Homework 1

David Trinh

September 17, 2024

- **Question 1**

$$r = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{k=1}^{n} 1 \tag{1}$$

$$r = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} n \tag{2}$$

$$r = n \sum_{i=1}^{n-1} (n-i) \tag{3}$$

$$r = n \left(\sum_{i=1}^{n-1} n - \sum_{i=1}^{n-1} i \right) \tag{4}$$

$$r = n^2 \sum_{i=1}^{n-1} 1 - n \sum_{i=1}^{n-1} i \tag{5}$$

$$r = n^2(n-1) - n \left(\frac{n(n-1)}{2} \right) \tag{6}$$

$$r = n^2(n-1) - \frac{n^2(n-1)}{2} \tag{7}$$

$$r = \frac{n^2(n-1)}{2} \tag{8}$$

$$r = \frac{n^3 - n^2}{2} \tag{9}$$

Big-O: $O(n^3)$

Big-Ω: $\Omega(n^3)$

Big-Θ: $\Theta(n^3)$

- **Question 2**

$$r = \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=j}^{i+j} 1 \tag{10}$$

$$r = \sum_{i=1}^{n} \sum_{j=1}^{i} (i+1) \tag{11}$$

$$r = \sum_{i=1}^{n} i(i+1) \tag{12}$$

$$r = \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i \tag{13}$$

$$r = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \tag{14}$$

$$r = \frac{(n^2+n)(2n+1) + 3(n^2+n)}{6} \tag{15}$$

$$r = \frac{2n^3 + n^2 + 2n^2 + n + 3n^2 + 3n}{6} \tag{16}$$

$$r = \frac{n^3 + 3n^2 + 2n}{3} \tag{17}$$

Big-O: $O(n^3)$

Big-Ω: $\Omega(n^3)$

Big-Θ: $\Theta(n^3)$