COMP 221 Homework 5

Due: Friday, October 25, by 11:59 pm

1 Guidelines

Please **type up** and submit a PDF of your solutions. I recommend LaTex, but you are certainly welcome to use Google Docs or some other word processing program. Just make sure to save it as a PDF before submission. If you have to draw any pictures for a question (such as trees, graphs), you can do so **neatly** and then add a picture of it to your PDF file.

Do your own work: Homework should be individual work. You may discuss problems with other people, but you must write up your solutions yourself. I will not say that the web is off limits, but handing in solutions you find online as if they are your own is also not acceptable. Speaking of, most of these questions are shamelessly stolen from Susan :)

Make sure you have your name on your homework!

- **Question 1**

  Show the **min-heap** that would be created with this list of numbers: [15, 42, 9, 55, 2, 4, 1], using *both* the graphical representation of a heap and the array representation. Show the heap at each step. You are welcome to use either heap creation method we used in class.

- **Question 2**

  Show the steps that would occur when using heapsort on the heap you created in question 1, sorting in **non-ascending** (or **descending**) order.

- **Question 3**

  Recurrence relations. Solve the following recurrence relation using **BOTH** backward and forward substitution, and find the big-O. Show your work for all the substitution steps. Then, check your work by using the Master Theorem (our version) to confirm the big-O you had found.

  $T(n) = 2T(\frac{n}{2}) + 5, T(1) = 0$

- **Question 4**

  For the recurrences below, use the Master Theorem to find the big-O, or say MT does not apply if it cannot be used.

  1. $9T(\frac{n}{3}) + 27n^3$

2. $0.5T(\frac{n}{0.9}) + n^n$
3. $-2T(\frac{n}{2}) + log^4 n$
4. $4T(\frac{n}{2}) + n^2$

- **Question 5**

Consider the array of data below (index numbers are on the top row, values in the array on the bottom row). Assuming that *left* is 0 and *right* is 9, show the indices and array for each pass through the Hoare Partition algorithm (one picture for each swap that happens). **Note:** Don't show the whole QuickSort, just one partition of the data! The pseudocode for the Hoare Partition is given below.

0	1	2	3	4	5	6	7	8	9
31	37	12	3	44	50	22	39	10	25

Algorithm HoarePartition ($A[left..right]$)
// if range is empty or has one value then algorithm does nothing
1. **let** $p = A[left]$) // pivot value is leftmost in range
2. **let** $i = left$ // start i at left end of range
3. **let** $j = right + 1$ // start j one past right end of range
4. **repeat**
5. **repeat** // move i right until it finds a value for right partition
6. **let** $i = i + 1$
7. **until** $A[i] \geq p$
8. **repeat** // move j left until it finds a value for left partition
9. **let** $j = j - 1$
10. **until** $A[j] \leq p$
11. **Swap** $A[i] and A[j]$)
12. **until** $i \geq j$
13. **Swap** $A[i] and A[j]$) // undo the last swap
14. **Swap** $A[left] and A[j]$) // move the pivot into the right position

- **Question 6**

Use the pseudocode for Mergesort given on the Notion page to perform the final merge step for the array given below. You don't need to show every step, but enough that it is clear you are following the pseudocode. Make sure you show the changes in i, j, and k, as well as the original array, as needed. Link to **Mergesort pseudocode**

low = 0 mid = 4 high = 9

Array A is given below:

0	1	2	3	4	5	6	7	8	9
3	18	25	44	51	10	12	21	36	39