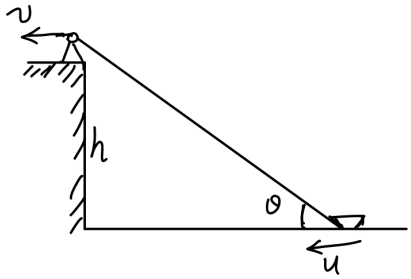
(1) ODE for kinematics

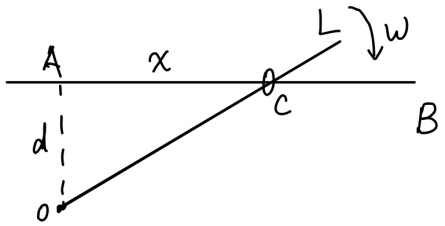
Q1. In a certain motion of a particle along a straight line, the acceleration turns out to be related to the position of the particle according to the formula $a_x = \sqrt{kx}$, where $k > 0$ is a constant and $x > 0$. How does the velocity depend on x, if we know that for $v_x(x_0) = v_0$?

(2) Relative Motion

Q2. A person standing on the bank is dragging rope with constant velocity v, when the angle between the rope and the water surface is θ , Find the velocity and the acceleration of the boat.



Q3. A rigid stick OL is rotating with constant angular velocity ω Around a fixed point O, and is pushing a ring C to slide on the string AB. Find the velocity and the acceleration of ring C.



(3) Polar Coordinate system

Q4. A particle moves along a hyperbolic spiral (i.e. a curve $r = c/\varphi$, where c is a positive constant), so that $\varphi(t) = \varphi_0 + \omega t$, where φ_0 and ω are positive constants. Find its velocity and acceleration (all components and magnitudes of both vectors).

Q5. Four spiders are initially placed at the four corners of a square with side length a. The spiders crawl counter-clockwise at the same speed and each spider crawls directly toward the next spider at all times. They approach the center of the square along spiral paths. Find

(a) polar coordinates of a spider at any instant of time, assuming the origin is at the center of the square.

(b) the time after which all spiders meet,

(c) the trajectory of a spider in polar coordinates.

(4) Trajectory and Curvature.

Q6. A particle moves in the x–y plane so that

$$x(t) = at, \qquad y(t) = bt^2,$$

where a, b are positive constants. Find its trajectory, velocity, and acceleration (its tangential and normal components).

Q7. (*solution provided*) A disc of radius R rotates about its axis of symmetry (perpendicular to the disk surface) with constant angular velocity $\dot{\varphi} = \omega = \text{const}$. At the instant of time $t = 0$ a beetle starts to walk with constant speed v_0 along a radius of the disk, from its center to the edge. Find

(a) the position of the beetle and its trajectory in the Cartesian and polar coordinate systems,

(b) its velocity both systems,

(c) its acceleration in both systems (Cartesian components, polar components, as well as tangential and normal components).

(d) What is the distance covered by the beetle (write down the integral only, do not evaluate it)?

(e) What is the curvature of the trajectory?