Homework 5

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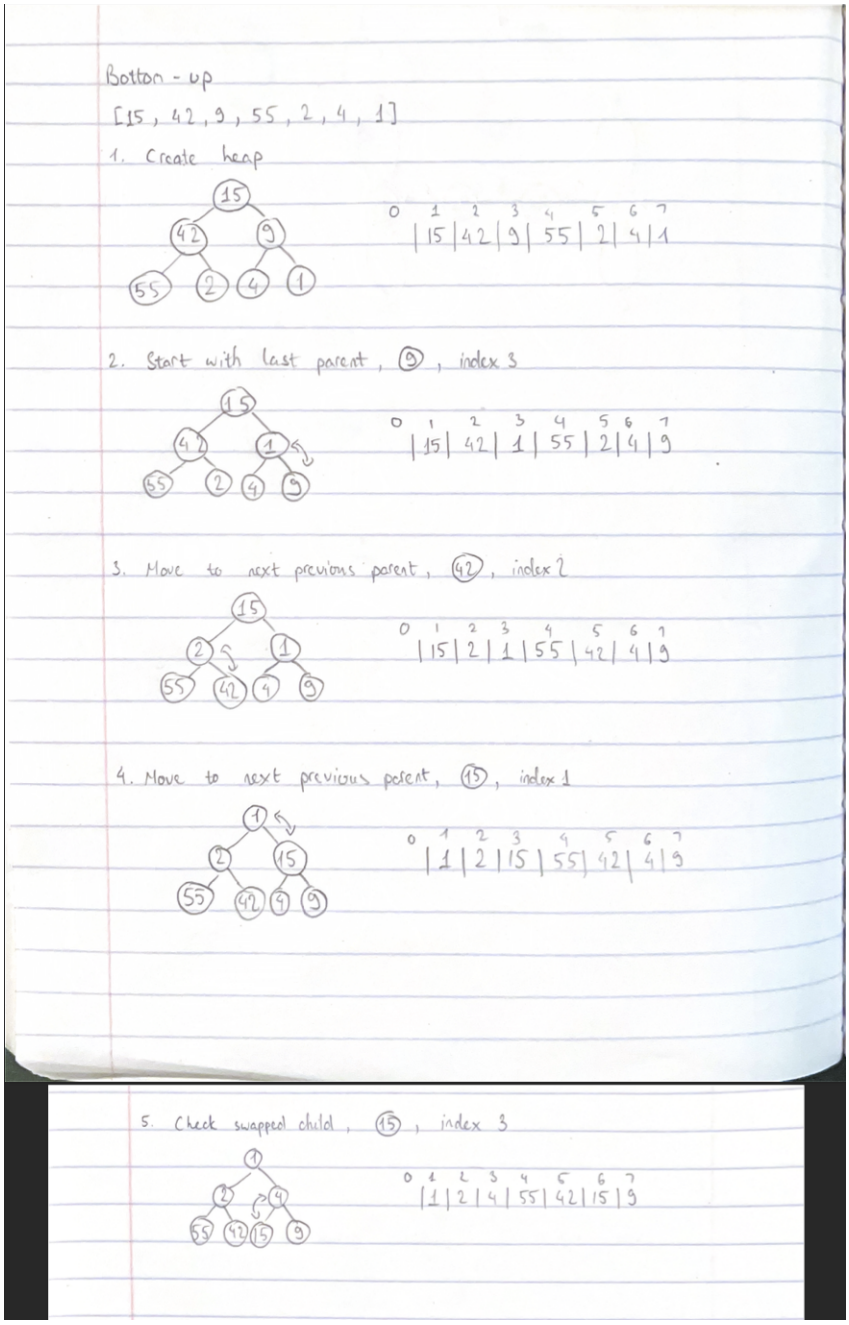
- **Question 1**



- **Question 2**

1. [1, 2, 3, 55, 42, 15, 9]
2. [9, 2, 3, 55, 42, 15] 1
3. [2, 9, 3, 55, 42, 15] 1
4. [15, 9, 3, 55, 42] 1, 2
5. [3, 9, 15, 55, 42] 1, 2
6. [42, 9, 15, 55] 1, 2, 3
7. [9, 42, 15, 55] 1, 2, 3
8. [55, 42, 15] 1, 2, 3, 9
9. [15, 42, 55] 1, 2, 3, 9
10. [55, 42] 1, 2, 3, 9, 15
11. [42, 55] 1, 2, 3, 9, 15
12. [55] 1, 2, 3, 9, 15, 42
13. 1, 2, 3, 9, 15, 42, 55

- **Question 3**

Backward

$T(n) = 2T(\frac{n}{2}) + 5,\ T(1) = 0$

$T(n/2) = 2T(\frac{n}{4}) + 5$

$T(n/4) = 2T(\frac{n}{8}) + 5$

$T(n/8) = 2T(\frac{n}{16}) + 5$

$T(n/16) = 2T(\frac{n}{32}) + 5$

...

Substitute

$T(n) = 2T(\frac{n}{2}) + 5$

$= 2(2T(\frac{n}{4}) + 5) + 5 = 4T(\frac{n}{4}) + 15$

$= 4(2T(\frac{n}{8}) + 5) + 15 = 8T(\frac{n}{8}) + 35$

$= 8(2T(\frac{n}{16}) + 5) + 35 = 16T(\frac{n}{16}) + 75$

$= 16(2T(\frac{n}{32}) + 5) + 75 = 32T(\frac{n}{32}) + 155$

...

Pattern

$2^iT(\frac{n}{2^i}) + (2^i - 1)5$

$T(1) = T(\frac{n}{2^i}) \rightarrow 1 = n/2^i, n = 2^i, i = \log_2 n$

Plug in and solve

$2^{\log_2 n}T(\frac{n}{2^{\log_2 n}}) + (2^{\log_2 n} - 1)5$

$nT(\frac{n}{n}) + (n - 1)5$

$0 + (n - 1)5$

$5n - 5$

$\Theta(n)$

Forward

$T(1) = 0$
$T(2) = 2T(1) + 5 = 5$
$T(4) = 2T(2) + 5 = 15$
$T(8) = 2T(4) + 5 = 35$
$T(16) = 2T(8) + 5 = 75$
$T(32) = 2T(16) + 5 = 155$
Pattern
$T(n) = 5n - 5$
$\Theta(n)$

In both the Backward and Forward steps

Master Theorem
$T(n) = 2T(\frac{n}{2}) + 5$
$a = 2, b = 2, d = 0$
$2 > 2^0$
$\Theta(n^{\log_2 2})$
$\Theta(n)$
Therefore all the steps above are correct.

- **Question 4**

1. $9T(\frac{n}{3}) + 27n^3$
   $a = 9, b = 3, d = 3$
   $9 < 3^3$
   $\Theta(n^3)$
2. $0.5T(\frac{n}{0.9}) + n^n$
   $a = 0.5, b = 0.9, d = n$
   $a < 1, b < 1$
   MT
3. $-2T(\frac{n}{2}) + \log_4 n$
   $a = -2, b = 2$
   $a < 1, f(n)$ not in $\Theta(n^d)$
   MT
4. $4T(\frac{n}{2}) + n^2$
   $a = 4, b = 2, d = 2$
   $4 = 2^2$
   $\Theta(n^2 logn)$

- **Question 5**

1. Index: 0, i, 2, 3, 4, 5, 6, 7, 8, j
   Value: 31, 37, 12, 3, 44, 50, 22, 39, 10, 25
2. Index: 0, i, 2, 3, 4, 5, 6, 7, 8, j
   Value: 31, 25, 12, 3, 44, 50, 22, 39, 10, 37

3. Index: 0, 1, 2, 3, i, 5, 6, 7, j, 9
   Value: 31, 25, 12, 3, 10, 50, 22, 39, 44, 37
4. Index: 0, 1, 2, 3, 4, i, j, 7, 8, 9
   Value: 31, 25, 12, 3, 10, 22, 55, 39, 44, 37
5. Index: 0, 1, 2, 3, 4, j, i, 7, 8, 9
   Value: 31, 25, 12, 3, 10, 55, 22, 39, 44, 37
6. Index: 0, 1, 2, 3, 4, j, i, 7, 8, 9
   Value: 31, 25, 12, 3, 10, 22, 55, 39, 44, 37
7. Index: j, 1, 2, 3, 4, left, i, 7, 8, 9
   Value: 22, 25, 12, 3, 10, 31, 55, 39, 44, 37

- **Question 6**

1. Low = 0, Mid = 4, High = 9, i = 0, j = 0, k = 0
   Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   Value: 3, 18, 25, 44, 51, 10, 12, 21, 36, 39
2. i = 0, j = 0, k = 0
   Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   Value: 3, 18, 25, 44, 51, 10, 12, 21, 36, 39
3. i = 1, j = 0, k = 1
   Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   Value: 3, 18, 25, 44, 51, 10, 12, 21, 36, 39
4. i = 1, j = 1, k = 2
   Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   Value: 3, 10, 25, 44, 51, 10, 12, 21, 36, 39
5. i = 1, j = 2, k = 3
   Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   Value: 3, 10, 12, 44, 51, 10, 12, 21, 36, 39
6. i = 2, j = 2, k = 4
   Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   Value: 3, 10, 12, 18, 51, 10, 12, 21, 36, 39
7. i = 2, j = 3, k = 5
   Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   Value: 3, 10, 12, 18, 21, 10, 12, 21, 36, 39
8. i = 3, j = 3, k = 6
   Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   Value: 3, 10, 12, 18, 21, 25, 12, 21, 36, 39
9. i = 3, j = 4, k = 7
   Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   Value: 3, 10, 12, 18, 21, 25, 36, 21, 36, 39
10. i = 4, j = 4, k = 8
    Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
    Value: 3, 10, 12, 18, 21, 25, 36, 44, 36, 39
11. i = 4, j = 5, k = 9
    Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
    Value: 3, 10, 12, 18, 21, 25, 36, 44, 39, 39
12. i = 5, j = 5, k = 10
    Index: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
    Value: 3, 10, 12, 18, 21, 25, 36, 44, 39, 51