VP160 Recitation Class 1

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Overview

Before We Start

My Style

1. Basically hand-written, especially for model's derivation.
2. A Brief review.
3. More focus on deeper/alternative understanding of formulas, useful and practical models that you may use in assignments and exams.
4. Exercise problems.

Asking Questions

You are encouraged to ask questions on the chat window of zoom during the RC class or ask directly in OH (just after the RC class). I will watch the chat window, and answer some good questions. By this way, I can fully use the 90 mins to talk about more things.

Before We Start

Extended Content

In the future I will also talk about some extended knowledge, which may not be required by VP160 course (Will be definitely less than 5% proportion), just try to improve your understanding for Physics and help you better understand vp160 course contents. I will use purple highlighter pen to mark them.

Scientific Notation

1. In the form of $a \times 10^n (1 \leq |a| < 10)$
2. Often been used in Physics, especially for some very large number or very small number. e.g. Planetary motion problem or Quantum Physics.
3. e.g. gravitation constant $G = 6.67384 \times 10^{-11}$

Unit Prefixes and Conversion

Add a prefix to the given unit to measure in a different scale.

p	n	μ	m	c	k	M
10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{3}	10^{6}

The procedure of a unit conversion is as follows:

$$1000\mathrm{m}^3 = 1000(\frac{\mathrm{m}}{\mathrm{km}})^3 \cdot \mathrm{km}^3 = 1 \times 10^3 \cdot 1 \times 10^{-9}\mathrm{km}^3 = 1 \times 10^{-6}\mathrm{km}^3$$

Dimension Analysis: System of Units

1. We can first select some physical quantities as the "basic quantities" and specify a "basic unit of measurement" for each basic quantity, **the other physical quantities' units can be derived from the relation (definition or law) between them and the fundamental quantities.** These physical quantities are called "derived quantities" and their units It's called derived unit.
2. A set of units formed in this way, is called a certain **"system of units"**.
3. For example, the **SI system of units**, which is most commonly used, contains seven basic quantities: L, m, t, I, T, n, lv; Seven basic units: m, kg, s, A, K, mol, cd. Force (F) is an derived quantity, N is the derived unit, and the relationship with the basic unit is $N = kg \cdot m/s^2$

Dimension Analysis: Method of Undetermined Coefficients

1. We often use capital letter to represent a "dimensional quantity", and use [x] to represent the "dimensional quantity" of physical quantity x. e.g. The dimensional quantity of mass m is written as: $M = [m]$
2. In this course, we use "Method of Undetermined Coefficients" to do exercises. (Although this method is not rigorous.)

Exercise 1

A simple pendulum consists of a light inextensible string AB with length l, with the end A fixed, and a point mass m attached to B. The pendulum oscillates with a small amplitude, and the period of oscillation is T. It is suggested that T is proportional to the product of powers of m, l and g, where g is the acceleration due to gravity. Use dimensional analysis to find this relationship.

Dimension Analysis: Method of Undetermined Coefficients

$g = 9.8\ m/s^2$

$$\begin{cases} [m] = M \\ [l] = L \\ [g] = L \cdot T^{-2} \end{cases}$$

$[T] = T$

We assume that $T = k \cdot m^{\alpha_1} \cdot l^{\alpha_2} \cdot g^{\alpha_3}$

k is a constant

$$\Longleftrightarrow \quad T = M^{\alpha_1} \cdot L^{\alpha_2} \cdot (L \cdot T^{-2})^{\alpha_3}$$

$$= M^{\alpha_1} \cdot L^{\alpha_2 + \alpha_3} \cdot T^{-2\alpha_3}$$

$$\begin{cases} \alpha_1 = 0 \\ \alpha_2 + \alpha_3 = 0 \\ -2\alpha_3 = 1 \end{cases} \Rightarrow \begin{cases} \alpha_1 = 0 \\ \alpha_2 = \frac{1}{2} \\ \alpha_3 = -\frac{1}{2} \end{cases}$$

$T \propto l^{\frac{1}{2}} \cdot g^{-\frac{1}{2}}$

$$\boxed{T \propto \sqrt{\frac{l}{g}}}$$

Dimension Analysis

Actually, There is an entire theory that describes dimension analysis. "Π Theorem"(NOT REQUIRED IN THIS COURSE) is the core essence of dimension analysis.
If you are interested, I have posted an extended reading file about "Π Theorem" that was written by me in 2019/09 on canvas.
For those students who wants to take UPC in November, personally I think it may be useful for you.

Uncertainty

1. Because of limitations of measurement devices, imperfect measurement procedures and randomness of environmental conditions, as well as human factors related to the experimenter himself, no measurement can ever be perfect. Its result may therefore only be treated as an estimate of what we call the "exact value" of a physical quantity. The experiment may both overestimate and underestimate the value of the physical quantity, and it is crucial to provide a measure of the error, or better uncertainty, that a result of the experiment carries (cited from "Introduction to Measurement Data Analysis" in VP141).
2. The detailed calculation will be encountered in VP141. The principles of uncertainty analysis will be explained in VE401

Significant Figures

1. The significant figure required in VP160 is not so strict.
2. However, in principle, the significant figure rule for this course is the same as VC210.

- Determine the number of significant figures:
  - 1.234 4
  - 1.02 3
  - 0.012 2
  - 0.100 3
  - 5000 1
  - 5000. 4
  - 5×10^3 1
  - 5.00×10^3 3
  - Planck Constant $h = 6.626 \times 10^{-34} J \cdot s$ ∞

Significant Figures

- Addition and Subtraction:
  - Round off the result to the leftmost decimal place.
  - $40.123 + 20.34 = 60.46$
- Multiplication and Division:
  - Round off the result to the smallest number of significant figures.
  - $1.23 \times 2.0 = 2.5$
- Logarithms:
  - Retain in the mantissa the same number of SF as there are in the number whose logarithm you are taking.
  - $\log 12.8 = 1.107$
- Exponents:
  - The number of SF is the same as in the mantissa.
  - $10^{1.23} = 17$ or 1.7×10^1

Significant Figures

1. The experiment measurement uncertainty should be rounded down to one significant figure.
2. The only exception is when the uncertainty (if written in scientific notation) has a leading digit of 1 and a second digit should be kept.
3. The total significant figure of any result in experiment should be determined by the uncertainty of this quantity. You should always calculate the uncertainty first in VP141.

Back-of-the-envelope Calculation

Definition

A quick estimation of some physical quantities.

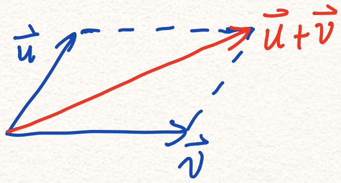
Comments

1. You should cultivate a common sense about the order of magnitude of quantities in everyday lives.
2. Tips: Try to remember the order of magnitude of some important constant.
3. This type of question will occur in your exams.

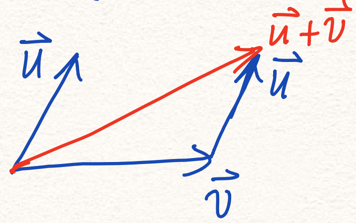
Vector Quantities

△ Vector Addition

① parallelogram rule.

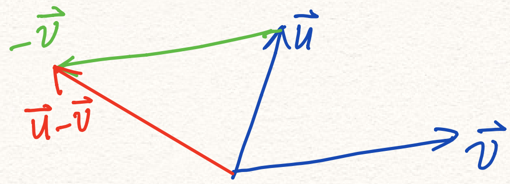


② Triangular rule



△ Vector subtraction

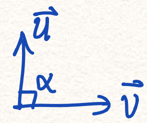
① Turn $\vec{v}$ to $-\vec{v}$
② Use vector addition



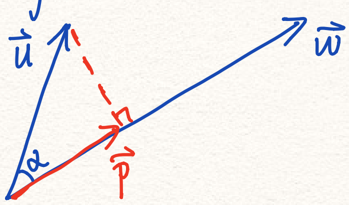
△ Scalar product (dot product)

$$\vec{u}\cdot\vec{v} := |\vec{u}|\cdot|\vec{v}|\cdot\cos\alpha$$

$$\vec{u}\cdot\vec{v} = 0 \Rightarrow \vec{u}\perp\vec{v}$$



△ Projection



$$\vec{p} = \text{projection of } \vec{u} = |\vec{u}|\cdot\cos\alpha\cdot\hat{w}$$

$$= \frac{\vec{u}\cdot\vec{w}}{|\vec{w}|}\cdot\frac{\vec{w}}{|\vec{w}|}$$

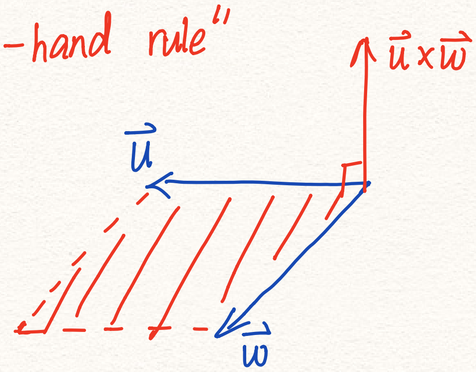
unit vector

$$\hat{w} = \frac{\vec{w}}{|\vec{w}|}$$

△ Vector (Cross) Product

If $\vec{b} = \vec{u}\times\vec{w}$, $|\vec{b}| = |\vec{u}|\cdot|\vec{w}|\sin\alpha$

"right-hand rule"



geometric meaning:

- The area of the parallelogram formed by $\vec{u}$ and $\vec{w}$.
- perpendicular to the parallelogram

- anticommutative : $\vec{u} \times \vec{w} = -\vec{w} \times \vec{u}$

△ In Cartesian Coordinate System :

Let $\vec{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$, $\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$

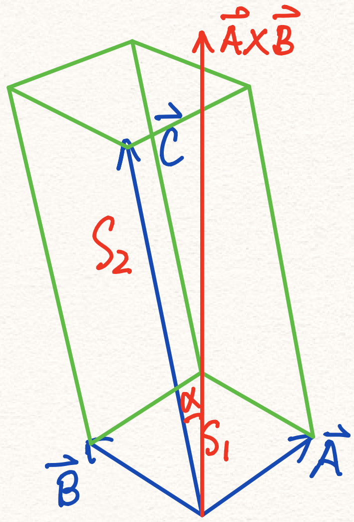
$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z \cdot v_z$ (since $\hat{n}_i \cdot \hat{n}_{j\,(i \neq j)} = 0$)

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

$$= (u_y v_z - v_y u_z) \cdot \hat{i} - (u_x v_z - u_z v_x)\hat{j} + (u_x v_y - u_y v_x) \cdot \hat{k}$$

An interesting rule : $(\vec{A} \times \vec{B}) \cdot \vec{C} = (\vec{C} \times \vec{A}) \cdot \vec{B} = (\vec{B} \times \vec{C}) \cdot \vec{A}$

explaining :



$|(\vec{A} \times \vec{B})| = S_1$

$(\vec{A} \times \vec{B}) \cdot \vec{C} = S_1 \cdot |\vec{C}| \cdot \cos\alpha$

$= S_1 \cdot h_1 = V$

$(\vec{B} \times \vec{C}) \cdot \vec{A} = S_2 \cdot \vec{A} \cdot \cos\alpha' = V$

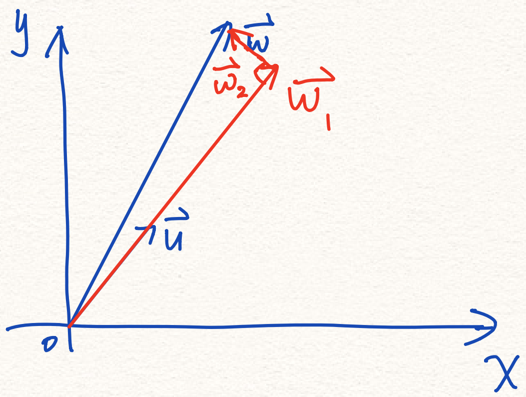
Exercise 2

Is it possible to find a vector $\mathbf{u}$, such that $(2\hat{n}_x - 3\hat{n}_y + 4\hat{n}_z) \times \mathbf{u} = (4\hat{n}_x + 3\hat{n}_y - \hat{n}_z)$? What is a quick way to check it?

No. $\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = 8 - 9 - 4 \neq 0$ So that's impossible.

Exercise 3

4. Consider two vectors $\mathbf{u} = 3\hat{n}_x + 4\hat{n}_y$ and $\mathbf{w} = 6\hat{n}_x + 16\hat{n}_y$. Find (a) the components of the vector $\mathbf{w}$ that are, respectively, parallel and perpendicular to the vector $\mathbf{u}$, (b) the angle between $\mathbf{w}$ and $\mathbf{u}$.



① Components: The projections of the vector.

$\vec{w}_1$ = projection of $\vec{w}$ on $\vec{u}$

$$= \frac{\vec{u}\cdot\vec{w}}{|\vec{u}|}\cdot\frac{\vec{u}}{|\vec{u}|}$$

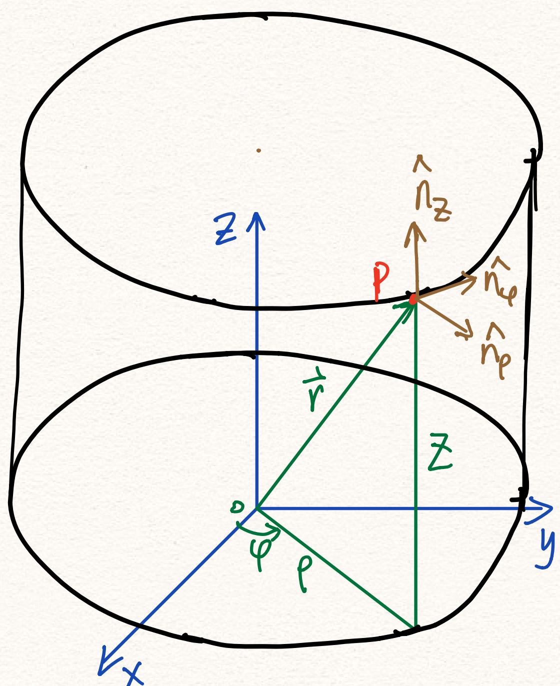
$$= \frac{\begin{pmatrix}3\\4\end{pmatrix}\cdot\begin{pmatrix}6\\16\end{pmatrix}}{\sqrt{3^2+4^2}}\cdot\begin{pmatrix}\frac{3}{5}\\ \frac{4}{5}\end{pmatrix}$$

$$= \frac{18+64}{5}\cdot\begin{pmatrix}\frac{3}{5}\\ \frac{4}{5}\end{pmatrix} = \begin{pmatrix}9.84\\13.12\end{pmatrix}$$

$$\vec{w}_2 = \vec{w} - \vec{w}_1 = \begin{pmatrix}6\\16\end{pmatrix} - \begin{pmatrix}9.84\\13.12\end{pmatrix} = \begin{pmatrix}-3.84\\2.88\end{pmatrix}.$$

② $$\cos\alpha = \frac{\vec{u}\cdot\vec{w}}{|\vec{u}|\cdot|\vec{w}|} = \frac{18+64}{5\times\sqrt{6^2+16^2}} = \frac{82}{10\sqrt{73}} = 0.9597$$

$$\alpha = \arccos(0.9597) = 16.314^\circ$$



Cylindrical Coordinate System (ρ, φ, z)

$$\vec{r}(\rho, \varphi, z) = \rho\cdot\hat{n}_\rho + z\cdot\hat{n}_z$$

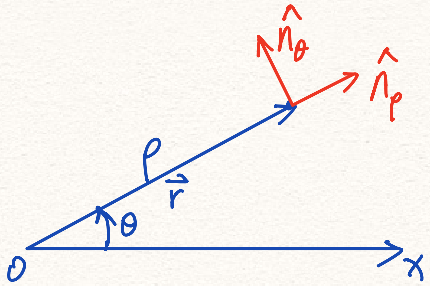
problem: where's the φ?

answer: $\hat{n}_\rho = \hat{n}_\rho(\varphi)$

And also: $\begin{cases}\hat{n}_\varphi = \hat{n}_\varphi(\varphi)\\ \hat{n}_z = \text{const vector.}\end{cases}$

$$\begin{cases} \rho = \sqrt{x^2+y^2} \\ \varphi = \arctan\left(\frac{y}{x}\right) \\ z = z \end{cases} \qquad \begin{cases} x = \rho\cos\varphi \\ y = \rho\sin\varphi \\ z = z \end{cases}$$

When $z=0 \Rightarrow$ Polar Coordinate System (ρ, θ)

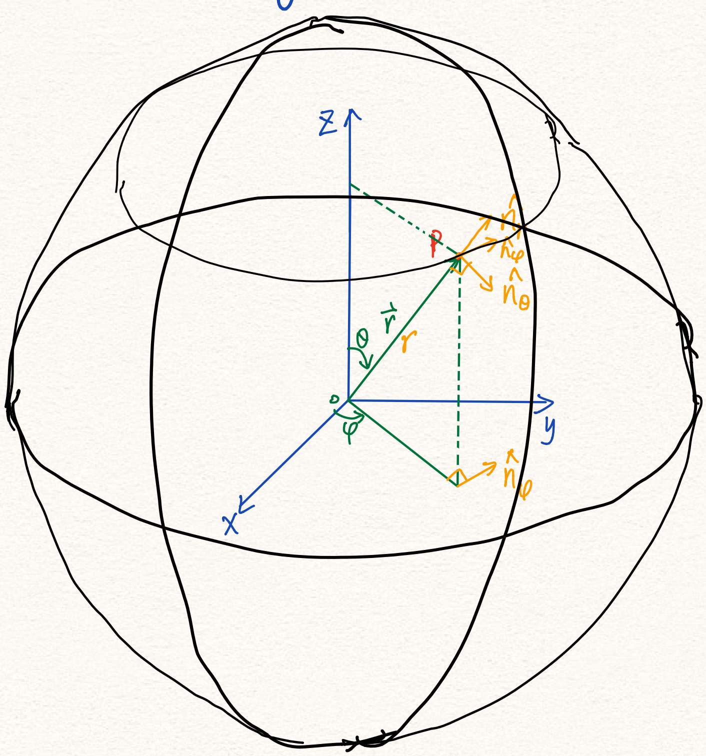


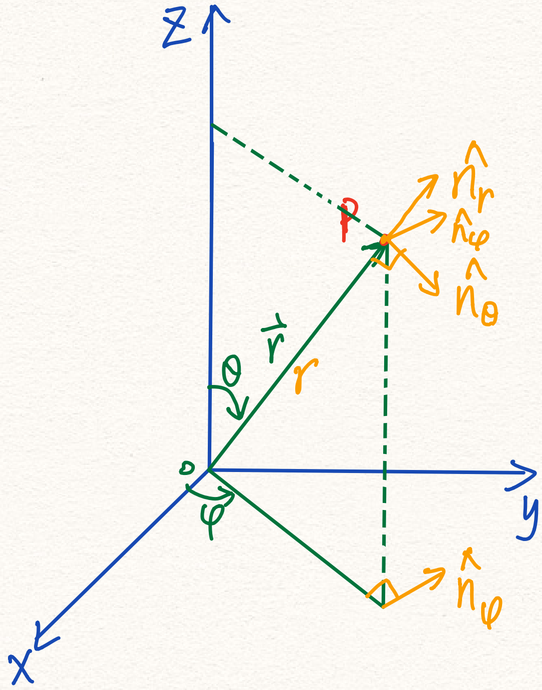
$$\vec{r}(\rho,\theta) = \rho \cdot \hat{n}_\rho$$

$$\begin{cases} \hat{n}_\rho = \hat{n}_\rho(\theta) \\ \hat{n}_\theta = \hat{n}_\theta(\theta) \end{cases}$$

$$\begin{cases} \rho = \sqrt{x^2+y^2} \\ \theta = \arctan\left(\frac{y}{x}\right) \end{cases} \qquad \begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

Spherical Coordinate System (r, θ, φ)





$$\vec{r}(r,\theta,\varphi) = r\cdot\hat{n}_r$$

with
$$\begin{cases} \hat{n}_r = \hat{n}_r(\theta,\varphi) \\ \hat{n}_\theta = \hat{n}_\theta(\theta,\varphi) \\ \hat{n}_\varphi = \hat{n}_\varphi(\theta,\varphi) \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \varphi = \arctan\left(\frac{y}{x}\right) \\ \theta = \arctan\left(\frac{\sqrt{x^2+y^2}}{z}\right) \end{cases}$$

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$$

1D Kinematics

- Average vs. Instantaneous Quantities

$$v_{xA} = \frac{x(t+\Delta t) - x(t)}{\Delta t} \qquad\qquad v_x = \dot{x}$$

$$a_{xA} = \frac{v_x(t+\Delta t) - v_x(t)}{\Delta t} \qquad\qquad a_x = \dot{v}_x = \ddot{x}$$

$\boxed{\text{Exercise 4}}$

A car is moving in one direction along a straight line. Find the average velocity of the car if: (a) it travels *half of the journey time* with velocity v_1 and the other half with velocity v_2, (b) it covers *half of the distance* with velocity v_1 and the other one with velocity v_2. Both v_1 and v_2 are constants.

(a) $x = \frac{1}{2}t_0\cdot v_1 + \frac{1}{2}t_0\cdot v_2$

$$v_A = \frac{x}{t_0} = \frac{1}{2}(v_1+v_2).$$

(b) $t = \dfrac{\frac{x_0}{2}}{v_1} + \dfrac{\frac{x_0}{2}}{v_2} = \frac{x_0}{2}\left(\frac{1}{v_1} + \frac{1}{v_2}\right)$

$v_A = \dfrac{x_0}{t} = 2 \cdot \dfrac{1}{\frac{1}{v_1} + \frac{1}{v_2}}$

Exercise 5

A particle is moving along a straight line with velocity $v_x(t) = -\beta A\omega e^{-\beta t}\cos\omega t$, where A, ω, and β are positive constants.

a) What are the units of these constants?

b) Find acceleration $a_x(t)$ and position $x(t)$ of the particle, assuming that $x(0) = 5$ [m].

c) Sketch $x(t)$, $v_x(t)$, and $a_x(t)$.

d) What kind of motion could these results refer to (qualitatively)?

(a) $\omega t = \theta \Leftrightarrow [\omega] = \dfrac{1}{[t]}$ unit of ω: s^{-1}

βt = dimensionless $\Rightarrow [\beta] = \dfrac{1}{[t]}$ unit of β : s^{-1}

$[-\beta A \omega] = [v_x] \Rightarrow$ unit of A: $m \cdot s$

(b) $\Delta x = \int_0^t v_x(t)\,dt = \int_0^t -\beta A \omega e^{-\beta t}\cos\omega t$

Integration by parts:

$(uv)' = u'v + v'u$

$\int u'v = uv - \int v'u$

$= -\beta A \omega \underbrace{\int_0^t e^{-\beta t}\cos\omega t}_{=A}$

$= -\beta A \omega \cdot \left[\frac{1}{\omega}\sin\omega t\, e^{-\beta t}\Big|_0^t - \int_0^t \frac{1}{\omega}\sin\omega t\,(-\beta) e^{-\beta t} \right]$

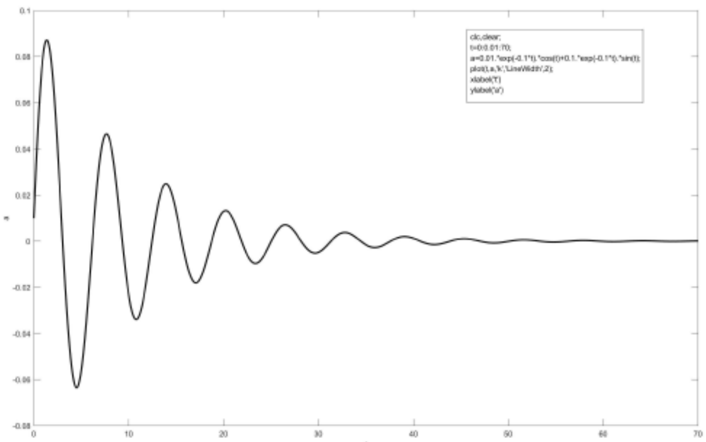
$= -\beta A \omega \cdot \left[\frac{1}{\omega}\sin\omega t\, e^{-\beta t} + \frac{\beta}{\omega}\int_0^t \sin\omega t\, e^{-\beta t} \right]$

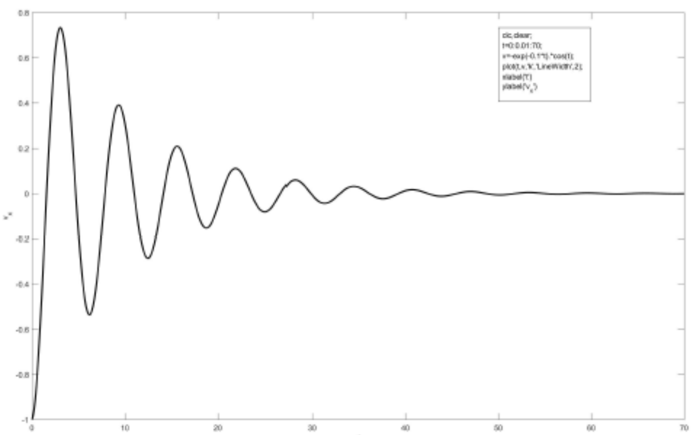
$$= -\beta A \omega \cdot \left[\frac{1}{\omega} \sin \omega t \, e^{-\beta t} + \frac{\beta}{\omega} \left[-\frac{1}{\omega} \cos \omega t \, e^{-\beta t} \Big|_0^t - \int_0^t -\frac{1}{\omega} \cos \omega t \cdot -\beta e^{-\beta t} \right] \right]$$

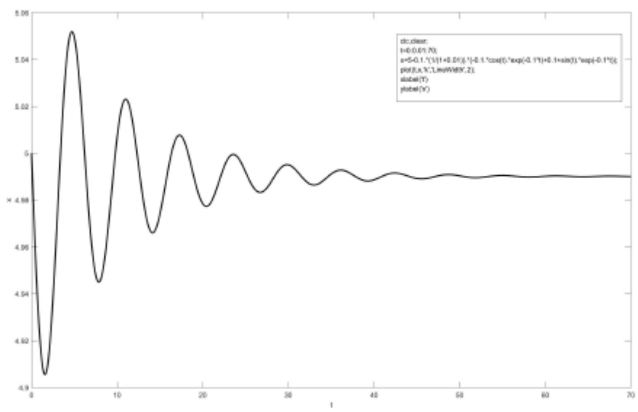
$$A = [a_1 + a_2 - a_3 A]$$

$$\hookrightarrow A = ?$$

$$X(t) = 5 - \frac{\beta A \omega}{\beta^2 + \omega^2} \left[\beta (1 - e^{-\beta t} \cos \omega t) + \omega e^{-\beta t} \sin \omega t \right]$$







Underdamped Oscillation