Non-Inertial FoR

<1> General derivation

"$F = m\vec{a}$" is wrong in non-inertial FoRs !

How can we use Newton's law in non-inertial FoR ?

① $\vec{a} = \vec{a}' + \vec{a}_c$

real acceleration

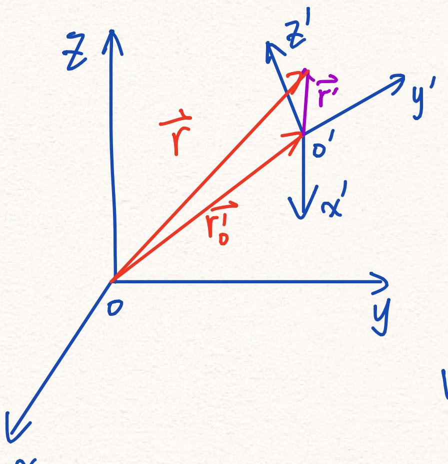
the acceleration in this FoR

The convected acceleration

② $\vec{F} = m\vec{a} = m(\vec{a}_c + \vec{a}')$

$$\Rightarrow m\vec{a}' = \vec{F} - m\vec{a}_c$$

fictitious force



xyz : inertial FoR

x'y'z' : non-inertial FoR

$$\vec{r} = \vec{r}_0' + \vec{r}'$$

We use Einstein's notation :

$$\vec{r}(t) = r_\alpha \hat{n}_\alpha$$

$$\vec{r}'(t) = r_\alpha' \cdot \hat{n}_\alpha'$$

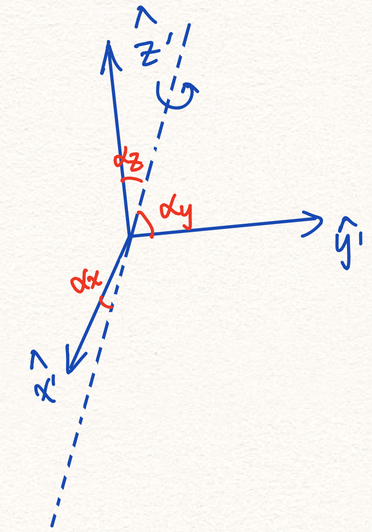
$$r_\alpha \hat{n}_\alpha = \sum_{x,y,z} r_i \hat{n}_i$$

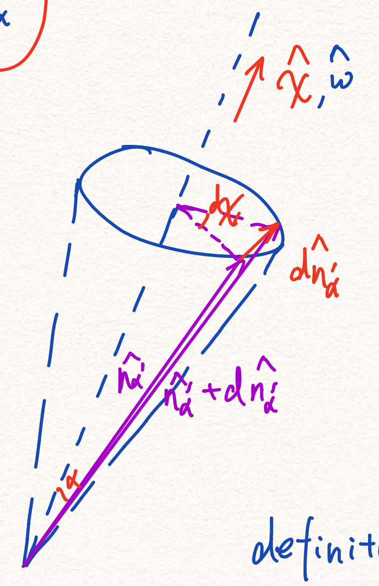
$$\frac{d\vec{r}}{dt} = \vec{v} = \frac{d\vec{r}_0'}{dt} + \frac{d\vec{r}'}{dt} = \vec{v}_0' + \frac{d\vec{r}'}{dt}$$

$$\frac{d\vec{r}'}{dt} = \frac{d}{dt}(r_\alpha' \cdot \hat{n}_\alpha')$$

$$= \dot{r}_\alpha' \cdot \hat{n}_\alpha + r_\alpha' \cdot \dot{\hat{n}}_\alpha'$$

$$= \vec{v}' + r'_\alpha \cdot \dot{\hat{n}}'_\alpha$$





$$|d\hat{n}'_\alpha| = d\chi \cdot |\hat{n}'_{\alpha'}| \cdot \sin\alpha$$

$$d\hat{n}'_\alpha = \vec{d\chi} \times \hat{n}'_\alpha$$

$$\frac{d\hat{n}'_\alpha}{dt} = \frac{d\vec{\chi}}{dt} \times \hat{n}'_\alpha$$

definition : $\vec{\omega} = \frac{d\vec{\chi}}{dt}$

$\vec{\omega}$: the new FoR's angular velocity in the inertial FoR.

So $$\frac{d\vec{r}}{dt} = \vec{v}' + r'_\alpha \cdot \dot{\hat{n}}'_\alpha$$
$$= \vec{v}' + r'_\alpha \cdot (\vec{\omega} \times \hat{n}'_\alpha)$$
$$= \vec{v}' + \vec{\omega} \times (r'_\alpha \cdot \hat{n}'_\alpha)$$
$$= \vec{v}' + (\vec{\omega} \times \vec{r}')$$

Finally, $$\vec{v} = \vec{v}_0' + \vec{v}' + (\vec{\omega} \times \vec{r}')$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}_0'}{dt} + \frac{d\vec{v}'}{dt} + \frac{d(\vec{\omega} \times \vec{r}')}{dt}$$
$$= \vec{a}_0' + \frac{d}{dt}(v'_\alpha \cdot \hat{n}'_\alpha) + \frac{d\vec{\omega}}{dt} \times \vec{r}' + \vec{\omega} \times \left(\frac{d\vec{r}'}{dt}\right)$$

$$\frac{d}{dt}(v'_\alpha \cdot \hat{n}'_\alpha) = \dot{v}'_\alpha \cdot \hat{n}'_\alpha + v'_\alpha \cdot \dot{\hat{n}}'_\alpha$$
$$= a' + v'_\alpha \cdot (\vec{\omega} \times \hat{n}'_\alpha)$$
$$= a' + \vec{\omega} \times \vec{v}'$$

$$\vec{\omega} \times \left(\frac{d\vec{r}'}{dt}\right) = \vec{\omega} \times (\dot{r}'_\alpha \cdot \hat{n}'_\alpha + r'_\alpha \cdot \dot{\hat{n}}'_\alpha)$$

$$= \vec{\omega} \times \vec{v'} + \vec{\omega} \times (r_\alpha' \cdot \vec{\omega} \times \vec{n}_{\alpha'})$$

$$= \vec{\omega} \times \vec{v'} + \vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

Finally, $\vec{a} = \vec{a_{o'}} + \vec{a'} + 2\vec{\omega} \times \vec{v'} + \frac{d\vec{\omega}}{dt} \times \vec{r'} + \vec{\omega} \times (\vec{\omega} \times \vec{r'})$

$$m\vec{a'} = m\vec{a} - m\vec{a_{o'}} - m\frac{d\vec{\omega}}{dt} \times \vec{r'} - 2m(\vec{\omega} \times \vec{v'}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

↳ "Newton's second law" in non-inertial FoR.

$$m\vec{a'} = \vec{F} \underbrace{- m\vec{a_{o'}} - m\frac{d\vec{\omega}}{dt} \times \vec{r'} - 2m(\vec{\omega} \times \vec{v'}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r'})}_{\text{pseudo forces!}}$$

① $-m\vec{a_{o'}}$: The fictitious force caused by the acceleration of the new FoR's origin.

(generally translational motion)

② $-m\frac{d\vec{\omega}}{dt} \times \vec{r'} = -m\vec{\beta} \cdot \vec{r'}$: The fictitious force caused by the angular acceleration of the new FoR.

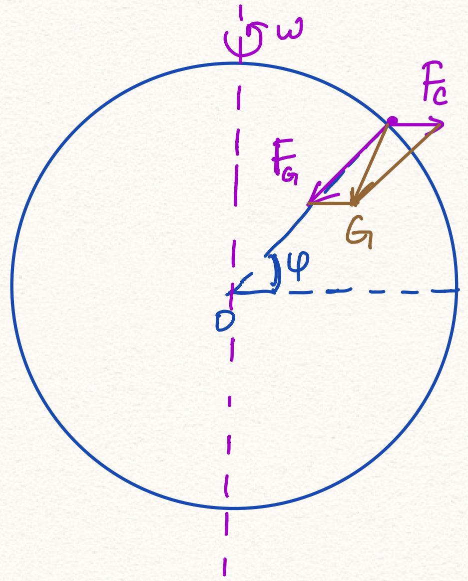
③ $-m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$: The centrifugal force. Direction: opposite from the origin O'.

(amplitute: $m\omega^2 r'$)

④ $-2m(\vec{\omega} \times \vec{v'}) = 2m(\vec{v'} \times \vec{\omega})$: Coriolis force

Earth as a non-inertial FoR.

<1> The real "gravity".



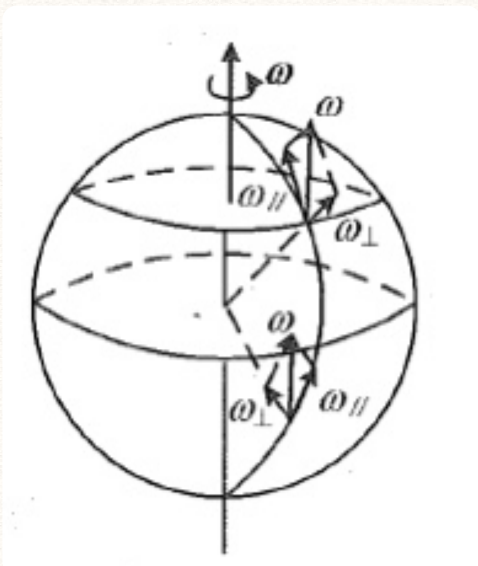
$$F_G = \frac{GmM}{R^2}$$

$$F_c = m\omega^2 R\cos\varphi$$

$$\vec{G} = \vec{F_G} + \vec{F_c}$$

$$|mg| \approx \frac{GmM}{R^2} - m\omega^2 R\cos^2\varphi$$

<2> The Coriolis Force

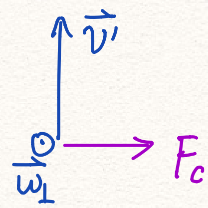


$$F_c = -2m\vec{\omega}\times\vec{v'}$$

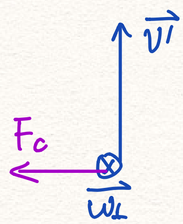
$$= -2m(\vec{\omega_{\parallel}} + \vec{\omega_{\perp}})\times\vec{v'}$$

$(-2m\vec{\omega_{\parallel}}\times\vec{v'})$ will be vertical

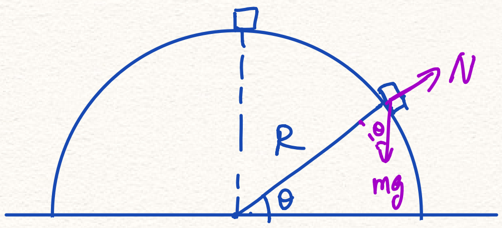
North : $(-2m\vec{\omega_{\perp}}\times\vec{v'})$



South : $(-2m\vec{\omega_{\perp}}\times\vec{v'})$



Q1. A particle with mass m slides from the top of a frictionless hemisphere with radius R. Find the place where the particle loses contact with the surface of the ball. What is its speed at this instant?



$$R(1-\sin\theta)mg = \frac{1}{2}mv^2$$

$$v^2 = 2Rg(1-\sin\theta)$$

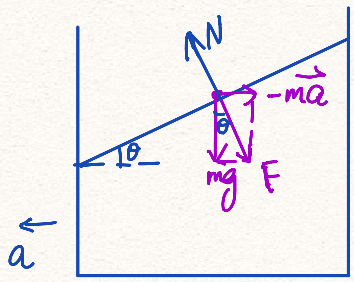
$$mg\sin\theta - N = ma_n \qquad N=0$$

$$a_n = g\sin\theta = \frac{v^2}{R} = 2g(1-\sin\theta)$$

$$3g\sin\theta = 2g \,, \quad \sin\theta = \frac{2}{3}$$

$$\theta = \arcsin\frac{2}{3}$$

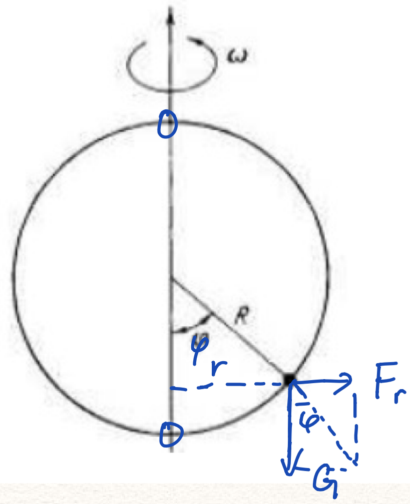
Q2. A box is filled with a liquid and is placed on a horizontal surface. Find the angle that the surface of the liquid forms with the horizontal if we pull the box with acceleration a.



$$\tan\theta = \frac{ma}{mg}$$

$$\theta = \arctan\frac{a}{g}$$

7. A small bead can slide without friction on a circular hoop that is in a vertical plane and has a radius R. Find points on the hoop, such that if we place the bead there it will remain at rest. Acceleration due to gravity is g.



$\varphi_1 = 0, \quad \varphi_2 = \pi$

$$F_r = m\omega^2 r$$

$$= m\omega^2 \cdot R\sin\varphi_3$$

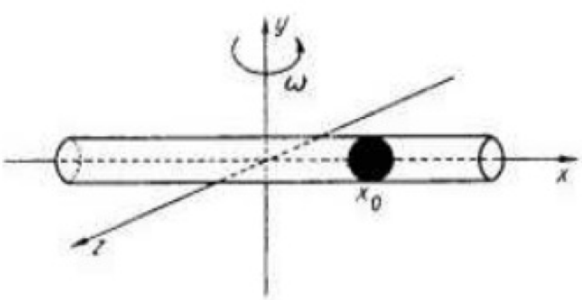
$$G = mg$$

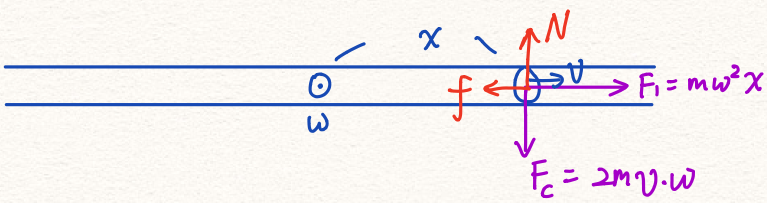
$$\tan\varphi = \frac{F_r}{G} = \frac{\cancel{m}\omega^2 R\sin\varphi_3}{\cancel{m}g} \Rightarrow \cos\varphi_3 = \frac{g}{\omega^2 R}$$

$$\varphi_3 = \arccos\frac{g}{\omega^2 R}$$

9. A particle with mass m is inside a pipe that rotates with constant angular velocity ω about an axis perpendicular to the pipe. The kinetic coefficient of friction is equal to μ_k. Write down (do not solve!) the equation of motion for this particle in the non-inertial frame of reference of the rotating pipe.

There is no gravitational force in this problem.





$N = 2mv\omega$

$f = \mu_k N = 2m\mu_k v\omega$

$F_1 - f = m\omega^2 x - 2m\mu_k \omega \dot{x} = m\ddot{x}$

Recommend Reading:

【练习 3-12】 双摆系统的初始起动。

一双摆系统由摆长分别为 l_1 和 l_2 的不可伸长的轻绳和质量分别为 m_1 和 m_2 的两小球 A、B 组成，并处于竖直平衡位置，如图 3-练 12(a)所示。现有质量同为 m_1 的 C 小球以水平速度 v_0 与 A 球相碰，碰撞是弹性的。求 C、A 两小球碰后瞬间两段绳中的张力 T_1 和 T_2。

分析与解 C、A 两小球碰后瞬间，三球速度分别为 $v_A = v_0$，$v_C = 0$，$v_B = 0$。所考察的两轻绳中的张力，取决于系统碰后初始起动状态：A 球(m_1)具有水平初速度 v_0，B 球(m_2)无初速；A 球(m_1)和 B 球(m_2)的位置未发生明显位移。

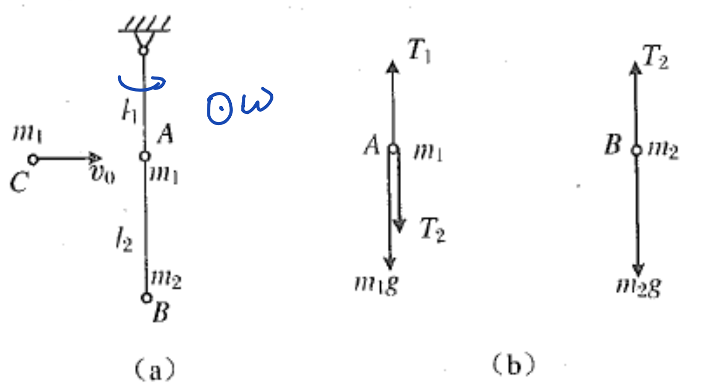


图 3-练 12

A 球(m_1)和 B 球(m_2)所受作用力如图 3-练 12(b)所示。其中由 A 球(m_1)的受力分析图，并考虑到 A 球(m_1)作圆周运动，可以得 A 球(m_1)的动力学方程

$$T_1 - T_2 - m_1 g = m_1 \frac{v_0^2}{l_1} \quad ①$$

在 A 球(m_1)作圆周运动的同时,B 球(m_2)相对 A 球(m_1)也作圆周运动。在地球系中不易立即写出 B 球(m_2)的动力学方程。如果选取 A 球(m_1)作为参照系将会带来方便。A 球(m_1)相对地球作半径为 l_1 的圆周运动,且在图 3-练 12(a)所示位置,A 球(m_1)只有向心加速度而无切向加速度。因此,我们作为练习可以选用三种不同的非惯性系写出 B 球(m_2)的动力学方程并求解:

1) 与 A 球(m_1)相对静止的平动参照系。

在此参照系中,B 球(m_2)以 v_0 向左绕 A 球作圆周运动,得 B 球(m_2)的动力学方程

$$T_2 - m_2 g - m_2 \frac{v_0^2}{l_1} = m_2 \frac{v_0^2}{l_2} \qquad ②$$

联立方程①、②,解得

$$T_1 = (m_1 + m_2)g + (m_1 + m_2)\frac{v_0^2}{l_1} + m_2 \frac{v_0^2}{l_2} \qquad ③$$

$$T_2 = m_2 g + m_2 \frac{v_0^2}{l_1} + m_2 \frac{v_0^2}{l_2} \qquad ④$$

2) 与 A 球(m_1)相对静止的绕悬挂点转动参照系。

此转动参照系的角速度为 $\omega = v_0/l_1$,B 球(m_2)在此转动系中的矢径大小为 $l_1 + l_2$,B 球(m_2)的相对速度为 $\left(\frac{v_0}{l_1}\right)(l_1 + l_2)$,方向向左。得 B 球(m_2)的动力学方程

$$T_2 - m_2 g - m_2\left(\frac{v_0}{l_1}\right)^2(l_1 + l_2) + 2m\left(\frac{v_0}{l_1}\right)\left[\left(\frac{v_0}{l_1}\right)(l_1 + l_2)\right] = m_2\frac{\left[\frac{v_0}{l_1}(l_1 + l_2)\right]^2}{l_2} \qquad ⑤$$

$\vec{v} = \vec{v_0'} + \vec{v'} + (\vec{\omega} \times \vec{r'})$

$v_B' = \vec{v_B} - \vec{v_0} - \vec{\omega} \times \vec{r'}$

$0 \quad \leftarrow v_0 \quad \leftarrow \left(\frac{v_0}{l_1}\right) l_2$

其中左式第三项为惯性离心力,左式第四项为科里奥利力。

3) 随 A 球(m_1)平动、绕 A 球(m_1)转动的平动转动参照系。

此转动系角速度取为 $\omega = v_0/l_1$,原点 A 球(m_1)的平动运动加速度为 v_0^2/l_1,方向竖直向上指向悬挂点。B 球(m_2)在此平动转动系中的位矢大小为 l_2,相对速度为 $\left[v_0 + \left(\frac{v_0}{l_1}\right) l_2\right]$。得 B 球(m_2)的动力学方程

$$T_2 - m_2 g - m_2\left(\frac{v_0^2}{l_1}\right) - m_2\left(\frac{v_0}{l_1}\right)^2 l_2 + 2m_2\left(\frac{v_0}{l_1}\right)\left[v_0 + \left(\frac{v_0}{l_1}\right) l_2\right] = m_2\frac{\left[v_0 + \left(\frac{v_0}{l_1}\right) l_2\right]^2}{l_2} \qquad ⑥$$

其中左式第三项是平动转动系原点 A 球(m_1)平动加速运动引起的惯性力,左式第四项是 B 球在平动转动系中的惯性离心力,左式第五项是 B 球在平动转动系中所受的科里奥利力。这里对 B 球在平动转动系中的初始相对速度 $\left[v_0 + \left(\frac{v_0}{l_1}\right) l_2\right]$ 给一点说明:相对速度与参照系无关。B 球在静系中初始速度为零,所以参照系随 A 球平动,给 B 球带来的相对速度是 v_0,方向向左;参照系绕 A 球转动,给 B 球带来的相对速度是 $\left(\frac{v_0}{l_1}\right) l_2$,方向也是向左。

将所得的 B 球(m_2)满足的动力学方程⑤、⑥与 A 球(m_1)满足的动力学方程①联立求解,便可得到相同的 T_1、T_2 的解③、④。

问题 3

1 / 1 分

A mass is suspended on a light spring to form a vertical harmonic oscillator and moves moves in a liquid which is a source of a linear drag $-b_1 v_x$. The mass is acted upon by a driving force, with a sinusoidal dependence of time. The angular frequency of the driving force is tuned so that the system is at resonance and the amplitude of steady state oscillations is A.

Suppose that we replace the liquid with another one, with the same density, but a different drag coefficient b_2 where $b_2 > b_1$. With the same driving force acting on the mass, what can we say about the amplitude of the new steady-state oscillations?

- ○ It will be greater than A.
- ◉ It will be smaller than A.
- ○ It will not change.
- ○ Not enough information to determine.

$$\boxed{A = \frac{F_0}{m\sqrt{\left(\omega_0^2-\omega_{dr}^2\right)^2+\left(\frac{b}{m}\omega_{dr}\right)^2}}}$$

问题 4

1 / 1 分

A harmonic oscillator system is subject to a linear drag, so that it is in the underdamped regime. The system may pass through the equilibrium position at most once.

- ○ True
- ◉ False

问题 5

1 / 1 分

The phase shift between the displacement and the driving force for a harmonic oscillator driven by a sinusoidal driving force (in steady state) does not depend on

- ○ the angular frequency of the driving force.
- ○ the natural angular frequency.
- ◉ the amplitude of the driving force.

$$\boxed{\tan\phi = \frac{b\omega_{dr}}{m(\omega_{dr}^2 - \omega_0^2)}}$$

问题 1

1 / 1 分

For a system that exhibits deterministic chaos, the word "chaos" refers to the fact that

- the system cannot be described by Newton's equations of motion.
- the forces acting in the system are chaotic.
- equations of motion of the system are random, that is chaotic.
- the evolution of the system is extremely sensitive to initial conditions.

问题 2

1 / 1 分

You are seated in a bus and notice that a hand strap that is hanging from the ceiling hangs away from the vertical in the backward direction. From this observation, you can conclude that

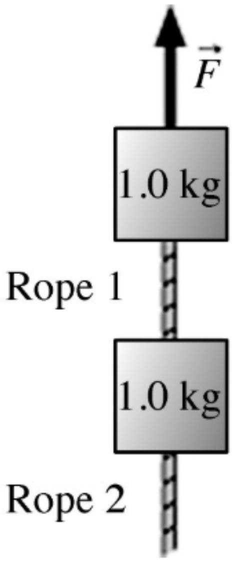
- the velocity of the bus is backward.
- You cannot conclude anything about the direction of the velocity of the bus.
- the velocity of the bus is forward.

问题 3

2 / 2 分

The figure shows two 1.0 kg-blocks connected by a rope. A second rope hangs beneath the lower block. Both ropes have a mass of 250 g. The entire assembly is accelerated upward at 2.3 m/s^2 by force $\vec{F}$. What is the magnitude of the tension at the top end of rope 1?

The acceleration due to gravity is 9.8 m/s^2.



- 15 N
- 3.5 N
- 2.9 N
- 18 N

问题 4

1 / 1 分

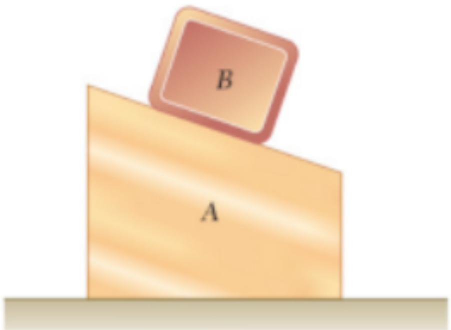
A projectile is launched at an angle to the horizontal, close to the earth surface. Assuming quadratic air drag, when is the magnitude of the projectile's total acceleration maximum?

- Just at the instant when the projectile is being launched.
- The projectile's acceleration is constant thoughout its motion.
- Not enough information to determine.
- At the instant when the projectile reaches the highest point of its trajectory.
- Just before the projectile lands on the ground.

问题 5

1 / 1 分

Block B is placed on the wedge A and released from rest in the position shown. There is no friction anywhere in the system. The normal force between the wedge A and the ground is



- Less than the weight of A plus the weight of B.
- Not enough information given to decide.
- Greater than the weight of A plus the weight of B.
- Equal to the weight of A plus the weight of B.