Simulation Assignment 6

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Exercise1

a)

Monte Carlo method uses many repetitive trials of draws of uniform distribution. Then it calculates the mean of the function and that could be a possible estimate for the value of theta. For the following code the result is 0.7490, using 10000 repetitive trials.

```
function[estimator]=exercise_1a(k)
x=rand(k,1);
g=x.^(1/3);
estimator = mean(g);
end
```

b)

The theoretical value of the variance is approximately equal to $0.0375/n$. The empirical value of it using Monte Carlo Method for k=10000 is $3.7304e-06$, which is very close to the theoretical one.

```
function[variance]=exercise_1b(k)
x=rand(k,1);
g_one=x.^(2/3);
g_two=x.^(1/3);
estimator_one= mean(g_one);
estimator_two= mean(g_two);
variance=(estimator_one - (estimator_two)^2)/k
end
```

c)

In order to choose U as a variable as a candidate control variable it should satisfy two conditions:

1. $Cov(\sqrt[3]{U}, U) > 0$
2. The E[U]=0.5

As we see U satisfies both of the conditions, hence U is a candidate for control variable.

d)

The empirical value for the variance is $3.0719e-07$,using k=10000.And the theoretical value is $0.0306/k$. We can see that the variance is way smaller comparing to question b.

```
function[variate_est , variance]=exercise_1d(k)
estimator=zeros(k,1);
var_1=zeros(k,1);
x=rand(k,1);
g_two=x.^(1/3);
c=-0.6428;
```

```
for i=1:k
  estimator(i) = estimator(i) + g_two(i)+c*(x(i)-0.5);
end
variate_est=mean(estimator);
var_1=var(estimator);
variance=(var_1)/k;
end
```

Exercise 2

a)

```
function [ revenue ] = ex_2a( S0, r , sigma_a, K,n )
T=1/3;
S=zeros(n+1,1);
S(1)=S0;
mu_x=(r-0.5*sigma_a^2)*T/n;
sigma_x=sigma_a*(sqrt(T/n));
for i=2:n+1;
  X(i)=normrnd(mu_x,sigma_x);
  S(i)=S(i-1)*exp(X(i-1));
end
A= sum(S)/(1+n);
revenue = max(A-K,0);

end
```

b)

```
function [ estimated_mean, estimated_var ] = ex_2b( S0, r , sigma_a, K,
    n )
T=1/3;
s=100000;
Y= zeros(1,s);
for j = 1:s
    Y(j) = ex_2a( S0, r , sigma_a, K,n );
end
estimated_mean=mean(Y);
estimated_var=var(Y);
end
```

- The estimated _mean is **5.3880** and the the estimated_var is **8.5698**.

c)

- The number of theoretical runs needed to obtain a 99% asymptotic confidence interval for $E[Y_T]$ that has length at most 0.01 is calculated as follows:

$$P[\overline{X}(n) - z_{1-a/2} * \sqrt[2]{S^2(n)/n} \leq \overline{\mu} \leq \overline{X}(n) + z_{1-a/2} * \sqrt[2]{S^2(n)/n}] \Longrightarrow$$
$$P[\overline{X}(n) - 2.58 * \sqrt[2]{S^2(n)/n} \leq \overline{\mu} \leq \overline{X}(n) + 2.58 * \sqrt[2]{S^2(n)/n}] \Longrightarrow$$
$$z_{1-a/2} * \sqrt[2]{S^2/n} \leq \frac{length}{2} \Longrightarrow z_{1-a/2} * \sqrt[2]{S^2/n} \leq 0.005 \Longrightarrow$$
$$2.58 * \frac{\sqrt[2]{8.5698}}{\sqrt[2]{n}} \leq 0.005 \Longrightarrow n = (\frac{2.58}{0.005})^2 * 8.5698 \Longrightarrow$$
$$n = 2,281,760.6688$$

h)

```
function [ revenue,revenue_mean] = ex_2h( S0, r , sigma_a, K,n )
T=1/3;
s=100000;
revenue= zeros(1,s);
mu_x=(r-0.5*sigma_a^2)*T/n;
sigma_x=sigma_a*(sqrt(T/n));
for j = 1:s
  S(1)=S0;
  X=normrnd(mu_x,sigma_x,1,n);
for i = 2:n+1;
  S(i)=S(i-1)*exp(X(i-1));
 end
 G = exp(sum(log(S))/(n+1));
 revenue(j) = max(G-K,0);
end
mu_g=0.5*mu_x*n+ log(S0);
sigma_g=sqrt(n*(2*n+1)/(6*(n+1)))*sigma_x;
f_1=normcdf(((mu_g-log(K)+sigma_g^2)/sigma_g));
f_2=normcdf(((mu_g-log(K))/sigma_g));
revenue_mean = mean(revenue);
theoretical_mean=exp(mu_g + sigma_g^2/2)*f_1-K*f_2
end
```

- `revenue_mean` = 5.3502.
  `theoretical_mean`=5.3497.

i)

```
function [opt_c,mean_est] = ex2_i(S0, r , sigma_a, K,n)
T=1/3;
s=100000;
%calculate Y
Y= zeros(1,s);
for j = 1:s
     Y(j) = ex_2a( S0, r , sigma_a, K,n );
end
mean_Y = mean(Y);
var_Y = var(Y);
%calculate U
[U,mean_U] = ex_2h( S0, r , sigma_a, K,n );
%Find optimal c
covariance=(mean(Y.*U) - mean_U*mean_Y);
opt_c = -1*(covariance/var(U));
Yc=zeros(1,s);
for i=1:s
  Yc(i) = Y(i) + opt_c*(U(i) - mean_U);
end
mean_est = mean(Yc);
var_est = var(Yc)/s
end
```

- `opt_c` is **6.9262e-04** and `mean_est` is **5.3776** and `var_est` is **8.5109e-05**
- The number of theoretical runs needed to obtain a 99% asymptotic confidence interval for $E[Y_T + c*(U_T - \mu U)]$ that has length at most 0.01 is calculated as follows:

$P[\overline{X}(n) - z_{1-a/2} * \sqrt[2]{S^2(n)/n} \leq \overline{\mu} \leq \overline{X}(n) + z_{1-a/2} * \sqrt[2]{S^2(n)/n}] \implies$
$P[\overline{X}(n) - 2.58 * \sqrt[2]{S^2(n)/n} \leq \overline{\mu} \leq \overline{X}(n) + 2.58 * \sqrt[2]{S^2(n)/n}] \implies$
$z_{1-a/2} * \sqrt[2]{S^2/n} \leq \frac{length}{2} \implies z_{1-a/2} * \sqrt[2]{S^2/n} \leq 0.005 \implies$
$2.58 * \frac{275.2199}{\sqrt[2]{n}} \leq 0.005 \implies n = (\frac{2.58}{0.005})^2 * 8.5109e - 05 \implies$
$n = 22.6608 \approx 23.$

j)

- In part i we use control variate (variance reduction method), hence the variance in part i is much smaller than that in c, but they approximately have the same mean. Consequently with a smaller variance the number of simulation runs will be smaller too.