

Homework 5

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- Question 1

Bottom-up

[15, 42, 9, 55, 2, 4, 1]

1. Create heap

```
graph TD; 15((15)) --- 42((42)); 15 --- 9((9)); 42 --- 55((55)); 42 --- 2((2)); 9 --- 4((4)); 9 --- 1((1))
```

0	1	2	3	4	5	6	7
15	42	9	55	2	4	1	

2. Start with last parent, 9, index 3

```
graph TD; 15((15)) --- 42((42)); 15 --- 1((1)); 42 --- 55((55)); 42 --- 2((2)); 1 --- 4((4)); 1 --- 9((9))
```

0	1	2	3	4	5	6	7
15	42	1	55	2	4	9	

3. Move to next previous parent, 42, index 2

```
graph TD; 15((15)) --- 2((2)); 15 --- 1((1)); 2 --- 55((55)); 2 --- 42((42)); 1 --- 4((4)); 1 --- 9((9))
```

0	1	2	3	4	5	6	7
15	2	1	55	42	4	9	

4. Move to next previous parent, 15, index 1

```
graph TD; 1((1)) --- 2((2)); 1 --- 15((15)); 2 --- 55((55)); 2 --- 42((42)); 15 --- 4((4)); 15 --- 9((9))
```

0	1	2	3	4	5	6	7
1	2	15	55	42	4	9	

5. Check swapped child, 15, index 3

```
graph TD; 1((1)) --- 2((2)); 1 --- 4((4)); 2 --- 55((55)); 2 --- 13((13)); 4 --- 15((15)); 4 --- 9((9))
```

0	1	2	3	4	5	6	7
1	2	4	55	42	15	9	

• **Question 2**

1. [1, 2, 3, 55, 42, 15, 9]
2. [9, 2, 3, 55, 42, 15] 1
3. [2, 9, 3, 55, 42, 15] 1
4. [15, 9, 3, 55, 42] 1, 2
5. [3, 9, 15, 55, 42] 1, 2
6. [42, 9, 15, 55] 1, 2, 3
7. [9, 42, 15, 55] 1, 2, 3
8. [55, 42, 15] 1, 2, 3, 9
9. [15, 42, 55] 1, 2, 3, 9
10. [55, 42] 1, 2, 3, 9, 15
11. [42, 55] 1, 2, 3, 9, 15
12. [55] 1, 2, 3, 9, 15, 42
13. 1, 2, 3, 9, 15, 42, 55

• **Question 3**

Backward

$$T(n) = 2T(\frac{n}{2}) + 5, T(1) = 0$$

$$T(n/2) = 2T(\frac{n}{4}) + 5$$

$$T(n/4) = 2T(\frac{n}{8}) + 5$$

$$T(n/8) = 2T(\frac{n}{16}) + 5$$

$$T(n/16) = 2T(\frac{n}{32}) + 5$$

...

Substitute

$$T(n) = 2T(\frac{n}{2}) + 5$$

$$= 2(2T(\frac{n}{4}) + 5) + 5 = 4T(\frac{n}{4}) + 15$$

$$= 4(2T(\frac{n}{8}) + 5) + 15 = 8T(\frac{n}{8}) + 35$$

$$= 8(2T(\frac{n}{16}) + 5) + 35 = 16T(\frac{n}{16}) + 75$$

$$= 16(2T(\frac{n}{32}) + 5) + 75 = 32T(\frac{n}{32}) + 155$$

...

Pattern

$$2^i T(\frac{n}{2^i}) + (2^i - 1)5$$

$$T(1) = T(\frac{n}{2^i}) \rightarrow 1 = n/2^i, n = 2^i, i = \log_2 n$$

Plug in and solve

$$2^{\log_2 n} T(\frac{n}{2^{\log_2 n}}) + (2^{\log_2 n} - 1)5$$

$$nT(\frac{n}{n}) + (n - 1)5$$

$$0 + (n - 1)5$$

$$5n - 5$$

$$\Theta(n)$$

Forward

$$T(1) = 0$$

$$T(2) = 2T(1) + 5 = 5$$

$$T(4) = 2T(2) + 5 = 15$$

$$T(8) = 2T(4) + 5 = 35$$

$$T(16) = 2T(8) + 5 = 75$$

$$T(32) = 2T(16) + 5 = 155$$

Pattern

$$T(n) = 5n - 5$$

$$\Theta(n)$$

In both the Backward and Forward steps

Master Theorem

$$T(n) = 2T\left(\frac{n}{2}\right) + 5$$

$$a = 2, b = 2, d = 0$$

$$2 > 2^0$$

$$\Theta(n^{\log_2 2})$$

$$\Theta(n)$$

Therefore all the steps above are correct.

• Question 4

$$1. \quad 9T\left(\frac{n}{3}\right) + 27n^3$$

$$a = 9, b = 3, d = 3$$

$$9 < 3^3$$

$$\Theta(n^3)$$

$$2. \quad 0.5T\left(\frac{n}{0.9}\right) + n^n$$

$$a = 0.5, b = 0.9, d = n$$

$$a < 1, b < 1$$

MT

$$3. \quad -2T\left(\frac{n}{2}\right) + \log_4 n$$

$$a = -2, b = 2$$

$$a < 1, f(n) \text{ not in } \Theta(n^d)$$

MT

$$4. \quad 4T\left(\frac{n}{2}\right) + n^2$$

$$a = 4, b = 2, d = 2$$

$$4 = 2^2$$

$$\Theta(n^2 \log n)$$

• Question 5

$$1. \text{ Index: } 0, i, 2, 3, 4, 5, 6, 7, 8, j$$

$$\text{Value: } 31, 37, 12, 3, 44, 50, 22, 39, 10, 25$$

$$2. \text{ Index: } 0, i, 2, 3, 4, 5, 6, 7, 8, j$$

$$\text{Value: } 31, 25, 12, 3, 44, 50, 22, 39, 10, 37$$

3. Index: 0, 1, 2, 3, i, 5, 6, 7, j, 9
Value: 31, 25, 12, 3, 10, 50, 22, 39, 44, 37
4. Index: 0, 1, 2, 3, 4, i, j, 7, 8, 9
Value: 31, 25, 12, 3, 10, 22, 55, 39, 44, 37
5. Index: 0, 1, 2, 3, 4, j, i, 7, 8, 9
Value: 31, 25, 12, 3, 10, 55, 22, 39, 44, 37
6. Index: 0, 1, 2, 3, 4, j, i, 7, 8, 9
Value: 31, 25, 12, 3, 10, 22, 55, 39, 44, 37
7. Index: j, 1, 2, 3, 4, left, i, 7, 8, 9
Value: 22, 25, 12, 3, 10, 31, 55, 39, 44, 37

• **Question 6**

1. Low = 0, Mid = 4, High = 9, i = 0, j = 0, k = 0
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 18, 25, 44, 51, 10, 12, 21, 36, 39
2. i = 0, j = 0, k = 0
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 18, 25, 44, 51, 10, 12, 21, 36, 39
3. i = 1, j = 0, k = 1
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 18, 25, 44, 51, 10, 12, 21, 36, 39
4. i = 1, j = 1, k = 2
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 10, 25, 44, 51, 10, 12, 21, 36, 39
5. i = 1, j = 2, k = 3
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 10, 12, 44, 51, 10, 12, 21, 36, 39
6. i = 2, j = 2, k = 4
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 10, 12, 18, 51, 10, 12, 21, 36, 39
7. i = 2, j = 3, k = 5
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 10, 12, 18, 21, 10, 12, 21, 36, 39
8. i = 3, j = 3, k = 6
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 10, 12, 18, 21, 25, 12, 21, 36, 39
9. i = 3, j = 4, k = 7
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 10, 12, 18, 21, 25, 36, 21, 36, 39
10. i = 4, j = 4, k = 8
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 10, 12, 18, 21, 25, 36, 44, 36, 39
11. i = 4, j = 5, k = 9
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 10, 12, 18, 21, 25, 36, 44, 39, 39
12. i = 5, j = 5, k = 10
Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Value: 3, 10, 12, 18, 21, 25, 36, 44, 39, 51