

Homework 2

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- **Question 1**

(a) $f(n) = n^2 + 3n + 2, f(n) = O(n^2)$

Let positive constants c and n_o , we have:

$$n^2 + 3n + 2 \leq c \cdot n^2 \text{ for all } n \geq n_o$$

$$1 + \frac{3}{n} + \frac{2}{n^2} \leq c$$

Let n be 1, we have:

$$1 + \frac{3}{1} + \frac{2}{1^2} \leq c$$

$$6 \leq c$$

As $n \rightarrow \infty$, the terms $\frac{3}{n}$ and $\frac{2}{n^2}$ tend to 0.

Thus, for all $n \geq 1$, $c \geq 6$.

Therefore, there exist $n_o = 1$ and $c = 6$.

(b) $f(n) = 4n^3 + n^2 + n \log n + 5, f(n) = \Theta(n^3)$

Let positive constants c_1 , c_2 , and n_o , we have:

$$c_1 \cdot n^3 \leq 4n^3 + n^2 + n \log n + 5 \leq c_2 \cdot n^3 \text{ for all } n \geq n_o$$

$$c_1 \leq 4 + \frac{1}{n} + \frac{\log n}{n^2} + \frac{5}{n^3} \leq c_2$$

Let n be 1, we have:

$$c_1 \leq 4 + \frac{1}{1} + \frac{\log 1}{1^2} + \frac{5}{1^3} \leq c_2$$

$$c_1 \leq 10 \leq c_2$$

As $n \rightarrow \infty$, the terms $\frac{1}{n}$, $\frac{\log n}{n^2}$, and $\frac{5}{n^3}$ tend to 0.

Thus, for all $n \geq 1$, $c_1 \leq 10 \leq c_2$.

Therefore, there exist $n_o = 1$ and $c = 10$.

(c) $f(n) = n^2 - 8n + 1, f(n) = \Omega(n)$

Let positive constants c and n_o , we have:

$$n^2 - 8n + 1 \geq c \cdot n \text{ for all } n \geq n_0$$

$$n - 8 + \frac{1}{n} \geq c$$

Let n be 9, we have:

$$9 - 8 + \frac{1}{9} \geq c$$

$$\frac{10}{9} \geq c$$

As $n \rightarrow \infty$, the term n tends to ∞ and $\frac{1}{n}$ tends to 0.

Thus, for all $n \geq 9$, $c \leq \frac{10}{9}$.

Therefore, there exist $n_0 = 9$ and $c = 1$.

- **Question 2**

```
def getTopology(A):
    n = len(A)
    isRing = True
    isStar = True
    isFullyConnectedMesh = True
    isCentralNode = False

    for i in range(n):
        totalAdjacent = 0
        for j in range(n):
            if A[i][j]:
                totalAdjacent += 1

    if isFullyConnectedMesh and totalAdjacent != n - 1:
        isFullyConnectedMesh = False
    if isStar:
        if totalAdjacent == n - 1:
            isCentralNode = True
        elif totalAdjacent != 1:
            isStar = False
    if isRing and totalAdjacent != 2:
        isRing = False

    if isRing:
        return "Ring"
    elif isFullyConnectedMesh:
        return "Fully Connected Mesh"
    elif isStar and isCentralNode:
        return "Star"
    else:
        return "None of the above"
```