

EXAMINATION

6 April 2005 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

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In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

- 1** A bond is priced at £95 per £100 nominal, has a coupon rate of 5% per annum payable half-yearly, and has an outstanding term of five years.

An investor holds a short position in a forward contract on £1 million nominal of this bond, with a delivery price of £98 per £100 nominal and maturity in exactly one year, immediately following the coupon payment then due.

The continuously compounded risk-free rates of interest for terms of six months and one year are 4.6% per annum and 5.2% per annum, respectively.

Calculate the value of this forward contract to the investor assuming no arbitrage. [5]

- 2** An investment fund had a market value of £2.2 million on 31 December 2001 and £4.2 million on 31 December 2004. It had received a net cashflow of £1.44 million on 31 December 2003.

The money weighted rate of return and the time weighted rate of return for the period from 31 December 2001 to 31 December 2004 are equal (to two decimal places).

Calculate the market value of the fund immediately before the net cashflow on 31 December 2003. [7]

- 3** A computer manufacturer is to develop a new chip to be produced from 1 January 2008 until 31 December 2020. Development begins on 1 January 2006. The cost of development comprises £9 million payable on 1 January 2006 and £12 million payable continuously during 2007.

From 1 January 2008 the chip will be ready for production and it is assumed that income will be received half yearly in arrear at a rate of £5 million per annum.

- (i) Calculate the discounted payback period at an effective rate of interest of 9% per annum. [6]
- (ii) Without doing any further calculations, explain whether the discounted payback period would be greater than, less than or equal to that given in part (i) if the effective interest rate were substantially greater than 9% per annum. [2]

[Total 8]

- 4** The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by

$$\delta(t) = \begin{cases} 0.07 - 0.005t & \text{for } t \leq 8 \\ 0.06 & \text{for } t > 8 \end{cases}$$

- (i) Calculate the accumulation at time $t = 10$ of £500 invested at time $t = 0$. [3]
- (ii) Calculate the present value at time $t = 0$ of a continuous payment stream at the rate of $\text{£}200e^{0.1t}$ paid from $t = 10$ to $t = 18$. [5]

[Total 8]

- 5** A university student receives a 3-year sponsorship grant. The payments under the grant are as follows:

- Year 1 £5,000 per annum paid continuously.
Year 2 £5,000 per annum paid monthly in advance.
Year 3 £5,000 per annum paid half yearly in advance.

Calculate the total present value of these payments at the beginning of the first year using a rate of interest of 8% per annum convertible quarterly. [8]

- 6** At time $t = 0$ an investor purchased an annuity-certain which paid her £10,000 per annum annually in arrear for three years. The purchase price paid by the investor was £25,000.

The value of the retail price index at various times was as shown in the table below:

Time t (years):	$t = 0$	$t = 1$	$t = 2$	$t = 3$
Retail price index:	170.7	183.3	191.0	200.9

- (i) Calculate, to the nearest 0.1%, the following effective rates of return per annum achieved by the investor from her investment in the annuity:
- (a) the real rate of return; and
(b) the money rate of return [7]
- (ii) By considering the average rate of inflation over the three-year period, explain the relationship between your answers in (a) and (b) of (i). [2]

[Total 9]

- 7** A loan of nominal amount £100,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 5% per annum. Capital is to be redeemed at 103 on a single coupon date between 15 and 20 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 20% and capital gains tax of 25% wishes to purchase the entire loan at the date of issue. Calculate the price which the investor should pay to ensure a net effective yield of at least 4% per annum. [9]

- 8** A small insurance fund has liabilities of £4 million due in 19 years time and £6 million in 21 years time. The manager of the fund has sold the assets previously held and is creating a new portfolio by investing in the zero-coupon bond market. The manager is able to buy zero-coupon bonds for whatever term he requires and has adequate monies at his disposal.

- (i) Explain whether it is possible for the manager to immunise the fund against small changes in the rate of interest by purchasing a single zero-coupon bond. [2]
- (ii) In fact, the manager purchases two zero-coupon bonds, one paying £3.43 million in 15 years time and the other paying £7.12 million in 25 years time. The current interest rate is 7% per annum effective.

Investigate whether the insurance fund satisfies the necessary conditions to be immunised against small changes in the rate of interest.

[8]
[Total 10]

- 9** The one-year forward rate of interest at time $t = 1$ year is 5% per annum effective.

The gross redemption yield of a two-year fixed interest stock issued at time $t = 0$ which pays coupons of 3% per annum annually in arrear and is redeemed at 102 is 5.5% per annum effective.

The issue price at time $t = 0$ of a three-year fixed interest stock bearing coupons of 10% per annum payable annually in arrear and redeemed at par is £108.9 per £100 nominal.

- (i) Calculate the one-year spot rate per annum effective at time $t = 0$. [4]
- (ii) Calculate the one-year forward rate per annum effective at time $t = 2$ years. [3]
- (iii) Calculate the two-year par yield at time $t = 0$. [3]
- [Total 10]

- 10** (i) In any year, the interest rate per annum effective on monies invested with a given bank has mean value j and standard deviation s and is independent of the interest rates in all previous years.

Let S_n be the accumulated amount after n years of a single investment of 1 at time $t = 0$.

(a) Show that $E[S_n] = (1 + j)^n$.

(b) Show that $\text{Var}[S_n] = (1 + 2j + j^2 + s^2)^n - (1 + j)^{2n}$.

[5]

- (ii) The interest rate per annum effective in (i), in any year, is equally likely to be i_1 or i_2 ($i_1 > i_2$). No other values are possible.

(a) Derive expressions for j and s^2 in terms of i_1 and i_2 .

- (b) The accumulated value at time $t = 25$ years of £1 million invested with the bank at time $t = 0$ has expected value £5.5 million and standard deviation £0.5 million.

Calculate the values of i_1 and i_2 .

[8]

[Total 13]

- 11** (i) A loan is repayable over 20 years by level instalments of £1,000 per annum made annually in arrear. Interest is charged at the rate of 5% per annum effective for the first 10 years, increasing to 7% per annum effective for the remaining term.

Show that the amount of the original loan is £12,033.56. (Minor discrepancies due to rounding will not be penalised). [2]

- (ii) The following are the details from the loan schedule for year x , i.e. the year running from exact duration $x - 1$ years to exact duration x years.

<i>Loan outstanding at the beginning of the year</i>	<i>Instalment paid at the end of the year</i>	
	<i>Interest</i>	<i>Capital</i>
Year x	£8,790.48	£439.52
		£560.48

Determine the value of x . [4]

- (iii) At the beginning of year 11, it is agreed that the increase in the rate of interest will not take place, so that the rate remains at 5% per annum effective for the remainder of the loan. The annual instalment will continue to be payable at the same level so that there may be a reduced term and a reduced final instalment.

- Calculate by how many years, if any, the repayment schedule is shortened.
- Calculate the amount of the reduced final instalment.
- Calculate the reduction in the total interest paid during the existence of the loan as a result of the interest rate not increasing.

[7]
[Total 13]

END OF PAPER

EXAMINATION

7 September 2005 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

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Graph paper is not required for this paper.

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1 Describe how cashflows are exchanged in an “interest rate swap”. [2]

2 An investor has earned a money rate of return from a portfolio of bonds in a particular country of 1% per annum effective over a period of ten years. The country has experienced deflation (negative inflation) of 2% per annum effective during the period.

Calculate the real rate of return per annum over the ten years. [2]

3 Calculate the time in days for £1,500 to accumulate to £1,550 at:

- (a) a simple rate of interest of 5% per annum
- (b) a force of interest of 5% per annum

[4]

4 The force of interest $\delta(t)$ at time t is $a + bt^2$ where a and b are constants. An amount of £200 invested at time $t = 0$ accumulates to £210 at time $t = 5$ and £230 at time $t = 10$.

Determine a and b . [5]

5 (i) Calculate the present value of £100 over ten years at the following rates of interest/discount:

- (a) a rate of interest of 5% per annum convertible monthly
- (b) a rate of discount of 5% per annum convertible monthly
- (c) a force of interest of 5% per annum

[4]

(ii) A 91-day treasury bill is bought for \$98.91 and is redeemed at \$100. Calculate the annual effective rate of interest obtained from the bill. [3]

[Total 7]

6 (i) State the features of a eurobond. [3]

(ii) An investor purchases a eurobond on the date of issue at a price of £97 per £100 nominal. Coupons are paid annually in arrear. The bond will be redeemed at par twenty years from the issue date. The rate of return from the bond is 5% per annum effective.

- (a) Calculate the annual rate of coupon paid by the bond.
- (b) Calculate the duration of the bond.

[6]

[Total 9]

7 A bank makes a loan to be repaid in instalments annually in arrear. The first instalment is 50, the second 48 and so on with the payments reducing by 2 per annum until the end of the 15th year after which there are no further payments. The rate of interest charged by the lender is 6% per annum effective.

- (i) Calculate the amount of the loan. [6]
- (ii) Calculate the interest and capital components of the second payment. [3]
- (iii) Calculate the amount of capital repaid in the instalment at the end of the fourteenth year. [3]

[Total 12]

8 An insurance company has just written contracts that require it to make payments to policyholders of £1,000,000 in five years' time. The total premiums paid by policyholders amounted to £850,000. The insurance company is to invest half the premium income in fixed interest securities that provide a return of 3% per annum effective. The other half of the premium income is to be invested in assets that have an uncertain return. The return from these assets in year t , i_t , has a mean value of 3.5% per annum effective and a standard deviation of 3% per annum effective. $(1 + i_t)$ is independently and lognormally distributed.

- (i) Deriving all necessary formulae, calculate the mean and standard deviation of the accumulation of the premiums over the five-year period. [9]
- (ii) A director of the company suggests that investing all the premiums in the assets with an uncertain return would be preferable because the expected accumulation of the premiums would be greater than the payments due to the policyholders.

Explain why this still may be a more risky investment policy. [2]

[Total 11]

9 (i) Explain what is meant by the “expectations theory” for the shape of the yield curve. [2]

(ii) Short-term, one-year annual effective interest rates are currently 8%; they are expected to be 7% in one years time, 6% in two years time and 5% in three years time.

(a) Calculate the gross redemption yields (spot rates of interest) from 1-year, 2-year, 3-year and 4-year zero coupon bonds assuming the expectations theory explanation of the yield curve holds.

(b) The price of a coupon paying bond is calculated by discounting individual payments from the bond at the zero-coupon bond yields in (a).

Calculate the gross redemption yield of a bond that is redeemed at par in exactly four years and pays a coupon of 5 per annum annually in arrear.

(c) A two-year forward contract has just been issued on a share with a price of 400p. A dividend of 4p is expected in exactly one year.

Calculate the forward price using the above spot rates of interest, assuming no arbitrage.

[12]

[Total 14]

10 An investor purchased a bond with exactly 15 years to redemption. The bond, redeemable at par, has a gross redemption yield of 5% per annum effective. It pays coupons of 4% per annum, half yearly in arrear. The investor pays tax at 25% on the coupons only.

(i) Calculate the price paid for the bond. [3]

(ii) After exactly eight years, immediately after the payment of the coupon then due, this investor sells the bond to another investor who pays income tax at a rate of 25% and capital gains tax at a rate of 40%. The bond is purchased by the second investor to provide a net return of 6% per annum effective.

(a) Calculate the price paid by the second investor.

(b) Calculate, to one decimal place, the annual effective rate of return earned by the first investor during the period for which the bond was held.

[10]

[Total 13]

- 11** (i) Explain what is meant by the following terms:
- (a) equation of value
(b) discounted payback period from an investment project [4]

- (ii) An insurance company is considering setting up a branch in a country in which it has previously not operated. The company is aware that access to capital may become difficult in twelve years time. It therefore has two decision criteria. The cashflows from the project must provide an internal rate of return greater than 9% per annum effective and the discounted payback period at a rate of interest of 7% per annum effective must be less than twelve years.

The following cashflows are generated in the development and operation of the branch.

Cash Outflows

Between the present time and the opening of the branch in three years time the insurance company will spend £1.5m per annum on research, development and the marketing of products. This outlay is assumed to be a constant continuous payment stream. The rent on the branch building will be £0.3m per annum paid quarterly in advance for twelve years starting in three years time. Staff costs are assumed to be £1m in the first year, £1.05m in the second year, rising by 5% per annum each year thereafter. Staff costs are assumed to be incurred at the beginning of each year starting in three years time and assumed to be incurred for 12 years.

Cash Inflows

The company expects the sale of products to produce a net income at a rate of £1m per annum for the first three years after the branch opens rising to £1.9m per annum in the next three years and to £2.5m for the following six years. This net income is assumed to be received continuously throughout each year. The company expects to be able to sell the branch operation 15 years from the present time for £8m.

Determine which, if any, of the decision criteria the project fulfils.

[17]

[Total 21]

END OF PAPER

EXAMINATION

4 April 2006 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

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AT THE END OF THE EXAMINATION

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- 1** An investment is discounted for 28 days at a simple rate of discount of 4.5% per annum. Calculate the annual effective rate of interest. [3]
- 2** An annuity certain with payments of £150 at the end of each quarter is to be replaced by an annuity with the same term and present value, but with payments at the beginning of each month instead. Calculate the revised payments, assuming an annual force of interest of 10%. [3]
- 3** At time $t = 0$ the n -year spot rate of interest is equal to $(2.25 + 0.25n)\%$ per annum effective ($1 \leq n \leq 5$).
- Calculate the 2-year forward rate of interest from time $t = 3$ expressed as an annual effective rate of interest.
 - Calculate the 4-year par yield.
 - Without performing any further calculations, explain how you would expect the gross redemption yield of a 4-year bond paying annual coupons of 3.5% to compare with the par yield calculated in (b).
- [7]
- 4** An investor, who is liable to income tax at 20% but is not liable to capital gains tax, wishes to earn a net effective rate of return of 5% per annum. A bond bearing coupons payable half-yearly in arrear at a rate 6.25% per annum is available. The bond will be redeemed at par on a coupon date between 10 and 15 years after the date of issue, inclusive. The date of redemption is at the option of the borrower. Calculate the maximum price that the investor is willing to pay for the bond. [5]
- 5** A share currently trades at £10 and will pay a dividend of 50p in one month's time. A six-month forward contract is available on the share for £9.70. Show that an investor can make a risk-free profit if the risk-free force of interest is 3% per annum. [4]
- 6** An actuarial student has created an interest rate model under which the annual effective rate of interest is assumed to be fixed over the whole of the next ten years. The annual effective rate is assumed to be 2%, 4% and 7% with probabilities 0.25, 0.55 and 0.2 respectively.
- Calculate the expected accumulated value of an annuity of £800 per annum payable annually in advance over the next ten years.
 - Calculate the probability that the accumulated value will be greater than £10,000.
- [4]

7 A company has entered into an interest rate swap. Under the terms of the swap the company makes fixed annual payments equal to 6% of the principal of the swap. In return, the company receives annual interest payments on the principal based on the prevailing variable short-term interest rate which currently stands at 5.5% per annum.

- (a) Describe briefly the risks faced by a counterparty to an interest rate swap.
- (b) Explain which of the risks described in (a) are faced by the company. [4]

8 An ordinary share pays annual dividends. A dividend of 25p per share has just been paid. Dividends are expected to grow by 2% next year and by 4% the following year. Thereafter, dividends are expected to grow at 6% per annum compound in perpetuity.

- (i) State the main characteristics of ordinary shares. [4]
- (ii) Calculate the present value of the dividend stream described above at a rate of interest of 9% per annum effective from a holding of 100 ordinary shares. [4]
- (iii) An investor buys 100 shares in (ii) for £8.20 each. He holds them for two years and receives the dividends payable. He then sells them for £9 immediately after the second dividend is paid.

Calculate the investor's real rate of return if the inflation index increases by 3% during the first year and by 3.5% during the second year assuming dividends grow as expected. [4]

[Total 12]

9 The force of interest $\delta(t)$ is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 & 0 < t \leq 5 \\ 0.008t & 5 < t \leq 10 \\ 0.005t + 0.0003t^2 & 10 < t \end{cases}$$

- (i) Calculate the present value of a unit sum of money due at time $t = 12$. [5]
- (ii) Calculate the effective annual rate of interest over the 12 years. [2]
- (iii) Calculate the present value at time $t = 0$ of a continuous payment stream that is paid at the rate of $e^{-0.05t}$ per unit time between time $t = 2$ and time $t = 5$. [3]

[Total 10]

- 10** A piece of land is available for sale for £5,000,000. A property developer, who can lend and borrow money at a rate of 15% per annum, believes that she can build housing on the land and sell it for a profit. The total cost of development would be £7,000,000 which would be incurred continuously over the first two years after purchase of the land. The development would then be complete.

The developer has three possible project strategies. She believes that she can sell the completed housing:

- in three years time for £16,500,000
- in four years time for £18,000,000
- in five years time for £20,500,000

The developer also believes that she can obtain a rental income from the housing between the time that the development is completed and the time of sale. The rental income is payable quarterly in advance and is expected to be £500,000 in the first year of payment. Thereafter, the rental income is expected to increase by £50,000 per annum at the beginning of each year that the income is paid.

- (i) Determine the optimum strategy if this is based upon using net present value as the decision criterion. [9]
 - (ii) Determine which strategy would be optimal if the discounted payback period were to be used as the decision criterion. [2]
 - (iii) If the housing is sold in six years' time, the developer believes that she can obtain an internal rate of return on the project of 17.5% per annum. Calculate the sale price that the developer believes that she can receive. [6]
 - (iv) Suggest reasons why the developer may not achieve an internal rate of return of 17.5% per annum even if she sells the housing for the sale price calculated in (iii). [2]
- [Total 19]

11 An actuarial student has taken out two loans.

Loan A: a five-year car loan for £10,000 repayable by equal monthly instalments of capital and interest in arrear with a flat rate of interest of 10.715% per annum.

Loan B: a five-year bank loan of £15,000 repayable by equal monthly instalments of capital and interest in arrear with an effective annual interest rate of 12% for the first two years and 10% thereafter.

The student has a monthly disposable income of £600 to pay the loan interest after all other living expenses have been paid.

Freeloans is a company which offer loans at a constant effective interest rate for all terms between three years and ten years. After two years, the student is approached by a representative of Freeloans who offers the student a 10-year loan on the capital outstanding which is repayable by equal monthly instalments of capital and interest in arrear. This new loan is used to pay off the original loans and will have repayments equal to half the original repayments.

- (i) Calculate the final disposable income (surplus or deficit) each month after the loan payments have been made. [5]
 - (ii) Calculate the capital repaid in the first month of the third year assuming that the student carries on with the original arrangements. [5]
 - (iii) Estimate the capital repaid in the first month of the third year assuming that the student has taken out the new loan. [5]
 - (iv) Suggest, with reasons, a more appropriate strategy for the student. [2]
- [Total 17]

- 12** A pension fund has liabilities of £3 million due in 3 years' time, £5 million due in 5 years' time, £9 million due in 9 years' time, and £11 million due in 11 years' time. The fund holds two investments, X and Y. Investment X provides income of £1 million payable at the end of each year for the next five years with no capital repayment. Investment Y is a zero coupon bond which pays a lump sum of £R at the end of n years (where n is not necessarily an integer). The interest rate is 8% per annum effective.

- (i) Investigate whether values of £R and n can be found which ensure that the fund is immunised against small changes in the interest rate.

You are given that $\sum_{t=1}^5 t^2 v^t = 40.275$ at 8%. [8]

- (ii) (a) The interest rate immediately changes to 3% per annum effective. Calculate the revised present values of the assets and liabilities of the fund.

- (b) Explain your answer to (ii)(a). [4]

[Total 12]

END OF PAPER

EXAMINATION

12 September 2006 (am)

Subject CT1 — Financial Mathematics Core Technical

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- 1** (a) Distinguish between a future and an option.
(b) Explain why convertibles have “option-like” characteristics.

[3]

- 2** An individual makes an investment of £4m per annum in the first year, £6m per annum in the second year and £8m per annum in the third year. The investments are made continuously throughout each year. Calculate the accumulated value of the investments at the end of the third year at a rate of interest of 4% per annum effective.

[3]

- 3** An individual has invested a sum of £10m. Exactly one year later, the investment is worth £11.1m. An index of prices has a value of 112 at the beginning of the investment and 120 at the end of the investment. The investor pays tax at 40% on all money returns from investment. Calculate:

- (a) The money rate of return per annum before tax.
(b) The rate of inflation.
(c) The real rate of return per annum after tax.

[4]

- 4** An investor is able to purchase or sell two specially designed risk-free securities, A and B. Short sales of both securities are possible. Security A has a market price of 20p. In the event that a particular stock market index goes up over the next year, it will pay 25p and, in the event that the stock market index goes down, it will pay 15p. Security B has a market price of 15p. In the event that the stock market index goes up over the next year, it will pay 20p and, in the event that the stock market index goes down, it will pay 12p.

- (i) Explain what is meant by the assumption of “no arbitrage” used in the pricing of derivative contracts. [2]
- (ii) Find the market price of B, such that there are no arbitrage opportunities and assuming the price of A remains fixed. Explain your reasoning. [2]

[Total 4]

- 5** (i) Calculate the time in days for £3,600 to accumulate to £4,000 at:
- (a) a simple rate of interest of 6% per annum
(b) a compound rate of interest of 6% per annum convertible quarterly
(c) a compound rate of interest of 6% per annum convertible monthly

[4]

- (ii) Explain why the amount takes longest to accumulate in (i)(a) [1]

[Total 5]

- 6** The rate of interest is a random variable that is distributed with mean 0.07 and variance 0.016 in each of the next 10 years. The value taken by the rate of interest in any one year is independent of its value in any other year. Deriving all necessary formulae calculate:
- (i) The expected accumulation at the end of ten years, if one unit is invested at the beginning of ten years. [3]
 - (ii) The variance of the accumulation at the end of ten years, if one unit is invested at the beginning of ten years. [5]
 - (iii) Explain how your answers in (i) and (ii) would differ if 1,000 units had been invested. [1]
- [Total 9]
- 7** A life insurance fund had assets totalling £600m on 1 January 2003. It received net income of £40m on 1 January 2004 and £100m on 1 July 2004. The value of the fund was:
- £450m on 31 December 2003;
 £500m on 30 June 2004;
 £800m on 31 December 2004.
- (i) Calculate, for the period 1 January 2003 to 31 December 2004, to three decimal places:
 - (a) The time weighted rate of return per annum.
 - (b) The linked internal rate of return, using sub intervals of a calendar year.
- [8]
- (ii) Explain why the linked internal rate of return is higher than the time weighted rate of return. [2]
- [Total 10]
- 8** The force of interest $\delta(t)$ at time t is $at + bt^2$ where a and b are constants. An amount of £100 invested at time $t = 0$ accumulates to £150 at time $t = 5$ and £230 at time $t = 10$.
- (i) Calculate the values of a and b . [5]
 - (ii) Calculate the constant force of interest that would give rise to the same accumulation from time $t = 0$ to time $t = 10$. [2]
 - (iii) At the force of interest calculated in (ii), calculate the present value of a continuous payment stream of $20e^{0.05t}$ paid between from time $t = 0$ to time $t = 10$. [4]
- [Total 11]

9 An individual took out a loan of £100,000 to purchase a house on 1 January 1980. The loan is due to be repaid on 1 January 2010 but the borrower can repay the loan early if he wishes. The borrower pays interest on the loan at a rate of 6% per annum convertible monthly, paid in arrears. The loan instalments only cover the interest on the loan. At the same time, the borrower took out a thirty-year investment policy, which was expected to repay the loan, and into which monthly premiums were paid, in advance, at a rate of £1,060 per annum. The individual was told that premiums in the investment policy were expected to earn a rate of return of 7% per annum effective. After twenty years, the individual was informed that the premiums had only earned a rate of return of 4% per annum effective and that they would continue to do so for the final ten years of the policy. The borrower agrees to increase his monthly payments into the investment policy to £5,000 per annum for the final ten years.

- (a) Calculate the amount to which the investment policy was expected to accumulate at the time it was taken out.
- (b) Calculate the amount by which the investment policy would have fallen short of repaying the loan had extra premiums not been paid for the final ten years.
- (c) Calculate the amount of money the individual will have, after using the proceeds of the investment policy to repay the loan, after allowing for the increase in premiums.
- (d) Suggest another course of action the borrower could have taken which would have been of higher value to him, explaining why this higher value arises.
- (e) Calculate the level annual instalment that the investor would have had to pay from outset if he had repaid the loan in equal instalments of interest and capital.

[11]

10 A financial regulator has brought in a new set of regulations and wishes to assess the cost of them. It intends to conduct an analysis of the costs and benefits of the new regulations in their first twenty years.

The costs are estimated to be as follows:

- The cost to companies who will need to devise new policy terms and computer systems is expected to be incurred at a rate of £50m in the first year increasing by 3% per annum over the twenty year period.
- The cost to financial advisers who will have to set up new computer systems and spend more time filling in paperwork is expected to be incurred at a rate of £60m in the first year, £19m in the second year, £18m in the third year, reducing by £1m every year until the last year, when the cost incurred will be at a rate of £1m.
- The cost to consumers who will have to spend more time filling in paperwork and talking to their financial advisers is expected to be incurred at a rate of £10m in the first year, increasing by 3% per annum over the twenty year period.

The benefits are estimated as follows:

- The benefit to consumers who are less likely to buy inappropriate policies is estimated to be received at a rate of £30m in the first year, £33m in the second year, £36m in the third year and so on, rising by £3m per year until the end of twenty years.
- The benefit to companies who will spend less time dealing with complaints from customers is estimated to be received at a rate of £12m per annum for twenty years.

Calculate the net present value of the benefit or cost of the regulations in their first twenty years at a rate of interest of 4% per annum effective. Assume that all costs and benefits occur continuously throughout the year.

[12]

11 (i) Describe the characteristics of an index-linked government bond. [3]

- (ii) On 1 July 2002, the government of a country issued an index-linked bond of term seven years. Coupons are paid half-yearly in arrears on 1 January and 1 July each year. The annual nominal coupon is 2%. Interest and capital payments are indexed by reference to the value of an inflation index with a time lag of eight months.

You are given the following values of the inflation index.

<i>Date</i>	<i>Inflation index</i>
November 2001	110.0
May 2002	112.3
November 2002	113.2
May 2003	113.8

The inflation index is assumed to increase continuously at the rate of 2½% per annum effective from its value in May 2003.

An investor, paying tax at the rate of 20% on coupons only, purchased the stock on 1 July 2003, just after a coupon payment had been made.

Calculate the price to this investor such that a real net yield of 3% per annum convertible half yearly is obtained and assuming that the investor holds the bond to maturity.

[10]

[Total 13]

- 12** A pension fund has the following liabilities: annuity payments of £160,000 per annum to be paid annually in arrears for the next 15 years and a lump sum of £200,000 to be paid in ten years. It wishes to invest in two fixed-interest securities in order to immunise its liabilities. Security A has a coupon rate of 8% per annum and a term to redemption of eight years. Security B has a coupon rate of 3% per annum and a term to redemption of 25 years. Both securities are redeemable at par and pay coupons annually in arrear.
- (i) Calculate the present value of the liabilities at a rate of interest of 7% per annum effective. [2]
- (ii) Calculate the discounted mean term of the liabilities at a rate of interest of 7% per annum effective. [4]
- (iii) Calculate the nominal amount of each security that should be purchased so that both the present value and discounted mean terms of assets and liabilities are equal. [7]
- (iv) Without further calculation, comment on whether, if the conditions in (iii) are fulfilled, the pension fund is likely to be immunised against small, uniform changes in the rate of interest. [2]
- [Total 15]

END OF PAPER

EXAMINATION

12 April 2007 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

- 1** An investor pays £400 every half-year in advance into a 25-year savings plan.

Calculate the accumulated fund at the end of the term if the interest rate is 6% per annum convertible monthly for the first 15 years and 6% per annum convertible half-yearly for the final 10 years. [5]

- 2** The force of interest $\delta(t)$ is a function of time and at any time, measured in years, is given by the formula:

$$\begin{aligned}\delta(t) &= 0.04 + 0.01t & 0 \leq t \leq 4 \\ \delta(t) &= 0.12 - 0.01t & 4 < t \leq 8 \\ \delta(t) &= 0.06 & 8 < t\end{aligned}$$

Calculate the present value at time $t = 0$ of a payment stream, paid continuously from time $t = 9$ to $t = 12$, under which the rate of payment at time t is $50e^{0.01t}$.

[6]

- 3** An ordinary share pays annual dividends. The next dividend is due in exactly eight months' time. This dividend is expected to be £1.10 per share. Dividends are expected to grow at a rate of 5% per annum compound from this level and are expected to continue in perpetuity. Inflation is expected to be 3% per annum. The price of the share is £21.50.

Calculate the expected effective annual real rate of return for an investor who purchases the share.

[7]

- 4** An investor entered into a long forward contract for a security five years ago and the contract is due to mature in seven years' time. The price of the security was £95 five years ago and is now £145. The risk-free rate of interest can be assumed to be 3% per annum throughout the 12-year period.

Assuming no arbitrage, calculate the value of the contract now if:

- (i) The security will pay dividends of £5 in two years' time and £6 in four years' time. [3]
- (ii) The security has paid and will continue to pay annually in arrear a dividend of 2% per annum of the market price of the security at the time of payment. [3]

[Total 6]

- 5** In a particular bond market, n -year spot rates per annum can be approximated by the function $0.08 - 0.04e^{-0.1n}$.

Calculate:

- (i) The price per unit nominal of a zero coupon bond with term nine years. [2]
- (ii) The four-year forward rate at time 7 years. [3]
- (iii) The three-year par yield. [3]

[Total 8]

- 6** A fund had a value of £21,000 on 1 July 2003. A net cash flow of £5,000 was received on 1 July 2004 and a further net cash flow of £8,000 was received on 1 July 2005. Immediately before receipt of the first net cash flow, the fund had a value of £24,000, and immediately before receipt of the second net cash flow the fund had a value of £32,000. The value of the fund on 1 July 2006 was £38,000.

- (i) Calculate the annual effective money weighted rate of return earned on the fund over the period 1 July 2003 to 1 July 2006. [3]
- (ii) Calculate the annual effective time weighted rate of return earned on the fund over the period 1 July 2003 to 1 July 2006. [3]
- (iii) Explain why the values in (i) and (ii) differ. [2]

[Total 8]

- 7** An insurance company has liabilities of £87,500 due in 8 years' time and £157,500 due in 19 years' time. Its assets consist of two zero coupon bonds, one paying £66,850 in four years' time and the other paying £ X in n years' time. The current interest rate is 7% per annum effective.

- (i) Calculate the discounted mean term and convexity of the liabilities. [5]
- (ii) Determine whether values of £ X and n can be found which ensure that the company is immunised against small changes in the interest rate. [5]

[Total 10]

8 A company has borrowed £800,000 from a bank. The loan is to be repaid by level instalments, payable annually in arrear for 10 years from the date the loan is made. The annual repayments are calculated at an effective rate of interest of 8% per annum.

- (i) Calculate the amount of the level annual payment and the total amount of interest which will be paid over the 10 year term. [3]
- (ii) At the beginning of the eighth year, immediately after the seventh payment has been made, the company asks for the term of the loan to be extended by two years. The bank agrees to do this on condition that the rate of interest is increased to an effective rate of 12% per annum for the remainder of the term and that payments are made quarterly in arrear.
 - (a) Calculate the amount of the new quarterly payment.
 - (b) Calculate the capital and interest components of the first quarterly instalment of the revised loan repayments.

[6]

[Total 9]

9 A property developer is constructing a block of offices. It is anticipated that the offices will take six months to build. The developer incurs costs of £40 million at the beginning of the project followed by £3 million at the end of each month for the following six months during the building period. It is expected that rental income from the offices will be £1 million per month, which will be received at the start of each month beginning with the seventh month. Maintenance and management costs paid by the developer are expected to be £2 million per annum payable monthly in arrear with the first payment at the end of the seventh month. The block of offices is expected to be sold 25 years after the start of the project for £60 million.

- (i) Calculate the discounted payback period using an effective rate of interest of 10% per annum. [7]
- (ii) Without doing any further calculations, explain whether your answer to (i) would change if the effective rate of interest were less than 10% per annum.

[3]

[Total 10]

10 A loan is issued bearing interest at a rate of 9% per annum and payable half-yearly in arrear. The loan is to be redeemed at £110 per £100 nominal in 13 years' time.

- (i) The loan is issued at a price such that an investor, subject to income tax at 25%, and capital gains tax at 30%, would obtain a net redemption yield of 6% per annum effective. Calculate the issue price per £100 nominal of the stock. [5]

(ii) Two years after the date of issue, immediately after a coupon payment has been made, the investor decides to sell the stock and finds a potential buyer, who is subject to income tax at 10% and capital gains tax at 35%. The potential buyer is prepared to buy the stock provided she will obtain a net redemption yield of at least 8% per annum effective.

- (a) Calculate the maximum price (per £100 nominal) which the original investor can expect to obtain from the potential buyer.
- (b) Calculate the net effective annual redemption yield (to the nearest 1% per annum effective) that will be obtained by the original investor if the loan is sold to the buyer at the price determined in (ii) (a).

[10]

[Total 15]

11 £80,000 is invested in a bank account which pays interest at the end of each year. Interest is always reinvested in the account. The rate of interest is determined at the beginning of each year and remains unchanged until the beginning of the next year. The rate of interest applicable in any one year is independent of the rate applicable in any other year.

During the first year, the annual effective rate of interest will be one of 4%, 6% or 8% with equal probability.

During the second year, the annual effective rate of interest will be either 7% with probability 0.75 or 5% with probability 0.25.

During the third year, the annual effective rate of interest will be either 6% with probability 0.7 or 4% with probability 0.3.

- (i) Derive the expected accumulated amount in the bank account at the end of three years. [5]
- (ii) Derive the variance of the accumulated amount in the bank account at the end of three years. [8]
- (iii) Calculate the probability that the accumulated amount in the bank account is more than £97,000 at the end of three years. [3]

[Total 16]

END OF PAPER

EXAMINATION

25 September 2007 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

- 1** A 90-day government bill is purchased for £96 at the time of issue and is sold after 45 days to another investor for £97.90. The second investor holds the bill until maturity and receives £100.

Determine which investor receives the higher rate of return. [2]

- 2** An investor purchases a share for 769p at the beginning of the year. Halfway through the year he receives a dividend, net of tax, of 4p and immediately sells the share for 800p. Capital gains tax of 30% is paid on the difference between the sale and the purchase price.

Calculate the net annual effective rate of return the investor obtains on the investment. [4]

- 3** An insurance company offers a customer two payment options in respect of an invoice for £456. The first option involves 24 payments of £20 paid at the beginning of each month starting immediately. The second option involves 24 payments of £20.50 paid at the end of each month starting immediately. The customer is willing to accept a monthly payment schedule if the annual effective interest rate per annum he pays is less than 5%.

Determine which, if any, of the payment options the customer will accept. [4]

- 4** State the characteristics of an equity investment. [4]

- 5** A one-year forward contract is issued on 1 April 2007 on a share with a price of 900p at that date. Dividends of 50p per share are expected on 30 September 2007 and 31 March 2008. The 6-month and 12-month spot, risk-free rates of interest are 5% and 6% per annum effective respectively on 1 April 2007.

Calculate the forward price at issue, stating any assumptions. [4]

- 6** The annual effective forward rate applicable over the period t to $t + r$ is defined as $f_{t,r}$ where t and r are measured in years. $f_{0,1} = 4\%$, $f_{1,1} = 4.25\%$ $f_{2,1} = 4.5\%$, $f_{2,2} = 5\%$. Calculate the following:

(i) $f_{3,1}$ [1]

(ii) All possible zero coupon (spot) yields that the above information allows you to calculate. [4]

(iii) The gross redemption yield of a four-year bond, redeemable at par, with a 3% coupon payable annually in arrears. [6]

(iv) Explain why the gross redemption yield from the four-year bond is lower than the one-year forward rate up to time 4, $f_{3,1}$ [2]

[Total 13]

- 7** The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by

$$\delta(t) = \begin{cases} 0.04 + 0.01t & \text{for } 0 \leq t \leq 10 \\ 0.05 & \text{for } t > 10 \end{cases}$$

- (i) Derive, and simplify as far as possible, expressions for $v(t)$ where $v(t)$ is the present value of a unit sum of money due at time t . [5]
 - (ii) (a) Calculate the present value of £1,000 due at the end of 15 years.
 - (b) Calculate the annual effective rate of discount implied by the transaction in (a). [4]
 - (iii) A continuous payment stream is received at a rate of $20e^{-0.01t}$ units per annum between $t = 10$ and $t = 15$. Calculate the present value of the payment stream. [4]
- [Total 13]

- 8** A pension fund makes the following investments (£m):

<i>1 January 2004</i>	<i>1 July 2004</i>	<i>1 January 2005</i>	<i>1 January 2006</i>
12.5	6.6	7.0	8.0

The rates of return earned on money invested in the fund were as follows:

<i>1 January 2004 to 30 June 2004</i>	<i>1 July 2004 to 31 December 2004</i>	<i>1 January 2005 to 31 December 2005</i>	<i>1 January 2006 to 31 December 2006</i>
5%	6%	6.5%	3%

You may assume that 1 January to 30 June and 1 July to 31 December are precise half year periods.

- (i) Calculate the linked internal rate of return per annum over the three years from 1 January 2004 to 31 December 2006, using semi-annual sub-intervals. [3]
- (ii) Calculate the time weighted rate of return per annum over the three years from 1 January 2004 to 31 December 2006. [3]
- (iii) Calculate the money weighted rate of return per annum over the three years from 1 January 2004 to 31 December 2006. [4]
- (iv) Explain the relationship between your answers to (i), (ii) and (iii) above. [2]

[Total 12]

- 9** The expected effective annual rate of return from a bank's investment portfolio is 6% and the standard deviation of annual effective returns is 8%. The annual effective returns are independent and $(1+i_t)$ is lognormally distributed, where i_t is the return in year t .

Deriving any necessary formulae:

- (i) calculate the expected value of an investment of £2 million after ten years. [6]
- (ii) calculate the probability that the accumulation of the investment will be less than 80% of the expected value. [3]

[Total 9]

- 10** A government is holding an inquiry into the provision of loans by banks to consumers at high rates of interest. The loans are typically of short duration and to high risk consumers. Repayments are collected in person by representatives of the bank making the loan. Campaigners on behalf of the consumers and campaigners on behalf of the banks granting the loans are disputing one particular type of loan. The initial loans are for £2,000. Repayments are made at an annual rate of £2,400 payable monthly in advance for two years.

The consumers' association case

The consumers' association asserts that, on this particular type of loan, consumers who make all their repayments pay interest at an annual effective rate of over 200%.

The banks' case

The banks state that, on the same loans, 40% of the consumers default on all their remaining payments after exactly 12 payments have been made. Furthermore half of the consumers who have not defaulted after 12 payments default on all their remaining payments after exactly 18 payments have been made. The banks also argue that it costs 30% of each monthly repayment to collect the payment. These costs are still incurred even if the payment is not made by the consumer. Furthermore, with inflation of 2.5% per annum, the banks therefore assert that the real rate of interest that the lender obtains on the loan is less than 1.463% per annum effective.

- (i) (a) Calculate the flat rate of interest paid by the consumer on the loan described above.
- (b) State why the flat rate of interest is not a good measure of the cost of borrowing to the consumer. [4]
- (ii) Determine, for each of the cases above, whether the assertion is correct. [10]

[Total 14]

- 11** A pension fund has liabilities to pay pensions each year for the next 60 years. The pensions paid will be £100m at the end of the first year, £105m at the end of the second year, £110.25m at the end of the third year and so on, increasing by 5% each year. The fund holds government bonds to meet its pension liabilities. The bonds mature in 20 years time and pay an annual coupon of 4% in arrears.
- (i) Calculate the present value of the pension fund's liabilities at a rate of interest of 3% per annum effective. [4]
 - (ii) Calculate the nominal amount of the bond that the fund needs to hold so that the present value of the assets is equal to the present value of the liabilities. [3]
 - (iii) Calculate the duration of the liabilities. [6]
 - (iv) Calculate the duration of the assets. [4]
 - (v) Using your calculations in (iii) and (iv), estimate by how much more the value of the liabilities would increase than the value of the assets if there were a reduction in the rate of interest to 1.5% per annum effective. [4]
- [Total 21]

END OF PAPER

EXAMINATION

15 April 2008 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** An eleven month forward contract is issued on 1 March 2008 on a stock with a price of £10 per share at that date. Dividends of 50 pence per share are expected to be paid on 1 April and 1 October 2008.

Calculate the forward price at issue, assuming a risk-free rate of interest of 5% per annum effective and no arbitrage. [4]

- 2** Describe the characteristics of the following investments:

- (a) Eurobonds
- (b) Certificates of deposit

[4]

- 3** A mortgage company offers the following two deals to customers for twenty-five year mortgages.

Product A

A mortgage of £100,000 is offered with level repayments of £7,095.25 made annually in arrear. There are no arrangement or exit fees.

Product B

A mortgage of £100,000 is offered whereby a monthly payment in advance is calculated such that the customer pays an effective rate of return of 4% per annum ignoring arrangement and exit fees. In addition the customer also has to pay an arrangement fee of £6,000 at the beginning of the mortgage and an exit fee of £5,000 at the end of the twenty-five year term of the mortgage.

Compare the annual effective rates of return paid by customers on the two products.

[8]

- 4** A loan of nominal amount £100,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 7% per annum. Capital is to be redeemed at 108% on a coupon date between 15 and 20 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 25% and capital gains tax at 35% wishes to purchase the entire loan at the date of issue.

Calculate the price which the investor should pay to ensure a net effective yield of at least 5% per annum. [8]

- 5** The n –year spot rate of interest, i_n , is given by:

$$i_n = a - bn$$

for $n = 1, 2$ and 3 , and where a and b are constants.

The one-year forward rates applicable at time 0 and at time 1 are 6.1% per annum effective and 6.5% per annum effective respectively. The 4 –year par yield is 7% per annum.

Stating any assumptions:

- (i) calculate the values of a and b . [4]
- (ii) calculate the price per £ 1 nominal at time 0 of a bond which pays annual coupons of 5% in arrear and is redeemed at 103% after 4 years. [5]
- [Total 9]

- 6** (i) An investor is considering the purchase of an annuity, payable annually in arrear for 20 years. The first payment is £ 500 . Using a rate of interest of 8% per annum effective, calculate the duration of the annuity when:

- (a) the payments remain level over the term.
(b) the payments increase at a rate of 8% per annum compound. [6]
- (ii) Explain why the answer in (i)(b) is higher than the answer in (i)(a). [2]
- [Total 8]

- 7** The shares of a company currently trade at £ 2.60 each, and the company has just paid a dividend of $12p$ per share. An investor assumes that dividends will be paid annually in perpetuity and will grow in line with a constant rate of inflation. The investor estimates the assumed inflation rate from equating the price of the share with the present value of all estimated future gross dividend payments using an effective interest rate of 6% per annum.

- (i) Calculate the investor's estimation of the effective inflation rate per annum based on the above assumptions. [4]
- (ii) Suppose that the actual inflation rate turns out to be 3% per annum effective over the following twelve years, but that all the investor's other assumptions are correct.

Calculate the investor's real rate of return per annum from purchase to sale, if she sold the shares after twelve years for £ 5 each immediately after a dividend has been paid. You may assume that the investor pays no tax. [6]

[Total 10]

- 8** An investor is considering investing in a capital project.

The project requires an outlay of £500,000 at outset and further payments at the end of each of the first 5 years, the first payment being £100,000 and each successive payment increasing by £10,000.

The project is expected to provide a continuous income at a rate of £80,000 in the first year, £83,200 in the second year and so on, with income increasing each year by 4% per annum compound. The income is received for 25 years.

It is assumed that, at the end of 15 years, a further investment of £300,000 will be required and that the project can be sold to another investor for £700,000 at the end of 25 years.

- (i) Calculate the net present value of the project at a rate of interest of 11% per annum effective. [9]
 - (ii) Without doing any further calculations, explain how the net present value would alter if the interest rate had been greater than 11% per annum effective. [3]
- [Total 12]

- 9** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 & 0 \leq t \leq 4 \\ 0.10 - 0.01t & 4 < t \leq 7 \\ 0.01t - 0.04 & 7 < t \end{cases}$$

- (i) Calculate the value at time $t = 5$ of £1,000 due for payment at time $t = 10$. [5]
 - (ii) Calculate the constant rate of interest per annum convertible monthly which leads to the same result as in (i) being obtained. [2]
 - (iii) Calculate the accumulated amount at time $t = 12$ of a payment stream, paid continuously from time $t = 0$ to $t = 4$, under which the rate of payment at time t is $\rho(t) = 100e^{0.02t}$. [6]
- [Total 13]

- 10** An insurance company holds a large amount of capital and wishes to distribute some of it to policyholders by way of two possible options.

Option A

£100 for each policyholder will be put into a fund from which the expected annual effective rate of return from the investments will be 5.5% and the standard deviation of annual returns 7%. The annual effective rates of return will be independent and $(1+i_t)$ is lognormally distributed, where i_t is the rate of return in year t . The policyholder will receive the accumulated investment at the end of ten years.

Option B

£100 will be invested for each policyholder for five years at a rate of return of 6% per annum effective. After five years, the accumulated sum will be invested for a further five years at the prevailing five-year spot rate. This spot rate will be 1% per annum effective with probability 0.2, 3% per annum effective with probability 0.3, 6% per annum effective with probability 0.2, and 8% per annum effective with probability 0.3. The policyholder will receive the accumulated investment at the end of ten years.

Deriving any necessary formulae:

- (i) Calculate the expected value and the standard deviation of the sum the policyholders will receive at the end of the ten years for each of options A and B. [17]
 - (ii) Determine the probability that the sum the policyholders will receive at the end of ten years will be less than £115 for each of options A and B. [5]
 - (iii) Comment on the relative risk of the two options from the policyholders' perspective. [2]
- [Total 24]

END OF PAPER

EXAMINATION

23 September 2008 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** A 91-day government bill is purchased for £95 at the time of issue and is redeemed at the maturity date for £100. Over the 91 days, an index of consumer prices rises from 220 to 222.

Calculate the effective real rate of return per annum. [3]

- 2** (i) State the strengths and weaknesses of using the money-weighted rate of return as opposed to the time-weighted rate of return as a measure of an investment manager's skill. [3]

- (ii) An investor had savings totalling £41,000 in an account on 1 January 2006. He invested a further £12,000 in this account on 1 August 2006. The total value of the account was £45,000 on 31 July 2006 and was £72,000 on 31 December 2007.

Assuming that the investor made no further deposits or withdrawals in relation to this account, calculate the annual effective time-weighted rate of return for the period 1 January 2006 to 31 December 2007. [2]

[Total 5]

- 3** (i) A forward contract with a settlement date at time T is issued based on an underlying asset with a current market price of B .

The annualised risk-free force of interest applying over the term of the forward contract is δ and the underlying asset pays no income. Show that the theoretical forward price is given by $K = Be^{\delta T}$, assuming no arbitrage. [3]

- (ii) An asset has a current market price of 200p, and will pay an income of 10p in exactly three months' time.

Calculate the price of a forward contract to be settled in exactly six months, assuming a risk-free rate of interest of 8% per annum convertible quarterly. [3]

[Total 6]

- 4** Describe the characteristics of commercial property (i.e. commercial real estate) as an investment. [5]

- 5** A bank offers two repayment alternatives for a loan that is to be repaid over ten years. The first requires the borrower to pay £1,200 per annum quarterly in advance and the second requires the borrower to make payments at an annual rate of £1,260 every second year in arrears.

Determine which terms would provide the best deal for the borrower at a rate of interest of 4% per annum effective. [5]

6 A pension fund holds an asset with current value £1 million. The investment return on the asset in a given year is independent of returns in all other years. The annual investment return in the next year will be 7% with probability 0.5 and 3% with probability 0.5. In the second and subsequent years, annual investment returns will be 2%, 4% or 6% with probability 0.3, 0.4 and 0.3, respectively.

- (i) Calculate the expected accumulated value of the asset after 10 years, showing all steps in your calculations. [3]
- (ii) Calculate the standard deviation of the accumulated value of the asset after 10 years, showing all steps in your calculations. [4]
- (iii) Without doing any further calculations explain how the mean and variance of the accumulation would be affected if the returns in years 2 to 10 were 1%, 4%, or 7%, with probability 0.3, 0.4 and 0.3 respectively. [2]

[Total 9]

7 The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by

$$\delta(t) = \begin{cases} 0.05 + 0.02t & \text{for } 0 \leq t \leq 5 \\ 0.15 & \text{for } t > 5 \end{cases}$$

- (i) Calculate the present value of £1,000 due at the end of 12 years. [5]
- (ii) Calculate the annual effective rate of discount implied by the transaction in (i). [2]

[Total 7]

8 A tax advisor is assisting a client in choosing between three types of investment. The client pays tax at 40% on income and 40% on capital gains.

Investment A requires the investment of £1m and provides an income of £0.1m per year in arrears for ten years. Income tax is deducted at source. At the end of the ten years, the investment of £1m is returned.

In Investment B, the initial sum of £1m accumulates at the rate of 10% per annum compound for ten years. At the end of the ten years, the accumulated value of the investment is returned to the investor after deduction of capital gains tax.

Investment C is identical to Investment B except that the initial sum is deemed, for tax purposes, to have increased in line with the index of consumer prices between the date of the investment and the end of the ten-year period. The index of consumer prices is expected to increase by 4% per annum compound over the period.

- (i) Calculate the net rate of return expected from each of the investments. [7]
- (ii) Explain why the expected rate of return is higher for Investment C than for Investment B and is higher for Investment B than for Investment A. [3]

[Total 10]

- 9** Three bonds, paying annual coupons in arrears of 6%, are redeemable at £105 per £100 nominal and reach their redemption dates in exactly one, two and three years' time respectively. The price of each of the bonds is £103 per £100 nominal.
- (i) Calculate the gross redemption yield of the three-year bond. [3]
- (ii) Calculate to three decimal places all possible spot rates, implied by the information given, as annual effective rates of interest. [4]
- (iii) Calculate to three decimal places all possible forward rates, implied by the information given, as annual effective rates of interest. [4]
- [Total 11]

- 10** An insurance company is considering two possible investment options.

The first investment option involves setting up a branch in a foreign country. This will involve an immediate outlay of £0.25m, followed by investments of £0.1m at the end of one year, £0.2m at the end of two years, £0.3m at the end of three years and so on until a final investment is made of £1m in ten years' time. The investment will provide annual payments of £0.5m for twenty years with the first payment at the end of the eighth year. There will be an additional incoming cash flow of £5m at the end of the 27th year.

The second investment option involves the purchase of 1 million shares in a bank at a price of £4.20 per share. The shares are expected to provide a dividend of 21p per share in exactly one year, 22.05p per share in two years and so on, increasing by 5% per annum compound. The shares are expected to be sold at the end of ten years, just after a dividend has been paid, for £5.64 per share.

- (i) Determine which of the options has the higher net present value at a rate of interest of 7% per annum effective. [9]
- (ii) Without doing any further calculations, determine which option has the higher discounted mean term at a rate of interest of 7% per annum effective. [2]
- [Total 11]

- 11** A company has a liability of £400,000 due in ten years' time.

The company has exactly enough funds to cover the liability on the basis of an effective interest rate of 8% per annum. This is also the interest rate on which current market prices are calculated and the interest rate earned on cash.

The company wishes to hold 10% of its funds in cash, and to invest the balance in the following securities:

- a zero-coupon bond redeemable at par in twelve years' time
 - a fixed-interest stock which is redeemable at 110% in sixteen years' time bearing interest at 8% per annum payable annually in arrear
- (i) Calculate the nominal amounts of the zero-coupon bond and the fixed-interest stock which should be purchased to satisfy Redington's first two conditions for immunisation. [10]
- (ii) Calculate the amount which should be invested in each of the assets mentioned in (i). [2]
- (iii) Explain whether the company would be immunised against small changes in the rate of interest if the quantities of stock in part (i) are purchased. [2]
- [Total 14]

- 12** An individual takes out a 25-year bank loan of £300,000 to purchase a house.

The individual agrees to pay only the interest payments, monthly in arrear, for the first 15 years whereupon he repays half of the capital as a lump sum. He then pays only the interest for the remaining 10 years, quarterly in arrear, and repays the other half of the capital as a lump sum at the end of the term.

- (i) Calculate the total amount of interest paid by the individual, assuming an effective rate of interest of 8½% p.a. [5]

- (ii) The individual believes that he can earn a nominal rate of interest convertible half-yearly of 9% p.a. from a separate savings account.

Calculate the level contribution he must make monthly in advance to the savings account in order to repay half the capital after 15 years. [4]

- (iii) The individual made the monthly contributions calculated in (ii) to the savings account. However, over the first 15 years, the effective rate of return earned on the savings account was 10% per annum.

The individual used the proceeds at that time to repay as much of the loan as possible and then decided to repay the remainder of the loan by level instalments of interest and capital. After the first 15 years, the effective rate of interest changed to 7% per annum.

Calculate the level payment he must make, payable monthly in arrear, to repay the loan over the final 10 years of the loan. [5]

[Total 14]

END OF PAPER

EXAMINATION

21 April 2009 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Describe the characteristics of Government Bills. [3]

2 Describe the characteristics of:

- (a) an interest-only loan (or mortgage); and
- (b) a repayment loan (or mortgage).

[4]

3 A loan is to be repaid by an annuity payable annually in arrear. The annuity starts at a rate of £300 per annum and increases each year by £30 per annum. The annuity is to be paid for 20 years.

Repayments are calculated using a rate of interest of 7% per annum effective.

Calculate:

(i) The amount of the loan. [3]

(ii) The capital outstanding immediately after the 5th payment has been made. [2]

(iii) The capital and interest components of the final payment. [2]

[Total 7]

4 (i) Explain what is meant by the “no arbitrage” assumption in financial mathematics. [2]

An investor entered into a long forward contract for £100 nominal of a security eight years ago and the contract is due to mature in four years’ time. The price per £100 nominal of the security was £94.50 eight years ago and is now £143.00. The risk-free rate of interest can be assumed to be 5% per annum effective throughout the contract.

(ii) Calculate the value of the contract now if it were known from the outset that the security will pay coupons of £9 two years from now and £10 three years from now. You may assume no arbitrage. [5]

[Total 7]

- 5** A company's required return for a particular investment project can be expressed as a force of interest, $\delta(t)$. This force of interest is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = 0.05 + 0.002t \quad 0 \leq t \leq 5$$

$$\delta(t) = 0.06 \quad 5 < t$$

The expenditure required for this project is a payment of £100,000 at $t = 0$ and a further payment of £80,000 at $t = 2$.

The income received from the project is a payment stream paid continuously from $t = 8$ to $t = 12$ under which the annual rate of payment at time t is $\text{£}100,000e^{0.001t}$.

Calculate the discounted payback period for this project. [8]

- 6** A pension fund purchased an office block nine months ago for £5 million.

The pension fund will spend a further £900,000 on refurbishment in two months time.

A company has agreed to occupy the office block six months from now. The lease agreement states that the company will rent the office block for fifteen years and will then purchase the property at the end of the fifteen year rental period for £6 million.

It is further agreed that rents will be paid quarterly in advance and will be increased every three years at the rate of 4% per annum compound. The initial rent has been set at £800,000 per annum with the first rental payment due immediately on the date of occupation.

Calculate, as at the date of purchase of the office block, the net present value of the project to the pension fund assuming an effective rate of interest of 8% per annum.

[8]

- 7** A fund had a value of £150,000 on 1 July 2006. A net cash flow of £30,000 was received on 1 July 2007 and a further net cash flow of £40,000 was received on 1 July 2008. The fund had a value of £175,000 on 30 June 2007 and a value of £225,000 on 30 June 2008. The value of the fund on 1 January 2009 was £280,000.

(i) Calculate the time-weighted rate of return per annum earned on the fund between 1 July 2006 and 1 January 2009. [3]

(ii) Calculate the money-weighted rate of return per annum earned on the fund between 1 July 2006 and 1 January 2009. [4]

(iii) Explain why the time-weighted rate of return is more appropriate than the money-weighted rate of return when comparing the performance of two investment managers over the same period of time. [2]

[Total 9]

- 8** An insurance company has liabilities consisting of eleven annual payments of £1 million, with the first payment due to be made in 10 years' time and the last payment due to be made in 20 years' time. The rate of interest is 6% per annum effective.
- (i) Show that the discounted mean term of these liabilities, to four significant figures, is 14.42 years. [3]
- The insurance company holds two zero-coupon bonds, one paying £ X in 10 years' time and the other paying £ Y in 20 years' time.
- (ii) Find values of X and Y such that Redington's first two conditions for immunisation from small changes in the rate of interest are satisfied. [6]
- (iii) Explain, without making any further calculations, whether you would expect Redington's third condition for immunisation to be satisfied for the values of X and Y calculated in (ii). [2]
- [Total 11]

- 9** Two bonds paying annual coupons of 5% in arrear and redeemable at par have terms to maturity of exactly one year and two years, respectively.
- The gross redemption yield from the 1-year bond is 4.5% per annum effective; the gross redemption yield from the 2-year bond is 5.3% per annum effective. You are informed that the 3-year par yield is 5.6% per annum.
- Calculate all zero-coupon yields and all one-year forward rates implied by the yields given above. [12]

- 10** A loan pays coupons of 11% per annum quarterly on 1 January, 1 April, 1 July and 1 October each year. The loan will be redeemed at 115% on any 1 January from 1 January 2015 to 1 January 2020 inclusive, at the option of the borrower. In addition to the redemption proceeds, the coupon then due is also paid.

An investor purchased a holding of the loan on 1 January 2005, immediately after the payment of the coupon then due, at a price which gave him a net redemption yield of at least 8% per annum effective. The investor pays tax at 30% on income and 25% on capital gains.

On 1 January 2008 the investor sold the holding, immediately after the payment of the coupon then due, to a fund which pays no tax. The sale price gave the fund a gross redemption yield of at least 9% per annum effective.

Calculate the following:

- (i) The price per £100 nominal at which the investor bought the loan. [6]
- (ii) The price per £100 nominal at which the investor sold the loan. [4]
- (iii) The net yield per annum convertible quarterly that was actually obtained by the investor during the period of ownership of the loan. [5]
- [Total 15]

- 11** An individual wishes to receive an annuity which is payable monthly in arrears for 15 years. The annuity is to commence in exactly 10 years at an initial rate of £12,000 per annum. The payments increase at each anniversary by 3% per annum. The individual would like to buy the annuity with a single premium 10 years from now.

- (i) Calculate the single premium required in 10 years' time to purchase the annuity assuming an interest rate of 6% per annum effective. [5]

The individual wishes to invest a lump sum immediately in an investment product such that, over the next 10 years, it will have accumulated to the premium calculated in (i). The annual effective returns from the investment product are independent and $(1 + i_t)$ is lognormally distributed, where i_t is the return in the t th year. The expected annual effective rate of return is 6% and the standard deviation of annual returns is 15%.

- (ii) Calculate the lump sum which the individual should invest immediately in order to have a probability of 0.98 that the proceeds will be sufficient to purchase the annuity in 10 years' time. [9]

- (iii) Comment on your answer to (ii). [2]
[Total 16]

END OF PAPER

EXAMINATION

30 September 2009 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 A 182-day government bill, redeemable at £100, was purchased for £96 at the time of issue and was later sold to another investor for £97.89. The rate of return received by the initial purchaser was 5% per annum effective.

- (a) Calculate the length of time in days for which the initial purchaser held the bill.

- (b) Calculate the annual simple rate of return achieved by the second investor.

[4]

2 List the characteristics of an equity investment. [4]

3 An investor bought a number of shares at 78 pence each on 31 December 2005. She received dividends on her holding on 31 December 2006, 2007 and 2008. The rate of dividend per share is given in the table below:

<i>Date</i>	<i>Rate of dividend per share</i>	<i>Retail price index</i>
31.12.2005	-----	147.7
31.12.2006	4.1 pence	153.4
31.12.2007	4.6 pence	158.6
31.12.2008	5.1 pence	165.1

On 31 December 2008, she sold her shares at a price of 93 pence per share.

Calculate, using the retail price index values shown in the table, the effective annual real rate of return achieved by the investor [7]

4 A fixed-interest security has just been issued. The security pays half-yearly coupons of 5% per annum in arrear and is redeemable at par 20 years after issue.

- (i) Calculate the price to provide an investor with a net redemption yield of 6% per annum effective. The investor pays tax at a rate of 20% on income and is not subject to capital gains tax. [3]

- (ii) Determine the annual effective gross redemption yield of this security assuming the price calculated in (i) is paid. [5]

- (iii) Determine the real annual effective gross redemption yield of this security if the rate of inflation is constant over the twenty years at 3% per annum. [2]

[Total 10]

5 The force of interest $\delta(t)$ at time t is $a + bt^2$ where a and b are constants. An amount of £100 invested at time $t = 0$ accumulates to £130 at time $t = 5$ and £200 at time $t = 10$.

- (i) Calculate the values of a and b . [6]
 - (ii) Calculate the constant rate of interest per annum convertible monthly that would give rise to the same accumulation from time $t = 0$ to time $t = 5$. [2]
 - (iii) Calculate the constant force of interest that would give rise to the same accumulation from time $t = 5$ to time $t = 10$. [2]
- [Total 10]

6 (i) Distinguish between a future and an option. [2]

An investor wishes to purchase a one year forward contract on a risk-free bond which has a current market price of £97 per £100 nominal. The bond will pay coupons at a rate of 7% per annum half yearly. The next coupon payment is due in exactly six months and the following coupon payment is due just before the forward contract matures. The six-month risk-free spot interest rate is 5% per annum effective and the 12-month risk-free spot interest rate is 6% per annum effective.

- (ii) Stating all necessary assumptions:
 - (a) Calculate the forward price of the bond.
 - (b) Calculate the six-month forward rate for an investment made in six months' time.
 - (c) Calculate the purchase price of a risk-free bond with exactly one year to maturity which is redeemed at par and which pays coupons of 4% per annum half-yearly in arrears.
 - (d) Calculate the gross redemption yield from the bond in (c).
 - (e) Comment on why your answer in (d) is close to the one-year spot rate.

[10]
[Total 12]

7 A member of a pensions savings scheme invests £1,200 per annum in monthly instalments, in advance, for 20 years from his 25th birthday. From the age of 45, the member increases his investment to £2,400 per annum. At each birthday thereafter the annual rate of investment is further increased by £100 per annum. The investments continue to be made monthly in advance for 20 years until the individual's 65th birthday.

- (i) Calculate the accumulation of the investment at the age of 65 using a rate of interest of 6% per annum effective. [6]

At the age of 65, the scheme member uses his accumulated investment to purchase an annuity with a term of 20 years to be paid half-yearly in arrear. At this time the interest rate is 5% per annum convertible half-yearly.

- (ii) Calculate the annual rate of payment of the annuity. [3]
- (iii) Calculate the discounted mean term of the annuity, in years, at the time of purchase. [3]

[Total 12]

8 A bank offers a customer two different repayment options on a loan of £50,000 as follows:

Option 1 – level instalments of capital and interest are paid annually in arrear over a period of 20 years.

Option 2 – over the 20-year term the customer pays only interest on the loan, annually in arrear at a rate of 5.5% per annum with the whole of the capital amount payable at the end of the term. The customer will take out a separate savings policy which involves making monthly payments in advance such that the proceeds will be sufficient to repay the loan at the end of its term. The payments into the savings policy accumulate at a rate of interest of 4% per annum effective.

- (i) Determine the effective rate of interest per annum that would be paid by the customer on the loan under Option 1, given that the level annual instalment on this loan is £4,012.13. [3]
- (ii) Determine the annual effective rate of interest paid by a customer under Option 2. [7]

[Total 10]

9 A life insurance company is issuing a single premium policy which will pay out £20,000 in twenty years time. The interest rate the company will earn on the invested funds over the first ten years of the policy will be 4% per annum with a probability of 0.3 and 6% per annum with a probability of 0.7. Over the second ten years the interest rate earned will be 5% per annum with probability 0.5 and 6% per annum with probability 0.5.

- (i) Calculate the premium that the company would charge if it calculates the premium using the expected annual rate of interest in each ten year period. [2]
- (ii) Calculate the expected profit to the company if the premium is calculated as in (i). The rate of interest in the second ten year period is independent of that in the first ten year period. [3]
- (iii) Explain why, despite the company using the expected rate of interest to calculate the premium, there is a positive expected profit. [2]
- (iv) By considering each possible outcome in (ii):
 - (a) Find the range of possible profits.
 - (b) Calculate the standard deviation of the profit to the company. [7]

[Total 14]

- 10** A group of experts is analysing options to try to avert problems caused by climate change. They agree on the following expected costs and benefits of climate change over the next 50 years, starting from the current time. All figures are given in 2009 dollars.

Costs of climate change:

- Serious events will occur once every three years, in arrear, each giving rise to costs of \$30bn, incurred immediately on the date of the event.
- Communities affected by climate change will incur costs of \$20bn per annum incurred continuously, increasing at a continuous rate of 1% per annum.
- Other costs, assumed to be \$40bn per annum, will be incurred annually in arrear.

Benefits arising from climate change:

- Benefits from higher crop yields and lower heating costs are assumed to be \$10bn per annum, incurred annually in arrear.

The experts are considering whether to recommend investment in a carbon storing technology which, it is believed, will reduce all the costs and benefits listed above to zero. The technology requires a one-off investment immediately of \$440bn. Costs are then assumed to be \$50bn per annum incurred annually in arrear for 50 years.

The experts do not agree about the appropriate rate of interest at which to evaluate the options available. One group believes that the net present value of using the carbon storage technology should be evaluated at a real rate of return of 4% per annum effective. A second group believe that it should be evaluated at a real rate of return of 1% per annum effective.

- (i) Define what is meant by the discounted payback period of an investment and indicate its main disadvantage as an investment decision criterion. [3]
 - (ii) Explain why the project must have a discounted payback period when the interest rate is 1.5% and the internal rate of return is higher than 1.5%. [2]
 - (iii) Calculate the net present value of the carbon storing technology at a real rate of interest of 1% per annum effective. [5]
 - (iv) Calculate the net present value of the carbon storing technology at a real rate of interest of 4% per annum effective. [5]
 - (v) Comment on whether the investment in the carbon storing technology should go ahead. [2]
- [Total 17]

END OF PAPER

EXAMINATION

27 April 2010 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** (i) Explain the difference
- (a) between options and futures
 - (b) between call options and put options
- [4]

A security is priced at £60. Coupons are paid half-yearly. The next coupon is due in two months' time and will be £2.80. The risk-free force of interest is 6% per annum.

- (ii) Calculate the forward price an investor should agree to pay for the security in three months' time assuming no arbitrage. [3]
- [Total 7]

- 2** In January 2008, the government of a country issued an index-linked bond with a term of two years. Coupons were payable half-yearly in arrear, and the annual nominal coupon rate was 4%. Interest and capital payments were indexed by reference to the value of an inflation index with a time lag of six months.

A tax-exempt investor purchased £100,000 nominal at issue and held it to redemption. The issue price was £98 per £100 nominal.

The inflation index was as follows:

<i>Date</i>	<i>Inflation Index</i>
July 2007	110.5
January 2008	112.1
July 2008	115.7
January 2009	119.1
July 2009	123.2

- (i) Calculate the investor's cashflows from this investment and state the month when each cashflow occurs. [3]
- (ii) Calculate the annual effective money yield obtained by the investor to the nearest 0.1% per annum. [3]
- [Total 6]

- 3** A company issues ordinary shares to an investor who is subject to income tax at 20%.

Under the terms of the ordinary share issue, the investor is to purchase 1,000,000 shares at a purchase price of 45p each on 1 January 2011.

No dividend is expected to be paid for 2 years. The first dividend payable on 1 January 2013 is expected to be 5p per share. Dividends will then be paid every 6 months in perpetuity. The two dividend payments in any calendar year are expected to be the same, but the dividend payment is expected to increase at the end of each year at a rate of 3% per annum compound.

Calculate the net present value of the investment on 1 January 2011 at an effective rate of interest of 8% per annum. [5]

- 4** An investor is considering purchasing a fixed interest bond at issue which pays half-yearly coupons at a rate of 6% per annum. The bond will be redeemed at £105 per £100 nominal in 10 years' time. The investor is subject to income tax at 20% and capital gains tax at 25%.

The inflation rate is assumed to be constant at 2.8571% per annum.

Calculate the price per £100 nominal if the investor is to obtain a net real yield of 5% per annum. [7]

- 5** Let f_t denote the one-year forward rate of interest over the year from time t to time $(t+1)$.

The current forward rates in the market are:

time, t	0	1	2	3
one-year forward rate, f_t	4.4% p.a.	4.7% p.a.	4.9% p.a.	5.0% p.a.

A fixed-interest security pays coupons annually in arrear at the rate of 7% per annum and is redeemable at par in exactly four years.

- (i) Calculate the price per £100 nominal of the security assuming no arbitrage. [3]
- (ii) Calculate the gross redemption yield of the security. [3]
- (iii) Explain, without doing any further calculations, how your answer to part (ii) would change if the annual coupon rate on the security were 9% per annum (rather than 7% per annum). [2]
- [Total 8]

- 6** The annual returns, i , on a fund are independent and identically distributed. Each year, the distribution of $1 + i$ is lognormal with parameters $\mu = 0.05$ and $\sigma^2 = 0.004$, where i denotes the annual return on the fund.
- (i) Calculate the expected accumulation in 25 years' time if £3,000 is invested in the fund at the beginning of each of the next 25 years. [5]
- (ii) Calculate the probability that the accumulation of a single investment of £1 will be greater than its expected value 20 years later. [5]
- [Total 10]

- 7** A pension fund has to pay out benefits at the end of each of the next 40 years. The benefits payable at the end of the first year total £1 million. Thereafter, the benefits are expected to increase at a fixed rate of 3.8835% per annum compound.
- (i) Calculate the discounted mean term of the liabilities using a rate of interest of 7% per annum effective. [5]
- The pension fund can invest in both coupon-paying and zero-coupon bonds with a range of terms to redemption. The longest-dated bond currently available in the market is a zero-coupon bond redeemed in exactly 15 years.
- (ii) Explain why it will not be possible to immunise this pension fund against small changes in the rate of interest. [2]
- (iii) Describe the other practical problems for an institutional investor who is attempting to implement an immunisation strategy. [3]
- [Total 10]

- 8** A loan is repayable by annual instalments paid in arrear for 20 years. The first instalment is £4,650 and each subsequent instalment is £150 greater than the previous instalment.

Calculate the following, using an interest rate of 9% per annum effective:

- (i) the amount of the original loan [3]
(ii) the capital repayment in the tenth instalment [4]
(iii) the interest element in the last instalment [2]
(iv) the total interest paid over the whole 20 years [2]
- [Total 11]

- 9** A company is undertaking a new project. The project requires an investment of £5m at the outset, followed by £3m three months later.

It is expected that the investment will provide income over a 15 year period starting from the beginning of the third year. Net income from the project will be received continuously at a rate of £1.7m per annum. At the end of this 15 year period there will be no further income from the investment.

Calculate at an effective rate of interest of 10% per annum:

- (i) the net present value of the project [3]
(ii) the discounted payback period [4]

A bank has offered to loan the funds required to the company at an effective rate of interest of 10% per annum. Funds will be drawn from the bank when required and the loan can be repaid at any time. Once the loan is paid off, the company can earn interest on funds from the venture at an effective rate of interest of 7% per annum.

- (iii) Calculate the accumulated profit at the end of the 17 years. [4]
[Total 11]

- 10** A pension fund's assets were invested with two fund managers.

On 1 January 2007 Manager A was given £120,000 and Manager B was given £100,000. A further £10,000 was invested with each manager on 1 January 2008 and again on 1 January 2009.

The values of the funds were:

	<i>31 December 2007</i>	<i>31 December 2008</i>	<i>31 December 2009</i>
Manager A	£130,000	£135,000	£180,000
Manager B	£140,000	£145,000	£150,000

- (i) Calculate the time-weighted rates of return earned by Manager A and Manager B over the period 1 January 2007 to 31 December 2009. [4]
- (ii) Show that the money-weighted rate of return earned by Manager A over the period 1 January 2007 to 31 December 2009 is approximately 9.4% per annum. [2]
- (iii) Explain, without performing further calculations, whether the money-weighted rate of return earned by Manager B over the period 1 January 2007 to 31 December 2009 was higher than, lower than or equal to that earned by Manager A. [3]
- (iv) Discuss the relative performance of the two fund managers. [3]
[Total 12]

- 11** The force of interest $\delta(t)$ is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.04 + 0.02t & 0 \leq t < 5 \\ 0.05 & 5 \leq t \end{cases}.$$

- (i) Derive and simplify as far as possible expressions for $v(t)$, where for $v(t)$ is the present value of a unit sum of money due at time t . [5]
- (ii) (a) Calculate the present value of £1000 due at the end of 17 years.
(b) Calculate the rate of interest per annum convertible monthly implied by the transaction in part (ii)(a). [4]

A continuous payment stream is received at a rate of $10e^{0.01t}$ units per annum between $t = 6$ and $t = 10$.

- (iii) Calculate the present value of the payment stream. [4]
[Total 13]

END OF PAPER

EXAMINATION

7 October 2010 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** A bond pays coupons in perpetuity on 1 June and 1 December each year. The annual coupon rate is 3.5% per annum. An investor purchases a quantity of this bond on 20 August 2009.

Calculate the price per £100 nominal to provide the investor with an effective rate of return per annum of 10%. [3]

- 2** A bond is redeemed at £110 per £100 nominal in exactly four years' time. It pays coupons of 4% per annum half-yearly in arrear and the next coupon is due in exactly six months' time. The current price is £110 per £100 nominal.

- (i) (a) Calculate the gross rate of return per annum convertible half-yearly from the bond.
(b) Calculate the gross effective rate of return per annum from the bond. [2]
- (ii) Calculate the net effective rate of return per annum from the bond for an investor who pays income tax at 25%. [2]
- [Total 4]

- 3** The annual rates of return from an asset are independently and identically distributed. The expected accumulation after 20 years of £1 invested in this asset is £2 and the standard deviation of the accumulation is £0.60.

- (a) Calculate the expected effective rate of return per annum from the asset, showing all the steps in your working.
(b) Calculate the variance of the effective rate of return per annum. [6]

- 4** A six-month forward contract was issued on 1 April 2009 on a share with a price of 700p at that date. It was known that a dividend of 20p per share would be paid on 1 May 2009. The one-month spot, risk-free rate of interest at the time of issue was 5% per annum effective and the forward rate of interest from 1 May to 30 September was 3% per annum effective.

- (i) Calculate the forward price at issue, assuming no arbitrage, explaining your working. [3]

It has been suggested that the forward price cannot be calculated without making a judgement about the expected price of the share when the forward contract matures.

- (ii) Explain why this statement is not correct. [2]
- (iii) Comment on whether the method used in part (i) would still be valid if it was not known with certainty that the dividend due on 1 May 2009 would be paid. [1]
- [Total 6]

- 5** (a) Describe the characteristics of Eurobonds.
(b) Describe the characteristics of convertible bonds.

[6]

- 6** On 1 January 2001 the government of a particular country bought 200 million shares in a particular bank for a total price of £2,000 million. The shares paid no dividends for three years. On 30 June 2004 the shares paid dividends of 10 pence per share. On 31 December 2004, they paid dividends of 20 pence per share. Each year, until the end of 2009, the dividend payable every 30 June rose by 10% per annum compound and the dividend payable every 31 December rose by 10% per annum compound. On 1 January 2010, the shares were sold for their market price of £3,500 million.

- (i) Calculate the net present value on 1 January 2001 of the government's investment in the bank at a rate of interest of 8% per annum effective. [5]
- (ii) Calculate the accumulated profit from the government's investment in the bank on the date the shares are sold using a rate of interest of 8% per annum effective. [1]

[Total 6]

- 7** (i) State the three conditions that are necessary for a fund to be immunised from small, uniform changes in the rate of interest. [2]
- (ii) A pension fund has liabilities of £10m to meet at the end of each of the next ten years. It is able to invest in two zero-coupon bonds with a term to redemption of three years and 12 years respectively. The rate of interest is 4% per annum effective.

Calculate:

- (a) the present value of the liabilities of the pension fund
- (b) the duration of the liabilities of the pension fund
- (c) the nominal amount that should be invested in the zero-coupon bonds to ensure that the present values and durations of the assets and liabilities is the same [7]
- (iii) One year later, just before the pension payment then due, the rate of interest is 5% per annum effective.
- (a) Determine whether the duration of the assets and the liabilities are still equal.
- (b) Comment on the practical usefulness of the theory of immunisation in the context of the above result. [6]

[Total 15]

- 8** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.05 + 0.001t & 0 \leq t \leq 20 \\ 0.05 & t > 20 \end{cases}$$

- (i) Derive and simplify as far as possible expressions for $v(t)$, where $v(t)$ is the present value of a unit sum of money due at time t . [5]
- (ii) (a) Calculate the present value of £100 due at the end of 25 years.
(b) Calculate the rate of discount per annum convertible quarterly implied by the transaction in part (ii)(a). [4]
- (iii) A continuous payment stream is received at rate $30e^{-0.015t}$ units per annum between $t = 20$ and $t = 25$. Calculate the accumulated value of the payment stream at time $t = 25$. [4]
- [Total 13]

- 9** The government of a particular country has just issued three bonds with terms to redemption of exactly one, two and three years respectively. Each bond is redeemed at par and pays coupons of 8% annually in arrear. The annual effective gross redemption yields from the one, two and three year bonds are 4%, 3% and 3% respectively.

- (i) Calculate the one-year, two-year and three-year spot rates of interest at the date of issue. [8]
- (ii) Calculate all possible forward rates of interest from the above spot rates of interest. [4]

An index of retail prices has a current value of 100.

- (iii) Calculate the expected level of the retail prices index in one year, two years' and three years' time if the expected real spot rates of interest are 2% per annum effective for all terms. [5]
- (iv) Calculate the expected rate of inflation per annum in each of the next three years. [2]
- [Total 19]

- 10** On 1 April 2003 a company issued securities that paid no interest and that were to be redeemed for £70 after five years. The issue price of the securities was £64. The securities were traded in the market and the market prices at various different dates are shown in the table below.

<i>Date</i>	<i>Market price of securities (£)</i>
1 April 2003	64
1 April 2004	65
1 April 2005	60
1 April 2006	65
1 April 2007	68
1 April 2008	70

- (i) Explain why the price of the securities might have fallen between 1 April 2004 and 1 April 2005. [1]

Two investors bought the securities at various dates. Investor X bought 100 securities on 1 April 2003 and 1,000 securities on 1 April 2005. Investor Y bought 100 securities every year on 1 April from 2003 to 2007 inclusive. Both investors held the securities until maturity.

- (ii) Construct a table showing the nominal amount of the securities held and the market value of the holdings for X and Y on 1 April each year, just before any purchases of securities. [5]

- (iii) (a) Calculate the effective money weighted rate of return per annum for X for the period from 1 April 2003 to 1 April 2008.

- (b) Calculate the effective time weighted rate of return per annum for X for the period from 1 April 2003 to 1 April 2008.

[6]

- (iv) (a) Determine whether the effective money weighted rate of return for Y is lower or higher than that for X for the period from 1 April 2003 to 1 April 2008.

- (b) Determine the effective time weighted rate of return per annum for Y for the period from 1 April 2003 to 1 April 2008.

[7]

- (v) Discuss the relationship between the different rates of return that have been calculated. [3]

[Total 22]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

19 April 2011 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.04 + 0.003t^2 & \text{for } 0 < t \leq 5 \\ 0.01 + 0.03t & \text{for } 5 < t \end{cases}$$

- (i) Calculate the amount to which £1,000 will have accumulated at $t = 7$ if it is invested at $t = 3$. [4]
- (ii) Calculate the constant rate of discount per annum, convertible monthly, which would lead to the same accumulation as that in (i) being obtained. [3]

[Total 7]

- 2** A one-year forward contract on a stock is entered into on 1 January 2011 when the stock price is £68 and the risk-free force of interest is 14% per annum. The stock is expected to pay an annual dividend of £2.50 with the next dividend due in eight months' time.

On 1 April 2011, the price of the stock is £71 and the risk-free force of interest is 12% per annum. The dividend expectation is unchanged.

Calculate the value of the contract to the holder of the long forward position on 1 April 2011. [6]

- 3** An investment trust bought 1,000 shares at £135 each on 1 July 2005. The trust received dividends on its holding on 30 June each year that it held the shares.

The rate of dividend per share was as given in the table below:

<i>30 June in year</i>	<i>Rate of dividend per share (£)</i>	<i>Retail price index</i>
2005	...	121.4
2006	7.9	125.6
2007	8.4	131.8
2008	8.8	138.7
2009	9.4	145.3
2010	10.1	155.2

On 1 July 2010, the investment trust sold its entire holding of the shares at a price of £151 per share.

- (i) Using the retail price index values shown in the table, calculate the real rate of return per annum effective achieved by the trust on its investment. [6]
- (ii) Explain, without doing any further calculations, how your answer to (i) would alter (if at all) if the retail price index for 30 June 2008 had been greater than 138.7 (with all other index values unchanged). [2]

[Total 8]

- 4** The n -year spot rate of interest y_n , is given by:

$$y_n = 0.03 + \frac{n}{1000} \quad \text{for } n = 1, 2, 3 \text{ and } 4$$

(i) Calculate the implied one-year and two-year forward rates applicable at time $t = 2$. [3]

(ii) Calculate, assuming no arbitrage:

(a) The price at time $t = 0$ per £100 nominal of a bond which pays annual coupons of 4% in arrear and is redeemed at 115% after 3 years.

(b) The 3-year par yield.

[6]

[Total 9]

- 5** A loan of nominal amount £100,000 was issued on 1 April 2011 bearing interest payable half-yearly in arrear at a rate of 6% per annum. The loan is to be redeemed with a capital payment of £105 per £100 nominal on any coupon date between 20 and 25 years after the date of issue, inclusive, with the date of redemption being at the option of the borrower.

An investor who is liable to income tax at 20% and capital gains tax of 35% wishes to purchase the entire loan on 1 June 2011 at a price which ensures that the investor achieves a net effective yield of at least 5% per annum.

(i) Determine whether the investor would make a capital gain if the investment is held until redemption. [3]

(ii) Explain how your answer to (i) influences the assumptions made in calculating the price the investor should pay. [2]

(iii) Calculate the maximum price the investor should pay. [5]
[Total 10]

6 The value of the assets held by a pension fund on 1 January 2010 was £10 million. On 30 April 2010, the value of the assets had fallen to £8.5 million. On 1 May 2010, the fund received a contribution payment of £7.5 million and paid out £2 million in benefits. On 31 December 2010, the value of the fund was £17.1 million.

(i) Calculate the annual effective money-weighted rate of return (MWRR) for 2010. [3]

(ii) Calculate the annual effective time-weighted rate of return (TWRR) for 2010. [3]

(iii) Explain why the MWRR is higher than the TWRR for 2010. [2]

The fund manager's bonus for 2010 is based on the return achieved by the fund over the year.

(iv) State, with reasons, which of the two rates of return calculated above would be more appropriate for this purpose. [2]

[Total 10]

7 A loan of £60,000 was granted on 1 July 1998.

The loan is repayable by an annuity payable quarterly in arrear for 20 years. The amount of the quarterly repayment increases by £100 after every four years. The repayments were calculated using a rate of interest of 8% per annum convertible quarterly.

(i) Show that the initial quarterly repayment is £1,370.41. [5]

(ii) Calculate the amount of capital repaid that was included in the payment made on 1 January 1999. [3]

(iii) Calculate the amount of capital outstanding after the quarterly repayment due on 1 July 2011 has been made. [4]

[Total 12]

8 A company has liabilities of £10 million due in three years' time and £20 million due in six years' time. The investment manager for the company is able to buy zero-coupon bonds for whatever term he requires and has adequate monies at his disposal.

(i) Explain whether it is possible for the investment manager to immunise the fund against small changes in the rate of interest by purchasing a single zero-coupon bond. [2]

The investment manager decides to purchase two zero-coupon bonds, one for a term of four years and the other for a term of 20 years. The current interest rate is 4% per annum effective.

(ii) Calculate the amount that must be invested in each bond in order that the company is immunised against small changes in the rate of interest. You should demonstrate that all three Redington conditions are met. [10]

[Total 12]

- 9** A company is considering investing in a project. The project requires an initial investment of three payments, each of £105,000. The first is due at the start of the project, the second six months later, and the third payment is due one year after the start of the project.

After 15 years, it is assumed that a major refurbishment of the infrastructure will be required, costing £200,000.

The project is expected to provide a continuous income stream as follows:

- £20,000 in the second year
- £23,000 in the third year
- £26,000 in the fourth year
- £29,000 in the fifth year

Thereafter the continuous income stream is expected to increase by 3% per annum (compound) at the start of each year. The income stream is expected to cease at the end of the 30th year from the start of the project.

- (i) Show that the net present value of the project at a rate of interest of 8% per annum effective is £4,000 (to the nearest £1,000). [7]
- (ii) Calculate the discounted payback period for the project, assuming a rate of interest of 8% per annum effective. [5]

[Total 12]

- 10** The annual rates of return from a particular investment, Investment A, are independently and identically distributed. Each year, the distribution of $(1+i_t)$, where i_t is the rate of interest earned in year t , is log-normal with parameters μ and σ^2 .

The mean and standard deviation of i_t are 0.06 and 0.03 respectively.

- (i) Calculate μ and σ^2 . [5]

An insurance company has liabilities of £15m to meet in one year's time. It currently has assets of £14m. Assets can either be invested in Investment A, described above, or in Investment B which has a guaranteed return of 4% per annum effective.

- (ii) Calculate, to two decimal places, the probability that the insurance company will be unable to meet its liabilities if:
- (a) All assets are invested in Investment B.
- (b) 75% of assets are invested in Investment A and 25% of assets are invested in Investment B. [6]
- (iii) Calculate the variance of return from each of the portfolios in (ii)(a) and (ii)(b). [3]

[Total 14]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

27 September 2011 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** A 91-day treasury bill is issued by the government at a simple rate of discount of 8% per annum.

Calculate the annual effective rate of return obtained by an investor who purchases the bill at issue. [3]

- 2** State the characteristics of index-linked government bonds. [3]

- 3** An individual intends to retire on his 65th birthday in exactly four years' time. The government will pay a pension to the individual from age 68 of £5,000 per annum monthly in advance. The individual would like to purchase an annuity certain so that his income, including the government pension, is £8,000 per annum paid monthly in advance from age 65 until his 78th birthday. He is to purchase the annuity by a series of payments made over four years quarterly in advance starting immediately.

Calculate the quarterly payments the individual has to make if the present value of these payments is equal to the present value of the annuity he wishes to purchase at a rate of interest of 5% per annum effective. Mortality should be ignored. [6]

- 4** A pension fund makes the following investments (£m):

<i>1 January 2009</i>	<i>1 July 2009</i>	<i>1 January 2010</i>
1.5	6.0	4.0

The rates of return earned on money invested in the fund were as follows:

<i>1 January 2009 to 30 June 2009</i>	<i>1 July 2009 to 31 December 2009</i>	<i>1 January 2010 to 31 December 2010</i>
1%	2%	5%

Assume that 1 January to 30 June and 1 July to 31 December are precise half-year periods.

- (i) Calculate the time-weighted rate of return per annum effective over the two years from 1 January 2009 to 31 December 2010. [3]
- (ii) Calculate the money-weighted rate of return per annum effective over the two years from 1 January 2009 to 31 December 2010. [3]

[Total 6]

- 5** A nine-month forward contract is issued on 1 March 2011 on a stock with a price of £9.56 per share at that date. Dividends of 20 pence per share are expected on both 1 April 2011 and 1 October 2011.
- (i) Calculate the forward price, assuming a risk-free rate of interest of 3% per annum effective and no arbitrage. [4]
 - (ii)
 - (a) Explain why the expected price of the share in nine months' time is not needed to calculate the forward price.
 - (b) Explain why the price of an option would be explicitly dependent on the variance of the share price but the price of a forward would not be.
- [4]
[Total 8]
- 6** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is $a + bt$ where a and b are constants. An amount of £45 invested at time $t = 0$ accumulates to £55 at time $t = 5$ and £120 at time $t = 10$.
- (i) Calculate the values of a and b . [5]
 - (ii) Calculate the constant force of interest per annum that would give rise to the same accumulation from time $t = 0$ to time $t = 10$. [2]
- [Total 7]
- 7** An investment manager is considering investing in the ordinary shares of a particular company.
- The current price of the shares is 12 pence per share. It is highly unlikely that the share will pay any dividends in the next five years. However, the investment manager expects the company to pay a dividend of 2 pence per share in exactly six years' time, 2.5 pence per share in exactly seven years' time, with annual dividends increasing thereafter by 1% per annum in perpetuity.
- In five years' time, the investment manager expects to sell the shares. The sale price is expected to be equal to the present value of the expected dividends from the share at that time at a rate of interest of 8% per annum effective.
- (i) Calculate the effective gross rate of return per annum the investment manager will obtain if he buys the share and then sells it at the expected price in five years' time. [6]
 - (ii) Calculate the net effective rate of return per annum the investment manager will obtain if he buys the share today and then sells it at the expected price in five years' time if capital gains tax is payable at 25% on any capital gains. [3]
 - (iii) Calculate the net effective real rate of return per annum the investment manager will obtain if he buys the share and then sells it at the expected price in five years' time if capital gains tax is payable at 25% on any capital gains and inflation is 4% per annum effective. There is no indexation allowance. [3]
- [Total 12]

- 8** (i) State the conditions that are necessary for an insurance company to be immunised from small, uniform changes in the rate of interest. [2]

An insurance company has liabilities to pay £100m annually in arrear for the next 40 years. In order to meet these liabilities, the insurance company can invest in zero coupon bonds with terms to redemption of five years and 40 years.

- (ii) (a) Calculate the present value of the liabilities at a rate of interest of 4% per annum effective.
- (b) Calculate the duration of the liabilities at a rate of interest of 4% per annum effective. [5]
- (iii) Calculate the nominal amount of each bond that the fund needs to hold so that the first two conditions for immunisation are met at a rate of interest of 4% per annum effective. [5]
- (iv) (a) Estimate, using your calculations in (ii) (b), the revised present value of the liabilities if there were a reduction in interest rates by 1.5% per annum effective.
- (b) Calculate the present value of the liabilities at a rate of interest of 2.5% per annum effective.
- (c) Comment on your results to (iv) (a) and (iv) (b). [6]
- [Total 18]

- 9** (i) Describe the information that an investor can obtain from the following yield curves for government bonds:
- (a) A forward rate yield curve.
(b) A spot rate yield curve.
(c) A gross redemption yield curve. [6]

An investor is using the information from a government bond spot yield curve to calculate the present value of a corporate eurobond with a term to redemption of exactly five years. The investor will value each payment that is due from the bond at a rate of interest equal to $j = i + 0.01 + 0.001t$ where:

- t is the time in years at which the payment is due
- i is the annual t -year effective spot rate of interest from the government bond spot yield curve and $i = 0.02t$ for $t \leq 5$

The eurobond pays annual coupons of 10% of the nominal amount of the bond and is redeemed at par.

- (ii) Calculate the present value of the eurobond. [6]
- (iii) Calculate the gross redemption yield from the eurobond. [3]

- (iv) Explain why the investor might use such a formula for j to determine the interest rates at which to value the payments from the corporate eurobond. [3]
[Total 18]

- 10** A country's football association is considering whether to bid to host the World Cup in 2026. Several countries aspiring to host the World Cup will be making bids. Regardless of whether the bid is successful, the association will incur various costs. For two years, starting on 1 January 2012, the association will incur costs at a rate of £2m per annum, assumed to be paid continuously, to prepare the bid.

If the football association is successful, the following costs will be incurred from 1 January 2016 until 31 December 2025:

- One stadium will be built each year for ten years. The first stadium will be built in 2016 and is expected to cost £200m; the stadium built in 2017 is expected to cost £210m; and so on, with the cost of each stadium rising by 5% each year. The costs of building each stadium are assumed to be incurred halfway through the relevant year.
 - Administration costs at a rate of £100m per annum will be incurred, payable monthly in advance from 1 January 2025 until 31 December 2026.
 - Revenues from television, ticket receipts, advertising and so on are expected to be £3,300m and are assumed to be received continuously throughout 2026.
- (i) Explain why the payback period is not a good indicator of whether this project is worthwhile. [3]

The football association decides to judge whether to go ahead with the bid by calculating the net present value of the costs and revenues from a successful bid on 1 January 2012 at a rate of interest of 4% per annum effective.

- (ii) Determine whether the association should make the bid. [13]

The football association is discussing how it might factor into its calculations the fact that it is not certain to win the right to host the World Cup because other countries are also bidding.

- (iii) Explain how you might adjust the above calculations if the probability of winning the right to host the World Cup is 0.1 and whether this adjustment would make it more likely or less likely that the bid will go ahead. [3]
[Total 19]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

24 April 2012 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** In a particular bond market, n -year spot rates can be approximated by the function $0.06 - 0.02e^{-0.1n}$.
- (i) Calculate the gross redemption yield for a 3-year bond which pays coupons of 3% annually in arrear, and is redeemed at par. Show all workings. [6]
 - (ii) Calculate the 4-year par yield. [3]
- [Total 9]

- 2** The value of the assets held by an investment fund on 1 January 2011 was £2.3 million.

On 30 April 2011, the value of the assets had risen to £2.9 million and, on 1 May 2011, there was a net cash inflow to the fund of £1.5 million. On 31 December 2011, the value of the assets was £4.2 million.

- (i) Calculate the annual effective time-weighted rate of return (TWRR) for 2011. [2]
 - (ii) Calculate, to the nearer 0.1%, the annual effective money-weighted rate of return (MWRR) for 2011. [4]
 - (iii) Explain why the TWRR is significantly higher than the MWRR for 2011. [2]
- [Total 8]

- 3** A company has borrowed £500,000 from a bank. The loan is to be repaid by level instalments, payable annually in arrear for ten years from the date the loan is made. The annual instalments are calculated at an effective rate of interest of 9% per annum.

- (i) Calculate:
 - (a) the amount of the level annual instalments.
 - (b) the total amount of interest which will be paid over the ten-year term.
- [3]

At the beginning of the eighth year, immediately after the seventh instalment has been made, the company asks for the loan to be rescheduled over a further four years from that date. The bank agrees to do this on condition that the rate of interest is increased to an effective rate of 12% per annum for the term of the rescheduled instalments and that repayments are made quarterly in arrear.

- (ii) (a) Calculate the amount of the new quarterly instalment.
 - (b) Calculate the interest content of the second quarterly instalment of the rescheduled loan repayments.
- [5]
- [Total 8]

- 4** (i) Explain what is meant by the “no arbitrage” assumption in financial mathematics. [2]

An investor entered into a long forward contract for a security four years ago and the contract is due to mature in five years’ time. The price of the security was £7.20 four years ago and is now £10.45. The risk-free rate of interest can be assumed to be 2.5% per annum effective throughout the nine-year period.

- (ii) Calculate, assuming no arbitrage, the value of the contract now if the security will pay dividends of £1.20 annually in arrear until maturity of the contract. [3]

- (iii) Calculate, assuming no arbitrage, the value of the contract now if the security has paid and will continue to pay annually in arrear a dividend equal to 3% of the market price of the security at the time of payment. [3]

[Total 8]

- 5** An investor is considering two projects, Project A and Project B. Project A involves the investment of £1,309,500 in a retail outlet. Rent is received quarterly in arrear for 25 years, at an initial rate of £100,000 per annum. It is assumed that the rent will increase at a rate of 5% per annum compound, but with increases taking place every five years. Maintenance and other expenses are incurred quarterly in arrear, at a rate of £12,000 per annum. The retail outlet reverts to its original owner after 25 years for no payment.

Project B involves the purchase of an office building for £1,000,000. The rent is to be received quarterly in advance at an initial rate of £85,000 per annum. It is assumed that the rent will increase to £90,000 per annum after 20 years. There are no maintenance or other expenses. After 25 years the property reverts to its original owner for no payment.

- (i) Show that the internal rate of return for project A is 9% per annum effective. [5]

- (ii) Calculate the annual effective internal rate of return for Project B. Show your working. [4]

- (iii) Discuss the extent to which the answers to parts (i) and (ii) above will influence the investor’s decision over which project to choose. [3]

[Total 12]

- 6** A fixed-interest bond pays annual coupons of 5% per annum in arrear on 1 March each year and is redeemed at par on 1 March 2025.

On 1 March 2007, immediately after the payment of the coupon then due, the gross redemption yield was 3.158% per annum effective.

- (i) Calculate the price of the bond per £100 nominal on 1 March 2007. [3]

On 1 March 2012, immediately after the payment of the coupon then due, the gross redemption yield on the bond was 5% per annum.

- (ii) State the new price of the bond per £100 nominal on 1 March 2012. [1]

A tax-free investor purchased the bond on 1 March 2007, immediately after payment of the coupon then due, and sold the bond on 1 March 2012, immediately after payment of the coupon then due.

- (iii) Calculate the gross annual rate of return achieved by the investor over this period. [2]

- (iv) Explain, without doing any further calculations, how your answer to part (iii) would change if the bond were due to be redeemed on 1 March 2035 (rather than 1 March 2025). You may assume that the gross redemption yield at both the date of purchase and the date of sale remains the same as in parts (i) and (ii) above. [3]

[Total 9]

- 7** The annual yields from a fund are independent and identically distributed. Each year, the distribution of $1 + i$ is log-normal with parameters $\mu = 0.05$ and $\sigma^2 = 0.004$, where i denotes the annual yield on the fund.

- (i) Calculate the expected accumulation in 20 years' time of an annual investment in the fund of £5,000 at the beginning of each of the next 20 years. [5]

- (ii) Calculate the probability that the accumulation of a single investment of £1 made now will be greater than its expected value in 20 years' time. [5]

[Total 10]

- 8** The force of interest, $\delta(t)$, at time t is given by:

$$\delta(t) = \begin{cases} 0.04 + 0.003t^2 & \text{for } 0 < t \leq 5 \\ 0.01 + 0.03t & \text{for } 5 < t \leq 8 \\ 0.02 & \text{for } t > 8 \end{cases}$$

- (i) Calculate the present value (at time $t = 0$) of an investment of £1,000 due at time $t = 10$. [4]
 - (ii) Calculate the constant rate of discount per annum convertible quarterly, which would lead to the same present value as that in part (i) being obtained. [2]
 - (iii) Calculate the present value (at time $t = 0$) of a continuous payment stream payable at the rate of $100e^{0.01t}$ from time $t = 10$ to $t = 18$. [4]
- [Total 10]

- 9** An ordinary share pays dividends on each 31 December. A dividend of 35p per share was paid on 31 December 2011. The dividend growth is expected to be 3% in 2012, and a further 5% in 2013. Thereafter, dividends are expected to grow at 6% per annum compound in perpetuity.

- (i) Calculate the present value of the dividend stream described above at a rate of interest of 8% per annum effective for an investor holding 100 shares on 1 January 2012. [4]

An investor buys 100 shares for £17.20 each on 1 January 2012. He expects to sell the shares for £18 on 1 January 2015.

- (ii) Calculate the investor's expected real rate of return.

You should assume that dividends grow as expected and use the following values of the inflation index:

Year:	2012	2013	2014	2015	
Inflation index at start of year:	110.0	112.3	113.2	113.8	[5] [Total 9]

10 A company has the following liabilities:

- annuity payments of £200,000 per annum to be paid annually in arrear for the next 20 years
- a lump sum of £300,000 to be paid in 15 years

The company wishes to invest in two fixed-interest securities in order to immunise its liabilities.

Security A has a coupon rate of 9% per annum and a term to redemption of 12 years. Security B has a coupon rate of 4% per annum and a term to redemption of 30 years.

Both securities are redeemable at par and pay coupons annually in arrear. The rate of interest is 8% per annum effective.

- Calculate the present value of the liabilities. [3]
 - Calculate the discounted mean term of the liabilities. [4]
 - Calculate the nominal amount of each security that should be purchased so that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this company. [8]
 - Describe the further calculations that will be necessary to determine whether the company is immunised against small changes in the rate of interest. [2]
- [Total 17]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

3 October 2012 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** An investor is considering two investments. One is a 91-day deposit which pays a rate of interest of 4% per annum effective. The second is a treasury bill.
- Calculate the annual simple rate of discount from the treasury bill if both investments are to provide the same effective rate of return. [3]
- 2** The nominal rate of discount per annum convertible quarterly is 8%.
- (i) Calculate the equivalent force of interest. [1]
- (ii) Calculate the equivalent effective rate of interest per annum. [1]
- (iii) Calculate the equivalent nominal rate of discount per annum convertible monthly. [2]
- [Total 4]
- 3** An investment fund is valued at £120m on 1 January 2010 and at £140m on 1 January 2011. Immediately after the valuation on 1 January 2011, £200m is paid into the fund. On 1 July 2012, the value of the fund is £600m.
- (i) Calculate the annual effective time-weighted rate of return over the two-and-a-half year period. [3]
- (ii) Explain why the money-weighted rate of return would be higher than the time-weighted rate of return. [2]
- [Total 5]
- 4** A ten-month forward contract was issued on 1 September 2012 for a share with a price of £10 at that date. Dividends of £1 per share are expected on 1 December 2012, 1 March 2013 and 1 June 2013.
- (i) Calculate the forward price assuming a risk-free rate of interest of 8% per annum convertible half-yearly and no arbitrage. [4]
- (ii) Explain why it is not necessary to use the expected price of the share at the time the forward matures in the calculation of the forward price. [2]
- [Total 6]

5 (i) State the characteristics of a Eurobond [4]

- (ii) (a) State the characteristics of a certificate of deposit.
- (b) Two certificates of deposit issued by a given bank are being traded. A one-month certificate of deposit provides a rate of return of 12 per cent per annum convertible monthly. A two-month certificate of deposit provides a rate of return of 24 per cent per annum convertible monthly.

Calculate the forward rate of interest per annum convertible monthly in the second month, assuming no arbitrage. [4]

[Total 8]

6 A loan is to be repaid by an increasing annuity. The first repayment will be £200 and the repayments will increase by £100 per annum. Repayments will be made annually in arrear for ten years. The repayments are calculated using a rate of interest of 6% per annum effective.

(i) Calculate the amount of the loan [2]

- (ii) (a) Calculate the interest component of the seventh repayment.
(b) Calculate the capital component of the seventh repayment.

[4]

- (iii) Immediately after the seventh repayment, the borrower asks to have the original term of the loan extended to fifteen years and wishes to repay the outstanding loan using level annual repayments. The lender agrees but changes the interest rate at the time of the alteration to 8% per annum effective.

Calculate the revised annual repayment. [3]

[Total 9]

7 An individual wishes to make an investment that will pay out £200,000 in twenty years' time. The interest rate he will earn on the invested funds in the first ten years will be either 4% per annum with probability of 0.3 or 6% per annum with probability 0.7. The interest rate he will earn on the invested funds in the second ten years will also be either 4% per annum with probability of 0.3 or 6% per annum with probability 0.7. However, the interest rate in the second ten year period will be independent of that in the first ten year period.

- (i) Calculate the amount the individual should invest if he calculates the investment using the expected annual interest rate in each ten year period. [2]
- (ii) Calculate the expected value of the investment in excess of £200,000 if the amount calculated in part (i) is invested. [3]
- (iii) Calculate the range of the accumulated amount of the investment assuming the amount calculated in part (i) is invested. [2]

[Total 7]

8 The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.03 + 0.01t & \text{for } 0 \leq t \leq 9 \\ 0.06 & \text{for } 9 < t \end{cases}$$

- (i) Derive, and simplify as far as possible, expressions for $v(t)$ where $v(t)$ is the present value of a unit sum of money due at time t . [5]
- (ii) (a) Calculate the present value of £5,000 due at the end of 15 years.
- (b) Calculate the constant force of interest implied by the transaction in part (a). [4]

A continuous payment stream is received at rate $100e^{-0.02t}$ units per annum between $t = 11$ and $t = 15$.

- (iii) Calculate the present value of the payment stream. [4]

[Total 13]

- 9** (i) Describe three theories that have been put forward to explain the shape of the yield curve. [7]

The government of a particular country has just issued five bonds with terms to redemption of one, two, three, four and five years respectively. The bonds are redeemed at par and have coupon rates of 4% per annum payable annually in arrear.

- (ii) Calculate the duration of the one-year, three-year and five-year bonds at a gross redemption yield of 5% per annum effective. [6]
- (iii) Explain why a five-year bond with a coupon rate of 8% per annum would have a lower duration than a five-year bond with a coupon rate of 4% per annum. [2]

Four years after issue, immediately after the coupon payment then due the government is anticipating problems servicing its remaining debt. The government offers two options to the holders of the bond with an original term of five years:

Option 1: the bond is repaid at 79% of its nominal value at the scheduled time with no final coupon payment being paid.

Option 2: the redemption of the bond is deferred for seven years from the original redemption date and the coupon rate reduced to 1% per annum for the remainder of the existing term and the whole of the extended term.

Assume the bonds were issued at a price of £95 per £100 nominal.

- (iv) Calculate the effective rate of return per annum from Options 1 and 2 over the total life of the bond and determine which would provide the higher rate of return. [6]
- (v) Suggest two other considerations that bond holders may wish to take into account when deciding which options to accept. [2]

[Total 23]

10 Two investment projects are being considered.

- (i) Explain why comparing the two discounted payback periods or comparing the two payback periods are not generally appropriate ways to choose between two investment projects. [3]

The two projects each involve an initial investment of £3m. The incoming cash flows from the two projects are as follows:

Project A

In the first year, Project A generates cash flows of £0.5m. In the second year it will generate cash flows of £0.55m. The cash flows generated by the project will continue to increase by 10% per annum until the end of the sixth year and will then cease. Assume that all cash flows are received in the middle of the year.

Project B

Project B generates cash flows of £0.64m per annum for six years. Assume that all cash flows are received continuously throughout the year.

- (ii) (a) Calculate the payback period from Project B.
(b) Calculate the discounted payback period from Project B at a rate of interest of 4% per annum effective. [5]
- (iii) Show that there is at least one “cross-over point” for Projects A and B between 0% per annum effective and 4% per annum effective where the cross-over point is defined as the rate of interest at which the net present value of the two projects is equal. [6]
- (iv) Calculate the duration of the incoming cash flows from Projects A and B at a rate of interest of 4% per annum effective. [6]
- (v) Explain why the net present value of Project A appears to fall more rapidly than the net present value of Project B as the rate of interest increases. [2]
- [Total 22]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

15 April 2013 (pm)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** The value of the assets held by an investment fund on 1 January 2012 was £1.3 million.

On 30 September 2012, the value of the assets was £1.9 million.

On 1 October 2012, there was a net cash outflow from the fund of £0.9 million.

On 31 December 2012, the value of the assets was £0.8 million.

- (i) Calculate the annual effective time-weighted rate of return (TWRR) for 2012. [2]
- (ii) Calculate the annual effective money-weighted rate of return (MWRR) for 2012 to the nearest 1%. [3]
- (iii) Explain why the MWRR is significantly higher than the TWRR. [2]
- [Total 7]

- 2** (i) Explain the main difference:

- (a) between options and futures.
(b) between call options and put options.

[4]

- (ii) A one-year forward contract is issued on 1 April 2013 on a share with a price at that date of £10.50. Dividends of £1.10 per share are expected on 30 September 2013 and 31 March 2014. On 1 April 2013, the 6-month risk-free spot rate of interest is 4.5% per annum convertible half-yearly and the 12-month risk-free spot rate of interest is 5% per annum convertible half-yearly.

Calculate the forward price at issue, stating any further assumptions made. [4]
[Total 8]

- 3** Three bonds each paying annual coupons in arrear of 6% and redeemable at £103 per £100 nominal reach their redemption dates in exactly one, two and three years' time, respectively. The price of each bond is £97 per £100 nominal.

- (i) Calculate the gross redemption yield of the 3-year bond. [3]
- (ii) Calculate the one-year and two-year spot rates implied by the information given. [3]
- [Total 6]

- 4** An investor is interested in purchasing shares in a particular company.

The company pays annual dividends, and a dividend payment of 30 pence per share has just been made.

Future dividends are expected to grow at the rate of 5% per annum compound.

- (i) Calculate the maximum price per share that the investor should pay to give an effective return of 9% per annum. [4]
- (ii) Without doing any further calculations, explain whether the maximum price paid will be higher, lower or the same if:
- (a) after consulting the managers of the company, the investor increases his estimate of the rate of growth of future dividends to 6% per annum.
 - (b) as a result of a government announcement, the general level of future price inflation in the economy is now expected to be 2% per annum higher than previously assumed.
 - (c) general economic uncertainty means that, whilst the investor still estimates future dividends will grow at 5% per annum, he is now much less sure about the accuracy of this assumption.

You should consider the effect of each change separately.

[6]

[Total 10]

- 5** The force of interest per unit time at time t , $\delta(t)$, is given by:

$$\delta(t) = \begin{cases} 0.1 - 0.005t & \text{for } t < 6 \\ 0.07 & \text{for } t \geq 6 \end{cases}$$

- (i) Calculate the total accumulation at time 10 of an investment of £100 made at time 0 and a further investment of £50 made at time 7. [4]
- (ii) Calculate the present value at time 0 of a continuous payment stream at the rate $\text{£}50e^{0.05t}$ per unit time received between time 12 and time 15. [5]

[Total 9]

- 6** A cash sum of £10,000 is invested in a fund and held for 15 years. The yield on the investment in any year will be 5% with probability 0.2, 7% with probability 0.6 and 9% with probability 0.2, and is independent of the yield in any other year.
- (i) Calculate the mean accumulation at the end of 15 years. [2]
- (ii) Calculate the standard deviation of the accumulation at the end of 15 years. [5]
- (iii) Without carrying out any further calculations, explain how your answers to parts (i) and (ii) would change (if at all) if:
- (a) the yields had been 6%, 7% and 8% instead of 5%, 7%, and 9% per annum, respectively.
- (b) the investment had been held for 13 years instead of 15 years. [4]
- [Total 11]
- 7** An insurance company has liabilities of £6 million due in 8 years' time and £11 million due in 15 years' time. The assets consist of two zero-coupon bonds, one paying £X in 5 years' time and the other paying £Y in 20 years' time. The current interest rate is 8% per annum effective. The insurance company wishes to ensure that it is immunised against small changes in the rate of interest.
- (i) Determine the values of £X and £Y such that the first two conditions for Redington's immunisation are satisfied. [8]
- (ii) Demonstrate that the third condition for Redington's immunisation is also satisfied. [2]
- [Total 10]

- 8** A car manufacturer is to develop a new model to be produced from 1 January 2016 for six years until 31 December 2021. The development costs will be £19 million on 1 January 2014, £9 million on 1 July 2014 and £5 million on 1 January 2015.

It is assumed that 6,000 cars will be produced each year from 2016 onwards and that all will be sold.

The production cost per car will be £9,500 during 2016 and will increase by 4% each year with the first increase occurring in 2017. All production costs are assumed to be incurred at the beginning of each calendar year.

The sale price of each car will be £12,600 during 2016 and will also increase by 4% each year with the first increase occurring in 2017. All revenue from sales is assumed to be received at the end of each calendar year.

- (i) Calculate the discounted payback period at an effective rate of interest of 9% per annum. [9]
- (ii) Without doing any further calculations, explain whether the discounted payback period would be greater than, equal to, or less than the period calculated in part (i) if the effective rate of interest were substantially less than 9% per annum. [2]
- [Total 11]

- 9** A fixed-interest security pays coupons of 8% per annum half yearly on 1 January and 1 July. The security will be redeemed at par on any 1 January from 1 January 2017 to 1 January 2022 inclusive, at the option of the borrower.

An investor purchased a holding of the security on 1 May 2011, at a price which gave him a net yield of at least 6% per annum effective. The investor pays tax at 30% on interest income and 25% on capital gains.

On 1 April 2013 the investor sold the holding to a fund which pays no tax at a price to give the fund a gross yield of at least 7% per annum effective.

- (i) Calculate the price per £100 nominal at which the investor bought the security. [5]
- (ii) Calculate the price per £100 nominal at which the investor sold the security. [3]
- (iii) Show that the effective net yield that the investor obtained on the investment was between 8% and 9% per annum. [6]
- [Total 14]

- 10** A loan is repayable by annual instalments in arrear for 20 years. The initial instalment is £5,000, with each subsequent instalment decreasing by £200.

The effective rate of interest over the period of the loan is 4% per annum.

(i) Calculate the amount of the original loan. [3]

(ii) Calculate the capital repayment in the 12th instalment. [3]

After the 12th instalment is paid, the borrower and lender agree to a restructuring of the debt.

The £200 reduction per year will no longer continue. Instead, future instalments will remain at the level of the 12th instalment and the remaining term of the debt will be shortened. The final payment will then be a reduced amount which will clear the debt.

(iii) (a) Calculate the remaining term of the revised loan.
(b) Calculate the amount of the final reduced payment.
(c) Calculate the total interest paid during the term of the loan.

[8]

[Total 14]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

23 September 2013 (pm)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 The rate of interest is 4.5% per annum effective.

(i) Calculate:

- (a) the annual effective rate of discount.
- (b) the nominal rate of discount per annum convertible monthly.
- (c) the nominal rate of interest per annum convertible quarterly.
- (d) the effective rate of interest over a five year period.

[5]

(ii) Explain why your answer to part (i)(b) is higher than your answer to part (i)(a).

[2]

[Total 7]

2 A nine-month forward contract is issued on 1 March 2012 on a share with a price of £1.80 at that date. Dividends of 10p per share are expected on 1 September 2012.

Calculate the forward price at issue assuming a risk-free rate of interest of 4% per annum effective and no arbitrage.

[3]

3 A fixed-interest security pays coupons of 4% per annum, half-yearly in arrear and will be redeemed at par in exactly ten years.

- (i) Calculate the price per £100 nominal to provide a gross redemption yield of 3% per annum convertible half-yearly.
- (ii) Calculate the price, 91 days later, to provide a net redemption yield of 3% per annum convertible half-yearly if income tax is payable at 25%.

[2]

[2]

[Total 4]

4 Describe the characteristics of the cash flows that are paid and received in respect of:

- (i) an index-linked security.
- (ii) an equity.

[2]

[3]

[Total 5]

- 5** An investor is considering the purchase of two government bonds, issued by two countries A and B respectively, both denominated in euro.

Both bonds provide a capital repayment of €100 together with a final coupon payment of €6 in exactly one year. The investor believes that he will receive both payments from the bond issued by Country A with certainty. He believes that there are four possible outcomes for the bond from Country B, shown in the table below.

<i>Outcome</i>	<i>Probability</i>
No coupon or capital payment	0.1
Capital payment received, but no coupon payment received	0.2
50% of capital payment received, but no coupon payment received	0.3
Both coupon and capital payments received in full	0.4

The price of the bond issued by Country A is €101.

- (i) Calculate the price of the bond issued by Country B to give the same expected return as that for the bond issued by Country A. [3]
 - (ii) Calculate the gross redemption yield from the bond issued by Country B assuming that the price is as calculated in part (i). [1]
 - (iii) Explain why the investor might require a higher expected return from the bond issued by Country B than from the bond issued by Country A. [2]
- [Total 6]

- 6** A pension fund is considering investing in a major infrastructure project. The fund has been asked to make an investment of £2m for a 1% share in revenues from building a road. No other costs will be incurred by the pension fund. The following revenues are expected to arise from the project:

In the first year, 40,000 vehicles a day will use the road, each paying a toll of £1.

In the second year, 50,000 vehicles a day will use the road, each paying a toll of £1.10.

In the third year, both the number of vehicles using the road and the level of tolls will rise by 1% from their level in the second year. They will both continue to rise by 1% per annum compound until the end of the 20th year.

At the end of the 20th year, it is assumed that the road has no value as it will have to be completely rebuilt.

You should assume that all revenue is received continuously throughout the year and that there are 365 days in all years.

Calculate the net present value of the investment in the road at a rate of interest of 8% per annum effective. [10]

7 An insurance company has just written contracts that require it to make payments to policyholders of £10 million in five years' time. The total premiums paid by policyholders at the outset of the contracts amounted to £7.85 million. The insurance company is to invest the premiums in assets that have an uncertain return. The return from these assets in year t , i_t , has a mean value of 5.5% per annum effective and a standard deviation of 4% per annum effective. $(1+i_t)$ is independently and lognormally distributed.

- (i) Calculate the mean and standard deviation of the accumulation of the premiums over the five-year period. You should derive all necessary formulae. [Note: You are not required to derive the formulae for the mean and variance of a lognormal distribution.] [9]

A director of the insurance company is concerned about the possibility of a considerable loss from the investment strategy suggested in part (i). He therefore suggests investing in fixed-interest securities with a guaranteed return of 4 per cent per annum effective.

- (ii) Explain the arguments for and against the director's suggestion. [3]
[Total 12]

8 Mrs Jones invests a sum of money for her retirement which is expected to be in 20 years' time. The money is invested in a zero coupon bond which provides a return of 5% per annum effective. At retirement, the individual requires sufficient money to purchase an annuity certain of £10,000 per annum for 25 years. The annuity will be paid monthly in arrear and the purchase price will be calculated at a rate of interest of 4% per annum convertible half-yearly.

- (i) Calculate the sum of money the individual needs to invest at the beginning of the 20-year period. [5]

The index of retail prices has a value of 143 at the beginning of the 20-year period and 340 at the end of the 20-year period.

- (ii) Calculate the annual effective real return the individual would obtain from the zero coupon bond. [2]

The government introduces a capital gains tax on zero coupon bonds of 25 per cent of the nominal capital gain.

- (iii) Calculate the net annual effective real return to the investor over the 20-year period before the annuity commences. [3]

- (iv) Explain why the investor has achieved a negative real rate of return despite capital gains tax only being a tax on the profits from an investment. [2]
[Total 12]

- 9** A bank makes a loan to be repaid by instalments paid annually in arrear. The first instalment is £400, the second is £380 with the payments reducing by £20 per annum until the end of the 15th year, after which there are no further repayments. The rate of interest charged is 4% per annum effective.

(i) Calculate the amount of the loan. [3]

(ii) Calculate the capital and interest components of the first payment. [2]

At the beginning of the ninth year, the borrower can no longer make the scheduled repayments. The bank agrees to reduce the capital by 50 per cent of the loan outstanding after the eighth repayment. The bank requires that the remaining capital is repaid by a 10-year annuity paid annually in arrear, increasing by £2 per annum. The bank changes the rate of interest to 8% per annum effective.

(iii) Calculate the first repayment under the revised loan. [5]

[Total 10]

- 10** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = 0.05 + 0.002t$$

Calculate the accumulated value of a unit sum of money:

- (i) (a) accumulated from time $t = 0$ to time $t = 7$.
(b) accumulated from time $t = 0$ to time $t = 6$.
(c) accumulated from time $t = 6$ to time $t = 7$.

[5]

(ii) Calculate, using your results from part (i) or otherwise:

(a) the seven-year spot rate of interest per annum from time $t = 0$ to time $t = 7$.

(b) the six-year spot rate of interest per annum from time $t = 0$ to time $t = 6$.

(c) $f_{6,1}$ where $f_{6,1}$ is the one-year forward rate of interest per annum from time $t = 6$. [3]

(iii) Explain why your answer to part (ii)(c) is higher than your answer to part (ii)(a). [2]

(iv) Calculate the present value of an annuity that is paid continuously at a rate of $30e^{-0.01t+0.001t^2}$ units per annum from $t = 3$ to $t = 10$. [5]

[Total 15]

- 11** A pension fund has liabilities to meet annuities payable in arrear for 40 years at a rate of £10 million per annum.

The fund is invested in two fixed-interest securities. The first security pays annual coupons of 5% and is redeemed at par in exactly ten years' time. The second security pays annual coupons of 10% and is redeemed at par in exactly five years' time. The present value of the assets in the pension fund is equal to the present value of the liabilities of the fund and exactly half the assets are invested in each security. All assets and liabilities are valued at a rate of interest of 4% per annum effective.

- (i) Calculate the present value of the liabilities of the fund. [1]
- (ii) Calculate the nominal amount held of each security purchased by the pension fund. [6]
- (iii) Calculate the duration of the liabilities of the pension fund. [3]
- (iv) Calculate the duration of the assets of the pension fund. [4]
- (v) Without further calculations, explain whether the pension fund will make a profit or loss if interest rates fall uniformly by 1.5% per annum effective. [2]

[Total 16]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

22 April 2014 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 You are given the following information in respect of a pension fund:

<i>Calendar Year</i>	<i>Value of fund at 1 January</i>	<i>Value of fund at 30 June</i>	<i>Net cash flow received on 1 July</i>
2011	£870,000	£872,000	£26,000
2012	£914,000	£902,000	£27,000
2013	£953,000	£962,000	£33,000
2014	£990,000		

Calculate, to the nearest 0.1%, the annual effective money-weighted rate of return earned by the fund during the period from 1 January 2011 to 1 January 2014. [4]

2 Describe the main features of:

- (a) debenture stocks.
- (b) unsecured loan stocks.

[5]

3 £900 accumulates to £925 in four months.

Calculate the following:

- (i) the nominal rate of interest per annum convertible half-yearly [2]
- (ii) the nominal rate of discount per annum convertible quarterly [2]
- (iii) the simple rate of interest per annum [2]

[Total 6]

4 A company issues a loan stock bearing interest at a rate of 8% per annum payable half-yearly in arrear. The stock is to be redeemed at 103% on any coupon payment date in the range from 20 years after issue to 25 years after issue inclusive, to be chosen by the company.

An investor, who is liable to income tax at 30% and tax on capital gains at 40%, bought the stock at issue at a price which gave her a minimum net yield to redemption of 6% per annum effective.

Calculate the price that the investor paid. [7]

- 5** On 25 October 2008 a certain government issued a 5-year index-linked stock. The stock had a nominal coupon rate of 3% per annum payable half-yearly in arrear and a nominal redemption price of 100%. The actual coupon and redemption payments were index-linked by reference to a retail price index as at the month of payment.

An investor, who was not subject to tax, bought £10,000 nominal of the stock on 26 October 2012. The investor held the stock until redemption.

You are given the following values of the retail price index:

	2008	-----	2012	2013
April	-----	-----	-----	171.4
October	149.2	-----	169.4	173.8

- (i) Calculate the coupon payment that the investor received on 25 April 2013 and the coupon and redemption payments that the investor received on 25 October 2013. [3]
- (ii) Calculate the purchase price that the investor paid on 25 October 2012 if the investor achieved an effective real yield of 3.5% per annum effective on the investment. [4]
- [Total 7]

- 6** An insurance company has liabilities of £10 million due in 10 years' time and £20 million due in 15 years' time. The company's assets consist of two zero-coupon bonds. One pays £7.404 million in 2 years' time and the other pays £31.834 million in 25 years' time. The current interest rate is 7% per annum effective.

- (i) Show that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this insurance company. [6]
- (ii) Calculate the present value of profit that the insurance company will make if the interest rate increases immediately to 7.5% per annum effective. [2]
- (iii) Explain, without any further calculation, why the insurance company made a profit as a result of the change in the interest rate. [2]
- [Total 10]

- 7** Six months ago, an investor entered into a one-year forward contract to purchase a non-dividend paying stock. The risk-free force of interest was 4% per annum. The value of the stock is now 98% of its original value.

Calculate the minimum value for the risk-free force of interest at which the original forward contract still has a positive value to the investor. [6]

- 8** An insurance company borrows £50 million at an effective interest rate of 9% per annum. The insurance company uses the money to invest in a capital project that pays £6 million per annum payable half-yearly in arrear for 20 years. The income from the project is used to repay the loan. Once the loan has been repaid, the insurance company can earn interest at an effective interest rate of 7% per annum.

(i) Calculate the discounted payback period for this investment. [4]

(ii) Calculate the accumulated profit the insurance company will have made at the end of the term of the capital project. [5]

[Total 9]

- 9** The effective n -year spot rate of interest y_n , is given by:

$$y_n = 0.035 + \frac{n}{1000} \quad \text{for } n = 1, 2 \text{ and } 3$$

(i) Determine the implied one-year forward rates applicable at times $t = 1$ and $t = 2$ to four significant figures. [4]

(ii) Calculate, assuming no arbitrage:

(a) The price at time $t = 0$ per £100 nominal of a bond which pays annual coupons of 4% in arrear and is redeemed at 105% per £100 nominal after three years.

(b) The two-year par yield. [6]

[Total 10]

- 10** A loan of £20,000 is repayable by an annuity payable annually in arrear for 25 years. The annual repayment is calculated at an effective interest rate of 8% per annum and increases by £50 each year.

(i) Calculate the amount of the first payment. [3]

(ii) Calculate the capital outstanding after the first three payments have been made. [2]

(iii) Explain your answer to part (ii). [2]

(iv) Calculate the total amount of interest paid over the term of the loan. [3]

[Total 10]

- 11** An individual can obtain a force of interest per annum at time t , measured in years, as given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.01t & 0 \leq t < 4 \\ 0.07 & 4 \leq t < 6 \\ 0.09 & 6 \leq t \end{cases}$$

- (i) Calculate the amount the individual would need to invest at time $t = 0$ in order to receive a continuous payment stream of \$3,000 per annum from time $t = 4$ to $t = 10$. [6]
 - (ii) Calculate the equivalent constant annual effective rate of interest earned by the individual in part (i). [3]
- [Total 9]

- 12** An investor is considering investing £18,000 for a period of 12 years. Let i_t be the effective rate of interest in the t^{th} year, $t \leq 12$. Assume, for $t \leq 12$, that i_t has mean value of 0.08 and standard deviation 0.05 and that $1+i_t$ is independently and lognormally distributed.

- (i) Determine the distribution of S_{12} where S_t is the accumulation of £1 over t years. [5]

At the end of the 12 years the investor intends to use the accumulated amount of the investment to purchase a 12-year annuity certain paying:

£4,000 per annum monthly in advance during the first four years;
 £5,000 per annum quarterly in advance during the second four years;
 £6,000 per annum continuously during the final four years.

The effective rate of interest will be 7% per annum in years 13 to 18 and 9% per annum in years 19 to 24 where the years are counted from the start of the initial investment

- (ii) Calculate the probability that the investor will meet the objective. [12]
- [Total 17]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

22 September 2014 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Describe how cash flows are exchanged in an interest rate swap. [2]

2 A life insurance company is issuing a single premium policy which will pay out £200,000 in 20 years' time. The interest rate the company will earn on the invested fund throughout the 20 years will be 4% per annum effective with probability 0.25 or 7% per annum effective with probability 0.75. The insurance company uses the expected annual interest rate to determine the premium.

(i) Calculate the premium. [2]

(ii) Calculate the expected profit made by the insurance company at the end of the policy. [2]

[Total 4]

3 A 91-day treasury bill is bought for £98.83 and is redeemed at £100.

(i) Calculate the annual effective rate of interest from the bill. [3]

(ii) Calculate the annual equivalent simple rate of interest. [2]

[Total 5]

4 A fund had a value of £2.0 million on 1 January 2013. On 1 May 2013, £2.5 million was invested. Immediately before this investment, the value of the fund was £2.1 million. At the close of business on 31 December 2013, the value of the fund was £4.2 million.

(i) Calculate the annual effective time-weighted rate of return for 2013. [2]

(ii) Calculate the annual effective money-weighted rate of return for 2013. [3]

(iii) Comment on your answers to parts (i) and (ii). [2]

[Total 7]

5 Calculate, at a rate of interest of 5% per annum effective:

(i) $a_{\overline{5}}^{(12)}$ [1]

(ii) ${}^4|a_{\overline{15}}$ [1]

(iii) $(I\bar{a})_{\overline{10}}$ [1]

(iv) $(\bar{I}\bar{a})_{\overline{10}}$ [1]

- (v) the present value of an annuity that is paid annually in advance for 10 years with a payment of 12 in the first year, 11 in the second year and thereafter reducing by 1 each year. [2]
 [Total 6]

- 6** A Eurobond has been issued by a company that pays annual coupons of 5% per annum annually in arrear and is redeemable at par in exactly 10 years' time.
- (i) Calculate the purchase price of the bond at issue at a rate of interest of 4% per annum effective assuming that tax is paid on the coupon payments at a rate of 20%. [2]
 - (ii) Calculate the discounted mean term of the bond at a rate of interest of 4% per annum effective, ignoring tax. [3]
 - (iii)
 - (a) Explain why the discounted mean term of the gross payments from the bond is lower than the discounted mean term of the net payments.
 - (b) State two factors other than the size of the coupon payments that would affect the discounted mean term of the bond. [3]
 - (iv) Calculate the price of the bond three months after issue at a rate of interest of 4% per annum effective assuming tax is paid on the coupon payments at a rate of 20%. [1]
 [Total 9]

- 7** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & \text{for } 0 < t \leq 10 \\ 0.003t & \text{for } 10 < t \leq 20 \\ 0.0001t^2 & \text{for } t > 20 \end{cases}$$

- (i) Calculate the present value of a unit sum of money due at time $t = 28$. [7]
- (ii)
 - (a) Calculate the equivalent constant force of interest from $t = 0$ to $t = 28$.
 - (b) Calculate the equivalent annual effective rate of discount from $t = 0$ to $t = 28$. [3]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per unit time between $t = 3$ and $t = 7$.

- (iii) Calculate the present value of the payment stream. [4]
 [Total 14]

- 8** (i) Explain what is meant by the following theories of the shape of the yield curve:
- (a) market segmentation theory
 (b) liquidity preference theory [4]

Short-term, one-year annual effective interest rates are currently 6%; they are expected to be 5% in one year's time; 4% in two years' time and 3% in three years' time.

- (ii) Calculate the gross redemption yields from one-year, two-year, three-year and four-year zero coupon bonds using the above expected interest rates. [4]

The price of a coupon-paying bond is calculated by discounting individual payments from the bond at the zero-coupon yields in part (ii).

- (iii) Calculate the gross redemption yield of a bond that pays a coupon of 4% per annum annually in arrear and is redeemed at 110% in exactly four years. [5]
- (iv) Explain why the gross redemption yield of a bond that pays a coupon of 8% per annum annually in arrear and is redeemed at par would be greater than that calculated in part (iii). [2]

The government introduces regulations that require banks to hold more government bonds with very short terms to redemption.

- (v) Explain, with reference to market segmentation theory, the likely effect of this regulation on the pattern of spot rates calculated in part (ii). [2]

[Total 17]

- 9** A government issued a number of index-linked bonds on 1 June 2012 which were redeemed on 1 June 2014. Each bond had a nominal coupon of 2% per annum, payable half yearly in arrear and a nominal redemption price of 100%. The actual coupon and redemption payments were indexed according to the increase in the retail price index between three months before the issue date and three months before the relevant payment dates. No adjustment is made to allow for the actual date of calculation of the price index within the month or the precise coupon payment date within the month.

The values of the retail price index in the relevant months were:

<i>Date</i>	<i>Retail Price Index</i>
March 2012	112
June 2012	113
September 2012	116
December 2012	117
March 2013	117
June 2013	118
September 2013	120
December 2013	121
March 2014	121
June 2014	122

An investor purchased £3.5m nominal of the bond at the issue date and held it until it was redeemed. The investor was subject to tax on coupon payments at a rate of 25%.

- (i) Calculate the incoming net cash flows the investor received. [5]
- (ii) Express the cash flows in terms of 1 June 2012 prices. [4]
- (iii) Calculate the purchase price of the bond per £100 nominal if the real net redemption yield achieved by the investor was 1.5% per annum effective. [3]

When the investor purchased the security, he expected the retail price index to rise much more slowly than it did in practice.

- (iv) Explain whether the investor's expected net real rate of return at purchase would have been greater than 1.5% per annum effective. [2]

In September 2012, the government indicated that it might change the price index to which payments were linked to one which tends to rise more slowly than the retail price index.

- (v) Explain the likely impact of such a change on the market price of index-linked bonds. [2]

[Total 16]

- 10** A student is considering whether to attend university or enter a profession immediately upon leaving school. If he enters the profession immediately, his salary is expected to be as follows.

Year 1: £15,000
Year 2: £18,000
Year 3: £20,000

In each subsequent year the expected salary would rise by 1% per annum compound. The salary is assumed to be received monthly in arrear for 40 years.

If he attends university, the fees and other costs will be £15,000 per annum for three years, paid annually in advance. After attending university, the student's potential earnings will rise. Immediately after leaving university, he expects to earn £22,000 in the first year, £25,000 in the second year and £28,000 in the third year. Thereafter, his salary is expected to rise each year by 1.5% per annum compound. The salary would be paid monthly in arrear for 37 years.

- (i) Calculate the present value of the student's earnings if he enters the profession immediately at a rate of interest of 7% per annum effective. [7]
- (ii) Calculate the net present value of the decision to attend university at a rate of interest of 7% per annum effective and hence determine whether attending university would be a more attractive option. [9]
- (iii) Explain why attending university would be relatively more attractive at lower interest rates. [2]

The student wishes to consider the effect of taxation on earnings.

- (iv) Determine the rate of income tax above which the option of attending university would be less attractive financially than that of entering the profession immediately. [2]

[Total 20]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

20 April 2015 (pm)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** Explain why the running yield from property investments tends to be greater than that from equity investments. [3]
- 2** Calculate the time in days for £3,000 to accumulate to £3,800 at:
- (a) a simple rate of interest of 4% per annum.
 - (b) a compound rate of interest of 4% per annum effective.
- [4]
- 3** A 182-day treasury bill, redeemable at \$100, was purchased for \$96.50 at the time of issue and later sold to another investor for \$98 who held the bill to maturity. The rate of return received by the initial purchaser was 4% per annum effective.
- (i) Calculate the length of time in days for which the initial purchaser held the bill. [2]
 - (ii) Calculate the annual simple rate of return achieved by the second investor. [2]
 - (iii) Calculate the annual effective rate of return achieved by the second investor. [2]
- [Total 6]
- 4** (i) Describe what is meant by the “no arbitrage” assumption in financial mathematics. [2]
- A 9-month forward contract is issued on 1 April 2015 on a stock with a price of £6 per share on that date. Dividends are assumed to be received continuously and the dividend yield is 3.5% per annum.
- (ii) Calculate the theoretical forward price per share of the contract, assuming no arbitrage and a risk-free force of interest of 9% per annum. [2]
 - (iii) Outline how an investor could make an arbitrage profit. [2]
- The actual forward price per share of the contract is £6.30 and the risk-free force of interest is as in part (ii).
- (iii) Outline how an investor could make an arbitrage profit. [2]
- [Total 6]
- 5** An investor pays £120 per annum into a savings account for 12 years. In the first four years, the payments are made annually in advance. In the second four years, the payments are made quarterly in advance. In the final four years, the payments are made monthly in advance.
- The investor achieves a yield of 6% per annum convertible half-yearly on the investment.
- Calculate the accumulated amount in the savings account at the end of 12 years. [7]

- 6** An ordinary share pays annual dividends. The next dividend is expected to be 6p per share and is due in exactly six months' time. It is expected that subsequent dividends will grow at a rate of 6% per annum compound and that inflation will be 4% per annum. The price of the share is 175p and dividends are expected to continue in perpetuity.

Calculate the expected effective real rate of return per annum for an investor who purchases the share. [6]

- 7** In a particular country, insurance companies are required by regulation to value their liabilities using spot rates of interest derived from the government bond yield curve.

Over time t (measured in years), the spot rate of interest is equal to:

$$i = 0.02t \text{ for } t \leq 5$$

An insurance company in this country has a group of annuity policies which involve making payments of £1m per annum for four years and £2m per annum in the fifth year. All payments are assumed to be paid halfway through the year.

- (i) Calculate the value of the insurance company's liabilities. [3]
- (ii) Outline two reasons why the spot yield curve might rise with term to redemption. [3]
- (iii) Calculate the forward rate of interest from time $t = 3.5$ to time $t = 4.5$. [2]
[Total 8]

- 8** A fixed-interest security, redeemable at par in 10 years, pays annual coupons of 9% in arrear and has just been issued at a price to give an investor who does not pay tax a rate of return of 7% per annum effective.

- (i) Calculate the price of the security at issue. [2]
- (ii) Calculate the discounted mean term (duration) of the security at issue. [3]
- (iii) Explain how your answer to part (ii) would differ if the annual coupons on the security were 3% instead of 9%. [2]
- (iv) (a) Calculate the effective duration (volatility) of the security at the time of issue.
(b) Explain the usefulness of effective duration for an investor who expects to sell the security over the next few months. [3]
[Total 10]

- 9** A property development company has just purchased a retail outlet for \$4,000,000. A further \$900,000 will be spent refurbishing the outlet in six months' time.

An agreement has been made with a prospective tenant who will occupy the outlet beginning one year after the purchase date. The tenant will pay rent to the owner for five years and will then immediately purchase the outlet from the property development company for \$6,800,000. The initial rent will be \$360,000 per annum and this will be increased by the same percentage compound rate at the beginning of each successive year. The rental income is received quarterly in advance.

Calculate the compound percentage increase in the annual rent required to earn the company an internal rate of return of 12% per annum effective. [9]

- 10** The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t \leq 4 \\ 0.12 - 0.01t & \text{for } 4 < t \leq 9 \\ 0.05 & \text{for } t > 9 \end{cases}$$

- (i) Determine the discount factor, $v(t)$, that applies at time t for all $t \geq 0$. [5]
- (ii) Calculate the present value at $t = 0$ of a payment stream, paid continuously from $t = 10$ to $t = 12$, under which the rate of payment at time t is $100e^{0.03t}$. [4]
- (iii) Calculate the present value of an annuity of £1,000 paid at the end of each year for the first three years. [3]

[Total 12]

- 11** On 1 January 2016, a student plans to take out a five-year bank loan for £30,000 that will be repayable by instalments at the end of each month. Under this repayment schedule, the instalment at the end of January 2016 will be X , the instalment at the end of February 2016 will be $2X$ and so on, until the final instalment at the end of December 2020 will be $60X$. The bank charges a rate of interest of 15% per annum convertible monthly.

(i) Prove that $(Ia)_{\overline{n}} = \frac{\ddot{a}_{\overline{n}} - nv^n}{i}$. [3]

(ii) Show that $X = \text{£}26.62$. [4]

The student is concerned that she will not be able to afford the later repayments and so she suggests a revised repayment schedule. The student would borrow £30,000 on 1 January 2016 as before. She would now repay the loan by 60 level monthly instalments of $36X = \text{£}958.32$ but the first repayment would not be made until the end of January 2019 and hence the final instalment is paid at the end of December 2023.

- (iii) Calculate the APR on the revised loan schedule and hence determine whether you believe the bank should accept the student's suggestion. [5]
- (iv) Explain the difference in the total repayments made under the two arrangements. [2]

[Total 14]

- 12** In any year, the yield on investments with an insurance company has mean j and standard deviation s and is independent of the yields in all previous years.

- (i) Derive formulae for the mean and variance of the accumulated value after n years of a single investment of 1 at time 0 with the insurance company. [5]

Each year the value of $(1+i_t)$, where i_t is the rate of interest earned in the t^{th} year, is lognormally distributed. The rate of interest has a mean value of 0.04 and standard deviation of 0.12 in all years.

- (ii) (a) Calculate the parameters μ and σ^2 for the lognormal distribution of $(1+i_t)$.
- (b) Calculate the probability that an investor receives a rate of return between 6% and 8% in any year. [8]
- (iii) Explain whether your answer to part (ii) (b) looks reasonable. [2]

[Total 15]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

30 September 2015 (pm)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all nine questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** An investor wishes to obtain a rate of interest of 3% per annum effective from a 91-day treasury bill.

Calculate:

- (a) the price that the investor must pay per £100 nominal.
- (b) the annual simple rate of discount from the treasury bill.

[3]

- 2** The nominal rate of discount per annum convertible monthly is 5.5%.

- (i) Calculate, giving all your answers as a percentage to three decimal places:

- (a) the equivalent force of interest.
 - (b) the equivalent effective rate of interest per annum.
 - (c) the equivalent nominal rate of interest per annum convertible monthly.

[3]

- (ii) Explain why the nominal rate of interest per annum convertible monthly calculated in part (i)(c) is less than the equivalent annual effective rate of interest calculated in part (i)(b)

[1]

- (iii) Calculate, as a percentage to three decimal places, the effective annual rate of discount offered by an investment that pays £159 in eight years' time in return for £100 invested now.

[1]

- (iv) Calculate, as a percentage to three decimal places, the effective annual rate of interest from an investment that pays 12% interest at the end of each two-year period.

[1]

[Total 6]

- 3** An insurance company has sold a pension product to an individual. Under the arrangement, the individual is to receive an immediate annuity of £500 per year annually in arrear for 12 years. The insurance company has invested the premium it has received in a fixed-interest bond that pays coupons annually in arrear at the rate of 5% per annum and which is redeemable at par in exactly eight years.

- (i) Calculate the duration of the annuity at an interest rate of 4% per annum effective.

[2]

- (ii) Calculate the duration of the bond at an interest rate of 4% per annum effective.

[3]

- (iii) State with reasons whether the insurance company will make a profit or a loss if there is a small increase in interest rates at all terms.

[2]

[Total 7]

- 4** A nine-month forward contract was issued on 1 October 2015 on a share with a price at that date of £10. Dividends of 50 pence per share are expected on 1 November 2015 and 1 May 2016. The risk-free force of interest is 5% per annum.
- (i) Calculate the forward price at issue, stating any further assumptions made and showing all workings. [4]
 - (ii) Explain why the expected price of the share nine months after issue does not have to be taken into account when pricing the forward. [2]
- [Total 6]

- 5** An individual can obtain a force of interest per annum at time t , measured in years, as given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \leq t \leq 3 \\ 0.005 & t > 3 \end{cases}$$

- (i) Determine the amount the individual would need to invest at time $t = 0$ in order to receive a continuous payment stream of £5,000 per annum from time $t = 3$ to time $t = 6$. [5]
 - (ii) Determine the equivalent constant annual effective rate of interest earned by the individual in part (i). [3]
 - (iii) Determine the amount an individual would accumulate from the investment of £300 from time $t = 0$ to time $t = 50$. [2]
- [Total 10]

- 6** Three bonds, each paying annual coupons in arrear of 3% and redeemable at £100 per £100 nominal, reach their redemption dates in exactly one, two and three years' time, respectively.

The price of each bond is £101 per £100 nominal.

- (i) Determine the gross redemption yield of the three-year bond. [3]
- (ii) Calculate the one-year, two-year and three-year spot rates of interest implied by the information given. [5]
- (iii) Calculate the one-year forward rate starting from the end of the second year, $f_{2,1}$. [2]

The pattern of spot rates is upward sloping throughout the yield curve.

- (iv) Explain, with reference to the various theories of the yield curve, why the yield curve might be upward sloping. [4]
- [Total 14]

- 7 A special type of loan is to be issued by a company. The loan is made up of 100,000 bonds, each of nominal value €100. Coupons will be paid semi-annually in arrear at a rate of 4% per annum. The bonds are to be issued on 1 October 2015 at a price of €100 per €100 nominal. Income tax will be paid by the bond holders at a rate of 25% on all coupon payments.

Exactly half the bonds will be redeemed after ten years at €100 per €100 nominal. The bonds that are redeemed will be determined by lot (i.e. the bonds will be numbered and half the numbered bonds will be chosen randomly for redemption). Coupon payments on the remaining bonds will be increased to 7% per annum and these bonds will be redeemed 20 years after issue at €130 per €100 nominal.

An individual buys a single bond.

Calculate, as an effective rate of return per annum:

- (i) the maximum rate of return the individual can obtain from the bond. [5]
- (ii) the minimum rate of return the individual can obtain from the bond. [2]
- (iii) the expected rate of return the individual will obtain from the bond [2]

An investor is considering buying the whole loan.

- (iv) Show that the rate of return that the investor will obtain is greater than the expected rate of return that the above individual who buys a single bond will receive. [5]
- [Total 14]

8 (i) State the characteristics of an equity. [4]

An investor was considering investing in the shares of a particular company on 1 August 2014. The investor assumed that the next dividend would be payable in exactly one year and would be equal to 6 pence per share.

Thereafter, dividends will grow at a constant rate of 1% per annum and are assumed to be paid in perpetuity. All dividends will be taxed at a rate of 20%. The investor requires a net rate of return from the shares of 6% per annum effective.

(ii) Derive and simplify as far as possible a general formula which will allow you to determine the value of a share for different values of:

- the next expected dividend.
- the dividend growth rate.
- the required rate of return.
- the tax rate.

(iii) Calculate the value of one share to the investor. [5]

The company announces some news that makes the shares more risky.

(iv) Explain what would happen to the value of the share, using the formula derived in part (ii). [2]

The investor bought 1,000 shares on 1 August 2014 for the price calculated in part (iii). He received the dividend of 6 pence on 1 August 2015 and paid the tax due on the dividend. The investor then sold the share immediately for 120 pence. Capital gains tax was charged on all gains of at a rate of 25%. On 1 August 2014, the index of retail prices was 123. On 1 August 2015, the index of retail prices was 126.

(v) Determine the net real return earned by the investor. [3]
[Total 14]

- 9** A student has inherited £1m and is considering investing the money in two projects, A and B.

Project A requires the investment of the whole sum in properties that are to be let out to tenants. The details are:

- The student expects to receive an income from rents at an annual rate of £60,000 a year for four years after an initial period of one year in which no income will be received.
- Rents are expected to rise thereafter at the start of each year at a rate of 0.5% per annum.
- The income will be received monthly in advance.
- The project involves costs of £10,000 per annum in the first year, rising at a constant rate of 0.5% per annum.
- The costs will be incurred at the beginning of each year.
- At the end of 20 years, the student expects to be able to sell the properties for £2m after which there will be no further revenue or costs.

Project B involves the investment of the whole sum in an investment fund.

- The fund is expected to pay an income of £60,000 per annum annually in advance and return the whole invested sum at the end of 20 years.
 - (i) (a) Calculate the payback period for project B.
 - (b) Show, by general reasoning or otherwise, that the payback period from project A is longer than that from project B. [5]
 - (ii) (a) Define the discounted payback period.
 - (b) Determine the discounted payback period from project B at a rate of interest of 1% per annum effective.
 - (c) Show, by general reasoning or otherwise, that the discounted payback period from project A is longer than that from project B. [5]
 - (iii) Determine the internal rate of return from project B expressed as an annual effective return. [3]
 - (iv) Show that the internal rate of return from project A is higher than that from project B. [10]
 - (v) Discuss which project is the better project given your answers to parts (i)–(iv) above. [3]
- [Total 26]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

12 April 2016 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 List the characteristics of convertible bonds. [3]

2 An insurance company has liabilities of £6 million due in exactly 8 years' time and a further £11 million due in exactly 15 years' time.

The assets held by the insurance company consist of:

- a 5-year zero-coupon bond of nominal amount £5.5088 million; and
- a 20-year zero-coupon bond of nominal amount £13.7969 million.

The current rate of interest is 8% per annum effective at all durations.

- (i) Show that the first two conditions of Redington's theory for immunisation against small changes in the rate of interest are satisfied. [5]
- (ii) Explain, without doing any further calculations, whether the insurance company will be immunised against small changes in the rate of interest. [2]
[Total 7]

3 At time $t = 0$, the one-year zero-coupon yield is 4% per annum effective and the one-year forward rate per annum effective at time t ($t = 1, 2, \dots$) is given by:

$$f_{t,1} = (4 + t)\%.$$

- (i) Determine the issue price per £100 nominal of a three-year 4% coupon bond issued at time $t = 0$, paying coupons annually in arrear and redeemable at 105%. [4]
- (ii) Determine the three-year par yield at time $t = 0$. [3]
[Total 7]

4 A loan of nominal amount £100,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 7% per annum. Capital is to be redeemed at £108 per £100 nominal on a coupon date between 15 and 20 years inclusive after the date of issue. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 25% and capital gains tax at 40% wishes to purchase the entire loan at the date of issue.

- (i) Determine the price which the investor should pay to ensure a net effective yield of at least 5% per annum. [5]
- (ii) Explain the significance of the redemption date being at the option of the borrower in relation to your calculation in part (i). [2]
[Total 7]

- 5** A loan is to be repaid by a series of instalments payable annually in arrear for 15 years. The first instalment is £1,200 and payments increase thereafter by £250 per annum.

Repayments are calculated using a rate of interest of 6% per annum effective.

Determine:

- (i) the amount of the loan. [3]
- (ii) the capital outstanding immediately after the 9th instalment has been made. [2]
- (iii) the capital and interest components of the final instalment. [2]
- [Total 7]

- 6** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 & 0 \leq t \leq 4 \\ 0.10 - 0.01t & 4 < t \leq 7 \\ 0.01t - 0.04 & 7 < t \end{cases}$$

- (i) Calculate, showing all working, the value at time $t = 5$ of £10,000 due for payment at time $t = 10$. [5]
- (ii) Calculate the constant rate of discount per annum convertible monthly which leads to the same result as in part (i). [2]
- [Total 7]

- 7** A one-year forward contract on a share was agreed on 1 September 2015 when the share price was £8.70 and the risk-free force of interest was 7% per annum. The stock was expected to pay a dividend of £1.10 eight months after the date of issue.

The price of the share was £9.90 on 1 February 2016 and the risk-free force of interest was 6.5% per annum. The dividend expectation was unchanged.

Calculate, showing all working, the value of the contract to the holder of the long forward position on 1 February 2016. [7]

- 8** An individual is planning to purchase £100,000 nominal of a bond on 1 June 2016 which will be redeemable at 110% on 1 June 2020. The bond will pay coupons of 3% per annum at the end of each year.

The individual wishes to invest the coupon payments on deposit until the bond is redeemed. It is assumed that, in any year, there is a 55% probability that the rate of interest will be 6% per annum effective and a 45% probability that it will be 5.5% per annum effective. It is also assumed that the rate of interest in any one year is independent of that in any other year.

- (i) Derive the necessary formula to determine the mean value of the total accumulated investment on 1 June 2020. [4]
- (ii) Calculate the mean value of the total accumulated investment on 1 June 2020. [2]
- [Total 6]

- 9** In January 2014, the government of a country issued an index-linked bond with a term of two years. Coupons were payable half-yearly in arrear, and the annual nominal coupon rate was 6%. The redemption value, before indexing, was £100 per £100 nominal. Interest and capital payments were indexed by reference to the value of an inflation index with a time lag of six months.

A tax-exempt investor purchased £100,000 nominal at issue and held it to redemption. The issue price was £97 per £100 nominal.

The inflation index was as follows:

<i>Date</i>	<i>Inflation Index</i>
July 2013	120.0
January 2014	122.3
July 2014	124.9
January 2015	127.2
July 2015	129.1
January 2016	131.8

- (i) Set out a schedule of the investor's cashflows, showing the amount and month of each cashflow. [3]
- (ii) Determine the annual effective real yield obtained by the investor to the nearest 0.1% per annum. [5]
- [Total 8]

- 10** The following table gives information concerning a fund held by an investment manager:

Year	2012	2013	2014	2015
Value of fund at 30 June	–	12,700,000	13,000,000	14,100,000
Net cash flow received on 1 July	–	2,600,000	–3,700,000	1,800,000
Value of fund at 31 December	12,000,000	13,500,000	12,900,000	17,200,000

- (i) Calculate, to the nearest 0.1% and showing all working, the annual effective time-weighted rate of return (TWRR) achieved by the fund during the period from 31 December 2012 to 31 December 2015. [3]
- (ii) Show that the annual effective money-weighted rate of return (MWRR) achieved by the fund over the same period is less than the answer obtained in part (i) above. [2]
- (iii) Explain why you would expect the outcome described in part (ii) for this fund. [2]
- (iv) Explain which of the two measures referred to in parts (i) and (ii) is a better indicator of the investment manager's performance over the period. [2]
- [Total 9]

- 11** An investor is considering the purchase of 10,000 ordinary shares in Enterprise plc.

Dividends from the shares are payable half-yearly in arrear. The next dividend is due in exactly six months and is expected to be 6.5 pence per share.

The required rate of return is 6% per half-year effective and an estimated rate of future dividend growth is 2% per half-year.

- (i) Calculate, showing all working, the maximum price that the investor should pay for the shares. [4]

As a result of a recently announced expansion plan, the investor increases the estimated rate of future dividend growth to 2.5% per half-year.

- (ii) (a) Calculate, showing all working, the maximum price the investor should now pay for the shares.
- (b) Explain the difference between your answers to part (i) and part (ii)(a). [2]

It is rumoured that new legislation may affect the operation of Enterprise plc.

As a result, the investor decides to increase her required rate of return to 7% per half-year effective. The estimated dividend growth rate remains at 2% per half-year

- (iii) (a) Explain why it might be appropriate for the investor to increase her required rate of return.
- (b) Calculate the maximum price that the investor should now pay for the shares.
- (c) Explain the difference between your answers to part (i) and part (iii)(b). [3]

In the prevailing economic circumstances, investors are expecting lower inflation in the wider economy.

As a result, the investor decides to reduce both the assumed rate of dividend growth and her required rate of return to 1% and 5% per half-year effective respectively.

- (iv) (a) Explain why it is appropriate for the investor to reduce both the future dividend growth rate and the required rate of return in this case.
- (b) Calculate the maximum price that the investor should now pay for the shares.
- (c) Explain the difference between your answers to part (i) and part (iv)(b). [5]

[Total 14]

12 (i) Show that:

$$\left(\bar{Ia}\right)_{\bar{n}} = \frac{\bar{a}_{\bar{n}} - nv^n}{\delta}. \quad [4]$$

A company is considering the purchase of a gold mine which has recently ceased production.

The company forecasts that:

- the cost of re-opening the mine will be \$900,000, which will be incurred continuously over the first twelve months.
- additional costs are expected to be constant throughout the term of the project at \$200,000 per annum, excluding the first year. These are also incurred continuously.
- after the first twelve months, the rate of revenue will grow continuously and linearly from zero per annum to \$3,600,000 per annum at a constant rate of \$300,000 per annum.
- when the rate of revenue reaches \$3,600,000 per annum it will then decline continuously and linearly at a constant rate of \$150,000 per annum until it reaches \$600,000 per annum.
- when the rate of revenue declines to \$600,000 per annum production will stop and the mine will have zero value.

(ii) Determine the overall term of the project. [2]

(iii) Calculate, showing all working, the price that the company should pay in order to earn an internal rate of return (IRR) of 25% per annum effective. [12]

[Total 18]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

27 September 2016 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 12 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 The nominal rate of interest per annum convertible quarterly is 5%.

Calculate, giving all the answers as a percentage to three decimal places:

(i) the equivalent annual force of interest. [1]

(ii) the equivalent effective rate of interest per annum. [1]

(iii) the equivalent nominal rate of discount per annum convertible monthly. [2]
[Total 4]

2 The nominal rate of interest per annum convertible quarterly is 2%.

Calculate the present value of a payment stream paid at a rate of €100 per annum, monthly in advance for 12 years. [4]

3 Describe the characteristics of a repayment loan (or repayment mortgage). [3]

4 The following table shows the cashflows paid into a fund on three different dates, together with the value of the fund on each date immediately before the cash flow takes place. There were no other cashflows except on the dates shown.

	<i>1 January 2014</i>	<i>1 January 2015</i>	<i>1 January 2016</i>
Value of fund (£m)	112	X	160
Cash flow (£m)	23	43	32

During 2014, the rate of return on the fund was 10% per annum effective.

(i) Calculate X . [1]

(ii) Calculate, showing all workings, the annual effective time weighted rate of return on the fund over the two-year period from 1 January 2014 to 1 January 2016. [3]

[Total 4]

5 A zero-coupon bond was issued on 1 January 1975 with a redemption date of 1 January 2015. An investor bought the bond to provide a yield to maturity of 5% per annum convertible half yearly. On a particular date the borrower defaulted, repaying 80% of the capital to all bondholders. The investor obtained a rate of return until the date of default which was equivalent to a force of interest of 4.8% per annum.

Determine the date on which the borrower defaulted. [5]

- 6** At the beginning of 2015 a 182-day commercial bill, redeemable at £100, was purchased for £96 at the time of issue and later sold to a second investor for £97.50. The initial purchaser obtained a simple rate of interest of 3.5% per annum before selling the bill.
- (i) Calculate the annual simple rate of return which the initial purchaser would have received if they had held the bill to maturity. [2]
- (ii) Calculate the length of time in days for which the initial purchaser held the bill. [2]
- The second investor held the bill to maturity.
- (iii) Calculate the annual effective rate of return achieved by the second investor. [2]
- [Total 6]
- 7** A nine-month forward contract was issued on 1 April 2015 on a share with a price of £1.10 at that date. Dividends of £0.10 per share were expected on 1 July 2015, 1 October 2015 and 1 January 2016.
- (i) Calculate, showing all workings, the forward price assuming a risk-free rate of interest of 8% per annum convertible half-yearly and no arbitrage. [4]
- (ii) Explain why you do not need to use the expected price of the share at the time the forward matures in the calculation of the forward price. [2]
- [Total 6]
- 8** Three bonds, each paying annual coupons in arrear of 4% and redeemable at par, reach their redemption dates in exactly one, two and three years' time, respectively. The price of each bond is £96 per £100 nominal.
- (i) Calculate the gross redemption yield of the three-year bond. [3]
- (ii) Calculate, showing all workings, the one-year and two-year spot rates of interest implied by the information given. [3]
- (iii) Calculate the forward rate of interest applicable over the second year. [2]
- (iv) Explain whether the three-year spot rate will be higher than or lower than the three-year gross redemption yield. [2]
- [Total 10]

- 9** An insurance company has just written single premium contracts that require it to make payments to policyholders of £10,000,000 in five years' time. The total single premiums paid by policyholders amounted to £8,000,000.

The insurance company is to invest the premiums in assets that have an uncertain return. The return from these assets in year t , i_t , is independent of the returns in all previous years with a mean value of 5.5% per annum effective and a standard deviation of 4% per annum effective. $(1 + i_t)$ is lognormally distributed.

- (i) Calculate, deriving all necessary formulae, the mean and standard deviation of the accumulation of the premiums over the five-year period. [9]

A director of the company is concerned about the possibility of a considerable loss from the investment in the assets suggested in part (i). Instead, the director suggests investing in fixed interest securities with a guaranteed return of 4% per annum effective.

- (ii) Set out the arguments for and against the director's position. [3]
[Total 12]

- 10** A particular charity invests its assets in a fund on which it has a target rate of return of 8% per annum effective. From time-to-time, the charity also invests in projects that help achieve its charitable objectives whilst providing a rate of return. Projects that are accepted by the charity must fulfil each of the following criteria:

1. a minimum annual effective internal rate of return of 2% less than the target return on the investment fund.
2. a payback period of no more than ten years.
3. a positive cash flow during the fifth year or earlier.

The charity is considering investing in a social enterprise project that involves providing loans to farmers in low-income countries to help them develop better resilience against poor weather conditions. The details are as follows:

- The project involves making loans of £1m at the start of each year for three years, the first loan being made at the beginning of 2017.
- The loans will be paid back from the extra income obtained by the farmers from the beginning of 2020.
- The repayments in each year will be through level monthly instalments paid in advance with the rate of payment of the instalments increasing by 1% per year for 10 years after which the payments stop.
- The annual rate of repayment in 2020 will be £495,000.
- The charity will also incur costs at the end of each of the years in which income is received of £50,000 per annum.

- (i) Explain why, in general, the payback period is not an appropriate decision criterion for an investment project. [2]
- (ii) Determine which of the three criteria used by the charity are met in this case. [12]
[Total 14]

11 The government of a heavily indebted country has a range of bonds currently in issue. These include bonds with nominal amounts outstanding of £4bn and £5bn with terms to redemption of exactly three years and ten years respectively from the current time. Both bonds pay annual coupons in arrear of 4%. The government is negotiating a restructuring of its debt portfolio and proposes to transform the three and ten year bonds into perpetuities paying an annual coupon of 5% in arrear. The yield curve is currently flat with gross redemption yields at 6% per annum effective.

- (i) Calculate, showing all workings, the duration of the current portfolio of three-year and ten-year bonds. [7]
- (ii) Calculate, showing all workings, the duration of the proposed portfolio of bonds. [4]

The government's objective is that the present value of the proposed portfolio of bonds will be 80% of the present value of the current portfolio of three-year and ten-year bonds.

- (iii) Determine the nominal amount of the new bonds that the government will have to issue to achieve the objective. [2]
[Total 13]

- 12** The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by:

$$\delta(t) = \begin{cases} 0.03 & \text{for } 0 \leq t \leq 10 \\ at & \text{for } 10 < t \leq 20 \\ bt & \text{for } t > 20 \end{cases}$$

where a and b are constants.

The present value of £100 due at time 20 is 50.

- (i) Calculate a . [5]

The present value of £100 due at time 28 is 40.

- (ii) Calculate b . [4]

- (iii) Calculate the equivalent annual effective rate of discount from time 0 to time 28. [2]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per annum between $t = 3$ and $t = 7$.

- (iv) (a) Calculate, showing all workings, the present value of the payment stream.

- (b) Determine the level continuous payment stream per annum from time $t = 3$ to time $t = 7$ that would provide the same present value as the answer in part (iv)(a) above. [8]

[Total 19]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

18 April 2017 (pm)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 10 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** Calculate the nominal rate of discount per annum convertible monthly which is equivalent to:
- (i) an effective rate of interest of 1% per quarter. [2]
 - (ii) a force of interest of 5% per annum. [2]
 - (iii) a nominal rate of discount of 4% per annum convertible every three months. [2]
- [Total 6]

- 2** A bank offers two repayment alternatives for a loan that is to be repaid over sixteen years:

Option 1: the borrower pays £7,800 per annum quarterly in arrear.

Option 2: the borrower makes payments at an annual rate of £8,200 every second year in arrear.

Determine which option would provide the better deal for the borrower at a rate of interest of 5% per annum effective. [5]

- 3** A one-year forward contract on a stock was agreed on 1 March 2017 when the stock price was £78 and the risk-free force of interest was 14% per annum. The stock was expected to pay dividends of £3.20 on 1 June and 1 December 2017.

On 1 April 2017, the price of the stock was £80 and the risk-free force of interest was 11% per annum. The dividend expectations were unchanged.

Determine the value of the contract to the holder of the long forward position on 1 April 2017. [7]

- 4** An investor borrows money from a bank in order to invest in a business venture. The initial loan is £500,000, with further loans of £250,000 made in 6 months' time and £250,000 made in 12 months' time.

The business venture will provide the investor with an income of £2 million in exactly 10 years' time and £3 million in exactly 15 years' time.

The bank offers a force of interest, $\delta(t)$, as a function of time t (measured in years) which is given by:

$$\delta(t) = \begin{cases} 0.04 & \text{for } 0 \leq t \leq 2 \\ 0.02 + kt & \text{for } t > 2 \end{cases}$$

- (i) Derive expressions for $v(t)$ which cover all values of t . [5]

- (ii) Determine the minimum value of k that would ensure that the discounted payback period is exactly 10 years. [4]
[Total 9]

5 An investment fund has liabilities of £11 million due in 7 years' time and £8.084 million in 11 years' time.

The manager of the fund will meet the liabilities by investing in zero-coupon bonds. The manager is able to buy zero-coupon bonds for whatever term is required and there are adequate funds at the manager's disposal.

- (i) Explain whether it is possible for the manager to immunise the fund against small changes in the rate of interest by purchasing a single zero-coupon bond. [2]

The manager decides to purchase two zero-coupon bonds, one paying £15.363 million in 7.5 years' time and the other paying £3.787 million in 14.25 years' time. The current interest rate is 5.5% per annum effective.

- (ii) Determine whether the investment fund satisfies the necessary conditions to be immunised against small changes in the rate of interest. [7]
[Total 9]

6 Exactly three months ago an investor purchased an office building for £5.8 million with the intention of renting it out. In three months' time the investor will spend £850,000 on necessary refurbishments and improvements.

A tenant has agreed to lease the building in six months' time for 35 years. The tenant will pay an initial rent of £1.250 million per annum payable monthly in arrear. The rent will be increased at five-yearly intervals at a rate of 4.2% per annum compound. It has further been agreed that at the end of the lease period the tenant will buy the building from the investor for £11.5 million.

The investor pays income tax at a rate of 35% and is expecting a net effective rate of return of 8% per annum.

Calculate, showing all workings, the net present value of the project to the investor at the time of purchase. [11]

- 7 A fixed interest bond was issued on 1 January 2017 with a term of 20 years and is redeemable at 105%. The security pays a coupon of 4% per annum, payable half-yearly in arrear.

An investor is liable to income tax at the rate of 30% and capital gains tax at the rate of 40%. Income tax and capital gains tax are both collected on 1 June each year in relation to gross payments made during the previous 12 months.

The investor bought £10,000 nominal of the stock at an issue price of £9,800.

- (i) Show that the net redemption yield obtained by the investor will be between 3% and 4% per annum effective. [7]

The inflation rate over the term of the bond is assumed to be 2% per annum.

- (ii) Calculate the net effective annual real redemption yield that would be obtained by the investor. [3]

- (iii) Explain, without doing any further calculations, how your answers to parts (i) and (ii) would alter if the tax were collected on 1 April instead of 1 June each year. [2]

[Total 12]

- 8 Two investment funds A and B are administered by different managers. The initial values of the two funds on 1 January 2015 were £1.5 million and £2.3 million, respectively. The funds received additional net cash flows at the beginning of 2015 and 2016, as follows:

	<i>Fund Net Cash Flows</i>	
	<i>1 January 2015</i>	<i>1 January 2016</i>
Fund A	£300,000	£1,700,000
Fund B	£2,000,000	£200,000

The fund managers achieved the following annual returns during 2015 and 2016:

	<i>Fund Annual Returns</i>	
	<i>2015</i>	<i>2016</i>
Fund A	42%	3%
Fund B	36%	2%

- (i) Calculate the annual effective time weighted rate of return for each fund for the period 1 January 2015 to 31 December 2016. [3]
- (ii) Calculate the annual effective money weighted rate of return per annum for each fund for the period 1 January 2015 to 31 December 2016. [8]
- (iii) Comment on your answers to parts (i) and (ii) by explaining which of the two measures is the better indicator of the comparative performance of the managers for the given two-year period. [3]

[Total 14]

- 9** Let f_t denote the one-year effective forward rate of interest over the year from time t to $(t + 1)$. Let i_t be the t -year effective spot rate over the period 0 to t .

The annual effective gross redemption yield from an n -year bond which pays coupons of 5% annually in arrear is given by:

$$g_n = 0.07 + 0.001n \quad \text{for } n = 1, 2 \text{ and } 3$$

Each bond is redeemed at par and is exactly one year from the next coupon payment. It is assumed that no arbitrage takes place.

- (i) Calculate i_1 , i_2 and i_3 as percentages to three decimal places. [7]
- (ii) Calculate f_0 , f_1 and f_2 as percentages to three decimal places. [4]
- (iii) Explain why the one-year forward rates increase more quickly with term than the spot rates. [2]
- [Total 13]

- 10** An individual aged exactly 65 intends to retire in five years' time and receive an annuity-certain. The annuity will be payable monthly in advance and will cease after 20 years. The annuity will increase at each anniversary of the commencement of payment at the rate of 3% per annum.

The individual would like the initial level of annuity to be £20,000 per annum. The price of the annuity will be the present value of the payments on the date it commences using an interest rate of 7% per annum effective.

- (i) Calculate the price of the annuity. [4]

In order to purchase the annuity described in part (i), the individual invests £200,000 on his 65th birthday in a particular fund.

The investment return on the fund in any given year is independent of returns in all other years and the annual return is:

- 4% with a probability of 60%.
- 7% with a probability of 40%.

- (ii) Calculate, showing all workings, the expected accumulation of the investment at the time of retirement. [3]
- (iii) Calculate, showing all workings, the standard deviation of the investment at the time of retirement. [4]
- (iv) Determine the probability that the individual will have sufficient funds to purchase the annuity. [3]
- [Total 14]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

25 September 2017 (pm)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 11 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** (i) Calculate the time in days for £6,000 to accumulate to £7,600 at:
- (a) a simple rate of interest of 3% per annum.
 - (b) a compound rate of interest of 3% per annum effective.
 - (c) a force of interest of 3% per annum. [6]

Note: You should assume there are 365 days in a year.

- (ii) Calculate the effective rate of interest per half year which is equivalent to a force of interest of 3% per annum. [1]
[Total 7]

- 2** Describe how cash flows are exchanged in an interest rate swap. [2]

- 3** An investor is considering two investments. One is a 91-day deposit which pays a compound rate of interest of 3% per annum effective. The second is a government bill.

Calculate the annual simple rate of discount from the government bill if both investments are to provide the same effective rate of return. [3]

- 4** A one-year forward contract was issued on 1 April 2016 on a share with a price of \$4.00 at that date. Dividends of \$0.10 per share were expected on 30 September 2016 and 31 March 2017. The 6-month and 12-month risk-free spot rates of interest were 5% and 6% per annum effective respectively on 1 April 2016.

Calculate the forward price at issue, stating any assumptions. [4]

- 5** An individual invests £100 in an asset. The expected accumulation of this asset after 20 years is £200 and the standard deviation of the accumulation after 20 years is £50.

- (i) Calculate the expected effective rate of return per annum. [1]
- (ii) Calculate the standard deviation of the effective rate of return per annum. [4]
[Total 5]

- 6** An investor has a choice of two 15-year savings plans, A and B, issued by a company. In both plans, the investor pays contributions of \$100 at the start of each month and the contributions accumulate at an effective rate of interest of 4% per annum before any allowance is made for expenses.

In plan A, the company charges for expenses by deducting 1% from the annual effective rate of return.

In plan B, the company charges for expenses by deducting \$15 from each of the first year's monthly contributions before they are invested. In addition it deducts 0.3% from the annual effective rate of return.

Calculate the percentage by which the accumulated amount in Plan B is greater than the accumulated amount in Plan A, at the end of the 15 years. [6]

7 Two investors, A and B, value corporate bonds using different models.

- Investor A uses the average gross redemption yield from all government securities with the addition of a risk premium of 1% per annum effective.
- Investor B uses the spot rates of interest derived from the government bond yield curve also with the addition of a risk premium of 1% per annum effective to value each payment.

The investors are valuing a particular corporate bond which has half-yearly coupon payments paid at a rate of 5% per annum and a term to redemption of exactly two years. The bond is redeemed at 110% and tax is payable on coupons only at a rate of 20%.

The average gross redemption yield from all government securities is 3% per annum effective.

(i) Calculate the price that investor A would pay for the corporate bond. [3]

Over time t , the spot rate of interest from the yield curve of government securities, y_t is given by $y_t = 0.015t$ per annum effective for $t \leq 2$.

(ii) Calculate the price that investor B would pay for the corporate bond. [3]

(iii) Calculate the forward rate of interest from government securities from $t = 1$ to $t = 2$. [2]

(iv) Giving two reasons, explain why the spot yield curve might rise with term to redemption. [3]

[Total 11]

- 8** A loan is to be repaid by an increasing annuity. The first payment will be £100 and the payments will increase by £50 per annum. Payments will be made annually in arrear for ten years. The repayments are calculated using a rate of interest of 5% per annum effective.

- (i) Calculate the amount of the loan. [2]
- (ii) Calculate:
- (a) the interest component of the sixth instalment.
- (b) the capital component of the sixth instalment. [4]

Immediately after the sixth instalment, the borrower asks to repay the remaining loan using level annual instalments. The lender agrees, but changes the interest rate at the time of the alteration to 6% per annum effective.

- (iii) Calculate the revised instalment. [3]
- [Total 9]

- 9** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.09 - 0.003t & 0 \leq t \leq 10 \\ 0.06 & t > 10 \end{cases}$$

- (i) Calculate the corresponding constant effective annual rate of interest for the period from $t = 0$ to $t = 10$. [4]
- (ii) Express the rate of interest in part (i) as a nominal rate of discount per annum convertible half-yearly. [1]
- (iii) Calculate the accumulation at time $t = 15$ of £1,500 invested at time $t = 5$. [3]
- (iv) Calculate the corresponding constant effective annual rate of discount for the period $t = 5$ to $t = 15$. [1]
- (v) Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $10e^{0.01t}$ from time $t = 11$ to time $t = 15$. [6]
- [Total 15]

- 10** An insurance company has liabilities of £100 million due in 10 years' time and £200 million due in 20 years' time.

The company's assets consist of a zero-coupon bond and a level annuity paid annually in arrear. The zero coupon bond will pay £144.054 million in 15 years' time. The current interest rate is 3% per annum effective at all terms to redemption.

Redington's first two conditions for immunisation against small changes in the rate of interest have been satisfied for this insurance company.

- (i) (a) Calculate the present value of the liabilities. [4]
- (b) Calculate the discounted mean term of the liabilities. [4]
- (ii) Show that the term of the annuity is 41 years. [6]
- (iii) Determine the annual rate of payment of the annuity. [1]
- (iv) State Redington's third condition for immunisation, explaining whether you think it is fulfilled. [2]
- (v) Explain the risks of implementing this decision. [2]
- [Total 15]

- 11** A university offers its students three financing options for a degree course that lasts exactly three years.

Option A

Fees are paid during the term of the course monthly in advance. The fees are £10,000 per annum in the first year and rise by 5% on the first and second anniversaries of the start of the course.

Option B

The university makes a loan to the students which is repaid in instalments after the end of the course. The instalments are determined as follows:

- No payments are made until three years after the end of the course.
- Over the following 15 years, students pay the university £1,300 per year, quarterly in advance.
- After 15 years of payments, the quarterly instalments are increased to £1,500 per year, quarterly in advance.

- After a further 15 years of payments, the quarterly instalments are increased to £1,800 per year, quarterly in advance, for a further 15-year period after which there are no more payments.

Option C

- Students pay to the university 3% of all their future earnings from work, with the payments made annually in arrear.

A particular student wishes to attend the university. He expects to leave university at the end of the three-year course and immediately obtain employment. The student expects that his earnings will rise by 3% per annum compound at the end of each year for 10 years and then he will take a five-year career break.

After the career break, he expects to restart work on the salary he was earning when the career break started. He then expects to receive salary increases of 1% per annum compound at the end of each year until retiring 45 years after graduating.

The student wishes to take the financing option with the lowest net present value at a rate of interest of 3% per annum effective.

- (i) Calculate the present value of the payments due under option A. [4]
- (ii) Calculate the present value of the payments due under option B. [5]
- (iii) Calculate the initial level of salary that will lead the payments under option C to have the lowest present value of the three options. [8]
- (iv) Comment on whether the student should use the same interest rate to evaluate all three options. [2]

The university is concerned that this scheme exposes it to considerable financial risk.

- (v) Explain three risks which the university faces. [4]
- [Total 23]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

17 April 2018 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 11 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 State the characteristics of a Eurobond. [4]

2 (i) Describe what is meant by the term “ex-dividend”. [1]

An individual purchased 10,000 shares on 1 December 2017. Dividends are payable on 1 January and 1 July each year, and are assumed to be payable in perpetuity. The next dividend, due on 1 January 2018, is \$0.07 per share.

The two dividend payments in any calendar year are expected to be the same, but the dividend payment is expected to increase at the end of each year at a rate of 2% per annum compound.

Assume that the share is ex-dividend on 1 December 2017 and use an effective rate of interest of 7% per annum.

(ii) Calculate the present value of the investment at the date of purchase. [5]
[Total 6]

3 An investor pays £80 at the start of each month into a 25-year savings plan.

The contributions accumulate at an effective rate of interest of 3% per half-year for the first 10 years, and at a force of interest of 6% per annum for the final 15 years.

Calculate the accumulated amount in the savings plan at the end of 25 years. [6]

4 The annual investment return achieved by an insurance company in year t is i_t .

Returns in successive years are assumed to be independent and:

$$\ln(1+i_t) \sim N(\mu, \sigma^2), \text{ where } \mu = 0.08 \text{ and } \sigma = 0.15.$$

The insurance company has a liability of €800,000 payable at the end of year 10.

The company wishes to invest an amount now so that there is a 95% probability that the accumulated amount at the end of year 10 will be sufficient to meet this liability.

(i) Calculate the amount of money that the insurance company should invest. [5]

(ii) Explain, without doing any further calculations, how your answer to part (i) would change if each of the following occurs separately, with all other parameters as in part (i):

- (a) The value of μ is increased to 0.1.
- (b) The value of σ is increased to 0.2.
- (c) The desired probability of meeting the liability is increased to 99%.

[3]
[Total 8]

- 5** (i) Describe what is meant by the “no arbitrage” assumption in financial mathematics. [2]

An investor entered into a long forward contract for a security three years ago and the contract is due to mature in six years’ time. The price of the security was £7.10 three years ago and is now £10.20. The risk-free rate of interest can be assumed to be 2% per annum effective throughout the nine-year period.

- (ii) Calculate the current value of the contract with the following dividend payments, assuming no arbitrage:
- (a) The security will pay dividends of £1.10 annually in arrear from now until the maturity of the contract.
 - (b) The security has paid and will continue to pay annually in arrear a dividend equal to 2.5% of the market price of the security at the time of payment.

[6]

[Total 8]

- 6** On 1 April 2018 a government issued a 10-year bond redeemable at £105 per £100 nominal and paying coupons at the rate of 3% per annum half-yearly in arrear. The price of the bond was £102 per £100 nominal.

An investor subject to income tax of 25% and capital gains tax of 35% purchased £10,000 nominal of the bond at issue.

The investor assumes that inflation will be constant over the term of the bond at a rate of 2% per annum.

- (i) Calculate the net effective real redemption yield which the investor expects to earn on the investment. [6]
- (ii) Explain how your answer to part (i) would change if inflation were less than 2% per annum throughout the term. [2]

[Total 8]

- 7** A retailer is considering opening a new store as a business venture. The purchase price of the store will be £2 million and there will be a further investment required of £0.5 million 6 months after purchase.

The store will open 12 months after purchase. Revenues less running costs are expected to occur continuously and will be £0.2 million in the first year of operation, £0.25 million in the second year of operation and thereafter increasing at yearly intervals by 4% per annum compound.

Eight years after purchase, a major refit costing £0.8 million will be required. Fifteen years after purchase, it is assumed that the store will be closed and sold for £6.4million.

The retailer requires a rate of return on its investment of 10% per annum effective.

- (i) Calculate the net present value of the venture. [8]

It is now assumed that the revenue less running costs will be received mid-way through each year, rather than continuously.

- (ii) Explain how your answer to part (i) would change. [2]
[Total 10]

- 8** An investment fund has liabilities of £20 million due in 8 years' time and £15 million due in 12 years' time.

The manager wishes to immunise the fund against small changes in the rate of interest and seeks to achieve this by purchasing two zero-coupon bonds. One bond is for a term of exactly 7 years and the other bond is for a term of exactly 14 years. The current interest rate is 4.5% per annum effective.

- (i) Calculate the amount that should be invested in each bond, demonstrating that all three Redington conditions are met. [9]
- (ii) Explain, without performing any further calculations, how the relative values of the assets and the liabilities will change if the interest rate changes immediately to 4.7% per annum effective. [2]
[Total 11]

- 9** Two bonds paying annual coupons of 6% in arrear and redeemable at par have terms to maturity of exactly one year and two years.

The gross redemption yield from the 1-year bond is 5.2% per annum effective. The gross redemption yield from the 2-year bond is 6.1% per annum effective. The 3-year par yield is 6.6% per annum.

Calculate the following as a percentage to three decimal places:

- (i) the annual effective spot yields for each of the three years [8]
- (ii) the annual effective one-year forward rates for each of the three years [4]
- [Total 12]

- 10** The force of interest $\delta(t)$ is a function of time, and at any time t , measured in years is given by the formula:

$$\delta(t) = \begin{cases} 0.24 - 0.02t & 0 < t \leq 6 \\ 0.12 & 6 < t \end{cases}$$

- (i) Derive, and simplify as far as possible, expressions in terms of t for the present value of a unit investment made at any time, t . You should derive separate expressions for each time interval $0 < t \leq 6$ and $6 < t$. [5]
- (ii) Determine the discounted value at time $t = 4$ of an investment of £1,000 due at time $t = 10$. [2]
- (iii) Calculate the constant nominal annual interest rate convertible monthly implied by the transaction in part (ii). [2]
- (iv) Calculate the present value of a continuous payment stream invested from time $t = 6$ to $t = 10$ at a rate of $\rho(t) = 20e^{0.36+0.32t}$ per annum. [4]
- [Total 13]

- 11** An n -year decreasing annuity is payable annually in arrear where the payment at the end of the first year is n , the payment at the end of the second year is $(n - 1)$, and so on until the final payment at the end of year n is 1.

(i) Show that the present value of this annuity is $\frac{n - a_n}{i}$ [3]

A loan is to be repaid over 25 years by means of annual instalments payable in arrear.

The amount of the first instalment is £8,000 and each subsequent instalment reduces by £200.

The effective rate of interest charged by the lender is 5.5% per annum.

(ii) Calculate the initial amount of the loan. [3]

(iii) Determine the interest and capital components of the 10th instalment. [6]

(iv) Calculate the total amount of interest payable over the term of the loan. [2]

[Total 14]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

28 September 2018 (pm)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 10 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** An investor is considering two investments. One investment is a 91-day bond issued by a bank which pays a rate of interest of 4% per annum effective. The second is a 91-day treasury bill which pays out €100.
- (i) Calculate the price of the treasury bill and the annual simple rate of discount from the treasury bill if both investments are to provide the same effective rate of return. [3]
- (ii) Suggest one factor, other than the rate of return, which might determine which investment is chosen. [1]
- [Total 4]

- 2** The effective rate of discount per annum is 5%.

Calculate:

- (i) the equivalent force of interest; [1]
- (ii) the equivalent rate of interest per annum convertible monthly; [2]
- (iii) the equivalent rate of discount per annum convertible monthly. [1]
- [Total 4]

- 3** An investment fund is valued at £60m on 1 January 2016 and at £70m on 1 January 2017. Immediately after the valuation on 1 January 2017, £100m is paid into the fund. On 1 July 2018, the value of the fund is £300m.

- (i) Calculate the effective time-weighted rate of return per annum over the whole period. [3]
- (ii) Explain why the money-weighted rate of return per annum would be higher than the time-weighted rate of return per annum. [2]
- [Total 5]

- 4** A company issues a loan stock which pays coupons at a rate of 6% per annum half-yearly in arrears. The stock is to be redeemed at 103% after 25 years.
- (i) (a) Calculate the price per £100 nominal at issue which would provide a gross redemption yield of 3% per annum convertible half yearly.
- (b) Calculate the price per £100 nominal three months after issue which would provide a gross redemption yield of 3% per annum convertible half-yearly.

[3]

An investor, who is liable to income tax at 30% and capital gains tax at 40%, bought the stock at issue at a price which gave him a net redemption yield of 10% per annum effective.

- (ii) Calculate the price the investor paid. [4]
[Total 7]

- 5** (i) Explain the main difference:
- (a) between options and futures;
- (b) between call options and put options.

[4]

A 12-month forward contract is issued on 1 March 2017 on a share with a price of £1.10 at that date. Dividends of £0.10 per share are expected on 1 June, 1 September and 1 December 2017.

- (ii) Calculate the forward price at issue assuming a risk-free rate of interest of 5% per annum convertible half-yearly and no arbitrage. [4]
[Total 8]

- 6** In a particular investment fund, i_t is the effective rate of return in the t^{th} year. Let S_n be the accumulation of £1 invested over a period of n years.

Assume the mean of i_t is 0.08, the standard deviation of i_t is 0.07 and that $1 + i_t$ is independently and lognormally distributed.

- (i) Determine the distribution of S_{10} . [5]

An investor is considering investing £6,000 in the fund for 10 years.

- (ii) Determine the amount of the accumulated value after 10 years such that there is a 97.5% probability of the investor actually achieving an amount greater than this. [3]
[Total 8]

- 7 The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & 0 \leq t \leq 10 \\ 0.003t & t > 10 \end{cases}$$

- (i) Calculate the present value of a unit sum of money due at time $t = 20$. [4]
- (ii) Calculate the equivalent constant force of interest from $t = 0$ to $t = 20$. [2]
- (iii) Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $e^{-0.06t}$ from time $t = 4$ to time $t = 8$. [4]

[Total 10]

- 8** Two countries have recently signed a free-trade treaty and an insurance company in one of the countries is considering establishing a subsidiary in the other. The country in which the investment will take place currently has a small insurance market, but it is expected to grow slowly over the next ten years and then rapidly thereafter.

The company expects to make investments of £15m in each of the next five years to establish the subsidiary. These costs are assumed to be incurred at the end of each year.

The subsidiary will start business immediately. Upon starting business, the following costs and revenues are expected.

- Costs at a rate of £3m per year will be incurred continuously throughout the first 30 years of the subsidiary's life.
- Revenues of £3.1m per year will be received continuously throughout the first 10 years of the subsidiary's life.
- In the 11th year, revenues will be received continuously at a rate of £3.2m. The rate at which revenues will be received is then expected to increase at a rate of 5% per annum from the end of the 11th year to the end of the 30th year with increases occurring at the end of each year from the end of the 11th year.

At the end of the 30th year, the company assumes that it will sell the subsidiary.

- (i) (a) Define the term "payback period".
(b) State two reasons why the payback period is a poor decision-making criterion in the above circumstances.

[4]

- (ii) Calculate the amount for which the company will have to sell the subsidiary at the end of 30 years so that the project breaks even at a rate of interest of 6% per annum effective.

[9]

Some directors are concerned that the project is too risky.

- (iii) Suggest two ways in which risk could be taken into account when appraising the project.

[2]

[Total 15]

- 9** (i) Describe the cash flows which are paid and received in respect of an index-linked security. [2]

An investor bought £1m nominal of an index-linked bond on 31 December 2015 for £100 per £100 nominal. Nominal coupon payments of 1% were received on 30 June and 31 December each year. The bond was sold for £101 per £100 nominal on 31 December 2017 immediately after the coupon due on that date had been received.

The coupon payments from the bond were linked to the retail prices index (RPI) with a three-month lag with cash payments being rounded to the nearest pound. RPI inflation was 2% per annum effective from three months before the bond was issued until three months before it was sold.

Assume that all months are of equal length.

- (ii) Calculate the cash payments received by the investor from the index-linked bond. [3]
- (iii) Calculate, to the nearest 0.1%, the effective rate of return per annum obtained from the bond over the holding period (before allowing for inflation). [5]

The real rate of return obtained from the bond over the holding period was 1% per annum convertible half-yearly.

- (iv) Calculate the rate of inflation in the three months to 31 December 2017, expressing your answer as an annual effective rate. [7]

[Total 17]

- 10** (i) Describe the characteristics of a repayment mortgage. [3]

A bank has just granted a loan of \$10,000 to a business to be repaid in ten equal instalments, annually in arrears. The rate of interest is 4% per annum effective.

- (ii) (a) Calculate the amount of the annual repayment.
(b) Calculate the duration (discounted mean term) of the repayments.

[5]

The bank wishes to immunise itself from changes in interest rates in relation to this particular asset. For this purpose, the bank has issued two zero-coupon bonds. The first bond is of nominal amount \$5,000 and has a term to redemption of two years.

- (iii) Determine the nominal amount of the second zero-coupon bond and its term to redemption such that the present value and durations of the assets and liabilities are equal. [6]

Immediately upon the loan being granted, the bank agrees to a request to change the terms of the loan. The loan is now to be repaid monthly in arrears over 25 years and the rate of interest remains unchanged.

- (iv) (a) Calculate the revised monthly instalment.
(b) Explain, without further calculation, the main risk to the bank of a change in interest rates.
(c) Determine the interest and capital portions of the 121st repayment under this new arrangement.

[8]

[Total 22]

END OF PAPER