year single premium temporary immediate annuities with an annual benefit payable in arrear of £25,000

On 1 January 2002, the company sold 5,000 term assurance policies and 2,000 pure endowment policies to male lives aged 45 exact and 1,000 temporary immediate annuity policies to male lives aged 55 exact. For the term assurance and pure endowment policies, premiums are payable annually in advance. During the first two years, there were fifteen actual deaths from the term assurance policies written and five actual deaths from each of the other two types of policy written.

- (a) Calculate the death strain at risk for each type of policy during 2004.
- (b) During 2004, there were eight actual deaths from the term assurance policies written and one actual death from each of the other two types of policy written. Calculate the total mortality profit or loss to the office in the year 2004.

Basis:

Interest 4% per annum

Mortality AM92 Ultimate for term assurances and pure endowments
PMA92C20 for annuities

[13]

[Total 19]

END OF PAPER

EXAMINATION

7 September 2005 (pm)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

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5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

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In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 Describe what is meant by adverse selection in the context of a life insurance company's underwriting process and give an example. [2]

2 Describe how occupation affects morbidity and mortality. [3]

3 A graph of $f_0(t)$, the probability density function for the random future lifetime, T_0 , is plotted on the vertical axis, with t plotted on the horizontal axis, for data taken from the English Life Table No. 15 (Males).

You are given that $f_0(t) = {}_t p_0 \mu_t$. You observe that the graph rises to a peak at around $t = 80$ and then falls.

Explain why the graph falls at around $t = 80$. [3]

4 Calculate the value of ${}_{1.75} p_{45.5}$ on the basis of mortality of AM92 Ultimate and assuming that deaths are uniformly distributed between integral ages. [3]

5 A population is subject to a constant force of mortality of 0.015.

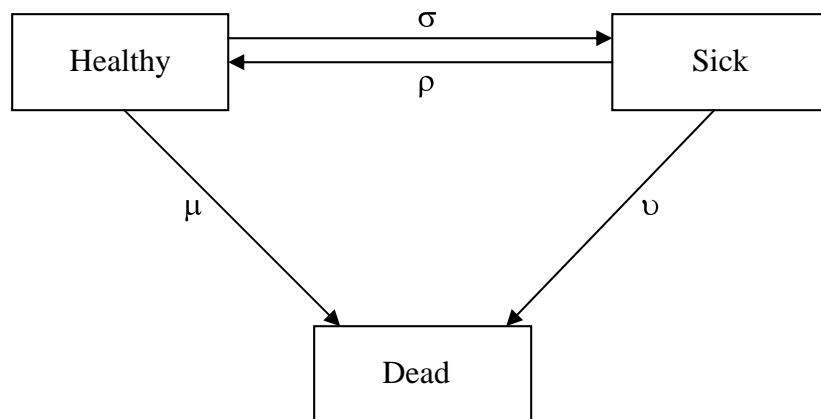
Calculate:

- (a) The probability that a life aged 20 exact will die before age 21.25 exact.
- (b) The curtate expectation of a life aged 20 exact.

[4]

6 Define $\ddot{a}_{60:50:20}^{(12)}$ fully in words and calculate its value using PMA92C20 and PFA92C20 tables for the two lives respectively at 4% interest. [5]

- 7 A life insurance company prices its long-term sickness policies using the following three-state continuous-time Markov model, in which the forces of transition σ , ρ , μ and ν are assumed to be constant:



The company issues a particular long-term sickness policy with a benefit of £10,000 per annum payable continuously while sick, provided that the life has been sick continuously for at least one year. Benefit payments under this policy cease at age 65 exact.

Write down an expression for the expected present value of the sickness benefit for a healthy life aged 20 exact. Define the symbols that you use.

[5]

- 8 A life insurance company issues an annuity contract to a man aged 65 exact and his wife aged 62 exact. Under the contract, an annuity of £20,000 per annum is guaranteed payable for a period of 5 years and thereafter during the lifetime of the man. On the man's death, an annuity of £10,000 per annum is payable to his wife, if she is then alive. This annuity commences on the monthly payment date next following, or coincident with, the date of his death or from the 5th policy anniversary, if later and is payable for the lifetime of his wife. Annuities are payable monthly in advance.

Calculate the single premium required for the contract.

Basis:

Mortality	PMA92C20 for the male and PFA92C20 for the female
Interest	4% per annum
Expenses	none

[9]

9 A life insurance company issues an annuity policy to two lives each aged 60 exact in return for a single premium. Under the policy, an annuity of £10,000 per annum is payable annually in advance while at least one of the lives is alive.

- (i) Write down an expression for the net future loss random variable at the outset for this policy. [2]

- (ii) Calculate the single premium, using the equivalence principle.

Basis:

Mortality	PMA92C20 for the first life, PFA92C20 for the second life
Interest	4% per annum
Expenses	ignored

[3]

- (iii) Calculate the standard deviation of the net future loss random variable at the outset for this policy, using the basis in part (ii).

You are given that $\ddot{a}_{\overline{60}:60} = 11.957$ at a rate of interest 8.16% per annum. [4]

[Total 9]

10 A life insurance company issued a with profits whole life policy to a life aged 20 exact, on 1 July 2002. Under the policy, the basic sum assured of £100,000 and attaching bonuses are payable immediately on death. The company declares simple reversionary bonuses at the start of each year. Level premiums are payable annually in advance under the policy.

- (i) Give an expression for the gross future loss random variable under the policy at the outset. Define symbols where necessary. [3]

- (ii) Calculate the annual premium, using the equivalence principle.

Basis:

Mortality	AM92 Select	
Interest	6% per annum	
Bonus loading	3% simple per annum	
Expenses	Initial	£200
Renewal	5% of each premium payable in the second and subsequent years	

Assume bonus entitlement earned immediately on payment of premium.

[4]

- (iii) On 30 June 2005 the policy is still in force. A total of £10,000 has been declared as a simple bonus to date on the policy.

The company calculates provisions for the policy using a gross premium prospective basis, with the following assumptions:

Mortality	AM92 Ultimate
Interest	4%
Bonus loading	4% per annum simple
Renewal expenses	5% of each premium

Calculate the provision for the policy as at 30 June 2005.

[4]

[Total 11]

- 11** A life insurance company issues a three-year unit-linked endowment assurance contract to a male life aged 62 exact under which level annual premiums of £10,000 are payable in advance throughout the term of the policy or until earlier death. 85% of each year's premium is invested in units at the offer price.

There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

There is an annual management charge of 1.25% of the bid value of units. Management charges are deducted at the end of each year, before death or maturity benefits are paid.

On the death of the policyholder during the term of the policy, there is a benefit payable at the end of the year of death of £20,000, or the bid value of the units allocated to the policy, if greater. On maturity, 115% of the full bid value of the units is payable.

The company holds unit provisions equal to the full bid value of the units. It sets up non-unit provisions to zeroise any negative non-unit fund cashflows, other than those occurring in the first year.

The life insurance company uses the following assumptions in carrying out profit tests of this contract:

Mortality	AM92 Ultimate
Expenses	Initial £600 Renewal £100 at the start of each of the second and third policy years
Unit fund growth rate	8% per annum
Non-unit fund interest rate	4% per annum
Non-unit fund provision basis	AM92 Ultimate mortality, interest 4% per annum
Risk discount rate	15% per annum

Calculate the profit margin on the contract.

[14]

- 12** On 1 January 2000, a life insurance company issued joint life whole life assurance policies to couples. Each couple comprised one male and one female life and both were aged 50 exact on 1 January 2000. Under each policy, a sum assured of £200,000 is payable immediately on the death of the second of the lives to die.

Premiums under each policy are payable annually in advance while at least one of the lives is alive.

- (i) Calculate the annual premium payable under each policy.

Basis:

Mortality	PMA92C20 for the male PFA92C20 for the female	
Interest	4% per annum	
Expenses	Initial	£1,000
	Renewal	5% of each premium payment

[5]

- (ii) On 1 January 2004, 5,000 of these policies were still in force. Under 100 of these policies only the female life was alive. Both lives were alive under the other 4,900 policies.

The company calculates provisions for the policies on a net premium basis, using PMA92C20 and PFA92C20 mortality for the male and female lives respectively and 4% per annum interest.

During the calendar year 2004, there was one claim for death benefit, in respect of a policy where the female life only was alive at the start of the year. In addition, one male life died during the year under a policy where both lives were alive at the start of the year. 4,999 of the policies were in force at the end of the year.

Calculate the mortality profit or loss for the group of 5,000 policies for the calendar year 2004.

[9]

[Total 14]

- 13** Under the rules of a pension scheme, a member may retire due to age at any age from exact age 60 to exact age 65.

On age retirement, the scheme provides a pension of $1/60^{\text{th}}$ of Final Pensionable Salary for each year of scheme service, subject to a maximum of $40/60^{\text{ths}}$ of Final Pensionable Salary. Only complete years of service are taken into account.

Final Pensionable Salary is defined as the average salary over the three-year period before the date of retirement.

The pension scheme also provides a lump sum benefit of four times Pensionable Salary on death before retirement. The benefit is payable immediately on death and Pensionable Salary is defined as the annual rate of salary at the date of death.

You are given the following data in respect of a member:

Date of birth	1 January 1979
Date of joining the scheme	1 January 2000
Annual rate of salary at 1 January 2005	£50,000
Date of last salary increase	1 April 2004

- (i) Derive commutation functions to value the past service and future service pension liability on age retirement for this member as at 1 January 2005. State any assumptions that you make and define all the symbols that you use.

[12]

- (ii) Derive commutation functions to value the liability in respect of the lump sum payable on death before retirement for this member as at 1 January 2005. State any assumptions that you make and define all the symbols that you use.

[6]

[Total 18]

END OF PAPER

EXAMINATION

5 April 2006 (pm)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

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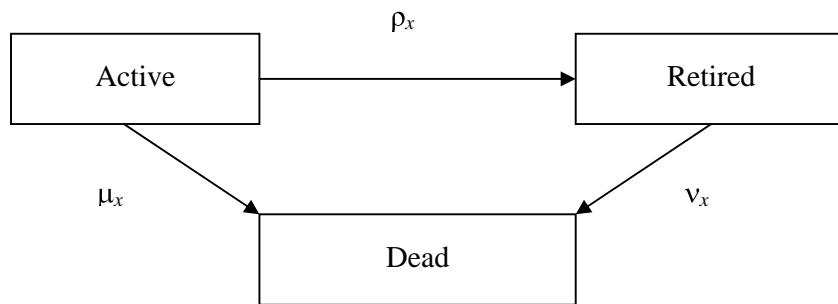
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In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

- 1** It is possible to model the mortality of current active members of a pension scheme using the following three-state continuous-time Markov model, with age-dependent forces of transition ρ_x , μ_x and v_x :



A pension scheme provides a benefit of £10,000 payable on death regardless of whether death occurs before or after retirement. Give an expression to value this benefit for an active life currently aged x . [2]

- 2**
- (i) In the context of with-profit policies, describe the super compound method of adding bonuses. [2]
 - (ii) Suggest a reason why a life insurance company might use the super compound method of adding bonuses as opposed to the compound method. [1]
- [Total 3]

- 3** Using the PMA92C20 table for both lives calculate:

(a) $\mu_{65:60}$

(b) ${}_5P_{65:60}$

(c) ${}_2q_{65:65}^1$

[4]

- 4** State the main difference between an overhead expense and a direct expense incurred in writing a life insurance policy and give an example of each. [4]

- 5** A life office issues term assurance policies to 500 lives all aged 30 exact with a term of 25 years. The benefit of £10,000 is payable at the end of the year of death of any of the lives into a special fund. Calculate the expected share of this fund for each survivor after 25 years.

Basis:

Mortality AM92 Select
Interest 4% per annum

[4]

- 6** A life office has issued for a number of years whole-life regular premium policies to a group of lives through direct advertising. Assured lives are only required to complete an application form with no further evidence of health. Outline the forms of selection that the insurer should expect to find in the mortality experience of the lives.

[5]

- 7** (i) Show that:

$$\frac{\partial}{\partial t} {}_s p_{x+t} = {}_s p_{x+t} (\mu_{x+t} - \mu_{x+t+s}) \quad [2]$$

- (ii) Prove Thiele's differential equation for a whole-life assurance issued to a life aged x to be as follows:

$$\frac{\partial}{\partial t} {}_t \bar{V}_x = -(1 - {}_t \bar{V}_x) \mu_{x+t} + \delta_t \bar{V}_x + \bar{P}_x \quad [4]$$

[Total 6]

- 8** (i) Calculate the expected present value of an annuity-due of 1 per annum payable annually in advance until the death of the last survivor of two lives using the following basis:

First life: male aged 70, mortality table PMA92C20

Second life: female aged 67, mortality table PFA92C20

Rate of interest: 4% per annum

[2]

- (ii) Give an expression for the variance of the annuity-due in terms of annuity functions. [5]

[Total 7]

- 9** (i) Express fully in words:

$$\bar{a}_{\overline{xy:n}}$$

[3]

- (ii) Express $\bar{a}_{\overline{xy:n}}$ as the expected value of random variables and hence show that

$$\bar{a}_{\overline{xy:n}} = \frac{1 - \bar{A}_{\overline{xy:n}}}{\delta} \quad [4]$$

[Total 7]

- 10** A 20-year special endowment assurance policy is issued to a group of lives aged 45 exact. Each policy provides a sum assured of £10,000 payable at the end of the year of death or £20,000 payable if the life survives until the maturity date. Premiums on the policy are payable annually in advance for 15 years or until earlier death.

You are given the following information:

Number of deaths during the 13 th policy year	4
Number of policies in force at the end of the 13 th policy year	195

- (i) Calculate the profit or loss arising from mortality in the 13th policy year. [7]
 (ii) Comment on your results. [2]

Basis:

Mortality	AM92 Ultimate
Interest	4% per annum
Expenses	none

[Total 9]

- 11** An employer wishes to introduce a lump-sum retirement benefit payable immediately on retirement at 65 or earlier other than on the grounds of ill-health. The amount of the benefit is £1,000 for each year of an employee's service, with proportionate parts of a year counting.

- (i) Give a formula to value this benefit for an employee currently aged x with n years of past service, defining all terms used. [5]
 (ii) Using the Pension Scheme Tables from the Actuarial Formulae and Tables, calculate the value for an employee currently aged 30 exact with exactly 10 years past service. [2]
 (iii) Calculate the level annual contribution payable continuously throughout this employee's service to fund the future retirement benefit. [3]

[Total 10]

12 (i) Define the following terms without giving detailed formulae:

- (a) Crude Mortality Rate
- (b) Directly Standardised Mortality Rate
- (c) Indirectly Standardised Mortality Rate

[3]

(ii) The data in the following table are taken from data published by the Office of National Statistics in 2001.

	England and Wales		Tyne and Wear	
	<i>Population</i>	<i>Number of births</i>	<i>Population</i>	<i>Number of births</i>
<i>Under 25</i>	3,149,000	153,000	71,000	4,000
<i>25–35</i>	3,769,000	339,000	74,000	6,000
<i>35+</i>	3,927,000	103,000	82,000	1,000

- (a) Using the population for England and Wales as the standard population calculate crude birth rates and the directly and indirectly standardised birth rates for Tyne and Wear.
- (b) State an advantage of using the Indirectly Standardised Birth Rate and comment briefly on the answers you have obtained.

[8]

[Total 11]

- 13** A life aged 35 exact purchases a 30-year with-profit endowment assurance policy. Level premiums are payable monthly in advance throughout the duration of the contract. The sum assured of £250,000 plus declared reversionary bonuses are payable at maturity or at the end of the year of death if earlier.
- (i) Show that the monthly premium is £647.47 if the life insurance company assumes that future simple reversionary bonuses will be declared at the rate of 2% per annum and vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

Basis:

mortality	AM92 Select
interest	4% per annum
initial expenses	£250 plus 50% of the gross annual premium
renewal expenses	3% of the second and subsequent monthly premiums
claims expenses	£300 on death; £150 on maturity

[7]

- (ii) At age 60 exact, immediately before the premium then due, the life wishes to surrender the policy. The life insurance company calculates a surrender value equal to the gross retrospective policy value, assuming the same basis as in (i) above.

Calculate the surrender value using the retrospective policy value at the end of the 25th policy year immediately before the premium then due and just after the declared bonus has increased the sum assured plus reversionary bonuses to £375,000. Assume that the life insurance company has declared a simple bonus throughout the duration of the policy consistent with the bonus loading assumption used to derive the premium in (i) above. [6]

- (iii) State with a reason whether the surrender value would have been larger, the same or smaller than in (ii) above if the office had used the prospective gross premium policy value, on the same basis. [1]

[Total 14]

- 14** A life insurance company issues a 3-year unit linked endowment policy to a life aged 45 exact under which level premiums are payable yearly in advance. In the 1st year, 35% of the premium is allocated to units and 105% in the 2nd and 3rd years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.5% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death and surrender benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of the bid value of the units is payable at the end of the year of death. The policyholder may surrender the policy only at the end of each year. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the year of exit.

The company uses the following assumptions in its profit test of this contract:

Rate of growth on assets in the unit fund	5% per annum
Rate of interest on non-unit fund cash flows	4% per annum
Independent rates of mortality	AM92 Ultimate
Independent rates of withdrawal	5% per annum
Initial expenses	£250
Renewal expenses	£50 per annum on the 2nd and 3rd premium dates
Initial commission	20% of 1st premium
Renewal commission	2.5% of the 2nd and 3rd years' premiums

The company sets premiums so that the net present value of the profit on the policy is 15% of the annual premium.

- (i) Using a risk discount rate of 8% per annum, calculate the premium for the policy on the assumption that the company does not zeroise future expected negative cash flows. [12]
 - (ii) Explain why the company might need to zeroise future expected negative cash flows on the policy. [2]
- [Total 14]

END OF PAPER

EXAMINATION

12 September 2006 (pm)

Subject CT5 — Contingencies Core Technical

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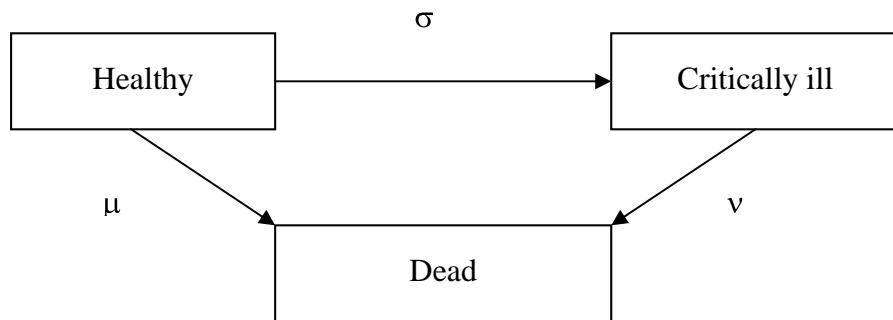
In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

- 1** In a certain country, pension funds always provide pensions to retiring employees. At the point of retirement, the fund can choose to buy an annuity from a life insurance company, or pay the pension directly themselves on an ongoing basis.

A mortality study of pensioners has established that the experience of those whose pension is received through annuities paid by insurance companies is lighter than the experience of those being paid directly by pension funds.

Explain why the mortality experiences of the two groups differ. Your answer should include reference to some form of selection. [4]

- 2** A life insurance company uses the following three-state continuous-time Markov model, with constant forces of transition, to price its stand-alone critical illness policies:



Under these policies, a lump sum benefit is payable on the occasion that a life becomes critically ill during a specified policy term. No other benefits are payable.

A 20-year policy with sum assured £200,000 is issued to a healthy life aged 40 exact.

- (i) Write down a formula, in integral terms, for the expected present value of benefits under this policy. [2]
- (ii) Calculate the expected present value at outset for this policy.

Basis: μ : 0.01
 σ : 0.02
 v : 3μ
 Interest: 8% per annum

[3]

[Total 5]

- 3** Calculate the exact value of $\bar{A}_{70:\overline{1}}^1$ assuming the force of mortality is constant between consecutive integer ages.

Basis: Mortality: ELT15 (Males)
 Interest: 7.5% per annum

[5]

- 4** A life insurance company issues a reversionary annuity contract. Under the contract an annuity of £20,000 per annum is payable monthly for life, to a female life now aged 60 exact, on the death of a male life now aged 65 exact. Annuity payments are always on monthly anniversaries of the date of issue of the contract.

Premiums are to be paid monthly until the annuity commences or the risk ceases.

Calculate the monthly premium required for the contract.

Basis: Mortality: PFA92C20 for the female
PMA92C20 for the male
Interest: 4% per annum
Expenses: 5% of each premium payment
1.5% of each annuity payment

[6]

- 5** T_x and T_y are the complete future lifetimes of two lives aged x and y respectively:

Let the random variable $g(T)$ take the following values

$$g(T) = \begin{cases} \bar{a}_{\overline{T_x]} & \text{if } T_x \leq T_y \\ \bar{a}_{\overline{T_y]} & \text{if } T_x > T_y \end{cases}$$

(i) Describe the benefit which has present value equal to $g(T)$. [2]

(ii) Express $E[g(T)]$ as an integral. [2]

(iii) Write down an expression for the variance of $g(T)$ using assurance functions. [2]

[Total 6]

- 6** A member of a pension scheme is aged 55 exact, and joined the scheme at age 35 exact. She earned a salary of £40,000 in the 12 months preceding the scheme valuation date.

The scheme provides a pension on retirement for any reason of $1/80^{\text{th}}$ of final pensionable salary for each year of service, with fractions counting proportionately. Final pensionable salary is defined as the average salary over the three years prior to retirement.

Using the functions and symbols defined in, and assumptions underlying, the Example Pension Scheme Table in the Actuarial Tables:

- (i) Calculate the expected present value now of this member's total pension. [4]
 - (ii) Calculate the contribution rate required, as a percentage of salary, to fund the future service element of the pension. [2]
- [Total 6]

- 7** The following data relate to a certain country and its biggest province:

Age-group	Country		Province
	Population	Deaths	Population
0–19	2,900,000	580	800,000
20–44	3,500,000	2,450	1,000,000
45–69	2,900,000	20,300	900,000
70 and over	700,000	49,000	300,000
Total	10,000,000	72,330	3,000,000

The population figures are from a mid-year census along with the deaths that occurred in that year.

There were 25,344 deaths in the province in total.

Calculate the Area Comparability Factor and a standardised mortality rate for the province. [6]

- 8** A pure endowment policy for a term of n years payable by single premium is issued to lives aged x at entry.

- (i) Derive Thiele's differential equation for \overline{V}_t , the reserve for this policy at time t ($0 < t < n$). [5]
 - (ii) Explain the effect of each term in your answer in (i). [2]
 - (iii) State the boundary condition needed to solve the equation in (i). [2]
- [Total 9]

- 9** A life insurance company issues a 3-year unit-linked endowment assurance contract to a female life aged 60 exact under which level premiums of £5,000 per annum are payable in advance. In the first year, 85% of the premium is allocated to units and 104% in the second and third years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.75% of the bid value of the units is deducted at the end of each year.

If the policyholder dies during the term of the policy, the death benefit of £20,000 or the bid value of the units after the deduction of the management charge, whichever is higher, is payable at the end of the year of death. On survival to the end of the term, the bid value of the units is payable.

The company holds unit reserves equal to the full bid value of the units but does not set up non-unit reserves.

It uses the following assumptions in carrying out profit tests of this contract:

Mortality:	AM92 Ultimate
Surrenders:	None
Expenses:	Initial: £600 Renewal: £100 at the start of each of the second and third policy years
Unit fund growth rate:	6% per annum
Non-unit fund interest rate:	4% per annum
Risk discount rate:	10% per annum

- (i) Calculate the expected net present value of the profit on this contract. [10]
- (ii) State, with a reason, what the effect would be on the profit if the insurance company did hold non-unit reserves to zeroise negative cashflows, assuming it used a discount rate of 4% per annum for calculating those reserves. (You do not need to perform any further calculations.) [2]
- [Total 12]

- 10** A life insurance company is reviewing the 2005 mortality experience of its portfolio of whole life assurances.

You are given the following information:

<i>Age exact on 1 Jan 2005</i>	<i>Sum assured in force on 1 Jan 2005</i>	<i>Reserves at 31 Dec 2005 of policies in force on 31 Dec 2005</i>
	£	£
69	500,000	175,000
70	400,000	150,000

There were 2 death claims during 2005 arising from these policies as follows:

<i>Date of issue of policy</i>	<i>Age exact at issue of policy</i>	<i>Sum assured £</i>
1 Jan 1980	45	12,000
1 Jan 1982	46	10,000

All premiums are payable annually on 1st January throughout life.

Sums assured are payable at the end of the year of death.

Net premium reserves are held, based on mortality of AM92 Ultimate and interest of 4% per annum.

- (i) Calculate the mortality profit or loss for 2005 in respect of this group of policies. [8]
- (ii) (a) Calculate the amount of expected death claims for 2005 and compare it with the amount of actual claims.
- (b) Suggest a reason for this result compared with that obtained in (i). [4]
- [Total 12]

- 11** A life insurance company issues identical deferred annuities to each of 100 women aged 63 exact. The benefit is £5,000 per annum payable continuously from a woman's 65th birthday, if still alive at that time, and for life thereafter.
- (i) Write down an expression for the random variable for the present value of future benefits for one policy at outset. [3]
 - (ii) Calculate the total expected present value at outset of these annuities.
 Basis: Mortality: PFA92C20
 Interest: 4% per annum [2]
 - (iii) Calculate the total variance of the present value at outset of these annuities, using the same basis as in part (ii). [8]
 [Total 13]
- 12** A life insurance company issues a 10-year decreasing term assurance to a man aged 50 exact. The death benefit is £100,000 in the first year, £90,000 in the 2nd year, and decreases by £10,000 each year so that the benefit in the 10th year is £10,000. The death benefit is payable at the end of the year of death.
- Level premiums are payable annually in advance for the term of the policy, ceasing on earlier death.
- (i) Calculate the annual premium.
 Basis:
 Interest: 6% per annum
 Mortality: AM92 Select
 Initial expenses: £200 and 25% of the total annual premium (all incurred on policy commencement)
 Renewal expenses: 2% of each premium from the start of the 2nd policy year and £50 per annum, inflating at 1.923% per annum, at the start of the second and subsequent policy years
 Claim expenses: £200 inflating at 1.923% per annum
 Inflation: For renewal and claim expenses, the amounts quoted are at outset, and the increases due to inflation start immediately. [8]
 - (ii) Write down an expression for the gross future loss random variable at the end of the ninth year, using whatever elements of the basis in (i) that are relevant. [3]
 - (iii) Calculate the gross premium reserve at the end of the ninth year, using the premium basis. [3]
 - (iv) Comment on any unusual aspect of your answer. [2]
 [Total 16]

END OF PAPER

EXAMINATION

17 April 2007 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 Calculate

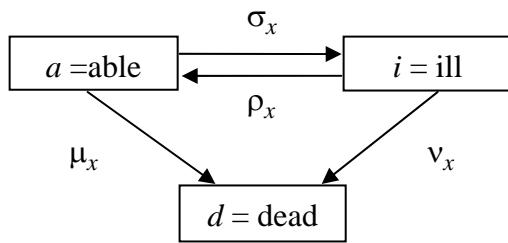
- (i) ${}_{5|10}q_{[52]}$
- (ii) $p_{[50]:[60]}$ for two independent lives

Basis:

Mortality: AM92 Select [3]

2 State, with examples, three distinct types of selection in the membership of a pension scheme. [3]

3 A three-state transition model is shown in the following diagram:



Assume that the transition probabilities are constant at all ages with $\sigma = 2\%$, $\nu = 6\%$, $\rho = 1\%$ and $\mu = 3\%$.

An able life age 55 exact takes out a 10-year sickness contract that provides a “no-claim” bonus of £100 if the insured remains able for the full duration of the contract. Calculate the expected present value of the bonus at the beginning of the contract with a force of interest of 0.04. [4]

4 (i) In the context of net premiums and reserves, state the conditions necessary for equality of prospective and retrospective reserves. [2]

(ii) Give two reasons why, in practice, these conditions may not hold. [2]
[Total 4]

- 5** An assurance contract provides a death benefit of £1,000 payable immediately on death, with a savings benefit of £500 payable on every fifth anniversary of the inception of the policy.

The following basis is used:

Force of mortality: $\mu_x = 0.05$ for all x

Force of interest: $\delta = 0.04$

Expenses: None

Calculate the level premium payable annually in advance for life. [5]

- 6** A pension scheme provides a benefit on death in service of 4 times the member's salary at the date of death. Normal Pension Age is 65. State a formula, without using commutation functions, for the present value of this benefit to a life aged 35 exact with salary of £25,000 who has just received a salary increase. Define all symbols used. [5]

- 7** A term assurance contract for a life aged 50 exact for a term of 10 years provides a benefit of £10,000 payable at the end of the year of death. Calculate the expected present value and variance of benefits payable under this contract.

Basis:

Mortality: AM92 Select

Interest: 4% per annum

[6]

- 8** You are given the following statistics in relation to the mortality experience of Actuaria and its province Giro:

Age	Actuaria		Giro	
	Exposed to risk	Number of deaths	Exposed to risk	Number of deaths
0–19	300,000	25	12,000	2
20–39	275,000	35	10,000	3
40–59	200,000	100	9,000	6
60–79	175,000	500	8,000	50

(i) Explain, giving a formula, the term Standardised Mortality Ratio (SMR). Define all the symbols that you use. [2]

(ii) Comment on the relative mortality of the province, by calculating the SMR for Giro. [4]

[Total 6]

- 9** A life insurance company issues an annuity to a life aged 60 exact to provide an annual income of £15,000. The annuity is payable monthly in advance and is guaranteed to be paid for a period of 5 years and for the whole of life thereafter. On the annuitant's death a survivor's pension is paid at the rate £7,500 per annum for the remainder of life for the spouse of the annuitant who is currently aged 55 exact under the following circumstances:

- (a) If the life dies within the guarantee period then the survivor's pension commences with the first payment immediately after the end of the guarantee period.
- (b) If the life dies after the guarantee period has expired then the survivor's pension commences with the first payment immediately after the death of the first life.

Calculate the single premium:

Basis:

Annuitant mortality: PMA92C20
 Spouse mortality: PFA92C20
 Interest: 4% per annum

[6]

- 10** Let X be a random variable representing the present value of the benefits of a whole of life assurance, and Y be a random variable representing the present value of the benefits of a temporary assurance with a term of n years. Both assurances have a sum assured of 1 payable at the end of the year of death and were issued to the same life aged x .
- (i) Describe the benefits provided by the contract which has a present value represented by the random variable $X - Y$. [1]
 - (ii) Show that

$$\text{Cov}[X, Y] = {}^2A_{x:n}^1 - A_x * A_{x:n}^1$$

and hence or otherwise that

$$\text{Var}(X - Y) = {}^2A_x - ({}_n|A_x)^2 - {}^2A_{x:n}^1$$

where the functions A are determined using an interest rate of i , and functions 2A are determined using an interest rate of $i^2 + 2i$. [7]
 [Total 8]

- 11** A five-year unit-linked policy issued to a life aged 50 exact has the following pattern of end of year cashflows per policy in force at the start of each year:

(-95.21, -30.18, -20.15, 77.15, 120.29)

- (i) Explain why a life office might need to set up non-unit reserves in respect of a unit-linked life assurance policy. [2]
- (ii) Calculate the non-unit reserves required for the policy in order to zeroise negative cashflows assuming AM92 Ultimate mortality and that reserves earn interest at the rate of 5% per annum. [2]
- (iii) Determine the net present value of the profits before and after zeroisation assuming the risk discount rate used is 8% per annum and state with reasons which of these figures you would expect to be higher. [6]

[Total 10]

- 12** A life office issued 750 identical 25-year temporary assurance policies to lives aged 30 exact each with a sum assured of £75,000 payable at the end of year of death. Premiums are payable annually in advance for 20 years or until earlier death.

- (i) Show that the annual net premium for each policy is approximately equal to £104 using the basis given below. [2]
- (ii) Calculate the net premium reserve per policy at the start and at the end of the 20th year of the policy. [4]
- (iii) Calculate the mortality profit or loss to the life office during the 20th year if twelve policyholders die during the first nineteen years of the policies and two policyholders die during the 20th year. [4]

Basis:

Mortality: AM92 Ultimate

Interest: 4% per annum

[Total 10]

- 13** A life office issues with-profit whole of life contracts, with the sum assured payable immediately on death of the life assured. Level premiums are payable monthly in advance to age 65 or until earlier death.

The life office markets two versions of this policy, one assumed to provide simple bonuses of 4% per annum of the sum assured vesting at the end of each policy year and the other assumed to provide compound bonuses of 4% of the sum assured, again vesting at the end of each policy year. The death benefit under each version does not include any bonus relating to the policy year of death.

The following basis is assumed to price these contracts:

Mortality	AM92 Select
Interest	4% per annum
Initial expenses	£300
Renewal expenses	2.5% of the second and subsequent monthly premiums
Initial commission	50% of the gross annual premium
Renewal commission	2.5% of the second and subsequent monthly premiums
Claims expenses	£250 at termination of the contract

Calculate the level monthly premium required for each version of this policy issued to a life aged 30 exact at outset for an initial sum assured of £50,000. [12]

- 14** A life office issues a 4-year non profit endowment assurance policy to a male life aged 61 exact for a sum assured of £100,000 payable on survival to the end of the term or at the end of the year of death if earlier. Premiums are payable annually in advance throughout the term of the policy.

There is a surrender benefit payable equal to a return of premiums paid, with no interest. This benefit is payable at the end of the year of surrender.

The life office uses the following assumptions to price this contract:

Mortality	AM92 Select
Surrenders	None
Interest	4% per annum
Initial expenses	£500
Renewal expenses (on the second and subsequent premium dates)	£50 per annum plus 2.5% of the premium

In addition, the company holds net premium reserves, calculated using AM92 Ultimate mortality and interest of 4% per annum.

In order to profit test this contract, the life office assumes the same mortality and expense assumptions as per the pricing basis above. In addition, it assumes it earns 5% per annum on funds and that 5% of all policies still in force at the end of 1, 2, and 3 years then surrender.

Calculate, using a risk discount rate of 8% per annum, the expected profit margin on this contract. [18]

END OF PAPER

EXAMINATION

28 September 2007 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

- 1** Calculate ${}_{t+1}V_x$ given the following:

$$\begin{aligned}P_x &= 0.017 \\{}_tV_x &= 0.468 \\i &= 0.03 \\q_{x+t} &= 0.024\end{aligned}$$

[2]

- 2** In a special mortality table with a select period of one year, the following relationships are true for all ages:

$$\begin{aligned}0.5 q_{[x]} &= (0.33)q_x \\0.5 q_{[x]+0.5} &= (0.5)q_x\end{aligned}$$

Express $p_{[x]}$ in terms of p_x . [3]

- 3** A twelve-year life insurance contract has the following profit signature before any non-unit reserves are created:

$$(+1, -1, +1, +1, +1, -1, 0, -1, +1, -1, +1, +1)$$

Non-unit reserves are to be set up to zeroise the negative cash flows.

Write down the revised profit signature, ignoring interest. [3]

- 4** An annuity makes monthly payments in arrear to a life aged 65 exact where each payment is 1.0039207 times greater than the one immediately preceding. The first monthly amount is £1,000.

Calculate the expected present value of the annuity using the following basis:

Mortality: PFA92C20

Interest: 9% per annum

[4]

- 5** (i) Write down the formula for a directly standardised mortality rate. [2]
- (ii) State the main disadvantage of this rate and outline how it is overcome in practice. [2]

[Total 4]

- 6** For a certain group of pensioners, $q_{75} = 0.05$ and $q_{76} = 0.06$.
- Calculate the probability that a pensioner aged 75 exact will die between ages 75.5 and 76.5 assuming:
- (a) a uniform distribution of deaths between consecutive birthdays
(b) a constant force of mortality between consecutive birthdays. [5]

- 7** A life insurance company sells two whole life contracts to lives aged 40 exact at entry. Level monthly premiums are payable in advance until the death of the life assured. Death benefits are paid at the end of the year of death.

Under policy A, the sum assured is £100,000 during the first year and it increases by £5,000 at the end of each year for surviving policyholders.

Policy B is a with profit policy with initial sum assured of £100,000. The company intends to declare simple annual reversionary bonuses of 5% of the original sum assured each year, vesting at the end of each policy year.

After ten years, the total declared bonuses under the with profit policy amount to £50,000.

Calculate the net premium reserve required for each policy after ten years.

Basis:

Mortality: AM92 Select
Interest: 4% per annum [6]

- 8** Explain the following terms and give an example of each:
- (a) class selection
(b) spurious selection
(c) time selection. [6]

- 9** A life office issues an annuity to a woman aged 65 exact and a man aged 68 exact. The annuity of £20,000 per annum is payable annually in arrears for as long as either of the lives is alive.

The office values this benefit using the following basis:

Interest: 4% per annum
Mortality: Female: PFA92C20
Males: PMA92C20

- (i) Calculate the expected present value of this benefit. [2]
- (ii) Calculate the probability that the life office makes a profit in this case if it charges a single premium of £320,000. [4]

[Total 6]

- 10** A policy provides a benefit of £500,000 immediately on the death of (y) if she dies after (x).

- (i) Write down an expression in terms of T_x and T_y (random variables denoting the complete future lifetimes of (x) and (y) respectively) for the present value of the benefit under this policy. [2]
- (ii) Write down an expression for the expected present value of the benefit in terms of an integral. [2]
- (iii) Suggest, with a reason, the most appropriate term for regular premiums to be payable under this policy. [2]

[Total 6]

- 11** Let X be a random variable representing the present value of the benefits of a pure endowment contract and Y be a random variable representing the present value of the benefits of a term assurance contract which pays the death benefit at the end of the year of death. Both contracts have unit sum assured, a term of n years and were issued to the same life aged x .

- (i) Derive and simplify as far as possible using standard actuarial notation an expression for the covariance of X and Y . [4]
- (ii) Hence or otherwise, derive an expression for the variance of $(X+Y)$ and simplify it as far as possible using standard actuarial notation. [4]

[Total 8]

- 12** On 1 January 1992 a life insurance company issued a number of 20-year pure endowment policies to a group of lives aged 40 exact. In each case, the sum assured was £75,000 and premiums were payable annually in advance.

On 1 January 2006, 500 policies were still in force. During 2006, 3 policyholders died, and no policy lapsed for any other reason.

The office calculates net premiums and net premium reserves on the following basis:

Interest: 4% per annum
Mortality: AM92 Select

- (i) Calculate the profit or loss from mortality for this group for the year ending 31 December 2006. [7]
- (ii) Explain why the mortality profit or loss has arisen. [2]

[Total 9]

- 13** A life insurance company issues a 35-year endowment assurance contract to a life aged 30 exact. The sum assured of £200,000 is payable at maturity or at the end of the year of death if earlier. Level premiums are payable annually in advance for the duration of the contract.

- (i) Show that the annual premium is approximately £2,007, using the following basis:

Interest: 6% p.a.
Mortality: AM92 Ultimate
Expenses: Initial: £300 plus 50% of the annual premium
Renewal: 2% of the second and subsequent annual premiums
Claim: £600 on death; £200 on maturity [6]

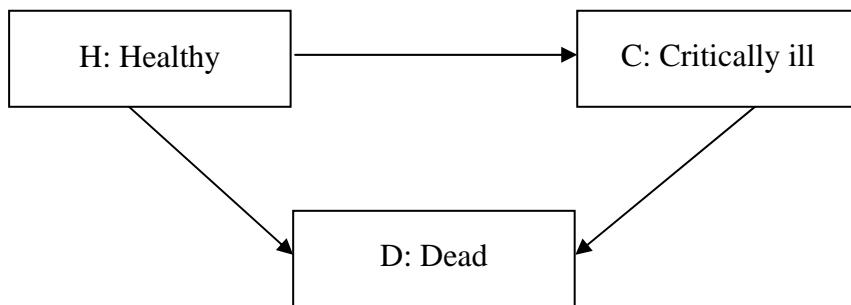
- (ii) Write down the gross premium future loss random variable after 25 years, immediately before the premium then due is paid. [3]

- (iii) Calculate the retrospective policy reserve after 25 years, using the same basis as in (i), but with 4% p.a. interest. [6]

- (iv) Explain whether the reserve in (iii) would have been smaller, the same or greater than in (iii) if the office had used the prospective gross premium reserve, on the same basis. [3]

[Total 18]

- 14** A life office uses the following three-state model to calculate premiums for a 2-year accelerated critical illness policy issued to healthy policyholders aged 63 exact at entry.



In return for a single premium payable at entry, the office will pay benefits of:

£100,000 if the policyholder dies from the healthy state;
 £60,000 if he is diagnosed as having a critical illness;
 £40,000 if he dies from the critically ill state.

All benefits are payable at the end of the relevant policy year.

Let S_t represent the state of the policyholder at age $63 + t$, so that $S_0 = H$ and for $t = 1, 2$, $S_t = H, C$ or D . The transition probabilities are defined as follows:

$$p_{63+t}^{ij} = \Pr(S_{t+1}=j \mid S_t=i).$$

Their values are:

t	p_{63+t}^{HC}	p_{63+t}^{HD}	p_{63+t}^{CD}
0	0.04	0.02	0.25
1	0.06	0.03	0.33

- (i) Identify all 6 possible outcomes under this policy. [3]
 - (ii) Calculate the net present value at entry of the benefits assuming a rate of interest of 10% per annum for each of the outcomes in (i). [3]
 - (iii) Calculate the probability that each outcome occurs. [3]
 - (iv) Calculate the mean and variance of the present value at entry of the total benefits per policy. [5]
 - (v) The office expects to sell 10,000 of these policies. The single premium is set at a level which will ensure that the probability that the office makes a profit is 0.95. Calculate the amount of the single premium, assuming the profit is normally distributed. [6]
- [Total 20]

END OF PAPER

EXAMINATION

14 April 2008 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 13 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

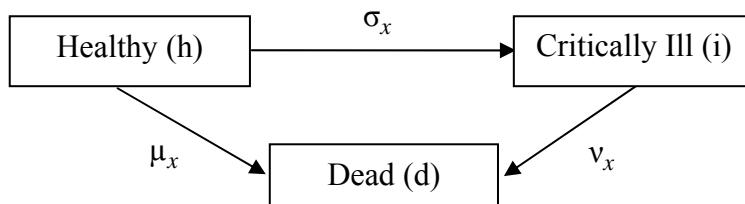
In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** (a) Express ${}_{5|10}q_{40}$ in words.
 (b) Calculate its value using AM92 mortality.
- [2]

- 2** Describe three types of reversionary bonus that may be given to a with-profits contract.
- [4]

- 3** Explain why a life insurance company will need to set up reserves for the endowment assurance contracts it has sold.
- [4]

- 4** A life insurance company sells a term assurance and critical illness policy with a 20 year term to a life aged 40 exact. The policy provides a benefit of £50,000 payable immediately on death or earlier diagnosis of critical illness. No further benefit is paid in the event of death within the term after a prior critical illness claim has been paid. The company prices the policy using the following multiple state model:



Calculate the expected present value of the benefits under the policy.

Basis: $i = 5\%$ per annum
 $\mu_x = 0.005$ at all ages
 $\nu_x = 0.006$ at all ages
 $\sigma_x = 0.003$ at all ages

[5]

- 5** A reversionary annuity is payable continuously beginning on the death of a life aged x to an annuitant aged y .
- (a) Derive an expression for the present value of the reversionary annuity using random variables for the future lifetimes.
- (b) Derive an expression for the expected present value of the reversionary annuity in terms of assurance functions.
- [5]

- 6** A parent who has just died left a bond in their will that provides a single payment of £15,000 in 10 years' time. The payment of £15,000 will be shared equally between the local cats' home and such of the parent's two sons (currently aged 25 and 30 exact) who are then still alive. Calculate the expected present value of the share due to the cats' home.

Basis: Mortality AM92 Ultimate
Interest 3% per annum

[5]

- 7** A defined benefit pension scheme provides a pension on retirement for any reason of one-sixtieth of final pensionable salary for each year of service (with proportion for part years of service). Final pensionable salary is average salary over the three years immediately preceding retirement. Calculate the cost of providing future service benefits for a new member aged 40 exact as a percentage of salary.

Basis: Example Pension Scheme Table in the Formulae and Tables for Examinations Handbook

[6]

- 8** (i) Show that

$${}_{t-s}q_{x+s} = \frac{(t-s)q_x}{(1-sq_x)}, \quad (0 \leq s < t \leq 1)$$

using an assumption of a uniform distribution of deaths. [4]

- (ii) Calculate the value of ${}_0.5q_{62.25}$ using assumptions of:

- (a) a uniform distribution of deaths
- (b) a constant force of mortality

Basis: Mortality PMA92C20

[3]

[Total 7]

- 9** A life insurance company prices annuities using a basis which incorporates the location of the proposing annuitants as an additional rating factor.
- (i) Identify three factors that influence mortality and would cause the insurance company to adopt location as a rating factor. State which form of selection is demonstrated by the use of location as a rating factor. [4]
- (ii) The company has produced the following data in respect of two locations. Calculate the standardised mortality ratio for each location based on the standard mortality table ELT15(Males).

Age	<i>Location A</i>		<i>Location B</i>	
	<i>Initial exposed to risk</i>	<i>Number of deaths</i>	<i>Initial exposed to risk</i>	<i>Number of deaths</i>
60	100	1	200	3
61	175	3	150	3
62	190	2	170	3
63	210	3	100	2

[4]
[Total 8]

- 10** A male life aged 60 exact wants to buy the following benefits within one policy:
- (a) an annuity of £5,000 per annum payable monthly in arrear to his wife currently aged 55 exact commencing on his death and for the rest of her life, and
- (b) an annuity of £2,000 per annum payable monthly in arrear to his grandson currently aged 13 exact commencing on the death of either grandparent and ceasing when the grandson reaches age 21

Calculate the overall single premium.

Basis:

Mortality Male life – PMA92C20
 Wife – PFA92C20
 Grandson – ignore

Interest 4% per annum

[10]

- 11** A life insurance company issues a 10-year with-profits endowment policy to a life then aged 50 exact. Under the policy, the basic sum assured of £75,000 and attaching bonuses are payable at maturity or immediately on death, if earlier. The company declares compound reversionary bonuses vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death). Level premiums are payable annually in advance under the policy.

- (i) Show that the annual premium, using the equivalence principle, is approximately £7,487.

Basis:

Mortality	AM92 Select
Interest	6% per annum
Bonus loading	1.92308% of the sum assured, compounded and vesting at the end of each policy year
Expenses	
Initial	£350 plus 50% of the annual premium
Renewal	5% of each premium payable in the second and subsequent years

[7]

At aged 55 exact, immediately before the premium then due and just after the declared bonus relating to the 5th policy year has been added to the policy, the policy is still in force.

- (ii) Calculate the reserve for the policy at this point in time using a gross premium prospective basis assuming the same basis as in (i) above. You should also assume that the life insurance company has declared a compound bonus throughout the duration of the policy consistent with the bonus loading assumption used to derive the premium in (i) above.

[5]

[Total 12]

12 A life assurance company issues the following policies:

- 10-year term assurances with a sum assured of £50,000 where the death benefit is payable at the end of the policy year of death
- 10-year pure endowment assurances with a sum assured of £50,000 payable on maturity

For the term assurance and pure endowment policies, premiums are paid annually in advance.

The company sold 5,000 policies of each type to lives then aged 50 exact. During the first policy year, there were five actual deaths from each of the two types of policies written.

- (i) Assuming each type of policy was sold to a distinct set of lives (i.e. no life buys more than one type of policy).
- Calculate the death strain at risk for each type of policy at the end of the second policy year of the policies.
 - During the second policy year, there were ten deaths from each of the two types of policy written. Calculate the total mortality profit or loss to the company during the second policy year.

Basis:

Interest 4% per annum
Mortality AM92 Ultimate for term assurance and pure endowment
Expenses Nil

[11]

- (ii) The company now discovers that 5,000 lives had bought one of each type of policy.

- State whether the mortality profit or loss calculated would now be higher, lower or unchanged to that calculated in (i)(b).
- State whether the variance of the benefits paid out by the company in future years would be higher, lower or unchanged to that in (i). Explain your answer by general reasoning.

[3]

[Total 14]

- 13** A life insurance company issues a 4-year unit-linked endowment policy to a life aged 50 exact under which level premiums of £750 are payable yearly in advance throughout the term of the policy or until earlier death. In the first policy year, 25% of the premium is allocated to units and 102.5% in the second and subsequent years. The units are subject to a bid-offer spread of 5% and an annual management charge of 1% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of £3,000 or the bid value of the units, whichever is higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of each policy year. On surrender, the bid value of the units is payable at the end of the policy year of exit. On maturity, 110% of the bid value of the units is payable.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	6.5% per annum
Rate of interest on non-unit fund cash flows	5.5% per annum
Mortality	AM92 Select
Initial expenses	£150
Renewal expenses	£65 per annum on the second and subsequent premium dates
Initial commission	10% of first premium
Renewal commission	2.5% of the second and subsequent years' premiums
Risk discount rate	8.5% per annum

In addition assume that at the end of each of the first 3 years, 10% of all policies still in force then surrender.

- (i) Calculate the profit margin for the policy on the assumption that the company does not zeroise future expected negative cash flows. [13]
- (ii) Suppose the company sets up reserves in order to zeroise future expected negative cash flows.
- (a) Calculate the expected reserve that must be set up at the end of each policy year, per policy in force at the start of each policy year.
- (b) Calculate the profit margin allowing for the cost of setting up these reserves.

[5]

[Total 18]

END OF PAPER

EXAMINATION

22 September 2008 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate (to the nearest integer) the lower quartile of the complete future lifetime of a person aged 25 exact who is subject to mortality according to ELT15 (Females). [3]

2 The profit signature of a 3-year assurance contract issued to a life aged 57 exact, with a premium payable at the start of each year of £500 is (-250, 150, 200).

Calculate the profit margin of the contract.

Basis:

Mortality	AM92 Ultimate
Lapses	None
Risk discount rate	12% per annum

[3]

3 In order to value the benefits in a final salary pension scheme as at 1 January 2008, a salary scale, s_x , has been defined so that $\frac{s_{x+t}}{s_x}$ is the ratio of a member's total earnings between ages $x + t$ and $x + t + 1$ to the member's total earnings between ages x and $x + 1$. Salary increases take place on 1 July every year. One member, whose date of birth is 1 April 1961, has an annual salary rate of £75,000 on the valuation date.

Write down an expression for the member's expected earnings during 2008. [3]

4 Write down an alternative expression for each of the following statements. Use notation as set out in the "International Actuarial Notation" section of the "Formulae and Tables for Examinations" where appropriate and express your answer as concisely as possible.

(i) Probability[maximum $\{T_x, T_y\} \leq n\}]$

[1]

(ii) $E[g(K_x)]$ where $g(K_x) = v^{K_x+1}$ for $K_x < n$ and 0 for $K_x \geq n$

[1]

(iii) Probability $\{n < T_x \leq m\}$

[1]

(iv) Limit $_{dt \rightarrow 0} \frac{1}{dt}$ Probability[minimum $\{T_x, T_y\} \leq t + dt | T_x > t, T_y > t\}]$

[1]

(v) $E[a_{\overline{\min(n-1, K_x)}}] + 1]$

[1]

[Total 5]

5 (i) Explain what is meant by $\ddot{s}_{x:\overline{n}}$ [2]

(ii) Calculate $\ddot{s}_{50:\overline{20}}$. [3]

Basis:

Mortality: AM92 Ultimate

Interest: 4% per annum

[Total 5]

6 A select life aged 62 exact purchases a 3-year endowment assurance with sum assured £100,000. Premiums of £30,000 are payable annually in advance throughout the term of the policy or until earlier death. The death benefit is payable at the end of the policy year of death.

Calculate the expected value of the present value of the profit or loss to the office on the contract, using the following basis:

Interest 7.5% per annum

Expenses Ignore

Mortality $q_{[x-t]+t} = \frac{1}{4-t} q_x$ for all x and for $t = 0, 1$ or 2 .

$$q_{62} = 0.018, q_{63} = 0.02 \text{ and } q_{64} = 0.022$$

[6]

7 A certain population is subject to three modes of decrement: α , β and γ .

(i) Write down an expression for $(aq)_x^\alpha$ in terms of the single decrement table probabilities q_x^α , q_x^β , and q_x^γ , assuming each of the three modes of decrement is uniformly distributed over the year of age x to $x + 1$ in the corresponding single decrement table. [2]

(ii) Suppose now that in the single decrement table α , ${}_t p_x^\alpha = 1 - t^2 q_x^\alpha$ ($0 \leq t \leq 1$), while decrements β and γ remain uniformly distributed. Derive a revised expression for $(aq)_x^\alpha$ in terms of the single decrement table probabilities q_x^α , q_x^β , and q_x^γ . [4]

[Total 6]

- 8** A life insurance company sells 1,000 whole life annuities on 1 January 2007 to policyholders aged 65 exact. Each annuity is for £25,000 payable annually in arrear. 5 annuitants die during 2007.

The office holds reserves using the following basis:

Mortality PFA92C20
Interest 4% per annum

- (i) Calculate the profit or loss from mortality for this group for the year ending 31 December 2007. [4]
- (ii) Explain why the mortality profit or loss has arisen. [2]
- [Total 6]

- 9** A new member aged 35 exact, expecting to earn £40,000 in the next 12 months, has just joined a pension scheme. The scheme provides a pension on retirement for any reason of 1/60th of final pensionable salary for each year of service, with fractions counting proportionately. Final pensionable salary is defined as the average salary over the three years prior to retirement.

Members contribute a percentage of salary, the rate depending on age. Those under age 50 contribute 4% and those age 50 exact and over contribute 5%.

The employer contributes a constant multiple of members' contributions to meet exactly the expected cost of pension benefits.

Calculate the multiple needed to meet this new member's benefits.

All elements of the valuation basis are contained in the Example Pension Scheme Table in the Formulae and Tables for Examinations. [6]

- 10** Calculate the variance of the present value of benefits under an annuity payable to a life aged 35 exact. The annuity has payments of 1 per annum payable continuously for life.

Basis:

Mortality $\mu = 0.02$ throughout
Interest $\delta = 0.05$ [7]

- 11** A life insurance company has reviewed its mortality experience. For each age, it has pooled all the deaths and corresponding exposures from its entire portfolio over the previous ten years, and derived a single mortality table.

List three types of selection which might be likely to produce heterogeneity in this particular investigation. In each case, explain the nature of the heterogeneity and how it could be caused, and state how the heterogeneity could be reduced. [9]

- 12** A life insurance company is considering selling with-profit endowment policies with a term of twenty years and initial sum assured of £100,000. Death benefits are payable at the end of the policy year of death. Bonuses will vest at the end of each policy year.

The company is considering three different bonus structures:

- (1) Simple reversionary bonuses of 4.5% per annum.
 - (2) Compound reversionary bonuses of 3.84615% per annum.
 - (3) Super compound bonuses where the original sum assured receives a bonus of 3% each year and all previous bonuses receive an additional bonus of 6% each year.
- (i) Calculate the amount payable at maturity under the three structures. [4]
- (ii) Calculate the expected value of benefits under structure (2) for an individual aged 45 exact at the start, using the following basis:
- | | |
|-----------|--------------|
| Interest | 8% per annum |
| Mortality | AM92 Select |
| Expenses | ignore |
- [4]
- (iii) Calculate the expected value of benefits, using the same policy and basis as in (ii) but reflecting the following changes:
- (a) Bonuses vest at the start of each policy year (the death benefit is payable at the end of the policy year of death).
 - (b) The death benefit is payable immediately on death (bonuses vest at the end of each policy year).
 - (c) The death benefit is payable immediately on death, and bonuses vest continuously. [3]
- [Total 11]

- 13** Two lives, a female aged 60 exact and a male aged 65 exact, purchase a policy with the following benefits:
- (i) an annuity deferred ten years, with £20,000 payable annually in advance for as long as either of them is alive
 - (ii) a lump sum of £100,000 payable at the end of the policy year of the first death, should this occur during the deferred period

Level premiums are payable monthly in advance throughout the deferred period or until earlier payment of the death benefit.

Calculate the monthly premium.

Basis:

Mortality	Female	PFA92C20
	Male	PMA92C20
Interest	4% per annum	
Expenses	Initial	£350
	Renewal	2.5% of each monthly premium excluding the first.

[14]

- 14** A life insurance company issues a decreasing term assurance policy to a life aged 55 exact. The death benefit, which is payable immediately on death, is £100,000 in the first policy year, £90,000 in the second year thereafter reducing by £10,000 each year until the benefit is £10,000 in the 10th year, with cover ceasing at age 65.

The policy is paid for by level annual premiums payable in advance for 10 years, ceasing on earlier death.

The life office uses the following basis for calculating premiums and reserves:

Basis:

Mortality	AM92 Select
Interest	4% per annum
Expenses	Initial £300 plus 25% of the first premium
	Renewal 5% of all premiums excluding the first and £50*(1.04) ^t on each policy anniversary where t is the exact duration of the policy on the anniversary
Claim	£200*(1.04) ^u where u is the exact duration of the policy at death, measured in years with fractions counting

- (i) Write down the gross premium future loss random variable at the start of the policy. Use P for the annual premium. [4]
- (ii) Calculate the premium, using the equivalence principle. [10]
- (iii) Calculate the gross premium prospective reserve after 9 years. [2]
[Total 16]

END OF PAPER

EXAMINATION

24 April 2009 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Define and calculate ${}_5|{}_{10}q_{[40]+1}$.

Basis: AM92 Select [4]

2 Calculate the following functions:

(i) $A_{40:\overline{20}}^1$ [3]

(ii) $A_{40:\overline{20}}$ [1]

Basis: $l_x = 110 - x$ (for $x \leq 110$).

Interest 4% per annum. [Total 4]

3 Employee contributions to a pension fund are paid continuously at the rate of 4% of salary per annum after a fixed deduction from salary of £5,000 per annum paid continuously.

Determine an expression using commutation functions for the present value of the future contributions by a member aged x with salary S in the previous 12 months.

[4]

4 Explain, in the context of the lapse rates of life insurance policies, what is meant by:

- (a) class selection
- (b) temporary initial selection
- (c) time selection

Give an example in each case. [5]

5 A population is subject to two modes of decrement α and β where $q_x^\beta = \frac{1}{3} + \frac{1}{4}q_x^\alpha$.

Derive from first principles $(aq)_x^\beta$.

State clearly any assumptions you make. [5]

6 The random variable T_{xy} represents the time to failure of the joint-life status $(x:y)$. (x) is subject to a constant force of mortality of 0.02 and (y) is subject to a constant force of mortality of 0.03. (x) and (y) are independent with respect to mortality.

Calculate the value of $E[T_{xy}]$. [5]

- 7** A life insurance company issues a special annuity contract to a male life aged 70 exact and a female life aged 60 exact. Annuity payments are due on the first day of the month.

Under the contract an annuity of £50,000 per annum is payable monthly to the female life, provided that she survives at least 5 years longer than the male life. The annuity commences on the monthly policy anniversary next following the fifth anniversary of the death of the male life and is payable for the balance of the female's lifetime.

Calculate the single premium required for the contract.

Basis: Mortality: PMA92C20 for males, PFA92C20 for females

Interest: 4% per annum

Expenses: Nil

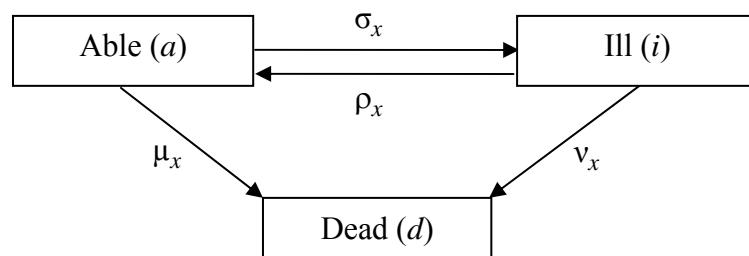
[5]

- 8**
- (i) Describe three distinct methods of averaging salary that might be defined in the scheme rules of a pension fund. [3]
 - (ii) Define s_x and z_x in the context of a pension fund. [2]
- [Total 5]

- 9** A life insurance company sells a policy with a 10 year term to a healthy life aged 55 exact. The policy provides the following benefits:

- £25,000 payable immediately on death
- £1,000 per annum payable continuously during illness

The company prices the policy using the following multiple state model:



Give a formula for the expected present value of the benefits under the policy. [5]

10 A life insurance company issues a term assurance policy for a term of 10 years to two lives whose ages are x and y , in return for the payment of a single premium. The following benefits are payable under the contract:

- In the event of either of the lives dying within 10 years, a sum assured of £100,000 is payable immediately on the first death if it is the life aged x or £50,000 if the life aged y .
- In the event of the second death within the remainder of the 10 year term, a further sum assured of twice the original claim previously paid is payable immediately on the second death.

Calculate the single premium.

Basis: Mortality: $\mu_x = 0.02$ constant throughout life and $\mu_y = 0.03$ constant throughout life

Interest: $\delta = 4\%$ per annum

Expenses: Nil

[8]

11 A life insurance company issues the following policies:

- 20-year endowment assurance with a sum assured of £75,000 payable at maturity or at the end of the policy year of death if earlier. Level premiums for this contract are paid annually in advance.
- 20-year single premium temporary immediate annuity with an annual benefit payable in advance of £18,000.

On 1 January 2001, the company sold 5,000 endowment assurance policies and 2,500 temporary immediate annuity policies, all to lives aged 45 exact.

- (i) Calculate the death strain at risk for each type of policy during 2008.

Basis: Mortality: AM92 Select

Interest: 4% per annum

Expenses: Nil

[4]

During the first seven policy years, there were 65 deaths from the endowment assurance policies and 30 deaths from the temporary immediate annuity policies. During 2008, there were 10 deaths from the endowment assurance policies and 5 deaths from the temporary immediate annuity policies.

- (ii) Calculate the total mortality profit or loss to the company during 2008 using the basis in (i) above.

[5]

[Total 9]

- 12**
- (i) Explain the terms “unit fund” and “non-unit fund” in the context of a unit-linked life assurance contract. [4]
 - (ii) Explain why a life insurance company might need to set up reserves in order to zeroise future expected negative cashflows in respect of a unit-linked life assurance contract. [2]
 - (iii) A life insurance company issues 4-year unit-linked contracts to a male lives aged 50 exact. The following non-unit fund cash flows, $NUCF_t$, ($t = 1, 2, 3, 4$) are obtained at the end of each year t per contract in force at the start of the year t :

Year t	1	2	3	4
$NUCF_t$	375.4	-152.0	-136.2	-118.0

The rate of interest earned on non-unit reserves is 5.5% per annum and mortality follows the AM92 Select table.

Calculate the reserves required at times $t = 1, 2$ and 3 in order to zeroise future negative cash flows. [4]
[Total 10]

- 13** A life insurance company issues a 3-year savings contract to unmarried male lives that offers the following benefits:

- On death during the 3 years, a sum of £15,000 payable immediately on death.
- On surrender during the 3 years, a return of premiums paid, payable immediately on surrender.
- On marriage during the 3 years, a return of premiums paid accumulated with compound interest at 4% per annum, payable immediately on marriage.
- On survival to the end of the 3 years, a sum of £5,000.

The contract ceases on payment of any benefit.

Calculate the level premium payable annually in advance for this contract for a life aged 40 exact.

Basis:	Independent rate of mortality	AM92 Ultimate
	Independent rate of surrender	10% per annum
	Independent rate of marriage	5% per annum
	Interest	5% per annum
	Expenses	0.5% of each premium

[12]

- 14** A life insurance company issues a 5-year with profits endowment assurance policy to a life aged 60 exact. The policy has a basic sum assured of £10,000. Simple reversionary bonuses are added at the start of each year, including the first. The sum assured (together with any bonuses attaching) is payable at maturity or at the end of year of death, if earlier. Level premiums are payable annually in advance throughout the term of the policy.

- (i) Show that the annual premium is approximately £2,476.

Basis:	Mortality:	AM92 Select
Interest:	6% per annum	
Initial expenses:	60% of the first premium	
Renewal expenses:	5% of the second and subsequent premiums	
Bonus Rates:	A simple reversionary bonus will be declared each year at a rate of 4% per annum	

[5]

The office holds net premium reserves using a rate of interest of 4% per annum and AM92 Ultimate mortality.

In order to profit test this policy, the company assumes that it will earn interest at 7% per annum on its funds, mortality follows the AM92 Ultimate table and expenses and bonuses will follow the premium basis.

- (ii) Calculate the expected profit margin on this policy using a risk discount rate of 9% per annum.

[14]

[Total 19]

END OF PAPER

EXAMINATION

5 October 2009 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** Evaluate ${}_20q_{[45]:[45]}^2$

Basis: AM92 Select for both lives.

[3]

- 2** Give an expression for the expected present value of a benefit of 1 under a critical illness assurance contract for a healthy life aged x for term of n years. [3]

- 3** Calculate ${}_0.5q_{72.25}$ using the assumption of a constant force of mortality.

Basis: PMA92Base

[3]

- 4** Using the following data:

<i>Age</i>	<i>Population</i>	<i>Number of deaths</i>
40	121,376	104
41	134,292	127
42	133,277	132

(i) Calculate the crude mortality rate for the total population. [1]

(ii) Calculate the standardised mortality ratio for this population using AM92 Ultimate. [3]

[Total 4]

- 5** Derive an expression for the variance of the present value of a temporary annuity-due in terms of assurance functions for a life aged x with a term of n years. [4]

- 6** A life insurance company sells annual premium whole life assurance policies with benefits payable at the end of the year of death. Renewal expenses are incurred at the start of each year, and claim expenses are nil.

(a) Write down a recursive relationship between the gross premium reserves at successive durations, calculated on the premium basis. Define all symbols used.

(b) Explain the meaning of this formula in words.

[4]

7 (a) State what is meant by direct expenses incurred by a life insurance company in respect of a life insurance contract.

(b) Describe three different categories of direct expenses and give an example of each.

[5]

8 (i) Identify three classes of pensioner in receipt of a benefit from a pension fund.

[3]

(ii) Give two examples of selection that might be exhibited by these pensioners.[2]
[Total 5]

9 (i) Describe how insurance companies use responses to questions from prospective policyholders to ensure the probability of a profit is set at an acceptable level.

[5]

(ii) Explain why an insurance company might not use questions requesting genetic information from prospective policyholders? [3]
[Total 8]

10 A pension fund provides a pension from normal retirement age of £1,000 per annum for each complete year of service. The pension is payable monthly in advance for 5 years certain and for the whole of life thereafter and is only paid if the life remains in service to normal retirement age of 65.

Calculate the expected present value of the pension for a new entrant aged 62 exact.

Basis: Interest: 4% per annum

Mortality after retirement: PMA92C20

Independent decrement rates before retirement

Age x	q_x^d	q_x^w
62	0.005650	0.015672
63	0.006352	0.078441
64	0.007140	0.055654

[8]

- 11** A life insurance company offers special endowment contracts that mature at age 65. Premiums are payable annually in advance on 1 January each year. The sum assured payable at the end of year of death during the term is one half of the sum assured that will be paid if the policyholder survives until maturity.

Details of these contracts in force on 31 December 2007 are:

<i>Exact age</i>	<i>Total sums assured payable on maturity (£)</i>	<i>Total annual premiums (£)</i>
60	12,250,000	440,000

The claims in 2008 were on policies with the following total sums assured and annual premiums:

<i>Total sums assured payable on maturity (£)</i>	<i>Total annual premiums (£)</i>
200,000	7,000

Calculate the mortality profit or loss in 2008 given that the company calculates reserves for these contracts using the gross prospective method.

Basis: Mortality: AM92 Ultimate
 Interest: 4% per annum
 Expenses: Nil

[9]

- 12** (i) Define in words $1000\bar{A}_{x:y}^2$. [3]

(ii) Calculate:

(a) $1000\bar{A}_{30:40}^2$

(b) The annual premium payable continuously until the 2nd death for the above assurance in (a) with a sum assured of £1,000.

Basis: $\mu = .02$ for a life aged 30 exact at entry level throughout their life
 $\mu = .03$ for a life aged 40 exact at entry level throughout their life
 $\delta = .05$ throughout
 Expenses: Nil

[7]

- (iii) Outline the main deficiency of the above premium paying scheme and suggest an alternative. [3]

[Total 13]

- 13** A life insurance company issues a 35-year non profit endowment assurance policy to a life aged 30 exact. Level premiums are payable monthly in advance throughout the term of the policy. The sum assured of £75,000 is payable at maturity or at the end of year of death of the life insured, if earlier.

- (i) Show that the monthly premium is approximately £74.

Basis:	Mortality:	AM92 Select
Interest:	6% per annum	
Initial expenses:	£250 plus 50% of the gross annual premium	
Renewal expenses:	£75 per annum, inflating at 1.92308% per annum, at the start of the second and subsequent policy years and 2.5% of the second and subsequent monthly premiums	
Claims expense:	£300 inflating at 1.92308% per annum	
Inflation:	For renewal and claim expenses, the amounts quoted are at outset, and the increases due to inflation start immediately.	

[7]

- (ii) The insurance company calculates a surrender values equal to the gross retrospective policy value, assuming the same basis as in (i) above.

Calculate the surrender value at the end of the 30th policy year immediately before the premium then due.

[7]

[Total 14]

- 14** A life insurance company issues a special term assurance policy for a 3-year term. Under the policy, a sum assured of £10,000 is paid at the end of the year of death. In addition on survival to the end of the term 50% of total premiums paid are returned.

Basis:	Initial expenses:	20% of the first year's premium
	Renewal expenses:	5% of 2 nd and 3 rd years' premiums
	Reserves:	Net Premium using AM92 Ultimate at 4% interest (allowing for return of 50% of <u>net</u> premiums paid on survival)
	Mortality experience:	80% AM92 Select
	Withdrawals:	20% in year 1, 10% in year 2 (with all withdrawals assumed to occur at end of year)
	Surrender Value:	Nil
	Interest earned:	6% per annum
	Risk discount rate:	10% per annum

- (i) On the basis of the above information, calculate the level annual premium payable in advance for a life aged 57 exact to achieve the required rate of return. [12]
- (ii) Discuss the effect of increased withdrawal rates on the rate of return to the company from this policy. [2]

Following comments from the marketing department, it has been decided to allow a surrender value at the end of years 1 and 2 equal to 25% of total premiums paid.

- (iii) Calculate the revised annual premium using the basis above. [3]
[Total 17]

END OF PAPER

EXAMINATION

26 April 2010 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** Explain what the following represent:
- $l_{[x]+r}$
 - $n|m q_x$
 - d_x
- [3]

- 2** Define spurious selection, giving two distinct examples. [3]

- 3** Calculate the standardised mortality ratio for the population of Urbania using the following data:

Age	Standard Population		Urbania	
	Population	Deaths	Population	Deaths
60	2,500,000	26,170	10,000	130
61	2,400,000	29,531	12,000	145
62	2,200,000	32,542	11,000	173

[3]

- 4** A life insurance company offers an increasing term assurance that provides a benefit payable at the end of the year of death of 10,000 in the first year, increasing by 100 on each policy anniversary.

Calculate the single premium for a five year policy issued to a life aged 50 exact.

Basis:

Rate of interest	4% per annum
Mortality	AM92 Select
Expenses	Nil

[4]

- 5** A population is subject to the force of mortality $\mu_x = e^{0.0002x} - 1$.

Calculate the probability that a life now aged 20 exact:

- survives to age 70 exact [2]
 - dies between ages 60 exact and 70 exact [3]
- [Total 5]

- 6** You are provided with the following extract from a life table:

x	l_x
50	99,813
51	97,702
52	95,046

Calculate ${}_0.75p_{50.5}$ using two different methods.

[5]

- 7** A company is about to establish a pension scheme that will provide an age retirement benefit of $n/60$ ths of final pensionable salary where n is total number of years of service. Final pensionable salary is the average salary in the three years before retirement.

An employee who will become a member of the pension scheme is currently aged 55 exact has and will be granted exactly 20 years of past service. The employee's salary in the year before the valuation date was £40,000.

- (i) Calculate the present value of benefits for this member (including future service). [3]
- (ii) Calculate the contribution required to fund this benefit as a percentage of future salaries. [3]

Basis:

Pension Scheme from the Formulae and Tables for Actuarial Examinations

[Total 6]

- 8** 100 graduates aged 21 exact decide to place the sum of £1 per week into a fund to be shared on their retirement at age 66 exact.

- (i) Show that each surviving member can expect to receive on retirement a fund of approximately £7,240. [4]

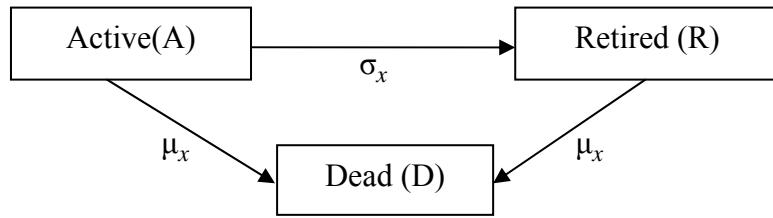
Basis:

Rate of interest 4% per annum
Mortality AM92 Ultimate

One of the survivors uses the accumulated fund to buy a weekly annuity payable for 10 years certain. After 10 years the annuity is payable at two-thirds of the initial level for the rest of life.

- (ii) Calculate the weekly amount of the annuity on the basis used in part (i). [2]
[Total 6]

- 9** A life insurance company models the experience of its pension scheme contracts using the following three-state model:



- (i) Derive the dependent probability of a life currently Active and aged x retiring in the year of age x to $(x + 1)$ in terms of the transition intensities. [2]
 - (ii) Derive a formula for the independent probability of a life currently Active and aged x retiring in the year of age x to $(x + 1)$ using the dependent probabilities. [4]
- [Total 6]

- 10** The decrement table extract below is based on the historical experience of a very large multinational company's workforce.

Age (x)	Number of employees (al) $_x$	Deaths (ad) $_x^d$	Withdrawals (ad) $_x^w$
40	10,000	25	120
41	9,855	27	144
42	9,684		

Recent changes in working conditions have resulted in an estimate that the annual independent rate of withdrawal is now 75% of that previously used.

Calculate a revised table assuming no changes to the independent death rates, stating your results to one decimal place. [7]

- 11** Thiele's differential equation for the policy value at duration t ($t > 0$), ${}_t\bar{V}_x$, of an immediate life annuity payable continuously at a rate of £1 per annum from age x is:

$$\frac{\partial}{\partial t} {}_t\bar{V}_x = \mu_{x+t} \times {}_t\bar{V}_x - 1 + \delta \times {}_t\bar{V}_x$$

- (i) Derive this result algebraically showing all the steps in your working. [5]
 - (ii) Explain this result by general reasoning. [3]
- [Total 8]

- 12** On 1 January 2005, a life insurance company issued 1,000 10-year term assurance policies to lives aged 55 exact. For each policy, the sum assured is £50,000 for the first five years and £25,000 thereafter. The sum assured is payable immediately on death and level annual premiums are payable in advance throughout the term of this policy or until earlier death.

The company uses the following basis for calculating premiums and reserves:

Mortality	AM92 Select
Interest	4% per annum
Expenses	Nil

- (i) Calculate the net premium retrospective reserve per policy as at 31 December 2009. [6]
- (ii) (a) Give an explanation of your numerical answer to (i) above.
- (b) Describe the main disadvantage to the insurance company of issuing this policy.
- (c) Give examples of how the terms of the policy could be altered so as to remove this disadvantage. [3]

There were, in total, 20 deaths during the years 2005 to 2008 inclusive and a further 8 deaths in 2009.

- (iii) Calculate the total mortality profit or loss to the company during 2009. [3]
[Total 12]

- 13** A life insurance company issues a 3-year unit-linked endowment assurance policy to a male life aged 45 exact.

Level premiums of £4,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death. 95% of the premium is allocated to units in the first policy year, 100% in the second and 105% in the third. A policy fee of £50 is deducted from the bid value of units at the start of each year. The units are subject to a bid-offer spread of 5% on purchase. An annual management charge of 1.75% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of 125% of the bid value of the units is payable at the end of the policy year of death. On maturity, 100% of the bid value of the units is payable.

The policyholder may surrender the policy only at the end of the first and second policy years. On surrender, the bid value of the units less a surrender penalty is payable at the end of the policy year of exit. The surrender penalty is £1,000 at the end of the first policy year and £500 at the end of the second policy year.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	5.5% per annum in year 1 5.25% per annum in year 2 5.0% per annum in year 3
Rate of interest on non-unit fund cash flows	4.0% per annum
Mortality	AM92 Select
Initial expenses	£200
Renewal expenses	£50 per annum on the second and third premium dates
Initial commission	15% of first premium
Renewal commission	2.0% of the second and third years' premiums
Rate of expense inflation	2.0% per annum
Risk discount rate	7.0% per annum

For renewal expenses, the amount quoted is at outset and the increases due to inflation start immediately. In addition, you should assume that at the end of the first and second policy years, 12% and 6% respectively of all policies still in force then surrender immediately.

- (i) Calculate the profit margin for the policy. [13]
- (ii) Calculate the expected present value of profit for the policy if the company assumed that there were no surrenders at the end of each of the first and second policy years. [3]
- [Total 16]

- 14** A life insurance company issues a 30-year with profits endowment assurance policy to a life aged 35 exact. The sum assured of £100,000 plus declared reversionary bonuses are payable on survival to the end of the term or immediately on death if earlier.

- (i) Show that the quarterly premium payable in advance throughout the term of the policy or until earlier death is approximately £616.

Pricing basis:

Mortality:	AM92 Select
Interest:	6% per annum
Initial commission:	100% of the first quarterly premium
Initial expenses:	£250 paid at policy commencement date
Renewal commission:	2.5% of each quarterly premium from the start of the second policy year
Renewal expenses:	£45 at the start of the second and subsequent policy years
Claim expense:	£500 on death; £250 on maturity
Future reversionary bonus:	1.92308% of the sum assured, compounded and vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death)

[10]

At the end of the 25th policy year, the actual past bonus additions to the policy have been £145,000.

- (ii) Calculate the gross prospective policy reserve at the end of that policy year immediately before the premium then due.

Policy reserving basis:

Mortality:	AM92 Ultimate
Interest:	4% per annum
Bonus loading:	4% of the sum assured and attaching bonuses, compounded and vesting at the end of each policy year
Renewal commission:	2.5% of each quarterly premium
Renewal expenses:	£90 at the start of each policy year
Claim expense:	£1,000 on death; £500 on maturity

[6]

[Total 16]

END OF PAPER

EXAMINATION

6 October 2010 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate:

(a) $20|10 q_{[45]}$

(b) $30 P_{[45]:[50]}$

Basis: AM92 Select

[3]

2 Calculate $0.5 p_{45.75}$ using the Uniform Distribution of Deaths assumption.

Basis: AM92 Ultimate

[3]

3 Calculate the single premium payable for a temporary reversionary annuity of £12,000 per annum payable monthly in arrear to a female life currently aged 55 exact on the death of a male life currently aged 50 exact. No payment is made after 20 years from the date of purchase.

Basis:

Rate of interest	4% per annum
Mortality of male life	PMA92C20
Mortality of female life	PFA92C20
Expenses	Nil

[4]

4 A gymnasium offers membership for a three-year period at a fixed fee of £240 per annum payable monthly in advance. The contract may only be cancelled at a renewal anniversary. Monthly premiums cease immediately on the death of the member.

Calculate the expected present value of membership fees if the gymnasium sells 120 memberships:

Basis:

Rate of interest	6% per annum
Rate of mortality	1% per annum
Probability of renewal	80% at each anniversary
Expenses	Nil

[5]

- 5** A pension scheme provides an age retirement benefit of $n/80$ ths of final pensionable salary where n is total number of years of service. Final pensionable salary is the average salary in the three years before retirement. Normal retirement age is 65 and age retirement is only permitted between ages 60 and 65 exact.

A member of the pension scheme currently aged 45 exact has 12 years of service and their salary in the year before the valuation date was £25,000.

Give a formula for the expected cashflows between the 66th and 67th birthdays as a result of entitlement from this past service. [5]

- 6** Calculate:

(a) $\bar{A}_{\overline{30}:40}$

(b) $\bar{a}_{30:40:\overline{20}}$

Basis:

$\mu = 0.01$ throughout for the life aged 30 now

$\mu = 0.02$ throughout for the life aged 40 now

$\delta = 4\%$ per annum

[6]

- 7** A life insurance company issues a 10-year term assurance policy to a life aged 55 exact. The sum assured which is payable immediately on death is given by the formula:

$$50,000 \times (1 + 0.1t) \quad t = 0, 1, 2, \dots, 9$$

where t denotes the curtate duration in years since the inception of the policy.

Level premiums are payable monthly in advance throughout the term of the policy or until earlier death.

Calculate the monthly premium for this policy using the following basis:

Mortality AM92 Select

Interest 4% per annum

Expenses Nil

[6]

- 8** Describe the causal factors that explain observed differences in mortality and morbidity. [6]

- 9** The actuary advising a pension scheme has decided that the independent mortality in the standard table for pension schemes (PEN) from page 142 of the Formulae and Tables for Actuarial Examinations is no longer appropriate for that pension scheme.

Calculate the revised row of the service table for age 61, assuming that the revised independent mortality rate at that age is 80% of the previous independent mortality rate.

[7]

- 10** Define the following terms, giving formulae and defining all notation used:

- (a) Crude mortality rate
- (b) Indirectly standardised mortality rate

[7]

- 11** A life insurance company issues a four-year unit-linked policy to a male life. The following non-unit cash flows, $NUCF_t$ ($t = 1,2,3,4$), are obtained at the end of each year t per policy in force at the start of the year t :

Year t	1	2	3	4
$NUCF_t$	-50.2	-43.1	-32.1	145.5

Assume that the annual mortality rate for the male life is constant at 1% at all ages.

- (i) Show that the annual internal rate of return is 6%. [3]

The company sets up reserves in order to zeroise future negative cash flows. The rate of interest earned on non-unit reserves is 2.5% per annum.

- (ii) Calculate the net present value of the profits after zeroisation using a risk discount rate of 6% per annum. [3]
- (iii) Comment on the results obtained in (i) and (ii) above. [1]

[Total 7]

- 12** A life insurance company issued a with profits whole life policy to a life aged 40 exact on 1 January 2000. Under the policy, the basic sum assured of £50,000 and attaching bonuses are payable immediately on death. Level premiums are payable annually in advance under the policy until age 65 or earlier death.

The company declares simple reversionary bonuses at the start of each year including the first year and the bonus entitlement on the policy is earned immediately the bonus is declared.

- (i) Give an expression for the gross future loss random variable under the policy at the outset, defining symbols where necessary. [4]

- (ii) Calculate the annual premium using the following assumptions:

Mortality	AM92 Select
Interest	6% per annum
Bonus loading	2.5% per annum simple
Initial expenses	£300
Renewal expenses	£25 at the start of the second and subsequent policy years while the policy is in force
Claim expenses	£250

[4]

On 31 December 2009, the policy is still in force. Bonuses declared to date total £13,750.

- (iii) Calculate the gross premium prospective reserve for the policy as at 31 December 2009 using the following assumptions:

Mortality	AM92 Ultimate
Interest	4% per annum
Bonus loading	3% per annum simple
Renewal expenses	£35 at the start of each policy year while the policy is in force
Claim expenses	£250

[4]

[Total 12]

- 13** On 1 January 2009, a life insurance company issued 10,000 joint life whole life assurance policies to couples. Each couple comprised one male life aged 60 exact and one female life aged 55 exact when the policy commenced. Under each policy, a sum assured of £100,000 is payable immediately on the death of the second of the lives to die.

Premiums under each policy are payable annually in advance while at least one of the lives is alive.

The life insurance company uses the following basis for calculating premiums and net premium reserves:

Mortality	PMA92C20 for the male PFA92C20 for the female
Interest	4% per annum
Expenses	Nil

- (i) Calculate the annual premium payable under each policy. [4]

During the calendar year 2009, there was one claim for death benefit, in respect of a policy where both the male and the female life died during the year. In addition, there were 20 males and 10 females who died during the year.

- (ii) Calculate the mortality profit or loss for the group of 10,000 policies for the calendar year 2009. [10]
[Total 14]

- 14** A life insurance company issues four-year without profits endowment assurance policies to male lives aged 56 exact. The sum assured is £21,500 payable on maturity or at the end of the year of death if earlier. Premiums of £5,000 are payable annually in advance throughout the term of the policy.

The company holds net premium reserves for these policies, calculated using AM92 Ultimate mortality and interest of 4% per annum.

Surrenders occur only at the end of a year immediately before a premium is paid. The surrender value is 70% of the net premium reserve calculated at the time the surrender value is payable.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of interest on cash flows	4% per annum
Mortality	AM92 Select
Surrenders	10% of all policies still in force at the end of each of the first, second and third policy years
Initial expenses	£600
Renewal expenses	£45 per annum on the second and subsequent premium dates
Risk discount rate	6% per annum

Calculate the expected profit margin for this contract.

[15]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

26 April 2011 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 13 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Give a different example of selection shown by each of the following mortality tables:

- (a) ELT15
- (b) PMA92
- (c) AM92

[3]

2 Calculate:

- (a) ${}_{23}P_{65}$
- (b) ${}_{10|5}q_{60}$
- (c) $\ddot{s}_{65:10}$

Basis:

Mortality PMA92C20
Rate of interest 4% per annum

[4]

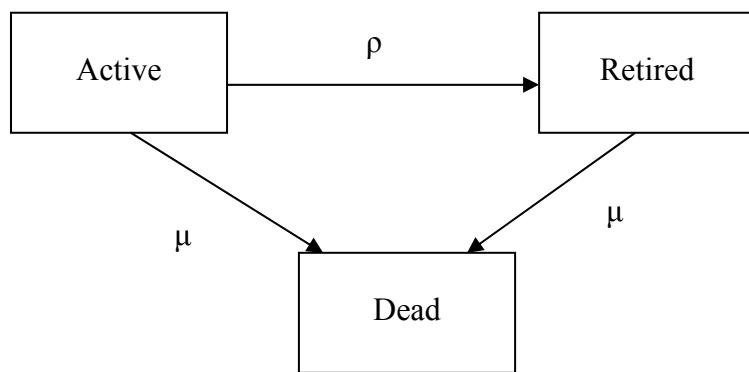
3 Calculate $(I\bar{a})_x$

Basis: $\mu_x = 0.02$ for all x
 $\delta = 4\%$ per annum

[4]

4 Outline the benefits that are usually provided by a pension scheme on retirement due to ill health. [5]

5 A pension scheme uses the following model to calculate probabilities, where the transition intensities are $\mu = 0.05$ and $\rho = 0.08$.



Calculate:

- (a) the dependent probability of retirement
- (b) the independent probability of death from active service

using the Kolmogorov equations.

[5]

- 6** (i) Define uniform distribution of deaths [2]

- (ii) Using the method in (i) above calculate $1.25q_{65.5}$ [4]

Basis:

Mortality ELT15(Males)

[Total 6]

- 7** Explain how education influences morbidity. [6]

- 8** A life insurance company issues a with profits whole life assurance policy to a life aged 40 exact. The sum assured of £100,000 plus declared reversionary bonuses are payable immediately on death. Level premiums are payable annually in advance to age 65 or until earlier death.

A simple bonus, expressed as a percentage of the sum assured, is added to the policy at the start of each year (i.e. the death benefit includes the bonus relating to the policy year of death).

The following basis is used to price this policy:

Mortality AM92 Select

Rate of Interest 4% per annum

Initial expenses £300 plus 50% of the first annual premium, incurred at the policy commencement date

Renewal commission 2.5% of each premium from the start of the second policy year

Claim expense £350 at termination of the contract

Using the principle of equivalence, calculate the level simple bonus rate that can be supported each year on this policy if the annual premium is £3,212. [6]

- 9** A male life aged 52 exact and a female life aged 50 exact take out a whole life assurance policy. The policy pays a sum assured of £100,000 immediately on first death. Premiums are payable for a period of five years, monthly in advance.

Calculate the monthly premium payable.

Basis:

Mortality PMA92C20 (male life), PFA92C20 (female life)
 Rate of interest 4% per annum
 Expenses Nil

[7]

- 10** Calculate the expected present value and variance of the present value of an endowment assurance of 1 payable at the end of the year of death for a life aged 40 exact, with a term of 15 years.

Basis:

Mortality AM92 Select
 Rate of interest 4% per annum
 Expenses Nil

[8]

- 11** A life insurance company issues a 4-year unit-linked endowment policy to a life aged 61 exact under which level premiums of £2,500 are payable yearly in advance throughout the term of the policy or until earlier death. In the first policy year 40% of the premium is allocated to units, while in the second and subsequent policy years 110% of the premium is allocated to units. The unit prices are subject to a bid-offer spread of 5%.

If the policyholder dies during the term of the policy, the death benefit of £10,000 or the bid value of the units, whichever is higher, is payable at the end of the policy year of death.

The policyholder may surrender the policy, in which case a value equal to a fixed percentage of the total premiums paid on the policy is payable at the end of the policy year of surrender. The percentage is based on the policy year of surrender as follows:

<i>Policy year</i>	<i>% of total premiums payable as a surrender value</i>
1	0
2	25
3	50
4	75

On maturity, 105% of the bid value of units is payable.

An annual management charge of 0.5% of the bid value of units is deducted at the end of each policy year before death, surrender and maturity benefits are paid.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	4.25% per annum
Rate of interest on non-unit fund cash-flows	3.5% per annum
Independent rate of mortality	AM92 Select
Independent rate of surrender	6% per annum
Initial expenses	£325
Renewal expenses	£74 per annum on the second and subsequent premium dates
Initial commission	10% of first premium
Renewal commission	2.5% of the second and subsequent years' premiums
Risk discount rate	5.5% per annum
(i)	Construct a multiple decrement table for this policy assuming that there is a uniform distribution of both decrements over each year of age in the single decrement table. [3]
(ii)	Construct tables showing the growth of the unit fund and the non-unit fund. Include all commissions in the non-unit fund. [7]
(iii)	Calculate the profit margin for this policy on the assumption that the company does not zeroise future expected negative cashflows. [3]
	[Total 13]

- 12** On 1 April 1988, a life insurance company issued a 25-year term assurance policy to a life then aged 40 exact. The initial sum assured was £75,000 which increased by 4% per annum compound at the beginning of the second and each subsequent policy year. The sum assured is payable immediately on death and level monthly premiums are payable in advance throughout the term of the policy or until earlier death.

The company uses the following basis for calculating premiums and reserves:

Mortality	AM92 Select
Rate of interest	4% per annum
Initial commission	50% of the total premium payable in the first policy year
Initial expenses	£400 paid at the policy commencement date
Renewal commission	2.5% of each premium from the start of the second policy year
Renewal expenses	£75 per annum, inflating at 4% per annum compound, at the start of the second and subsequent policy years (the renewal expense quoted is as at the start of the policy and the increases due to inflation start immediately)
Claim expense	£300 on termination (the claim expense is fixed over the duration of the policy)

- (i) Show that the monthly premium for the policy is approximately £56. [10]
- (ii) Calculate the gross premium prospective reserve as at 31 March 2011. [6]
[Total 16]

- 13** (i) Explain, including formulae, the following expressions assuming that the sum assured is payable at the end of the year of death:
- death strain at risk
 - expected death strain
 - actual death strain
- [6]

- (ii) A life insurance company issues the following policies:

- 25-year term assurances with a sum assured of £200,000
- 25-year endowment assurances with a sum assured of £100,000

The death benefit under each type of policy is payable at the end of year of death.

On 1 January 2000, the company sold 10,000 term assurance policies to male lives then aged 40 exact and 20,000 endowment assurance policies to male lives then aged 35 exact. For each type of policy, premiums are payable annually in advance.

During the first ten years, there were 145 actual deaths from the term assurance policies written and 232 actual deaths from the endowment assurance policies written.

- (a) Calculate the death strain at risk for each type of policy during 2010.

During 2010, there were 22 actual deaths from the term assurance policies and 36 actual deaths from the endowment assurance policies.

Assume that there were no lapses/withdrawals on each type of policy during the first eleven years.

- (b) Calculate the total mortality profit or loss to the office in the year 2010.

- (c) Comment on the results obtained in (b) above.

Basis:

Mortality	AM92 Ultimate
Rate of interest	4% per annum
Expenses	Nil

[11]

[Total 17]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

4 October 2011 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate:

- (a) $10|q_{[50]}$
- (b) $10P_{[60]+1}$
- (c) $\ddot{a}_{[40]:20}^{(12)}$

Basis:

Mortality AM92
Rate of interest 6% per annum

[3]

2 Calculate ${}_0.5q_{75.25}$ using the assumption of a constant force of mortality.

Basis:

Mortality AM92

[3]

3 In a special mortality table with a select period of one year, the following relationships are true for all ages:

$$0.5q_{[x]} = 0.25q_x$$

$$0.5q_{[x]+0.5} = 0.45q_x$$

Express $p_{[x]}$ in terms of p_x .

[3]

4 A term assurance contract with a term of 20 years pays a sum assured of 1 immediately on death to a life now aged 30 exact.

Calculate the expected value and variance of this contract.

Basis:

Mortality AM92 Ultimate
Rate of interest 4% per annum

[4]

5 (a) Write down the random variable form of $\bar{A}_{x:y}^1$.

(b) Calculate $\bar{A}_{x:y}^1$ on the following assumptions:

$$\mu_x = 0.02 \text{ for all } x$$

$$\mu_y = 0.03 \text{ for all } y$$

$$\delta = 4\% \text{ per annum}$$

[5]

- 6** Explain why it is necessary to have different mortality tables for different classes of lives. [6]

- 7** A special joint life last survivor annuity of £10,000 per annum is payable continuously in respect of a male and female life each aged 60 exact. Payments commence on the first death and continue for 5 years after the second death.

Calculate the expected present value of this annuity.

Basis:

Mortality PMA92C20 (male life), PFA92C20 (female life)

Rate of interest 4% per annum

Expenses Nil

[6]

- 8** The following data is extracted from a population census:

Age	All Professions		Profession A	
	Population	Deaths	Population	Deaths
20–29	120,000	256	12,500	30
30–39	178,000	458	15,000	40
40–49	156,000	502	16,000	50
50–64	123,000	600	14,000	60

- (a) Calculate the area comparability factor for Profession A using the data for All Professions as the standard population.
- (b) Hence or otherwise derive the standardised mortality ratio and the indirectly standardised mortality rate.

[6]

- 9** Members of a pension scheme are subject to three decrements:

- (a) Deaths - with independent decrement rates that are assumed to follow ELT15(Males)
- (b) Ill-health retirement - with an independent decrement rate of 0.01 at age 50 exact increasing by 0.005 for each additional year of age (so the ill-health independent decrement at age 53 exact is 0.025)
- (c) Age retirement - with an independent decrement rate of 0.2 at each age from 60 to 64 all exact.

Age retirements are assumed to take place on the attainment of the exact age, whilst other decrements act uniformly across the year of age.

Calculate the probability that a member currently aged 59 exact will retire at age 62 exact.

[6]

- 10** (i) Five years ago a with profits whole life assurance policy was sold to a life then aged 30 exact.

The sum assured is £150,000 payable at the end of year of death and premiums are payable annually in advance throughout life. The super compound method of adding bonuses to the policy is used as follows:

- each year there is a simple bonus of 2.5% on the sum assured
- and an additional bonus of 5% on all existing bonuses (excluding the simple bonus relating to that policy year)

Assume that bonuses vest at the start of each policy year and that the actual past bonus additions have followed the assumptions stated above.

Calculate the net premium policy value just before payment of the 6th premium.

Basis:

Mortality AM92 Select
Rate of interest 4% per annum [5]

- (b) Suggest two reasons why a life insurance company might use the super compound method of adding bonuses to with profits policies, as opposed to the compound method. [2]
[Total 7]

- 11** A pension scheme provides a pension on retirement of 1% of final pensionable salary for each completed year of pensionable service. On retirement due to ill-health, pensionable service is calculated as service that would have been completed by the normal retirement age of 65. Final pensionable salary is defined as the average salary in the last three years before retirement.

Derive an expression, without using commutation functions, for the present value of the benefits for a new member age 30 with salary of £20,000 in the year after entry to the scheme. Define all symbols used. [8]

- 12** (i) List the main features of a unit-linked policy. [4]

A four-year unit-linked policy issued by a life insurance company to a life aged 56 exact has the following profit vector:

$$(1525.89, -334.08, -292.05, -933.82)$$

- (ii) Determine the net present value of the profits of this policy, assuming that the company sets up reserves in order to zeroise future negative expected cash flows on the policy.

Basis:

Mortality	AM92 Ultimate
Rate of interest on non-unit fund cash flows	4.5% per annum
Risk discount rate	7.5% per annum
	[5]

[Total 9]

- 13** A life insurance company issues a 3-year without profits endowment assurance policy to a male life aged 57 exact for a sum assured of £15,000 payable on maturity or at the end of the year of death if earlier. Premiums of £4,700 are payable annually in advance throughout the term of the policy.

The office holds net premium reserves for these policies, calculated using AM92 Ultimate mortality and interest of 4% per annum.

Surrenders occur only at the end of a year immediately before a premium is paid. The surrender value payable is 75% of total premiums paid on the contract at the time the surrender value is payable. Assume that at the end of the first and second policy years, 10% and 5% respectively of all policies still in force at that time then surrender.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of interest on cash flows	
and Reserves	5% per annum
Mortality	AM92 Select
Initial expenses	10% of the annual premium
Renewal expenses	£65 per annum on the second and subsequent premium dates
Risk discount rate	7% per annum

- (i) Calculate the net present value of profits for this contract. [10]
- (ii) Calculate the internal rate of return for this contract. [2]

The company weakens the reserving basis by assuming that net premium reserves for these policies are now calculated using AM92 Ultimate mortality and interest of 6% per annum.

- (iii) Calculate the revised net present value of profits and comment on your answer. [4]

[Total 16]

- 14** On 1 January 2001, a life insurance company issued a number of 30-year endowment assurance policies that pay £100,000 at maturity, or £50,000 at the end of the year of earlier death to lives then aged 35 exact. Premiums are payable annually in advance.

The company uses the following basis for calculating premiums and reserves:

Mortality	AM92 Select
Interest	4% per annum
Initial commission	50% of the premium payable in the first policy year
Initial expenses	£300 paid at policy commencement date
Renewal expenses	2.5% of each premium from the start of the second policy year

- (i) Write down the recursive relationship between the gross premium reserves at successive durations of these policies, defining all symbols used. [4]
- (ii) Show that the annual premium for each policy is approximately £1,803. [4]
- There were 385 policies in force on 1 January 2010. During 2010, there were three actual deaths, actual interest earned by the company was 5% and expenses were as expected.
- (iii) Calculate the profit or loss made by the company from both mortality and interest in respect of these policies for the year 2010 based on the formula stated in (i) above. [10]
- [Total 18]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

23 April 2012 (am)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 15 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1**
- (a) Define ${}_{4|5}q_{[60]+1}$ in words.
 - (b) Calculate its value.

Basis:

Mortality AM92

[3]

- 2** Under a policy issued by a life insurance company, the death benefit payable at the end of year of death is a return of premiums paid without interest. A level premium of £3,000 is payable annually in advance throughout the term of the policy.

For a policy in force at the start of the 12th policy year, you are given the following information:

Reserve at the start of the policy year	£25,130
Reserve at the end of the policy year per survivor	£28,950
Probability of death during the policy year	0.03
Expenses incurred at the start of the policy year	£90
Rate of interest earned	4% per annum

Reserves given above are immediately before payment of the premium due.

Calculate the profit/loss expected to emerge at the end of the 12th policy year per policy in force at the start of that year. [3]

- 3** Calculate:

(a) $a_{50:\overline{15}}$

(b) $(IA)_{50:\overline{15}}^1$

Basis:

Mortality AM92
Rate of interest 6% per annum

[4]

- 4** A joint life assurance contract provides a death benefit of £100,000 payable immediately on the second death of two lives, a male life currently aged 60 exact and a female life currently aged 55 exact.

Calculate the expected present value of the contract.

Basis:

Mortality PMA92C20 (male life), PFA92C20 (female life)
Rate of interest 4% per annum
Expenses Nil

[4]

- 5** A 10-year unit-linked policy has the following profit vector:

$$(-40, -12, -6, -1, 5, -4, 8, 20, 25, 30)$$

Determine the revised profit vector if reserves are set up to zeroise future negative cash flows on the following basis:

Mortality 0.5% per annum (i.e. probability of death at each age)
Interest 2.5% per annum

[4]

- 6** (a) Calculate the constant force of mortality applicable to a life aged between 67 and 68 exact.

- (b) Calculate the value of ${}_0.5 q_{67.25}$ using the assumption of a constant force of mortality and the value derived in (a) above.

Basis: AM92 Ultimate

[4]

- 7** Describe the benefits typically provided by a salary-related pension scheme for active members on age retirement. [6]

- 8** Explain the impact of occupation on mortality and morbidity. [6]

- 9** (i) List the main categories of expenses incurred by life insurance companies. [2]

- (ii) Give one example of each category in part (i) and indicate the manner in which it is usually allowed for in the calculation of premiums. [4]

[Total 6]

- 10** An insurance company writes policies that provides benefits of £1,000 in the event of becoming disabled due to accident and £10,000 on death.

- (a) Construct a multiple state transition model for these policies.
(b) Give a formula for the expected present value of the benefits.

[6]

- 11** (i) State the advantages and disadvantages of using crude mortality rates and directly standardised mortality rates as the comparison measure of mortality in two or more different populations [4]

You are given the following data in respect of a sub-population:

<i>Age</i>	<i>Population</i>
50	100,000
55	95,000
60	80,000
Number of deaths in sub-population	1,250

- (ii) Calculate the Standardised Mortality Ratio using ELT15 (Males) as the mortality rate for the standard population. [3]

[Total 7]

- 12** An endowment assurance contract with a term of 10 years pays a sum assured of £100,000 immediately on death and a sum of £50,000 on survival for 10 years.

Calculate the expected present value and variance of this contract.

Basis:

Mortality	$\mu_x = 0.03$ throughout	
Rate of interest	5% per annum	[8]

- 13** A life insurance company issues a 40-year with profit endowment assurance policy to a life aged 20 exact. The sum assured of £85,000 plus declared reversionary bonuses is payable on survival to the end of the term or immediately on death if earlier.

The company assumes that future annual bonuses will be declared at a rate of 1.92308% of the sum assured, compounded and vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

Calculate the monthly premium payable in advance throughout the term of the policy.

Basis:

Mortality	AM92 Select	
Interest	6% per annum	
Initial commission	480% of the first monthly premium	
Initial expenses	£325	
Renewal commission	2.5% of each monthly premium excluding the first	
Renewal expenses	£75 per annum at the start of the second and subsequent policy years. The renewal expense is assumed to increase by £5 per annum from the start of the third policy year.	

[10]

- 14** A life insurance company issues 20-year decreasing term assurance policies to single lives aged 40 exact. The death benefit, which is payable at the end of the year of death, is £200,000 in the first policy year, £190,000 in the second policy year thereafter reducing by £10,000 each year until the benefit is £10,000 in the twentieth and final policy year. Premiums on the policies are payable annually in advance for 20 years or until earlier death.

The company calculates its reserves on a net premium basis and negative reserves are permitted.

- (i) Show that the annual net premium for each policy is approximately equal to £204 using the basis below. [4]

625 policies were in force at the start of the 10th policy year and 3 policyholders died during that policy year.

- (ii) Calculate the mortality profit or loss to the life insurance company during the 10th policy year using the basis below. [6]

- (iii) Comment briefly on the results obtained in part (ii) above. [2]

Basis:

Mortality	AM92 Ultimate
Interest	4% per annum
Expenses	Nil

[Total 12]

- 15** A life insurance company issues a three-year term assurance policy to a male life aged 57 exact under which level premiums are payable annually in advance throughout the term of the policy or until earlier death. The sum assured is £150,000 payable at the end of year of death.

The company uses the following assumptions to calculate the premium for this policy:

Rate of interest on cash flows	6% per annum
Mortality	AM92 Select
Initial expenses	£350
Renewal expenses	£50 per annum on the second and third premium dates
Initial commission	15% of first premium
Renewal commission	2.5% of the second and third years' premiums
Risk discount rate	6% per annum

- (i) Write down the gross future loss random variable at the outset of the policy. [5]
- (ii) Calculate the office premium using assurance and annuity functions, setting the expected value of the gross future loss random variable to zero. [4]
- (iii) Derive the office premium using a discounted cash flow projection, assuming no withdrawals and using the same profit criterion as in part (ii). [6]
- (iv) Without further calculation explain the effect of:
- (a) setting up reserves within the calculation of part (iii).
- (b) having set up the reserves in part (a), increasing the risk discount rate to 8% per annum. [2]
- [Total 17]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

2 October 2012 (am)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 15 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate:

(a) ${}_{12}P_{43}$

(b) ${}_{10|5}Q_{55}$

(c) $\ddot{a}_{45:\overline{10}}$

Basis:

Mortality AM92

Rate of interest 6% per annum

[3]

2 Give three different forms of selection that would be expected in a group of lives purchasing immediate annuities with an example of each. [3]

3 Explain how nutrition affects mortality and morbidity. [4]

4 Calculate ${}_3P_{55.75}$ using the assumption of Uniform Distribution of Deaths.

Basis:

Mortality ELT15 (Females)

[4]

5 The Area Comparability Factor is defined as:

$$F = \frac{\sum_x {}^sE_{x,t} {}^s m_{x,t}}{\sum_x {}^sE_{x,t}^c} \quad \left/ \quad \frac{\sum_x {}^sE_{x,t} {}^s m_{x,t}}{\sum_x {}^sE_{x,t}^c} \right.$$

(a) Define the notation used.

(b) Explain what is measured by the Area Comparability Factor by considering the ratio of the numerator to the denominator.

[4]

6 A life insurance company issues a with profit whole life assurance policy to a life aged 40 exact, under which the sum assured S and any attaching bonuses, are payable immediately on death. Compound bonuses are added annually in advance. Premiums are payable annually in advance ceasing at exact age 85 or on earlier death.

Write down an expression for the net future loss random variable at outset for this policy defining all symbols that are used. [4]

- 7** On 1 January 2007, a life insurance company sold a large number of 30-year pure endowment policies to lives then aged 35 exact. The sum assured under each policy is £125,000 payable on maturity. Premiums are payable annually in advance throughout the term of the policy.

There were 3521 pure endowment policies still in force on 1 January 2011 and 8 policyholders died during 2011.

Calculate the total mortality profit or loss to the life insurance company during 2011 assuming the company calculates net premium reserves on the following basis:

Mortality	AM92 Select
Interest	4% per annum
Expenses	Nil

[4]

- 8** Examine the column of d_x shown in the English Life Table No. 15 (Males) in the Formulae and Tables for Examinations (Pages 68–69).

Describe the key characteristics of this mortality table using the data to illustrate your points.

[6]

- 9** (i) Explain what is meant by the following in the context of life insurance policies:

- (a) gross premium prospective reserve
(b) gross premium retrospective reserve

[4]

- (ii) State the conditions necessary for gross premium prospective and gross premium retrospective reserves to be equal.

[3]

[Total 7]

- 10** A pension scheme provides a lump sum benefit to members on reaching retirement at age 65 equal to one month's pensionable salary for each complete year of service. Pensionable salary is defined as average annual salary in the last two years before retirement.

Calculate the cost of this benefit as a percentage of salary for a new member of the scheme aged 35 exact, with salary in the next year of £20,000.

Basis:

Pension Scheme tables in the Formulae and Tables for Examinations
Interest 4% per annum

[8]

- 11** A special joint life annuity of £500 per week is payable in arrear in respect of a male life aged 65 exact and a female life aged 62 exact. The annuity has the following features:
- The annuity is guaranteed in any event for the first 5 years at the level of £500 per week.
 - At the end of the guarantee period if both lives are still surviving the annuity continues at the same level until one life dies at which time it reduces to two-thirds of the initial level and continues at this reduced level until the second life dies.
 - At the end of the guarantee period if only one life has survived the annuity reduces to two-thirds of the initial level and continues at this reduced level until the second life dies.
 - At the end of the guarantee period if both lives have previously died then the annuity ceases.

Calculate the expected present value of this annuity.

Basis:

Mortality	PMA92C20 (male life), PFA92C20 (female life)	
Rate of interest	4% per annum	
Expenses	Nil	[8]

- 12** A life insurance company issues a special endowment assurance policy for a 25 year term to two lives x and y . Under this policy, a sum assured of £100,000 is paid immediately on the second death within the 25 year term. At the end of 25 years a sum of £50,000 is paid to each survivor.

Calculate the annual premium paid continuously under this policy assuming this is paid throughout the term or until the second death if earlier.

Basis:

Mortality	Life x : $\mu_x = 0.02$ for all x	
	Life y : $\mu_y = 0.03$ for all y	
Force of interest	5% per annum	
Expenses	Nil	[10]

- 13** A life insurance company issues a with profit whole life assurance policy to a life aged 55 exact. The sum assured is £75,000 together with any attaching bonuses and is payable immediately on death. Level premiums are payable monthly in advance ceasing on the policyholder's death or on reaching age 85 if earlier.

Simple annual bonuses are added at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

The company calculates the premium on the following basis:

Mortality	AM92 Select
Interest	4% per annum
Expenses	
Initial	£275
Renewal	£65 at the start of the second and subsequent policy years and payable until death
Claim	£200 on death
Commission	
Initial	75% of the total premium payable in the first policy year
Renewal	2.5% of the second and subsequent monthly premiums
Bonuses	Simple bonus of 2.0% of basic sum assured per annum

- (i) Calculate the monthly premium for this policy. [6]
- (ii) Calculate the gross prospective policy value at the end of the 30th policy year given that the total actual past bonus additions to the policy have followed the assumptions stated in the premium basis above (including the bonus just vested).

Policy value basis:

Mortality	AM92 Ultimate
Interest	4% per annum
Expenses	
Renewal	£80 at the start of each policy year and payable until death
Claim	£250 on death
Commission	
Renewal	2.5% of the monthly premiums
Bonuses	Simple bonus of 2.5% of basic sum assured per annum

[4]
[Total 10]

- 14** A life insurance company issues a four-year policy to a male life aged 30 exact that offers the following benefits:
- On death during the term of the policy or on survival to the end of the term, a sum of £60,000.
 - On redundancy during the term of the policy, a return of 100% of total premiums paid.
 - On surrender during the term of the policy, a return of 50% of total premiums paid.

Premiums of £14,000 are payable annually in advance throughout the term of the policy or until earlier claim. The death, surrender and redundancy benefits are payable immediately on claim. The contract ceases on payment of any claim.

The company uses the following basis to profit test this contract:

Interest earned on cash flows	3% per annum
Expenses	5% of each premium paid
Reserves	Ignore

The company has also calculated the following dependent rates of mortality, surrender and redundancy which are used to profit test this contract:

Year t	$(aq)_{[30]+t-1}^d$	$(aq)_{30+t-1}^s$	$(aq)_{30+t-1}^r$
1	.000447	.098727	.023744
2	.000548	.049361	.024368
3	.000602	.024680	.024680
4	.000636	0	0

Calculate the expected profit margin to the company on this policy using a risk discount rate of 5% per annum. [10]

- 15** A life insurance company issues a three-year unit-linked endowment assurance policy to a male life aged 45 exact. The main features of the contract are:

Premiums:	£3,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death
Allocation rates:	75% of premium is allocated to units in the first policy year, 100% in the second and 105% in the third
Policy fee:	£35 is deducted from the bid value of units at the start of each policy year
Death benefit:	150% of the bid value of the units is payable at the end of the policy year of death
Maturity benefit:	100% of the bid value of the units is payable

Bid-offer spread:	5%
Annual management charge:	1.5% of the bid value of units is deducted at the end of each policy year (management charges are deducted from the unit fund before death and maturity benefits are paid).

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	5.0% per annum in year 1 4.5% per annum in year 2 4.0% per annum in year 3
Rate of interest on non-unit fund cash flows	3.0% per annum
Mortality	AM92 Select
Withdrawals	None
Initial expenses	£275
Renewal expenses	£80 per annum on the second and subsequent premium dates
Initial commission	20% of first premium
Renewal commission	2.5% of the second and subsequent years' premiums
Rate of expense inflation	2.0% per annum
Risk discount rate	6.5% per annum

For renewal expenses, the amount quoted is at outset, and the increases due to inflation start immediately.

- (i) Calculate the non-unit fund cash flows in each year of the contract and hence the expected present value of profit assuming that the policyholder dies in the third year of the contract. [9]
- (ii) Calculate the expected present value of profit for the policy if the policyholder dies in the:
 - (a) first year of the contract.
 - (b) second year of the contract.
[4]
- (iii) Hence calculate the expected present value of the contract allowing for the possibility that the policyholder survives to the end of the contract. [2]

[Total 15]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

19 April 2013 (pm)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate:

(a) ${}_{10|5}q_{40}$

(b) \bar{a}_{65}

(c) ${}_{15}P_{[46]}$

Basis:

Mortality AM92

Interest 4% per annum

[3]

2 Calculate $(aq)_x^\alpha$.

Basis:

Mortality: $\mu_x^\alpha = 0.1$ and $\mu_x^\beta = 0.2$ for all x

α, β are independent decrements

[3]

3 Describe how climate and geography affect mortality and morbidity. [3]

4 Describe the use of terminal bonus within the reversionary bonus system. [3]

5 A pension scheme provides a pension on retirement of one-sixtieth of final pensionable salary for each year of service. Final pensionable salary is average salary received in the three years before retirement. Normal retirement age is 65 exact. The same level of pension is payable on retirement on the grounds of ill-health or otherwise prior to age 65.

Calculate the expected present value of past and future benefits for a life currently aged 30 exact with 10 years of past service and salary in the previous year of £40,000.

Basis:

PEN Tables in Formulae and Tables for Actuarial Examination.

[4]

- 6** A life insurance company issues a 20-year increasing endowment assurance policy which provides a sum assured given by the formula:

$$\text{£} [50,000 + 1,500t] \quad t = 1, 2, \dots, 20$$

where t denotes the policy year.

The sum assured is payable on maturity at age 50 exact or at the end of year of death if earlier. Premiums on the policy are payable annually in advance.

Write down an expression for:

- (a) the net premium for the policy.
- (b) the net premium prospective policy reserve for the policy immediately before the tenth premium is paid.

[4]

- 7** Explain why it is necessary to have different mortality tables for different classes of lives. [6]

- 8** (i) Define the measures of crude mortality rate and directly standardised mortality rate. You should include a definition of all symbols used. [5]

The data in the table below is for a sub-population for the year 2012.

<i>Age</i>	<i>Number of lives</i>	<i>Number of deaths</i>
65	125,000	2,937
66	130,000	3,301
67	140,000	3,756

- (ii) Calculate the standardised mortality ratio for this sub-population using ELT15 (Males) as the standard population. [2]

[Total 7]

- 9** A male life currently aged 65 exact purchases a special joint life annuity of £10,000 per annum payable monthly in advance together with additional benefits detailed below.

On the death of the male life, the annuity reduces to £5,000 per annum payable monthly in advance to a female life until her death, assuming she survives him. The female life is currently aged 62 exact.

The policy additionally provides benefits of:

- An annuity certain (extra to the above and not dependent on the survival status of each life) of £10,000 per annum payable monthly in advance and paid only for ten years, and
- £10,000 payable immediately on the death of each life.

Calculate the expected present value of the total benefits.

Basis:

Mortality Male life PMA92C20
 Female life PFA92C20

Interest 4% per annum

Expenses Nil

[7]

- 10** A special whole life assurance policy issued to a life aged 40 exact provides a benefit of £1,000 on death within 20 years of inception, £2,000 on death between 20 and 40 years from inception and £3,000 on death thereafter. Benefits are payable at the end of the year of death.

Calculate the expected present value and variance of the present value of this policy.

Basis:

Mortality AM92 Ultimate
Interest 4% per annum

[8]

11 Two lives are both aged 45 exact.

Calculate:

- (i) The probability of both lives surviving to age 65 exact. [1]
- (ii) The present value of an annuity of £1,000 per annum increasing by 3% each year payable annually in advance so long as both lives survive. [3]
- (iii) The present value of a 20-year term assurance with a benefit of £100,000 payable immediately on the second death. [5]

Basis:

Mortality $\mu_x = 0.05$ for all x for both lives

Interest 4% per annum

[Total 9]

12 A life insurance company issues whole life assurance policies to lives aged 50 exact for a sum assured of £75,000 payable at the end of the year of death. Premiums are payable annually in advance.

- (i) Calculate the annual gross premium for each policy using the basis below. [4]
- (ii) Calculate the minimum annual gross premium that the company should charge in order that the probability of making a loss on any one policy would be 10% or less. [6]

Basis:

Mortality AM92 Select

Interest 6% per annum

Initial commission 100% of the annual gross premium

Initial expenses £325

Renewal commission 2.5% of each annual gross premium excluding the first

Renewal expenses £75 per annum at the start of the second and subsequent policy years

[Total 10]

- 13** A life insurance company issues 5,000 four-year decreasing term assurance policies on 1 January 2012 to a group of male lives aged 56 exact at that date.

Premiums are payable annually in advance on each policy. The initial annual gross premium P reduces to $.75P$, $.5P$ and $.25P$ at the beginning of the second, third and fourth policy year respectively.

The sum assured on each policy is payable at the end of year of death and is given by the formula:

$$\text{£}100,000 \times [1 - 0.25t] \quad t = 0, 1, 2, 3$$

where t denotes the curtate duration in years since the inception of the policy.

- (i) Calculate the initial annual gross premium P for each policy using the basis below. [7]
- (ii) Determine the prospective gross premium reserve for each policy in force at the end of the first policy year using the same basis. [5]
- (iii) Calculate the mortality profit or loss for this portfolio of business for the calendar year 2012 given that 27 policyholders died during that year. [2]

Actual expenses incurred and interest earned by the company on this portfolio of business during 2012 was the same as that assumed in the premium basis.

- (iv) Derive the mortality profit or loss for the calendar year 2012 using the recursive relationship between the opening and closing prospective reserves in the first policy year. [2]

Basis:

Mortality	AM92 Ultimate
Interest	6% per annum
Initial commission	25% of the first annual premium
Initial expenses	£125
Renewal commission	3% of each annual premium excluding the first
Renewal expenses	£35 per annum at the start of the second and subsequent policy years. The renewal expense is assumed to increase by 1.92308% compound per annum from inception of the policy. [Total 16]

- 14** A life insurance company issues a three-year unit-linked endowment assurance policy to a life aged 67 exact. Level premiums are payable yearly in advance throughout the term of the policy or until earlier death. In the first year, 50% of the premium is allocated to units and 110% in the second and third years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.75% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death and surrender benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of the bid value of the units is payable at the end of the year of death. The policyholder may surrender the policy only at the end of each year immediately before a premium is paid. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the year of exit.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	4% per annum
Rate of interest on non-unit fund cash flows	3% per annum
Mortality	90% AM92 Ultimate
Surrenders	8% at end of first year, 4% at end of second year based on policies in force at that time.
Initial expenses	£235
Renewal expenses	45 per annum on the second and third premium dates
Initial commission	12.5% of first premium
Renewal commission	2.5% of the second and third years' premiums
Claim expense	£75 on deaths and surrenders only

The company sets premiums so that the net present value of the profit for the policy is 10% of the annual premium, using a risk discount rate of 6% per annum.

- (i) Calculate the premium for the policy on the assumption that the company **does not** zeroise future expected negative cash flows. [12]
- (ii) Calculate the net present value of the profit on the policy on the assumption that the company **does** set up reserves in order to zeroise future expected negative cash flows. [5]
- [Total 17]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

27 September 2013 (pm)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional booklets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

11 Calculate:

(a) ${}_{10}q_{63}$

(b) $\ddot{a}_{63}^{(2)}$

(c) $s_{55:\overline{10}}$

Basis:

Mortality PFA92C20

Interest 4% per annum

[2]

12 Define temporary initial selection, giving a distinct example of the process. [2]

13 A whole life assurance policy was issued to a life aged x exact for a sum assured of S payable at the end of year of death. A premium of P is payable annually in advance until death. The following expense assumptions were used to derive the gross premium payable on the policy:

Initial commission $a\%$ of the annual premium

Initial expenses B

Renewal commission $c\%$ of each annual premium excluding the first

Renewal expenses D per annum at the start of the second and subsequent policy years

Claim expenses $e\%$ of the sum assured

Let ${}_tV_x$ represent the gross premium reserve on this policy at duration t .

Write down an equation linking the gross premium reserve at the beginning and the end of:

(a) the first policy year

(b) the t^{th} policy year where $t > 1$

[3]

14 Calculate ${}_{2.25}p_{90.25}$ using the method of Uniform Distribution of Deaths.

Basis:

Mortality AM92

[3]

- 15** A three-year unit-linked endowment assurance policy is sold to a male life aged 40 exact. The profit signature for this policy, calculated using AM92 Select mortality and making no allowance for surrenders, is:

$$(-209.80, 253.55, 109.85)$$

It is now assumed for the cash flows for this policy that 15% of all policies in force at the end of the first policy year are surrendered at that time. The surrender value payable at that time is the bid value of units at the end of the policy year less a surrender penalty of £500. There are no other changes to the policy.

- (a) Calculate the revised profit signature in the first policy year.
- (b) Comment on the impact on the profit signature in the second and third policy years.

[4]

- 16** A life aged 75 exact purchases a ten-year temporary annuity of an initial amount of £1,200 per annum. This annuity increases on each policy anniversary by £100 per annum, the last increase being at the beginning of the tenth policy year. All annuity payments are annual in advance.

Calculate the expected present value of the annuity benefits.

Basis:

Mortality AM92 Ultimate
Interest 6% per annum
Expenses Nil

[4]

- 17** Calculate $(\bar{A})_{20}$ (the present value of a whole life assurance issued to a life aged 20 exact payable immediately on death where the benefit paid on death at time t is t) using the following basis:

Basis:

Mortality $\mu_x = 0.03$ for $x < 40$ inclusive and 0.04 for $x \geq 40$
Force of interest 5% per annum

[6]

- 18** Show, using the random variable approach, that the expected present value of an annuity of 1 per annum payable annually in arrears to a life now aged x , deferred for n years is equal to $a_x - a_{x:n}$. [7]

- 19** The following statistics have been provided in relation to a particular country and one of its regions:

Age band	Region A		Country	
	Population exposed	Number of Deaths	Population exposed	Number of Deaths
18–35	25,000	25	500,000	1,000
36–50	50,000	80	125,000	375
51–70	70,000	170	110,000	500

- (i) Calculate:
- (a) the mortality rates for each age band both for Region A and Country
 - (b) the crude mortality rate for each of Region A and Country
 - (c) the directly standardised mortality rate for Region A by reference to the Country
 - (d) the standardised mortality ratio for Region A by reference to the Country
- [5]
- (ii) Comment on the results.
- [2]
- [Total 7]

- 20** The following is an extract of a decrement table assumed for a funeral plan, showing deaths (d) and withdrawals (w):

Age x	$(al)_x$	$(ad)_x^d$	$(ad)_x^w$
85	10,000	1,400	2,300
86	6,300	1,000	1,100
87	4,200		

It has been established that the independent rates of decrement of withdrawal are now only 50% of those assumed in the table above for the ages of 85 and 86. The underlying independent mortality rates are unchanged.

Construct a revised decrement table to reflect this change.

[7]

21 A pension scheme provides a lump sum benefit on death in service of three times salary in the 12 months before death. Normal retirement age is 65 exact.

- (i) Calculate the expected cashflow between ages 40 and 41 in respect of the death benefit for a life now aged 35 exact with salary in the previous 12 months of 25,000.

Basis: Pension scheme tables in the Formulae and Tables for Examinations [3]

- (ii) (a) Give a formula for the expected present value of the death benefit.
(b) Hence express this value using commutation factors.

[5]

[Total 8]

22 At the beginning of 2004, a life insurance company issued a number of 20-year “special” endowment assurance policies to male lives then aged 40 exact. Each policy provides a death benefit of £75,000 payable at the end of year of death and a maturity benefit of £150,000.

Premiums on each policy are payable annually in advance for the term of the policy, ceasing on earlier death.

- (i) Calculate the annual gross premium for each policy using the following premium basis: [4]

Mortality	AM92 Select
Interest	4% per annum
Initial commission	25% of the first annual premium
Initial expenses	£400
Renewal expenses	£45 per annum at the start of the second and subsequent policy years

- (ii) Determine the gross premium reserve for each policy in force at the end of the eighth policy year **and** for each policy in force at the end of the ninth policy year, using the same basis as above. [6]

At the beginning of 2012, there were 625 policies in force. Actual experience for this portfolio of business during 2012 was as follows:

Number of deaths	3
Interest earned	4.5%
Expense incurred per policy in force at beginning of policy year	£45

- (iii) Derive, using the recursive relationship between the opening and closing reserves, the profit/loss from this portfolio of business in 2012 separately from:

- mortality
- interest
- expenses

[4]

[Total 14]

- 23** A life insurance company issues a 15-year increasing term assurance policy to a life aged 50 exact.

The death benefit on the policy, payable immediately on death, is given by the formula:

$$\text{£}10,000 \times [6+t] \quad t = 0, 1, 2, \dots, 14$$

where t denotes the curtate duration in years since the inception of the policy.

Level premiums on the policy are payable monthly in advance for the term of the policy, ceasing on death if earlier.

- (i) Calculate the monthly premium for the policy using the following premium basis:

Mortality	AM92 Select
Interest	6% per annum
Expenses	
Initial	£225
Renewal	£65 per annum inflating at 1.92308% per annum, at the start of the second and subsequent policy years
Commission	
Initial	30% of the total premium payable in the first policy year
Renewal	4% of the second and subsequent monthly premiums
Claim	£275 on termination, inflating at 1.92308% per annum
Inflation	For renewal and claim expenses, the amounts quoted are at outset, and the increases due to inflation start immediately.

[8]

- (ii) Calculate the gross prospective reserve for the policy at the end of the 14th policy year using the elements of the premium basis that are relevant. [3]
- (iii) Write down an expression for the gross future loss random variable at the end of the 14th policy year, again using the elements of the premium basis that are relevant. [4]

[Total 15]

- 24** A life insurance company issues a four-year with profits endowment assurance policy for a basic sum assured of £25,000 to a life aged 56 exact. Level premiums are payable annually in advance throughout the term of the policy.

Compound reversionary bonuses are added to the policy at the start of each year, including the first. The basic sum assured (together with any bonuses attaching) is payable at maturity or at the end of year of death, if earlier.

- (i) Show that the annual premium is approximately £6,483 using the following premium basis:

Mortality	AM92 Select
Interest	6% per annum
Initial expenses	£100 plus 25% of the first premium (all incurred on policy commencement)
Renewal expenses	2.5% of the second and subsequent premiums plus £40 at the start of the second and subsequent policy years
Bonus rates	A compound reversionary bonus will be declared each year at a rate of 1.92308% per annum

[5]

- (ii) The insurance company holds net premium reserves using a rate of interest of 4% per annum and AM92 Ultimate mortality.

Calculate the expected profit margin on this policy using the following profit test basis:

Mortality	80% AM92 Select
Interest earned on funds	7.5% per annum
Initial expenses	as per premium basis
Renewal expenses	as per premium basis
Bonus rates	as per premium basis
Risk discount rate	9.5% per annum

[13]
[Total 18]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

29 April 2014 (am)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 13 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Define Class Selection giving two distinct examples. [2]

2 Calculate:

(a) $\ddot{a}_{25:20}^{(4)}$

(b) $(\bar{IA})_{25:20}^1$

Basis: Mortality AM92
Interest 4% per annum

[4]

3 For a particular species of animal the mortality and rate of interest are shown according to the Basis below.

Calculate $A_{3:\overline{5}}^1$.

Basis: Mortality $l_x = l_0 e^{-0.15x}$
Interest 5% per annum

[4]

4 Outline with examples the advantages of “cash flow” or “discounted emerging costs” techniques for product pricing, compared with the use of traditional commutation functions. [4]

5 A pension scheme provides a pension of 1/40 of final pensionable salary for each year of service, limited to a maximum of 2/3 of final pensionable salary, upon retirement at age 65. No other retirement age is allowed.

Final pensionable salary is defined as average annual salary over the 3 years immediately preceding retirement.

A member is now aged exactly 45 and has 16 years of past service. He earned £40,000 in the previous 12 months.

Calculate the expected present value now of this member’s expected total pension on retirement.

Basis:

PEN Tables in the Formulae and Tables for Actuarial Examinations – Interest 4% per annum

[4]

- 6**
- (a) Describe the Method of Constant Force of Mortality.
 - (b) Calculate ${}_2.75q_{85.5}$ using the method given in (a) above.
- Basis: Mortality ELT15 (Males) [5]

- 7** A Joint Life Annuity is issued to a male aged 65 exact and a female aged 62 exact. The annuity is payable quarterly in arrear the first payment commencing 3 months after issue.

The annuity has the following conditions:

- £10,000 per annum whilst both lives survive.
- If the male life predeceases the female life the annuity reduces to £7,500 per annum payable for the remainder of her lifetime.
- If the female life predeceases the male life the annuity reduces to £6,000 per annum payable for the remainder of his lifetime.

Calculate the expected present value of the annuity.

Basis: Mortality PMA92C20 (male) and PFA92C20 (female)
 Interest 4% per annum
 Expenses Ignore

[6]

- 8** A double decrement table is to be constructed from two single decrement tables. The modes of decrement are α and β . The basis for each of the single decrement tables is shown below:

Basis:

In the table for single decrement α : $l_{x+t}^\alpha = l_x^\alpha - t^3 d_x^\alpha$ for $0 \leq t \leq 1$

In the table for single decrement β : $l_{x+t}^\beta = l_x^\beta - t^5 d_x^\beta$ for $0 \leq t \leq 1$

The l function represents the number of lives and the d function the number of decrements in the appropriate table.

- (i) Show that

$${}_t p_x^\beta \mu_{x+t}^\beta = 5t^4 q_x^\beta \text{ for } 0 \leq t \leq 1 \quad [3]$$

- (ii) Hence or otherwise show that

$$(aq)_x^\beta = q_x^\beta \left(1 - \frac{5}{8} q_x^\alpha \right) \quad [5]$$

[Total 8]

- 9** Calculate the expected present value and the variance of $\bar{A}_{x:20}^1$ given the basis below.

Basis: Mortality $\mu_x = 0.03$ for all x
 Force of interest $\delta = 5\%$ per annum throughout

[8]

- 10** (i) Define the following terms in words without giving any formulae:

- (a) crude mortality rate
- (b) directly standardised mortality rate
- (c) indirectly standardised mortality rate
- (d) area comparability factor

[4]

The following table gives a summary of mortality for a particular occupational group compared to the whole population.

Age Group	Occupational Group		Whole Population	
	Number	Deaths	Number	Deaths
20–34	20,000	67	1,000,000	3,500
35–49	15,000	92	1,500,000	7,800
50–64	11,000	125	700,000	8,000
TOTAL	46,000	284	3,200,000	19,300

- (ii) Calculate the crude mortality rate, the directly standardised mortality rate and the indirectly standardised mortality rate for the occupational group. [5]
 [Total 9]

- 11** On 1 January 2008, a life insurance company issued a number of without profit endowment policies maturing at age 60 to lives then aged 40 exact. The sum assured is payable at the end of year of death or on survival to the end of the term and level premiums are payable annually in advance throughout the term of the contract.

Premiums and reserves on each policy are both calculated on the following basis:

Mortality	AM92 Select
Interest	4% per annum
Initial commission	60% of the first premium
Renewal commission	6% of each annual premium excluding the first

- (i) Calculate the annual office premium per £1,000 sum assured for each policy. [2]
- (ii) Calculate the gross premium prospective reserve per £1,000 sum assured for each policy in force at 31 December 2012. [2]

(iii) Calculate the profit or loss to the company in 2013 in respect of these policies given the following information:

- The total sums assured in force on 1 January 2013 were £15,500,000
- The company incurred expenses relating to these policies of £76,500 on 1 January 2013 (including renewal commission).
- The total sums assured paid on 31 December 2013 in respect of deaths during 2013 were £295,000.
- The total sums assured surrendered during 2013 were £625,000. The surrender value on each policy (which was paid on 31 December 2013) was calculated as 85% of the gross premium prospective reserve applicable at the date of payment of the surrender value.
- The company earned interest of 3.5% per annum on its assets during 2013.

[10]

[Total 14]

- 12** A life assurance company issues a policy which provides a three-year temporary annuity of £15,000 per annum payable annually in arrear to a male life aged 65 exact. The single premium payable at outset on the policy is £42,000.

The company uses the following basis to profit test the policy:

Mortality	PMA92C20
Interest earned on cash flow and reserves	5% per annum
Initial commission	1% of the single premium
Initial expenses	£350
Renewal expenses	£55 per annuity payment which is assumed to increase by 3% per annum from inception of the policy
Risk discount rate	7% per annum

In addition, the company establishes reserves on the policy at the beginning and end of each policy year where:

$${}_tV = 15,000 \times (3-t) \text{ for } t = 1 \text{ and } 2$$

$${}_tV = 0 \text{ otherwise}$$

- (i) Calculate the net present value of the expected profits on the policy:
 - (a) allowing for reserves
 - (b) ignoring reserves[10]
 - (ii) Briefly comment on the reason for the difference in the two values calculated in part (i). [2]
 - (iii) Describe briefly how the net present value calculated in part (i)(a) and part (i)(b) would change if a risk discount rate of 4% per annum had been used (instead of 7% per annum) and state the reasons for the difference. [3]
- [Total 15]

- 13** On 1 January 2003, an insurance company issued a 35 year with profit endowment assurance policy to a life aged 30 exact for a sum assured of £60,000. The sum assured (together with any bonuses attaching) is payable at maturity or immediately on death, if earlier. Level premiums are payable annually in advance throughout the policy term or until earlier death. Compound reversionary bonuses vest at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

The company calculates the premium on the following basis:

Mortality	AM92 Ultimate
Interest	6% per annum
Initial expenses	£250 plus 60% of the first year's premium, incurred at outset
Renewal expenses	2.5% of the second and each subsequent year's premium, incurred at the beginning of the respective policy years
Bonuses:	1.92308% per annum

- (i) Show that the annual premium is approximately £1,146. [8]
- (ii) Express, in stochastic form, the gross future loss random variable for this policy at duration t , where t is an integer and $0 < t < 35$.

You should use T_x , K_x or both, together with the elements of the premium basis that are relevant.

Assume bonuses declared follow the assumptions stated in the premium basis. [3]

On 31 December 2012, and just after the 10th bonus has been declared, the life wishes to surrender the policy. The insurance company calculates a surrender value equal to the gross prospective policy reserve, using the premium basis above.

- (iii) Calculate the surrender value payable by the insurance company given that the total actual past bonus additions to the policy have followed the assumptions stated in the premium basis (including the bonus just vested). [6]

[Total 17]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

1 October 2014 (am)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Define Time Selection giving a distinct example. [2]

2 With respect to life assurance contracts, discuss the possible effects of policy lapses on subsequent mortality experience.

[3]

3 Calculate ${}_{2.5}q_{75.75}$ using the method of Uniform Distribution of Deaths.

Basis: Mortality PMA92C20 [4]

4 Calculate ${}_{5|3}q_{40:40}^1$.

Basis: Mortality AM92 [4]

5 (i) Define in words and give a formula for the area comparability factor F. [2]

(ii) Mortality levels for a certain country have been studied at national and regional level.

Explain the circumstances under which a particular region may have an Area Comparability Factor of 1.5. [2]

[Total 4]

6 A pension scheme provides an ill-health retirement pension of 1/80 of Final Pensionable Salary for each year of company service limited to a maximum of 50% of Final Pensionable Salary, with fractions of a year to count proportionately. Retirement due to ill-health may take place at any time before age 65 the normal retirement age.

Final Pensionable Salary is defined as the average annual salary over the three-year period preceding retirement.

Calculate the present expected value of past and future service ill-health benefits for a male now aged 30 exact who has a current salary of £30,000 per annum and has 10 years past service. His salary will increase in 1 year's time.

Basis:

PEN Tables in the Formulae and Tables for Actuarial Examinations – Interest 4% per annum

[4]

- 7** A life aged 40 exact purchases an endowment assurance policy whereby the sum assured on survival at age 60 exact is £20,000 and the benefit payable on death during the term is £10,000. Death benefits are payable at the end of the year of death.

Calculate the expected present value and variance of the benefits under this policy.

Basis: Mortality AM92 Select
Interest 4% per annum
Expenses Ignore

[6]

- 8** (i) In the context of random variables define T_x and K_x . [2]

(ii) State the random variable for the following expected values:

(a) \bar{A}_x

(b) a_x

(c) $A_{[x]:\overline{n}}$

(d) ${}_5|\ddot{a}_x$

[5]

[Total 7]

- 9** A life aged 60 exact purchases a special deferred term assurance policy for an overall term of 20 years.

Under this policy a sum assured of £100,000 is paid on death but only on death from age 65 exact up to the end of the term. On death between age 60 and 65 the benefit is equal to the total premiums paid without interest.

All payments on death are made at the end of the year of death. An annual premium paid in advance is payable for the full 20 year term.

Calculate the annual premium payable.

[7]

Basis:

Mortality AM92 Ultimate
Interest 4% per annum
Expenses Ignore

- 10** A life insurance company issues a non-profit assurance policy for a term of n years to a life aged x exact.

For $t = 1, 2, \dots, n$:

- The level annual premium payable at the start of year t is P .
- The expense at the start of policy year t is E_t .
- The benefits payable at the end of the t^{th} policy year on death, surrender and survival are D_t , B_t and S_t respectively.
- The rate of interest earned on net cash flows during the t^{th} policy year is i_t .
- The dependent rates of mortality and surrender at age $x+t$ are $(aq)_{x+t}^d$ and $(aq)_{x+t}^w$ respectively.

Assume that the insurance company **does not** set up a reserve for the policy.

- (i) Write down an expression for $(CF)_t$, the accumulation to the end of the t^{th} policy year of the expected net cash flow arising during the t^{th} policy year per policy in force at the start of that year. [2]
- (ii) Derive an expression which could be used to calculate the level annual premium that the company should charge if the company requires the expected net present value of profit on the policy to be zero assuming a risk discount rate of $j\%$ per annum defining any notation used. [3]

Assume that the insurance company **does** set up a reserve ${}_{t-1}V$ for the policy at the start of the t^{th} policy year.

- (iii) Write down an expression for the expected profit at the end of the t^{th} policy year for each policy in force at the start of that year. [2]
[Total 7]

- 11** A life assurance company has issued whole of life assurance policies over a number of years. Premiums on these policies are payable annually in advance and the sums assured are payable at the end of the year of death.

You are given the following information relating to a group of policies within the portfolio of whole of life assurance policies:

<i>Age exact on 1 January 2013</i>	<i>Sums assured in force on 1 January 2013</i>	<i>Reserves held on 31 December 2013 for policies in force at that date</i>
69	£740,000	£371,000

During 2013, there was 1 death claim (on a policy which was issued on 1 January 2000 for a sum assured of £15,000) arising from this group of policies.

- (i) Calculate the mortality profit or loss for 2013 to the company in respect of this group of policies assuming net premiums are held on the following basis:

Mortality AM92 Ultimate
Interest 4% per annum

[5]

- (ii) Calculate the amount of expected death claims in 2013 for this group of policies. [1]
- (iii) Compare your answer in part (ii) with the amount of actual claims and comment on your answer with reference to your answer in part (i) above. [2]
- [Total 8]

- 12** (i) Calculate the probability that a life now aged 30 exact will die between the ages of 55 and 65 both exact.

Basis: Mortality ELT15 (Males)

[2]

- (ii) Calculate the above probability again assuming the basis below

Basis: Mortality $\mu_x = 0.005e^{0.09(x-20)}$ for $20 \leq x \leq 70$

[7]

[Total 9]

- 13** A life insurance company issues 3-year policies to lives aged 55 exact who are employees of a manufacturer. These policies offer the following benefits during the term of the policy:

- On death whilst still an employee, £200,000 paid at the end of year of death.
- On ill-health retirement, £100,000 paid at the end of the year of retirement.
- On leaving their employer other than on death or ill-health retirement, a return of all premiums paid accumulated with interest at a rate of 2% per annum payable at the end of the year of leaving.
- On survival as an employee at the end of 3 years, £10,000 is payable.

The company uses the following basis to calculate annual premiums for this policy:

Independent rate of mortality	110% of AM92 Ultimate
Interest earned on cash flows	5% per annum
Initial expenses	£150
Renewal expenses	£25 at the start of the second and third policy year
Reserves	None held

In addition, you are given the following independent rates of ill-health retirement and withdrawal. You can assume that the decrements operate uniformly over each year of age in each single decrement table.

Age	<i>Ill-health retirement</i>	<i>Withdrawal</i>
55	0.04	0.10
56	0.05	0.08
57	0.06	0.06

- (i) Calculate the dependent rates of mortality, ill-health retirement and withdrawal for each policy year. [3]

The company sets premiums so that the net present value of the profit for the policy is 5% of the annual premium, using a risk discount rate of 5% per annum.

- (ii) Calculate the level premium payable annually in advance for this policy. [9]
- (iii) Discuss briefly whether the life insurance company needs to hold reserves at the beginning and end of each policy year for this policy. [2]

Assume that the company does hold reserves at the beginning and end of each policy year for this policy and that reserves earn interest at 5% per annum.

- (iv) Explain, without doing further calculations, whether the premium would be higher, the same or lower than that calculated in part (ii) above. [2]
- [Total 16]

- 14** A life insurance company, is proposing to launch a “Low Start” unit-linked endowment policy for a term of 3 years under which premiums increase by a fixed monetary amount each year and are payable yearly in advance throughout the term of the policy or until earlier death. The premium payable and the amount of premium allocated to units in each policy year are as follows:

<i>Policy Year</i>	<i>Premium Payable</i>	<i>Allocation Rate</i>
	£	%
1	1500	50
2	2250	105
3	3000	115

If the policyholder dies during the term of the policy, the death benefit of £6,750 (i.e. the total amount of premiums due to be paid on the policy if held to maturity) or the bid value of the units, whichever is higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of each policy year. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the policy year of exit.

The units are subject to a bid-offer spread of 6% and an annual management charge of 1% of the bid value of units is deducted at the end of each policy year. Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

You should use the following assumptions in carrying out profit tests of this policy:

Rate of growth on assets in the unit fund	4.5% per annum
Rate of interest on non-unit fund cash flows	2.5% per annum
Mortality	90% AM92 Ultimate
Surrender	7.5% of policies in force at the end of year 1 and 2.5% of policies in force at the end of year 2 then surrender £200
Initial expenses	
Renewal expenses	£55 per annum on the second and third premium dates
Initial commission	5% of first premium
Renewal commission	2.5% of the second and third years' premiums
Claim expense	£75 (payable only on death and surrender)
Risk discount rate	6.5% per annum

- (i) Calculate the profit margin for the policy issued to a life aged 61 exact on the assumption that the company does not set up sterling reserves for this policy. [13]
 - (ii) Explain why a life insurance company might need to set up non-unit reserves in respect of a unit-linked life assurance policy. [2]
 - (iii) Calculate the profit margin for the policy on the assumption that the company does set up reserves for this policy. [4]
- [Total 19]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

27 April 2015 (pm)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate $A_{50:\overline{4}}[1]$.

Basis:

Mortality $q_{50} = 0.05$
 $q_{51} = 0.06$
 $q_{51+t} = 1.1q_{50+t}$ for $t \geq 1$

Interest 6% per annum [3]

2 Describe how the quality of housing affects mortality and morbidity. [3]

3 Suppose α and β are the only two independent modes of decrement and $\mu_x^\beta = \frac{1}{4}\mu_x^\alpha$

Express $(aq)_x$ in terms of μ_x^β . [3]

4 Calculate:

(a) ${}_{10|15}q_{60}$

(b) ${}_{12}P_{[50]+1}$

(c) $a_{40:\overline{10}}^{(4)}$

Basis:

Mortality AM92
Interest 6% per annum

[4]

5 Draw a multiple state model diagram for the parameters involved using the service table from the PEN tables on Page 142 of the Formulae and Tables for Examinations and labelling your diagram clearly. [5]

- 6** A life aged 55 exact purchases a 3-year term assurance with sum assured of £150,000 payable if death occurs during the term of the policy. Level premiums of £900 are payable annually in advance throughout the term of the policy or until earlier death. The death benefit is payable at the end of the policy year of death.

Calculate the expected present value of the profit or loss to the office on the contract.

Basis:

Mortality	AM92 Select
Interest	3% per annum
Initial expenses	£260
Renewal expenses	£70 per annum incurred at the start of both the second and third policy year

Assume no reserves are required for this policy.

[6]

- 7** Calculate ${}_{1.75}P_{82.75}$.

- (a) Using the method of Uniform Distribution of Deaths.
(b) Using the method of Constant Force of Mortality.

Basis:

Mortality ELT15(Males)

[6]

- 8** (a) Explain why lives are subdivided into separate groups for the analysis of mortality.

- (b) Suggest three types of selection with an example for each.

[7]

- 9** On 1 January 1999, an insurance company issued a without profit whole life policy to a life aged 45 exact. The sum assured on the policy is £125,000 which is payable at the end of the year of death. Level premiums are payable annually in advance to age 65 or until earlier death. The company calculated the premium on the following basis:

Mortality	AM92 Select
Interest	6% per annum
Initial expenses	75% of the first year's premium, incurred at outset
Renewal expenses	5% of the second and each subsequent year's premium, incurred at the beginning of the respective policy years
Claims expense	£325 payable at the end of the year of death

- (i) Show that the annual premium is approximately £1,883. [4]

On 31 December 2013, immediately before the premium then due, the life wishes to surrender the policy. The insurance company calculates a surrender value equal to the gross prospective policy reserve, using the following basis:

Mortality	AM92 Ultimate
Interest	6% per annum
Expenses	Ignore

- (ii) Calculate the surrender value payable by the insurance company. [3]
 [Total 7]

- 10** (i) Calculate: $\bar{A}_{40:50}^1$. [2]

Basis:

Mortality	$\mu_x = 0.04$ throughout life for the life aged 40
	$\mu_x = 0.06$ throughout life for the life aged 50
Rate of interest	5% per annum

Two lives aged 40 and 50 exact purchase a policy with the benefit in part (i) above and a sum assured of 75,000. The benefit is funded by a premium payable continuously for a 30-year period or until the first death if earlier. The premium is paid at a level annual rate for the first 20 years, then reduces by 25% to be paid at the lower level annual rate for the remainder of the period.

- (ii) Calculate the initial level annual premium using the basis in part (i) above. [6]
 [Total 8]

11 A special joint life annuity is issued to a male life now aged 65 exact and a female life now aged 62 exact. The annuity is payable monthly in arrear and is subject to the following conditions:

- The amount of the annuity while both lives survive is 100,000 per annum.
- If the male life dies first leaving the female life surviving the annuity reduces to 50,000 per annum payable until she dies.
- If the female life dies first leaving the male life surviving the annuity reduces to 75,000 per annum payable until he dies.
- In addition if either life is alive at the 10th and 20th anniversaries of the policy a cash lump sum of 20,000 is paid at each date.

Calculate the expected present value of the annuity.

Basis:

Mortality	PMA92C20 and PFA92C20	
Interest	4% per annum	
Expenses	Nil	[9]

12 An insurance company issues a 25-year with-profit endowment assurance policy to a life aged 40 exact. The sum assured of £75,000 plus declared reversionary bonuses are payable on survival to the end of the term or immediately on death if earlier.

The insurance company assumes that future reversionary bonuses will be declared at a rate of 3% of the sum assured, simple and vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death). Premiums are payable in advance throughout the term of the policy or until earlier death.

Calculate the monthly premium.

Basis:

Mortality	AM92 Select
Interest	6% per annum
Initial commission	115% of the first monthly premium
Initial expenses	£210
Renewal commission	2.5% of each monthly premium payable excluding the first £85 per annum, inflating at 1.92308% per annum
Renewal expenses	compound, at the start of the second and subsequent policy years.
Inflation	For renewal expenses, the amount quoted is at outset, and the increases due to inflation start immediately. [9]

- 13** On 1 January 2004, an insurance company issued 15-year temporary assurance policies to 3,000 lives then aged 45 exact. For each policy, the sum assured is £100,000 for the first 10 years, and £40,000 thereafter. The sum assured is payable immediately on death and level annual premiums are payable in advance throughout the term or until earlier death.

Basis:

Mortality AM92 Ultimate
Interest 4% per annum

- (i) Show that the annual premium payable for each policy is approximately £233 using the basis above. [4]
- (ii) Calculate the reserve per policy as at 31 December 2013, assuming the reserving basis is the same as the premium basis. [3]
- (iii) (a) Describe the disadvantages to the insurance company of issuing this policy.
(b) Suggest two examples of how the terms of the policy could be altered so as to remove these disadvantages. [4]

There were 122 deaths between 2004 and 2012 inclusive and a further 12 deaths in 2013.

- (iv) Calculate the mortality profit or loss to the insurance company in 2013 on the basis above. [2]
[Total 13]

- 14** A life insurance company issues a three-year unit-linked endowment assurance policy to a life aged 58 exact under which level premiums of £3,000 are payable annually in advance throughout the term of the policy or until earlier death. The premium allocation rate (%) at time t is given by:

$$[75 + 20t] \text{ where } t = 0, 1 \text{ and } 2.$$

The units are subject to a bid-offer spread of 5%. An annual management charge of 0.75% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before any death, surrender or maturity benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of £9,000 or the bid value of the units if higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of each policy year. On surrender at the end of the policy year or on survival to the end of the term, the current bid value of the units is payable.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	4% per annum
Rate of interest on non-unit fund cash flows	2% per annum
Mortality	AM92 Select
Surrender	10% at the end of first, second and third policy years only
Initial expenses	£275
Renewal expenses	£70 per annum on the second and subsequent premium dates
Initial commission	5% of first premium
Renewal commission	2% of the second and subsequent years' premiums
Risk discount rate	6% per annum

- (i) Calculate the profit margin for the policy on the assumption that the company does not zeroise future expected negative cash flows. [13]

Suppose the company sets up reserves in order to zeroise future negative expected cash flows.

- (ii) Calculate the profit margin for the policy allowing for the cost of setting up these reserves. [4]
[Total 17]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

8 October 2015 (pm)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate:

(a) ${}_{25}P_{40}$

(b) ${}_{10|}q_{[53]}$

(c) $\bar{a}_{55:\overline{10}}$

Basis:

Mortality AM92
Interest 4% per annum

[3]

2 Derive (to the nearest integer) the median of the complete future lifetime of a person aged 30 exact who is subject to the force of mortality shown below:

$$\mu_{30+t} = \begin{cases} .01 & 0 \leq t < 10 \\ .02 & 10 \leq t < 20 \\ .03 & 20 \leq t \end{cases}$$

[3]

3 (i) Describe the difference between an overhead expense and a direct expense in the context of calculating premiums for a life assurance policy. [1]

(ii) (a) State an example of an overhead expense and a direct expense.

(b) Describe the manner in which each example in (a) is usually allowed for in the calculation of premiums. [3]
[Total 4]

4 (i) Describe how education may affect mortality. [1]

(ii) List three examples of the influence of education on mortality. [3]
[Total 4]

- 5** A special annuity pays 5,000 per annum for five years increasing to 6,000 per annum for the next five years and increasing further to 7,000 thereafter. The payments for the first five years are guaranteed and thereafter are contingent on survival. The annuity is payable monthly in advance.

Calculate the expected present value of this annuity for a life aged 60 exact. Show all your workings.

Basis:

Mortality PMA92C20
Interest 4% per annum

[5]

- 6** The employees of a manufacturing company are subject to two modes of decrement, mortality and withdrawal from employment.

The independent forces of mortality and withdrawal for employees aged 50 and 51 are given in the following table:

Age	μ_x^d	μ_x^w
50	0.0010	0.15
51	0.0015	0.10

Calculate, showing all your workings, the probability that a new employee aged 50 exact will die as an employee at age 51 last birthday. State any assumptions that you make.

[5]

- 7** A critical illness scheme provides a benefit of 100,000 on death or earlier diagnosis of a critical illness.

(i) Draw and label the appropriate transition diagram. [3]

(ii) Set out an expression for the expected present value of this benefit. [3]

[Total 6]

- 8** Calculate, showing all your workings, $a_{73.25}^{(4)}$.

Basis:

Mortality PFA92C20 (assume that the force of mortality is constant between ages 73 and 74 only)
Interest 4% per annum

[7]

9 A pension scheme provides a pension of one-sixtieth of final pensionable salary on retirement, due to age or ill-health, for each year of service (part years included). Final pensionable salary is average salary over the three years before retirement. Normal retirement age is 65. Members contribute 5% of pensionable salary each year.

- (i) Calculate the expected present value of the combined past and future benefits for a member aged 45 exact with 10 years of past service and salary in the previous year of 25,000. [5]
- (ii) Calculate the present value of the member's future contributions. [2]

Basis:

Pension Scheme Table in the Formulae and Tables for Examinations

[Total 7]

10 (i) Define in words the Area Comparability Factor. [2]

The table below shows an extract from a study of mortality for Country A and Area N:

Age	Country A		Area N	
	Population	Number of deaths	Population	Number of deaths
60	100,235	566	25,366	125
61	95,666	621	22,159	121
62	92,386	635	21,864	135

- (ii) Calculate, showing all your workings, the Area Comparability Factor for Area N using Country A as the standard population. [3]
 - (iii) Calculate, showing all your workings, the directly standardised mortality rate for Area N. [2]
- [Total 7]

11 An assurance policy provides a benefit of 10,000 payable immediately on the death of the last survivor of a male life aged 55 exact and a female life aged 50 exact.

- (i) Calculate, showing all your workings, the expected present value for this policy. [5]
- (ii) Derive an expression for the variance of the value of this policy. [3]

Basis:

Mortality PFA92C20
Interest 4% per annum

[Total 8]

- 12** A life insurance company issues a two year unit-linked endowment assurance policy to a male life aged 45 exact. Level premiums of 6,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death with 98% of each premium being allocated to units. A policy fee of 50 is deducted from the bid value of units at the start of each policy year. The units are subject to a bid-offer spread of 6%. An annual management charge of 1.25% of the bid value of units is deducted at the end of each policy year.

If the policyholder dies during the term of the policy, the death benefit of 200% of the bid value of the units is payable at the end of the policy year of death.

The policyholder may only surrender the policy at the end of the first policy year. On surrender, the bid value of units less a surrender value penalty of 500 is payable.

On maturity, 100% of the bid value of the units is payable.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	5.0% per annum in year 1 4.5% per annum in year 2
Rate of interest on non-unit fund cash flows	3.0% per annum in both years 1 & 2
Mortality	AM92 Select
Surrenders	2.5% of all policies in force at the end of policy year 1
Initial expense	225
Renewal expense	80 on the second premium date
Initial commission	7.5% of first premium
Renewal commission	2.5% of the second premium
Death claim expense	90
Maturity claim expense	55
Risk discount rate	6% per annum

(i) Calculate, showing all your workings, the non-unit fund cash flows in the first and second years of the policy if the policyholder:

- (a) dies in the first year of the policy.
- (b) surrenders in the first year of the policy.
- (c) dies in the second year of the policy.
- (d) survives to the end of the policy.

[7]

(ii) Derive the expected present value of profit for the policy in the event that the policyholder:

- (a) dies in the first year of the policy.
- (b) surrenders in the first year of the policy.
- (c) dies in the second year of the policy.
- (d) survives to the end of the policy.

[5]

(iii) Calculate, showing all your workings, the expected present value of the profit for the policy.

[1]

[Total 13]

13 A life assurance company issues a policy to a male life aged 40 exact which provides the following benefits:

- An annuity of 30,000 per annum, payable annually in advance starting on the policyholder's 65th birthday and continuing for life thereafter. The annuity increases by 1,500 each year, with the first increase given on the policyholder's 66th birthday.
- A decreasing term assurance with a death benefit, payable immediately on death, which is given by the formula:

$$10,000 \times (25 - t) \quad t = 0, 1, 2, \dots, 24$$

where t denotes the curtate duration in years since inception of the policy. Death benefit cover ceases at age 65.

The policy is paid for by level monthly premiums payable in advance from the date of issue for 25 years, but ceasing on earlier death.

The company uses the following premium basis for the policy:

Mortality	AM92 Select
Interest	4% per annum
Initial commission	35% of the total premiums payable in the first policy year
Initial expenses	225
Renewal commission	5% of the second and subsequent monthly premiums
Renewal expense	55 per annum at the start of the second and subsequent policy years
Death benefit claim expense	275
Annuity payment expense	2.5% of each annuity payment

The renewal expense and the death benefit claim expense are both assumed to increase continuously at 4% compound per annum from inception of the policy and to cease at age 65, or earlier death.

Calculate, showing all your workings, the monthly premium for the policy. [13]

- 14** (i) Write down in the form of symbols, and also describe, the expression “death strain at risk”. [2]

On 1 January 2011, a life insurance company issued the following three types of policies to male lives aged 55 exact:

- A 5-year pure endowment assurances with a sum assured of 75,000.
- B 5-year term assurances with a sum assured of 75,000, where the death benefit is payable at the end of the year of death.
- C 5-year single premium temporary immediate annuities with an annual benefit payable in arrear of 15,000.

For policies A and B, premiums are payable annually in advance throughout the policy term or until earlier death.

- (ii) Calculate, showing all your workings, the death strain at risk for each type of policy during 2014. [8]

Basis:

Mortality	AM92 Ultimate for policies A and B PMA92C20 for policy C
Expenses	Ignore
Interest	4% per annum

At the beginning of 2014, the numbers of policies in force were:

Pure endowment assurances	984
Term assurances	3,950
Temporary immediate annuities	495

During 2014, the actual deaths were 5 from policy A, 22 from policy B and 2 from policy C.

- (iii) Calculate, showing all your workings, the total mortality profit or loss to the company for 2014 using the same basis as in (ii). [5]
[Total 15]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

13 April 2016 (am)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 13 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** Calculate ${}_0.5 p_{90.25}$ using the method of Uniform Distribution of Deaths. Show all your workings.

Basis:

Mortality ELT15 (Males)

[3]

- 2** (i) State the two conditions under which the net premium prospective reserve will equal the net premium retrospective reserve. [2]

(ii) Describe two reasons why these conditions are unlikely to hold in practice. [2]

[Total 4]

- 3** Calculate:

(a) ${}_{25} p_{30}$

(b) $\ddot{a}_{[40]:\overline{15}}^{(4)}$

(c) $A_{50:\overline{20}}^1$

Basis:

Mortality AM92

Rate of interest 4% per annum

[4]

- 4** Describe with examples how selection applies to decrements experienced by members of a pension scheme. [6]

- 5** The table below shows an extract from a study of mortality for a small country:

<i>Age</i>	<i>Population</i>	<i>Number of deaths</i>
60	9,950	52
61	8,020	68
62	6,997	73

- (i) Show that the standardised mortality ratio using ELT15 (Males) as the standard population is 0.5. It can be assumed that the age definition of the country matches that of ELT15 (Males). [4]
- (ii) Explain what the result in part (i) indicates. [2]
- [Total 6]

- 6** (i) Prove that $\bar{A}_{x:n} = 1 - \delta \bar{a}_{x:n}$ for the following basis. [3]

Basis:

Force of mortality	μ_x is constant for all x
Force of interest	δ throughout

An endowment assurance pays a sum assured of 10,000 immediately on death or on survival to the end of the term of the policy.

- (ii) Calculate, showing all your workings, the premium payable continuously for a life aged 40 exact for an endowment assurance with a term of 20 years. [4]

Basis:

Mortality	$\mu_x = .01$ for all x
Rate of interest	5% per annum

[Total 7]

- 7** A five year unit-linked policy issued by an insurance company to a life aged 60 exact has the following profit vector:

$$(751.25, -321.06, -267.57, -192.05, 201.75)$$

- (i) Define the meaning of zeroisation in the context of this unit linked policy. [1]
- (ii) Explain why an insurance company might choose to zeroise the above profit vector. [1]
- (iii) Calculate, showing all your workings, the net present value of the profits of this policy after zeroisation.

Basis:

Mortality	AM92 Ultimate
Rate of interest on non-unit fund cash flows	3.5% per annum
Risk discount rate	6.0% per annum
[5]	
[Total 7]	

- 8** A company provides its employees with a benefit on disability before age 65. The benefit is a life annuity of 50% of salary at the date of disability.

- (i) Draw and label a transition state diagram for this benefit. [4]
 - (ii) Derive a formula for the expected present value of this benefit for a life aged x with a current annual salary of 20,000. [3]
- [Total 7]

- 9** A company provides a cash benefit of five times salary on disability before retirement where normal retirement age is 65.

Determine the expected present value of this benefit for a life aged 63 exact with current annual salary of 50,000 stating all your assumptions.

Basis:

Independent force of mortality	ELT15 (Males)
Independent force of disability	0.03
Discount rate	5% per annum
Salary increase	3% at age 64
[8]	

- 10** A life insurance policy for a male life aged 55 exact provides the following benefits:
- 50,000 payable immediately on his death, if this occurs before the age of 65 exact.
 - On survival to age 65 exact, a refund of 25% of total premiums paid without interest.
 - On death of the male at any time, a pension of 5,000 per annum is payable monthly in advance to his widow (who is 5 years younger than him) for the remainder of her life, should she survive him. (This benefit is available throughout the lifetime of the male.)

The policy is funded by premiums payable annually in advance for five years, or until the death of the male life, if earlier.

Basis:

Male mortality	PMA92C20
Female mortality	PFA92C20
Rate of interest	4 % per annum
Expenses	Nil

Calculate, showing all your workings, the premium for this policy. [9]

- 11** On 1 January 2012, a life insurance company issued joint life whole life assurance policies. Each policy was issued to a male life aged 65 exact and a female life aged 60 exact. A sum assured of 75,000 is payable immediately on the death of the second of the lives to die.

Premiums of 1,395.11 are payable annually in advance for each policy while at least one of the lives is alive.

At the beginning of 2014, there were 5997 policies in force. For all of these policies, both lives were still alive. During 2014, the following experience was observed:

- for 2 policies, both lives died
- for 12 policies, only the male life died
- for 8 policies, only the female life died

Calculate, showing all your workings, the mortality profit or loss for the group of policies for the calendar year 2014.

Basis:

Mortality	PMA92C20 for the male PFA92C20 for the female
Rate of interest	4% per annum
Expenses	Ignore

[10]

- 12** On 1 March 1997, a life insurance company issued a whole life with profit policy to a life then aged 45 exact. The basic sum assured was 150,000. The sum assured (together with any bonuses attaching) is payable immediately on death. Level premiums are payable monthly in advance to age 85 or until earlier death. Compound reversionary bonuses vest at the beginning of each policy year (i.e. the death benefit includes any bonus relating to the policy year of death).

The company calculates the premium on the following basis:

Mortality	AM92 Select
Rate of interest	6% per annum
Initial expenses	70% of the first year's premium, incurred at the outset
Renewal expenses	5% of the second and each subsequent year's premium, incurred at the beginning of the respective policy years
Bonuses:	1.92308% per annum compound

- (i) Show that the monthly premium is approximately 276. [7]

On 28 February 2015, the company alters the policy at the request of the policyholder to a paid-up policy with no future premiums payable. The sum assured under the policy is reduced, with no further bonuses payable.

The company calculates the reduced sum assured after alteration by equating prospective gross premium policy reserves immediately before and after alteration, allowing for an expense of alteration of 175.

The company calculates prospective gross premium policy reserves for the purpose of the alteration using the following basis:

Mortality	AM92 Ultimate
Rate of interest	6% per annum
Expenses	ignore
Future bonuses	ignore

Bonuses have vested at a rate of 2% per annum compound at the beginning of each policy year from the date of issue of the policy.

- (ii) Calculate, showing all your workings, the sum assured for the policy after alteration. [6]
 [Total 13]

- 13** On 1 January 2015, a life insurance company issued four year increasing term assurance policies with level premiums payable annually in advance for the term of the policy, but ceasing on earlier death. The initial sum assured is 140,000, increasing by 20,000 at each policy anniversary (the first increase taking place at the beginning of the second policy year). The death benefit is payable at the end of the year of death. If the policyholder survives to the end of the term of the contract, 50% of the total premiums paid (accrued with no interest) is payable.

The company calculates the premium on the following basis:

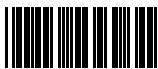
Mortality	AM92 Select
Rate of interest	6% per annum
Initial expense	275
Initial commission	30% of the first year's premium, incurred at the outset
Renewal expense	55 per annum, incurred at the time of payment of the second and subsequent years' premiums
Renewal commission	2.5% of the second and subsequent years' premiums

For a male life aged 56 exact at inception of the policy:

- (i) Set out, in stochastic form, the gross future loss random variable at the outset of this policy using where applicable, T_x , K_x and elements of the premium basis that are relevant. [3]
 - (ii) Calculate, showing all your workings, the office premium using annuity and assurance functions and setting the expected present value of the gross future loss random variable equal to zero. [4]
 - (iii) Calculate, showing all your workings, the office premium using a discounted cash flow projection, assuming no surrenders, ignoring reserves and using the same profit criterion as in part (ii) above. The discount rate is assumed to be 6% per annum. [6]
 - (iv) Explain, without further calculations, the effect of:
 - (a) allowing for the setting up of reserves for the calculation in (iii).
 - (b) having set up reserves in (iv) (a), increasing the discount rate to 8% per annum.
- [3]
[Total 16]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

7 October 2016 (pm)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 13 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** A whole life assurance policy provides a benefit of 100,000 payable immediately on the death of a male life who is now aged 45 exact.

Calculate, showing all your workings:

- (a) the expected present value of this policy.
- (b) the variance of the value of this policy.

Basis:

Mortality AM92 Ultimate
Rate of interest 4% per annum

[4]

- 2** A 10-year unit-linked policy has the following profit vector:

$$(-50, -10, -10, 5, 5, 5, -3, 15, 40, 60)$$

Reserves are set up to zeroise future negative cash flows on the following basis:

Basis:

Mortality The probability of death at each age is a constant 0.25% per annum
Rate of interest 1.5% per annum

Determine the revised profit vector.

[4]

- 3** A pension scheme provides the following benefits:

- a pension of two-thirds of Final Pensionable Salary on ill-health retirement, where Final Pensionable Salary is the average pensionable salary over the 3 years before retirement.
- a lump sum immediately on death before retirement of four times pensionable salary in the year before death.

Normal Pension Age is age 65.

A life aged 42 exact has pensionable salary in the previous year of 100,000.

- (i) Calculate, showing all your workings, the present value of the ill-health retirement pension for this life.

Basis: Pension Scheme Table in the Formulae and Tables for Examinations [2]

- (ii) Write down an expression for the value of the lump sum death benefit for the same life using commutation functions. [2]
[Total 4]

4 Calculate, showing all your workings:

(a) ${}_{10|5}q_{65}$.

(b) $\ddot{a}_{[30]:15}^{(12)}$.

Basis:

Mortality AM92

Rate of interest 4% per annum

[4]

5 Describe how a life insurance company uses risk classification to manage the probability of making a loss from selling its products.

[6]

6 (i) Show, using the method of Uniform Distribution of Deaths, that:

$${}_{2.5}q_{80.75} = \frac{10}{117}$$

[3]

(ii) Calculate, showing all your workings, $\ddot{a}_{80:\overline{4}}$.

Basis:

Mortality $l_x = 110 - x$ for all x

Rate of interest 5% per annum

[3]

[Total 6]

7 A life insurance company writes policies that provide income during periods of disability.

(i) Draw the transition state model for these policies labelling your diagram carefully.

[5]

(ii) Describe two examples of selection that might apply for these policies.

[2]

[Total 7]

- 8** (i) Describe one advantage and one disadvantage of using single figure indices to summarise and compare mortality levels. [2]

The table below shows an extract from a study of mortality for a country and a given sub-population:

Age	Country		Sub-population	
	Population	Number of deaths	Population	Observed mortality rate
40–44	834,561	3,510	123,978	0.0029
45–49	779,862	3,153	116,853	0.0033
50–54	750,234	3,620	102,800	0.0051

- (ii) Calculate, showing all your workings, the Standardised Mortality Ratio for the sub-population, using the country as the standard population. [4]
- (iii) Comment on what the value of the Standardised Mortality Ratio shows about the sub-population. [2]

[Total 8]

- 9** A life insurance company issues a 35-year with-profits endowment assurance policy to a life aged 30 exact. The sum assured of 125,000 plus declared reversionary bonuses are payable at the end of the year of death or on maturity.

Level premiums of 3,090 are payable annually in advance for 35 years or until earlier death.

A simple bonus, expressed as a percentage of the sum assured, is added to the policy at the end of each year. The death benefit does not include the bonus relating to the policy year of death.

The following basis is used to price this policy:

Mortality	AM92 Select
Rate of interest	4% per annum
Initial expenses	325 plus 75% of the first annual premium, incurred at the policy commencement date
Renewal commission	2.5% of each premium from the start of the second policy year
Claim expense	375 at the point of claim payment

Calculate, showing all your workings, the level simple bonus rate that can be supported each year on this policy, using the principle of equivalence. [9]

- 10** A 25-year “double” endowment assurance policy is issued to a group of lives aged 40 exact. Each policy provides a sum assured of 25,000 payable at the end of the year of death or 50,000 payable if the life survives until the maturity date.

Premiums are payable annually in advance throughout the term of the policy or until earlier death.

The following information has been provided:

Number of deaths during the 17th policy year: 24
Number of policies in force at the end of the 17th policy year: 5,350

- (i) Calculate, showing all your workings, the profit or loss for the group arising from mortality in the 17th policy year. [7]
- (ii) Comment on your result. [2]

Basis:

Mortality AM92 Select
Rate of interest 4% per annum
Expenses Ignore

[Total 9]

- 11** A life insurance company issues a joint annuity policy to a male aged 60 exact and a female aged 62 exact. Under the policy:

- an annuity of 50,000 per annum is guaranteed to be payable for a period of 10 years and thereafter for the lifetime of the male.
- on the death of the male, an annuity of 20,000 per annum is payable to the female, if she is still alive. This annuity commences on the monthly payment date next following, or coincident with, the date of his death or from the 10th policy anniversary, if later. It is payable for the lifetime of the female.
- all annuities are payable monthly in arrear.

Determine the expected present value of the policy.

Basis:

Mortality PMA92C20 for the male and PFA92C20 for the female
Rate of interest 4% per annum
Expenses Ignore

[10]

- 12** A life insurance company issues a 15-year decreasing term assurance policy to a life aged 50 exact. The initial sum assured is 450,000, decreasing by 30,000 at each policy anniversary (the first decrease taking place at the beginning of the second policy year). The death benefit is payable immediately on death.

Level annual premiums are payable in advance for 15 years, ceasing on earlier death.

The life insurance company uses the following basis for calculating premiums and reserves:

Mortality	AM92 Select								
Rate of interest	4% per annum								
Expenses	<table> <tr> <td>Initial</td><td>275 plus 30% of the first premium</td></tr> <tr> <td>Renewal</td><td>5% of all premiums excluding the first plus 68 per annum inflating at 4% per annum compound at the start of the second and subsequent policy years.</td></tr> <tr> <td>Claim</td><td>315 inflating at 4% per annum compound</td></tr> <tr> <td>Inflation</td><td>For renewal and claim expenses, the amounts quoted are at outset of the policy and the increases due to inflation start immediately.</td></tr> </table>	Initial	275 plus 30% of the first premium	Renewal	5% of all premiums excluding the first plus 68 per annum inflating at 4% per annum compound at the start of the second and subsequent policy years.	Claim	315 inflating at 4% per annum compound	Inflation	For renewal and claim expenses, the amounts quoted are at outset of the policy and the increases due to inflation start immediately.
Initial	275 plus 30% of the first premium								
Renewal	5% of all premiums excluding the first plus 68 per annum inflating at 4% per annum compound at the start of the second and subsequent policy years.								
Claim	315 inflating at 4% per annum compound								
Inflation	For renewal and claim expenses, the amounts quoted are at outset of the policy and the increases due to inflation start immediately.								
(i)	Write down the Gross Premium Future Loss Random Variable at the start of the policy. Use P for the annual premium. [4]								
(ii)	Calculate, showing all your workings, the premium, using the principle of equivalence. [8]								
(iii)	Calculate, showing all your workings, the gross premium prospective reserve after 14 years. [2]								
	[Total 14]								

- 13** A life insurance company issues a 3-year endowment assurance policy to an unmarried life that offers the following benefits:

- On marriage, a return of 107.5% of total premiums paid.
- On surrender, a return of 50% of total premiums paid.
- On death, a benefit which is given by the formula:

$$10,000 \times (1 + t) \quad t = 0, 1 \text{ and } 2$$

where t denotes the curtate duration in years since the inception of the policy.

- On survival, 30,000 is payable immediately.

The marriage, surrender and death benefits are payable at the end of the policy year of claim.

Premiums of 9,516 are payable annually in advance throughout the term of the policy or until earlier claim.

The policy ceases on payment of any benefit.

The company uses the following basis to profit test this policy:

Independent force of marriage	15%
Independent force of surrender	7.5% in years 1 and 2 only
Independent force of mortality	1%
Interest earned on cash flows	3.5% per annum
Expenses	1.5% of each premium paid
Reserves	None held

The company assumes that:

- each force of decrement is independent and constant over each year of age.
 - surrenders only occur in policy years 1 and 2.
- (i) Determine for each policy year the dependent rates of mortality, marriage and surrender. [4]
- (ii) Derive the expected cash flows for the policy for each policy year. [7]
- (iii) Calculate, from part (ii) the expected present value of the profit or loss to the company for each policy year and in total. Use a risk discount rate of 4% per annum. [2]
- (iv) Discuss the consequences for the company of the profit vector derived in part (iii). [2]
- [Total 15]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

19 April 2017 (am)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 13 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** Calculate ${}_{2.75} p_{77.4}$ assuming a Uniform Distribution of Deaths.

Basis:

Mortality PMA92C20

[3]

- 2** Under a 20-year policy issued by a life insurance company, the benefit payable on death, at the end of the year of death, is a return of premiums paid without interest.

A premium of 631 is payable annually in advance, throughout the term of the policy.

The following information is available for a policy in force at the start of the 19th year:

Reserves at the start of the year, ${}_{18}V$: 17,095

Reserves at the end of the year per survivor, ${}_{19}V$: 18,510

Probability of death during the year: 0.015

Rate of interest earned: 4.5% per annum

Determine the profit which is expected to emerge at the end of the 19th year for each policy in force at the start of that year. Ignore expenses and all decrements other than death.

[3]

- 3** Calculate:

(a) ${}_{12} p_{73}$.

(b) ${}_{10|} a_{56}$.

(c) $A_{64:\overline{10}}$.

Basis:

Mortality AM92

Rate of interest 4% per annum

[3]

- 4** Describe the principal features of a non-unitised accumulating with-profits contract.

[4]

- 5** A small country has just two major cities A and B of approximately equal size.

The following statistics are available for the two cities in relation to the working population:

Age Band	City A		City B	
	Exposed to Risk	Observed Mortality Rate	Exposed to Risk	Observed Mortality Rate
20–29	200,000	0.00115	250,000	0.00125
30–39	350,000	0.00187	325,000	0.00171
40–49	400,000	0.00402	375,000	0.00358
50–59	250,000	0.01386	200,000	0.01515
60–69	100,000	0.04271	125,000	0.04368

Calculate:

- (a) the directly standardised mortality rate for City B
- (b) the indirectly standardised mortality rate for City B

using the combined population of both cities as the standard population. [6]

- 6** A pension scheme provides an age retirement pension of 1% of Final Pensionable Salary for each year of service. Final Pensionable Salary is defined as the average Pensionable Salary over three years before retirement less 1,000.

Age retirement in normal health follows the principles in the Pension Scheme Table for age retirement functions in the Formulae and Tables for Examinations.

A life is aged 54 exact with 10 years of past service and pensionable salary in the previous year of 30,000.

Set out the expected present value of the past service part of this life's pension in a formula not using commutation functions. Define all symbols used. [7]

- 7** (i) Describe, in the context of underwriting for life insurance, the following selection processes:

- (a) adverse selection
- (b) spurious selection

[5]

- (ii) Describe an example of each selection process in part (i). [2]
[Total 7]

- 8** (i) Describe in words the difference between the functions $(\bar{IA})_{x:\overline{n}}$ and $(\bar{IA})_{x:\overline{n}}$. [2]

(ii) Determine, showing all your working, $(\bar{IA})_{x:\overline{15}}$.

Basis:

Force of mortality $\mu_x = 0.02$ for all x

Force of interest 3%

[5]
[Total 7]

- 9** An assurance policy provides a benefit of 1 payable immediately on the death of the last survivor of a male life aged 55 exact and a female life aged 50 exact.

Determine:

(i) the expected present value of this policy. [4]

(ii) the variance of the present value of this policy. [4]

Basis:

Force of mortality Male life – a constant force of 0.03

Female life – a constant force of 0.02

Force of interest 4%

[Total 8]

- 10** A special joint-life deferred annuity policy provides the following benefits:

- 20,000 payable immediately on each death at any age
- a pension payable monthly in advance after 10 years at a rate of 10,000 per annum if both lives are alive and 5,000 per annum if only one life is alive

Premiums are payable monthly in advance until the first death for a maximum of 10 years.

Show that the monthly premium payable for a male life aged 55 exact and a second female life aged 50 exact is approximately 1,114.

Basis:

First life mortality PMA92C20

Second life mortality PFA92C20

Rate of interest 4% per annum

Expenses Ignore

[9]

- 11** On 1 January 2000 a life insurance company issued a number of 20-year pure endowment policies to a group of lives aged 40 exact. In each case, the sum assured was 60,000 and premiums were payable annually in advance throughout the term or until earlier death.

On 1 January 2016, 18,230 policies were still in force. During 2016, 86 policyholders died, and no policy lapsed for any other reason.

- (i) Calculate the profit or loss from mortality for this group for the year ending 31 December 2016. [7]

Basis:

Mortality	AM92 Select
Rate of interest	4% per annum
Expenses	Ignore

- (ii) Comment on your answer in part (i). [2]
[Total 9]

- 12** A life insurance company issues a 30-year with-profits endowment assurance policy to a life aged 35 exact. The sum assured is 100,000 together with any attaching bonuses and is payable immediately on death.

Level premiums are payable monthly in advance, ceasing on maturity or on the policyholder's death if earlier.

Simple annual bonuses are added at the end of each policy year. The death benefit does not include any bonus relating to the policy year of death.

The company calculates the premium on the following basis:

Mortality	AM92 Select
Rate of interest	4% per annum
Expenses	
Initial	325
Renewal	70 at the start of the second and subsequent policy years and payable until death
Claim	275 on death
Commission	
Initial	70% of the total premium payable in the first policy year
Renewal	2.5% of the second and subsequent monthly premiums
Bonuses	Simple bonus of 2.5% of basic sum assured per annum

- (i) Show that the monthly premium for this policy is approximately 292. [9]

As at the end of the 28th policy year, the total actual past bonus additions to the policy have followed the assumptions stated in the premium basis above.

- (ii) Calculate the gross prospective policy value at the end of the 28th policy year.

Policy value basis:

Mortality	AM92 Ultimate	
Rate of interest	4% per annum	
Expenses		
Renewal	85 at the start of each policy year and payable until death	
Claim	300 on death	
Commission		
Renewal	2.5% of the monthly premiums	
Bonuses	Simple bonus of 2.75% of basic sum assured per annum	[6]
		[Total 15]

- 13 A life insurance company issues a 3-year unit-linked endowment assurance policy to a male life aged 60 exact. The details are:

- Level premiums of 9,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death.
- 80% of the premium is allocated to units in the first policy year and 100% in the second and third policy years.
- A policy fee of 25 is deducted from the annual premium before the allocation to units.
- The units are subject to a bid-offer spread of 5%.
- An annual management charge of 1.5% of the bid value of the units is deducted at the end of each policy year.
- Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.
- If the policyholder dies during the term of the policy, the death benefit of 125% of the bid value of the units is payable at the end of the policy year of death.
- On maturity, 100% of the bid value of the units is payable.
- The policyholder may surrender the policy at any time during the first and second policy years.

- On surrender, the bid value of the units less a surrender penalty is payable at the end of the policy year of exit as follows:

<i>Year</i>	<i>Penalty</i>
1	600
2	300

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	4.5% per annum in year 1 4.0% per annum in year 2 3.5% per annum in year 3
Rate of interest on non-unit fund cash flows	2.0% per annum
Mortality	AM92 Select
Initial expenses	220
Renewal expenses	75 per annum on the second and subsequent premium dates
Initial commission	30% of first premium
Renewal commission	1.5% of the second and subsequent years' premiums
Rate of expense inflation	2.0% per annum
Risk discount rate	6.5% per annum

For renewal expenses, the amount quoted is at the commencement of the policy, and the increases due to inflation start immediately.

The company assumes that the force of decrement due to surrender is:

- 0.1 in policy year 1.
- 0.05 in policy year 2.

It also assumes that each force of decrement is independent and constant over each year of age.

- Determine for each policy the dependent rates of mortality and surrender. [4]
- Calculate the profit margin for the policy. [11]

The company now assumes that there are no surrenders.

- Calculate the expected present value of profit for the policy. [4]
[Total 19]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

27 September 2017 (pm)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 13 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 List the four main categories of costs which are affected by inflation in the context of allocating expenses to life assurance contracts. [2]

2 Describe what is meant by spurious selection, including an example. [3]

3 Calculate ${}_2.25q_{85.5}$ using the method of Uniform Distribution of Deaths.

Basis: ELT15 (Males)

[4]

4 Explain why a life insurance company will need to set up reserves for the level premium conventional whole life assurance contracts it has sold. [4]

5 (i) Calculate $\bar{A}_{47:\overline{11}}$. [3]

(ii) Calculate $\ddot{a}_{[53]:\overline{13}}^{(4)}$. [2]

Basis: Mortality AM92

Interest 4% per annum

[Total 5]

6 (i) Calculate $\ddot{a}_{40:\overline{4}}$. [2]

(ii) Derive the value of $A_{40:\overline{4}}^1$, using your result from part (i). [4]

Basis:

From the following life table extract

x	l_x
40	100,000
41	99,200
42	98,100
43	96,700
44	94,700

Interest 5% per annum

[Total 6]

- 7** A population is subject to two modes of decrement, α and β as defined below:

$$\mu_x^\alpha = 1/(110 - x) \quad \text{for } 0 \leq x < 110; \text{ and}$$

$$\mu_x^\beta = 0.03 \quad \text{for } 0 \leq x < 110$$

You are given that $\int_0^1 te^{-0.03t} dt = 0.490112$

Determine the value of $(aq)_{40}^\beta$. [7]

- 8** A company is about to establish a pension scheme that will provide retirement benefits to its members of $n/80$ ths of final pensionable salary at age 65 or on earlier ill health, where n is the total number of years of service to retirement. Age retirement in normal health follows the principles in the Pension Scheme Table for age retirement functions in the Formulae and Tables for Actuarial Examinations. Final pensionable salary is the average salary in the three years before retirement.

An employee joins the scheme aged 45 exact and is granted exactly 15 years of past service. The employee's salary in the year before joining was 35,000.

- (i) Calculate the present value of benefits for this member (including future service). [5]
- (ii) Calculate the contribution required, as a percentage of future salary, to fund this benefit. [3]

Basis:

Pension Scheme from the Formulae and Tables for Actuarial Examinations

[Total 8]

- 9** A special whole life assurance policy is issued on a life aged 50 exact.

Under this policy the sum assured, payable at the end of the year of death, is 1 unit for the first 10 years decreasing to 0.75 units thereafter.

- (i) Calculate the expected present value of the benefit. [3]
- (ii) Determine the variance of the present value of the benefit. [5]

Basis:

Mortality AM92 Ultimate
Interest 4% per annum

[Total 8]

10 (i) Explain each of the following terms in words, without giving any formulae:

- (a) crude mortality rate
- (b) directly standardised mortality rate
- (c) indirectly standardised mortality rate
- (d) standardised mortality ratio

[4]

The following table summarises the mortality experience of a particular region of a country compared to the whole population of the country.

<i>Age Group</i>	<i>Regional Group</i>		<i>Whole Population</i>	
	<i>Number of lives</i>	<i>Deaths</i>	<i>Number of lives</i>	<i>Deaths</i>
20–34	40,000	42	1,000,000	1,300
35–49	75,000	135	1,600,000	3,200
50–64	35,000	110	900,000	2,500
TOTAL	150,000	287	3,500,000	7,000

(ii) Calculate the value of each of the terms defined in part (i), for the regional group. [5]
[Total 9]

- 11** A life insurance company issues a large number of 4-year unit-linked endowment assurance policies to lives aged 65 exact. Level premiums are payable annually in advance until maturity or earlier death.

The company has performed a profit test on these policies and the profit vector per policy sold, ignoring surrenders, is as follows:

$$(185.21, -121.52, -5.28, 12.95)$$

- (i) Calculate the profit signature per policy sold if negative non-unit fund cash flows are zeroised. [3]

The company now wishes to allow for surrenders in its calculations. It assumes that at the end of the first and second policy years only, 3% of the surviving policyholders will surrender. Surrender values are equal to the bid value of units held (after deduction of the fund management charge) less a surrender penalty of 50.

- (ii) Calculate the revised profit signature per policy sold after allowing for surrenders if negative non-unit cash flows are zeroised. [6]

- (iii) Calculate the net present value of the revised profit signature in part (ii), using a risk discount rate of 8% per annum. [1]

Basis:

Mortality	AM92 Ultimate
Interest earned on non-unit cash flows	5% per annum fund
Expenses	Ignore
[Total 10]	

- 12** A life insurance company issues a 3-year guaranteed bonus endowment assurance policy to a life aged 62 exact with a basic sum assured of 75,000. The basic sum assured, together with any attaching bonuses, is payable at the end of the year of death or maturity if earlier.

Level premiums are payable annually in advance throughout the term of the policy or until earlier death.

Simple annual bonuses are added at the beginning of each policy year (i.e. the death benefit does include any bonus relating to the policy year of death).

The company uses the following basis for carrying out profit tests of this policy:

Mortality	AM92 Ultimate
Withdrawals	Ignore
Interest earned	5% per annum on cash flows
Expenses	Initial 15% of the first premium Renewal 5% of subsequent premiums
Bonuses	Simple bonus of 4% of basic sum assured per annum

The company holds net premium reserves for the policy using the following basis:

Mortality	AM92 Ultimate
Interest	4% per annum

- (i) Calculate the net premium reserve for the policy at policy duration $t = 1$ and $t = 2$ years immediately before the premium then due. [8]
- (ii) Determine the annual premium required for the policy to achieve an internal rate of return of 6% per annum to the company. [9]
- [Total 17]

- 13** On 1 January 2000, a life insurance company issued 25-year increasing term assurance policies to single lives aged 40 exact.

The death benefit, payable at the end of the year of death, was 50,000 in the first policy year and increased at the beginning of each policy year at a rate of 1.92308% per annum compound. The first increase was at the start of the second policy year.

A return of premiums paid, with no interest, is payable on survival to the end of the term of the policy.

Level premiums on the policies are payable annually in advance for 25 years or until earlier death.

The company calculates its reserves on a net premium basis and negative reserves are permitted.

- (i) Show that the annual net premium for each policy is approximately equal to 323 using the basis below. [4]

At the start of 2016, there were 1425 policies in force. 10 policyholders died during 2016.

- (ii) Calculate the mortality profit or loss to the company during 2016 using the basis below. [7]

After an analysis of surplus investigation by the company, it found that it had made neither a profit nor a loss in 2016 in respect of the policies in part (ii).

- (iii) Determine the rate of interest the company earned in 2016. [6]

Basis:

Mortality AM92 Select
Interest 6% per annum
Expenses Ignore

[Total 17]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

23 April 2018 (am)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 13 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** Calculate ${}_2.75q_{84.5}$ using the method of Constant Force of Mortality.
 Basis: AM92 [3]
- 2** Describe three types of reversionary bonus that may be awarded to a with-profits contract. [4]
- 3** A life aged 50 exact purchases a single-premium temporary annuity. The annuity pays 7,500 annually in arrears for a term of 10 years, ceasing on death, if earlier.
 Calculate the reserve for the annuity at the end of the first policy year, using the net retrospective method and the following basis:
 Mortality AM92 Select
 Interest 4% per annum
 Expenses Ignore [4]
- 4** Describe the impact of occupation on mortality and morbidity. [5]
- 5** On 1 January 2010, a life insurance company issued single life annuities to policyholders then aged 65 exact.
 Each annuity is for 30,000 payable annually in arrears.
 At the beginning of 2017, there were 5650 policyholders alive and during 2017, 80 policyholders died.
 The company calculates its reserves using the following basis:
 Mortality PMA92C20
 Interest 4% per annum
 Expenses Ignore
 (i) Calculate the mortality profit or loss for these annuities for the year 2017. [4]
 (ii) Comment on your answer in part (i). [2]
 [Total 6]

- 6** (i) Give expressions, using random variables, for the expected present value of:
- an annual temporary annuity-due.
 - a joint whole-life insurance payable immediately on first death.

Define any symbols that you use.

[4]

A life insurance company issues continuous reversionary annuities based on two lives, x and y . Payments begin on the death of the first life, x , provided the second life, y , is alive, and cease on the death of the second life, y .

- (a) State, using random variables, the present value of this annuity.
- (b) Give an expression for the expected present value of this annuity in terms of life table functions.

[4]

[Total 8]

- 7** An employer wishes to introduce a lump sum retirement benefit scheme.

Employees will receive a lump sum benefit of 1,500 for each year of service, payable immediately on retirement at age 65 or earlier in normal health. In the event of ill-health retirement the benefit is 1,000 for each year of service payable immediately. Service includes proportionate parts of a year.

- Give a formula in terms of commutation functions to value the total benefits (past and future service) for an employee currently aged x with n years of past service. Definition of terms is not required. [3]
- Calculate the expected present value of the benefits for an employee currently aged 45 exact with exactly 15 years' past service. [3]
- Calculate the level annual contribution payable continuously throughout this employee's future service to fund the benefits. [2]

Basis: Pension Scheme from the Formulae and Tables for Actuarial Examinations

[Total 8]

- 8** A term assurance policy is issued on a life aged x for a term of 20 years.

Under this policy a sum assured, payable immediately on death, is 10,000 for the first 10 years increasing to 20,000 for the subsequent 10 years.

(i) Calculate the expected present value of the benefits. [3]

(ii) Determine the variance of the present value of the benefits. [5]

Basis:

Mortality $\mu_x = 0.03$ for all x
Force of Interest $\delta = 5\%$ throughout

[Total 8]

- 9** A life insurance company issues 20-year critical illness term assurance policies. The benefits, payable during the policy term, are a lump sum of 50,000 payable immediately on diagnosis of a critical illness, or a benefit of 100,000 immediately on death if earlier.

Premiums are payable monthly in advance during the term of the policy, ceasing on any claim.

(i) Draw a transition state model for this policy labelling your diagram. [3]

A policy is issued to a life aged 45 exact, with an annual premium rate of 750.

(ii) Calculate the value of the policy to the company. [6]

Basis:

Mortality $m_x = 0.004$ for all x
Critical Illness $s_x = 0.002$ for all x
Force of Interest $\delta = 4\%$ throughout

[Total 9]

- 10** A life insurance company sells a special deferred annuity policy to a life aged 60.75 years exact (i.e. 60 years and 9 months). The policy is funded by quarterly premiums in advance ceasing at age 65 (no premium is payable at age 65 exact).

The benefits under the policy are as follows:

- At age 65, an annuity of 12,000 a year is payable half-yearly in advance. Payment is guaranteed for five years and then continues to the age of 75 when it reduces to 10,000 a year under the same payment method until the death of the policyholder.
- At age 75, an additional lump sum benefit of 10,000 is payable. The annuity then reduces to 10,000 a year, ceasing on the death of the policyholder.

Determine that the quarterly premium is approximately 9,030.

Basis:

Mortality	PFA92C20 (assume the uniform distribution of deaths method)
Rate of Interest	4% per annum
Expenses	Ignore

[9]

- 11** A life insurance company issues 30-year term assurance policies to lives aged 35 exact. For each policy, the initial sum assured is 90,000 which increases by 45,000 at the start of the 16th policy year. The sum assured is payable immediately on death. Level monthly premiums are payable in advance throughout the term of these policies or until earlier death.

The company uses the following basis for calculating premiums and reserves:

Mortality	AM92 Select
Interest	4% per annum
Initial commission	50% of the total premium payable in the first policy year
Initial expenses	375 at policy commencement date
Renewal commission	2.5% of each premium from the start of the second policy year
Renewal expenses	72 per annum, inflating at 4% per annum compound, at the start of the second and subsequent policy years (the renewal expense quoted is at outset and the increases due to inflation start immediately)
Claim expense	295 on death

Calculate the monthly premium for the policy.

[10]

- 12** On 1 January 2013, a life insurance company issued a with-profits whole-life policy to a life then aged 25 exact. Under the policy, the basic sum assured of 150,000 and attaching bonuses are payable immediately on death. The company declares simple reversionary bonuses at the start of each year (i.e. the death benefit includes the bonus relating to the policy year of death). Level premiums are payable annually in advance under the policy.

- (i) Give an expression for the gross future loss random variable under the policy at the outset using the basis below. [3]
- (ii) Calculate the gross annual premium, using the equivalence principle and the same basis.

Basis:

Mortality	AM92 Select
Interest	6% per annum
Bonus loading	4% per annum simple
Expenses	Initial 265
	Renewal 5% of each premium payable in the second and subsequent years
	Claim 315

[4]

On 31 December 2017 the policy is still in force. The actual bonuses added to the policy have been as follows:

<i>Year</i>	<i>Simple bonus per annum</i>
2013	4.0%
2014	4.0%
2015	3.75%
2016	3.5%
2017	3.0%

- (iii) Determine the gross premium prospective reserve for the policy as at 31 December 2017 using the following basis:

Mortality	AM92 Ultimate
Interest	4% per annum
Bonus loading	3% per annum simple
Renewal expenses	5% of each premium
Claim expenses	325

[4]

[Total 11]

- 13** A life insurance company issues a three-year unit-linked endowment assurance contract to a male life aged 62 exact under which level annual premiums of 6,000 are payable in advance throughout the term of the policy or until earlier death. 90% of each year's premium is invested in units at the offer price.

There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

There is an annual management charge of 1% of the bid value of units. Management charges are deducted at the end of each year, before death or maturity benefits are paid.

On the death of the policyholder during the term of the policy, the benefit, payable at the end of the year of death, is 12,000, or the bid value of the units if greater. The policyholder may surrender the policy only at the end of each year immediately before a premium is payable. On surrender, the bid value of the units is payable at the end of the year of exit. On maturity, 110% of the bid value of the units is payable.

The company holds unit reserves equal to the full bid value of the units. It sets up non-unit reserves to zeroise any negative non-unit fund cash flows, other than those occurring in the first year.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	5% per annum
Rate of interest on non-unit fund cash flows	3% per annum
Mortality	AM92 Ultimate
Surrenders	10% at the end of the first policy year, 5% at the end of the second policy year based on policies in force at that time
Initial expenses	225 plus 5% of the first premium (all incurred on policy commencement)
Renewal expenses	65 at the start of each of the second and third policy years plus 2.5% of the second and third premiums
Risk discount rate	7% per annum

Calculate the profit margin on the contract.

[15]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

17 September 2018 (pm)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 13 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate:

(a) ${}_{12}q_{[54]}$

(b) $\ddot{a}_{65}^{(6)}$

(c) $\overline{s}_{43:\overline{10}}$

Basis:

Mortality	AM92
Rate of Interest	4% per annum

[4]

2 A reversionary continuous annuity begins on the death of life x , if a second life y is then alive. Payment continues during the lifetime of y .

(a) State, using random variables, the present value of this annuity.

(b) Give an expression for the expected present value of this annuity in terms of assurance functions.

[4]

3 A life insurance company sells a special immediate annuity policy to a life aged 65 exact. The policy provides an annuity of 30,000 a year payable monthly in advance. Payment is guaranteed for the first five years and thereafter ceases immediately on the death of the policyholder.

Calculate the expected present value of this annuity.

Basis:

Mortality	PFA92C20
Rate of Interest	4% per annum
Expenses	Ignore

[4]

- 4**
- (i) Define temporary initial selection and give an explanation as to how it might arise. [3]
 - (ii) Comment on how the absence of medical questions on a proposal form might affect the impact of temporary initial selection in the context of a life assurance policy. [2]
- [Total 5]

- 5** A two-year term assurance policy is issued to a life aged x . Under this policy an immediate payment of 100,000 is made if death occurs in the first year, rising to 150,000 if death occurs in the second year.

Calculate the expected present value of this policy.

Basis:

Mortality $p_x = 0.99$ and $p_{x+1} = 0.975$

The force of mortality can be assumed to be constant over each year of age

Force of Interest 5%

[7]

- 6** (i) Define, showing notation:

- (a) the directly standardised mortality rate;
- (b) the indirectly standardised mortality rate.

[4]

- (ii) Calculate the directly and indirectly standardised mortality rates for Occupation A using All Occupations as the standard population.

[4]

Age	All Occupations		Occupation A	
	Population	Mortality rate	Population	Number of deaths
40–49	140,000	0.00169	10,236	125
50–59	156,000	0.00220	11,256	156
60–69	168,000	0.00277	10,633	166
Total	464,000		32,125	447

[Total 8]

- 7** Calculate $\bar{A}_{70.75}$ assuming a constant force of mortality between ages 70 and 71 only.

Basis:

Mortality PMA92C20
Rate of Interest 4% per annum

[8]

- 8** (i) State the main features of a unit-linked policy. [4]

A life insurance company issues a four-year unit-linked policy to a life aged 51 exact. The policy has the following profit vector:

$$(1798.01, -401.56, -355.10, -1075.23)$$

- (ii) Determine the net present value of the profits of this policy, assuming that the company sets up reserves in order to zeroise future negative expected cash flows on the policy.

Basis:

Mortality	AM92 Ultimate
Rate of interest on non-unit fund cash flows	2.5% per annum
Risk discount rate	4.5% per annum
	[5]

[Total 9]

- 9** A life insurance company issues a three-year policy to a life that offers the following benefits:

- On death during the term of the policy, a sum of 37,500.
- On redundancy during the term of the policy, a return of 105% of total premiums paid.
- On surrender during the term of the policy, a return of 33% of total premiums paid.
- On survival to the end of the term, a sum of 39,000.

Premiums of 12,500 are payable annually in advance throughout the term of the policy or until earlier claim. The death, redundancy and surrender benefits are payable immediately on claim. The policy ceases on payment of any claim.

The company uses the following basis to profit test this policy:

Independent force of mortality	1.5%
Independent force of redundancy	2%
Independent force of surrender	5% in years 1 and 2 only
Interest earned on cash flows	2.5% per annum
Expenses	2.5% of each premium paid
Reserves	Ignore

The company assumes that each force of decrement is constant over each year of age.

- (i) Calculate the dependent rates of mortality, redundancy and surrender for each policy year. [3]
- (ii) Calculate the expected profit margin to the company on this policy using a risk discount rate of 4% per annum. [7]

[Total 10]

10 A life insurance company issues whole of life assurance policies to lives aged 35 exact for a sum assured of 85,000 payable at the end of year of death. Premiums are payable annually in advance.

- (a) Calculate the annual office premium for each policy using the basis below.
- (b) Calculate the minimum office premium the company should charge in order that the probability of making a loss on any one policy would be 5% or less.

Basis:

Mortality	AM92 Select
Interest	6% per annum
Initial commission	75% of the annual premium
Initial expenses	350
Renewal commission	2.5% of each annual premium excluding the first
Renewal expenses	85 per annum at the start of the second and subsequent policy years.

[10]

11 A pension scheme provides a pension on retirement (through age or ill-health) of one-eightieth of Final Pensionable Salary for each year of pensionable service.

Normal Pension Age is age 65. Final Pensionable Salary is the average annual salary in the three years before retirement.

- (i) Draw a transition state model for the pension scheme fully labelling the diagram. [6]
- (ii) Calculate the expected present value of the total retirement benefits for a life aged 50 exact with salary in the previous year of 30,000 and 10 years of past service.

Basis: Pension Scheme Table in the Formulae and Tables for Examinations. [4]
[Total 10]

- 12** A life insurance company issues with-profits whole of life policies to lives aged 35 exact with the sum assured of 100,000 together with any attaching bonuses payable immediately on death of the life assured. Level premiums are payable monthly in advance to age 65 or until earlier death.

The company markets two versions of this policy as follows:

Version A – assumed to provide compound bonuses of 4% of the sum assured vesting at the end of each policy year.

Version B – assumed to provide simple bonuses of $b\%$ per annum of the sum assured, again vesting at the end of each policy year.

The death benefit under each version does not include any bonus relating to the policy year of death.

The following basis is used to price these contracts:

Mortality	AM92 Select
Rate of Interest	4% per annum
Initial expenses	275
Renewal expenses	2.5% of the second and subsequent monthly premiums
Initial commission	40% of the total premiums payable in the first policy year, all incurred at the policy commencement date
Renewal commission	2.5% of the second and subsequent monthly premiums
Claim expenses	225 at payment of death claim

- (a) Show that the monthly premium under version A of this policy is approximately 511.
- (b) Calculate the level simple bonus rate b that can be supported each year under version B of this policy if the monthly premium calculated in part (a) is charged.

[10]

- 13** A life insurance company issued 25-year decreasing term assurance policies on 1 January 2001 to lives then aged 40 exact. The death benefit, payable at the end of the year of death, is 500,000 in the first policy year, 480,000 in the second policy year thereafter reducing by 20,000 each year until the benefit is 20,000 in the twenty-fifth and final policy year. Premiums on the policies are payable annually in advance for 25 years or until earlier death.
- (a) Show that the annual net premium per policy is approximately 643 using the basis below.
- (b) Calculate the mortality profit or loss to the life insurance company during 2017 using the basis below if 1,527 policies were in force at the start of that year and 9 policyholders died during the year.

Basis:

Mortality AM92 Ultimate
Rate of Interest 4% per annum

[11]

END OF PAPER