Homework 2

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- **Question 1**

(a) $f(n) = n^2 + 3n + 2, f(n) = O(n^2)$
Let positive constants c and n_o, we have:

$$n^2 + 3n + 2 \leq c \cdot n^2 \text{ for all } n \geq n_0$$

$$1 + \frac{3}{n} + \frac{2}{n^2} \leq c$$

Let n be 1, we have:

$$1 + \frac{3}{1} + \frac{2}{1^2} \leq c$$

$$6 \leq c$$

As $n \to \infty$, the terms $\frac{3}{n}$ and $\frac{2}{n^2}$ tend to 0.
Thus, for all $n \geq 1$, $c \geq 6$.
Therefore, there exist $n_0 = 1$ and $c = 6$.

(b) $f(n) = 4n^3 + n^2 + nlogn + 5, f(n) = \Theta(n^3)$
Let positive constants c_1, c_2, and n_o, we have:

$$c_1 \cdot n^3 \leq 4n^3 + n^2 + nlogn + 5 \leq c_2 \cdot n^3 \text{ for all } n \geq n_0$$

$$c_1 \leq 4 + \frac{1}{n} + \frac{logn}{n^2} + \frac{5}{n^3} \leq c_2$$

Let n be 1, we have:

$$c_1 \leq 4 + \frac{1}{1} + \frac{log1}{1^2} + \frac{5}{1^3} \leq c_2$$

$$c_1 \leq 10 \leq c_2$$

As $n \to \infty$, the terms $\frac{1}{n}$, $\frac{logn}{n^2}$, and $\frac{5}{n^3}$ tend to 0.
Thus, for all $n \geq 1$, $c_1 \leq 10 \leq c_2$.
Therefore, there exist $n_0 = 1$ and $c = 10$.

(c) $f(n) = n^2 - 8n + 1, f(n) = \Omega(n)$
Let positive constants c and n_o, we have:

$$n^2 - 8n + 1 \geq c \cdot n \text{ for all } n \geq n_0$$

$$n - 8 + \frac{1}{n} \geq c$$

Let n be 9, we have:

$$9 - 8 + \frac{1}{9} \geq c$$

$$\frac{10}{9} \geq c$$

As $n \to \infty$, the term n tends to ∞ and $\frac{1}{n}$ tends to 0.
Thus, for all $n \geq 9$, $c \leq \frac{10}{9}$.
Therefore, there exist $n_0 = 9$ and $c = 1$.

- **Question 2**

$$r = \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=j}^{i+j} 1 \tag{1}$$

$$r = \sum_{i=1}^{n} \sum_{j=1}^{i} (i+1) \tag{2}$$

$$r = \sum_{i=1}^{n} i(i+1) \tag{3}$$

$$r = \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i \tag{4}$$

$$r = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \tag{5}$$

$$r = \frac{(n^2+n)(2n+1) + 3(n^2+n)}{6} \tag{6}$$

$$r = \frac{2n^3 + n^2 + 2n^2 + n + 3n^2 + 3n}{6} \tag{7}$$

$$r = \frac{n^3 + 3n^2 + 2n}{3} \tag{8}$$

Big-O: $O(n^3)$
Big-Ω: $\Omega(n^3)$
Big-Θ: $\Theta(n^3)$